

Exercises #10

Instructions

Exercises #10 are due on Wednesday, April 20th.

Exercises may be presented for credit as a hard copy at the end of the class meeting on the due date, or may be submitted electronically on Blackboard by the following Monday. If submitted on Blackboard, exercises should be attached as a Portable Document Format (*.pdf) file. It is possible to convert handwritten work to *.pdf using scanner or a camera-equipped device with Microsoft Office Lens (Android, iOS, or Windows), Google Drive (Android), or Apple Notes (iOS).

Exercises are “collaborative and open book” assignments. You are encouraged to make use of help from your peers, textbook, notes, and me, but you must submit your own answers. There is no penalty for incorrect answers; the expectation is simply for you to progress as far as you can on each question. Complete answers with explanations will be provided in recitation.

Questions

- 12.B.1 Suppose that the inverse demand function in a monopolist market is $p(q) = a - bq$ and the monopolist's cost function is $c(q) = cq$, where $a > c \geq 0$ (so that $p(0) > c'(0)$) and $b > 0$. Calculate the monopolist's optimal quantity and price and contrast this to the socially optimal (competitive) output level and price.
- 12.B.7 Consider the widget market. The total demand by men for widgets is given by $x_m(p) = a - \theta_m p$ and the total demand by women is $x_w(p) = a - \theta_w p$, where $\theta_w < \theta_m$. The cost of production is c per widget.
- (a) Suppose the widget market is competitive. Find the equilibrium price and quantity sold.
 - (b) Suppose, instead, that firm A is a monopolist of widgets. (Also make this assumption in c and d.) If firm is prohibited from charging different prices to men and women, what is the profit-maximizing price? Under what conditions do both men and women consume a positive level of widgets in this solution?
 - (c) If firm A has produced some level of output X , what is the welfare-maximizing way to distribute it between the men and women?
 - (d) Suppose that firm A is allowed to charge men and women different prices. What prices does it charge? In the case where the nondiscriminatory solution in b has positive consumption of widgets by both men and women, does aggregate welfare as measured by Marshallian aggregate surplus rise or fall relative to when discrimination is allowed? What if the nondiscriminatory solution in b has only one type of consumers being served?
- 12.C.1 Show that in any Nash equilibrium of the Bertrand model with $J > 2$ firms, all sales take place at a price equal to cost.
- 12.C.6 Consider a Cournot duopoly in which firms have a cost per unit produced of c and the inverse demand function is $p(q) = a - bq$, with $a > c \geq 0$ and $b > 0$. Find firm j 's best response function and find the output pair (q_1^*, q_2^*) that supports a Nash equilibrium. Verify that the Nash equilibrium price is between the monopolist's and socially optimal (competitive) price calculated in 12.B.1.
- 12.C.7 Derive the Nash equilibrium price and quantity levels in the Cournot model with J firms where each firm has a constant unit production cost of c and the inverse demand function in the market is $p(q) = a - bq$,

with $a > c \geq 0$ and $b > 0$. Verify that when $J = 1$, we get the monopoly outcome; that output rises and price falls as j increases; and that as $J \rightarrow \infty$ and price and aggregate output in the market approach their competitive levels.

- 12.C.15 Derive the Nash equilibrium prices of the linear city model where a consumer's travel cost is quadratic in distance, that is, where the total cost of purchasing from firm j is $p_j + td^2$, where d is the consumers' distance from firm j . Restrict attention to the case where v is large enough that the possibility of nonpurchase can be ignored.
- 12.C.18 There are two firms in a market. Firm 1 is the "leader" and picks its quantity first. Firm 2, the "follower", observes firm 1's choice and then chooses its quantity. Profits for firm j given quantity choices q_1 and q_2 are $p(q_1 + q_2)q_j - cq_j$, where $p'(q) < 0$ and $p'(q) + p''(q)q < 0$ at all $q \geq 0$.
- (a) Prove formally that firm 1's quantity choice is larger than its quantity choice would be if the firms chose quantities simultaneously and that its profits are larger as well. Also show that aggregate output is larger and that firm 2's profits are smaller.
 - (b) Draw a picture of this outcome using best-response functions and isoprofit contours.