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ARE THERE LAWS OF PRODUCTION?*

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I. *Introduction*

A century and a third ago, in 1815, Malthus[1] and Sir Edward West[2] simultaneously pointed out that if successive combined doses of labor and capital were applied to a given piece of land, the amount of the product would increase by diminishing increments. Two years later this principle was adopted by Ricardo in his *Principles of Political Economy* as the basis for his theory of distribution. The joint return to labor and capital was declared by Ricardo to be governed by and to be equal to the amount of product added by the last combined dose of labor and capital, while the owners of land received as rent all sums in excess of these amounts. Since the quantities of labor and capital were not supposed to vary in relation to each other but were instead bound together in fixed and unvarying proportions, there was no way of isolating the specific contributions of these two factors as a means of determining the rates of wages and of interest. These rates were instead presumed to be regulated by cost-of-supply factors, namely, the Malthusian forces governing population which would keep wages down to a fixed minimum which was at least close to basic subsistence and the low minimum needed to compensate savers and investors. Such was the classical theory of distribution which dominated economic thinking for over sixty years.

Meanwhile, in Germany, during the 1840's, Von Thünen had theoretically broken up the combined dose of labor and capital and had pointed out that when each of the factors was separately increased but the others held constant, the product increased by diminishing

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[1] T. R. Malthus, *Nature and Progress of Rent,* p. 61.

[2] Edward West, *The Application of Capital to Land* (Hollander ed., 1815) p. 54. Prior to Malthus and West, Turgot had pointed out in 1768 that successive applications of labor to land yielded diminishing increments to product. Turgot, *Oeuvres* (1844 ed.), pp. 420-21.

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Number 49 of a series of photographs of past presidents of the Association.

increments.[3] Von Thünen went on to state that the rates of wages and of interest were equal to the amounts of the product added by the last increments of each. He was thus the real discoverer of marginal productivity. Nor was this all. He reasoned that the product added by each equal increment of a factor was a constant fraction of the preceding increment of product, namely two-thirds, in the case of labor and nine-tenths, in the case of capital. This meant that it would be necessary to increase a factor in a given geometric ratio in order to increase the product by equal arithmetic amounts. This is precisely the law of the soil which Mitscherlich and W. J. Spillman later discovered,[4] and it is strikingly similar to the so-called Weber-Fechner law of physiological response. It would be most interesting to find out whether these conclusions of Von Thünen were merely happy hypotheses or whether, like so much of his work, they were based upon experimentation. Von Thünen's work, unfortunately, never had the influence which it deserved. The British, with their customary insularity of thought, virtually ignored it. The Germans, dominated by the fact-grubbing historical school, while lavishing attention upon Von Thünen's theory of location and his advocacy of $\sqrt{ap}$ as a just wage, almost completely neglected his discovery of the curve of the diminishing increment as the guiding principle for both production and distribution. Indeed, schooled as they were to believe in the relativity of economic principles, they naturally averted their gaze from what gave every evidence of being an economic law, which was independent of time and place.

It is to the glory of American economics that it was one of our own number, John Bates Clark, who at a meeting of our Association in 1888, fifty-nine years ago tonight, announced what was in effect the rediscovery of the marginal productivity principle. Clark, who had studied in Germany, had possibly been unconsciously influenced by Von Thünen, but certainly he was not consciously following him when he stated:[5]

An increasing amount of labor applied to a fixed amount of pure capital goods yields a smaller and smaller rate of return. . . . Let there be ten thousnd dollars worth of productive instruments and ten men to use them. Let each man be supposed to create by the operation a product worth three dollars

[3] J. H. Von Thünen, *Der Isolirte Staat; Zweiter Teil,* pp. 507-557-59.

[4] W. J. Spillman, *The Law of Diminishing Returns* (1924).

[5] John Bates Clark, "The Possibility of a Scientific Law of Wages," *Publications, American Economic Association,* Vol. IV (March, 1889). pp. 39-63. It was at this same session that Stuart Wood, the economist-businessman, also developed the marginal productivity theory of wages and interest and indeed went somewhat further by developing the principle of elasticity of substitution. Stuart Wood, "The Theory of Wages," *op. cit.*, pp. 5-35.

a day. Raise now the number of workmen to twenty and let the capital remain the same and each man will create less than before. A day's product will be 3—X dollars. Each successive unit of labor employed in connection with a fixed amount of pure capital produces less than any of its predecessors. . . . General wages tend to be equal to the actual product created by the last laborer that is added to the social working force.[6]

The earnings of capital are subject to identically the same law as those of labor; they are fixed by the product of the last increment that is brought into the field. . . . Let the labor supply remain fixed and let capital increase and each increment of the latter, as it enters the productive field finds that it can create less than any of its predecessors. The general law of diminishing returns is two-sided.[7]

During the next decade Clark completed his theory in a series of subtle articles, and in 1899 gave it final expression in his book *The Distribution of Wealth.*

In the meantime, in 1894, the extraordinarily gifted Philip Wicksteed showed in his pathbreaking little essay, *The Coordination of the Laws of Distribution,* that if production were characterized by a homogeneous linear function of the first degree (that is, if when each and all of the factors of production were doubled or tripled, product would increase in the same proportion), then with each factor receiving its marginal product, the total product would be absorbed in payments to the factors without either surplus or deficit. This essay of Wicksteed's fluttered the mathematical dovecotes. Edgeworth, who in his *Mathematical Psychics,* had attempted to prove, by quotations from Owen Meredith's *Lucille,* that men should receive larger incomes than women, now dismissed with elegant irony the theory that production followed a homogeneous linear function. Pareto's attempted refutation was almost pure sophistry in which, by limiting the market, he sought to prove that product would not increase in proportion to the factors. It remained for Wicksell to give the most sensible treatment of this subject when he pointed out that while the homogeneous production function could not be expected to apply over the whole range of output within a plant, nevertheless under perfect competition, each firm would tend to carry its scale of output to the point where neither increasing nor decreasing returns prevailed but where instead the rate of return was constant.[8] Since industries were merely aggregates of firms and the economy as a whole was an aggregate of industries, it was presumed that the linear function tended, therefore, to be true of society

[6] *Publications, American Economic Association* (Mar., 1889), p. 49.

[7] *Ibid.,* p. 53.

[8] Knut Wicksell, *Lectures on Political Economy,* Vol. I, pp. 101-33.

as a whole at its growing points. Under these conditions, Wicksteed's conclusion held that the payment of the marginal products to each unit of the respective factors of production exactly distributed the product.

At this point, the theoretical discussion of marginal productivity was largely allowed to lapse, except for the clarifications and refinements which were introduced by our chairman, Professor Carver and by F. M. Taylor.

Over the course of the decades which followed, two tendencies in economic teaching became fairly evident. The first was a form of split personality or scientific schizophrenia, which developed in our economics departments. In the classes in economic theory, the principles of pure marginal productivity were taught, uncontaminated by any idea that there might be imperfect competition in either the product or the factor markets, or that there might be unemployment for reasons other than a wage rate in excess of social marginal productivity. This group taught that labor received the amount which its last unit added to the total product multiplied by the number of workers, while the return to capital was similarly determined. Neither trade union nor governmental action was needed to give to labor its own marginal product under conditions of full employment. All that was required was for the employers to bid competitively against each other for labor and this condition was commonly assumed to exist. But if government and unions disturbed the system of *laissez-faire* by raising wage rates above the social margin, this could only be effected by decreasing the numbers employed and hence creating unemployment.

In the classes which dealt with labor economics, however, a different doctrine was taught. Here marginal productivity was muted and the theory of the Webbs, as developed in *Industrial Democracy,* was stressed. It was the pressure of the market competition for lower prices which, weighing more heavily upon the successive levels of sellers because of their heavier overhead costs, tended to drive down wages and to worsen working conditions. Unions and governmental legislation operating through the imposition of common rules, could not only protect the workers from this competition but could raise the general standard of living.

It would be a fascinating task to analyze the differences between these two sets of theories; one dealing primarily with real and the other, with money wages; one assuming the relative absence and the other, the presence of unemployment; one postulating free and perfect competition between employers and between workers; the other absorbed by the power struggles of combinations of employers and workers. But such is not our present task. It is enough to point out that both of these two widely conflicting doctrines have been taught within our economics departments with little effort made by either set of protagonists to determine the relative truth of either, or their compatability. So far has this confusion of doctrine gone, that I have known professors, who teaching both theory and labor economics, have instilled the pure doctrine of John Bates Clark during one hour, and then during the next hour have taught as economic gospel the bargain theories of Sidney and Beatrice Webb!

The effect upon our students of this dualism in the winds of doctrine has been most unfortunate. It has caused some to shrug their shoulders and to dismiss all economic teaching with the words of Omar, so beloved by sophomores,

> Myself when young, did eagerly frequent
> Doctor and Saint, and heard great argument
> About it and about; but evermore
> Came out by the same door where in I went.

Others, like chameleons, have given diametrically different answers to identical questions, depending on which instructor asked them. But to every candidate for the Ph.D. degree, there has loomed the nightmare of that dreaded hour when in his oral examination, he must face both sets of teachers and know that the answers which would be judged right by one school would be judged wrong by the other. Such a state of affairs is at once both ridiculous and scandalous, and as long as it continues, there is little hope for scientific progress or even sound mental health among economists.

But within the ranks of the theorists themselves, a serious intellectual slovenliness, unfortunately, set in. Convinced that the marginal productivity curves of the factors were negatively inclined, they contented themselves with drawing the curves as sloping downwards and to the right, but took apparently little interest in trying to determine what the positions and slopes of these curves actually were. Thus I have seen an experienced instructor on successive days draw widely differing marginal productivity curves for labor, one declining very gradually, one at an angle of 45°, and the third plunging sharply downward. Moreover, this instructor gave every evidence of not realizing that there was any significant difference between these curves nor did he indicate whether he was drawing the curves upon an arithmetic or a double logarithmic scale. Indeed, the slope of the curve seemed to be determined partly by chance, partly by the stance of the instructor, and partly by the degree to which he happened to bend his arm!

The orthodox theorists may urge in self defense that they do not have the statistical information which would permit them to approximate the production function, the elasticities of the marginal productivity curves, or to determine the degree to which the actual distribution of the product conforms to what one would expect from the nature of the production function itself. But the sad truth of the matter is that they have made little effort to find out and have instead turned their backs upon inductive research and have, in effect, been school men living within ivy-clad towers.

II. *The Early Studies of the Cobb-Douglas Production Function*

It was twenty years ago last spring that, having computed indexes for American manufacturing of the numbers of workers employed by years from 1899 to 1922, as well as indexes of the amounts of fixed capital in manufacturing deflated to dollars of approximately constant purchasing power, and then plotting these on a log scale together with the Day index of physical production for manufacturing, I observed that the product curve lay consistently between the two curves for the factors of production and tended to be approximately a quarter of the relative distance between the curve of the index for labor, which showed the least increase over the period, and that of the index for capital which showed the most. Since I was lecturing at Amherst College at the time, I suggested to my friend, Charles W. Cobb, that we seek to develop a formula which would measure the relative effect of labor and capital upon product during this period. We were both familiar with the Wicksteed analysis and Cobb was, of course, well versed in the history of the Euler theorem. At his suggestion, therefore, the sum of the exponents was tentatively made equal to unity in the formula

$$P = bL^kC^{1-k} \tag{1}$$

Here it was only necessary to find the values of b and k. This was done by the method of least squares and the value of k was found to be .75. This was almost precisely what we had expected because of the relative distance of the product curve from those of the two factors. The value of the capital exponent, or $1\text{-}k$, was, of course, then taken as .25. Using these values, we then computed indexes of what we would theoretically have expected the product to be in each of the years had it conformed precisely to the formula. We found that the divergencies between the actual and theoretical product were not great since in only one year did they amount to more than 11 per cent, and that except for two years, the deviation of the differences was precisely what we would expect from the imperfect nature of the indexes of capital and labor. Since our index of capital measured the quantities which were *available for,* rather than their relative *degree of use,* it did not make allowance for the idle capital in periods of depression nor for the more intensive use of capital during years of prosperity. Similarly, our index of labor did not make allowance for failures to work full time in the bad years, nor for overtime hours which were worked in the good years. It was, therefore, to be expected that the actual product (P) would exceed the theoretical product (P') in years of prosperity and would fall below it in years of depression. So in fact it did in every year except the war years of 1918 and 1919. Professor Cobb and I, therefore, regarded these deviations as additional evidence of the general validity of the formula for normal times.

Still another striking bit of evidence was found in the fact that under perfect competition with a production formula of this type we would expect a factor to receive as its share of the product, the proportion indicated by its exponent. From the income studies of the National Bureau of Economic Research, we found that labor's share of the net value product of manufacturing during the decade 1909-1918, was estimated at 74.1 per cent, or almost precisely the value of the exponent for labor.

Professor Cobb and I embodied the results of our inquiries in a paper which we read before this Association exactly twenty years ago tonight.[9] We then determined to analyze more of such time series. Cobb computed indexes of labor, capital, and product in Massachusetts manufacturing for the period 1890-1926, and found the value of k to be .743. Interestingly enough, it was also found that the average of labor's share of the net value of the product in that state for that period was .74, or a virtual identity with the value of k.[10] A similar study, which was made in Chicago by Mr. Director for New South Wales manufacturing for the period 1901-1927, found k to have a value of .65.

There the matter more or less rested when my book, *The Theory of Wages,* appeared in 1934. Three years later, with the aid of Mrs. Marjorie Handsaker, I resumed our analysis of time series, and after working up data for Victorian manufacturing for the period 1907-1929, we found the value of k under the k and $1\text{-}k$ formula to be .71.[11] Labor's share of the net product or W/P was found to be .61 for this period.

We then introduced two important new features into our investiga-

[9] C. W. Cobb and Paul H. Douglas, "A Theory of Production," *Am. Econ. Rev.,* Suppl., Vol. XVIII (Mar., 1928), pp. 139-65.

[10] See Douglas, *The Theory of Wages,* pp. 159-66.

[11] Handsaker and Douglas, "The Theory of Marginal Productivity as Tested by Data for Manufacturing in Victoria," *Quart. Jour. Econ.,* Vols. LII and LIII (Nov., 1937 and Feb., 1938), pp. 1-36 and 215-54.

tions. An able young American scholar, David Durand,[12] had published in 1937, an excellent critical article of the earlier material, and had urged that the restricted function of

$$P = bL^kC^{1-k} \tag{1}$$

be abandoned for one in which the exponent for capital was independently determined. As he correctly pointed out, the use of the k and 1-k function assumed the existence of an economic law which it should be one of the tasks of science to test, namely, the assumption of true constant returns. If we permitted the exponent for capital to be independently determined, it would then be possible for the sum of the exponents to be either greater or less than unity and hence to show true increasing and decreasing as well as constant returns to scale. We therefore decided that Durand's suggestion should be adopted and that we should try to find the values in terms of the formula:

$$P = bL^kC^j \tag{2}$$

The second change was to broaden our fields of investigation. Hitherto, we had dealt only with time studies and had found the values of our exponents from index numbers of labor, capital, and product within a given economy, with each year serving as a separate observation. Here we measured the effect upon total physical product, of changes in the physical quantities of labor and of capital and from these we derived curves of diminishing incremental physical productivity of the classical type. We now determined to open up another field for investigation and to make cross-section analyses between industries in a given economy for specific years. Thus the annual statistics of manufacturing for the British Dominions (although not the British Census of Production itself) and the decennial and quinquennial *Censuses for Manufacturing* for the United States up until 1921 showed aggregates for each of a wide variety of industries from which it was possible to compute: (1) aggregates of the average numbers employed, including wage earners, clerical and salaried employees, officials and generally firm members and working proprietors (L), (2) aggregates of capital (C) expressed in terms of dollars including both fixed and working capital, and (3) aggregates of the *net* value of product added by manufacturing expressed in terms of dollars (P).

In these studies, differences between industries in the quantities of their net value product were presumed to be a function of the total

[12] David Durand, "Some Thoughts on Marginal Productivity with Special Reference to Professor Douglas' Analysis," *Jour. Pol. Econ.*, Vol. XLV (Dec., 1937), pp. 740-58.

number of employees and of the total quantities of fixed and working capital with each industry serving as a separate observation. This is obviously a somewhat different production function from that which is based on the time series. The quantities of labor used are physical quantities and though capital is expressed in value terms, these are also rough measurements of relative physical amounts. But since product is also expressed in value terms, this is the result not only of: (a) changes in the increments to the total physical product but also of (b) changes in the exchange value, or the relative price per unit of the products of an industry. The net values turned out by the respective industries will, therefore, be affected in these cases not only by the quantities produced but also by the respective demand curves for the products. Changes in each of these variables will affect the total exchange value produced.

Some critics will, of course, object that this second type of study, since it includes both quantities and prices, does not measure production at all and is in no sense a test of marginal productivity theory. It is certainly a somewhat different type of production function from that which is based on index numbers of quantities. But marginal productivity theory has always implicitly dealt in terms of values as well as of physical quantities since it assumes that the supplies of labor and capital in each of the various industries are regulated by the principle that the respective marginal laborers will produce equal amounts of value as will the marginal units of capital. In the apportionment of resources within an economy, therefore, the principle of diminishing incremental value productivity is an essential part of economic theory and is worthy of consideration. There is no reason why a production function which deals with it should not also be worthy of consideration and treatment.

Although interrupted by the war, we now have completed six cross-section or inter-industry studies for American manufacturing, namely, for the years 1889, 1899, 1904, 1909, 1914, and 1919; four cross-section studies for Canada covering the years 1923, 1927, 1935, and 1937; three studies for Victoria for the years 1910-11, 1923-24, and 1927-28; one study for New South Wales for 1933-34, and five studies for the Commonwealth of Australia, namely, 1912, 1922-23, 1926-27, 1934-35, and 1936-37. Two of my students, Messrs. G. Brinegar and K. O. Campbell, have just finished such a study for Queensland for 1937-38, and two more, Messrs. B. Solomon and N. A. Deif, are completing another study for New Zealand for 1926-27. In all, therefore, twenty-one cross-section studies have been carried through by our Chicago group to add to our previous four time studies, namely, for the United States, Massachusetts, New South Wales, and Victoria. In addition, two New Zealand economists, Max Brown[13] and J. W. Williams,[14] have carried through two time studies for New Zealand while the latter has also carried through a cross-section study for that country, as has G. W. G. Browne[15] for South Africa. We have, therefore, records for a total of twenty-nine inductive studies of the production function instead of the three which were reported upon thirteen years ago in *The Theory of Wages*.

In these investigations which have been carried out over the last two decades, we have had the assistance of a devoted and, I believe, competent group of associates, and in the aggregate many tens of thousands of hours have been spent upon the work. I am deeply indebted to this group, and while I am solely responsible for any errors which may lie within the work, my associates are chiefly entitled to any credit which may be forthcoming.[16]

Since these studies were carried out over a period of many years and since there were differences between countries and between years within a country in the basic data used, and since we were also constantly trying to improve our methods, it was inevitable that some dissimilarities should have developed in the precise content of the categories used and in the methods of attack. We have now ironed out a great many of these differences, and I believe that with a few exceptions which will be later noted, the results have now been made roughly comparable. It is hoped that in the next few months they may be made completely so.

III. *The Main Results of the Study of the Production Function in Manufacturing*

We can summarize the main results of these studies in three tables. Table I brings together the main results for manufacturing in the

[13] See an unpublished Ph.D. thesis at Cambridge by Max Brown, *The Relation Between Capital and Labour in New Zealand.*

[14] J. W. Williams, "Professor Douglas' Production Function," *Econ. Record,* Vol. XXV (1945), pp. 55-63.

[15] G. W. G. Browne, "The Production Function for the South African Manufacturing Industry," *So. African Jour. Econ.,* Vol. XI (Dec., 1943), pp. 258-68.

[16] First, of course, I am indebted to my chief associate during this period, namely, Grace Gunn, and after her to Marjorie L. Handsaker, Patricia Ogburn, Martin Bronfenbrenner, Ernest Olson, and Estelle Mass. But we have also had the faithful aid of numerous research assistants, computers, and draftsmen, among whom have been Yetta Abend, Helen Butcher, Julia Elliott Lewis, Oscar Seltzer, K. Sanow, H. Minsky, B. Nimer, William L. Slayton, Betty Roth, Donna Allen, Mitchell Locks, Y. K. Wong and Margaret Labadie. My colleague H. G. Lewis has also been most helpful in his criticisms and suggestions, as have John H. Smith and Colin Clark.

United States, as does Table II, for Australia; while Table III covers the investigations for the three British Dominions of New Zealand, South Africa, and Canada.

We may properly begin with a consideration of the American results, which include four time series studies for the period 1899-1922, and six cross-section or inter-industry investigations for the various census years from 1889 to 1919. It is unfortunate that the statistics of capital were omitted from the *United State Census of Manufactures* after 1919 and that we have been unable to continue our analysis of American data beyond the dates stated. Fortunately, the British Dominions in their admirable annual *Censuses of Production* have continued to collect statistics on the amounts of capital invested and this has permitted us to carry on studies for these countries in more recent times.

The four sets of time studies for the United States show somewhat differing results because of the differences which exist between the series of index numbers for labor and product. The precise nature of these series is described in the footnotes to Table I. It is believed that Series II, III, and IV, are appreciable improvements over the original Cobb-Douglas series or Series I. It will be observed that Series II and III, which use total man years (including clerical employees as well as wage earners) and total standard man hours respectively, as the measure of labor, show k's with values of .78 and .73 respectively. Series IV, which eliminates secular trends from each of the three basic series and expresses each observation as a percentage of its respective trend, gives k a value of .63. The value of j varies from .15 under Series II to .30 under Series IV. On the whole, Series II is the one in which the definition of the factors of production is most comparable to that of the cross-section or inter-industry studies, but Series IV avoids the dangers connected with the downward bias of the index of production and also eliminates the factor of time.

Five of the six inter-industries studies show lower values for k than do Series I, II, and III of the time studies. The k's average .63 for the six cross-section years with the average of the j's amounting to .34. The values of k and j for the initial and terminal years of 1889 and 1919, however, deviate appreciably from this average. Those for the earlier years have lower values of k and higher values of j, while in 1919 this tendency is reversed. The values of the exponents during the four middle years of 1899, 1904, 1909, and 1914, however, do exhibit a marked stability around the general average, with the k's ranging between .61 and .65, and the j's between .31 and .37.

It will be observed that in three cross-section studies, the values of k and j are many times their respective standard errors, the k's from

TABLE I.—THE VALUES OF THE PRODUCTION FUNCTION FOR AMERICAN MANUFACTURING 1889–1922

Years		$P=bL^kC^j$						$P=bL^kC^{1-k}$	
		k	σ_k	j	σ_j	$k+j$	b	k	σ_k
A. Time series									
Series I[a]	1899–1922	.81	±.15	.23	±.06	1.04	.84	.75	±.04
Series II[b]	1899–1922	.78	±.14	.15	±.08	.93	1.38	.90	±.04
Series III[c]	1899–1922	.73	±.12	.25	±.05	.98	1.12	.76	±.04
Series IV[d]	1899–1922	.63	±.15	.30	±.05	.93	1.35	.69	±.05
B. Cross-section or inter-industry studies based on industry aggregates[e]									
Year	N								
1889	363	.51	±.03	.43	±.03	.94	58.34	.53	±.03
1899	332	.62	±.02	.33	±.02	.95	106.43	.66	±.02
1904	336	.65	±.02	.31	±.02	.96	107.40	.68	±.21
1909	258	.63	±.02	.34	±.02	.97	90.99	.66	±.02
1914	340	.61	±.03	.37	±.02	.98	81.66	.63	±.02
1919	556	.76	±.02	.25	±.02	1.01	244.21	.75	±.02
Average		.63		.34		.97		.65	

[a] The original Cobb-Douglas series of Labor, Capital, and Product as published in the original paper in 1927, were as follows: (1) Labor (L) = average number of employed wage earners only. Salaried employees, officials, working proprietors, etc., were *not* included; (2) Capital (C) = value of plant, buildings, and tools and machinery reduced to dollars of constant purchasing power with annual increments of investment divided by a specially constructed index of the relative cost of capital goods in which the wholesale prices of metals and metal products, of building materials, and of wages were given the respective weights of 4, 2, and 3; (3) Product (P) = the original Day index of physical production as published in the *Review of Economic Statistics*, Vol. II (1920), pp. 328–29, and Vol. VI (1923), p. 201. For a fuller description see Cobb and Douglas, "A Theory of Production," *Am. Econ. Rev.*, Vol. XVIII, Suppl. (Mar., 1928), pp. 139–65.

[b] The basic data used in Series II differ from those in Series I in that (1) labor now includes clerical and salaried workers as well as wage earners; (2) The basic index of physical production used was the Day-Thomas revision as it appeared in *The Growth of Manufactures, 1899–1923*, Census Monograph VIII, instead of the earlier Day study which was used in Series I. Values for the intercensal years were interpolated by the use of the earlier Day series, while I constructed a new index for leather. See Douglas, *The Theory of Wages*, pp. 174–76. The Day-Thomas index gave slightly lower values for P for the terminal years than the earlier index.

[c] The main difference which distinguishes Series III from Series II is that Labor (L) is defined to be the *relative total standard man hours* worked in the various years by the combined force of wage earners and clerical and salaried employees. This was obtained by multiplying (a) the indexes of employment for the various years by (b) the indexes of the length in manufacturing of the standard working week. For data and methods see Douglas, *Real Wages in the United States*, pp. 546–47.

[d] The essential distinguishing feature of Series IV is that the factor of time was eliminated from the basic data, not from the logs of data. This was done by fitting trends to each of the three series and expressing each index for a given year as a percentage of its trend. The basic data themselves were, however, identical with those used in Series II.

[e] The series of Labor, Capital, and Product in the six cross-section studies have now been reduced to an almost completely comparable basis: (1) Labor (L) = average number of wage

17 to 38 times their standard errors and the *j*'s from 12 to 18 times as great as their standard errors.

On the whole, we should not be surprised by the fact that we obtain higher values for *k* and lower values for *j* in our first three time series studies than we do for the cross-section studies. For as we have pointed out, the two functions are, in fact, somewhat different and we should not necessarily expect identical results. Moreover, in the time studies, there tends to be a systematic downward bias to the index of production which keeps it closer to the index of labor than it should be in reality and hence gives an excessive value to *k*. This downward bias is caused by two factors: (1) Since the indexes are primarily based on the quantities of raw material produced, they do not include the increased fabrication and reworking of these materials which is a pronounced, although not a universal, tendency of industry. (2) It is in practice not possible to include with sufficient rapidity the new products which are continually pushing themselves forward, nor to drop in adequate time the products which are becoming obsolete. The net result is to keep the index numbers of Product (*P*), particularly during the latter years of a given period, closer to Labor (*L*) than in reality they should be and hence *k* is raised above and *j* is depressed below their "true" values.

This weakness is absent from the cross-section or inter-industry studies, which are made for a given year, and we would, therefore, expect the *k*'s to be lower and the *j*'s to be higher in this group of studies. Such is, in fact, exactly the case.

It may be of some significance that when the factor of time is eliminated from each of the three basic series of Labor, Capital, and Product, and the deviations from the trends are studied (as in Series IV), that the true value of *k* for the period of 1899-1922 is reduced to .63. This is identical with the average value of *k* for the six years for which inter-industry studies were made.

It may also be of some significance that in three of the four time

earners, salaried employees, supervisory officials, firm members, and working proprietors; (2) Capital (*C*) = total fixed and working capital; (3) Product (*P*) = gross sales value minus (a) cost of raw materials, (b) cost of fuel, heat, power, and rent, (c) taxes and insurance payments, (d) amounts paid to contractors, (e) cost of repairs, (f) sundries. No deduction has been made for the depreciation of fixed capital except that included under the heading of "repairs." For earlier studies on four of these years see Gunn and Douglas, "The Production Function for American Manufacturing for 1919," *Am. Econ. Rev.*, Vol. XXXI (Mar., 1941) pp. 67–80; Gunn and Douglas, "The Production Function for American Manufacturing for 1914," *Jour. Pol. Econ.*, Vol. L (Aug., 1942), pp. 595–602; Bronfenbrenner and Douglas, "Cross-Section Studies in the Cobb-Douglas Function," *Jour. Pol. Econ.*, Vol. XLVII (Dec., 1939), pp. 761–85; Daly, Olson and Douglas, "The Production Function for Manufacturing in the United States in 1904," *Jour. Pol. Econ.*, Vol. LI (Feb., 1943), pp. 61–65. A more complete description of the methods and results for 1889 will shortly be published by Miss Estelle Mass.

studies and in five of the six cross-section studies, the sum of k and j is slightly less than unity. While this by no means establishes the reality of true diminishing returns, since the differences between $k + j$ and unity are well within the range of the standard errors, there is at least a faint suggestion to that effect. It is possible that American manufacturing industry during this period may have exceeded the optimum size and that the desire for the power and prestige which is attached to bigness may have caused firms to be conducted on a larger scale than that which was justified by the most efficient combination of the factors of production.

While all due caution in drawing conclusions should be observed, it would seem that the most likely long-run norm for k during the period covered was between .63 and .64; and for j was approximately .34. This would mean that a change of one per cent in the quantity of labor (unaccompanied by any change in the quantity of capital) would normally result during this period in a change in the same direction of about sixty-three or sixty-four hundredths of one per cent in the quantity of product, and that similarly a change of one per cent in quantity of capital (unaccompanied by any change in the quantity of labor) would normally result, during this period, in a change in the same direction of about thirty-four hundredths of one per cent. If both factors of production were increased by one per cent, then the total product would normally increase during this period by from ninety-seven to ninety-eight hundredths of one per cent.

If we disregard the slight suggestion of decreasing returns and treat the most probable sum of the exponents as equal to unity, then an increase of one per cent in the quantities of both labor and capital would normally result in a corresponding increase of one per cent in product. A one per cent increase in the quantity of labor alone would normally be accompanied, during this period, by an increase of approximately two-thirds of one per cent in product and an increase of one per cent in the quantity of capital alone would normally be accompanied by an approximate increase of one-third of one per cent in the product. Perhaps this is as close a tentative conclusion as we should draw for this period although further studies may lead to some revision of these results.

Since under these conditions (*i.e.*, $k + j = 1.0$) the elasticity of the marginal productivity curves for a given factor is equal to the reciprocal of the exponent for the other factors, that is $e_L = \frac{1}{1\text{-}k}$ and $e_C = \frac{1}{1\text{-}j}$, it follows that the approximate elasticity of the normal marginal productivity curve for labor during this period would seem to be not far from 3.0, and for capital, to be around 1.5[17]

Let us now turn to the examination of the two time series and nine cross-section studies which have been made for Australia, and which are summarized in Table II, with its accompanying notes. In the Victorian time study, k has a value of .84 and j of .23, while in the New South Wales study, the value of k is .78, and that of j is .20. It should be noticed, however, that the respective standard errors of k and j are quite high and that the values of k under the original formula ($P = bL^kC^{1-k}$) vary somewhat from those obtained under the second formula.

As we would expect from the reasons which have been given, the values of k in the nine cross-section studies for Australia are somewhat lower. The combined average of the k's was .60 and of the j's was .37. Their average sum was, therefore, .97. Here it will be observed that we get identical results with either formula since the average of the k's under formula (1) is also .60. It should also be noted that the values of k are from 8 to 14 times and the j's from 3 to 10 times their standard errors.

The results for the five Commonwealth studies differ somewhat from those for the separate states, having somewhat lower k's and higher j's. Thus, in the Commonwealth, the average of the k's is .55, with a spread in individual years between .49 and .64, while the average for the four state studies is .65. On the other hand, the j's average .43 in the Commonwealth, as contrasted with .20 in the state studies.[18] After the text

[17] The marginal productivity of labor is

$$\frac{\partial P}{\partial L} = \frac{k}{L} P = \frac{k}{L} bL^{k}C^{j} = MP_L.$$

The elasticity of the marginal productivity curve for labor is then defined as

$$\eta = \frac{1}{\frac{(MP_L)}{L}} \cdot \frac{MP_L}{L} = \frac{1}{\frac{k(k-1)P}{L^2}} \cdot \frac{\frac{kP}{L}}{L},$$

then

$$\eta = \frac{1}{k-1}.$$

The flexibility of the marginal productivity curve for labor is defined as the reciprocal of the elasticity of this curve or

$$\phi_l = \frac{1}{\eta} = k - 1.$$

[18] The values of k and j are each relatively large in relation to their standard errors.

TABLE II.—THE VALUES OF THE PRODUCTION FUNCTION FOR MANUFACTURING IN AUSTRALIA

Years		N	$P=bL^kC^j$						$P=bL^kC^{1-k}$	
			k	σ_k	j	σ_j	$k+j$	b	k	σ_k
A. Time series								.71		
Victoria[a]	1907–29	22	.84	±.34	.23	±.17	1.07	.71	.71	±.07
New South Wales[b]	1901–27	26	.78	±.12	.20	±.08	.98	1.14	.86	±.05
B. Cross-section or inter-industry studies										
Australia[c]	1912	85	.52	±.05	.47	±.05	.99	15.87	.52	±.04
Australia[c]	1922–23	87	.53	±.05	.49	±.05	1.02	16.49	.52	±.05
Australia[c]	1926–27	85	.59	±.05	.34	±.04	.93	77.26	.64	±.05
Australia[c]	1934–35	138	.64	±.04	.36	±.04	1.00	39.79	.64	±.04
Australia[c]	1936–37	87	.49	±.04	.49	±.04	.98	21.57	.50	±.04
Victoria[d]	1910–11	34	.74	±.08	.25	±.11	.99	42.87	.75	±.08
Victoria[d]	1923–24	38	.62	±.08	.31	±.10	.93	96.93	.61	±.08
Victoria[d]	1927–28	35	.59	±.07	.27	±.09	.86	207.49	.60	±.05
New South Wales[d]	1933–34	125	.65	±.04	.34	±.03	.99	53.70	.66	±.03
Average all Commonwealth and state studies			.60		.37		.97		.60	
Average Commonwealth studies only			.55		.43		.98		.56	
Average state studies only			.65		.29		.94		.66	

[a] The Victorian index numbers for Labor, Capital, and Product from which the results were computed, were constituted as follows: (1) Labor (L) = average number of persons employed including wage earners, salaried employees, supervisory officials, and working employers; (2) Capital (C) = fixed capital reduced to dollars of constant purchasing power but excluding land values and working capital; (3) Product (P) = index of physical production using 1911 value weights. See Handsaker and Douglas, "The Theory of Marginal Productivity Tested by Data for Manufacturing in Victoria," *Quart. Jour. Econ.*, Vol. LII (Nov., 1937), pp. 1–36.

[b] The New South Wales study is based on series for Labor, Capital, and Product, which are virtually identical in their definition with those of Victoria. The capital index differs from that constructed earlier by Mr. Director in that it does not provide for the replacement at differing price levels for the estimated depreciation on capital. For the Director study, see Douglas, *The Theory of Wages*, pp. 167–172.

[c] In the Commonwealth cross-section studies, the terms are defined as follows: (1) Labor (L) = average number employed of wage earners and salaried employees but generally excluding working proprietors (except in 1934–35); (2) Capital (C) = value of plant and machinery, buildings, and land, but excluding working capital. The exclusion of working capital is the chief dissimilarity between this series and the definition of total capital used in the case of the United States and Canadian cross-section studies. We will try to revise the Australian capital figures to include working capital but this may be difficult. (3) Product (P) = value added by manufacturing. For all the years except 1934–35, this was defined as gross sales value minus (a) cost of materials used, (b) cost of fuel and light, and (c) cost of replacing tools and repairs to plants. In the 1934–35 study estimated deductions were made for fire insurance and workmen's compensation premiums, and also for estimated depreciation rates on the various types of capital goods used based on the rates estimated in the Production Bulletin, No. 29, *Commonwealth Bureau of Census and Statistics*. For a further discussion of these issues, see Gunn

for this article had been prepared, Keith Campbell and George Brinegar, in their Queensland study for 1937-38, found *k* to have a value of .58 and *j* one of .45.[19]

Since the sum of the exponents tends to be slightly less than unity, there is an added slight suggestion of true diminishing returns. But here again, since the difference is less than the standard errors of estimate, we should be chary about drawing definite conclusions.

If we choose .60 as the most probable "normal" value of *k* and .37 as the corresponding value of *j*, this would mean that the approximate elasticity of the marginal productivity curve in Australia for labor was somewhere around 2.7 and for capital of about 1.7.

The third set of results which we should consider are those for New Zealand, Canada, and South Africa. These are shown in Table III. Using the formula of *k* and 1-*k*, Max Brown found for New Zealand a value of .51 for *k* for the period 1915-1935,[20] and when we reworked the Brown series with the second formula, we found values of .42 for *k* and .49 for *j*. It is interesting that in the single cross-section study which has been made for New Zealand, namely, that made by Williams for 1938-39, *k* has a value of .46 and *j* of .51.[20a] In the four Canadian studies which we made for so-called "normal" years, the *k*'s range between .43 and .50, with an average of .47, and the *j*'s between .48 and .58, with an average of slightly more than .52. There is a considerable degree of steadiness in these results, which seem to indicate an elasticity of the marginal productivity curve for labor in that country as slightly less than 2.0 for the years studied and of capital as slightly more than that figure.

One of the most interesting studies which has been made is that by G. W. G. Browne for South Africa for 1937-38. Taking the seventeen main groups of industry and treating all labor as homogeneous, Browne

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and Douglas, "The Production Function for Australian Manufacturing," *Quart. Jour. Econ.*, Vol. LVI (Nov., 1941), pp. 108–129.

[d] The definitions of terms in the studies for the separate Australian states were substantially similar to those in the four Commonwealth studies, except that (1) working proprietors were included in the definition of labor; (2) the estimated value of land was deducted from the capital figures; (3) deductions were made from the value of product for (a) the estimated cost of local and state and federal taxes, (b) estimated fire insurance and workmen's compensation premiums, and (c) allowances for depreciation upon buildings, plant, and machinery. These were also the methods used in the Commonwealth Study of 1934–35. See Gunn and Douglas, "Further Measurements of Marginal Productivity," *Quart. Jour. Econ.*, Vol. LIV (May 1940), pp. 399–428.

[19] See an unpublished manuscript study by Keith O. Campbell and George K. Brinegar, *The Production Function for Queensland Manufacturing in 1937-38* (1947).

[20] Brown omitted the war years of 1916-17 and 1917-18.

[20a] In the cross-section study just completed for New Zealand, 1926-27, *k* has a value of .48 and *j* of .53.

TABLE III.—THE VALUES OF THE PRODUCTION FUNCTION FOR MANUFACTURING IN NEW ZEALAND, CANADA, AND SOUTH AFRICA

Years		N	$P=bL^kC^j$						$P=bL^kC^{1-k}$	
			k	σ_k	j	σ_j	$k+j$	b	k	σ_k
A. Time studies										
New Zealand[a]	1915–16	18	.42	±.11	.49	±.03	.91	2.03	.51	±.03
(Brown)	1918–35									
New Zealand[b]										
(Williams)	1923–40	18	—	—	—	—	—	—	.54	±.02
B. Cross-section or inter-industry studies										
South Africa[c]										
(Browne)	1937–38	17	.66	±.08	.32	±.08	.98	54.48	—	—
South Africa[d]										
(Browne)	1937–38	85	.65	—	.37	±.08	1.02	55.25	—	—
Canada[e]	1923	167	.48	±.04	.48	±.04	.96	48.53	.52	±.04
Canada[e]	1927	163	.46	±.04	.52	±.04	.98	33.04	.48	±.04
Canada[e]	1935	165	.50	±.04	.52	±.04	1.02	22.23	.48	±.04
Canada[e]	1937	164	.43	±.04	.58	±.04	1.01	15.42	.42	±.04
New Zealand[f]										
(Williams)	1938–39	61	.46	—	.51	—	.97	.73	—	—

[a] Dr. Max Brown, in his study, *The Relation Between Capital and Labour in New Zealand* (an unpublished doctoral dissertation at Cambridge University), defined Product (P) as the total money value of production divided by the price index of locally produced goods. His index of Labor (L) was one of total *man-hours* worked, *i.e.*, numbers employed multiplied by the length of the standard working week plus or minus the hours of overtime worked or short time suffered in the various years. The index of Capital (C) was the value of buildings, plant, and machinery (*i.e.*, fixed capital) with the annual increments of investment deflated by a price index of the cost of capital goods. We have fitted the function $P=bL^kC^j$ to the Brown data as well as the $P=bL^kC^{1-k}$ formula which Brown originally used.

[b] Professor Williams computed his indexes as follows: (1) Product (P) = (a) value added by manufacturing (*i.e.*, gross value minus cost of materials, fuel, and power), divided by (b) the price index of locally produced goods; (2) Capital (C) = initial value (1919–20) of land, buildings, machinery, and plant *minus* depreciation actually written off each year, and *plus* the money value of additions to capital in each year adjusted for changes in the index, number of prices for buildings and construction; (3) Labor (L) = number of persons employed. See J. W. Williams, "Professor Douglas' Production Function," *Econ. Record*, Vol. XXI (1945), pp. 55–63.

[c] Professor Browne defined his units as follows: (1) Product (P) = net value added by manufacturing or the gross value of output minus cost of materials, fuel, light, and power; (2) Labor (L) = average number of employees, including wage earners, salaried staff, managers, accountants, working proprietors, and persons regularly employed in their homes; (3) Capital (C) = the value of land, buildings, machinery, plant, and tools (*i.e.*, *fixed* capital only, with working capital excluded). See G. W. G. Browne, "The Production Function for South African Manufacturing Industry," *South African Jour. Econ.*, Vol. XI (1943), p. 259.

[d] This study differed from the former in that it was based on a more minute classification of industries and that it also separated white and black laborers and treated each as a separate factor of production. The value of k given is the sum of the k for white labor (.45) and for native labor (.20). See Browne, *op. cit.*, pp. 260–61. The value of σ_k could not be obtained by adding the σ_k of the exponents for native and white labor.

[e] The statistical series used were: (1) Labor (L) = average number of employed wage earn-

found the value of k was .66, and that for j, .32. When he made white and black labor separate factors of production and broke manufacturing down into eighty-five industries, the sum of the two exponents for labor amounted to .65. While we cannot rely too much upon only one study, it is of interest that his results were substantially the same as those which we obtained on the average for the United States for the period 1889-1919 and not far from the Australian results. This would be equivalent to an elasticity of approximately 3.0 for the marginal productivity curve for labor and of 1.5 for the marginal productivity curve for capital.

If we try to summarize our results, we do find a relatively close agreement between the values of k and j which we obtain from the cross-section studies for the United States, Australia, and South Africa. But we also find differences in the values of k and j (1) between Canada and New Zealand, on the one hand, and the United States, upon the other, with the former having lower k's and higher j's than the United States, and (2) between years within the same country. This is to be expected, as I pointed out long ago in a section of my *Theory of Wages*.[21] But underneath all these differences, it is submitted that there has been *for the periods studied*, a substantial core of stability within countries and that differences in technique, differences in the relative importance of given industries, and differences in the ratios of capital to labor may account for such deviations in the values of the exponents as exist.[22]

[21] Douglas, *The Theory of wages*, pp. 203-4.

[22] The economic and statistical meaning of b deserves to be considered.

A. In the four time series for the United States, the values of b under formula (1) are closely approximate to unity. For the United States the values are the same for each of the four series.

	b		b
Series I	1.01	Series III	1.01
Series II	1.01	Series IV	1.01

(Footnote 22 continued on next page)

ers, salaried workers, etc.; (2) Capital (C) = total capital used, *i.e.*, (a) fixed capital in the form of land, buildings, plant, tools, and machinery plus (b) working capital including materials, goods in process, and goods in storage. The inclusion of working capital makes the results comparable with the cross-section studies for the United States but differentiates them from the series used in Australia, New Zealand, and South Africa. (3) Product (P) = gross sales value *minus* cost of materials, fuel, electricity, etc., used. See Patricia Daly (Ogburn) and Paul H. Douglas, "The Production Function for Canadian Manufactures," *Jour. Am. Stat. Assoc.*, Vol. XXXVIII (1943), pp. 178–86.

[f] Professor Williams defined his terms as follows: (1) Labor (L) = number of persons engaged; (2) Capital (C) = value of land, buildings, plant, and machinery, *i.e.*, *fixed* capital. (3) Product (P) = gross sales value of product minus cost of materials, fuel, and power. See J. W. Williams, *op. cit.*, p. 59.

It is submitted that the results are, on the whole, corroborative. If they were purely accidental, as some have charged, they would show widely varying results. The fact that on the basis of fairly wide studies there is an appreciable degree of uniformity, and that the sum of the

For Victoria, the value of b was .97 and for New South Wales, 1.02. In all these cases, b represents the value of the intercept with the functional plane of theoretical product merely moved up or down by the small difference between the values of b and unity. In the time series, of course, we are dealing with index numbers which show relative changes, not absolute values.

B. Under formula (2) (*i.e.*, $P = bL^kC^j$) b deviates in a greater degree from unity because of the greater degree of freedom given to the exponent for capital. Here the values for the United States are:

	b		b
Series I	.84	Series III	1.12
Series II	1.38	Series IV	1.35

For Victoria b is .71 and for New South Wales, .97.

C. In the cross-section or inter-industry studies, b still represents the intercept but it is also a conversion factor which translates the number of employees and dollars of invested capital into *dollars* of net value product. As in the case of the time series based on index numbers, b is generally higher under formula (2) when the values of j are independently determined than under formula (1). It also tends to move in some direct ratio with changes in the general price level, being generally higher in those years when the price level is higher and *vice versa*. There are, however, occasional exceptions to this rule. These tendencies are shown in the following tables for the United States:

	Values of b	
Year	Formula (1)	Formula (2)
1889	28.58	58.34
1899	69.66	106.43
1904	79.62	120.23
1909	98.63	90.99
1914	66.22	81.66
1919	258.82	244.21

For the Commonwealth of Australia the corresponding values are

	Values of b	
Year	Formula (1)	Formula (2)
1912	14.79	15.87
1922-23	19.72	16.49
1926-27	41.50	77.26
1934-35	37.15	39.79
1936-37	17.99	21.57

It will, of course, be remembered that Australian prices and values are expressed in terms of pounds.

For Canada, the values are

	Values of b	
Year	Formula (1)	Formula (2)
1923	38.55	48.53
1927	28.51	33.04
1935	24.38	22.23
1937	16.48	15.42

I hope to give a fuller treatment of the significance of the b term in the regression equations in a book on *The Theory of Production* which I hope shortly to publish with Miss Grace Gunn.

exponents approximates unity, fairly clearly suggests that there are laws of production which can be approximated by inductive studies and that we are at least approaching them.

And yet it is proper to chronicle the fact that we have obtained some negative results. One persistent area of difficulty in these last months has been the Massachusetts time series. We tried to improve on Professor Cobb's series of capital and product with the result that the more we refined the basic series, the more nonsensical the results became. We are still working on this problem, but at the moment we certainly do not see the light. Secondly, it is disconcerting to observe that if we shorten our time periods by dropping off a number of terminal years, we appreciably alter our results. We observed this fact earlier, as did Professor Williams in New Zealand, but this paradox has been most manifest when we omit the war years from 1916 on, in our United States time series. Finally, we have attempted various inter-spatial studies in which we use individual states as separate observations. We have personally had no success with these attempts. The most ambitious study of this latter nature has, however, been made by my friend and former associate, Ernest Olson, and will be presented to this Association later in these meetings. I do not wish to anticipate the results of his paper, but I think it is proper to say that Mr. Olson has been able to develop a formula which makes differences between countries in their *real* national income a mathematical function of (1) the total energy used, (2) the numbers of the working population, (3) the quantity of livestock reduced to comparable units, and (4) the amount of land—and he has derived exponents which indicate the comparative importance of each. There is still much to be done in this direction and some hard puzzles remain to be solved, but Mr. Olson's comparative success offers us some hope that we may not face a completely blank wall in working with this third method for deriving the laws of production.

Finally, I should like to point out that in the case of the United States, we were compelled because of lack of capital figures, to stop with 1922 in our time series and with 1919 in our cross-section studies. We have, therefore, not been able to cover the very perplexing period of 1920-40. I am doubtful for two reasons whether we can develop a satisfactory production function for the United States during this period: (a) In spite of the excellent work of the National Bureau of Economic Research, we still lack adequate data for this period on the capital *available* for use; and (b) there was wide variation between the decades in the degree to which the available capital was *actually used.* During the 'twenties, capital was quite fully employed, but during the 'thirties a large proportion of this equipment lay idle. Variations in the degree to which available capital was utilized created some difficulties within the ordinary business cycles which prevailed between 1899 and 1919 when each of the four phases of the cycle did not last for more than one or two years. But the period between the two wars was quite extraordinary in that we had high prosperity from 1922 to the fall of 1929 and that we did not fully recover from the collapse which then set in until 1941. It was this fundamental difficulty which prevented one of my students, Mr. Leonard Felsenthal, from developing a satisfactory production function for Germany in the inter-war period. I shall, therefore, await with sympathetic interest the paper on this subject which Mr. Burton Wall is to give tomorrow.

IV. *The Production Function as Based on Plant Averages Rather than Industry Aggregates*

The inductive values which have thus far been developed in the cross-section or inter-industry studies have been based on industry aggregates, namely the totals of workers, capital, and net values added by manufacturing in each industry. This method is somewhat disconcerting to those who are accustomed in their *a priori* reasoning to start with the theory of production for the individual firm and who then move to a model for a given industry but who shy away from developing a theory of production for the economy as a whole or from the manufacturing sector of that economy. Such theorists probably believe that we are starting at the wrong end and that we should begin instead with the individual firm rather than the whole manufacturing sector of the economy and that we should consider the production function within these units rather than deal with inter-industry and aggregate functions.

There are two answers to this position. The first is that I should be very glad indeed to make studies of individual firms if the necessary data were available. But statistics on the changing quantities of labor and capital which are used over a period of time by individual firms, and the amounts of product which are thus turned out by them, are some of the most carefully guarded secrets of business. I am reluctant to believe that we should stop all our investigations until all of these facts are forthcoming for a multitude of firms.

Secondly, I personally see no reason why we cannot approach this problem from either end and study the macrocosm as well as the microcosm. No one, for example, in the physical sciences would propose that we give up using the telescope because the microscope had not yielded all its secrets. Why should we not, therefore, study the economy as a whole as well as speculate about the individual firm, particularly since a knowledge of the former throws a great deal of light upon the problems of the latter?

In the meantime, however, we should all welcome such brilliant studies of the production function for individual firms as that which will shortly be published by my friend and colleague, Professor William H. Nicholls, for a meat-packing plant.[22a] Moreover, if we could get the figures for specific firms and plants *within* given industries for a specific year, we would then be able to develop production functions for each of the main industries with each firm serving as an observation. But the census has always been obligated to conceal the identity of the specific firms which report to it and can only publish totals by industries and geographical subdivisions. This fact prevents us, at present, from developing such studies, although it is barely possible that either the Census Bureau itself or employers' associations could carry them on, were they once convinced of their value. This cannot, however, be done at present.

To my mind, therefore, we are at present forced to work primarily with industry aggregates. But there is one important refinement which we can and should introduce. That is to divide the total number of workers, the aggregate amounts of capital, and the total net value of the product in each of the various industries by the number of plants in that industry. This will give us *plant averages* for given industries rather than industry aggregates as the individual observations[23] and from these we can derive another variant of the production function.

We have made such studies for each of the six years which were covered for American manufacturing and for two of the Australian studies and these results are embodied in Table IV.

It will thus be seen that while we obtained closely similar results in Australia by the two methods, nevertheless, clear differences developed in the case of the United States. In every year the value of k in our American studies was substantially less under the method of plant averages than it was under the method of industry aggregates. The amount of this difference ranged between 5 and 6 points, as in 1904 and 1909, to 15 points in 1889. On the other hand, the values of j were always higher under the methods of plant averages than under that of industry aggregates but the amounts of these differences were much less. As a result, the combined values of $k + j$ are less by from 3 to 8 points

[22a] William H. Nicholls, *Labor Productivity Functions in Meat Packing,* to be published by University of Chicago Press, 1948.

[23] This is not quite the same as the so-called "representative" firm because many firms operate multiple plants.

on the plant average basis than they are when industry aggregates are used and, indeed, average only .92. This gives an unmistakable indication of true diminishing returns so far as the size of individual

TABLE IV.—A COMPARISON OF THE VALUES OF k AND j OBTAINED BY THE METHOD OF PLANT AVERAGES WITH THOSE OBTAINED BY THE METHOD OF INDUSTRY AGGREGATES ($P=bL^kC^j$)[a]

Year	Values According to Method of Plant Averages			Difference (In Points) from Those Obtained by Methods of Industry Aggregates		
	k	j	$k+j$	k	j	$k+j$
United States						
1889	.36	.50	.86	−.15	+.07	−.08
1899	.52	.36	.88	−.10	+.03	−.07
1904	.60	.32	.92	−.05	+.01	−.04
1909	.57	.37	.94	−.06	+.03	−.03
1914	.52	.41	.93	−.09	+.04	−.05
1919	.66	.32	.98	−.10	+.07	−.03
Australia Commonwealth						
1934–35[b]	.60	.38	.98	+.04	−.04	.00
Victoria						
1910–11	.76	.26	1.02	+.02	+.01	+.03

[a] The formula, $P=bL^kC^{1-k}$, gives identical results using aggregate and per plant data.

(1) $$P = bL^kC^{1-k}$$

(2) $$\frac{P}{N} = b\left(\frac{L}{N}\right)^k \left(\frac{C}{N}\right)^{1-k} = b\,\frac{L^k}{N^k}\,\frac{C^{1-k}}{N^{1-k}}$$

$$\frac{P}{N} = \frac{b}{N}\,L^kC^{1-k}.$$

Multiplying both sides by N, we get formula (1).

[b] Figures refer to Commonwealth of Australia 1934-35 A, which used the studies of C. H. Wickens, "The Commonwealth Statistical Allocation of Factory Output," *Econ. Record* (1929), pp. 226-33. The values of k and j for the industry aggregates were .56 and .42, respectively. The results of the plant averages for the Commonwealth for the years 1912, 1922-23, 1926-27, 1936-37 were previously published. It was found that the values of k and j did not change greatly. Only one value of k, that for 1922-23, differed from the aggregate k, by an amount greater than σ_k. For further discussion, see Gunn and Douglas, "The Production Function for Australian Manufacturing," *Quart. Jour. Econ.*, Vol. LVI (Nov., 1941), pp. 108-29.

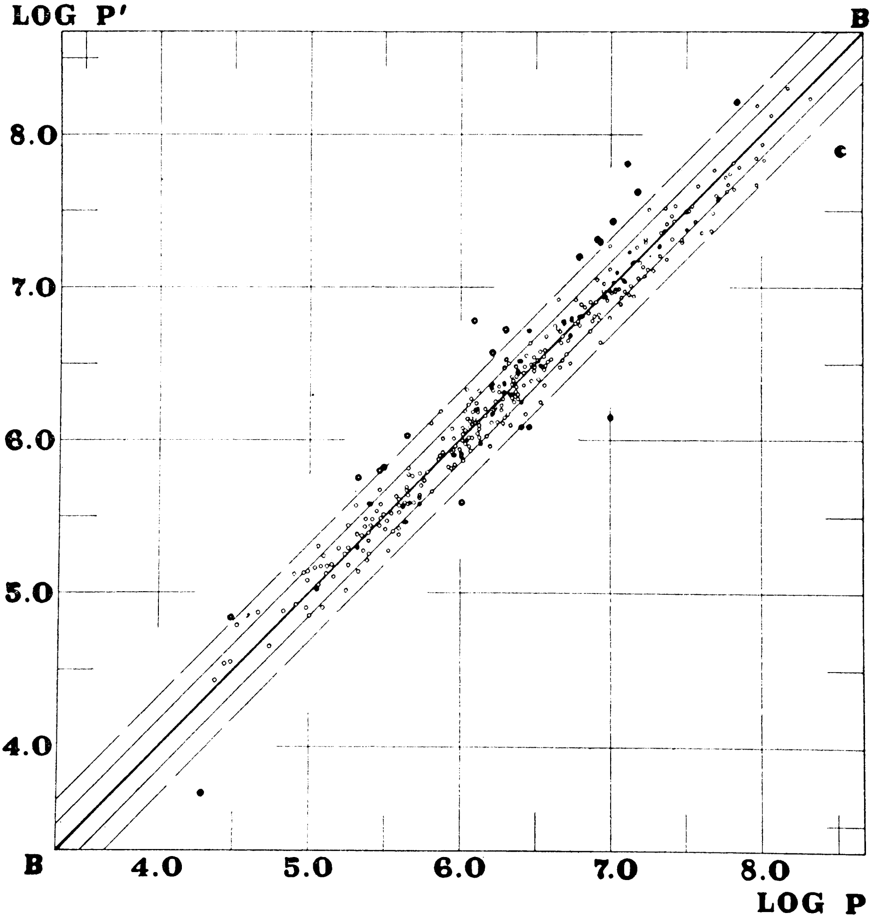
plants is concerned. While much more study is needed to develop and to clarify this point, it is suggested that quite possibly American plants during this period were in practice developed beyond the point of maximum efficiency. Whether or not the differences between the respective $k + j$'s can be taken as a coefficient of managerial megalomania, I shall have to leave to the psychiatrists.[24]

[24] If this is a psychiatric problem, we can take consolation in the fact that the disease was apparently less acute in 1919 than in 1889.

V. *Do the Deviations of the Actual Products from Those Which We Would Theoretically Expect from the Formula Tend to Strengthen or Weaken Belief in the Validity of the Production Function?*

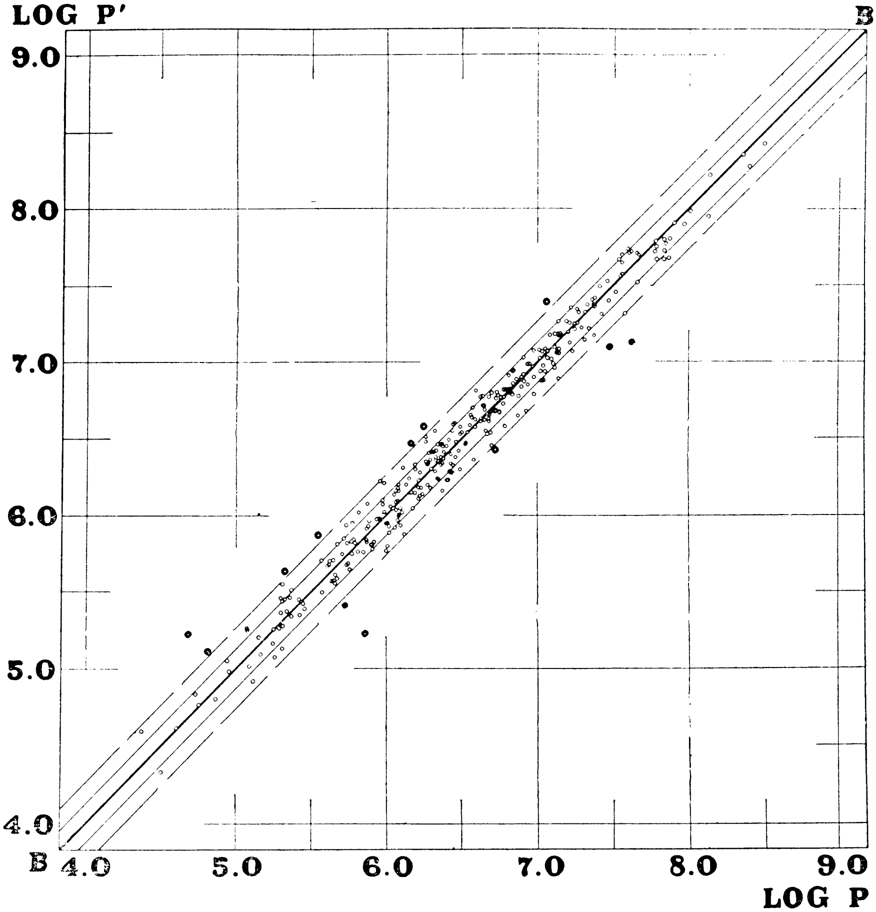
An important test of our function is the degree to which the values of the product which we would expect from the quantities of labor

CHART I—DEVIATIONS OF ACTUAL FROM THEORETICAL VALUES OF LOG P
UNITED STATES MANUFACTURING 1889



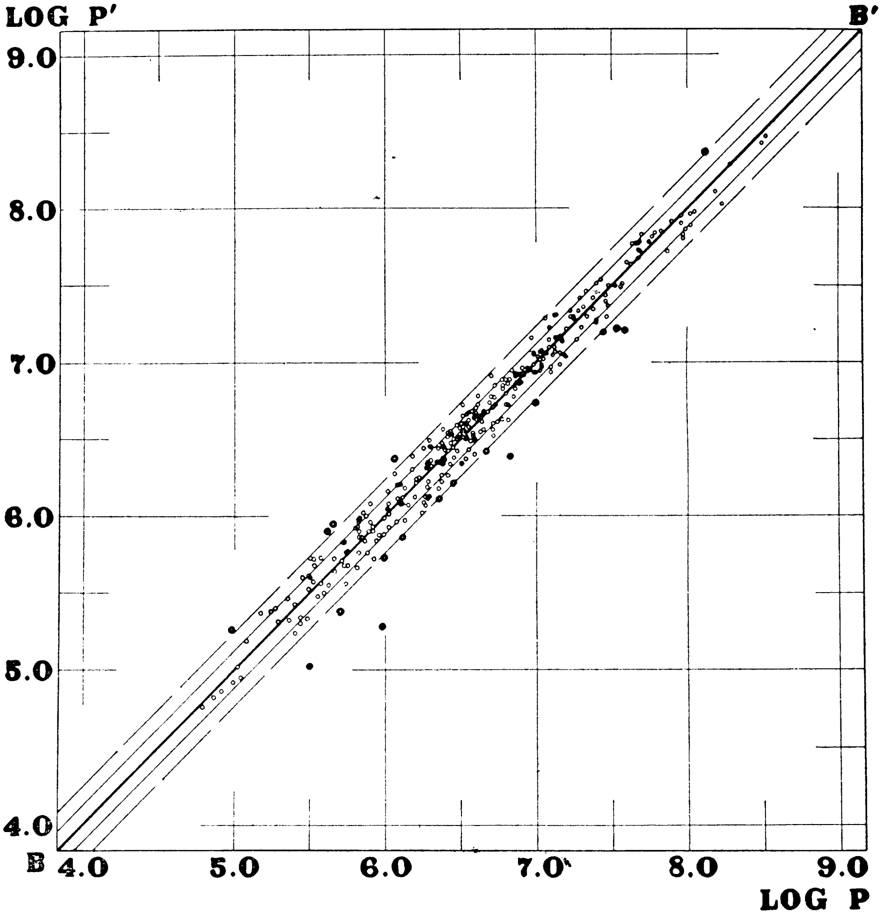
and capital available, tend in practice to be realized in terms of actual product in each of the various industries during the given years. We have made these tests and I should like to present our results in a series of charts and summary tables. As a first step, we computed the standard errors of estimate (S) for each study. Under a normal distribution of cases with the only departures of the actual from theoretical values being those caused by random errors of measurement and of sampling, we would expect that in 68.3 per cent of the cases

CHART II—DEVIATIONS OF ACTUAL FROM THEORETICAL VALUES OF LOG P
UNITED STATES MANUFACTURING 1899



the actual values would deviate from the theoretical values by less than one standard error of estimate, and that in 95 per cent of the cases the actual values would deviate from the theoretical values by less than two such standard errors. In only one per cent of the cases would the actual values deviate by more than three standard errors of estimate. Then in our charts of the cross-section or inter-industry studies, we have plotted the logs of the theoretical or expected products on the vertical scale and of the actual products on the horizontal scale. Since the values of scale are the same on both axes, the line BB′ (with a slope of unity) is the locus of all those points for which the theoretical

CHART III—DEVIATIONS OF ACTUAL FROM THEORETICAL VALUES OF LOG P
UNITED STATES MANUFACTURING 1904



values of the product are identical with the actual values. In these studies for a given year, it will be remembered that each industry constitutes a separate observation.

The degree of departure of the actual from the theoretical values is, therefore, shown by either the horizontal or the vertical distance of a given point from the line BB′. We have, therefore, marked out on each side of the line BB′ two other pairs of lines at the respective distances of one and two standard errors of estimate. An inspection of these charts for American manufacturing for the years 1889, 1899, 1904, 1909, 1914, and 1919, (Charts I, II, III, IV, V, and VI) show that in practice the actual values tend to be close to the line BB′, and a statistical analysis of these variations is given in Table V. Here it will be seen that in every year more than 70 per cent of the actual values were within one standard error of estimate of the values which we would

TABLE V.—DEGREE OF DEVIATION OF ACTUAL FROM THEORETICAL VALUES OF PRODUCT IN AMERICAN MANUFACTURING INDUSTRIES 1889–1919

Census Year	Number of Industries or Observations—N	Deviation of Actual Product (P) from Theoretical Product (P') in Terms of Standard Errors of Estimate					
		Number			Per Cent		
		Less than 1σ	1–2 σ	Over 2σ	Less than 1σ	1–2 σ	Over 2σ
1889	363	280	63	20	77.0	17.0	6.0
1899	332	250	70	12	75.0	21.0	4.0
1904	336	236	82	18	70.0	25.0	5.0
1909	258	215	38	5	83.0	15.0	2.0
1914	340	243	83	14	72.0	24.0	4.0
1919	556	453	85	18	82.0	15.0	3.0
Total	2185	1677	421	87	—	—	—
Averages	—	—	—	—	76.5	19.5	4.0

theoretically expect under the formula and that in two of the six years, over 80 per cent of the cases were within this range.

Taking the 2185 industry observations in the United States as a whole, we find that in 76.5 per cent of the cases, the actual products were within one standard error of estimate of the theoretical products, whereas under a normal distribution we would only expect a little over 68 per cent of the cases to lie within this range. Moreover, in only one year did the number of observations whose actual products varied from the theoretical values by more than two standard errors of estimate come to as much as 6 per cent of the total while the average for all 2185 observations was 4 per cent as compared with the 5 per cent which we would normally expect.[25]

In our American studies, the distribution of the actual values about

[25] Incidentally, instead of 22 cases or 1.0 per cent of the total, which would normally expect to deviate by more than three standard errors of estimate, we find only 16 observations or three-quarters of one per cent in this class.

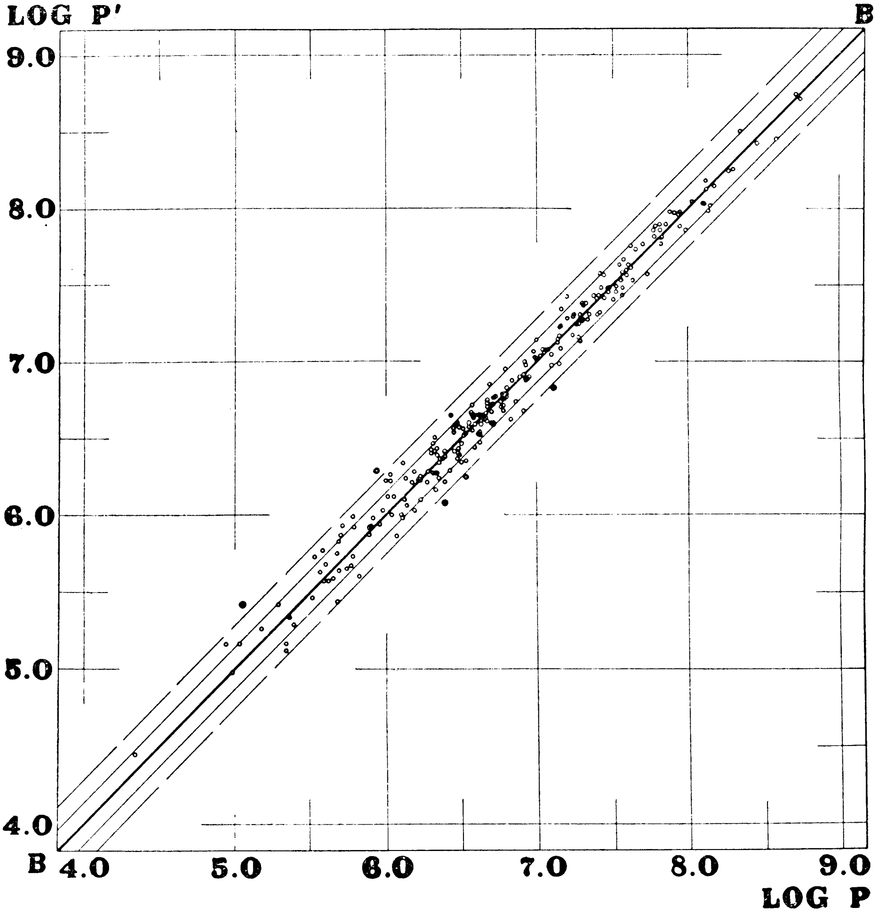
the theoretical values is, therefore, somewhat *closer* than what we would normally expect on the basis of random errors of sampling and of measurement. Belief in the reliability of the formula as a description of production during this period is, therefore, strengthened, rather than weakened.

TABLE VI.—DEGREE OF DEVIATION OF ACTUAL FROM THEORETICAL VALUES OF PRODUCT IN MANUFACTURING INDUSTRIES OF BRITISH DOMINIONS FOR SPECIFIC YEARS

Country and Year	Number of Industries (N)	Deviation of Actual Product (P) from Theoretical Product (P') in Terms of Standard Errors of Estimate (σ)					
		Number			Per Cent		
		Less than 1σ	1–2 σ	More than 2σ	Less than 1σ	1–2 σ	More than 2σ
Canada							
1923	167	116	41	10	69.0	25.0	6.0
1927	163	115	40	8	71.0	24.0	5.0
1935	165	113	45	7	69.0	27.0	4.0
1937	164	122	33	9	74.0	20.0	6.0
Commonwealth of Australia							
1912	85	66	13	6	78.0	15.0	7.0
1922–23	87	66	15	6	76.0	17.0	7.0
1926–27	85	65	17	3	76.0	20.0	4.0
1934–35	138	110	23	5	80.0	17.0	3.0
1936–37	87	70	9	8	81.0	10.0	9.0
Australian States							
New South Wales							
1933–34	125	98	22	5	78.0	18.0	4.0
Victoria							
1910–11	34	26	7	1	76.0	21.0	3.0
Victoria							
1923–24	38	32	4	2	84.0	11.0	5.0
Victoria							
1927–28	35	26	6	3	74.0	17.0	9.0
Total	1373	1025	275	73	—	—	—
Average	—	—	—	—	74.7	20.0	5.3

Let us see from Table VI if these results are confirmed by an analysis of the deviations of the actual from the theoretical values in the thirteen cross-section studies which we have thus far made for the Dominions within the British Commonwealth of Nations. It will be noticed that out of the total of 1373 observations, 1025, or over 74 per cent deviated by less than one standard error of estimate from the theoretical values, and that between 94 and 95 per cent deviated by less than two standard errors of estimate. The distribution of the observations in this sample is, therefore, somewhat better than that which we would expect

CHART IV—DEVIATIONS OF ACTUAL FROM THEORETICAL VALUES OF LOG P
UNITED STATES MANUFACTURING 1909



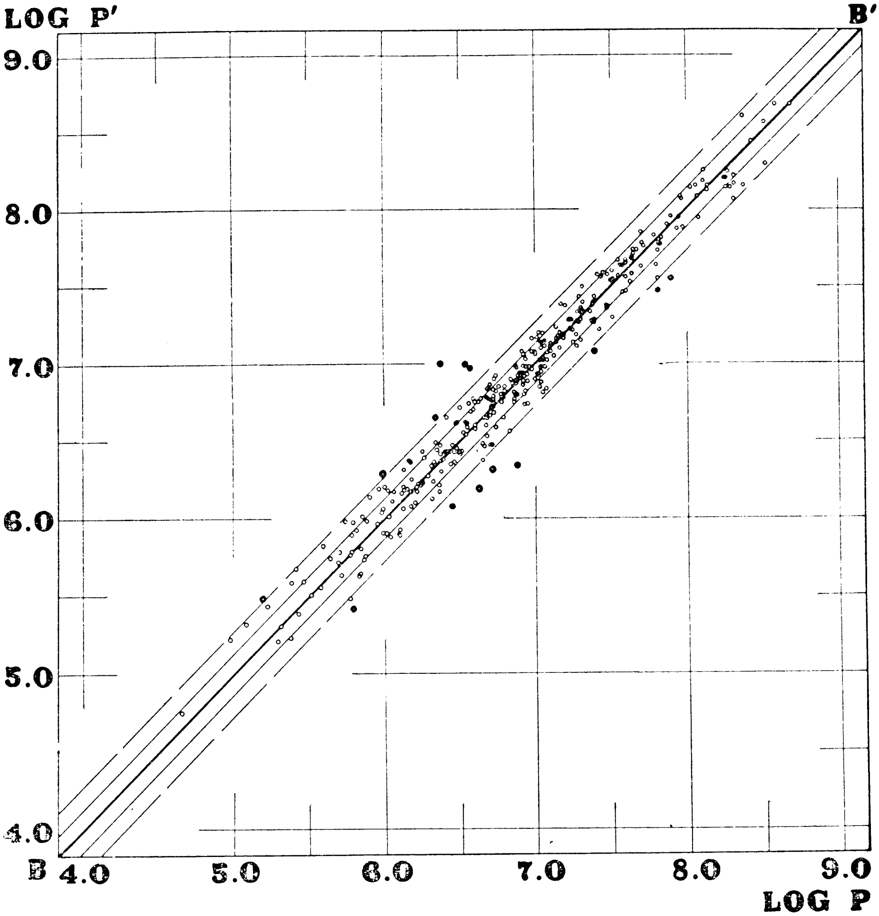
under normal conditions of random error. Credence in the production function would seem to be further reinforced.[25a]

The fact that we have, therefore, in practice, a somewhat closer distribution of the actual values about the line of the theoretical values

[25a] While charts showing the distribution of the individual products about the line of theoretical relationship have been prepared for the British Dominion, these are not published in this article because of considerations of space and expense. They were, however, shown in connection with the address.

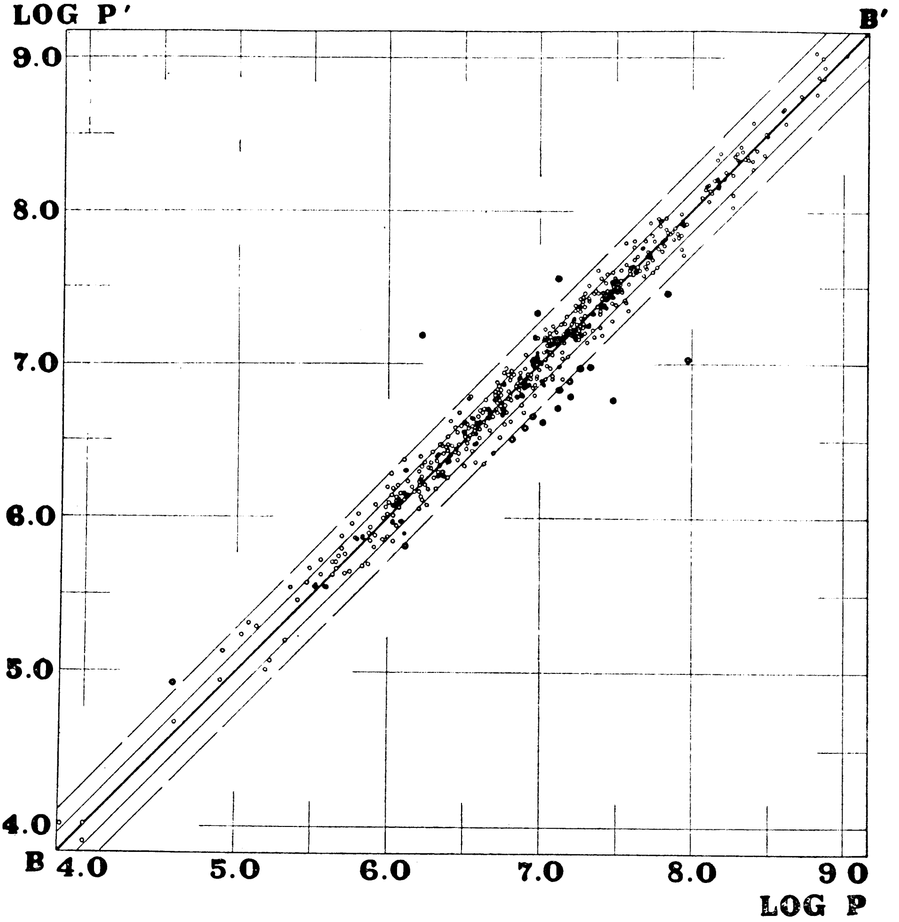
under the formula than we would normally expect, is all the more striking in view of the fact that the values of the production function need not be the same within all industries or allied groups of industries.

CHART V—DEVIATIONS OF ACTUAL FROM THEORETICAL VALUES OF LOG P
UNITED STATES MANUFACTURING 1914



As I have constantly pointed out during the last twenty years, there is no reason why the exponents of capital and labor should be constant for all periods and economies. As a matter of fact, we have already seen that they are not and that there is some variation between countries and years in the values of *k* and *j*. We would similarly expect some variation to exist as between groups of industries within a country at any given time. Thus the values of the production function for the textile industries need not be the same as for the clothing group, while these might well differ from those prevailing in the food industries and be appreciably different from those in the iron and steel and heavy metals industries, etc. If, therefore, we could compute separate values

CHART VI—DEVIATIONS OF ACTUAL FROM THEORETICAL VALUES OF LOG P
UNITED STATES MANUFACTURING 1919



of k and j for each of the various main groups of manufacturing and compare the actual with the resulting theoretical products, we could doubtless find the resulting deviations to be appreciably less than those obtained when we treat all manufacturing as a whole. The fact that we do get such a good fit when we treat all of the industries as homogeneous is, therefore, all the more remarkable. It seems, further, to suggest that to the degree that the values of k and j do differ between groups of industries, such differences tend to be more or less symmetrically distributed around the "normal" values which we have found in the given years for manufacturing as a whole.

A further analysis of the plus and minus deviations offers interesting suggestions. We would expect industries characterized by monopoly and by highly imperfect competition to have a value product which would be appreciably greater than that which we would expect from the production formula itself, and from the quantities of labor and capital which are available. This would be caused by the control over supplies and prices exercised by the dominant firms and by their ability to control or to limit entrance into the monopolized industries. Conversely, in the industries which may be characterized by "excessive" competition into which large numbers of workers and also in some cases, relatively large quantities of capital are forced and which consequently lower the marginal productivity of one or both factors appreciably below their general levels, we would expect the value product per unit of labor and possibly also of capital, to be below the general average for society as a whole.

Similarly, we would expect that the industries which are rapidly expanding because of an increase in demand or in the disposition of consumers income, or because of great technical progress, will have value products which are in excess of those derived from the formula. Conversely, again, we would expect that the contracting industries, which are suffering from a decrease in demand and an obsolescence of technique, would produce less value product than that which our formula would predict.

There are, moreover, a considerable number of industries which probably can best be described as "sweated." These are industries which have had large supplies of cheap labor, sometimes caused, as in the past, by an influx of immigrants or by the presence of a large number of women and juveniles who are forced to seek work because of the low earnings of the male heads of households. The average earnings in these industries tends to be appreciably below the national average, and if there is a normal degree of competition at work, these low earnings will commonly be translated into a lower sales price for the product than would normally be the case. The value product in these industries will, therefore, tend to be less than what would be shown under the formula for manufacturing as a whole.

We can test the relative truth of these hypotheses both by statistical analysis and also by identifying the specific industries where the deviations of the actual from the theoretical values are great. In analyzing the American deviations by years, as in Table VII, certain marked differences appear between the results for the earliest year of 1889 and the years from 1904 on. From 1904 on, and particularly in 1914 and 1919, the big deviations were predominantly on the plus, and the minor deviations on the minus, side. Thus, in 1919, of the 18 industries where the deviations amounted to more than two standard errors of estimate, 16 had their actual products in excess of the theoretical values, while in only two cases did they fall below. On the other hand, of the 538 industries in 1919 where the deviations amounted to less than two standard errors, 308 or nearly three-fifths, were below what would have been expected under the formula. For the three census years of 1909, 1914, and 1919, there were 29 industry observations where the actual products were more than two standard errors of estimate greater than the theoretical values and only 10 industry observations which were more than two standard errors less. Conversely, in these three years there were only 504 industries which had actual products which exceeded the theoretical values up to two standard errors as contrasted with 613 industries where the actual products fell below the theoretical by these amounts.

TABLE VII.—AN ANALYSIS OF THE COMPARATIVE DEGREES OF PLUS AND MINUS DEVIATIONS OF ACTUAL FROM THEORETICAL PRODUCTS IN AMERICAN MANUFACTURING INDUSTRIES 1889–1919

Year	Number of Industries with Deviations of Less Than Two Standard Errors of Estimate		Number of Industries with Deviations of More Than Two Standard Errors of Estimate	
	Plus	Minus	Plus	Minus
1889	190	153	4	16
1899	166	154	5	7
1904	144	174	11	7
1909	122	131	3	2
1914	152	174	10	4
1919	230	308	16	2

The general framework of these later results is approximately what we would expect on theoretical grounds. The monopolistic and expanding industries tend to absorb large quantities of purchasing power at the expense of the rest of the economy. They would, therefore, be expected to show wider profit margins than the general average and each combined dose of labor and capital would consequently tend to yield a greater dollar value than would normally be the case. The withdrawal of this purchasing power would, moreover, exert a slight depressing influence upon each of the remaining industries so that we would expect the number of industries where the actual product fell below the theoretical to exceed in number those where it was greater. This was exactly what happened from 1904 to 1919. Why the opposite result should have occurred in 1889 and to a much lesser degree in 1899, however, merits further study.

Even more important, however, is an analysis of each of the 87 American cases where the deviations were more than two standard errors of estimate from the theoretical values and I only regret that lack of time prevents a full analysis of these instances. Let us first consider the forty in which the deviations were of a minus nature. In no less than ten cases, these were in the flax, hemp, linen, jute and oakum family of industries, which has always been one of the most "sweated" groups in all industrial countries. Two were in allied "sweated" industries, namely nets and seines (1904) and hammocks (1889), while three more were connected with cotton which has generally been a sub-standard industry. Three more, charcoal, waste, and canning oysters, have been distinctly disagreeable and badly "sweated" industries, while several others, such as grindstones, millstones, hooks and eyes, etc., were instances of contracting demand.

On the other hand, the vast majority of the plus deviations which amounted to more than two standard errors of estimate can be explained as caused by (1) some form of quasi-monopoly or imperfect competition, or (2) by expanding demand, or (3) by both factors. Examples of the first are wood engraving, gold and silver reducing, lapidary work, music publishing, glucose, starch, linseed oil, patent medicines, tin plate, brass, and lead. These in themselves accounted for nineteen of the markedly plus deviations.

Illustrations of the second group, namely those caused by an expanding demand, were cordials and flavoring syrups (1909, 1914, 1919), oleomargarine (1914), perfumery (1919), and washing machines (1919).

There is also a third class of plus deviations which was probably affected both by imperfect competition and by expanding demand. Illustrations of this group are airplanes (1914), chewing gum (1919), cigars and cigarettes (1919), fountain pens (1914), photographic supplies and equipment (1904, 1909, 1914), cash registers (1889), smelting and refining copper (1899, 1904), typewriters and supplies (1889).

A very large majority of the major deviations so far as the United States is concerned, were, therefore, precisely what we would expect on *a priori* grounds. Belief in the function as a description of "normal" relationships is, therefore, still further strengthened.

VI. *To What Degree Do the Shares Which Labor and Capital Receive of the Product Approximate the Proportions Which We Would Expect from the Values of the Production Function?*

We now come to one of the most important features of the theory of production and of distribution, namely, the relative degree to which the actual shares received by labor and capital approximate those

TABLE VIII.—A COMPARISON BY YEARS OF THE VALUES OF THE EXPONENTS OF LABOR AND CAPITAL IN THE PRODUCTION FUNCTION FOR AMERICAN MANUFACTURING (k AND j) WITH THE UNWEIGHTED AVERAGE OF THE SHARES OF THE NET VALUE PRODUCT RECEIVED BY LABOR (W/P)

Year	N	k		$\frac{k}{k+j}$	$\frac{W}{P}$	Degree to which W/P differs from k and $\frac{k}{k+j}$ in terms of standard errors	
						$\dfrac{\frac{W}{P}-k}{\sigma_k}$	$\dfrac{\frac{W}{P}-\frac{k}{k+j}}{\sigma_k}$
1889	363	.51	.43	.54	.60	+ 3	+ 2
1899	332	.62	.33	.65	.58	− 2	−3–4
1904	336	.65	.31	.68	.64	−0–1	− 2
1909	258	.63	.34	.65	.63	0	− 1
1914	340	.61	.37	.62	.59	−0–1	− 1
1919	556	.76	.25	.75	.59	−8–9	− 8
Average	—	.63	.34	.65	.605	—	—

which we would expect from the values of the production function. As my associates and I have demonstrated mathematically a number of times, we would expect, under conditions of (1) true constant returns where the sum of the exponents is equal to unity and (2) perfect competition, that each factor of production would receive that fraction of the total product which is indicated by its exponent.[26]

[26] The share which labor receives:

Let $W =$ the amount of wages received.

Marginal productivity of labor $= k\dfrac{P}{L}$

$$W = L\,k\frac{P}{L} = kP$$

$$W = kP$$

$$k = W/P$$

Similarly, for capital.

Let us, therefore, compare the actual share which wages and salaries formed of the net value product (*i.e.*, W/P) in the various years with the values of k. It will also be instructive to compare W/P with the ratio of $\frac{k}{k+j}$ since the latter is a rough measure of what we would approximately expect to occur if the total product were to be divided between labor and capital so as to eliminate either net residual profits or losses.

This is shown for the United States cross-section studies in Table VIII. From an examination of this table, it will be seen that in five of the six years there was a very close agreement between the values of k and of W/P. In one year (1909), there was precise agreement between the two; in two of the years (1904 and 1914), the differences were approximately only one standard error, while in two more (1889 and 1899), they amounted to two to three standard errors. The biggest difference was in 1919 when W/P was less than k by over eight standard errors.[27]

Taking the average for the six years as a whole, we find that k averages .63, $\frac{k}{k+j}$ equals .65, and labor's actual share or W/P was .605. There was, therefore, a close average agreement for the period between what we would have theoretically expected the distribution of the product to be under conditions of perfect competition and that which actually occurred. It should be remembered, moreover, that due to our inability to deduct allowances for depreciation in specific industries, the true values of W/P are probably understated by approximately 3 percentage points,[28] and that, therefore, the average

[27] The year 1919 was one in which prices rose with great rapidity. It would be expected, therefore, that wages would lag behind in such a period.

[28] On the basis of Dr. Fabricant's estimate of depreciation totals for manufacturing as a whole in 1919, it appears that these amounted to approximately five per cent of the value added by manufacturing in that year. If this had been deducted, labor's share would, therefore, have been raised in 1919 by about three percentage points, or to approximately .62. Thus, Fabricant's careful allowance for depreciation in 1919 was 1151 millions of dollars. (Solomon Fabricant, *Capital Consumption and Adjustment*, pp. 260-61.) The total value added by manufacturing in that year, (*i.e.*, value of product minus cost of raw material minus rent and taxes minus cost of contract work) was 22,486 millions of dollars. This comes to a depreciation rate in terms of net value of product of 5.1 per cent. If we deduct such estimated charges, we would raise labor's share by almost precisely 3 points (*i.e.*, $^{59}/_{95} = .62$). Due to the smaller quantity of capital used per unit of product, in the earlier years the additional "loading" required to approximate labor's share would then have been somewhat less, and for 1889 and 1899 would probably have been nearer two percentage points.

ratio of W/P was probably very close to .63 or the exact average value of k.

I submit, therefore, that the degree of agreement between the values of k and of W/P is most striking and that the results conform to what normally would be expected to occur under competitive productivity theory. Hence, this constitutes a still further reinforcement to the productivity function itself.

It should, however, be frankly recognized that there is a further problem of reconciling these results with the known facts of imperfect competition, oligopoly and monopoly. Such conditions, as has been abundantly developed in our meetings, do exist, and, in fact, characterize a large sector of our economy. It is, therefore, puzzling to find labor's share approximately equal to that which we would expect under conditions of perfect competition. A further investigation of this subject is much needed. In the meantime, I would merely suggest that perhaps one answer to the paradox may be that the quasi-monopolies and oligopolies may have shared with their workers the excess gains which they have made at the expense of the consumers.

We can make a further test of the degree to which W/P approximates k and $\frac{k}{k+j}$ by examining the results for the British Dominions of Australia, New Zealand, and Canada. This is done in Table IX.

Taken in the large, the agreement in the cross-section studies for Australia between the values of W/P and k are indeed striking. In each and all of the five inter-industry studies for the Commonwealth, the differences never exceeded one standard error of k. For the five years as a whole, the average value of k and W/P were both .55. The average value of $\frac{k}{k+j}$ was .56. It would scarcely be possible to have a closer agreement than this.

In the case of the four studies for the Australian states, the differences in the case of Victoria were not great, never exceeding two standard errors of k and being slightly reduced if the comparisons are made between W/P and $\frac{k}{k+j}$.[29] For the three years as a whole, the differences are largely ironed out since the average values of k are .65 and of W/P .66. The average value of $\frac{k}{k+j}$ was .70.

[29] In the Queensland study the value of W/P was .614 or less than one standard error more than the value of k.

TABLE IX.—A COMPARISON BY YEARS OF THE VALUES OF THE EXPONENTS OF LABOR AND CAPITAL IN THE PRODUCTION FUNCTION FOR THE BRITISH DOMINIONS (k AND j) WITH THE UNWEIGHTED AVERAGE OF THE SHARES OF THE NET VALUE PRODUCT RECEIVED BY LABOR (W/P)

Dominion and Year	N	k	j	$\frac{k}{k+j}$	$\frac{W}{P}$	Differences between W/P and k in terms of standard error: $\frac{\frac{W}{P}-k}{\sigma_k}$	Differences between W/P and k in terms of standard error: $\frac{\frac{W}{P}-\frac{k}{k+j}}{\sigma_k}$
I. Australian time series							
Victoria							
1907–1929	22	.84	.23	.79	—	—	—
New Zealand (Brown)							
(1915–16)–(1934–35)	18	.42	.49	.46	.52[a]	+0–1	+0–1
New Zealand (Williams)							
1923–1940[b]	18	.54	—	—	.54	—	—
II. Cross-section studies							
Australia							
1912	85	.52	.47	.53	.54	+0–1	+0–1
Australia							
1922–23	87	.53	.49	.52	.54	+0–1	+0–1
Australia							
1926–27	85	.59	.34	.63	.57	−0–1	−1–2
Australia							
1934–35	138	.64	.36	.64	.61	+0–1	−0–1
Australia							
1936–37	87	.49	.49	.50	.51	+0–1	+0–1
Victoria							
1910–11	34	.74	.25	.75	.64	−1–2	−1–2
Victoria							
1923–24	38	.62	.31	.67	.65	+0–1	−0–1
Victoria							
1927–28	35	.59	.27	.69	.68	+1–2	−0–1
New South Wales							
1933–34	125	.65	.34	.66	.51	−3–4	−3–4
Average All Commonwealth and State Studies		.60	.37	.62	.58		
Average Commonwealth Studies Only		.55	.43	.56	.55		
New Zealand							
1938–39	61	.46	.51	.47	.57	—	—
Canada							
1923	167	.48	.48	.50	.50	+0–1	0
Canada							
1927	163	.46	.52	.47	.48	+0–1	+0–1
Canada							
1935	165	.50	.52	.49	.40	−2–3	−2–3
Canada							
1937	164	.43	.58	.43	.52	+2–3	+2–3
Average Canadian Studies	—	.47	.52	.47	.48	—	—

[a] For the years 1924–1935 only.
[b] In the Williams study, the values of k were computed using formula (1). All other values were computed under formula (2).

In the one cross-section study which was carried through for New South Wales, the differences were greater, amounting to between 3 and 4 standard errors of estimate. In the case of Canada, however, the *average* degree of agreement was very close. The average value of k for the four years was .47, and similarly, $\frac{k}{k+j}$ was .47; while the average ratio of W/P was .48. This is an almost precise agreement. This agreement was also true of the years 1923 and 1927 when they are considered individually. The years 1935 and 1937, however, exhibit opposing tendencies. In the former year, k exceeded W/P by an appreciable amount; in the latter year, which was marked by great wage advances in the United States, which were reflected to some degree in Canada, this situation was exactly reversed. The two differences, however, almost precisely offset each other. The case of South Africa does, however, merit special mention. As I have pointed out, Professor Browne found that the combined exponents for black and white labor in 1937-38 were .65, but he also found that both groups of labor received only a total of .46 per cent of the net value added. While Professor Browne does not draw such a conclusion, perhaps this is a case where a highly monopolized set of industries which are largely run by foreign employers or by men whose cultural interests are elsewhere, do not give to the laborers that which in a competitive society they would obtain.

VII. *Summary*

After working on this problem for the better part of twenty years, I think I am aware of the many difficulties which are involved. In a few cases, the method apparently breaks down and in other cases incongruous results are obtained. I should like to suggest, however, that the following tentative conclusions seem justified.

1. That within a given country for the periods studied, there is a substantial and indeed a surprising degree of agreement in the values of k and of j which we obtain for various years.

2. There is also a surprising degree of agreement between the results for the United States, Australia, and South Africa.

3. It is hard to believe that these results can be purely accidental, as some critics have maintained.[30] Time studies for the period between

[30] It would be interesting to work out the mathematical possibility that these results are purely accidental. I believe that it would only be one out of many millions. It is theoretically possible, as Bertrand Russell has pointed out, that all the books in the British Museum were written by monkeys pounding typewriters at random. But we know that they were not!

the two great wars are, however, likely to present difficulties.

4. The deviations of the actual or observed values from those which we would theoretically expect to prevail under the formula are not large and indeed are slightly less than we would expect under the random distribution of errors of sampling and of measurement. It is submitted that the total number of observations, namely over 3,500, is sufficiently large so that if the results had been purely accidental, this degree of agreement would not have occurred.

5. The instances of large deviations of the actual from the theoretical values can in most cases be explained as being caused by imperfect competition and by expanding demand in the case of the plus deviations and by contracting demand, "sweating," and possibly "excessive" competition in the case of the minus industries. This would indicate that if these complications could be eliminated, the agreement between the actual and theoretical products would be greater.

6. That, taken in the large, there is an almost precise degree of agreement between the actual share received by labor and that which, according to the theory of marginal productivity, we would expect labor to obtain.

In conclusion, may I emphasize again that there is much work which remains to be done on this question and that a lifetime would be all too short to probe the many problems which present themselves. Is it too much to hope that the succeeding twenty years may see further progress along this line and that if the older generation finds it impossible to carry on such studies, the younger economists may find such lines of inquiry a challenge to their ingenuity and abilities?[31] I have always been struck by the old Hindu saying, "This is no door but only a little window that opens out upon a great world." Since this is peculiarly applicable to the studies which I have attempted, upon that note I shall end.

[31] Some of the studies which badly need to be carried out are (1) to develop the production function for each and every year over a long period, say 1910-1940, for Australia, New Zealand, and Canada; (2) to carry on studies of the production function for a large number of specific firms and *within* specific industries and hence connect the theory of the firm and of the industry with that of the economy; (3) to carry on further studies, along the lines of Mr. Olson, on inter-spatial variations in real income and the factors affecting them; and (4) to develop production functions for agriculture, mining, and public utilities.

In order to bring our analysis down to date in the larger countries, it is highly desirable that statistics on the quantities of capital be collected for Great Britain, Sweden, and the United States.