Homework #0 Stochastic Calculus

Conditional Probability Distributons and Expectations

Due date: Nov 8, 2022 (in class)

Email HW (PDF) to dedicated address: yuxu@fordham.edu

Important: Homework should be done independently although discussion of relevant general knowledge is allowed. *No late homework will be accepted.*

Problem 0 [Reading Assignment, no need to handin]. Read the section on Conditional Expectations in the textbook.

Problem 1. Let X be a random variable distributed as $N\left(\mu, \sigma^2\right)$ with $\mu = 0.06$, $\sigma = 0.25$.

- (1) Find the density of $Y = \exp\left(X\right)$.

- (2) Compute $\mathbb{E}Y$ using the density of Y found in (1)

- (3) Compute $\mathbb{E}Y$ in another way:

$$\mathbb{E}Y = \mathbb{E}\exp(X) = \int e^x \phi(x) dx$$

where $\phi(x)$ is the density of X.

Comment on Solutions: This is to review an elementary probablity technique of how to compute the expectation of $Y = f(X)$, assuming you are given $f(x)$ and the distribution of X. There are essentially two ways to do it: (1). Compute $\mathbb{E}Y$ using the density of Y; (2). Compute $\mathbb{E}Y$ using the density of X. The second (2) way is usually easier, as demonstrated in the 3rd part of Problem 1.

Problem 2. Let $\Omega = \{a, b, c, d, e\}$. Denote by F the σ-algebra the collection of all subsets of Ω. We define probability P by the following

$$
\begin{aligned}
P\{a\} &= \frac{1}{6} \\
P\{b\} &= \frac{1}{6} \\
P\{c\} &= \frac{1}{4} \\
P\{d\} &= \frac{1}{4} \\
P\{e\} &= \frac{1}{6}
\end{aligned}
$$

Define random variable X and Y by

$$
\begin{aligned}
X(a) &= 2, X(b) = 2, X(c) = 2, X(d) = -1, X(e) = -1 \\
Y(a) &= 1, Y(b) = -2, Y(c) = 1, Y(d) = -2, Y(e) = -2
\end{aligned}
$$

(i). What are the sets in F?

(ii). What are the sets in $\sigma(X)$?

(ii). Determine the conditional expectation $E[Y|X]$.

(iii). Determine the conditional expectation $E[Y^2|X]$.

Comment on Solutions: This is essentially a sliight modification of the Example in class. The purpose is to ask you to review the lecture notes of that part — especially for arguments leading to answers to part (i) and (ii) in the problem. Read the lecture notes and make sure you understand the argument!