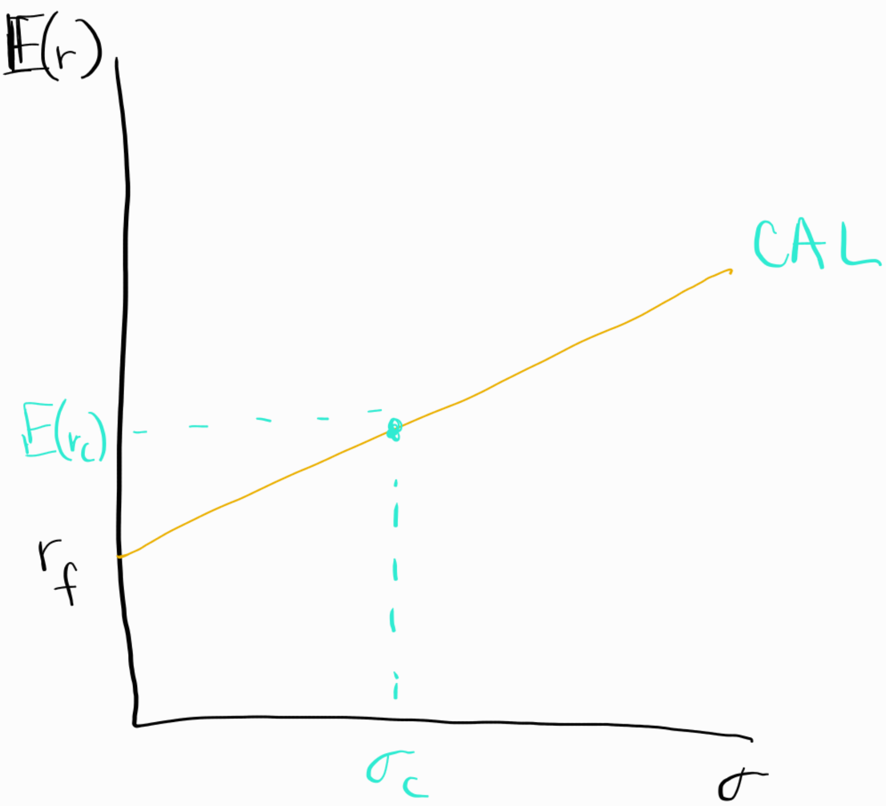
Security Allocation

So far, we have covered capital allocation and asset allocation.

Review:

→ Capital Allocation: we have a graph mapping Expected returns from our overall portfolio c (risky + fixed) to the volatility of the return. The resulting line is the capital assets line (CAL). The point along the line where agents invest depends on their utility function and risk aversion coefficient (A).



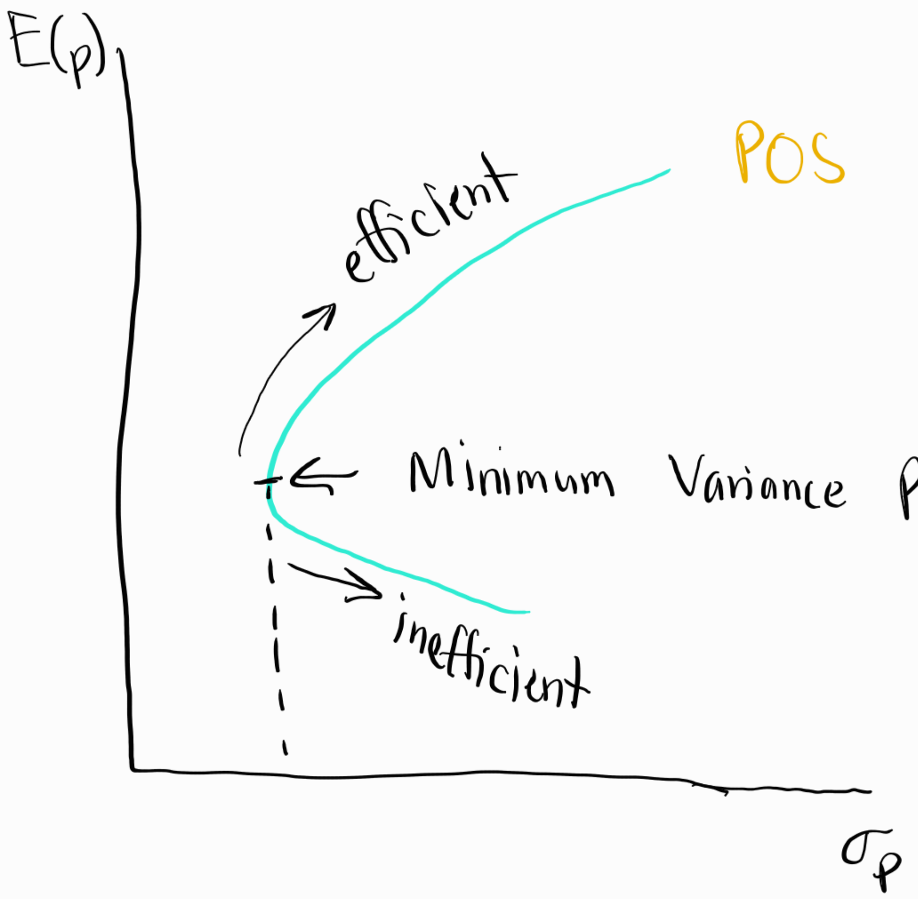
$$y^* = \frac{E(r_p) - r_f}{0.01\, A\, \sigma_p^2}$$

→ Asset Allocation: We work in a mean–variance framework. We defined 2 assets (debt & security) within our risky portfolio (p). We then find the weight we should apply to each (w_D & w_E) such that $\sum_i w_i = 1$ (where a w_i could be negative).

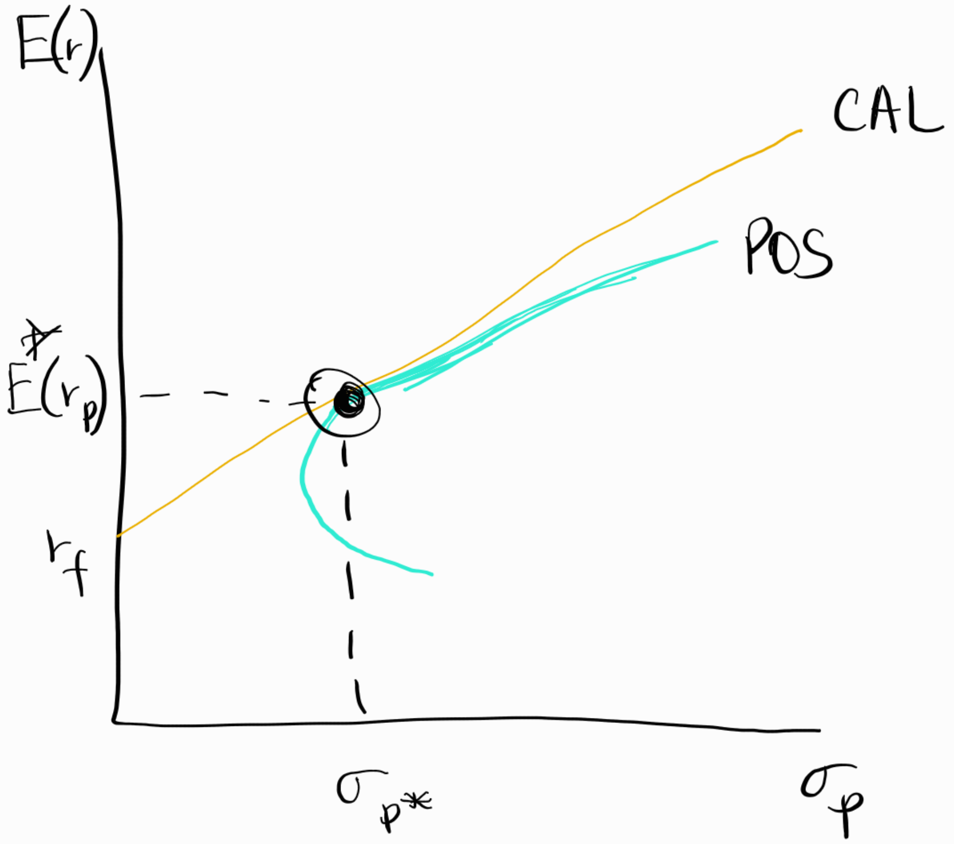
$$E(r_p) = w_D\, E(r_D) + w_E\, E(r_E)$$

$$\sigma_p^2 = w_D^2 \sigma_D^2 + w_E^2 \sigma_E^2 + 2 w_D w_E \operatorname{Cov}(r_D, r_E)$$

Using different values for the weights produces the portfolio opportunity set (POS):



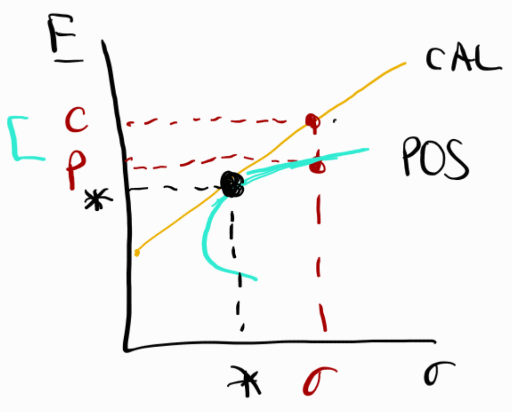
Mapping both the CAL and POS onto the same graph, we get:



where we want to maximize the slope of the CAL (the Sharpe ratio) with respect to the chosen weights.

$$\max_{w_D} \quad S = \frac{E(r_p) - r_f}{\sigma_p}$$

The maximizing point happens when the POS and CAL are tangent. Away from that point, you get a higher return from the CAL line than the POS line:



So any part of the POS that is below the CAL is inefficient in this context.

Security Allocation

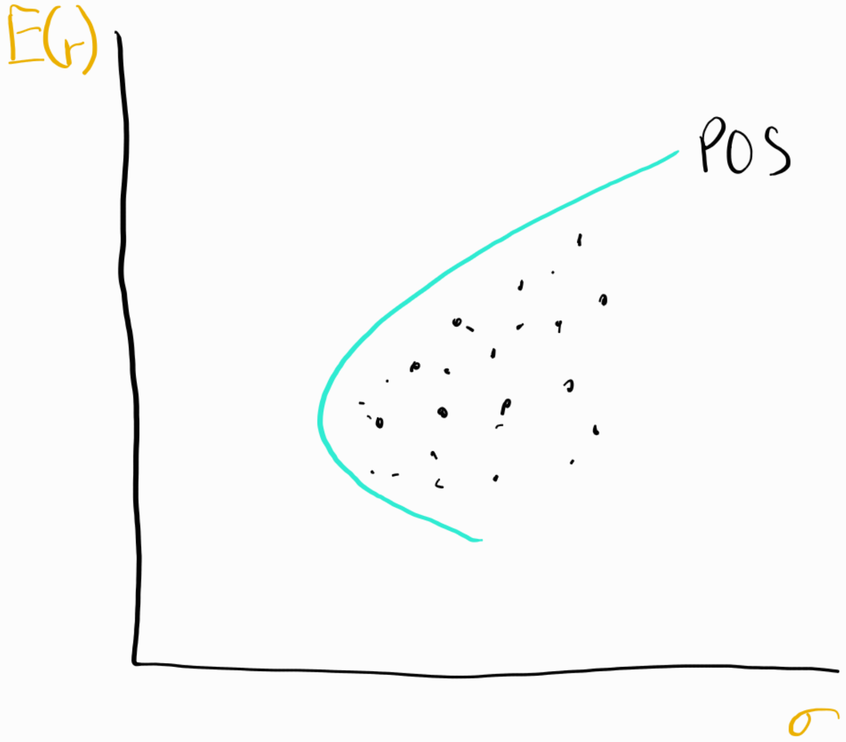
This is similar to asset allocation, except now we break down each asset class into a number of securities.

Asset → Equity (E) (weight w_E)

- Amazon → w_1
- Tesla → w_2
- Apple → w_3
- ⋮ → ⋮
- ⋮ → ⋮

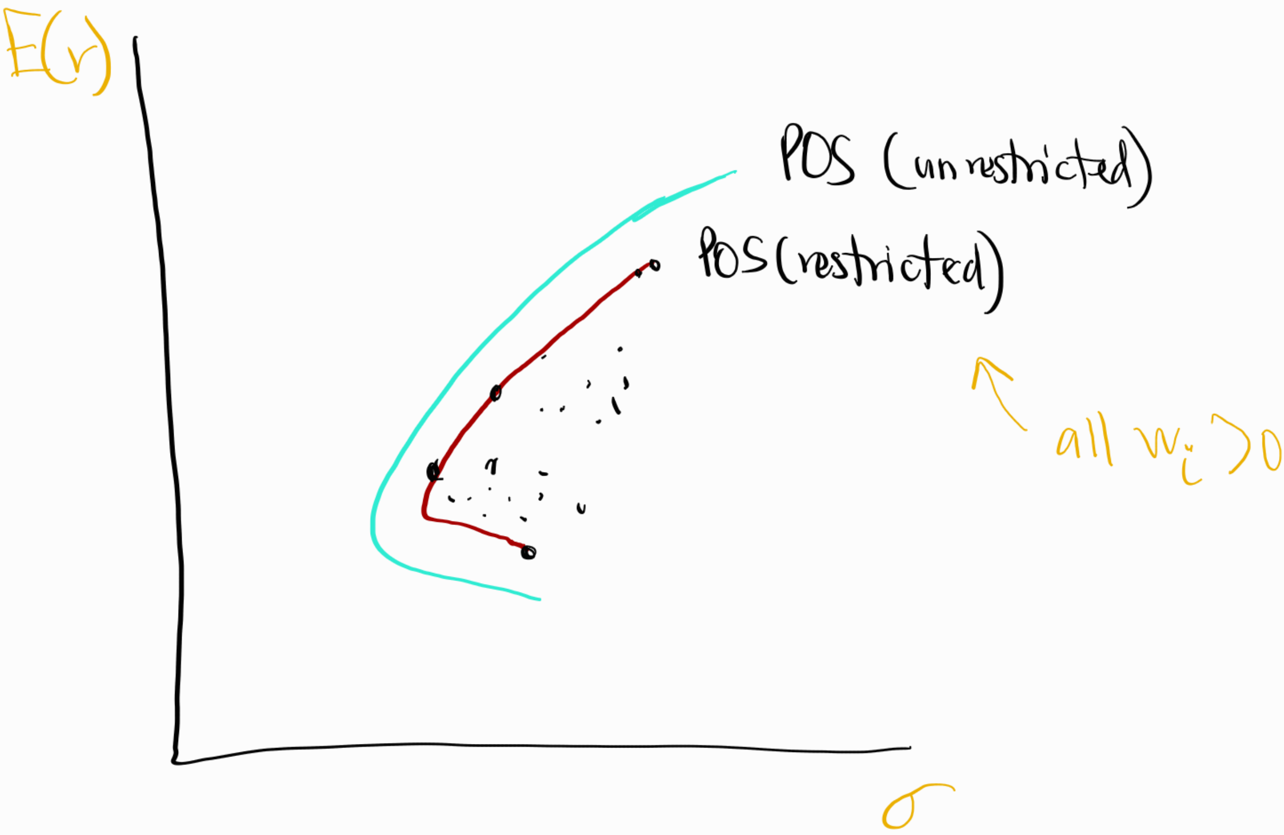
Like before, $\sum_{i=1}^{n} w_i = 1$ where short sales (negative weights) are allowed

Each security has its own expected return $E(r_i)$ and volatility (σ_i). Just as before, changing the weights on each of the assets forms a POS line.



when short sales are allowed, the POS will envelope all possible points like it does here.

when we restrict the weights to being positive (so we have no short sales), the area covered by the POS shrinks. The restricted and unrestricted POS is drawn below:



Notes for Excel

Correlation vs. Covariance Matrix

1	·	·	·
·	1	·	·
·	·	1	·
·	·	·	1
·	·	·	·

var	·	·	·
·	var	·	·
·	·	var	·
·	·	·	var

$-1 \leq \rho \leq 1$

correlation coefficient

$$cov(x,y) = \underbrace{\sigma_x \sigma_y}_{\text{st. dev.}} \rho_{xy}$$

Border-Multiplied Covariance Matrix

w_i	w_1	w_2	w_3	w_4	...
w_1	w_1^2 var	$w_1 w_2$ cov	$w_1 w_3$ cov	$w_1 w_4$ cov	...
w_2	$w_2 w_1$ cov	$w_2 w_2$ var	$w_2 w_3$ cov	$w_2 w_4$ cov	...
w_3	$w_3 w_1$ cov	$w_3 w_2$ cov	$w_3 w_3$ var	$w_3 w_4$ cov	...
w_4	$w_4 w_1$ cov	$w_4 w_2$ cov	$w_4 w_3$ cov	$w_4 w_4$ var	...
⋮	⋮	⋮	⋮	⋮	

For the border-multiplied matrix, the sum of the weights is always 1. The sum across all $w_i w_j cov$ & $w_i^2 var$ is the portfolio variance. The portfolio standard deviation is the square root of the variance.

Expected return of the portfolio is the weight for each security times its expected return all summed:

$$E(r_E) = \sum_{i=1}^{n} w_i E(r_i)$$

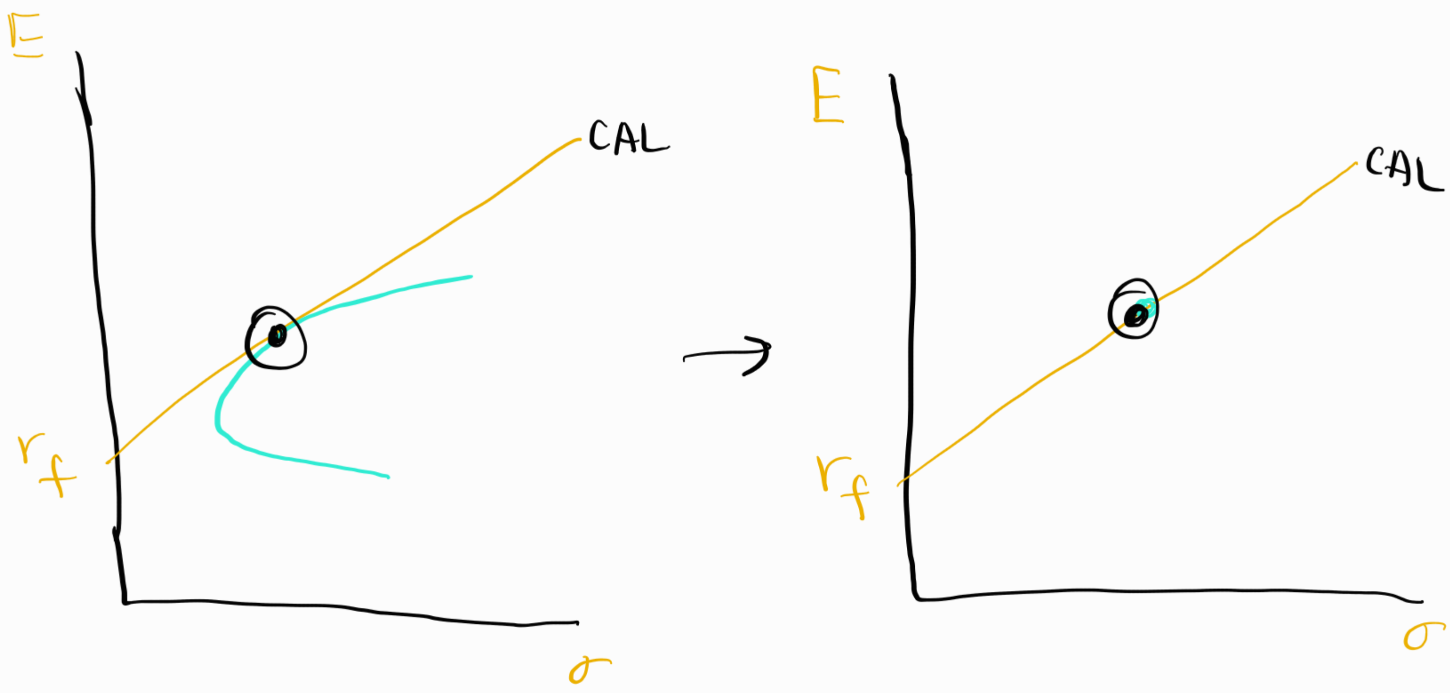
Using equal weights (so each weight equals 1 ÷ number of securities) gives us the benchmark portfolio.

Use solver on BM Cov Matrix to find min. variance & st. d. for a given expected portfolio return by changing the weights. Doing this multiple times gives a range of $E(r_p)$ and σ_p pairs. Graphing these will give the POS line. Note that the given $E(r_p)$ should show up as a constraint in the solver window.

Next we want to find the CAL that is tangent to the POS we find given a certain r_f.
Note that this is a unique solution.

Essentially, we use solver to maximize the Sharpe Ratio $\left(\frac{E(r_p)-r_f}{\sigma_p}\right)$ by changing our weights. Our restrictions are that the weights must add to 1 (for unrestricted). For restricted, we also add that each weight must be greater than or equal to zero.

The unique solution we get here gives us the weights we will use for our securities. We can now forget the rest of the POS:



Where we land on the CAL line depends on the agents risk aversion. From here, we can calculate y^*, which was the amount of capital we should invest in our risky portfolio.

$$\text{risky portfolio share} = y^* = \frac{E(r_p) - r_f}{0.01 \, A \, \sigma_p^2}$$

$$\text{fixed portfolio share} = 1 - y^*$$

Once we have the total share of capital going to the risky portfolio, we can use the security weights we found to calculate the share going to each security.

$$\text{share going to security } i = w_i y^*$$

and $$\sum_{i=1}^{n} w_i y^* = y^*$$

Finally, we can calculate the expected return and volatility of the entire porfolio C (including risky and fixed).

$$E(r_c) = r_f + y^*[E(r_p) - r_f]$$

which is also equation of CAL line

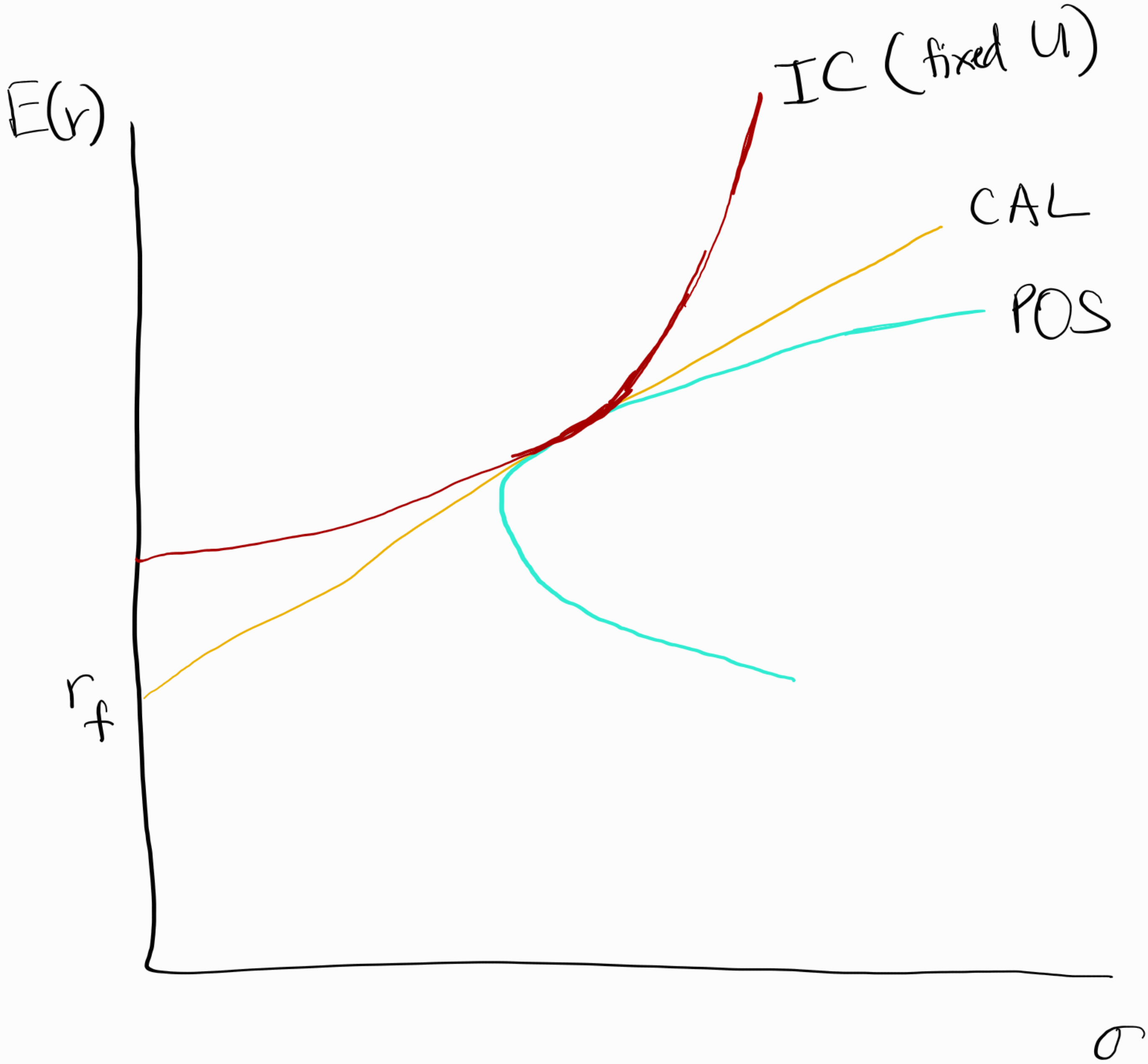
$$\sigma_c = y^* \sigma_p$$

share of capital going to the risky portfolio x the volatility of the risky portfolio. Remember that $\sigma_f = 0$ because it is fixed. So σ_c depends solely on σ_p

Then we can calulate utility where:

$$U = E(r_c) - 0.05A\sigma_c^2$$

The final graph in the Excel document looks at what happens to CAL and $E(r_c)$ when we change σ_c for a fixed U (indifference curve). The indifference curve takes a strange shape here since less of σ_c is preferred (so no monotonicity assumption). IC looks like  instead of  .


E(r)
IC (fixed U)
CAL
POS
r
f
σ