

Notes on Microeconomic Theory

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These notes are intended for use in courses in microeconomic theory taught at Harvard University. Consequently, much of the structure is inherited from the required text for the course, which is currently Mas-Colell, Whinston, and Green's *Microeconomic Theory* (referred to as MWG in these notes). They also draw on material contained in Silberberg's *The Structure of Economics*, as well as additional sources. They are not intended to stand alone or in any way replace the texts.

In the early drafts of this document, there will undoubtedly be mistakes. I welcome comments from students regarding typographical errors, just-plain errors, or other comments on how these notes can be made more helpful.

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Chapter 1

The Economic Approach

Economics is a social science.¹ Social sciences are concerned with the study of human behavior. If you asked the next person you meet while walking down the street what defines the difference between economics and other social sciences, such as political science or sociology, that person would most likely say that economics studies money, interest rates, prices, profits, and the like, while political science considers politicians, elections, etc., and sociology studies the behavior of groups of people. However, while there is certainly some truth to this statement, the things that can be fairly called economics are not so much defined by a subject matter as they are united by a common approach to problems. In fact, economists have written on topics spanning human behavior, from traditional studies of firm and consumer behavior, interest rates, inflation and unemployment to less traditional topics such as social choice, voting, marriage, and family.

The feature that unites these studies is a common approach to problems, which has become known as the “marginalist” or “neoclassical” approach. In a nutshell, the marginalist approach consists of four principles:

1. Economic actors have preferences over allocations of the world’s resources. These preferences remain stable, at least over the period of time under study.²
2. There are constraints placed on the allocations that a person can achieve by such things as wealth, physical availability, and social/political institutions.
3. Given the limits in (2), people choose the allocation that they most prefer.

¹See Silberberg’s *Structure of Economics* for a more extended discussion along these lines.

²Often preferences that change can be captured by adding another attribute to the description of an allocation.

More on this later.

4. Changes in the allocations people choose are due to changes in the limits on available resources in (2).

The marginalist approach to problems allows the economist to derive predictions about behavior which can then, in principle, be tested against real world data using statistical (econometric) techniques. For example, consider the problem of what I should buy when I go to the grocery store. The grocery store is filled with different types of food, some of which I like more and some of which I like less. Principle (1) says that the trade-offs I am willing to make among the various items in the store are well-defined and stable, at least over the course of a few months. An allocation is all of the stuff that I decide to buy. The constraints (2) put on the allocations I can buy include the stock of the items in the store (I can't buy more bananas than they have) and the money in my pocket (I can't buy bananas I can't afford). Principle (3) says that given my preferences, the amount of money in my pocket and the stock of items in the store, I choose the shopping cart full of stuff that I most prefer. That is, when I walk out of the store, there is no other shopping cart full of stuff that I could have purchased that I would have preferred to the one I did purchase. Principle (4) says that if next week I buy a different cart full of groceries, it is because either I have less money, something I bought last week wasn't available this week, or something I bought this week wasn't available last week.

There are two natural objections to the story I told in the last paragraph, both of which point toward why doing economics isn't a trivial exercise. First, it is not necessarily the case that my preferences remain stable. In particular, it is reasonable to think that my preferences this week will depend on what I purchased last week. For example, if I purchased chocolate chip cookies last week, this may make me less likely to purchase them this week and more likely to purchase some other sort of cookie. Thus, preferences may not be stable over the time period we are studying. Economists deal with this problem in two ways. The first is by ignoring it. Although widely applied, this is not the best way to address the problem. However, there are circumstances where it is reasonable. Many times changes in preferences will not be important relative to the phenomenon we are studying. In this case it may be more trouble than it's worth to address these problems. The second way to address the problem is to build into our model of preferences the idea that what I consumed last week may affect my preferences over what I consume this week. In other words, the way to deal with the cookie problem is to define an allocation as "everything I bought this week and everything I bought last week." Seen in this light, as long as the effect of having chocolate chip cookies last week on my preferences this week stay stable over time, my

preferences stay stable, whether or not I actually had chocolate chip cookies last week. Hence if we define the notion of preferences over a rich enough set of allocations, we can usually find preferences that are stable.

The second problem with the four-step marginalist approach outlined above is more troublesome: Based on these four steps, you really can't say anything about what is going to happen in the world. Merely knowing that I optimize with respect to stable preferences over the groceries I buy, and that any changes in what I buy are due to changes in the constraints I face does not tell me anything about what I will buy, what I should buy, or whether what I buy is consistent with this type of behavior.

The solution to this problem is to impose structure on my preferences. For example, two common assumptions are to assume that I prefer more of an item to less³ (monotonicity) and that I spend my entire grocery budget in the store (Walras' Law). Another common assumption is that only real opportunities matter. If I were to double all of the prices in the store and my grocery budget, this would not affect what I can buy, so it shouldn't affect what I do buy.

Once I have added this structure to my preferences, I am able to start to make predictions about how I will behave in response to changes in the environment. For example, if my grocery budget were to increase, I would buy more of at least one item (since I spend all of my money and there is always some good that I would like to add to my grocery cart).⁴ This is known as a **testable implication** of the theory. It is an implication because if the theory is true, I should react to an increase in my budget by buying more of some good. It is testable because it is based on things which are, at least in principle, observable. For example, if you knew that I had walked into the grocery store with more money than last week and that the prices of the items in the store had not changed, and yet I left the store with less of every item than I did last week, something must be wrong with the theory.

The final step in economic analysis is to evaluate the tests of the theories, and, if necessary, change them. We assume that people follow steps 1 - 4 above, and we impose restrictions that we believe are reasonable on their preferences. Based on this, we derive (usually using math) predictions about how they should behave and formulate testable hypotheses (or refutable propositions) about how they should behave if the theory is true. Then we observe what they really do. If

³Or, we could make the weaker assumption that no matter what I have in my cart already, there is something in the store that I would like to add to my cart if I could.

⁴The process of deriving what happens to people's choices (the stuff in the cart) in response to changes in things they do not choose (the money available to spend in the store) is known as comparative statics.

their behavior accords with our predictions, we rejoice because the real world has supported (but not proven!) our theory. If their behavior does not accord with our predictions, we go back to the drawing board. Why didn't their behavior accord with our predictions? Was it because their preferences weren't like we thought they were? Was it because they weren't optimizing? Was it because there was an additional constraint that we didn't understand? Was it because we did not account for a change in the environment that had an important effect on people's behavior?

Thus economics can be summarized as follows: It is the social science that attempts to account for human behavior as arising from consistent (often maximizing – more on that later) behavior subject to one or more constraints. Changes in behavior are attributed to changes in the constraints, and the test of these theories is to compare the changes in behavior predicted by the theory with the changes that actually occur.