Last Name__________, First Name ___________ (Please Print!!!)

Introduction to Stochastic Calculus
Final Fxam

Fall 2022

Note 1: The exam time is from 3:30PM to 5:30PM. You will have 15 minutes after the exam (until 5:45PM) to scan/take picture of your answers to email to the dedicated email address (please use copy-paste to insure no typing errors):

`msqfeconometrics2015@gmail.com`

[**Don't forget to cc yourself and check if the file is attached!!!**]

Note 2: Although this exam is openbook, you are only allowed to reference the lecture notes, HW, and your notes on your laptop. Nothing else is allowed: No use of the Internet or communicating to any other people or source is allowed.

Note 3: Details for your solutions. Solutions with missing arguments in calculations/derivations will NOT receive full credit.

Note 4: You can write your answers to this PDF le as markers/comments. Please provide the Necessary

Problem 1 [Calculating Stochastic Integrals using Ito Formula] [25 points]. Let W_t be the standard Brownian Motion. Then evaluate the following stochastic integral (simplify to the extent possible; Riemann integrals are considered done in the final answer):

$$\int_0^T (W_t)^5 \, dW_t.$$

Problem 2 [Ito Isometry] [25 points]. Let X_t be defined by stochastic integral

$$X_t = \int_0^t \sin(1 + 3W_s) dW_s.$$

(1) Find an expression for X_t using Ito's formula.

(2). Find the mean and variance for X_t (evaluate to the extent possible).

Problem 3 [Stochastic Differential Equations][25 points]. Assume that X_t satisfies the following SDE

$$dX_t = \alpha X_t dt + \beta \left(X_t\right)^2 dW_t, \text{ with } X_0 = 2.$$

Define

$$f(x) = \int_1^x \frac{1}{w^2} dw,$$

and set $Y_t = f(X_t)$. What SDE does Y_t satisfy? What is the initial condition fo Y_t process? **Note: The final SDE for** Y_t can not contain X_t. Hint: evaluate integral definining $f(x)$ first.

Problem 4 [Derivatives Pricing] [25 points]. Assume the assumptions of the Black-Scholes Model, where the stock price follows $dS_t = \mu S_t dt + \sigma S_t dW_t$ with $S_0 = S$, and the risk-free rate is constant r. Find the pricing formula for the European Style derivative whose payoff is given by the function $(K - S_T)^2$, where S_T is the stock price on date T.

Important Note: (a). If you want to use Feymann-Kac, you will need to run the Black-Scholes dynamic hedging/replication procedure to derive the Black-Scholes pricing PDE, invoke the Feyman-Kac, and then evaluate the expectation to get the pricing formula. I am looking for all the key steps of Black-Sholes method in the answer, so do not jump steps! (b). If you use risk-neutral pricing, you also need to clearly state all key steps, especially where change of measure is used, and how!

[More space for Problem 4]