S21 Final Key

Macroeconomic Theory I, Final Exam, Spring 2021.

You may consult the book, your notebook, or any other materials, but you may not work with another student or consult any other individual; all of the work must be yours and yours alone.

Answer any three of the following four questions. All questions receive equal weight.

Question 1. Part A: Consider a representative consumer whose current financial wealth is a_t. Assume that her income is exogenous and that she maximizes the present discounted value of utility over her (infinite) lifetime. Set up and solve the (infinite horizon) utility maximization problem. Making appropriate assumptions use the intertemporal Euler equation to **derive** the relationship between her current consumption and her current and future income.

Part B: Use your results from Part A to explain the Ricardian Equivalence Hypothesis. How is the Ricardian Equivalence Hypothesis related to the Permanent-Income/Life-Cycle Hypothesis?

Question 2. Consider the following simple New-Keynesian model:

$$x_t = -\alpha\left(R_t - E_t\pi_{t+1} - r\right) \qquad \alpha > 0 \qquad (1)$$

$$\pi_t = \beta E_t\pi_{t+1} + x_t + e_t, \qquad 0 < \beta < 1 \qquad (2)$$

$$R_t = r + \phi\pi_t, \qquad 0 < \phi < 1 \qquad (3)$$

where π_t denotes inflation, x_t denotes the GDP gap, R_t denotes the nominal federal funds rate, and r is the steady-state equilibrium real interest rate. Assume that e_t is white noise, that is, e_t is $i.i.d.\left(0,\sigma_e^2\right)$.

A.) What is the Taylor principle? How would you modify the specification of the parameters in this model, equations (1)-(3), so that the Taylor principle holds?

B.) Using your modification of the parameters from part A, derive the rational expectations equilibrium values of x_t and π_t, each as a function of e_t.

C.) Continuing from your results in part B, derive the policy that minimizes the variance of π_t

Question 3. Suppose that the social planner seeks to maximize

$$\sum_{s=0}^{\infty} \beta^s U\left(c_{t+s}, l_{t+s}\right) \tag{1}$$

subject to

$$l_t + n_t = 1 \tag{2}$$

$$k_{t+1} = (1-\delta)k_t + y_t - c_t \tag{3}$$

$$y_t = F\left(k_t, n_t\right) = A\left[\alpha k_t^\rho + (1-\alpha)n_t^\rho\right]^{\frac{1}{\rho}} \tag{4}$$

$$U\left(c_t, l_t\right) = \ln c_t + \varphi \ln l_t \tag{5}$$

where $\beta = \frac{1}{1+\theta}$, $\theta > 0$, $\delta \in (0,1)$, $\alpha \in [0,1]$.

A.) Set up the lagrangian and derive the first-order conditions.

B.) Use the first-order conditions to derive the intertemporal optimality condition and the intratemporal optimality condition.

C.) Derive the expression that gives the steady state equilibrium value of $\frac{y_t}{k_t}$ as a function of the model's parameters. Also, derive the expression that gives the steady state equilibrium real interest rate as a function of the model's parameters.

Question 4.

Part A: **DERIVE** the "New-Keynesian IS Function" from the household's intertemporal optimality condition. (In your notation you should use the usual $\beta = \left(\frac{1}{1+\theta}\right)$ for the discount factor.)

Part B: Write down the New-Keynesian Phillips curve (i.e., the Calvo Aggregate Supply Curve).

Suppose that the nominal interest rate is set according to the following Taylor rule:

$R_t = \theta + \psi\pi_t + v_t$ where v_t is $i.i.d\left(0, \sigma_v^2\right)$.

Use the New-Keynesian IS Function, from Part A, together with the New-Keynesian Phillips Curve and the Taylor Rule to derive a second-order expectional difference equation in equilibrium inflation. (You do not need to *solve* the difference equation, just derive it.)

Problem 1: Part A:

$$\text{Max} \sum_{s=0}^{\infty} \beta^s u(c_{t+s}) \qquad (1)$$

s.t. $a_{t+1} = (1+r_t)a_t + y_t - c_t \qquad (2)$ $\qquad y_t$ is exogenous.

Lagrangian

$$\mathcal{L}_t = \sum_{s=0}^{\infty} \left\{ \beta^s u(c_{t+s}) + \lambda_{t+s}\left[(1+r_{t+s})a_{t+s} + y_{t+s} - c_{t+s} - a_{t+s+1}\right] \right\} \qquad (3)$$

$$\frac{\partial \mathcal{L}_t}{\partial c_{t+s}} = \beta^s u'(c_{t+s}) - \lambda_{t+s} = 0 \quad \text{or}$$

$$\lambda_{t+s} = \beta^s u'(c_{t+s}) \qquad (4)$$

$$\frac{\partial \mathcal{L}_t}{\partial a_{t+s+1}} = -\lambda_{t+s} + (1+r_{t+s+1})\lambda_{t+s+1} = 0 \quad \text{or}$$

$$\lambda_{t+s} = (1+r_{t+s+1})\lambda_{t+s+1} \qquad (5)$$

use (4) in (5) to get

$$\beta^s u'(c_{t+s}) = \beta^{s+1} u'(c_{t+s+1})(1+r_{t+s+1}) \quad \text{or}$$

$$u'(c_{t+s}) = \beta(1+r_{t+s+1})u'(c_{t+s+1}) \qquad (6)$$

Let $\beta = \frac{1}{1+\theta}$. Assume $r_{t+s} = r$ a const, and $r \cong \theta$

so that $\beta(1+r) \cong 1$

...en (6) Gives

$$u'(c_t) = u'(c_{t+1}) = u'(c_{t+2}) = \cdots$$

So Consumption Smoothing: $c_t = c_{t+1} = c_{t+2} = \cdots$ (7)

Next, Consider (2) w/ Constant r:

$$a_{t+1} = (1+r)a_t + y_t - c_t \qquad (8)$$

Let $R \equiv 1+r > 1$ and write (8) as

$(1-RL)a_{t+1} = y_t - c_t$ or

$$a_{t+1} = \left(\frac{1}{1-RL}\right)(y_t - c_t) = \left[\frac{-R^{-1}L^{-1}}{1-R^{-1}L^{-1}}\right](y_t - c_t)$$

$$= \left(\frac{1}{1+r}\right)L^{-1}\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j}(c_{t+j} - y_{t+j})$$ or, multiplying through by L

$$a_t = \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j+1}(c_{t+j} - y_{t+j}) \qquad (9)$$

Use (7) and note that $\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j+1}c_t =$

$$\left(\frac{1}{1+r}\right)\left[\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j}\right]c_t = \left(\frac{1}{1+r}\right)\left[\frac{1}{1-\left(\frac{1}{1+r}\right)}\right]c_t = \left(\frac{1}{1+r}\right)\left(\frac{1+r}{r}\right)c_t$$

$$= \frac{1}{r}c_t$$

Collecting,

$$\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j+1} c_{t+j} = \frac{1}{r} c_t \qquad (10)$$

Use (10) in (9) to get

$$a_t = \frac{1}{r} c_t - \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j+1} y_{t+j} \quad \text{or}$$

$$\boxed{c_t = r\left[a_t + \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j+1} y_{t+j}\right] \qquad (11)}$$

Current Cons $= r$ [Current wealth + PDV of lifetime Income]

PART B: Suppose that the household pays taxes so that instead of y_{t+j} we use $y^d_{t+j} = (y_{t+j} - T_{t+j})$

Thus (11) becomes

$$c_t = r\left[a_t + \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j+1}(y_{t+j} - T_{t+j})\right] \quad \text{or}$$

$$c_t = r\left[a_t + \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j+1} y_{t+j} - \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j+1} T_{t+j}\right] \qquad (12)$$

Further, Assume That The Gov't Faces A Budget Constraint So That

$$\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j+1} G_{t+j} = \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j+1} T_{t+j} \qquad (13)$$

Use (13) in (12) to write

$$\boxed{C_t = r\left[a_t + \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j+1} y_{t+j} - \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j+1} G_{t+j}\right] \quad (14)}$$

Note Here That if The Gov't cuts taxes, Ceteris Paribus (i.e., w/o changing PDV of Gov't Expenditure), $\Delta T_t < 0$ Has No Effect on Consumption.

If There is No change in $\sum\left(\frac{1}{1+r}\right)^{j+1} G_{t+j}$ The pattern of taxation used to Finance it Doesn't Matter.

→ The Ricardian Equivalence Hypothesis is essentially The Same As The permanent income Hypothesis. ~~This~~ ~~Can Be Seen in (12) which Says The~~

QUESTION 2

PART A: The TAYLOR principle Requires That The Fed increase The Fed Funds Rate by an amount That is greater Than any increase in inflation, π_t. Thus it Requires That $\phi > 1$. This is The necessary Modification of The parameters - That $\phi > 1$.

PART B:

Use (3) in (1) to get

$$\cancel{x_t = -\alpha[R_t}$$

$$x_t = -\alpha\left[r + \phi\pi_t - E_t \pi_{t+1} - r\right] \quad \text{or}$$

$$x_t = \alpha E_t \pi_{t+1} - \alpha\phi\pi_t \qquad (4)$$

Use (4) in (2) to get

$$\pi_t = \beta E_t \pi_{t+1} + \alpha E_t \pi_{t+1} - \alpha\phi\pi_t + e_t \quad \text{or}$$

$$[1+\alpha\phi]\pi_t = (\beta+\alpha) E_t \pi_{t+1} + e_t \quad \text{or}$$

$$(\beta+\alpha) E_t \pi_{t+1} - [1+\alpha\phi]\pi_t = -e_t \qquad (5)$$

Divide (5) Through by $(\beta+\alpha)$ to get

$$\underset{t}{E}\,\Pi_{t+1} - \left[\frac{1+\alpha\phi}{\beta+\alpha}\right]\Pi_t = \left(\frac{-1}{\beta+\alpha}\right)e_t \qquad (6)$$

or

$$\underset{t}{E}\,\Pi_{t+1} - \lambda\,\Pi_t = \left(\frac{-1}{\beta+\alpha}\right)e_t \qquad (7)$$

where $\lambda = \left(\frac{1+\alpha\phi}{\beta+\alpha}\right)$ (8)

Note That, Since $\beta < 1$ and $\phi > 1$ so $\alpha\phi > \alpha$ it follows That $\lambda > 1$. So we Solve (8) Forward

$$\underset{t}{E}\,\Pi_{t+1} = \left[\frac{1}{1-\lambda L}\right]\left(\frac{-1}{\beta+\alpha}\right)e_t$$

$$= \left[\frac{-\lambda^{-1}L^{-1}}{1-\lambda^{-1}L^{-1}}\right]\left(\frac{-1}{\beta+\alpha}\right)e_t$$

$$\underset{t}{E}\,\Pi_{t+1} = \frac{1}{\lambda}\left(\frac{L^{-1}}{1-\lambda^{-1}L^{-1}}\right)\frac{1}{(\beta+\alpha)}e_t$$

Mult Through by L

$$\Pi_t = \frac{1}{\lambda}\left(\frac{1}{1-\lambda^{-1}L^{-1}}\right)\left(\frac{1}{\beta+\alpha}\right)e_t \quad \text{or}$$

$$\pi_t = \left[\frac{\beta+\alpha}{1+\alpha\phi}\right]\left(\frac{1}{\beta+\alpha}\right)\sum_{j=0}^{\infty}\lambda^{-j}\underset{t}{E}e_{t+j} \qquad (9)$$

Since $e_t \sim iid\,(0, \sigma_e^2)$

$\underset{t}{E}e_t = e_t$ but $\underset{t}{E}e_{t+j} = 0$ for $j = 1, 2, 3, \ldots$

Thus (9) Becomes

$$\boxed{\pi_t = \left(\frac{1}{1+\alpha\phi}\right)e_t \qquad (10)}$$

Note from (10) That

$$\underset{t}{E}\,\pi_{t+1} = \left(\frac{1}{1+\alpha\phi}\right)\underset{t}{E}e_{t+1} = 0 \qquad (11)$$

Use (10) and (11) in The RHS of (4) to get

$$\boxed{x_t = \left[\frac{-\alpha\phi}{1+\alpha\phi}\right]e_t \qquad (12)}$$

Eqns (10) and (12) give The REE values of π_t and x_t, Respectively.

PART C

From (10) it follows that

$$VAR(\pi_t) = \left[\frac{1}{1+\alpha\phi}\right]^2 \sigma_e^2 \quad (13)$$

From (13) and noting that $\lim_{\phi \to +\infty} \left[\frac{1}{1+\alpha\phi}\right] = 0$

The policy that minimizes the variance of π_t is

$\phi \to +\infty$.

QUESTION 3

A.) LAGRANGIAN

$$\mathcal{L}_t = \sum_{s=0}^{\infty} \Big\{ \beta^s U(C_{t+s}, l_{t+s}) + \mu_{t+s}(1 - l_{t+s} - n_{t+s})$$
$$+ \lambda_{t+s}\big[(1-\delta)K_{t+s} + F(K_{t+s}, n_{t+s}) - C_{t+s} - K_{t+s+1}\big]\Big\} \quad (6)$$

F.O.C. w.r.t. $C_{t+s}, n_{t+s}, l_{t+s}, K_{t+s+1}; \mu_{t+s}, \lambda_{t+s}$

$$\frac{\partial \mathcal{L}_t}{\partial C_{t+s}} = \beta^s U_c(C_{t+s}, l_{t+s}) - \lambda_{t+s} = 0 \quad (7)$$

$$\frac{\partial \mathcal{L}_t}{\partial n_{t+s}} = -\mu_{t+s} + \lambda_{t+s} F_n(t+s) = 0 \quad (8)$$

$$\frac{\partial \mathcal{L}_t}{\partial l_{t+s}} = \beta^s U_l(t+s) - \mu_{t+s} = 0 \quad (9)$$

$$\frac{\partial \mathcal{L}_t}{\partial K_{t+s+1}} = -\lambda_{t+s} + \lambda_{t+s+1}\big[1 - \delta + F_K(t+s+1)\big] = 0 \quad (10)$$

$\frac{\partial \mathcal{L}_t}{\partial \mu_{t+s}}$ gives (2) $\quad \frac{\partial \mathcal{L}_t}{\partial \lambda_{t+s}}$ gives (3)

B. OPTIMALITY CONDITIONS

INTRATEMPORAL Eqn (7) gives

$$\lambda_{t+s} = \beta^s U_c(t+s) \quad (11)$$

Use (11) for λ_{t+s} and λ_{t+s+1} in (10) to get

$$\beta^S U_c(t+s) = \beta^{S+1} U_c(t+s+1)\left[1-\delta+F_K(t+s+1)\right] \qquad (12)$$

From (5) $U_c(t) = \frac{1}{C_t}$ (13)

From (4) $F_K(t) = A\left[\alpha K_t^\rho + (1-\alpha)M_t^\rho\right]^{\frac{1}{\rho}-1} \alpha K_t^{\rho-1}$ or

$$F_K(t) = \alpha A^\rho A^{1-\rho}\left[\alpha K_t^\rho + (1-\alpha)M_t^\rho\right]^{\frac{1-\rho}{\rho}} K_t^{\rho-1} \text{ or}$$

$$F_K(t) = \alpha A^\rho \left[\frac{Y_t}{K_t}\right]^{1-\rho} \qquad (14)$$

In (12) set $S=0$ and use (13) and (14) to get

$$\boxed{\frac{1}{C_t} = \beta \frac{1}{C_{t+1}}\left\{1-\delta+\alpha A^\rho\left[\frac{Y_{t+1}}{K_{t+1}}\right]^{1-\rho}\right\} \qquad (15)}$$

Eqn (15) is the <u>inter</u> temporal optimality condition.

<u>Intratemporal</u> Eqn (9) gives

$$\mu_{t+s} = \beta^S U_\ell(t+s) \qquad (16)$$

use (16) and (11) in (8) to get

$$-\beta^S U_\ell(t+s) + \beta^S U_c(t+s) F_m(t+s) = 0 \quad \text{or}$$

$$F_m(t+s) = \frac{U_\ell(t+s)}{U_c(t+s)} \qquad (17)$$

From (5) $U_\ell(t) = \frac{\phi}{\ell_t}$. This w/(13) ~~in (17) gives~~

$$\frac{U_\ell(t)}{U_c(t)} = \frac{\phi C_t}{\ell_t} \qquad (18)$$

Also, from (4)

$$F_m(t) = A\left[\alpha K_t^{\rho} + (1-\alpha)m_t^{\rho}\right]^{\frac{1}{\rho}-1}(1-\alpha)m_t^{\rho-1} \quad \text{or}$$

$$F_m(t) = (1-\alpha)A^{\rho}A^{1-\rho}\left[\alpha K_t^{\rho} + (1-\alpha)m_t^{\rho}\right]^{\frac{1-\rho}{\rho}} m_t^{\rho-1} \quad \text{or}$$

$$F_m(t) = (1-\alpha)A^{\rho}\left[\frac{y_t}{m_t}\right]^{1-\rho} \qquad (19)$$

Use (18) and (19) in (17) w/ $s=0$ to get

$$(1-\alpha)A^{\rho}\left[\frac{y_t}{m_t}\right]^{1-\rho} = \frac{\phi c_t}{l_t} \qquad (20)$$

Eqn (20) is the <u>INTRA</u> temporal optimality condition.

C.) At the steady state $c_{t+1} = c_t = c_s$ and $\frac{y_t}{K_t} = \frac{y_s}{K_s}$

so (15) at the steady state gives

$$\frac{1}{c_s} = \beta\frac{1}{c_s}\left[1-\delta+\alpha A^{\rho}\left(\frac{y_s}{K_s}\right)^{1-\rho}\right] \quad \text{or}$$

$$1 = \beta\left[1-\delta+\alpha A^{\rho}\left(\frac{y_s}{K_s}\right)^{1-\rho}\right] \quad \text{or}$$

$$\frac{1}{\beta}-1+\delta = \alpha A^{\rho}\left(\frac{y_s}{K_s}\right)^{1-\rho} \quad \text{or}$$

$$\frac{\theta+\delta}{\alpha A^{\rho}} = \left(\frac{y_s}{K_s}\right)^{1-\rho} \quad \text{or}$$

$$\frac{y_s}{k_s} = \left[\frac{\theta + \delta}{\alpha A^\rho}\right]^{\frac{1}{1-\rho}} \qquad (21)$$

Note
$$\frac{y_s}{k_s} = \left[\frac{\frac{1}{\beta} - 1 + \delta}{\alpha A^\rho}\right]^{\frac{1}{1-\rho}} \qquad (21')$$

(21) or (21') Answers this part of the question

<u>Real interest rate</u> The implied real interest rate is equal to the marginal product of capital net of the depreciation rate. Thus

$$r_t = F_K(K_t, N_t) - \delta \qquad (22)$$

Use this in (12) to get

$$\beta^s U_c(t+s) = \beta^{s+1} U_c(t+s+1)\left[1 + r_{t+s+1}\right]$$

At the steady state $U_c(t+s) = U_c(t+s+1)$ so

$1 = \beta\left[1 + r_s\right]$ or

$1 + \theta = 1 + r_s$ Thus

$$\boxed{r_s = \theta \qquad (23)}$$

Problem 4 **Part A** The New-Keynesian IS function

is given by $\boxed{x_t = E_t x_{t+1} - \alpha(R_t - E_t \pi_{t+1} - \theta) + e_{xt} \quad (1)}$

Where $x_t = y_t - y_N$

$y_t = \ln GDP$, $y_N = \ln(\text{Nat Rate of GDP})$

$\pi_t =$ inflation

$R_t =$ Nominal interest rate

$\alpha > 0 \qquad \theta > 0$ where $\beta = \frac{1}{1+\theta}$ is disc Factor.

Let $e_{xt} \sim iid(0, \sigma_x^2)$

Derivation: The Household's Intertemporal Euler eqn is

$$U_c(C_{t+s}) = \beta(1+r_{t+s+1})U_c(C_{t+s+1}) \qquad (2)$$

Set $s=0$.

Use CRRA Utility so that $U(c) = \dfrac{c^{1-\sigma}}{1-\sigma}$

and, Thus, $U_c(C_t) = C_t^{-\sigma}$. Now, eqn (2) becomes

$$C_t^{-\sigma} = \beta(1+r_{t+1})C_{t+1}^{-\sigma} \qquad (3)$$

Take logs

$$-\sigma C_t = \ln\beta + \ln(1+r_{t+1}) + (-\sigma)\ln C_{t+1} \qquad (4)$$

Use $\ln(1+r_{t+1}) \cong r_{t+1}$ and

$\ln \beta = \ln\left(\frac{1}{1+\theta}\right) = -\ln(1+\theta) \cong -\theta$ so (4) becomes

$$-\sigma \ln C_t = -\theta + r_{t+1} + (-\sigma)\ln C_{t+1} \quad \text{or}$$

$$\ln C_t = \ln C_{t+1} + \frac{-1}{\sigma}\left[r_{t+1} - \theta\right] \qquad (5)$$

Now, in general, $Y_t = C_t + I_t + G_t + NX_t$.
Closed Economy, $NX = 0$. No (fiscal) Gov't, $G = 0$.
Labor is only input so $K_t = 0$ and $I_t = 0$.
Thus $Y_t = C_t$ and $\ln Y_t = \ln C_t$ so (5) becomes

$$y_t = y_{t+1} + \frac{-1}{\sigma}\left[r_{t+1} - \theta\right]$$

Subtract y_N both sides, use $r_{t+1} = R_t - E_t \Pi_{t+1}$,
Allow for uncertainty + take expectations to get

$$y_t - y_N = E_t(y_{t+1} - y_N) + \left(\frac{-1}{\sigma}\right)\left[R_t - E_t \Pi_{t+1} - \theta\right] + e_{xt}$$

OR

$$\boxed{x_t = E_t x_{t+1} - \alpha\left[R_t - E_t \Pi_{t+1} - \theta\right] + e_{xt} \qquad (6)}$$

where $\alpha \equiv \frac{1}{\sigma} > 0$

Note (6) is (1)

PART B

NKPC: $\pi_t = \beta E_t \pi_{t+1} + \gamma x_t + e_{\pi t}$ (7)

TR: $R_t = \Theta + \psi \pi_t + v_t$ (8)

Derive SODE: use (8) in (6) to get

$$x_t = E_t x_{t+1} - \alpha\left[\Theta + \psi \pi_t + v_t - E_t \pi_{t+1} - \Theta\right] + e_{xt}$$

$$x_t = E_t x_{t+1} - \alpha \psi \pi_t - \alpha v_t + \alpha E_t \pi_{t+1} + e_{xt} \quad (9)$$

use (9) in (7) to get

$$\pi_t = \beta E_t \pi_{t+1} + \gamma\left[E_t x_{t+1} - \alpha \psi \pi_t + \alpha E_t \pi_{t+1} + e_{xt} - \alpha v_t\right] + e_{\pi t}$$

or

$$\left[1 + \gamma \alpha \psi\right] \pi_t = (\beta + \gamma \alpha) E_t \pi_{t+1} + \gamma E_t x_{t+1} + \gamma e_{xt} - \gamma \alpha v_t + e_{\pi t} \quad (10)$$

Invert (7) to get

$$\gamma x_t = \pi_t - \beta E_t \pi_{t+1} - e_{\pi t} \quad \text{so}$$

$$\gamma x_{t+1} = \pi_{t+1} - \beta E_t \pi_{t+2} - e_{\pi t+1} \quad \text{and}$$

$$\gamma E_t x_{t+1} = E_t \pi_{t+1} - \beta E_t \pi_{t+2} \quad (11)$$

use (11) in (10) to get

$$\left[1+\gamma\alpha\psi\right]\pi_t = (\beta+\gamma\alpha)\underset{t}{E}\pi_{t+1} + \underset{t}{E}\pi_{t+1} - \beta\underset{t}{E}\pi_{t+2}$$

$$+ \gamma e_{xt} - \alpha\gamma v_t + e_{\pi t}$$

or

$$\beta\underset{t}{E}\pi_{t+2} - \left[1+\beta+\alpha\gamma\right]\underset{t}{E}\pi_{t+1} + \left[1+\alpha\gamma\psi\right]\pi_t = Z_t \quad (12)$$

$$\text{where } Z_t \equiv \gamma e_{xt} - \alpha\gamma v_t + e_{\pi t} \quad (13)$$

Eqn (12) is a second-order expectational Diff. Eqn. in π_t.