Chapter 1

Problem: 1.C.1

Consider a choice structure $(\beta, c(\cdot))$ with $\beta = (\{x, y\}, \{x, y, z\})$ and $C(\{x, y\}) = \{x\}$. Show that if $(\beta, c(\cdot))$ satisfies WARP, then we must have $C(\{x, y, z\}) = \{x\}, = \{z\}, or = \{x, z\}$.

Answer

To show any of the three possible answers, we need to show that $y \notin C(\{x, y, z\})$. If $y \in C(\{x, y, z\})$, then by WARP $y \in C(\{x, y\})$ but this contradicts the given equality $C(\{x, y\}) = \{x\}$. Therefore, $y \notin C(\{x, y, z\})$. Since $y \notin C(\{x, y, z\})$, and we do not know anything about the relationship between $x\ and\ z$, therefore $C(\{x, y, z\}) = \{x\}, = \{z\}, or = \{x, z\}$ are all possible.

Problem: 1.C.2

Show that WARP Definition 1.C.1 <<<If for some B ∈ β with $x, y \in B$, we have x ∈ C(B), then for any B' ∈ β with $x, y \in B'$ and y ∈ C(B'), we must also have x ∈ C(B')>>> is equivalent to the following property holding:

- Suppose that $B, B' \in \beta$, that $x, y \in B$, and that $x, y \in B'$. Then if $x \in C(B)$ and $y \in C(B')$, we must have $\{x, y\} \subset C(B)$ and $\{x, y\} \subset C(B')$.

Answer

You can answer this in two ways. First, I will show that the above statement is correct assuming WARP by Definition 1.C.1. Suppose that WARP is satisfied as stated in definition. Reminder that we are assuming that $B, B' \in \beta$ and $x, y \in B, B'$, $x \in C(B)$ and $y \in C(B')$. Since $x, y \in B$ and $x, y \in B'$ and $x \in C(B)$, if $y \in C(B')$, then $y \in C(B)$. And if $x \in C(B)$ and $y \in C(B')$, then $y \in C(B)$. Therefore, we have both $x \; and \; y \in C(B)$ and $x \; and \; y \in C(B')$ which is equivalent to $\{x, y\}x \subset C(B)$ and $\{x, y\}x \subset C(B')$.

On the other hand, we could have showed that the stated property holds and then prove Definition of WARP. So suppose that the stated property holds. Let $B, B' \in \beta$, $x, y \in B, B'$, $x \in C(B)$ and $y \in C(B')$. This implies that if $x \in C(B)$ and $y \in C(B')$, then we must have $\{x, y\}x \subset C(B)$ and $\{x, y\}x \subset C(B')$ according to the statement. It means that when both are available, and one is one chosen bundle, it must be in the other bundle as well (which works for both goods). Therefore, this implies WARP.

Problem: 1.C.3 (b, c)

Suppose that a choice structure $(\beta, c(\cdot))$ satisfies WARP. Consider 2 possible revealed preferred relations $\succ^* \ and \ \succ^{**}$.

- $x \succ^* y \Leftrightarrow$ there is some $B \in \beta$ such that $x, y \in B$, $x \in C(B)$ and $y \notin C(B)$.
- $x \succ^{**} y \Leftrightarrow x \succsim^* y$ but not $y \succsim^* x$.

where $\succsim^*$ is the revealed at least as good as relation as in Definition 1.C.2.

b) must $x \succ^* y$ be transitive?
c) Show that if β includes all 3 element subsets of X, then $\succ^*$ is transitive.

Answer

To part b. NO to part b. It is not required to be transitive. Define $X = \{x, y, z\}, \beta = (\{x, y\}\{y, z\})$ and $C(\{x, y\}) = \{x\}$ and $C(\{y, z\}) = \{y\}$. We see that $x \succ^* y$ and $y \succ^* z$. However, we don't have $x \succsim^* z$ because neither sets in β includes $\{x, z\}$ and hence we do not have $x \succ^* z$ either.

To part c. Let $x, y, z \in X, x \succ^* y$, and $y \succ^* z$. We have $\{x, y, z\} \in \beta$.
Suppose $B \in \beta$ such that $x, y \in B$ and $x \in C(B)$ and $y \notin C(B)$. Thus, $x \succ^* y$.
Now suppose, that $y \succsim^* x$, then there exists $B \in \beta$ such that $x \in B$, $y \in B$ and $x \in C(B)$. But then WARP implies that $y \in C(B)$ also but since it was already stated that $y \notin C(B)$ because of $x \succ^* y$. Contradiction. If we have $x \in C(B)$ and $y \notin C(B)$, thus $x \succ^{**} y$.

By same logic we get $y \succ^{**} z$. Hence we have neither $y \succ^* x$, nor $z \succ^* y$. Since $\succ^*$ rationalizes $(\beta, c(\cdot))$, this implies that $y \notin C(\{x, y, z\})$ and $z \notin C(\{x, y, z\})$. Since $C(\{x, y, z\}) \neq \emptyset$, $C(\{x, y, z\}) = \{x\}$.

Thus, $x \succ^* z$.

Problem: 1.D.3

Let $X = \{x, y, z\}$ and consider a choice structure $\left(\beta, c(\cdot)\right)$ with $\beta = (\{x, y\}, \{y, z\}, \{x, z\}, \{x, y, z\})$ and $C(\{x, y\}) = \{x\}$, $C(\{y, z\}) = \{y\}$, $C(\{x, z\}) = \{z\}$. Show that it violates WARP.

Answer

To show that it violates WARP, I will use contradiction. Let's suppose WARP holds in this scenario and see what happens.

$$\text{If } x \in C(\{x, y, z\}), \text{then it contradicts } C(\{x, z\}) = \{z\}$$

$$\text{If } y \in C(\{x, y, z\}), \text{then it contradicts } C(\{x, y\}) = \{x\}$$

$$\text{If } z \in C(\{x, y, z\}), \text{then it contradicts } C(\{y, z\}) = \{y\}$$

Therefore, the choice structure $\left(\beta, c(\cdot)\right)$ violates WARP.