LECTURES II & III

THEORY OF CONSUMER BEHAVIOR AND DEMAND

1. DERIVATION OF CONSUMER DEMAND CURVE
2. EXAMPLE
3. GENERALIZATION TO N GOODS
4. HOMOGENEITY OF CONSUMER DEMAND CURVE
5. CONSUMER SURPLUS
6. NO ADDITIVITY OF UTILITIES
7. NO INTERPERSONAL COMPARISON OF UTILITIES

Next (Lesson IV): Derivation of Hick’s and Slutsky’s Demand Curves
Read in R.L. -- II: 4 (ch.14) and II: 6 (both can be accessed electronically)

1. INTRO. AND DERIVATION OF CONSUMER DEMAND

1) CONSUMER UTILITY

U= U(X,Y)

where $\frac{\partial U}{\partial X}$ and $\frac{\partial U}{\partial Y} > 0$

$$\frac{\partial^2 U}{\partial X^2} \; and \; \frac{\partial^2 U}{\partial Y^2} < 0$$

2) CONSUMER INDIFFERENCE CURVE (I.C.)

$$dU = \frac{\partial U}{\partial X}\, dX + \frac{\partial U}{\partial Y} dY = 0$$

$$\frac{\partial U}{\partial X} dX = -\frac{\partial U}{\partial Y} dY$$

$$-\frac{\mathrm{d}Y}{dX} = \frac{\frac{\partial U}{\partial X}}{\frac{\partial U}{\partial Y}} = \frac{MU_{\{X\}}}{MU_{\{Y\}}} = MRS_{\{XY\}}$$

Also, $\frac{d^2\mathrm{Y}}{dX^2} > 0$, so I.C. is negatively sloped and convex

3) CONSUMER BUDGET CONSTRAINT:

$$P_X X + P_Y Y = I$$

4) CONSUMER EQUILIBRIUM AND DERIVATION OF DEMAND CURVE

$$V = U(X, Y) + \lambda(I - P_X X - P_Y Y)$$

$$\frac{\partial V}{\partial X} = \frac{\partial U}{\partial X} - \lambda P_X = 0$$

$$\frac{\partial V}{\partial Y} = \frac{\partial U}{\partial Y} - \lambda P_Y = 0$$

$$\frac{\partial V}{\partial \lambda} = I - P_X X - P_Y Y = 0$$

$$\therefore \lambda = \frac{MU_X}{P_X} = \frac{MU_Y}{P_Y} = MRS_{XY} = \frac{P_X}{P_Y}$$

The Second order condition for U maximization satisfied by I.C. convexity.
This gives one point on d_X and d_Y; other points by assuming other values of P_X and P_Y.

2. EXAMPLE

1) GIVEN: $U = XY$; and $P_X = \$2$; $P_Y = \$5$, $I = \$100$

DERIVE d_X and d_Y

2) $$V = XY + \lambda(I - P_X X - P_Y Y)$$

3) $$\frac{\partial V}{\partial X} = Y - \lambda P_X = 0$$

$$\frac{\partial V}{\partial Y} = X - \lambda P_Y = 0$$

$$\frac{\partial V}{\partial \lambda} = I - P_X X - P_Y Y = 0$$

4) $$\frac{Y}{X} = \frac{P_X}{P_Y}; \quad \therefore Y = \frac{P_X}{P_Y} X \ and$$

$$I - P_X X - P_Y \left(\frac{P_X}{P_Y}\right) X = 0$$

$$I - 2P_X X = 0 \ and \ X = \frac{I}{2\,P_X}$$

5) SUBSTITUTING GIVEN VALUES,

$$X = \frac{100}{2(2)} = 25 \text{ with } P_X = \$2$$

$$\text{If } P_X = \$1.50,\ X = 33\tfrac{1}{3},\ \text{ and if } P_X = \$1, X = 50, \text{and get } d_X$$

6) WE GET d_Y IN THE SAME WAY

$$\text{with } P_Y = \$5, Y = 10;\ \text{if } P_Y = \$10, Y = 5; \text{and if } P_Y = \$20, Y = 2.5$$

7) NOTE CONSUMER SPENDS $50 ON X AND Y WHATEVER THEIR PRICE.
This means that d_X and d_Y are rectangular hyperbolas and their price elasticity equals -1

3. GENERALIZATION TO N GOODS

1) GIVEN: $U = U(X_1, X_2, \dots, X_n)$

$$I = P_1X_1 + P_2X_2 + \dots + P_nX_n$$

DERIVE: $d_1, d_2, \dots, d_n$

2) $$V = U(X_1, X_2, \dots, X_n) - \lambda(M - P_1X_1 - P_2X_2 - \cdots - P_nX_n)$$

3) $$\frac{\partial V}{\partial X_1} = \frac{\partial U}{\partial X_1} - \lambda P_1 = 0$$

$$\frac{\partial V}{\partial X_2} = \frac{\partial U}{\partial X_2} - \lambda P_2 = 0$$

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$$\frac{\partial V}{\partial X_n} = \frac{\partial U}{\partial X_n} - \lambda P_n = 0$$

$$\frac{\partial V}{\partial \lambda} = I - P_1X_1 - P_2X_2 - \cdots - P_nX_n = 0$$

4) $$\lambda = \frac{MU_1}{P_1} = \frac{MU_2}{P_2} = \cdots = \frac{MU_n}{P_n}$$

5) SECOND ORDER CONDITION FOR MAXIMIZATION:

The bordered Hessian determinants must alternate in sign with $|\overline{H_i}| > 0$ if $i =$ even and $|\overline{H_i}| < 0$ for $i =$ odd, for $i = 1$ to n

6) THE ABOVE WILL GIVE ONE POINT ON

Repeating the process for alternative $P_1, P_2, \ldots, P_n$ will give $d_1, d_2, \ldots, d_n$

4. HOMOGENEITY OF THE DEMAND CURVE (D)

1) D IS HOMOGENEOUS OF DEGREE ZERO WITH RESPECT TO PRICE AND INCOME.

This means that if price and income increase in the same proportion (k), the quantity demanded of the good remains unchanged (no "money illusion")

2) TO PROVE THE ABOVE START WITH

$U = U(X, Y)$ and $kI = kP_X X + kP_Y Y$ so that,

$V^* = U(XY) + k\lambda^*(kI - kP_X X - kP_Y Y)$

and show that the first order condition for maximizing V^* is

$$\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y} = k\lambda^* = \lambda \text{ (the same as without k)}$$

5) CONSUMER SURPLUS (C.S.)

1. CONSUMER SURPLUS IS THE DIFFERENCE BETWEEN WHAT THE CONSUMER IS WILLING TO PAY FOR A GIVEN QUANTITY OF A GOOD AND WHAT HE/ SHE ACTUALLY PAYS
2. Start with $X = G(P_X)$ *cet. par.*, so that $P_X = H(X)$ and the consumer spends $P_X^* X^*$ to purchase X^*
3. $C.S. = \int_{\{0\}}^{X^*} H(X)dX - P_X^* X^*$
4. Uses and shortcoming of c.s.

6) NO ADDITIVITY OF UTILITIES

$U(X, Y) \neq G(X) + H(Y)$ BECAUSE MU_X IS NOT INDEPENDENT OF MU_Y

7) NO INTERPERSONAL COMPARISON OF UTILITIES BETWEEN INDIVIDUALS

But many economic policies (such as progressive taxation) are based on it.

QUESTIONS AND DISCUSSION