Math Camp
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Homework 2

Find an equation of the tangent plane to the given surface at the specified point.

$$z = 3y^2 - 2x^2 + x, \quad (2, -1, -3)$$

$$z = \sqrt{xy}, \quad (1, 1, 1)$$

$$z = x\sin(x + y), \quad (-1, 1, 0)$$

Explain why the function is differentiable at the given point. Then find the linearization of the function at that point.

$$f(x, y) = 1 + x\ln(xy - 5), \quad (2, 3)$$

Find the differential of the function.

$$z = e^{-2x}\cos 2\pi t$$

$$m = p^5 q^3$$

$$R = \alpha\beta^2 \cos\gamma$$

If $g(s, t) = f(s^2 - t^2, t^2 - s^2)$ and f is differentiable, show that g satisfies the equation

$$t\frac{\partial g}{\partial s} + s\frac{\partial g}{\partial t} = 0$$

For the following two sets of functions with a point in the direction vector **u.**

(a) Find the gradient of f.
(b) Evaluate the gradient at the point P.
(c) Find the rate of change of f at P in the direction of the vector $\mathbf{u}$.

$$f(x, y) = \sin(2x + 3y), \quad P(-6, 4), \quad \mathbf{u} = \tfrac{1}{2}\left(\sqrt{3}\,\mathbf{i} - \mathbf{j}\right)$$

$$f(x, y, z) = x^2yz - xyz^3, \quad P(2, -1, 1), \quad \mathbf{u} = \left\langle 0, \tfrac{4}{5}, -\tfrac{3}{5} \right\rangle$$

Find the directional derivative of $f(x, y) = \sqrt{xy}$ at $P(2, 8)$ in the direction of $Q(5, 4)$.

Find the maximum rate of change of f at the given point and the direction in which it occurs.

$$f(x, y) = \sin(xy), \quad (1, 0)$$

Find and classify the critical points of the function

$$f(x, y) = 10x^2y - 5x^2 - 4y^2 - x^4 - 2y^4$$

Find the local maximum and minimum values and saddle point(s) of the functions.

$f(x, y) = x^2 + xy + y^2 + y$

$f(x, y) = (x - y)(1 - xy)$

$f(x, y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2$

$f(x, y) = x^3 - 12xy + 8y^3$

EXERCISE 7.4

1. Find $\partial y/\partial x_1$ and $\partial y/\partial x_2$ for each of the following functions:

(a) $y = 2x_1^3 - 11x_1^2x_2 + 3x_2^2$

(b) $y = 7x_1 + 6x_1x_2^2 - 9x_2^3$

(c) $y = (2x_1 + 3)(x_2 - 2)$

(d) $y = (5x_1 + 3)/(x_2 - 2)$

EXERCISE 7.6

1. Use Jacobian determinants to test the existence of functional dependence between the paired functions.

(a) $y_1 = 3x_1^2 + x_2$

$y_2 = 9x_1^4 + 6x_1^2(x_2 + 4) + x_2(x_2 + 8) + 12$

(b) $y_1 = 3x_1^2 + 2x_2^2$

$y_2 = 5x_1 + 1$