Exercises #9

Instructions

Exercises #9 are due on Wednesday, April 13th.

Exercises may be presented for credit as a hard copy at the end of the class meeting on the due date, or may be submitted electronically on Blackboard by the following Monday. If submitted on Blackboard, exercises should be attached as a Portable Document Format (`*.pdf`) file. It is possible to convert handwritten work to `*.pdf` using scanner or a camera-equipped device with Microsoft Office Lens (Android, iOS, or Windows), Google Drive (Android), or Apple Notes (iOS).

Exercises are "collaborative and open book" assignments. You are encouraged to make use of help from your peers, textbook, notes, and me, but you must submit your own answers. There is no penalty for incorrect answers; the expectation is simply for you to progress as far as you can on each question. Complete answers with explanations will be provided in recitation.

Questions

11.B.5 Suppose that at fixed input prices of $\overline{w}$ a firm produces output with differentiable and strictly convex cost function $c(q, h)$, where $q \geq 0$ is its output level (whose price is $p > 0$) and h is the level of a negative externality generated by the firm. The externality affects a single consumer, whose derived utility function takes the form $\phi(h) + w$. The actions of the firm and consumer do not affect any market prices.

(a) Derive the first-order condition for the firm's choice of q and h.

(b) Derive the first-order conditions characterizing the Pareto optimal levels of q and h. (Since the consumer's utility function is quasilinear in wealth, the utility possibilities frontier is a linear line with slope -1; therefore the Pareto optimal level of q and h can be found by the sum of the profit function and utility function omitting wealth.)

(c) Suppose that the government taxes the firm's output level. Show that this cannot restore efficiency, unlike a direct tax on the externality.

(d) Show, however, that in the limiting case where h is necessarily produced in fixed proportions with q, so that $h(q) = \alpha q$ for some $\alpha > 0$, a tax on the firm's output *can* restore efficiency. What is the efficiency-restoring tax?

11.D.1 First-year graduate students are a hard-working group. Consider a typical class of I students. Suppose that each student i puts in h_i hours of work on his or her classes. This effort involves a disutility of $h_i^2/2$. His or her benefits depend upon how well he or she performs relative to his or her peers and take the form $\phi(h_i/\tilde{h})\ \forall i$, where $\tilde{h} = (1/I)\sum_i h_i$ is the average number of hours put in by all students in the class and $\phi(\cdot)$ is a differentiable concave function, with $\phi'(\cdot) > 0$ and $\lim_{h\to 0}\phi'(h) = \infty$. Characterize the symmetric (Nash) equilibrium. Compare it with the Pareto optimal symmetric outcome. Interpret.