Exercises #6

Instructions

Exercises #6 are due on Wednesday, March 2$^{\text{nd}}$.

Exercises may be presented for credit as a hard copy at the end of the class meeting on the due date, or may be submitted electronically on Blackboard by the following Monday. If submitted on Blackboard, exercises should be attached as a Portable Document Format (`*.pdf`) file. It is possible to convert handwritten work to `*.pdf` using scanner or a camera-equipped device with Microsoft Office Lens (Android, iOS, or Windows), Google Drive (Android), or Apple Notes (iOS).

Exercises are "collaborative and open book" assignments. You are encouraged to make use of help from your peers, textbook, notes, and me, but you must submit your own answers. There is no penalty for incorrect answers; the expectation is simply for you to progress as far as you can on each question. Complete answers with explanations will be provided in recitation.

Questions

6.B.2 Show that if the preference relation $\succsim$ on $\mathcal{L}$ is represented by a utility function $U(\cdot)$ that has the expected utility form, then $\succsim$ satisfies the independence axiom.

6.C.4 Suppose that there are N risky assets whose returns z_n $(n = 1, \ldots, N)$ per dollar invested are jointly distributed according to the function distribution function $F(z_1, \ldots, z_N)$. Assume also that all the returns are nonnegative with probability 1. Consider an individual who has a continuous, increasing, and concave Bernoulli utility function $u(\cdot)$ over $\mathbb{R}_+$. Define the utility function $U(\cdot)$ of this investor over $\mathbb{R}_+^N$, the set of all nonnegative portfolios, by

$$U(\alpha_1, \ldots, \alpha_N) = \int u(\alpha_1 z_1 + \cdots + \alpha_N z_N) dF(z_1, \ldots, z_N).$$

Prove that $U(\cdot)$ is (a) increasing and (b) concave.

6.C.15 Assume that, in a world with uncertainty, there are two assets. The first is a riskless asset that pays 1 dollar. The second pays amounts a and b with probabilities π and $1 - \pi$, respectively. Denote demand for the two assets by (x_1, x_2).

Suppose that a decision-makers preferences satisfy the axioms of expected utility theory and that she is a risk averter. The decision maker's wealth is 1, and so are the prices of the assets. Therefore, the decision maker's budget constraint is given by

$$x_1 + x_2 = 1, \quad x_1, x_2 \in [0, 1].$$

(a) Give a simple *necessary* condition (involving a and b only) for the demand for the riskless asset to be strictly positive.

(b) Give a simple *necessary* condition (involving a, b, and π only) for the demand for the risky asset to be strictly positive.

6.D.1 The purpose of this exercise is to prove prove Proposition 6.D.1 in a two-dimensional probability simplex.

> The distribution $F(\cdot)$ first-order stochastically dominates the distribution $G(\cdot)$ if and only if, for every nondecreasing function $F(x) \leq G(x)$ for every x.

Suppose there are three outcomes: 1 dollar, 2 dollars, and 3 dollars. Consider the two-dimensional probability simplex representation of the lottery.

(a) For a given lottery L over these outcomes, determine the region of the probability simplex in which lie the lotteries whose distributions first-order stochastically dominate the distribution of L.

(b) Given the lottery L, determine the region of the probability simplex in which lie the lotteries L' such that $F(x) \leq G(x)$ for every x, where $F(\cdot)$ is the distribution of L' and $G(\cdot)$ is the distribution of L. [Notice that we get the same region as in (a).]

6.D.2 Prove that if $F(\cdot)$ first-order stochastically dominates $G(\cdot)$, then the mean of x under $F(\cdot)$, $\int x dF(x)$, exceeds that under $G(\cdot)$, $\int x dG(x)$. Also provide an example where $\int x dF(x) > \int x dG(x)$, but $F(\cdot)$ does not first-order stochastically dominate $G(\cdot)$.