Homework Solution-Selected Questions

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ECON 6700-Math 2

2021 Fall, no due date

1 1.A Eigenvalues and Eigenvectors

No need to do any computation, so easy.

2 1.B Deterministic Difference Equations

For the following difference equations,(i) find the statinary state value of y_t, which you should denote by y_s, (ii) rewrite the difference equation in terms of $z_t = (y_t - y_s)$, (iii) give the general solution and, using the given initial condition, the definite solution to the FODE, (iv) evaluate whether the DE converges or diverges and whether it's oscillatory or non-oscillatory.

1. For $y_{t+1} + 3y_t = 4$, $y_0 = 4$
   (a) For stationary state value of y_t: $y_s + 3y_s = 4 \longrightarrow y_s = 1$
   (b) $y_{t+1} = -3y_t + 3 + 1$, which means $y_{t+1} = -3(y_t - 1) + 1$. Thus, $y_{t+1} - 1 = -3(y_t - 1)$. It's easy to get $z_{t+1} = y_{t+1} - 1$, $z_t = y_t - 1$ and $z_{t+1} = -3z_t$.
   (c) Since $z_{t+1} = -3z_t$, thus the general solution would be$z_t = (-3)^t z_0$, because $z_0 = y_0 - 1 = 4 - 1 = 3$. The definite solutioni is $z_t = (-3)^t \cdot 3$
   (d) To determine Whether DE converges or diverges, we need to justify the coefficient befor z_0. In this case, $|-3| > 1$ and $-3 < 0$, thus, it diverges and osillates as well.
2. $y_{t+1} = 0.2y_t + 4$, $y_0 = 4$
   (a) $y_s = 0.2y_s + 4 \longrightarrow y_s = 5$
   (b) $y_{t+1} = 0.2(y_t - m) + 0.2m + 4$. If $z_t = y_t - y_s$ exists, $0.2m + 4 = m \longrightarrow m = 5$ Thus, $y_{t+1} - 5 = 0.2(y_t - 5) \longrightarrow z_{t+1} = 0.2z_t \longrightarrow z_t = 0.2^t z_0$, and $z_0 = y_0 - 5 = 4 - 5 = -1$
   (c) The general solution is $z_{t+1} = 0.2^t z_t$, and definite solution is $z_t = -0.2^t$

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(d) Since $0.2 < 1$ it will diverges and is non-oscillatory.

For each of difference equations below (i) find the stationary-state value of y_t, which you should denote by y_s, (ii) rewrite the difference equation in terms of $z_t = \begin{bmatrix} y_t - y_s \\ y_{t-1} - y_s \end{bmatrix}$, (iii) give the general solution and, using the given initial conditions, the definite solution to SODE.

1. $y_{t+1} + 3y_t - \frac{7}{4}y_{t-1} = 9$. $y_0 = 3, y_{-1} = 1$

   (a) $y_s + 3y_s - \frac{7}{4}y_s = 9 \longrightarrow y_s = 4$