Econometrics I Review

Midterm Review

Chapter 2.

- PE: $\frac{\partial E(y|x_1,\cdots x_N)}{\partial x_j}$
- Elasticity: $\frac{\partial E(y|\underset{\sim}{x})}{\partial x_j} \frac{x_j}{E(y|\underset{\sim}{x})}$

  $E[y|\underset{\sim}{x}]>0$, $x>0$ $\Downarrow$ $\frac{\partial \log E(y|x)}{\partial \log x_i}$

- Semi-Elasticity: $100 \cdot \frac{\partial E(y|\underset{\sim}{x})}{\partial x_j} \frac{1}{E(y|\underset{\sim}{x})}$
- Decompose of y: $y = E[y|\underset{\sim}{x}] + u$
- $E[u|x]=0 \Rightarrow$ (i) $E(u)=0$

  (ii) $cov(u, x_i)=0$

  (iii) $cov(u, f(x_i))=0$
- (u,v) IND of $\underset{\sim}{x} \Rightarrow E[u|v,x]=E[u|v]$
- LIE (CE)

  $E(y|x) = E[E[y|\underset{\Updownarrow}{x}]|\underset{\Updownarrow}{x}]$ (w, w)

  $\exists f \ni x=f(w)$

  $E[y|x]=E[E[y|x,z]|x]$
- $E[y|x] = E[y|g(x)]$
- APE $\delta_j(x_0) = E_q\left[\frac{E[y|\underset{\sim}{x}]}{\partial x_j}\Big|_{x_j=x_0}\right]$

LP1. $E[y|x]=x\beta \Rightarrow L(y|x)=x\beta$

LP2. $u' \equiv y - L(\underset{\sim}{y}|\underset{\sim}{x})$ Residual

$\Rightarrow E(x'u)=0$

LP4. (LIE) $L(y|x)=L[L(y|x,z)|x]$

LP5. (LIE) $L(y|x)=L[E(y|x,z)|x]$

Problems

1. $E[y|x,q] = \mu_1(x,q)$. where q is an unobserved variable.

(1) Find the Partial Effect (P15)

Then $\theta(x,q) \overset{def}{=} \frac{\partial E[y|x,q]}{\partial x} = \frac{\partial \mu_1(x,q)}{\partial x}$

(2) Find the APE at x_j^* over q.

$E_q[\theta(x^*,q)] = \int_{\mathbb{R}} \theta(x^*,q)\, g(q)\, dq$

$g(q)$ is the PDF of q.

2. $y = \beta_0 + \beta_1 x + \gamma q + u$ (P22)

(1) What can we say about $E[u|x,q]$

$V(u|x,q)$

- $E[u|x,q]=0$
- Moreover, when consider consistency, x and q should be independent
- A weaker assumption is $D(q|x,w)=D(q|w)$ where w is observed variables, which can be taken as proxy for q.
- Redundancy

  $E[y|x,q,w] = E[y|x,q]$
- If u is invariant of $\underset{\sim}{x}$

  $var(u|\underset{\sim}{x}) = var[y|\underset{\sim}{x}] = E[u^2|x] = E[u^2]$

3. For random scalars u, q and random vector $\underset{\sim}{x}$, suppose

(1) $E[u|\underset{\sim}{x},q]$ is a linear function of $(\underset{\sim}{x},q)$

(2) $E[u]=E[q]=0$

(3) u,q are uncorrelated with $\underset{\sim}{x}$

Then

$E[u|\underset{\sim}{x},q] = E[u|q] = \rho_1 q$, for some ρ_1

Chapter 3

Convergence of different types

- P-converge: $\forall \varepsilon > 0,\ P(|x_N - a| > \varepsilon) \to 0 \quad (N \to \infty)$
- D-converge: $\forall \xi > 0,\ F_N(\xi) \to F(\xi) \quad (N \to \infty)$
- P-convergence $\Rightarrow$ D-convergence

- $O(N^\lambda)$: $\frac{a_N}{N^\lambda}$ bounded $\quad o(N^\lambda)$: $\frac{a_N}{N^a} \to 0 \ (N \to \infty)$
- Bounded in probability: $\forall \varepsilon > 0,\ \exists b_\varepsilon, N_\varepsilon, \forall N > N_\varepsilon$.
  $P(|x_N| \geq b_\varepsilon) < \varepsilon$
- $O^P(a_N)$: $\frac{x_N}{a_N}$ P-bounded. $\quad o^P(a_N)$: $\frac{x_N}{a_N} \xrightarrow{P} 0$.

Limiting theories

- Slutsky: $\operatorname{plim} g(x_N) = g(\operatorname{plim} x_N)$ $\left(g(x_N) \xrightarrow{P} g(x) \right)$
  $g(x_N) \xrightarrow{d} g(x)$
- CMT $\quad \underset{\sim}{x}_N \xrightarrow{d} \underset{\sim}{x} \Rightarrow g(\underset{\sim}{x}_N) \xrightarrow{d} g(\underset{\sim}{x})$, $g \in C(\mathbb{R}^k)$
- WLLN: $w_i \overset{iid}{}$ $\frac{\sum w_i}{N} \xrightarrow{P} E[w_i]$
- CLT: $w_i \overset{iid(0,?)}{}$, $\frac{\sum w_i}{N^{\frac{1}{2}}} \xrightarrow{d} N(0, E[w_i' w_i])$ PD.

Asymptotic analysis

- Consistent $\hat{\theta}_N \xrightarrow{P} \hat{\theta}$
- $\sqrt{N}$-asymptotic
  $\sqrt{N}(\hat{\theta}_N - \theta) \overset{a}{\sim} N(0, V)$
  $\operatorname{Avar} \sqrt{N}(\hat{\theta}_N - \theta) = V$
  $\operatorname{Avar}(\hat{\theta}_N) = \frac{\hat{V}}{N}$
  $\operatorname{Ase}(\hat{\theta}_{N_j}) = \left(\frac{\hat{V}_{N_{jj}}}{N} \right)^{\frac{1}{2}} \quad (V_N \xrightarrow{d} V)$
- $\sqrt{N}$ consistent est. if $\sqrt{N}(\hat{\theta}_N - \theta) = O_p(1)$
- Consistent test: $\lim\limits_{N \to \infty} P_N(\text{reject } H_0 \mid H_1 \text{ v}) = 1$
- Wald test $\quad H_0: R\underset{\sim}{\theta} = \underset{\sim}{r}$

$$W_N \equiv (R\hat{\theta}_N - r)' \left(R \frac{\hat{V}_N}{N} R' \right)^{-1} (R\hat{\theta}_N - r) \sim \chi^2_Q$$

Delta Method $\quad \gamma_N \equiv g(\theta_N)$

(1) $\sqrt{N}(\hat{\theta}_N - \theta) \overset{a}{\sim} N(0, \sigma^2)$

$\Rightarrow \sqrt{N}(g(\hat{\theta}_N) - g(\theta)) \overset{a}{\sim} N(0, \sigma^2 [g'(\theta)]^2)$

$\operatorname{Avar} g(\hat{\theta}_N) = [g'(\theta)]^2 \operatorname{Avar}(\theta_N)$ — CLT

$\operatorname{ASE}(\hat{\gamma}_N) = C'(\theta) \operatorname{ASE}(\hat{\theta}_N)$

(2) $C(\theta) = \nabla_\theta C(\theta) = \begin{bmatrix} \frac{\partial c_1}{\partial \theta_1} & \cdots & \frac{\partial c_Q}{\partial \theta_1} \\ \vdots & & \vdots \\ \frac{\partial c_1}{\partial \theta_P} & & \frac{\partial c_Q}{\partial \theta_P} \end{bmatrix}$

$\sqrt{N}(\hat{\theta}_N - \theta) \overset{a}{\sim} N(0, V)$

$\Rightarrow \sqrt{N}[c(\hat{\theta}_N) - c(\theta)] \overset{a}{\sim} N(0, C' V C')$

$\operatorname{Avar}(c(\hat{\theta}_N)) = C(\hat{\theta}_N)' \operatorname{Avar}(\hat{\theta}_N) C(\hat{\theta}_N)$

$\operatorname{Ase}(\hat{\gamma}_N) = [\nabla_\theta c(\hat{\theta}_N)]^{\frac{1}{2}} \operatorname{Avar}(\hat{\theta}_N) [\nabla_\theta c(\hat{\theta}_N)']^{\frac{1}{2}}$

- Wald $\quad H_0: c(\underset{\sim}{\theta}) = 0$
  $\text{Wald} = c(\hat{\theta}_N)' \{ C_N \frac{V}{N} C_N' \}^{-1} c(\hat{\theta}_N) \sim \chi^2_Q$

Problems

1. Find Var, $\operatorname{Avar} \sqrt{N}(\bar{y}_N - y)$, $\operatorname{Avar}(\bar{y}_N)$
   Asd, ASE

eg1. $\quad y_i \overset{iid}{\sim} N(\mu, \sigma^2)$

a. Find $\operatorname{Var}(\sqrt{N}(\bar{y}_N - \mu))$

$\because \hat{y}_i \overset{iid}{\sim} N(\mu, \sigma^2) \therefore \bar{y}_N = \frac{\sum y_i}{N} \sim N(\mu, \frac{N\sigma^2}{N^2})$ $\left(= \frac{\sigma^2}{N} \right)$

$\therefore \sqrt{N}(\bar{y}_N - \mu) \sim N(0, \sigma^2)$, $\operatorname{Var}(\sqrt{N}(\bar{y}_N - \mu)) = \sigma^2$.

b. Find $\operatorname{Avar} \sqrt{N}(\bar{y}_N - y)$

$\because E[\sqrt{N}(\bar{y}_N - \mu)] = 0$

By CLT $\frac{\sqrt{N}(\bar{y}_N - \mu)}{\sqrt{N}} \xrightarrow{d} N(0, B)$

$\therefore \bar{y}_N$ is $\sqrt{N}$-asym. where $B = \operatorname{Avar}(\sqrt{N}(\bar{y}_N - \mu)) = \sigma^2$.

c. $\operatorname{Avar}(\bar{y}_N) = \frac{\sigma^2}{N}$

d. $\operatorname{ASD}(\bar{y}_N) = \sqrt{\operatorname{Avar}(\bar{y}_N)} = \sqrt{\frac{\sigma^2}{N}} = \frac{\sigma}{\sqrt{N}}$

e. $ASE(\bar{y}_N)$: Step 1. Find unbiased consistent est. for $Avar\sqrt{N}(\bar{y}_N - y) = V$

Step 2. $\left(\frac{V_{jj}}{N}\right)^{\frac{1}{2}}$

Here $\because Avar\sqrt{N}(\bar{y}_N - y) = V = \sigma^2$

The unbiased est. for sample $V = \sigma^2$ is $\frac{\sum^N (y_i - \bar{y}_N)^2}{N-1}$

$$\therefore ASE = \left[\frac{\sum_{i=1}^N (y_i - \bar{y}_N)^2}{N-1} \Big/ N\right]^{\frac{1}{2}}$$

$$= \left[\frac{\sum^N (y_i - \bar{y}_N)^2}{N(N-1)}\right]^{\frac{1}{2}}.$$

eg2. $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2)$ $\sqrt{N}$-AND for (θ_1, θ_2)

$\hat{\gamma} = \hat{\theta}_1 / \hat{\theta}_2$ is an est. for $\gamma = \theta_1/\theta_2$.

a. show $\text{plim}\,\hat{\gamma} = \gamma$

$\hat{\gamma} = g(\hat{\theta}_1, \hat{\theta}_2) = \hat{\theta}_1 / \hat{\theta}_2$ is continuous.

By slutsky $\text{plim}\, g(\hat{\theta}_N) = g(\text{plim}\,\hat{\theta}_N)$

i.e. $\text{plim}\left(\frac{\hat{\theta}_1}{\hat{\theta}_2}\right) = g\left(\text{plim}\,\frac{\hat{\theta}_1}{\hat{\theta}_2}\right)$

$$\text{plim}\,\hat{\gamma} = \left(\frac{\text{plim}\,\hat{\theta}_1^N}{\text{plim}\,\hat{\theta}_2^N}\right) = \frac{\theta_1}{\theta_2}$$

$$= \gamma$$

b. Find $Avar(\hat{\gamma})$ by θ, $Avar(\hat{\theta})$

By Delta Method $Avar(\hat{\gamma}) = C(\theta)' Avar(\hat{\theta}) \cdot C(\theta)$

$$C(\theta) = \nabla_\theta C(\theta) = \begin{bmatrix} \frac{\partial C_1}{\partial \theta_1} \\ \frac{\partial C_1}{\partial \theta_2} \end{bmatrix} = \begin{bmatrix} \frac{1}{\theta_2} \\ -\theta_1/\theta_2^2 \end{bmatrix}$$

$$\therefore Avar(\hat{\gamma}) = \left[\frac{1}{\theta_2}, \ \frac{-\theta_1}{\theta_2^2}\right] Avar(\hat{\theta}) \begin{bmatrix} \frac{1}{\theta_2} \\ -\frac{\theta_1}{\theta_2^2} \end{bmatrix}$$

$$\equiv B$$

c. Derive the limiting Dist'n of $C(\hat{\theta}_1, \hat{\theta}_2)$

By b. $\sqrt{N}\left[C(\hat{\theta}_1, \hat{\theta}_2) - C(\theta_1, \theta_2)\right] \overset{a}{\sim} N(0, B)$

① $\left[C(\hat{\theta}_1, \hat{\theta}_2) - C(\theta_1, \theta_2)\right] \overset{a}{\sim} N\left(0, \frac{B}{(\sqrt{N})^2}\right)$

② $C(\hat{\theta}_1, \hat{\theta}_2) \overset{a}{\sim} N\left(C(\theta_1, \theta_2), \frac{B}{N}\right)$

2. Validate consistency

$\hat{\gamma}_N = g(\hat{\theta}_N) \xrightarrow{P} \hat{\gamma} = \log(\theta)$

Step 1. Pf $\hat{\theta}_N \xrightarrow{P} \theta$

Step 2. by slutsky

3. Describe the limiting behavior on β_1 if $y = \beta_0 + \beta_1 x + \gamma q + u$ and $q = \delta_1 + \delta_2 x + \delta_3 x^2 + v$.

Simply combine then

$$y = \beta_0 + \beta_1 x + \gamma(\delta_1 + \delta_2 x + \delta_3 x^2 + v) + u$$

$$= \beta_0 + (\beta_1 + \gamma\delta_2)x + \gamma\delta_3 x^2 + \gamma v + \gamma\delta_1 + u$$

If $E[\underset{\sim}{x}' u] = 0$ and full rank

Then $\text{plim}(\hat{\beta}_1) = \beta_1 + \gamma\delta_2$

Chapter 4.

1. OLS and AST properties $\hat{\beta}=(X'X)^{-1}X'Y$

OLS1. $E(u'\underset{\sim}{x})=0 \Leftarrow E[u|x]=0$

OLS2. $E(x'x)=k$ ($1\times k$) - full rank

OLS1 + OLS2 $\Rightarrow \hat{\beta}$ is consistent for β

OLS3. $B\equiv E(u^2x'x)=\sigma^2 E(x'x) \Leftarrow Var(u|x)=E(u^2|x)=\sigma^2$

OLS1 + OLS2 + OLS3 $\sqrt{N}(\hat{\beta}-\beta)\overset{a}{\sim}N(0,\sigma^2A^{-1})$

$A=E(x'x)$

2. Test for significance

(1) F-test $y=x_1\beta_1+x_2\beta_2+u$ (β_1,β_2) is $(k+1)\times 1$, β_2 is $Q\times 1$

$$F=\frac{(R^2_{ur}-R^2_r)}{1-R^2_{ur}}\cdot\frac{N-(k+1)}{Q}\sim F_{Q,N-(k+1)}$$

$H_0: \beta_2=0$

(2) HTSKD Robust test

$H_0: R\beta=r$

$$W=(R\hat{\beta}-r)'(R\hat{V}R')^{-1}(R\hat{\beta}-r) \overset{a}{\sim}\chi^2_Q$$

$$\hat{V}=AVar(\hat{\beta})=(X'X)^{-1}(\Sigma\hat{u}_i^2x_i'x_i)(X'X)^{-1}$$

$\Updownarrow$ $\frac{W}{Q}\sim F_{Q,N-k}$

(3) LM Test

For $y=\beta_0+\beta_1x_1+\beta_2x_2+u$, test $H_0: \underset{Q\times 1}{\beta_2}=0$

Step1. Run $L(y|x_1)$ to get $\tilde{u}$

Step2. Run $L(\tilde{u}|x_1,x_2)$ to get R^2_u.

Step3 $LM=NR_u^2\sim\chi^2_Q$

F and LM are equivalent

(4) LM under HTSKD.

$y=\beta_0+\beta_1x_1+\cdots+\beta_5x_5$. H_0: $\beta_4=\beta_5=0$.

Step1. Run $y=\tilde{\beta}_0+\tilde{\beta}_1x_1+\cdots\tilde{\beta}_3x_3+\tilde{u}$

Step2. Run $\begin{cases}x_4=\beta^*_{10}+\beta^*_{11}x_1+\cdots+\beta^*_{13}x_3+r_1\\ x_5=\beta^*_{20}+\beta^*_{21}x_1+\cdots+\beta^*_{23}x_3+r_2\end{cases}$

Step3 Run $1=\beta^\Delta_1\underset{\sim}{r_1u}+\beta^\Delta_2\underset{\sim}{r_2u}+u^\Delta$

Step4 $LM=N-\underset{\sim}{u}^{\Delta\prime}$

3. Ignoring—omitted variables

$E(y|x_1,\cdots x_k,q)=\beta_0+\beta_1x_1+\cdots+\beta_kx_k+\gamma q$

q is the omitted variable.

$$y=\beta_0+\beta_1x_1+\cdots+\beta_kx_k+\overbrace{\gamma q+v}^{u}. \quad ①$$

Structural error: ① $E(v|x_1\cdots x_k,q)=0$

$E[u|x]=\gamma E(q)+\underset{0}{E[v|x]}$

② $E(q)\equiv 0$

If $q=\delta_0+\delta_1x_1+\cdots+\delta_kx_k+r$

w/. $E(r)=0$, $Cov(x_j,r)=0$

$$y=(\beta_0+\gamma\delta_0)+(\beta_1+\gamma\delta_1)x_1+\cdots+(\beta_k+\gamma\delta_k)x_k+v+\gamma r \quad ②$$

4. Proxy Variable

(1) Proxy is redundant (z is proxy for q)

$E[y|\underset{\sim}{x},z,q]=E[y|\underset{\sim}{x},q]$

(2) Correlation between q and x_j are 0 when partialing out z.

$L(q|1,\underset{\sim}{x},z)=L(q|1,z)$

Let $q=\delta_0+\delta_1z+r$, we need ① $E[r]=0$

② $Cov(z,r)=0$ $\Rightarrow$ $Cov(\underset{\sim}{x},r)=0$

5. Models w/ Interactions in Unobservables: Random Coeff Models

$$y=\beta_0+\beta_1x_1+\cdots+\beta_kx_k+\gamma_1q+\gamma_2x_kq+v \quad ④$$

PE: $\frac{\partial E(y|x,q)}{\partial x_k}=\beta_k+\gamma_2q$

$E(q)=0 \Rightarrow APE=\beta_k$

6. Average treatment effect

$E[y|x_1\cdots x_{k-1},1,q]-E(y|x_1\cdots x_{k-1},0,q)$

$=\beta_k+\gamma_2q$.

$q=\theta_1z$

$E[q|x,z]\overset{Asm}{=}E[q|z]=\theta_1z$

$$y=\beta_0+\beta_1x_1+\cdots+\beta_kx_k+\gamma_1\theta_1z+\gamma_2\theta x_kz \quad ⑤$$

6. Measurement error

(1) ln y

$y^* = \beta_0 + \beta_1 x_1 + \cdots \beta_k x_k + v$

$y^* = y - e_0$

$Var(v + e_0) = \sigma_v^2 + \sigma_0^2$

(2) $y = \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k^* + v$

$x_k^* = x_k - e_k$

$Cov(x_k, e_k) = 0$

$\Rightarrow y = \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k + (v - \beta_k e_k)$

$Var(v - \beta_1 e_k) = \sigma_v^2 + \beta_1^2 \sigma_k^2$

Classic-errors-in-variable (CEV)

$Cov(x_k^*, e_k) = 0$

7. Hypothesis Test under asymp. theories

$$t = \frac{\hat{\beta}_1 - \beta_1^{Null}}{se(\hat{\beta}_1)}$$

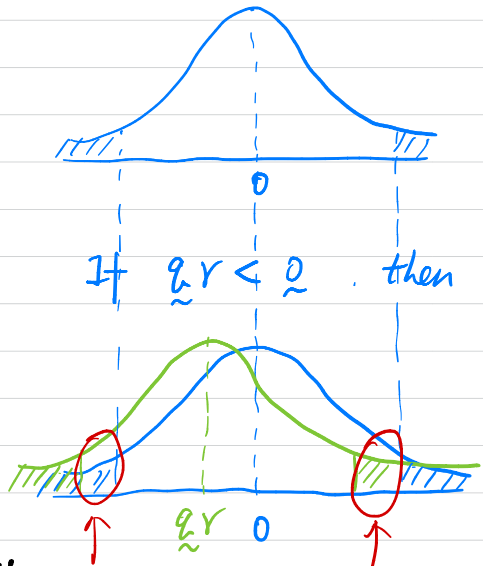
Plim(t-stat) under the null

$$plim(t\text{-stat}) = plim\left(\frac{\hat{\beta}_1 - \beta_1^{Null}}{se(\hat{\beta}_1)}\right)$$

$$= \frac{plim(\hat{\beta}_1) - \beta_1^{Null}}{plim\, se(\hat{\beta}_1)}$$

Ex. $y = \underset{\sim}{x}\beta + \underset{\sim}{q}\gamma + u$

a) t-stat under H_0 w/. $\underset{\sim}{q}\gamma = 0$



b) F-stat = t-stat²

Chapter 5

1. 2SLS – (1 IV)

$$y = \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k + u$$

① $\theta_1 \neq 0$

② $Cov(r_k, x_{k-1}) = 0$, $Cov(r_k, z) = 0$

③ $E(z'x)$, $E(z'z)$ full rank

Reduced: $x_k = \delta_0 + \delta_1 x_1 + \cdots + \delta_{k-1} x_{k-1} + \theta_1 z_1 + r_k$

Substitute, $V \equiv \beta_k r_k + u$

$$\Rightarrow y = \alpha_0 + \alpha_1 x_1 + \cdots + \alpha_{k-1} x_{k-1} + \lambda_1 z_1 + V$$

Now, v is uncorrelated w/ $x_1 \cdots x_{k-1}, z_1$,

thus our estimates on parameters are consistent.

Let $\underset{\sim}{z} = [x_1, x_2, \cdots x_{k-1}, z_1]$

$x = [1, x_1, \cdots x_k]$

Then $y = \underset{\sim}{x}\beta + u$, $\Rightarrow$ $\underset{\sim}{z}'y = \underset{\sim}{z}'\underset{\sim}{x}\beta + u$

$$E[\underset{\sim}{z}'y] = \underbrace{E[\underset{\sim}{z}'\underset{\sim}{x}]}_{\text{need to be full rank}}\beta \Rightarrow \beta = \left(E[\underset{\sim}{z}'\underset{\sim}{x}]\right)^{-1} E[\underset{\sim}{z}'y]$$

When z, x, y are given, expectation can be ignored

$$\Rightarrow \beta = (\underset{\sim}{z}'\underset{\sim}{x})^{-1}(\underset{\sim}{z}'y)$$

2. Multiple IV

$z \equiv (1, x_1 \cdots x_{k-1}, z_1, \cdots z_m)$

step1. $L(x_k | z) \Rightarrow \hat{x}_k$

step 2. $L(y | 1, x_1 \cdots x_{k-1}, \hat{x}_k)$

3. Pitfalls for 2SLS

① we rely on large-sample analysis to justify 2SLS.

② gets close to zero. Thus seemingly small correlations between z_1 and u can cause severe inconsistency—and therefore severe finite sample bias—if z_1 is only weakly correlated with x_1. In such cases it may be better to just use OLS, even if we only

③ 2SLS standard errors have a tendency to be "large."

Chapter 6

1. Testing for endogeneity. (1-V)

Step 1. Run the reduced regression

For $\begin{cases} z_1 \cdots z_m \text{ are IV's for } x_k \\ y = \beta_0 + \beta_1 x_1 + \cdots \beta_{k-1} x_{k-1} + \beta_k x_k + u \end{cases}$

Run $L(x_k | 1, x_1, x_2 \cdots x_{k-1}, z_1, \cdots z_m)$

obtain residual $\hat{V}_2$

Step 2. Run structural regression

$L(y | 1, x_1 \cdots x_k, \hat{V}_2)$

obtain t-stat on $\hat{V}_2$.

t-stat > CV $\Rightarrow$ there is endogeneity

2. Testing endogeneity (multi-variable

$\log(wage) = \alpha_1 educ + \alpha_2 educ \cdot black + z_1\delta_1 + u_1$

Let nearc4 be IV for edu.

To test endogeneity, we need to test

if educ & u_1, educ · black & u, are correlated

Step1 $L(educ | 1, z_1, nearc4, nearc4 \cdot black) \Rightarrow \hat{V}_{21}$

step2 $L(educ \cdot black | 1, z_1, nearc4, nearc4 \cdot black) \Rightarrow \hat{V}_{22}$

step3. $L(y | 1, z_1, educ, \hat{V}_{21}, \hat{V}_{22})$

$\Rightarrow$ F-stat

3. Test for HTSKD. ($H_0: E[u^2|x] = \sigma^2$)

Breusch – Pagan test (BP test)

(i) Estimate y on $\underset{\sim}{x}$, obtain $\hat{u}$, take $\hat{u}^2$

(ii) Run $\hat{u}^2 = \delta_0 + \delta_1 x_1 + \cdots \delta_k x_k + V$, keep its R^2, as $R^2_{\hat{u}^2}$

(iii) Form either $\underset{(F_{k, n-k-1})}{F}$ or $\underset{(\chi^2_k)}{LM}$ statistic

Problems

1. (5.9.) Suppose that the following wage equation is for working high school graduates:

$$\log(wage) = \beta_0 + \beta_1 exper + \beta_2 exper^2 + \beta_3 twoyr + \beta_4 fouryr + u$$

where *twoyr* is years of junior college attended and *fouryr* is years completed at a four-year college. You have distances from each person's home at the time of high school graduation to the nearest two-year and four-year colleges as instruments for *twoyr* and *fouryr*. Show how to rewrite this equation to test $H_0: \beta_3 = \beta_4$ against $H_0: \beta_4 > \beta_3$, and explain how to estimate the equation. See Kane and Rouse (1995) and Rouse (1995), who implement a very similar procedure.

$\theta_4 \stackrel{def}{=} \beta_4 - \beta_3 \Rightarrow \beta_4 = \beta_3 + \theta_4.$

Plug in the specification

$\log(wage) = \beta_0 + \beta_1 exper + \beta_2 exper^2 + \beta_3 (twoyr + fouryr) + \theta_4 fouryr + u$

$= \beta_0 + \beta_1 exper + \beta_2 exper^2 + \beta_3 totcoll + \theta_4 fouryr + u$

$totcoll = twoyr + fouryr$

Now. just estimate the latter equation by 2SLS.
w/. exper, exper², dist2yr, dist4yr as IV's

2. Propose an IV
Discuss if z is a good IV.

1) $E(z, u) = 0$

2) Need a F-stat > 10
From $H_0: \theta = 0$
$z = x\beta + \theta z + v.$
when controlling X