ECON 7020
Philip Shaw
Final Exam

Problem 1

Take model that is populated with an infinite number of agents with total mass equal to one. Agents differ only with respect to their employment status (ϵ) and their asset holdings (a). Households maximize their intertemporal utility: $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_t)$. At the beginning of each period agents observe their initial asset holdings and employment status $\epsilon_0 \in \{u, e\}$ where $\epsilon = e$ if an agent is employed and $\epsilon = u$ if the agent is unemployed. If an agent is unemployed at time t they receive unemployment compensation b_t and $n_t = 0$. Taxes are levied on wage and capital income to pay for unemployment compensation.[1] The government balances its period-by-period budget constraint. We also impose a borrowing constraint on agents such that $a \in [-2, \infty)$. If an agent is employed, they choose n_t that satifies the inequality $0 \le n_t < 1$. Production is given by: $Y_t = z_t K_t^{\alpha} N_t^{1-\alpha}$ with aggregate shocks to production z_t. Finally assume the conditional transition matrix for employment status is given by:

$$\pi(\epsilon'|\epsilon) = \begin{bmatrix} p_{uu} & p_{ue} \\ p_{eu} & p_{ee} \end{bmatrix} = \begin{bmatrix} .50 & .50 \\ .0435 & .9565 \end{bmatrix} \tag{1}$$

a. How does Aiyagari (1994) motivate the use of heterogenous agent models as an alternative to representative agent models?

b. What is the budget constraint for an unemployed agent? Employed agent? What percentage of agents are unemployed in equilibrium?

c. Formulate the Bellman equation, calculate the first-order conditions, and *derive* the Euler equation for consumption.

d. Define a stationary competitive equilibrium making the necessary assumptions.

[1] After tax wages are given by $(1-\tau)w_t$ and the gross after tax interest rate is given by $(1 + (1-\tau)r_t)$

e. Graph a hypothetical policy function for a' against a. Show how the optimal policy depends on ϵ. In what way does this depend on the equilibrium wage?

f. Describe how Krusell and Smith (1998) would reformulate the problem to be solved with aggregate uncertainty.

Problem 2

Take a two-period OLG model with production. The economy is populated with two types of agents: managers (m) and workers (ω). Managers are endowed with the ability level $a_m > 1$ and workers are endowed with ability $a_\omega = 1$. The distribution of each type is fully determined by a Markov chain $(a_h, P_h, \pi_{h,0})$ with transition matrix:

$$P_h = \begin{bmatrix} p_{mm} & p_{m\omega} \\ p_{\omega m} & p_{\omega\omega} \end{bmatrix} \tag{2}$$

Agents maximize their discounted stream of utility over the two periods of their life. The budget constraint of the young is given by:

$$\begin{aligned} c_t^h(t) &= I(h=m)(r^k(t)k(t) + w(t)a_m\Delta_t^m(t) - k^m(t+1) - l^m(t)) \\ &+ (1 - I(h=m))(r^k(t)k(t) + w(t)\Delta_t^\omega(t) - k^\omega(t+1) - l^\omega(t)) \end{aligned}$$

and the budget constraint of the old is given by:

$$\begin{aligned} c_t^h(t+1) &= I(h=m)(w(t+1)a_m\Delta_t^m(t+1) + r^k(t+1)k^m(t+1) + r^l(t)l^m(t)) \\ &+ (1 - I(h=m))(w(t+1)\Delta_t^\omega(t+1) + r^k(t+1)k^\omega(t+1) + r^l(t)l^\omega(t)) \end{aligned}$$

where $I(h=m)$ is an indicator function that takes the value one if the agent is a manager and zero otherwise. Assume that $\Delta_t^m(t) = \Delta_t^\omega(t)$ for all t but effective labor supply is $a_m\Delta_t^m(t)$ for managers and $\Delta_t^\omega(t)$ for laborers. Instantaneous utility is given by $u(c_t^h(t))$. Also assume that factor markets are perfectly competitive and the production function is given by $Y(t) = \gamma(t)L(t)^{1-\alpha}K(t)^\alpha$. In addition, let $N(t) = \pi_{m,t} + \pi_{\omega,t}$ and $N(t-1) = \pi_{m,t-1} + \pi_{\omega,t-1}$.

a. Formulate the Bellman equation for each type of agent. What are the relevant state variables? Explain why the Bellman equation converges in one iteration.

b. Define a perfect foresight competitive equilibrium.

c. Now assume that $\gamma(t) = 1$ for all t and that $p_{ij} = .5$ for $i \in \{m, \omega\}$ and $j \in \{m, \omega\}$. Using the utility function $c_t^h(t)[c_t^h(t+1)]$ solve for the individual savings functions for both types of agents in the economy.

d. Calculate aggregate savings across all managers and workers.

e. Using the expression for total aggregate savings, solve for the law of motion for aggregate capital for the time t generation $K_t(t+1)$. What is the expression for $K(t+1)$?

f. Under what conditions does aggregate capital converge to a steady-state value K^*?