Homework #1 Stochastic Calculus

Conditional Expectation/Brownian Motion

Due date: Nov15, 2022 (in class)

(Please note the deadline above)

`Email HW to the dedicated address: msqfeconometrics2015@gmail.com`

Important Note: Homework should be done independently although discussion is allowed. No late homework will be accepted.

Problem 1. Let $W(t)$ be the standard Brownian Motion on the interval $[0, T]$.

(a) Write down the density of $W(T)$.

(b) What is the joint density for $W(s)$ and $W(t)$ where $0 \leq s < t \leq T$. **Hint**: These two random variables have joint normal distribution. You can use the definition of Brownian Motion to figure out all the parameters of that distribution.

Problem 2. Let $W(t)$ be the standard Brownian Motion on the interval $[0, T]$.

(a) Compute the conditional expectation of $E[W(t)|W(s) = c]$, where $0 < s < t < T$ and c is a fixed constant. **Hint:** you can use the "traditional approach" to do this. First by finding the conditional density of $W(t)$ and given $W(s) : W(t)|W(s) = c$, which can be done using either the joint density in Problem 1(b), or using the defining properties of Brownian Motion. Then calculate the expectation of the conditional distribution.

(b) Compute the expectation $E[W(t)^2]$

(c) (Bonus question) Compute $E[W(t)^6]$

(c) (Bonus question) Compute Expectation $E[e^{1+2W(t)}]$. **Hint**: There are two ways to compute $Eh(X)$ where h is a known function: One way is to first find the density for $Y \equiv h(X)$ (which we denote by $g(y)$), and then $Eh(X) = \int yg(y)dy$. The other way is to use formula $E[h(X)] = \int h(x)f(x)dx$, where $f(x)$ is the density of X. The second way is usually easier. To see why, try both ways and compare.