HW5

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ECON 5700

Due on August 20, 2020.

1 Question 1

Solution:

1. $V = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$, $u = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. The projection of v onto u:

$$\begin{bmatrix} \frac{2}{5} \\ \frac{1}{5} \end{bmatrix}$$

2. $v = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$, $u = e_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, the projection of v onto u:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

3. $v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $u = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{bmatrix}$. The projection of v onto u is:

$$\begin{bmatrix} \frac{3+3\sqrt{2}}{4} & \frac{3+3\sqrt{2}}{4} & \frac{3\sqrt{2}+6}{4} \end{bmatrix}$$

2 Question 2

Solution:

At the points $P = (-1, 5, 0)$ and $Q = (2, 1, 1)$, a direction vector through the two points are $v =< 3, -4, 1 >$. Thus, the line of this vector is $r = r_0 + t(r_1 - r_0)$.

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$$
\begin{aligned}
< x, y, z > &= \overrightarrow{OP} + t(\overrightarrow{OQ} - \overrightarrow{OP}) \\
&= (-1, 5, 0) + t(3, -4, 1)
\end{aligned}
$$

This is one possible line. ☺

3 Question 3

Solution:

The reason why I think w is given variable is that if it's not, then 4 unknown variables, but 3 equations, which make our equations unsolvable.

$$
\left(\begin{array}{ccc|c}
-1 & -1 & 2 & 1 - w \\
-2 & -1 & 3 & 3 - 2w \\
1 & -1 & 0 & -3 + w
\end{array}\right)
$$

$$
\left(\begin{array}{ccc|c}
1 & -1 & 0 & 3 + w \\
0 & -1 & 1 & 1 \\
0 & 0 & 0 & -2
\end{array}\right)
$$

This question is unsolvable. ☹

4 Question 4

Solution:

$$
\left(\begin{array}{ccc|c}
1 & -1 & 2 & 3 \\
1 & 2 & -1 & -3 \\
0 & 2 & -2 & 1
\end{array}\right)
$$

$$
\left(\begin{array}{ccc|c}
1 & -1 & 3 & 3 \\
0 & 1 & -1 & -2 \\
0 & 0 & 0 & -3
\end{array}\right)
$$

Again, this system is inconsistent, thus unsolvable. ☹

5 Question 5

Solution:

$$
\left(\begin{array}{ccc|c}
1 & 1 & -2 & 4 \\
1 & 3 & -1 & 7 \\
2 & 1 & -5 & 7
\end{array}\right)
$$

After several elimination:

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

Thus, $x = 0, y = 2, z = -1$. ☺

6 Question 6

Solution:

$$\begin{pmatrix} 1 & -2 & 0 & 3 & 2 \\ 0 & 5 & 1 & -6 & -2 \\ 0 & 10 & 2 & -12 & -4 \\ 0 & -5 & -1 & 6 & 2 \end{pmatrix}$$

After sevral elimination process, the matrix is:

$$\begin{pmatrix} 1 & -2 & 0 & 3 & 2 \\ 0 & 5 & 1 & -6 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Therefore, the rank of this matrix is 2. ☺

7 Question 7

Solution:

To justify the two matrices are equivalent or not, we need to make some eliminations on these two matrices, respectively.

1. This matrix would be:
$$\begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & 17 \end{pmatrix}$$

2. The second matrix would be:
$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

Remark: If 17 in the first matrix can be extracted out, then after elimination, it becomes identity matrix. The second matrix can also be identity matrix without plugging anything out. But $17 * \mathbb{1}? = \mathbb{1}$? Check! Yes, it is. The two matrix is equivalent, because, they can be transformed into identity matrix.

8 Question 8

Solution:

$$\left(\begin{array}{ccc|c} 8 & -18 & 1 & 35-3w \\ 2 & -4 & 0 & 11-w \\ 3 & -7 & 1 & 10-w \end{array}\right)$$

After elimination of this matrix:

$$\left(\begin{array}{ccc|c} 1 & -2 & 0 & \frac{2}{11}-\frac{w}{2} \\ 0 & -1 & -2 & -\frac{13}{2}+\frac{w}{2} \\ 0 & 0 & 5 & 4+2w \end{array}\right)$$

Thus, the equation would be: $x = \frac{153}{10} - \frac{31}{10}w$, $y = \frac{49}{10} - \frac{13}{10}w$, and $z = \frac{4}{5} + \frac{2}{5}w$.The result is weird, check later. I decide not to check again. Need to check with the solution sheet later.

9 Question 9

Solution:

$u = \begin{bmatrix} 5 \\ 1 \\ 3 \end{bmatrix}$, $v = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$, $w = \begin{bmatrix} 7 \\ 5 \\ 8 \end{bmatrix}$, and $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$.

1.
$$uv' = \begin{bmatrix} 15 & 5 & -5 \\ 3 & 1 & -1 \\ 9 & 3 & -3 \end{bmatrix}$$

2.
$$uw' = \begin{bmatrix} 35 & 25 & 40 \\ 7 & 5 & 8 \\ 21 & 15 & 24 \end{bmatrix}$$

3.
$$xx' = \begin{bmatrix} x_1^2 & x_1x_2 & x_1x_3 \\ x_2x_1 & x_2^2 & x_2x_3 \\ x_3x_1 & x_3x_2 & x_3^2 \end{bmatrix}$$

4.
$$v'u = 13$$

5.
$$u'v = 13$$

6.
$$w'x = 7x_1 + 5x_2 + 8x_3$$

7.

$$u'u = 35$$

8.

$$x'x = x_1^2 + x_2^2 + x_3^2$$