Homework 1

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CISC5825 - Computer Algorithm

Due on Feb 6, 2023

Problem1

Express function below in terms of growth functions using the best fit.

a) $2^n + n^2 + n$

b) $(\frac{n+3}{5})^2$

Solution:

a) Since $2^n >= n^2$ in general, thus, $2^n + n^2 + n \leq 2^n + 2^n + 2^n = 3 \cdot 2^n$. $c_1 = 3$, $g(n) = 2^n$, and $n_0 = 1$. The big O notion is $O(2^n)$.

b)

$$\begin{aligned}(\frac{n+3}{5})^2 &= \frac{n^2+6n+9}{25} \\ &= \frac{n^2}{25} + \frac{6n}{25} + \frac{9}{25}\end{aligned}$$

Thus,

$$\frac{n^2}{25} \leq f(n) \leq \frac{n^2}{25} + \frac{6n^2}{25} + \frac{9n^2}{25} = \frac{16}{25}n^2$$

$c_1 = \frac{1}{25}$, $c_2 = \frac{16}{25}$, $n_0 = 1$, and $g(n) = n^2$. Thus it's $\Theta(n^2)$

Problem 2

Determine which of the two functions in the pair of of the function below grows faster: $(n)^{\sqrt{n}}$ vs $(\sqrt{n})^n$

Solution:

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Make monotone transformation for each of the functions. $\lg(n^{\sqrt{n}}) = \sqrt{n}\lg n$, $\lg((\sqrt{n})^n) = n\lg\sqrt{n}$. Make monotone transformation again, $\lg\sqrt{n}\lg n = \lg(\sqrt{n}\lg n) = \lg\sqrt{n} + \lg\lg n = \frac{1}{2}\lg n + \lg\lg n$, $\lg n\lg\sqrt{n} = \lg n + \lg\frac{1}{2}\lg n$.

WLOG let $\lg n = Z$, thus compare $\frac{1}{2}Z + \lg Z$ and $Z + \lg\frac{1}{2}Z$

Make substraction: $Z + \lg\frac{1}{2}Z - (\frac{1}{2}Z + \lg Z) = \frac{1}{2}Z + \lg\frac{\frac{1}{2}Z}{Z} = \frac{1}{2}Z + \lg\frac{1}{2} = M$.

When $M \geq 0 \rightarrow \frac{1}{2}Z \geq -\lg\frac{1}{2} \rightarrow Z \geq \lg(\frac{1}{2})^{-2} = \lg 4 \rightarrow \lg n \geq \lg 4$

When $n \geq 4$, $(\sqrt{n})^n \geq (n)^{\sqrt{n}}$, otherwise, $(\sqrt{n})^n < (n)^{\sqrt{n}}$

Thus, $(\sqrt{n})^n$ grows faster.