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2 Capital Accumulation in the Theory of Long-Run Growth

Paul M. Romer

After more than a decade of quiescence, growth theory may once again be entering a period of ferment. By the end of the 1960s there was a general consensus about the basic elements of growth theory, and this theory formed the basis for a great deal of empirical work in growth accounting. This agreeable state of affairs was achieved only by narrowing the range of the questions that growth theory was expected to address. From the point of view of classical economists, the two most interesting questions about growth were dropped from consideration. How can one reconcile extraordinary, continuing increases in average per capita income with the notion of diminishing returns? What determines the rate of growth of the population? Attention is once again turning to these issues.

During the 1960s the consensus view on these bothersome questions came to be that each should be assigned an exogenous, exponential trend term. Then the economic analysis of the other features of an economy could proceed. From a practical point of view, this finesse of the problems of endogenous per capita income growth and endogenous population growth was useful. Important theoretical progress in the understanding of dynamic models was made precisely because the most difficult questions about growth were set aside. But because it ignored the fundamental questions and concentrated on abstraction and formalism, growth theory came increasingly to be viewed as a sterile exercise. From the point of view of policy advice, growth theory had little to offer. In models with exogenous technological change and exogenous population growth, it never really mattered what the government did. Partly in reaction, development branched off as a separate endeavor, designed to offer the direct policy advice that growth theory could not.

The irony underlying the disrepute that growth theory fell into is that many economists missed the self-referential nature of the theory. Economics, like any science, has a two-sector technology. The final output good of economics, produced in one sector, is correct answers to questions that noneconomists care about. A separate investment sector produces the intellectual capital that is the key input into the final output sector. Development for the most part took 1960s vintage intellectual capital and has been producing policy advice with it ever since. In contrast, growth theory gave up any pretense of having anything to say about questions that a policymaker might care about and concentrated on intellectual capital accumulation.

This capital accumulation now shows signs of a significant payoff. One of the major themes of this chapter is that the substantive contribution of growth theory has so far been quite small but the methodological impact has been far-reaching and fundamental. The methodological advances have had their greatest impact in macroeconomics, where they can truly be said to have revolutionized accepted practice. To cite just one example from the other chapters in this volume—in the years between 1970 and 1980, the discussion of the theory of aggregate consumption moved from a point where it would have been impolite to mention Euler equations to a point where it was impossible to carry on a discussion without them. By itself, this methodological impact has long justified attention to developments in this area; but now may be an especially good time to tune into growth theory. The second theme developed here is that the tools have evolved to the point where growth theory is on the verge of having something interesting to say about growth.

Of all the policy questions concerning growth, the most fundamental is whether there are any policies that an omniscient, omnipotent, benevolent social planner could implement to raise the welfare of all the individuals in an economy. An affirmative answer to this question is of course a necessary condition (but not a sufficient condition) for justifying interventions by actual governments. In formal terms, the question is whether or not equilibria are Pareto optimal. To treat this question seriously, economists must have available models with Pareto optimal equilibria and models with Pareto suboptimal equilibria. Given a range of models, the answer to this question reduces to a choice of the type of model that best describes the data.

Until recently, virtually all explicit models of growth had Pareto optimal equilibria. [Overlapping-generations models, as used for example in Diamond (1965), are an exception, but one of questionable relevance for growth theory. The conclusion of these models is that an equilibrium may be suboptimal because the capital stock is too high.] In the last few years, the most important technical advance in the analysis of dynamic models is a recognition that it is almost as easy to characterize dynamic equilibrium models with a wide variety of distortions as it is to characterize the more familiar models with all of the requisites for perfect competition. Virtually all of the analysis of dynamic models is based on the analysis of a maximization problem, but the problem need not be a first best social planning problem.

Since this observation is fundamental to the expanded range of recent growth models, much of this chapter is devoted to a detailed discussion of the relationship between maximization problems and equilibria. In part, this discussion is intended as a self-contained introduction to the tools used in the study of dynamic models. Methods are described in the context of earlier work on growth theory in cases where this is convenient, but what is offered here is a synthesis, not a survey.[1] It takes advantage of hindsight and interprets the methods in the context of a unifying result from the mathematical theory of convex analysis, the abstract Kuhn-Tucker theorem.

The plan of the chapter is as follows. Section 2.1 sets the stage with a description of some of the basic data about economic growth. The data naturally suggest specific questions that a theory of growth should be able to address. Does international trade affect growth? What explains the negative correlation between income growth and population growth? What explains the dispersion in growth rates observed across countries? Are less-developed countries systematically catching up with developed countries? Why is there continuing pressure for migration from low-income countries to high-income countries? Is growth slowing down? Section 2.2, the bulk of the chapter, describes the tools that can be used to study questions of this kind. It covers a subset of the standard methods for solving dynamic maximization problems, with emphasis on the methods that translate most easily to models with distortions. Section 2.3 concludes with a discussion of recent models that use the methods outlined in Section 2.2 and that address one or more of the specific questions suggested by the data.

2.1. Data

In an influential article on growth written in 1961, Nicholas Kaldor stated his view that a theorist ought to start with a summary of the facts that are relevant to the problem of interest. This summary should be "stylized," he claimed, concentrating on broad tendencies. One could then construct hypotheses to explain these stylized facts. In the formative stages of a body of theory, this kind of informal treatment of the data can be quite useful, for without stylized facts to aim at, theorists would be shooting in the dark. When Kaldor wrote, the basic elements of a theory of growth seemed to be up for grabs in a developing debate between Cambridge, England and Cambridge, Massachusetts, and the facts he set out became the target for economists on both sides.

If, as is claimed in the introduction, growth is entering a similar phase—namely, one in which the basic questions about growth are being re-examined—it may be useful to review and update Kaldor's list of facts. To do this in a way that does not bias the outcome, it is important to make sure not only that the facts have some connection with measured data but also that the list be as inclusive as possible. Different theories can often explain different subsets of the facts. For example, depending on the set of countries one looks at, one can conclude either that per capita income across countries is converging rapidly or that no tendency toward convergence is present. (This point is discussed in greater detail below.) Another example: Solow (1970) observes that his model of growth can explain five of the six stylized facts described by Kaldor, but he acknowledges that the sixth—the wide dispersion in growth rates across countries—is something of a problem for his model. Subsequent advocates of the neoclassical model have sometimes been less forthcoming, listing only five facts that a model of growth should explain.

Here are Kaldor's six stylized facts:

1. Output per worker shows continuing growth "with no tendency for a *falling* rate of growth of productivity" (emphasis in the original).
2. Capital per worker shows continuing growth.
3. The rate of return on capital is steady.
4. The capital–output ratio is steady.
5. Labor and capital receive constant shares of total income.
6. There are wide differences in the rate of growth of productivity across countries.

It is readily seen that these statements are not all independent. Let Y, K, and L represent total output, capital, and labor, respectively. Let r denote the return on capital. If Y/L is growing and Y/K is constant, K/L must also be growing. Thus, fact 2 follows from facts 1 and 4. If Y/K is constant and rK/Y is constant, then r must also be constant. Thus, 4 and 5 imply 3. With no loss in generality, one can concentrate on 1, 4, 5, and 6. On the basis of the kind of data exhibited below, 1, 4, and 6 may still be reasonable stylized characterizations of the data. On the other hand, there is some evidence of a long-run trend in factor shares.

There are five other prominent features of the data:

7. In cross section, the mean growth rate shows no variation with the level of per capita income.
8. Growth in the volume of trade is positively correlated with growth in output.
9. Population growth rates are negatively correlated with the level of income.
10. The rate of growth of factor inputs is not large enough to explain the rate of growth of output; that is, growth accounting always finds a residual.
11. Both skilled and unskilled workers tend to migrate toward high-income countries.

Observation 7 can be discerned from the wider array of data that are now available. Observation 8 has been noted in discussions of export-led development and 9 has been the focus of considerable study among demographers. However, since formal theories of growth have until recently been silent on the determinants of population growth and of international trade, they have not been considered relevant parts of the target that growth theories should aim at. Observation 10 describes the widely noted conclusion from the work on growth accounting. Observation 11, concerning migration, may not appear to have any direct bearing on theories of income growth; however, recent theoretical work by Robert Lucas suggests that this may be a crucial piece of evidence in distinguishing between theories of growth based on constant returns to scale and those based on increasing returns.

Tables 2.1 and 2.2 bear on Kaldor's first observation. Viewed from a long-run perspective, there is no question that cumulative growth in output per worker has been truly remarkable and that the rate of growth increased over a long period of time. Table 2.1, taken from Maddison (1982), identifies the country with the highest level of output per hour worked during different historical epochs and estimates the rate of productivity growth for that country. The trend is clear, but the magnitudes may need some amplification. The natural logarithm of 2 is 0.69, so it follows that the observed productivity growth rate of 2.3% per year for the United States leads to a doubling of output per worker every 30 years.

Table 2.1 Productivity growth rates for leading countries

Leading country	Interval	Average annual growth rate of GDP per man-hour (%)
Netherlands	1700–1785	−0.07
United Kingdom	1785–1820	0.5
United Kingdom	1820–1890	1.4
United States	1890–1970	2.3

Source: Maddison (1982).

Table 2.2 Increases in output per man-hour

Country	Symbol[a]	Output per man-hour 1870	Output per man-hour 1979	Ratio
Australia	A	1.30	6.5	5
Austria	T	0.43	5.9	14
Belgium	B	0.74	7.3	10
Canada	C	0.64	7.0	11
Denmark	D	0.44	5.3	12
Finland	L	0.29	5.3	18
France	F	0.42	7.1	17
Germany	G	0.43	6.9	16
Italy	I	0.44	5.8	13
Japan	J	0.17	4.4	26
Netherlands	N	0.74	7.5	10
Norway	W	0.40	6.7	17
Sweden	S	0.31	6.7	22
Switzerland	Z	0.55	5.1	9
United Kingdom	K	0.80	5.5	7
United States	E	0.70	8.3	12

Source: Maddison (1982).
a. Country symbols are used in Figures 2.3 and 2.7.

Table 2.2 shows that growth rates of this magnitude are not unique to the United States. It lists the factor by which output per hour worked increased over the period 1870 to 1979 for 16 developed countries. These magnitudes speak for themselves. (Table 2.2 also introduces symbols used to identify different countries in subsequent figures.)

Figures 2.1 and 2.2 show the behavior of labor productivity in the United States in more detail and address the perception that growth rates are falling rather than increasing. Figure 2.1 shows the annual rate of change of output per man-hour for the private business sector in the postwar era. Careful examination reveals a lower average rate of labor productivity growth in the period since 1969. This reduction in the rate of productivity growth has been the source of much concern and attention and is indicative of the kind of evidence that has led to concern that growth rates are slowing. Because these data are sensitive to business cycle variation that does not seem to impinge uniformly on the two halves of the sample, it is not clear that one should draw strong inferences about secular trends from them yet.

The data plotted in Figure 2.2 show the long-run behavior of labor productivity.[2] For comparison, the figure also shows the behavior of per capita income. Year-to-year variation is smoothed by taking 20-year averages. The period 1919 to 1939 shows relatively strong growth in output per

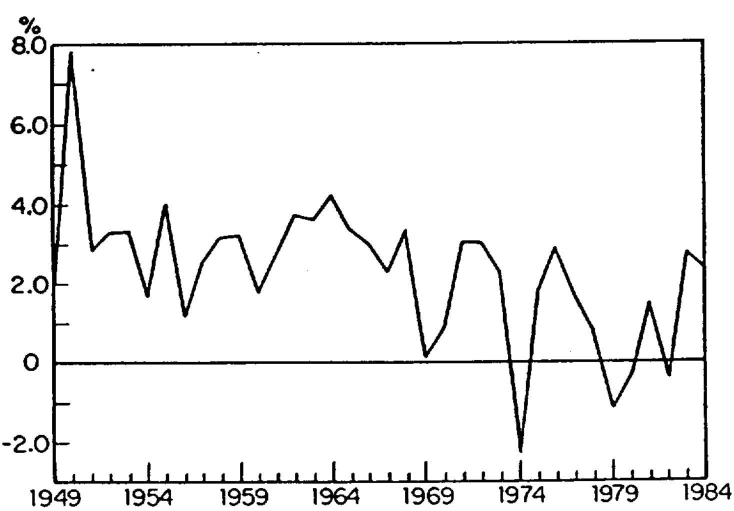

Figure 2.1 Postwar productivity growth. Data are from the Bureau of Labor Statistics figures published in the *Monthly Labor Review*.

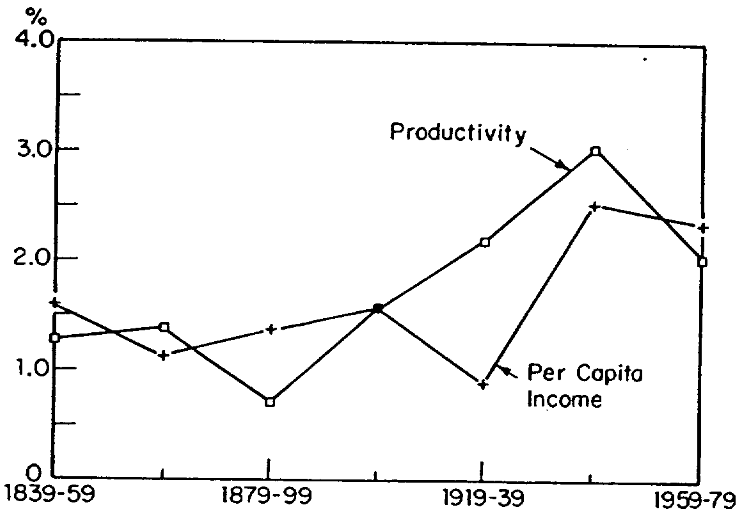

Figure 2.2 Long-run income and productivity growth.[2]

hour worked that masks a fall in employment and hours worked during the Depression. Similarly, the fall in productivity growth at the end of the sample masks a sizable increase in the fraction of the population at work. Using either series, one must judge the recent decline in productivity against the background of a general upward trend. Judging from the variability evident in the data, one cannot yet conclude that this trend has permanently reversed itself.

These impressions are reinforced by an examination of other developed countries. The outstanding feature in the long-run data is the unprecedented surge in growth after the Second World War. There has been some recent slowing of growth relative to that of the 1950s and 1960s, but growth has decreased only to levels that are still high by historical standards. Consequently, in a test for trend in per capita income for 11 developed countries described in Romer (1986), the evidence is in every case supportive of a positive trend; in most cases, the hypothesis of no trend can be rejected at conventional significance levels.

Overall, the data offer relatively strong support for Kaldor's first fact. Unless one is willing to draw very strong inferences from the few most recent observations on a relatively noisy time series and conclude that they represent a break with historical patterns, there is no reason for a theory of growth to aim for falling growth rates and stagnation.

Figure 2.3 bears on the constancy of the capital output ratio. Using data from Maddison (1982), it reports the growth rate of capital per hour worked and of output per hour worked for three time intervals and seven countries. Each country is represented by a letter listed in Table 2.2. The numbers refer to different periods: 1 is the period 1870 to 1913; 2 is the period 1913 to 1950; 3 refers to the period 1950 to 1979. (Like the other data reported here, this sample includes all the countries and time periods for which data are reported in the specified source. Maddison's study covers 16 countries, but these seven, for the specified intervals, are the only ones for which capital stock data are reported.) For the capital–output ratio to be constant, capital and output must grow at the same rate, so a scatter plot of the growth rates should line up on the 45-degree line. Subtracting a constant from each pair—the growth rate of hours worked—should leave each pair on the 45-degree line. In Figure 2.3 they cluster around this line to a surprising extent.

Further evidence on this result can be offered. Let i denote the fraction of total income devoted to investment and let δ denote the depreciation rate on capital. Then the equation for the evolution of the capital stock is $\dot{K} = iY - \delta K$. Let g denote the rate of growth of output, $g = \dot{Y}/Y$. If g, i,

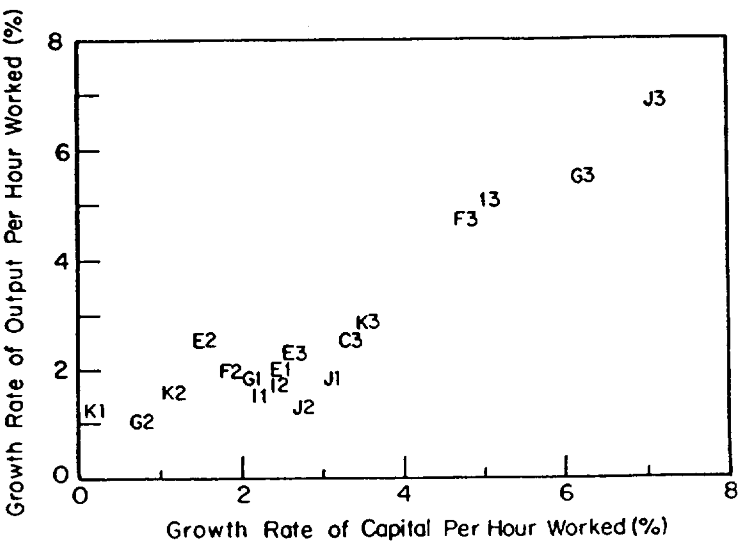

Figure 2.3 Output and capital per hour worked. Data from Maddison (1982).

Table 2.3 Investment, GDP growth, and the capital–output ratio

Country	Investment share (%)	GDP growth (%)	Capital–output ratio: Depreciation rate $\delta = 0.03$	$\delta = 0.04$
Japan	31	7.4	3.0	2.8
Germany	28	4.7	3.7	3.3
Canada	28	4.2	3.9	3.4
Italy	26	4.4	3.6	3.1
France	25	4.2	3.5	3.1
United States	24	3.0	3.9	3.4
United Kingdom	17	2.1	3.4	2.8

Source: Summers and Heston (1984).

and δ are constant, then the capital–output ratio will converge to the value

$$\frac{K}{Y} = \frac{i}{\delta + g},$$

with dynamics that behave like $e^{-(g+\delta)t}$. Table 2.3 reports average values for i and g using data from national income accounts for the seven countries in Figure 2.3 for the period 1950 to 1981. The data used here are from Summers and Heston (1984) and cover a slightly longer period than those from Maddison.[3] An estimate of the magnitude of δ can be derived as follows: Maddison (1987) reported that an average ratio of total depreciation D to income Y for a similar sample of industrialized countries is between 11% and 12%. Using this value for D/Y, one can solve for δ from

$$\delta = \frac{D}{Y}\frac{Y}{K} = \frac{D}{Y}\frac{g + \delta}{i}.$$

When one uses the values of g and i for the countries reported in Table 2.3, this calculation yields values of 3% or 4% for δ. The table reports estimates of the capital output ratio on the basis of this calculation. For these values of δ and for a value for g of 3% or 4%, the time for K/Y to converge halfway toward its steady-state value is about 10 years; for the 31-year interval covered by the table, this steady-state approximation may not be too misleading.

The interesting feature of the data in Table 2.3 is that the steady-state capital-output ratios are relatively similar. There is a relatively large amount of variation in the investment share i and the growth rate g, but there is no systematic variation in the estimated value of K/Y across countries. Note that this result is stronger than the finding from Figure 2.3 that K and Y increase in roughly equal proportions in a given country and time period. Equiproportionate increases in Y and K could arise in a world where output varies exogenously and the investment rate stays constant, but Table 2.3 shows that the investment share varies closely with the growth rate. The question suggested by these data is, Why do the share of investment and the rate of growth of output move together in such a way that the implied capital–output ratio shows little systematic variation?

These statements must be qualified to some extent because the findings are weaker if one significantly increases the sample of countries considered. The conventional wisdom is that the data for developing countries do not show a strong correlation between growth and the share of output devoted to investment. Figure 2.4 shows why. It uses data from Summers and Heston (1984) and is a plot of the average investment share and the average rate of growth of output for all 115 of the economies that they label (somewhat loosely) as market economies. For 50 of these countries—those denoted in the figure with an "x"—data are available only

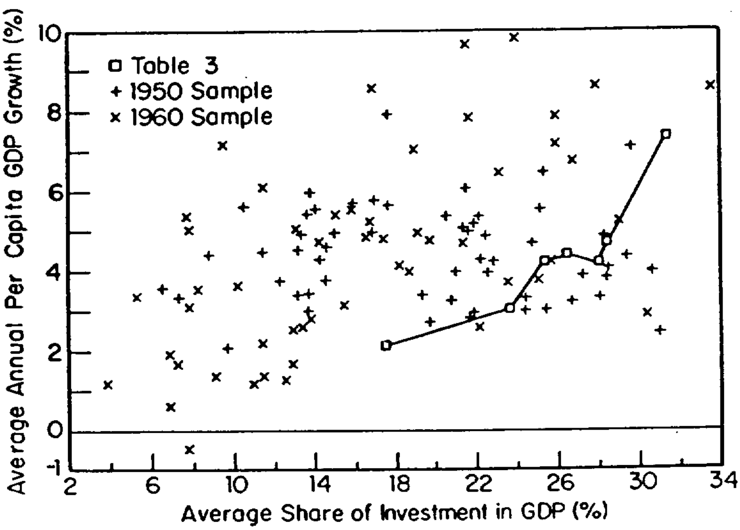

Figure 2.4 Investment share and GNP growth. Data from Summers and Heston (1984).

for the 20-year interval 1960 to 1981. For the other 65 countries, data cover the interval 1950 to 1981. Seven of these countries, the ones from Figure 2.3 and Table 2.3, are denoted with squares and are plotted from right to left in the order in which they appear in Table 2.3. For clarity they are connected by a line. The remaining 58 countries with data for the period 1950 to 1981 are denoted with plus signs. The growth rate is the average annual (continuously compounded) growth rate of gross domestic product, valued at international prices. The investment measure includes investment by both the private and government sectors.

Judging solely from the countries denoted with a plus sign, one finds no strong evidence of a positive association between investment and growth. With the addition of the countries denoted by an x, for which data are available only since 1960, a positive association is once again apparent, but it is not as tight as the relation observed among the seven developed countries connected by the line. Moreover, both the x countries and the + countries tend to lie systematically to the left of the locus for the developed countries. This pattern suggests either that the process of growth for the developing countries differs fundamentally from that for the developed countries, or that investment tends to be systematically underestimated in the less developed countries, or that the steady-state assumption used here is misleading. For example, countries with large recent additions to their capital stock may have less depreciation than the steady-state approximation would suggest. In this case, gross investment could be smaller, but net investment could be comparable to that in developed countries with similar growth rates. Overall, Kaldor's observation 4—the constancy of the capital–output ratio—can still be judged to be a useful target for theories of growth, but so also might the apparent departures from this tendency for low-income countries suggested by Figure 2.4.

Kaldor's fact 5—the assertion that the share of capital in total income has remained constant—has increasingly been disputed. Attempts to measure this share in a consistent fashion over time tend to show a fall in capital's share. Table 2.4 reports estimates from different country studies that are collected in Maddison (1987). There are a number of judgmental issues that must be resolved in deciding what constitutes income to capital, and different authors have taken different positions on how to handle them. Consequently, the estimates are not comparable across countries and authors. Looking only within countries, the trend is for the share of capital to decline from about 0.4 to 0.3. Of course, realistic standard

Table 2.4 Estimates of the share of capital in total income[a]

Country	Interval	Share of capital (%)	Reference
Japan	1913–1938	40	Ohkawa and Rosovsky (1973)
	1954–1964	31	
United Kingdom	1856–1873	41	Matthews, Feinstein, and Odling-Smee (1982)
	1873–1913	43	
	1913–1951	33	
	1951–1973	27	
United States	1899–1919	35	Kendrick (1961)
	1919–1953	25	
	1929–1953	29	Kendrick (1973)

a. Results collected in Maddison (1987).

errors for these estimates might be on the same order of magnitude as this decline. The kind of problem that adds to the uncertainty is a systematic and sizable reduction over time in the fraction of self-employed workers and sole proprietorships, for whom it is particularly difficult to distinguish returns to capital from returns to labor. Another source of uncertainty is the somewhat arbitrary methods for imputing income on capital like housing that is outside of the corporate sector. Given this uncertainty, Maddison argued that for some purposes it may not be too serious a distortion of the data to use identical shares for different countries and to assume that the weights are constant over time. Nonetheless, after acknowledging the uncertainty involved, one must give some credence to the assertion that capital's share is falling.

Kaldor's fact 6—that growth rates differ substantially across countries—and the added fact 7—that the growth rates do not vary systematically with the level of income—are both clearly evident from Figure 2.5. This figure is a plot of the data for all 115 market economies from Summers and Heston. The horizontal axis measures the ratio of per capita income in a country relative to that in the United States, with income in both countries measured in 1960. One of the major contributions made by Summers, Heston, and Kravis was to correct official exchange rates for departures from purchasing power parity so that this kind of comparison of levels is meaningful. The vertical axis measures the growth rate of per capita income for each country in the subsequent interval 1960 to 1981.

The main result here is that the growth rate shows no systematic varia-

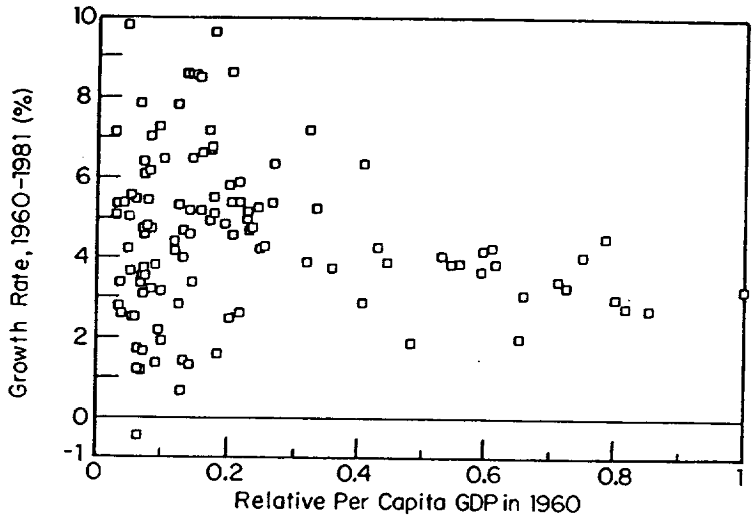

Figure 2.5 Growth vs. rank for per capita GDP, 1960 sample. Data from Summers and Heston (1984).

tion with the level of income. For countries with any initial level of income, the average growth rate is 3% to 4%. The variance does seem to vary systematically, falling rapidly with per capita income, but this may at least partially reflect the fact that the low-income countries are much more heavily sampled than the high-income countries. In a sample drawn from any distribution, the difference between the minimum and the maximum values will be monotonically increasing in the sample size.

It is perhaps worth emphasizing that the range of growth rates of over 10% is quite large. Over the span of a mere 21 years, the ratio of per capita income in the fastest growing country to that in the slowest growing country more than tripled. If even one-tenth of this variation in growth rates is due to forces that government policy can influence, the potential long-term gains from better policy are sizable.

The absence of any negative slope in this scatter plot is evidence against the assertion that low-income countries tend to grow more rapidly than high income countries and that convergence in per capita income is taking place.[4] One can of course select a set of countries where convergence has taken place. Figure 2.6 is based on data for the interval 1950 to 1981, the period when substantial convergence is typically alleged to have taken place. Overall, for this smaller sample of 65 countries with data

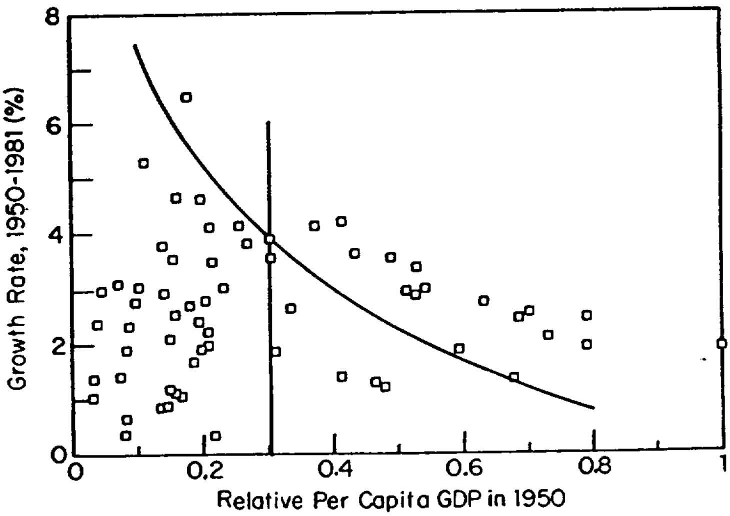

Figure 2.6 Growth vs. rank for per capita GDP, 1950 sample. Data from Summers and Heston (1984).

extending back to 1950, one finds the same pattern as for the larger sample in Figure 2.5—a triangular shape that is roughly symmetric about a horizontal line. This figure also has two lines that intersect at the point corresponding to Italy. These lines allow one to make income comparisons at the beginning and at the end of the period. The vertical line divides the sample into countries on the right, which had a higher per capita income than Italy in 1950, and those on the left, which had a lower per capita income. The downward sloping line divides the countries on the basis of income comparisons at the end of the period. Countries that lie above this line had per capita income in 1981 that was higher than that in Italy in 1981. It is downward sloping because there are two ways to end up richer than Italy. If a country starts out poorer, it must grow more rapidly. If it starts out richer, it can grow more slowly.

Although a selection criterion based on levels of per capita income at the end of the period seems suspect for purposes of testing for a downward slope, it is implicitly what one uses when one specifies a sample of countries that are now thought of as being industrialized. The list of countries that lie above the downward sloping line corresponds almost exactly with the list of countries that Maddison (1982) studied and that Baumol (1986) subsequently used in his analysis of convergence. The only difference is that New Zealand, Luxembourg, and Iceland have levels of income as high as those of Italy in 1981 but are omitted from Maddison's study, presumably because they are so small. If one had picked developed countries in 1950, Japan would not have made the list and Argentina would have.

Ex post, it is always possible to tell stories about why Japan should have been included and why Argentina should not have been included, but this seems like a risky methodology. Judging from Figures 2.5 and 2.6, one finds no obvious reason for treating some countries as being so different from the others that they must be excluded from the analysis of questions like convergence. Even if one did conclude that some truncation of the sample is called for—for example, because of concern about data reliability—the way to truncate the sample without biasing the inferences is to use the initial level of income rather than the terminal level. Regardless of the initial level of income chosen (that is, regardless of where one chooses to draw a vertical line), the remaining points will not have an obvious negative slope.[5]

Fact 8—the correlation between growth and trade for developed countries—is summarized by the three panels of Figure 2.7. Over time and across countries, income growth and trade growth are positively correlated, with trade growth varying more than income growth. The data are drawn from Maddison (1982). Each panel represents a different time period. The variation across countries in a particular period suggests the kind of concern that is voiced in current trade disputes, that somehow increases in trade by some countries may increase their rate of growth at the expense of growth in other countries. In contrast, the variation over time suggests that in terms of growth rates, trade may not be a zero sum game. The rate of growth in all countries may be positively related to the rate of growth of world trade.

Fact 9 refers to a negative correlation between per capita income and population growth. Data from Summers and Heston for the years 1960 to 1981 is presented in Figure 2.8, which shows a scatter plot of this relationship. A better test for the influence of per capita income on individual decisions would look at fertility rates, which correct for the age structure of the population and subtract out the effects of mortality and migration, but the gross correlation shown in the figure will almost surely survive any such refinement.

This cross-sectional variation has a time series counterpart that is re-

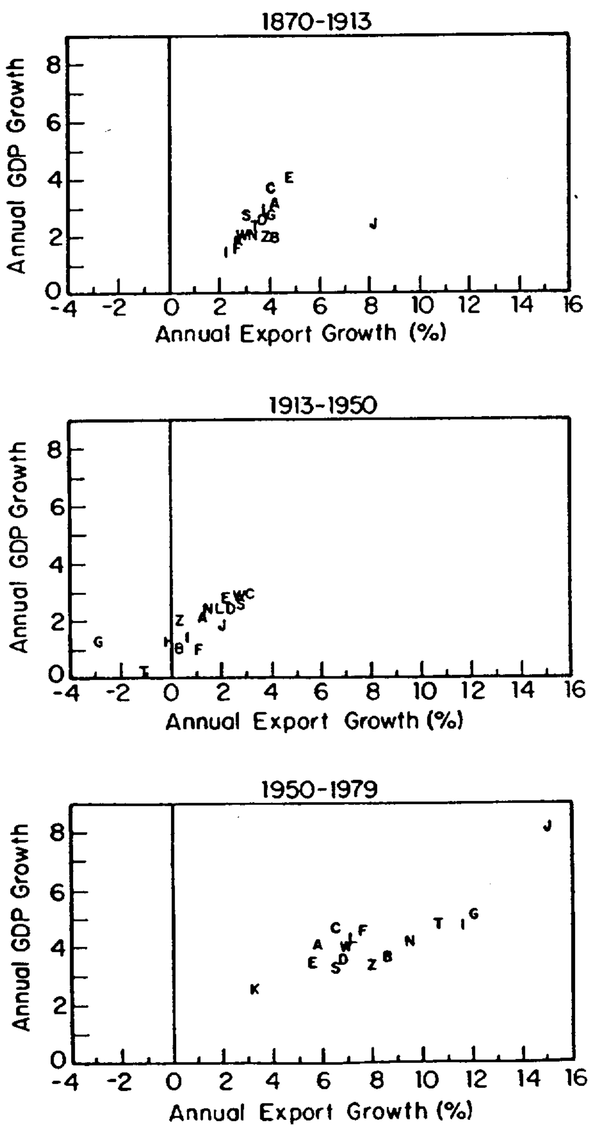

Figure 2.7 Growth of GDP and exports. Data from Maddison (1982).

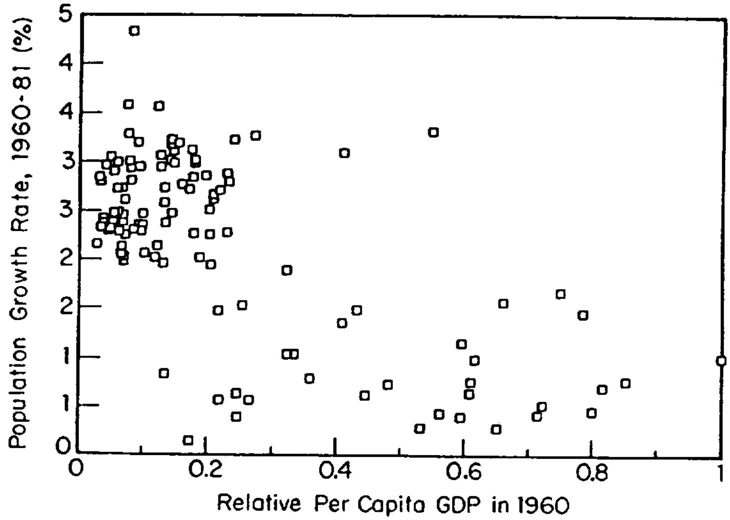

Figure 2.8 Population growth vs. per capita GDP. Data from Summers and Heston (1984).

ferred to as the demographic transition. All developed countries have gone through a transition from high fertility and mortality rates to low rates. This transition can be interpreted either as the response of fertility to an exogenous change in mortality rates, or as the common response of both mortality and fertility to increases in income, or both. The cross-sectional variation in population growth rates shown here is sometimes interpreted in these terms. The suggestion is that it reflects an exogenous fall in mortality that took place too recently and too rapidly for fertility to have yet responded, or that it is an example of a fall in mortality that has not been accompanied by an increase in income.

Fact 10 is an assertion about the growth accounting literature, which is far too vast to summarize here. A useful overview of the work of the three key participants, Edward Dennison, Dale Jorgenson, and John Kendrick, is given in Norsworthy (1984). A recent, particularly transparent application of the methodology is given in Maddison (1987). The basic reason for the persistent finding of a residual in growth accounting can be seen from Figure 2.3. Let $Y = F(K,L)$ denote the output from a constant returns to scale production function of aggregate capital and aggregate hours worked. Let $y = Y/L$ and $k = K/L$ denote output per hour worked and capital per hour worked. Then differentiating with respect to time and using the assumption of competitive markets so that $r = f'(k)$ gives

$$\frac{\dot{y}}{y} = \frac{rK}{Y}\frac{\dot{k}}{k}.$$

For values of capital's share rK/Y on the order of 0.3 or even 0.4, there is no way to match the data in Figure 2.3. Fitting a regression line of $\dot{y}/y$ on $\dot{k}/k$ to this data gives a coefficient on $\dot{k}/k$ that is very close to one. For a country like Japan with capital and output growth rates of over 7%, a share parameter of even 0.4 will imply unexplained growth of over 4% per year for nearly 30 years.

This analysis also casts doubt on simple arguments that capital deepening in the neoclassical model explains why low-capital countries like Japan and Germany grew faster and caught up with the leaders. This explanation is typically identified as one of the great successes of the model, but the numbers do not fit the story. If the growth of capital per worker was 4 percentage points higher in Germany than in the United States, the model would predict a growth rate that is higher by 0.3 or 0.4 times 4 percentage points, or less than 2 percentage points. In fact, the growth rate is higher by the same 4 percentage points. The difference between these two numbers is of course the same residual identified above for Japan. One can argue about what accounts for this difference. A die-hard neoclassicist could claim that the exogenous rate of technological change was higher in countries like Germany and Japan, but this position reduces the neoclassical model from a theory to a description of the data. Countries that grow fast are countries with fast exogenous growth in the technology. Whatever it is, something other than neoclassical physical capital accumulation was taking place.

One possible other factor is accumulation of skills and education, that is, of human capital as well as physical capital. To the extent that this kind of accumulation takes place, the correct measure of labor input is not man-hours, but man-hours adjusted for quality change due to better education or on-the-job experience. One can use the cross-sectional variation in wages, education, and experience in a country together with time series estimates of average experience and education in the labor force to construct an estimate of growth in quality-adjusted labor input. There is considerable latitude in how one goes about the details of this construction, with a corresponding variation in the resulting estimate of the unexplained residual. The consensus view seems to be that in long-run data for the United States, there is still a sizable component of growth, on the order of 1% or more, that is not explained by growth in capital or quality-adjusted labor input. Unless fast-growing countries in Figure 2.4 like Japan, Germany, France, and Italy had substantially more rapid growth in the level of education and experience than that observed in the United States, they will have even larger residuals.

Evidence on fact 11 concerning migration flows is heavily influenced by the constraints imposed on these flows. Historical evidence suggests that the unconstrained flows into industrialized countries could be quite large. Greenwood and McDowell (1986) report that during the late 1960s and early 1970s, quotas on immigration into the United States favored migration of skilled workers and professionals. Attention then turned most naturally to consideration of the brain drain. More recently, a policy shift in favor of applicants with refugee status or relatives in the United States combined with legislative debates about illegal immigration have focused attention on unskilled migrants. Potential flows from either source are apparently large.

2.2. The Kuhn-Tucker Theorem and Dynamic Equilibrium Theory

Growth is a general equilibrium process. All markets and all participants in an economy influence growth and are influenced by it. A growth theorist must therefore construct a dynamic general equilibrium model, starting with a specification of preferences and the technology and specifying an equilibrium concept. To be able to say anything about the properties of the model beyond an assurance that some equilibrium exists, the theorist must be able to explicitly solve the model or at least give a qualitative description of the solution.

Either explicitly or implicitly, the central tool used in the characterization of dynamic competitive equilibrium models is the Kuhn-Tucker theorem. It offers a general procedure for reducing the problem of calculating a competitive equilibrium to the problem of solving a maximization problem. All of the theory of growth can be understood in terms of the application of this theorem to models with tractable functional forms for preferences and the technology. To develop these claims, it is best to start with a simple Irving Fisher-type economy that has a finite number of choice variables and is assumed to have perfect markets. Next, this procedure for studying perfect markets equilibria is extended to the kind of infinite