HW 1

1a $f(n) = 2^n + n^2 + n$

$\underline{\Omega(g(n))}$

$0 \leq 2^n \leq 2^n + n^2 + n$ (2^n) blows up much faster than $n^2 + n$

$c = 1$

$n_0 = 1$ $\in \Omega(2^n)$

$\underline{O(g(n))}$

$0 \leq 2^n + n^2 + n \leq 2 \cdot 2^n$

$c = 2$

$n_0 = 4$ $\in O(2^n)$

$\underline{\Theta(g(n))}$

$0 \leq 2^n \leq f(n) \leq 2 \cdot 2^n$

$c_1 = 1$ $\in \Theta(2^n)$

$c_2 = 2$

$n_0 = 4$ (choose bigger n_0)

Best fit is $\Omega(2^n)$ because 2^n blows up way faster as $n \to \infty$ than other terms in $f(n)$

1b) $f(n) = \frac{(n+3)^2}{5} = \frac{1}{5}(n^2 + \underbrace{6n+9}_{< n^2 \text{ as } n \to \infty})$

$\underline{\Omega(g(n))}$

$0 \leq \frac{1}{5}n^2 \leq f(n)$

$c = \frac{1}{5}$ $\in \Omega(n^2)$

$n_0 = 1$

$\underline{O(g(n))}$

$0 \leq f(n) \leq \frac{2}{5}n^2$

$c = \frac{2}{5}$ $\in O(n^2)$

$n_0 = 8$

$\Theta(g(n))$

$0 \leq \frac{1}{5}n^2 \leq f(n) \leq \frac{2}{5}n^2$

$c_1 = \frac{1}{5}$ $\qquad \in \Theta(n^2)$

$c_2 = \frac{2}{5}$

$n_0 = 8$ (Choose bigger n_0)

$\Theta(n^2)$ is best fit

2 $\quad n^{\sqrt{n}} \qquad (\sqrt{n})^n$

$n^{n^{\frac{1}{2}}} \qquad n^{\frac{n}{2}}$

$n^{\frac{1}{2}} \log n \qquad \frac{n}{2}\log n$

$\sqrt{n}\log n < \frac{1}{2} n \log n \qquad (n > \sqrt{n})$