AE HW 1

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Chapter 2. Conditional Expectation

I. Chapter Summary

1. Role of Conditional Expectation

(1) Express Conditional Expectation w/ $\mu(\underset{\sim}{x})$

(i) Let $\underset{\sim}{x} = (x_1, \cdots x_k)$ be the explanatory variables

(ii) If $E[|y|] < \infty$

then $\exists\ \mu: \mathbb{R}^k \to \mathbb{R} \ni E[y | x_1 \cdots x_k] = \mu(x_1 \cdots x_k)$

Remark 1: $\mu(\underset{\sim}{x})$ is a random variable, i.e. $E(y|\underset{\sim}{x})$ is a function of $\underset{\sim}{x}$

Remark 2: $\mu(\underset{\sim}{x})$ is the average value of y given $\underset{\sim}{x}$

(2) Basics of Conditional Expectation (more in handout 1)

- Joint PDF of X, Y: $f_{X,Y}(x,y)$
- Joint CDF of X, Y: $F_{X,Y}(x,y) = \int_{-\infty}^{s}\int_{-\infty}^{x} f_{X,Y}(t,s)\,dt\,ds$
- Marginal PDF of X: $f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,t)\,dt$
- Conditional PDF of X $f_{Y|X=x}(y) = \dfrac{f_{X,Y}(x,y)}{f_X(x)}$
- Conditional CDF of X $F_{Y|X=x}(y) = \int_{-\infty}^{y} f_{Y|X=x}(s)\,ds$

One more key property $\Rightarrow$ $= \dfrac{\partial F_{X,Y}(x,y)/\partial x}{f_X(x)}$

- Conditional Expectation

$$E[Y|X=x] = \int_{-\infty}^{\infty} y\, f_{Y|X=x}(y)\,dy$$

(3) Basic Properties of Conditional Expectation

① Constancy: $E[a|Y] = a$

② Additivity: $E[aX+bZ|Y] = aE[X|Y] + bE[Z|Y]$

③ Independency: $E[X|Y] = E[X]$ iff. X & Y are independent

④ Scalar multiplier: $E[Xg(Y)|Y] = g(Y)E[X|Y]$ esp. $E[g(Y)|Y] = g(Y)$

⑤ Tediousness: $E[X|Y, g(Y)] = E[X|Y]$

eg: $E[wage|educ, exper, exper^2, \log(educ), exper \cdot educ] = E[wage|educ, exper]$

⑥ LIE: $E[E[Y|X]] = E[Y]$

A Times Series Version: $E\left[E[Y_{t+1}|I_t] \middle| I_{t-1}\right] = E[Y_{t+1}|I_{t-1}]$

(4) Stronger formats

- Mean independence: $E[Y|X=x] = E[Y]$

Remark 3: Independence $\Rightarrow$ Mean independence

eg: Let x be race, then if we know X, Y are independent ($E[Y|X]=E[Y]$)

we also know Y is independent at any level of X $E[Y|X=x] = E[Y]$

- Conditional vs unconditional

Remark 4: $E[u|\underset{\sim}{x}] = 0 \Rightarrow E[u] = 0$

proof: ($\Rightarrow$) $E[u] = E[E[u|\underset{\sim}{x}]] = 0$

($\not\Leftarrow$) when $E[u] = 0 \Rightarrow E[u] = E[u|\underset{\sim}{x}] = 0$

iff. x and u are independent

- Independence $\Rightarrow$ uncorrelated

uncorrelated $\not\Rightarrow$ Independence

(5) Advanced Properties

① Decompose y: (i) $\underset{\sim}{y} = E[\underset{\sim}{y}|\underset{\sim}{x}] + \underset{\sim}{u}$ (ii) $E[u|\underset{\sim}{x}] = 0$

Remark 4: $E[u|\underset{\sim}{x}] = 0 \Rightarrow$ (i) $E[u] = 0$

(ii) $cov(u, x_i) = 0$ $i = 1, 2, \cdots k$

(iii) $cov(u, g(x_i)) = 0$

② Functional Invariant LIE - I

$\boxed{CE3}$ (i) (2.19) $E[y|\underset{\sim}{x}] = E[E[y|\underset{\sim}{w}]|\underset{\sim}{x}]$, where $\underset{\sim}{x} = f(\underset{\sim}{w})$

ex: $x_1 = f_1(w_1, w_2)$ $x_2 = f_2(w_2)$

Then $E[y|x_1, x_2] = E\Big[[y|w_1, w_2]\Big| x_1, x_2\Big]$

(ii) (2.20) $E[y|\underset{\sim}{x}] = E[E[y|\underset{\sim}{x}]|\underset{\sim}{w}]$ $\Leftarrow$ Since $\underset{\sim}{x}$ is function of $\underset{\sim}{w}$ knowing $\underset{\sim}{w}$ implies knowing $\underset{\sim}{x}$

Remark 5: From 2.19 we find the position of $\underset{\sim}{x}$ and $\underset{\sim}{w}$ does not make difference to the values of iterated conditional expection.

Remark 6: Let us suppose y is the wage. w is all the related variables such as eduction years, experience, ability, family wealth $\cdots$.
But in real world, we only have the data for education, experience,
Which is our $\underset{\sim}{x} = (\text{educ}, \text{exper})$. i.e. we have limited information.
Suppose, fortunately, we know $edu = f_1(\underset{\sim}{w})$ $exp = f_2(\underset{\sim}{w})$. And it will be easy to get $E[y|\underset{\sim}{x}]$. Then the average effect of $E[y|\underset{\sim}{w}]$ given limited information $(\underset{\sim}{x})$ is the same as $E[y|\underset{\sim}{x}]$ if we estimate $E[y|\underset{\sim}{x}]$ w/. a parametric model, without necessarity of knowing all the information.

* Even if we may not know $E[y|\underset{\sim}{w}]$. but we know $E[E[y|\underset{\sim}{w}]|\underset{\sim}{x}]$

③ Functional Invariant of LIE - II.

$$E[y|\underset{\sim}{x}] = E\Big[E[y|\underset{\sim}{x}, \underset{\sim}{z}]\Big|\underset{\sim}{x}\Big]$$

where $\underset{\sim}{x}$ is observed variables, $\underset{\sim}{z}$ is unobserved ones

eg: $E[y|x_1, x_2, z] = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 z$. which is our target
But z is unobserved here.
By ③ $E[y|x_1, x_2] = E\Big[E[y|x_1, x_2, z]\Big| x_1, x_2\Big] = E[\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 z | x_1, x_2]$

$$= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 E[z|x_1, x_2] \quad [1]$$

In many cases, we can assume $E[z_i|x_1, x_2] = f(x_1, x_2)$. say $E[z|x_1, x_2] = \delta_0 + \delta_1 x_1 + \delta_2 x_2$ [2]

Then $[1]+[2] \Rightarrow$ $\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3(\delta_0 + \delta_1 x_1 + \delta_2 x_2)$

$$= (\beta_0 + \beta_2 \delta_0) + (\beta_1 + \beta_3 \delta_1) x_1 + (\beta_2 + \delta_2) x_2$$

This has nothing to do w/. z. it is purely function of x_1 and x_2.

④ $\boxed{\text{CES}}$ If (u, v) are independent of $\underset{\sim}{x}$, then $E(u \mid \underset{\sim}{x}, v) = E(u \mid v)$

2. Partial Effects, Elasticities, semielasticities.

(1) Partial Effects of x_i on $E[y \mid \underset{\sim}{x}]$: $PE(y \mid x_i) = \dfrac{\partial E[y \mid \underset{\sim}{x}]}{\partial x_i}$

w/. $x_1 \cdots x_{i-1}, x_{i+1} \cdots$ fixed

(2) Elasticity (partial w/. x_i): $\dfrac{E[y|x] \text{ \% change}}{x \text{ \% change}} = \dfrac{\Delta E[y|x] / E[y|x]}{\Delta x_i / x_i}$

$$= \frac{\partial E[y|x]}{\partial x_i} \cdot \frac{x_i}{E[y|x]} \quad ©$$

Since $d \log x = \dfrac{dx}{x}$, thus $© = \dfrac{\partial \log E[y \mid \underset{\sim}{x}]}{\partial \log x_i}$

Remark 7: Elasticity can be constant or variant. In production function framework, this means constant-return-to-scale or increasing (decreasing) -return-to-scale

(3) Semielasticity of $E[y \mid \underset{\sim}{x}]$ w.r.t. x_i: $100 \cdot \dfrac{\partial \log E[y \mid \underset{\sim}{x}]}{\partial x_i}$

3. APE

(1) Average partial effects: the partial effets of x on $E[y \mid \underset{\sim}{x}]$ averaged across the distribution of x's

i.e. $APE(y \mid x_i) = E\left[\dfrac{\partial \log E[y|x]}{\partial \log x_i} \,\middle|\, x \right]$

Remark 8: An important case is to consider $\underset{\sim}{x}$ w/. observal part x, and unobserval part q. the $APE(y \mid x_i)_{\underset{\sim}{x} = \underset{\sim}{x_0}} = E\left[\dfrac{\partial E[y \mid \underset{\sim}{x}, q]}{\partial x_i} \Big|_{\underset{\sim}{x} = \underset{\sim}{x_0}} \,\middle|\, q \right]$ At specific $\underset{\sim}{x}$ level $\underset{\sim}{x_0}$.

- q: unobserval heterogeneity.

(2) Proxy variables

(i) One key assumption to generate unbiased estimates on $\underset{\sim}{x}$ in a parametric model of $E[y|\underset{\sim}{x}]$ is the the observed ones ($\underset{\sim}{x}$) and unobserved ones (q) are independent.

(ii) Eliminate unobserved heterogeneity

[1] Good Proxy

$$D(q|\underset{\sim}{x},\underset{\sim}{w}) = D(q|\underset{\sim}{w})$$

Distribution — good proxies for q.

[2] Redundant proxy

$$E[y|\underset{\sim}{x},q,\underset{\sim}{w}] = E[y|\underset{\sim}{x},q]$$

When we have a good proxy w for q.

The $APE(y|x_i)\big|_{\underset{\sim}{x}=\underset{\sim}{x}_0} = E\left[\left[\frac{\partial E[y|\underset{\sim}{x},\underset{\sim}{w}]}{\partial x_i}\right]_{\underset{\sim}{x}=\underset{\sim}{x}_0} \Big| \underset{\sim}{w}\right]$ is free of q.

4. Linear Projections

(1) Basics of LP

By the definition of LP,

$$L(y|1,x_1\cdots x_k) = \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k = \beta_0 + \underset{\sim}{x}\underset{\sim}{\beta}$$

$$\underset{\sim}{\beta} = \mathrm{var}^{-1}(\underset{\sim}{x})\,\mathrm{cov}(y,\underset{\sim}{x})$$

which is exactly linear regression of y on $x_1\cdots x_k$.

Thus, we have

• $L(c|\underset{\sim}{x})=c$, $L(\underset{\sim}{x}|\underset{\sim}{x})=\underset{\sim}{x}$, $L(\underset{\sim}{x}|c,\underset{\sim}{x})=\underset{\sim}{x}$

where c is constant vector.

$$\underset{\sim}{\beta} = \mathrm{var}^{-1}(\underset{\sim}{x})\,\mathrm{cov}(y,\underset{\sim}{x})$$

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Thus, we have

• $L(c|\underset{\sim}{x})=c$, $L(\underset{\sim}{x}|\underset{\sim}{x})=\underset{\sim}{x}$, $L(\underset{\sim}{x}|c,\underset{\sim}{x})=\underset{\sim}{x}$

where c is constant vector.

• (2.44) $L(y|1,\underset{\sim}{x}) = L[L[y|1,\underset{\sim}{x},\underset{\sim}{z}]|1,\underset{\sim}{x}]$

$\Leftrightarrow$ LP.4

• (2.45) $L(y|1,\underset{\sim}{x}) = L[E(y|\underset{\sim}{x},\underset{\sim}{z})|1,\underset{\sim}{x}]$

$\Leftrightarrow$ LP.5 *here x does not include 1's.

(2) Properties of LP

- (2.44) $L(y|1,x) = L[L[y|1,x,z]|1,x]$

$\Leftrightarrow$ LP.4

- (2.45) $L(y|1,x) = L[E(y|x,z)|1,x]$

$\Leftrightarrow$ LP.5 * here x does not include 1's.

$\boxed{\text{LP.1}}$ If $E(y|x) = x\beta$, then

$L(y|x) = x\beta$

$\boxed{\text{LP.2}}$ $u \equiv y - L(y|x)$.

$\Rightarrow E(x'u) = 0$

$\boxed{\text{LP.4}}$ (LIE) * For all LP's, x include 1's.

$L(L(y|x,z)|x) = L(y|x)$

$\boxed{\text{LP.5}}$ $L(E(y|x,z)|x) = L(y|x)$

$\boxed{\text{LP.7}}$ If $L(y|x,z) = x\beta + z\gamma$

Then by constructing projections

$r \equiv x - L(x|z) \equiv x - \hat{x}_{OLS}$

which are the residuals from regression

and $v \equiv y - L(y|z) \equiv y - \hat{y}_{OLS}$

we have $L(v|r) = r\beta = L(y|r)$

II. HW Problem Set 1.

$\boxed{2.1}$ Given RV's y, x_1, x_2.

$$E(y|x_1, x_2) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_2^2 + \beta_4 x_1 x_2$$

$\boxed{a}$ Find the PE of x_1, x_2 on $E(y|x_1, x_2)$

$$\frac{\partial E(y|x_1, x_2)}{\partial x_1} = \beta_1 + \beta_4 x_2$$

$$\frac{\partial E(y|x_1, x_2)}{\partial x_2} = \beta_2 + 2\beta_3 x_2 + \beta_4 x_1$$

$\boxed{b}$ $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_2^2 + \beta_4 x_1 x_2 + u$

What can we say about $E(u|x_1, x_2)$ $E(u|x_1, x_2, x_2^2, x_1 x_2)$?

$E(u|x_1, x_2)$ assumed to be zero ($\Rightarrow E(u) = 0$ by LIE)

$E(u| x_1, x_2, f_1(x_1), g_1(x_2) \cdots) = 0$ by my CE note Part 1.

Thus u and $x_1, x_2, x_2^2, x_1 x_2$ are independent ($\Rightarrow$ uncorrelated)

$E[u| x_1, x_2] = 0$ is a stronger condition than $E[u] = 0$.

Since by LIE $E[E[u|x_1, x_2] | x_1, x_2] = E[0|x_1, x_2] = 0 = E[u]$

but $E[u]$ does not guarantee that $E[u|x_1, x_2] = 0$

$\boxed{C}$ Discuss $Var(u|x_1, x_2)$

Recap $V(\underset{\sim}{y}|\underset{\sim}{x}) \equiv \sigma^2(\underset{\sim}{x}) \equiv E[(y - E[\underset{\sim}{y}|\underset{\sim}{x}])^2 | \underset{\sim}{x}] = E[y^2|\underset{\sim}{x}] - (E[y|x])^2$

$\overset{\shortparallel}{V(\underset{\sim}{u}|\underset{\sim}{x})} \qquad = E[u^2|\underset{\sim}{x}]$ ①

$\because V(\underset{\sim}{u}|\underset{\sim}{x}) = E[u^2|x] - (\underset{\overset{\shortparallel}{0}}{E[u|x]})^2 = E[u^2|\underset{\sim}{x}]$ ②

$E[u^2] \overset{LIE}{=} E[E[u^2|X]] = E[\sigma^2(\underset{\sim}{x})] = \sigma^2$, if σ^2 is invariant of X.

Thus $\sigma^2(\underset{\sim}{x}) = V(\underset{\sim}{y}|\underset{\sim}{x}) = V(\underset{\sim}{u}|\underset{\sim}{x}) = E[u^2|\underset{\sim}{x}]$

$= E[u^2]$ if σ^2 is invariant of x

We can only claim that $V(u|x_1, x_2)$ is positive here.

* If we assume u_1 and x_1, x_2 are independent, then.

$SSE \equiv \underset{\sim}{u}'\underset{\sim}{u}$ where $\underset{\sim}{u} \equiv y - \hat{y}$ $\left\{ \begin{array}{l} \underset{\sim}{y} \text{ is } n \times 1 \text{ vector} \\ \underset{\sim}{X} \text{ is } p+1) \text{ vector} \end{array} \right.$

$\hat{\sigma}^2 \equiv \frac{MSE}{n-p}$

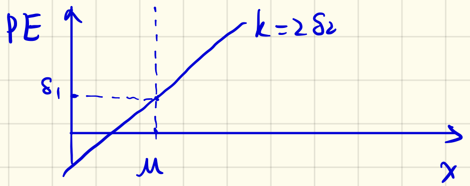
$\boxed{2.2}$ $E(y|x) = \delta_0 + \delta_1(x-\mu) + \delta_2(x-\mu)^2$

$\mu = E(X)$

$\boxed{a}$ Find $\partial E(y|x)/\partial x$; comment

PE: $\dfrac{\partial E(y|x)}{\partial x} = \delta_1 + 2\delta_2(x-\mu) \equiv \ell(x)$,

which is very flexible, depending on the values of δ_1, δ_2 and μ all, as shown in the graph.



$\boxed{b}$ Show that δ_1 is equal to $\dfrac{\partial E(y|x)}{\partial x}$ averaged across the distribution of x.

W.t.s. $E_x\left(\dfrac{\partial E(y|x)}{\partial x}\right) = \delta_1$

i.e. $E_x\left(\dfrac{\partial E(y|x)}{\partial x}\right) = E_x\left[\delta_1 + 2\delta_2(x-\mu)\right]$

$= \delta_1 + 2\delta_2 E[X-\underset{\downarrow \atop E(x)}{\mu}] = \delta_1 + 2\delta_2 \underbrace{E[X-E(x)]}_{=0}$

$= \delta_1$

$\boxed{C}$ Let $E[(x-\mu)^3]=0$. Show that

$L(y|1,x)=\alpha_0+\delta_1 x$, for some α_0

By 2.45 (or LP. 5)

Since $E[y|x]=\delta_0+\delta_1(x-\mu)+\delta_2(x-\mu)^2$, $E(y|x)$ has already include 1 (so as δ_0) as parameter, i.e. $E[y|x]=E[y|1,x]$

$$L(y|1,x)\equiv L[E[y|1,x]|1,x]\overset{\downarrow}{=}L[E[y|x]|1,x]$$

$$=L[\delta_0+\delta_1(x-\mu)+\delta_2(x-\mu)^2|1,x]$$

$$=\delta_0+\delta_1(x-\mu)+\delta_2 L[(x-\mu)^2|1,x]$$

Now let us check if $(x-\mu)^2$ and x are correlated.

Denote $Y\equiv(x-\mu)^2$, $Z=x-\mu=x-E[x]$

Thus $\mathrm{cov}((x-\mu)^2, x)=\mathrm{cov}(Y, Z+E[x])$

$$=\mathrm{cov}[Y, E[x]]+\mathrm{cov}(Y,Z) \quad \text{* By cov property}$$

$$=E[Y\cdot E[x]]-E[Y]E[x]+E[YZ]-E[Y]E[Z]$$

$$=\underbrace{\mu E[(x-\mu)^2]-\mu E[(x-\mu)^2]}_{=0}+\underbrace{E[(x-\mu)^3]}_{=0}-E[(x-\mu)^2]\underbrace{E[x-\mu]}_{=0}$$

$$=0$$

* Since $\mu=E[x]$ which is an arbitrary value instead of $\mu(x)$, it can be taken out from the bracket.

As $(x-\mu)^2$ is uncorrelated with x, thus, for $L[(x-\mu)^2|1,x]=\gamma_0+\gamma_1 x$

$\gamma_1=0$

Thus, $L(y|1,x)=\delta_0+\delta_1(x-\mu)+\gamma_0=(\delta_0-\delta_1\mu+\gamma_0)+\delta_1 x=\alpha_0+\delta_1 x$

where $\alpha_0=\delta_0-\delta_1\mu+\gamma_0$

2.4. For random scalars u, v, and random vector $\underset{\sim}{x}$, suppose

(1) $E[u|\underset{\sim}{x}, v]$ is a linear function of $(\underset{\sim}{x}, v)$ (2) $E[u] = E[v] = 0$

(3) u, v are uncorrelated with $\underset{\sim}{x}$. Show that

$$E[u|\underset{\sim}{x}, v] = E[u|v] = \rho_1 v, \text{ for some } \rho_1$$

Note here we only have uncorrelation assumption, which does not imply independence. thus, we cannot use CE.5.

Assume $E[u|\underset{\sim}{x}, v] = \underset{\sim}{x}\underset{\sim}{\beta} + \rho_1 v$, $x = [1, x_1, x_2 \cdots]$

We want to find $\underset{\sim}{\beta}$ to be zero

Since we are only given that $E[u|\underset{\sim}{x}, v]$ is linear of $(\underset{\sim}{x}, v)$, we have no idea if $\underset{\sim}{\beta}$ are equal to that by LP.

By LP.1 $E[u|\underset{\sim}{x}, v] = \underset{\sim}{\beta}\underset{\sim}{x} + \rho_1 v = \underbrace{(\underset{\sim}{x}, v)}_{\underset{\sim}{X}_{Aug}}\underbrace{\begin{pmatrix}\beta_1 \\ \rho_1\end{pmatrix}}_{\beta_{Aug}} = L[u|\underset{\sim}{x}, v]$

Now, we can use LP.7 to estimate the $\underset{\sim}{\beta}$ in CE, since they are consistent by LP.2

Let $\underset{\sim}{r} \equiv \underset{\sim}{x} - L(\underset{\sim}{x}|v)$, which is the residuals of x regressed on v. As $\underset{\sim}{x}, u$ are uncorrelated, thus, $L(\underset{\sim}{x}|v) = 0$ i.e. $\underset{\sim}{r} = \underset{\sim}{x}$

Thus by LP.7. $L(u|\underset{\sim}{r}) = \underset{\sim}{\beta}\underset{\sim}{r}$, and $u, \underset{\sim}{x}$ uncorrelated

$$\overset{L(u|\underset{\sim}{r})}{\|} \quad L(u|\underset{\sim}{x}) \Rightarrow \underset{\sim}{\beta} = 0$$

Thus, $E[u|x, v] = E[u|v] = \rho_1 v$, for some ρ_1

2.7. For conditional expectation $E(y|\underset{\sim}{x},\underset{\sim}{z}) = g(\underset{\sim}{x}) + \underset{\sim}{z}\underset{\sim}{\beta}$

where $g(\cdot)$ is a general function of $\underset{\sim}{x}$. $\underset{\sim}{\beta}$ is $1\times M$ vector. Show that $E[\tilde{y}|\tilde{\underset{\sim}{z}}] = \tilde{\underset{\sim}{z}}\underset{\sim}{\beta}$

where $\tilde{y} \equiv y - E[y|\underset{\sim}{x}]$, $\tilde{\underset{\sim}{z}} \equiv \underset{\sim}{z} - E[\underset{\sim}{z}|\underset{\sim}{x}]$

$\because y - u = E[y|\underset{\sim}{x},\underset{\sim}{z}] = g(\underset{\sim}{x}) + \underset{\sim}{z}\underset{\sim}{\beta}$

$\therefore y = g(\underset{\sim}{x}) + \underset{\sim}{z}\underset{\sim}{\beta} + u$ ①, u is the error w/ $E[u|\underset{\sim}{x},\underset{\sim}{z}]=0$

Take ① conditional expectation on $\underset{\sim}{x}$

$$E[y|\underset{\sim}{x}] = \underbrace{E[g(\underset{\sim}{x})|\underset{\sim}{x}]}_{g(\underset{\sim}{x})} + E[\underset{\sim}{z}|\underset{\sim}{x}]\underset{\sim}{\beta} + \underbrace{E[u|\underset{\sim}{x}]}_{0}$$

$$E[y|\underset{\sim}{x}] = g(\underset{\sim}{x}) + E[z|\underset{\sim}{x}]\underset{\sim}{\beta} \qquad ②$$

① − ② $\Rightarrow$

$$y - E[y|\underset{\sim}{x}] = (\underset{\sim}{z} - E[\underset{\sim}{z}|\underset{\sim}{x}])\beta + u$$

$$\Rightarrow \tilde{y} = \tilde{\underset{\sim}{z}}\underset{\sim}{\beta} + u$$

$$\Rightarrow E[\tilde{y}|\tilde{z}] = E[\tilde{z}|\tilde{z}]\beta + \underbrace{E[u|\tilde{z}]}_{0}$$

since $\tilde{\underset{\sim}{z}}$ is a function of z & $\underset{\sim}{x}$

$\therefore E[\tilde{y}|\tilde{z}] = \tilde{z}\beta$