Financial Economics
27177 ECON 6240-R01
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Department of Economics
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Exercise set N° 1

Part I: Time value of money, the Net Present Value and the Internal Rate of Return

1. Use excel to answer this question. If you have the following financial transactions:
   (10000, -6000,-5500)
   (10000, -3200,-8960)
   (10000, -6800,-4320)
   a. What is the interest rate for each of the cash flows?
   b. If your IRR is 15%, which one would you select? Why?
   c. If your IRR is 12%, which one would you select? Why?
   d. Generalize this analysis and present a graph that shows the regions in which one financial transaction is preferred to the other. Comment your results.
2. Assuming that the average annual return of the three-month T-bills is 3% and the average annual return for the SP500 is 9%. If you have the following investment transactions:
   (-10000, 11000)
   (-1000, 1200)
   a. Which project would you execute if the future cash flow is a sure thing?
   b. Discuss and present a possible solution: If the future cash flows are simply expected ones and you consider that the risk of these investments can be comparable to an investment in stock markets, would your decision of part a. change? Why?

Part II: Time value of money, the interest rate and the investment opportunity line

The following question needs to be solved **graphically**, explaining the economic and financial intuition of these graphs.

Assume that you have two cash flows, one for time t0 and the other for time t1:
F0=10000
F1=15000
Assume that the interest rate (your opportunity cost of capital) is 10%

1. If you decide to smooth your consumption borrowing money based on a 20% decrease of future consumption. What should be the **total** money available for consumption at t0?

2. If the interest rate is 14% or 6%, how these different rates would impact your smoothing process? Which of the three scenarios would be more advantageous for you?
3. Assume that you have the opportunity to invest in real markets and that you have the following investment opportunities with their respective returns:
   Investment opportunity 1 (I1): 500, 18%
   Investment opportunity 2 (I2): 500, 10%
   Investment opportunity 3 (I3): 500, 6%
   Assume that your initial endowment (at t0) is 2000.
   a. Assuming that the interest rate of the financial markets is 10% and that the return of each of the projects is a sure thing. Which project(s) should you do? Why? What is the technical criterion?
   b. Using your answer of part a., what should be your inter temporal cash flow for consumption?
   c. If you want to use the financial markets to save 30% of the money actually allocated for consumption (after the investment in real assets), what should be the total amount of money that you are going to have in the next period?
   d. Assuming that ignoring the technical rule you decide to do the three projects. Using a graph explain why this is not optimal. Can you confirm this intuition using numbers and the results of previous parts?
4. Explain, using the substitution and income effect, the effect of an increase of the capital market's interest rate. (tip: assume 2 type of agents: net borrowers and net lenders).
   a. According to the substitution effect, what happens with each type of agents' savings?
   b. According to the income effect, what happens with each type of agents' savings?
   c. What is the global (aggregate) result in terms of savings?

Part III: Preferences and utility functions

1. Explain the intuition behind the idea that the indifference curves must not cross if the agent is a rational one.
2. Explain the intuition behind the idea that the marginal rate of substitution should equal the ratio of marginal utilities. Use a graphical representation and mathematical derivation for this question.
3. Assume that an individual's utility function is $U(X, Y) = \sqrt{XY}$. Using the MRSy,x equation and knowing that at time zero X=25, Y=30. Assume that at time one, x increases by 0.1, what is the change on Y to keep the utility level constant? Estimate the time-one value of Y and verify that the utility values at time zero and one are effectively equal. What happens if x changes by 3 units? Repeat the previous exercise and write your findings in terms of utility values (at time zero and time 1).
4. Assume that the utility function of a given agent is given by: U(X,Y) = logX + 3logY. And that this agent faces the following vector of prices: (10, 8) and that his income is 100. What should be the optimal consumption of X and Y for this specific agent? (Assume perfect divisibility of goods)

Part IV: General competitive equilibrium

1. Assume that we have two period of time (t0 and t1) and three states of nature in period t1.
   a. Assume that you know that the contingent claims' prices are $q^1=0.8$; $q^2=0.4$ and $q^3=0.1$. Assume that we have three securities (i=1, 2, 3) with the following state contingent payoffs:

$$Z_1^1 = 4;\ Z_1^2 = 1;\ Z_1^3 = 0$$

$$Z_2^1 = 3;\ Z_2^2 = 2;\ Z_2^3 = 1$$

$$Z_3^1 = 5;\ Z_3^2 = 0;\ Z_3^3 = 2$$

   Estimate the price of each security (P^1, P^2 and P^3).
   b. Now, starting from a. assume that you know the price of each of the security, build the state contingent payoff table and from there estimate the contingent claims' prices (q^1, q^2 and q^3) and verify that these values are exactly the same as the ones you were presented in part a.
2. Assume an exchange economy with two goods and two agents. What would happen with the vector of prices if we have an excess demand of good 1 (horizontal line) and an excess supply of good 2 (vertical line), in order to achieve the equilibrium.
3. Why Hypothesis 3 (convexity of preferences) is important to find the general competitive equilibrium.
4. How the intuition of general equilibrium in a timeless economy should be adapted to consider time and uncertainty? What is a contingent commodity?
5. Present an example of a contingent claim (also known as Arrow-Debreu Security).
6. Assume that we have two states of nature (up and down). Assume that there is a 50%-50% probability of ending in any of the states; assume that the payments for each of these states are 1.1 and 0.9, respectively. If the state contingent prices are 2 and 2.5, respectively,
   a. What should be the value of this security?
   b. How should the state contingent prices change if the probability of the Up-state increase to 80% (as opposed to 50%)?
   c. Explain the reason why even when an agent has equal probabilities of occurrence and equal expected losses and gains, agent's preferences being convex implies that their marginal utilities are different (use figures to illustrate your point).
7. Suppose that each consumer has the Cobb-Douglas utility function $u_i(x_{1i}, x_{2i})=x_{1i}^{\alpha} x_{2i}^{1-\alpha}$. In addition the endowments are $w_1=(1,2)$ and $w_2=(2,1)$. What should be the vector of prices (p_1^*, p_2^*) in order to achieve equilibrium (supply=demand). [Note use an increasing transformation of the utility functions given by $\alpha \ln x_{1i} + (1-\alpha) \ln x_{2i}$].
8. Assume that you have two agents with the following utility functions: $u_1(x,y)=2\ln(x)+\ln(y)$ and $u_2(x,y)=\ln(x)+3\ln(y)$. The endowments are $w_1=(5,4)$ and $w_2=(2,6)$. What should be the vector of prices (p_x^*, p_y^*) in order to achieve

equilibrium (supply=demand). Assume that $p_x^*=2.5$, what is p_y^* and what should be the optimal quantities of x and y for each agent?

Part V: Time value of money, perpetuities and annuities

1. As a winner of a given competition, you can choose one of the following prices:
   a. $100,000 now
   b. $180,000 at the end of 5 years
   c. $11,400 a year forever
   d. $19,000 for each of 10 years
   e. $6500 next year and increasing thereafter by 3% forever.

   If the interest rate is 10%, which is the most valuable prize?

Part VI: Choices in risky situations

1. Given the following investment opportunities, with θ_1 and θ_2 representing two states of nature in Θ and assuming that the states probabilities Π_1 and Π_2 are equal to 0.6 and 0.4 respectively:

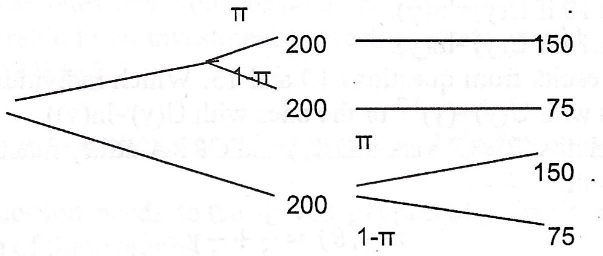
Investment No.	Initial investment	θ_1	θ_2
1	-1000	1250	1750
2	-1000	1300	1500
3	-1000	1300	1800

   a. Does any of this investments state-by-state dominate the others?
   b. Does any of this investments mean-variance dominate the others?
   c. Is there any contradiction between these two dominance measures? Which one is stronger?

2. Represent, using a tree diagram, the following lottery: $((x,y,p_1),(x,(x,y,p_3),p_2),\Pi)$. Can this lottery be written as $(x, y, \Pi p_1 + (1-\Pi)p_2 + (1-\Pi)(1-p_2)p_3)$. What is the probability associated to y?
3. Regarding the timing of uncertainty resolution, given the following tree diagram and assuming:

$$W(P_1, E(U)) = EU^{1/2},$$
$$U_1(P_1, P_2(\theta)) = (P_1 + P_2(\theta))^{1/2}$$

Does this agent prefer early or late resolution of the uncertainty? Prove it mathematically using the above functions.



4. An individual has a utility function $U(x)=\ln(x)$. He faces a risky decision $q=(2000,5000,0.5)$ and a risk-free alternative $s=3500$. Is this a risk averse individual? What is his certain equivalent? Answer graphically and mathematically.
5. Replicate question 4 and define the individual as risk-neutral or risk lover. $U(x)=e^x$. Also estimate individuals certain equivalent.
6. Replicate question 4 and define the individual as risk-neutral or risk lover. $U(x)=5x$. Also estimate individuals certain equivalent.

7. Using the concept of relative risk aversion, show that $U(x)=\ln(x)$ represents a more risk averse investor than $U(x)=(x)^{1/2}$. Is this measure of risk aversion invariant to affine (linear) transformations? Present and example using a linear transformation of $U(x)=\ln(x)$.
8. You have the following lotteries (500, 1200, 0.3). If the investor is risk averse and has a CRRA utility function, with $\gamma = 4$ (coefficient of risk aversion). What is his certain equivalent? Is he a risk averse individual? Why? (answer in terms of risk premium).
9. Using the concept of relative risk aversion, show that $U(x)=\ln(x)$ represents a more risk averse investor than $U(x)=(x)^{1/2}$. Is this measure of risk aversion invariant to affine (linear) transformations? Present and example using a linear transformation of $U(x)=\ln(x)$.
10. Based on the Absolute Risk Aversion (R_A) and the odds result:

$$\pi(y,h) = \frac{1}{2} + \frac{h}{4}(R_A(y))$$

Estimate the following probabilities $\pi(y,h)$, assuming that $y=100$ (wealth):
   a. $U(y)=(y)^{1/2}$, $h=1$.
   b. $U(y)=(y)^{1/2}$, $h=10$.
   c. What is the "h" that makes $\pi(y,h)=1$?
11. Based on the Relative Risk Aversion (R_R) and the odds result:

$$\pi(y,\theta) = \frac{1}{2} + \frac{\theta}{4}(R_R(y))$$

Estimate the following probabilities $\pi(y,\theta)$, assuming that $y=100$ (wealth):
   a. $U(y)=(y)^{1/2}$, $\theta=0.01$.
   b. $U(y)=(y)^{1/2}$, $\theta=0.10$.
   c. What is the "θ" that makes $\pi(y,\theta)=1$?
12. Compare your results from questions 10 and 11. Are they similar?
13. Repeat question 10 if $U(y)=\ln(y)$.
14. Repeat question 11 if $U(y)=\ln(y)$.
15. Compare your results from questions 10 and 13. Which individuals are more risk averse (the ones with $U(y)=(y)^{1/2}$ or the ones with $U(y)=\ln(y)$).
16. Based on the Relative Risk Aversion (R_R) and CRRA utility function (with $\gamma \neq 1$) and the odds result:

$$\pi(y,\theta) = \frac{1}{2} + \frac{\theta}{4}\gamma$$

   a. What is the coefficient of risk aversion (γ) that makes $\pi(y,\theta)=1$, when $y=100$ and $\theta=0.50$?
   b. What is the coefficient of risk aversion (γ) that makes $\pi(y,\theta)=1$, when $y=100$ and $\theta=0.25$?
   c. Which individual is more risk averse based on the coefficient of risk aversion?
   d. What is the coefficient of risk aversion of a risk neutral individual?