Homework 1

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ECON 7010- Microeconomics II

Due on Jan 26, 2022

1 Question 1 – 1.C.1

Solution:

Since $(\mathscr{B}, C(\cdot))$ satisfies WARP, and $C(\{x,y\}) = \{x\}$, which means $x \succ y$. In the meanwhile, $\{x,y\} \cap \{x,y,z\} = \{x,y\}$. We discuss by four scenarios:

1) If $C(\{x,y,z\}) = \{z\}$, this means $z \succ x$ and $z \succ y$ It doesn't violate WARP of $(\mathscr{B}, C(\cdot))$.

2) If $C(\{x,y,z\}) = \{x\}$, this means $x \succ y$ and $x \succ z$. By WARP, $\{x\}$ should be chosen in $C(\{x,y\})$,it is!

3) If $C(\{x,y,z\}) = \{y\}$, this means $y \succ x$ and $y \succ z$, by WARP, $\{y\}$ should be chosen in $C(\{x,y\})$, but it's not. So it's a contradiction.

4) If $C(\{x,y,z\}) = \{x,z\}$, this means $\{x,z\} \succ y$, by WARP, x should be in $C(\{x,y\})$, it is!

For other two scenarios, they are related to y, but y should not be chosen as it contradicts to the statement that our choice structure is WARP.So we eliminate other two scenarios. Therefore, the only possible situations are $C(\{x,y,z\}) = \{x\}, = \{z\}, or = \{x,z\}$ when the choice structure satisfies WARP.☺

2 Question 2– 1.C.2

Proof:

1): Prove ⇒:

Since WARP is satisfied, and from the condition, $B \cap B' = \{x,y\}$, $x \in C(B)$, by WARP, $x \in C(B')$. As $y \in C(B')$ in condition, by WARP, $y \in C(B)$. Thus, we can make a conclusion $\{x,y\} \subset C(B)$ and $\{x,y\} \subset C(B')$.

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2): Prove $\Leftarrow$:

When given listed conditions, as $\{x,y\} \cap \{x,y,z\} = \{x,y\}$, and $\{x,y\} \subset C(B)$ and $\{x,y\} \subset C(B')$, we can go back and get when $x \in C(B)$ and $x \in C(B')$ as we defined in Definition 1.C.1 in textbook. Thus, WARP is statisfied for the choice structure.

Combining there two direction proof, we can conclude WARP is equivalent to the property listed in the quesiton.☺

3 Question 3-1.C.3

Solution:

a). The answer is yes. If there are less than two elements in the choice sets, like the example in the question, it has transitive property. Because $x \in C(B)$ and $y \in C(B)$, it means $x \succ^* y$.

b). Proof:

Suppose $\{x,y,z\} \subset C(B)$, then $C(B) \neq \varnothing$. If $x \in C(\{x,z\})$, then $x \succ^* z$, if $y \in C(\{z,y\})$, then $y \succ^* z$, by the rationality, only x is chosen when the choice set is $\{x,y,z\}$, thus, $x \succ^* y \succ^* z$, and transitivity $x \succ^* y$ holds in this case.☺

4 Question 4– 1.D.3

Proof:

As in the question, $X = \{x,y,z\} \subset (\mathscr{B}, C(\cdot))$. We make our proof by contradiction, if $x \in C(X)$, since $\{x,z\} \subset X$, $x \in C(\{x,z\})$, but from the information we get, $C(\{x,z\}) = \{z\} \neq \{x\}$, it's a contradiction. Thus, the choice structure $(\mathscr{B}, C(\cdot))$ violates weak axiom. ☺