Financial Economics midterm.
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Problem 1.

$U_A(x_{1A}, x_{2A}) = 3\ln x_1 + \ln x_2$ $\quad W_A = (3, 8)$

$U_B(x_{1B}, x_{2B}) = \ln x_1 + 2\ln x_2$ $\quad W_B = (7, 2)$

set up the Lagrangian Eqn for A. $\quad \max U_A$ s.t. $P_1 x_1 + P_2 x_2 = W_A$

$$\mathcal{L} = 3\ln x_1 + \ln x_2 + \lambda(W_A - P_1 x_1 - P_2 x_2)$$

$[x_1]$: $\dfrac{3}{x_{1A}} = \lambda P_1 \Rightarrow \lambda = \dfrac{3}{x_{1A}P_1}$ ①

$[x_2]$: $\dfrac{1}{x_{2A}} = \lambda P_2 \Rightarrow \lambda = \dfrac{1}{x_{2A}P_2}$ ②

$[\lambda]$: $W_A = P_1 x_{1A} + P_2 x_{2A}$ ③

By ① & ②

$\therefore \dfrac{3}{x_{1A}P_1} = \dfrac{1}{x_2 P_2}$

$\therefore 3x_{2A}P_2 = x_{1A}P_1$

$\therefore x_{1A} = \dfrac{3x_{2A}P_2}{P_1}$ ④

put ④ to ③

$$W_A = P_1 \frac{3x_{2A}P_2}{P_1} + P_2 x_{2A}$$

$\therefore W_A = 3P_2 x_{2A} + P_2 x_{2A}$

$\therefore \boxed{x_{2A} = \dfrac{W_A}{4P_2}}$ $\Rightarrow$ $x_{1A} = \dfrac{3\frac{W_A}{4\cancel{P_2}}\cancel{P_2}}{P_1}$

$$= \frac{3W_A}{4P_1}$$

Now, do the same thing for B.

max U_B

s.t $P_1 X_{1B} + P_2 X_{2B} = W_B$

And set up the Lagrangian eqn:

$$\mathcal{L} = \ln(X_{1B}) + 2\ln(X_{2B}) + \lambda(W_B - P_1 X_{1B} - P_2 X_{2B})$$

$[X_{1B}]$: $\frac{1}{X_{1B}} = \lambda P_1$ ⑤

$[X_{2B}]$: $\frac{2}{X_{2B}} = \lambda P_2$ ⑥

$[\lambda]$: $P_1 X_{1B} + P_2 X_{2B} = W_B$ ⑦

From ⑤ & ⑥:

$\therefore \frac{1}{X_{1B} P_1} = \frac{2}{X_{2B} P_2}$

$\therefore P_2 X_{2B} = 2 P_1 X_{1B}$

$\therefore X_{2B} = \frac{2 P_1 X_{1B}}{P_2}$ ⑧

Put ⑧ into ⑦

$$P_1 X_{1B} + P_2 \frac{2P_1 X_{1B}}{P_2} = W_B$$

$$\therefore \quad P_1 X_{1B} + 2P_1 X_{1B} = W_B$$

$$\therefore \quad X_{1B} = \frac{W_B}{3P_1}$$

$$\therefore \quad X_{2B} = \frac{2P_1 \cdot \frac{W_B}{3P_1}}{P_2} = \frac{\frac{2}{3} W_B}{P_2}$$

$$= \frac{2W_B}{3P_2}$$

$$\therefore CC_A: \left[\frac{3W_A}{4P_1}, \frac{W_A}{4P_2}\right] = \left[\frac{3(3P_1+8P_2)}{4P_1}, \frac{3P_1+8P_2}{4P_2}\right]$$

$$CC_B = \left[\frac{W_B}{3P_1}, \frac{2W_B}{3P_2}\right] = \left[\frac{7P_1+2P_2}{3P_1}, \frac{2(7P_1+2P_2)}{3P_2}\right]$$

Sine for good 1, the total endowment is 10

2 10

$$\therefore \begin{cases} \dfrac{3(3P_1+8P_2)}{4P_1} + \dfrac{7P_1+2P_2}{3P_1} = 10 & (9) \\[2ex] \dfrac{3P_1+8P_2}{4P_2} + \dfrac{2(7P_1+2P_2)}{3P_2} = 10 & (10) \end{cases}$$

For (9)

$\Rightarrow$

$$\frac{9(3P_1+8P_2)}{12P_1} + \frac{4(7P_1+2P_2)}{12P_1} = 10$$

$$\Rightarrow 27P_1 + 72P_2 + 28P_1 + 8P_2 = 120P_1$$

$$\Rightarrow 55P_1 + 80P_2 = 120P_1$$

$$80P_2 = 65P_1$$

$$\therefore P_2 = \frac{65}{80}P_1$$

$$= \frac{13}{16}P_1$$

From ⑩

$$\frac{3(3P_1+8P_2)}{12P_2} + \frac{8(7P_1+2P_2)}{12P_2} = 10$$

$$9P_1 + 24P_2 + 56P_1 + 16P_2 = 120P_2$$

$$65P_1 + 40P_2 = 120P_2$$

$$80P_2 = 65P_1$$

$$\therefore P_2 = \frac{\cancel{65}\ 13}{\cancel{80}\ 16}P_1$$

Yes, it is general competitive equilibrium.

①: it satisfies nonsatiation,

② Mkt clearance, $X_{1A} + X_{1B} = X_1$ total endowment

$X_{2A} + X_{2B} = X_2$

③ there Budget Balance.
As they consume within their endorsement.

Yes. From the first fundamental theorem of welfare any CE allocation is Pareto optimum.

if $P_2 = 5$

$P_1 = \frac{16}{13} P_2 = \frac{16}{13} 5 = \frac{80}{13}$

$\therefore p^* [\frac{80}{13}, 5]$

$CC_A = [7.125, 2.923]$

$CC_B = [2.875, 7.076923]$

problem 2.

$$r_r = -\frac{u''(x)\cdot x}{u'(x)}$$

$u'_{I_1} = \frac{1}{x}$ $\quad u''_{I_1} = -\frac{1}{x^2}$

$u'_{I_2} = -3.2x + 8$ $\quad u''_{I_2} = -3.2$

$$r_{r_{I_1}} = \frac{-u''_{I_1}(x)\,x}{u'_{I_1}(x)} = -\frac{-\frac{1}{x^2}\,x}{\frac{1}{x}} = -\frac{-\frac{1}{x}}{\frac{1}{x}} = 1$$

$$r_{r_{I_2}} = \frac{-u''_{I_2}(x)x}{u'_{I_2}(x)} = \frac{3.2\,x}{-3.2x+8}$$

If

$r_{r_{I_1}} > r_{r_{I_2}} \Rightarrow I_1$ is more risk averse.

$$\Rightarrow 1 > \frac{3.2x}{-3.2x+8} = \frac{1}{-1+\frac{8}{3.2x}}$$

$\therefore -1 + \frac{8}{3.2x} > 1$

$\frac{8}{3.2x} > 2$, $\therefore$ $6.4x < 8$

$x < \frac{8}{6.4} = 1.25$

$\therefore$ when $x < 1.25$ I_1 is more risk averse.

when $x > 1.25$ I_2 more risk averse.

$\therefore$ I_1 is not alway more risk averse.

b.

when $x = 1.25$ from a, they have same risk aversion

c. From a. $x < 1.25$, I_1 is more risk averse

d. When $x > 1.25$, I_2 is more risk averse problem 4

lottery $(500, 1200, 0.3)$

The agent has CRRA utility function w/ $\gamma = 4$

$$u(y) = \frac{y^{1-\gamma}}{1-\gamma} = \frac{y^{-3}}{-3}$$

$$u'(y) = y^{-\gamma}$$

$$u''(y) = -\gamma y^{-\gamma-1}$$

$$\therefore r_A = -\frac{-\gamma y^{-\gamma-1}}{y^{-\gamma}} = \gamma y^{-1} \geq 0$$

as $\gamma = 4 > 0$, $y \geq 0$

$\therefore$ He is risk averse when $y > 0$

He is risk neutral when $y = 0$

$$u(CE) = 0.3\, u(500) + 0.7\, u(1200)$$

$$= \frac{3}{10} \frac{500^{-3}}{-3} + \frac{7}{10} \frac{1200^{-3}}{-3}$$

$$= \underbrace{\frac{500^{-3}}{-10} + \frac{7}{10} \frac{1200^{-3}}{-3}}$$

just a const number, Denote this number as A

$$\therefore \frac{y_{CE}^{-3}}{-3} = A$$

$$y_{CE}^{-3} = -3A$$

$$y_{CE}^3 = \frac{1}{-3A}$$

$$\therefore y_{CE} = \sqrt[3]{-\frac{1}{3A}} \approx 709.06$$

$$Ey = 0.3 \times 500 + 0.7 \times 1200$$

$$= \frac{3}{10} \times 500 + \frac{7}{10} \times 1200$$

$$= 150 + 840 = 990$$

$\because Ey > y_{CE}$ And $RP = Ey - y_{CE}$

$= 990 - 709.06 > 0$

$\therefore$ he is risk averse.

problem 6:

Since $W(P_1, E(u)) = EU^{\frac{1}{2}}$ $\quad u_1(P_1, P_2(0)) = (P_1 + P_2(0))^{\frac{1}{2}}$

$$E(u) = \frac{1}{2}(200+150)^{\frac{1}{2}} + \frac{1}{2}(200+75)^{\frac{1}{2}}$$

$$= \frac{1}{2} \times 18.71 + \frac{1}{2} \times 16.58$$

$$= 9.355 + 8.29$$

$$= 17.645.$$

For early decision maker:

His W would be:

$$\frac{1}{2} W(200, 18.71) + \frac{1}{2} W(200, 16.58)$$

$$= \frac{1}{2}\sqrt{18.71} + \frac{1}{2}\sqrt{16.58}$$

$$= 4.19868.$$

For late decision maker:

His W would be:

$$W(200, E(u)) = W(200, 17.645)$$

$$= \sqrt{17.645}$$

$$= 4.200595$$

$W_{\text{late decise}}$ is higher.

$\therefore$ He prefers late resolution.