ECON 7920
Econometrics II
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Problem Set 2
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Problem 1

Consider the population model that relates the price of a house sold (*price*) to the number of rooms in a house (*rooms*) and the number of bathrooms (*baths*). Assume the functional form for the conditional mean function is given by: $m(x, \theta_0) = exp(\theta_{01} + \theta_{02} rooms + \theta_{03} baths)$.

a. Under what conditions will the function $m(x, \theta_0)$ be identified for $\theta \in \Theta$?

Answer: Under assumptions NLS.1 and NLS.2 $m(x, \theta_0)$ is identified. Assumption NLS.1: For some $\theta_0 \in \Theta$, $E(y|x) = m(x, \theta_0)$. Assumption NLS.1 will hold if x is independent of u or if $E(u|x) = 0$ in the population model $y = m(x, \theta_0) + u$. In addition to this we also need NLS.2: $E\{[m(x, \theta_0) - m(x, \theta)]^2\} > 0$, all $\theta \in \Theta$, $\theta \neq \theta_0$.

b. Under the functional form assumption above, state the minimization problem clearly.

Answer: The mimimization problem given the functional form above can be expressed as:

$$\min_{\theta \in \Theta} N^{-1} \sum_{i=1}^{N} [y_i - exp(\theta_1 + \theta_2 rooms_i + \theta_3 baths_i))]^2 / 2 \qquad (1)$$

where the solution is $\hat{\theta}$.

c. Using the functional form above what is the analytical expression for the score function $s(w_i, \theta)$?

Answer: We know that the score function for any function $m(x_i, \theta)$, is given by: $s(w_i, \theta) = -\nabla_\theta m(x_i, \theta)'[y_i - m(x_i, \theta)]$. So for our functional form this is given by: $s(w_i, \theta) = [-exp(\theta_1 + \theta_2 rooms_i + \theta_3 baths_i) \times u_i, -exp(\theta_1 +$

$\theta_2 rooms_i + \theta_3 baths_i) rooms_i \times u_i, -exp(\theta_1 + \theta_2 rooms_i + \theta_3 baths_i) baths_i \times u_i]$.

d. What is the analytical expression for *expected* Hessian?

Answer: For our functional form we get the following: from equation (12.30) the expected Hessian is given by $\hat{A}_0 = N^{-1} \sum_{i=1}^{N} [exp(\theta_1 + \theta_2 rooms_i + \theta_3 baths_i), exp(\theta_1 + \theta_2 rooms_i + \theta_3 baths_i) rooms_i, exp(\theta_1 + \theta_2 rooms_i + \theta_3 baths_i) baths_i]' [exp(\theta_1 + \theta_2 rooms_i + \theta_3 baths_i), exp(\theta_1 + \theta_2 rooms_i + \theta_3 baths_i) rooms_i, exp(\theta_1 + \theta_2 rooms_i + \theta_3 baths_i) baths_i]$.

e. Using the *nls* command in R and the dataset *hprice.csv*, estimate the population model under consideration.[1]

Listing 1: R output

```
Formula: price ~ exp(b0 + b1 * rooms + b2 * baths)

Parameters:
   Estimate Std. Error t value Pr(>|t|)
b0 10.29226    0.15669  65.683   <2e-16 ***
b1  0.04148    0.02641   1.571    0.117
b2  0.36908    0.03552  10.392   <2e-16 ***
---
Signif. codes:  0  ***  0.001  **  0.01  *  0.05  .  0.1

Residual standard error: 33410 on 318 degrees of freedom

Number of iterations to convergence: 5
Achieved convergence tolerance: 3.377e-06
```

f. Now construct the estimated average partial effects (APE) for each of the explanatory variables in the model. Call the APE estimator $\hat{\gamma}_j$. Explain how you would test $H_0 : \gamma_j = 0$ versus $H_1 : \gamma_j \neq 0$ where j indexes the variable of interest.

[1] nlsout=nls(price~exp(b0 + b1*rooms+b2*baths), start = list(b0 = 10, b1 = 0.04321,b2=.9))

Answer: For each j, we can construct the APE as: $\hat{\gamma}_j = N^{-1}\hat{\theta}_j \sum_{i=1}^{N}(exp(\hat{\theta}_1 + \hat{\theta}_2 rooms_i + \hat{\theta}_3 baths_i))$ for $j = 2, 3$. In order to test $H_0 : \gamma_j = 0$ versus $H_1 : \gamma_j \neq 0$ we would need to construct a $t - stat = \frac{\hat{\gamma}_j}{se(\hat{\gamma}_j)}$ which is evaluated at the zero null. To get the t-stat, we would either construct the standard error using a Delta-method approximation or the appropriate bootstrap method.