Homework #4 Stochastic Calculus Solutions

Ito Formula for Ito Processes

Due Date: Dec 16, 2022

Please email HW to the dedicated email: msqfeconometrics2015@gmail.com

Note: *I have not double checked the answers, so there could be computational errors here. Use these solutions as a hint for how to solve the problems.*

Problem 1. Let X_t be Ito process

$$dX_t = W(t)dt + [W(t)]^2 dW(t).$$

Apply Ito's formula to express $Y_t = \log(1 + (X_t)^2)$ as an Ito process (i.e., as a stochastic integral plus a Riemann integral).

Problem 2. Let

$$dX_t = -\frac{1}{1+t}X_t dt + \frac{1}{1+t}dW(t); \ X(0) = 0.$$

Let $Y_t = (1+t)X_t$. Use Ito's formula to find out what stochastic differential equation Y_t satisfies? Identify Y_t as a Brownian Motion.

Problem 3. Solve $dX_t = X_t dt + dW(t)$. **Hint:** This is a special case of the O-U process used in the Vasicek model. Read the lecture notes about how you solved the Vascek model.

Problem 4. Solve $dX_t = -X_t dt + e^{-t}dW(t)$. **Hint:** Same hint as in Problem 3.