Homework 3

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CISC5825 - Computer Algorithm

Due on Feb 20, 2023

The algorithm of Random is as below:

Algorithm 1 Algorithm for RANDOM

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procedure RANDOM(n)
  if $n = 2$ **then**
    return (1)
  else
    assign $x = 0$ with probability $\frac{1}{2}$, or assign $x = 1$ with probability $\frac{1}{3}$, or assign $x = 2$ with probability $\frac{1}{6}$
    if $x = 0$ **then**
      return $(RANDOM(n) + RANDOM(n-1))$
    end if
    if $x = 1$ **then**
      return $(2 \cdot RANDOM(n-1) + RANDOM(n-1))$
    end if
    if $x = 2$ **then**
      (*)return $(2 \cdot RANDOM(n-2) + RANDOM(n-1))$
    end if
  end if
end procedure

(a) Give the Recurrence for the expected running time of RANDOM

Solution:

Let $T(n)$ be the expected running time of RANDOM. $T(1) = 1$

$$\begin{aligned} T(n) &= 1 + \frac{T(n) + T(n-1)}{2} + \frac{T(n-1) + T(n-1)}{3} + \frac{T(n-1) + T(n-1)}{6} \\ &= 1 + \frac{T(n)}{2} + \frac{4T(n-1)}{3} + \frac{T(n-2)}{6} \end{aligned}$$

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Thus $T(n) = 1 + \frac{8T(n-1)}{3} + \frac{T(n-2)}{3}$

All in all,

$$T(n) = \begin{cases} 1, & n = 1 \\ 1 + \frac{8T(n-1)}{3} + \frac{T(n-2)}{3}, & n > 1 \end{cases}$$

(b) Give the exact recurrence equation for the expected number of recursive calls executed by a call to RANDOM(n)

Solution:

Let $R(n)$ be the expected number of recursive calls executed by a call to RANDOM(n).

We know $R(1) = 0$.

$$\begin{aligned} R(n) &= \frac{1 + R(n) + 1 + R(n-1)}{2} + \frac{1 + R(n-1) + 1 + R(n-1)}{3} + \frac{1 + R(n-2) + 1 + R(n-1)}{6} \\ &= \frac{R(n)}{2} + \frac{4R(n)}{3} + \frac{R(n-2)}{6} + 2 \end{aligned}$$

Thus, $R(n) = \frac{8R(n-1)}{3} + \frac{R(n-2)}{3} + 4$

Therefore,

$$R(n) = \begin{cases} 0, & n = 1 \\ \frac{8R(n-1)}{3} + \frac{R(n-2)}{3} + 4, & n > 1 \end{cases}$$

(c) Give the exact recurrence equation for the expected number of times the return statement at time (*) is executed in all calls to RANDOM(n), recursive or not.

Solution:

Let $C(n)$ be the expected number of times.

$C(1) = 0$

$$\begin{aligned} C(n) &= \frac{C(n) + C(n-1)}{2} + \frac{C(n-1) + C(n-1)}{3} + \frac{1 + C(n-2) + C(n-1)}{6} \\ &= \frac{C(n)}{2} + \frac{4C(n-1)}{3} + \frac{C(n-2)}{6} + \frac{1}{6} \end{aligned}$$

Then, we can derive $C(n) = \frac{8}{3}C(n-1) + \frac{C(n-2)}{3} + \frac{1}{3}$

Therefore,

$$C(n) = \begin{cases} 0, & n = 1 \\ \frac{8C(n-1)}{3} + \frac{C(n-2)}{3} + \frac{1}{3}, & n > 1 \end{cases}$$