ECON 7020 Final Exam.

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Problem 1:

employment status: ε asset: $a. \in [-2, \infty)$ util $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_t)$,

$\varepsilon_0 = \{u, e\}$ — u: unemploy, e: employed.

$u \to \begin{cases} b_t & : \text{compensate to bt} \\ n_t = 0 \end{cases}$

$e: \{ 0 \le n_t < 1$.

$Y_t = Z_t k_t^{\alpha} N_t^{1-\alpha}$. — Z_t: Agg shock

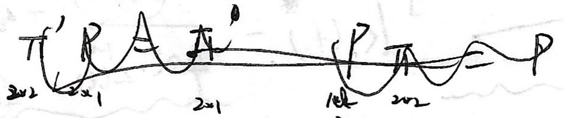
a. In Aiyagari (1994), he assumes individuals face different shock, in which they are heterogeneous. However, in our traditional representative model, individuals usually face the same shock, which is unrealistic in the real world. So, that's why Aiyagari (1994) uses heterogeneous agents model.

b. For the umployment agents:

$$c_t + a_{t+1} = b_t + (1 + (1-\tau) r_t) a_t.$$

For the employment agents:

$$c_t + a_{t+1} = (1-\tau) W_t n_t + (1 + (1-\tau) r_t) a_t.$$



$\pi' P = \pi$ — 202 201, 201 $P \cdot \pi' = \pi$ $P\pi = P$ — 1x2 202 KK.

$$(I - \Pi)p = 0$$

$$\begin{bmatrix} 0.5 & -0.5 \\ -0.0435 & 0.0435 \end{bmatrix} \begin{bmatrix} P_u \\ P_e \end{bmatrix} = 0$$

$\therefore$ $0.5 P_u - 0.5 P_e = 0$

$P_u + P_e = 1.$ $\quad \therefore \begin{cases} P_u = 0.5 \\ P_e = 0.5. \end{cases}$

Thus, in equilibrium, the percentage of agents is a ~~half~~. 0.5, 0.5. for unemployment and employment agents.

c. ~~Be~~ Set up a Bellman. Eqn:

$$V(a) = \max_{c,n} \left\{ u(c, 1-n) + \beta E_t \left[V(a') \underbrace{Q(\varepsilon, \varepsilon')}_{\text{Transition probability func.}} \right] \right\}$$

s.t. $\begin{cases} c + a' = b + (1 + (1-\tau) r) a. & \text{if } \varepsilon = u. \\ c + a' = (1-\tau) wn + (r(1-\tau) r) a. \end{cases}$

Now, solve the ~~Bellmay~~ Bellman Eqn:

(i) Take F.O.C w.r.t c, n, respectively.

[c]. $u_1(c, 1-n) + \beta E[V'(a') Q(\varepsilon, \varepsilon')] \cdot \frac{\partial a'}{\partial c} = 0$

$\Rightarrow u_1(c, 1-n) = \beta E[V'(a') Q(\varepsilon, \varepsilon')]$ ①

[n] $u_2(c, 1-n)(-1) + \beta E[V'(a')Q(\varepsilon,\varepsilon')] \cdot 0 \cdot \mathbb{1}_{\{\varepsilon'=u\}}$

$$+\beta E[V'(a')Q(\varepsilon,\varepsilon')] \cdot (1-\tau)w \cdot \mathbb{1}_{\{\varepsilon'=e\}} = 0$$

$$\therefore \quad u_2(c, 1-n) = \beta E[V'(a')Q(\varepsilon,\varepsilon')]\left[0\cdot\mathbb{1}_{\{\varepsilon'=u\}} + (1-\tau)w\cdot\mathbb{1}_{\{\varepsilon'=e\}}\right] \quad ②$$

ow use the Envelope Theorem:

$$V'(a) = \beta E[V'(a')Q(\varepsilon,\varepsilon')] \cdot (1+(1-\tau)r)$$

$$\therefore \quad \frac{V'(a)}{1+(1-\tau)r} = \beta E[V'(a')Q(\varepsilon,\varepsilon')] \quad ③$$

plug ③ into ①

$$u_1(c, 1-n) = \frac{V'(a)}{1+(1-\tau)r}$$

$$\therefore \quad V'(a) = u_1(c, 1-n)[1+(1-\tau)r] \quad ④$$

$$\therefore \quad V'(a') = u_1(c', 1-n')[1+(1-\tau)r']$$

refore,

$$u_1(c, 1-n) = \beta E[u_1(c', 1-n')(1+(1-\tau)r')\, Q(\varepsilon,\varepsilon')] \quad ⑤$$

The ⑤ is the Euler Eqn.

d. For the stationary competitive Equilibrium.

A stationary competitive equilibrium is when the price and quantity satisfies the following conditions:

① in labor market, labor supply = labor demand of firm.

② in capital market.

$$1+(1-\tau)r_t = MPK = \alpha Z_t K_t^{\alpha-1} N_t^{1-\alpha}.$$

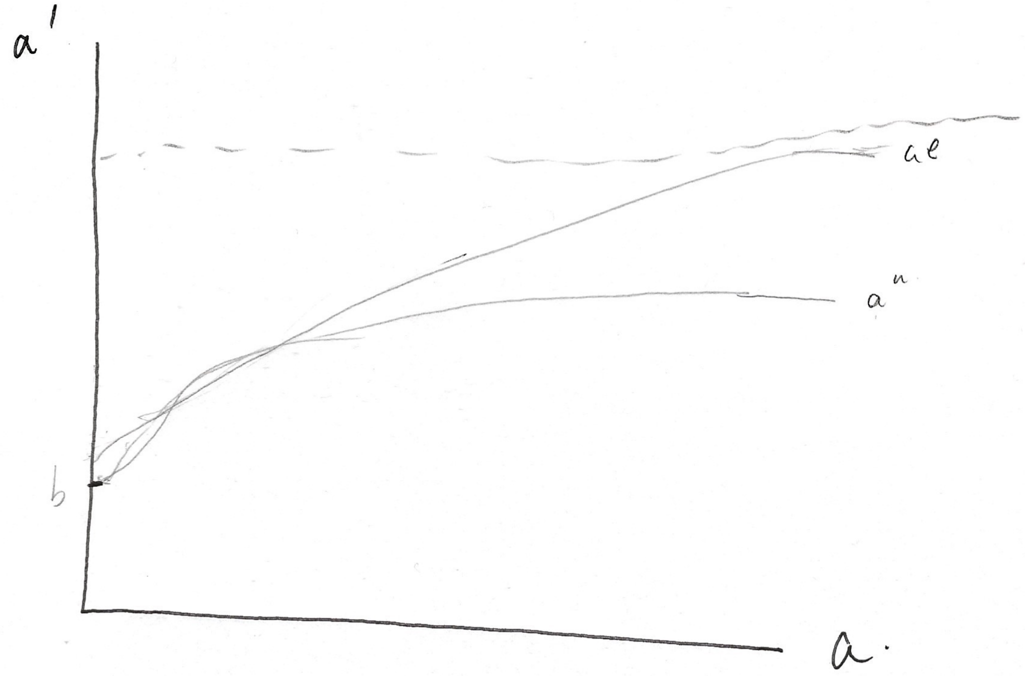
③ $(1-\tau)w_t = MPL = (1-\alpha)Z_t K_t^{\alpha} N_t^{-\alpha}.$

④ the consumers' quantity of consumptions is in the set of quantity that maximizes their utility given the price of the endowment.

⑤ Transfer = Tax on wage and capital.

⑥ fixed distribution of each type of agents.

e.



if $\varepsilon = e$, there will be more assets ~~for~~ in the next period.

if $\varepsilon = u$, there will be less.

it $a'_e \geq a'_u$.

$\boxed{b} \geq \cancel{(1-\tau)} w^* n$. $\quad a > \min\left\{ b, \frac{(1-\tau)w^* n}{1+(1-\tau)r} \right\}$.

f. Krusell and Smith (1998) assumes there ~~was~~ is bounded rationality regarding the ~~...~~ production. ~~Such th~~

And there exists an invariant distribution $f(a, n)$

such that $\cancel{K = \sum_{j=1}^{\infty} k_j f(a_j, n_j)}$

$$\cancel{L = \sum\sum l f(a, n)}$$

$$K = \int k_j f(a_j, n_j) \, d.\pi$$

$$N = \iint n_j f(a_j, n_j) \, d.\pi$$

in this case, even though individuals are heterogeneous, the aggregate level for K and N are deterministic

Problem 2 :

type: m , w.

$a_m > 1$ $a_w = 1$

a. Set up a Bellman Eqn :

First for the managers :

$$V(k, \ell,) = \max_{c_t} [u(c) + \cancel{\beta V} \beta E[V(k', \ell') \cdot \mathbb{1}_{\{h=m\}}]]$$

st

$$c = r^k k + w a_m s_t^m - k^{m\prime} - \ell.$$

For the workers.

$$V(k, \ell) = \max_{c_t} \{u(c) + \beta E[. V(k', \ell) \cdot \mathbb{1}_{\{h=w\}}]]$$

The state variables are k and ℓ.

Reasons to converge :

① By Feller Property. that r.v. are bdd.

②. Budget constraint. + No Ponzi scheme, => function will converge in a closed form given the resource is in a compact set.

③ $V(\cdot)$ and $u(\cdot)$ continuous.

b. perfect CE:

A PCE is the quantity and prices satisfies the following conditions:

① for the consumers in the set of quantities can maximize their utility given the price of their endowment.

② labor supply = Labor demand.

③ $r^K = MPK_t$

$w(t) = MPL$.

④ $S(r^l(t)) = K(t+1)$.

⑤ $r^l(t) = r^K(t)$

c. $\sigma(t) = 1$ $\quad P_{ij} = 0.5$.

First, for managers:

$$r(t) = MRS_t = \frac{MU_{t+1}}{MU_t} = \frac{C_t^h(t)}{C_t^h(t+1)}$$

Than By $C_t^h(t+1) = w(t+1)a_m q^m(t+1) + r^k(t+1)k^m(t+1) + r^l(t)l^m(t)$.

Replace it into above, and solve $C_t^h(t)$.

Then by $S_t^h(t) = W_t^h(t) - C_t^h(t)$.

$S_{mt}^{h}(t)$ for mangers: =

$$\frac{W(t)\,a_m\,\Delta_t^m(t)}{2} - \frac{W(t+1)\,a_m\,\Delta_t^m(t+1)}{2r^t(t)}$$

The same procedure goes to workers:

$S_{wt}^{h}(t)$ for workers:

$$\frac{W(t).\,\Delta_t^W(t)}{2} - \frac{W(t+1)\,\Delta_t^W(t+1)}{2r^t(t)}$$

d $\because P_{ij}=0.5$ $\quad \therefore P_h = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$

$\therefore$ $\Pi\ P_h =$ 2×2. $\quad \Pi : \begin{bmatrix} \Pi_{m,t} \\ \Pi_{w,t} \end{bmatrix}_{2\times 1}$

$\therefore \underset{2\times2}{P_h}\ \underset{2\times1}{\Pi} = \Pi.$

WLOG assume:
$N(t)=1$

$\therefore \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}\begin{bmatrix} \Pi_{m,t} \\ \Pi_{w,t} \end{bmatrix} = \begin{bmatrix} \Pi_{m,t} \\ \Pi_{w,t} \end{bmatrix}$

$\therefore 0.5\,\Pi_{m,t} + 0.5\,\Pi_{w,t} = \Pi_{m,w}$

$\Pi_{m,t} + \Pi_{w,t} = 1.$

$\therefore \Pi_{m,t} = \Pi_{w,t} = 0.5.$

$\therefore S(t) = \frac{1}{2} S^h_{m_t}(t) + \frac{1}{2} S^h_{w_t}(t)$

$$= \frac{w(t)\, a_m \delta^m_t(t)}{4} - \frac{w(t+1)\, a_m \delta^m_t(t+1)}{4r^l(t)} + \frac{w(t)\cdot \delta^w_t(t)}{4} - \frac{w(t+1)\, \delta^w_t(t+1)}{4r^l(t)}$$

$$= \frac{w(t)[a_m \delta^m_t(t) + \delta^w_t(t)]}{4} - \frac{w(t+1)[a_m \delta^m_t(t+1) + \delta^w_t(t+1)]}{4r^l(t)}$$

e. $\because$ in Equilibrium,

$$S(t) = K_t(t+1)$$

$$\therefore K_t(t+1) = \frac{w(t)[a_m \delta^m_t(t) + \delta^w_t(t)]}{4} - \frac{w(t+1)[a_m \delta^m_t(t+1) + \delta^w_t(t+1)]}{4r^l(t)}$$

In CE:

$$w(t) = MPL = (1-\alpha) L(t)^{-\alpha} K(t)^{\alpha}.$$

$$r^l(t) = MPK = \alpha L(t)^{1-\alpha} K(t)^{\alpha-1}.$$

$$\therefore K_t(t+1) = \underbrace{\frac{(1-\alpha)\left(\frac{1}{L(t)}\right)^{\alpha}[a_m \delta^m_t(t) + \delta^w_t(t)]}{4}}_{\text{Denote as } Q} K(t)^{\alpha} - \underbrace{\frac{(1-\alpha)\left(\frac{1}{L(t+1)}\right)^{\alpha}[a_m \delta^m_t(t+1) + \delta^w_t(t+1)]}{4\alpha \cdot L(t)^{1-\alpha}}}_{\text{Denote as } J} \cdot \frac{K(t+1)^{\alpha}}{K(t)^{1-\alpha}}$$

$\therefore$

$$K_t(t+1) = Q\, K(t)^{\alpha} - J\, K(t+1)^{\alpha}\, K_t^{1-\alpha}.$$

if in steady state for K

$\therefore K(t+1) = K(t)$

$$K(t+1) = QK^{\alpha}(t+1) - J\, K(t+1)$$

$$\Rightarrow (1+J)K(t+1) = QK^{\alpha}(t+1)$$

$$\therefore 1+J = QK^{\alpha-1}(t+1)$$

$$\therefore K^{\alpha-1}(t+1) = \frac{1+J}{Q}$$

$$\therefore K(t+1) = \left(\frac{1+J}{Q}\right)^{\frac{1}{\alpha-1}}$$

d. Conditions:

① ~~traders~~ agents neutralize the shocks to the economy in total.

② The path of $K(t+1)$ is a saddle path.

③ is CE condition

④ The Distribution of each type of agents is stable from the MC in the long run.

⑤