HW1

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ECON 7910 Econometrics

Due on Sep 23, 2021

1 Question 1– 2.1

Solution:

1. - $$\frac{\partial E(y|x_1,x_2)}{\partial x_1} = \beta_1 + \beta_4 x_2$$
   - $$\frac{\partial E(y|x_1,x_2)}{\partial x_2} = \beta_2 + 2\beta_3 x_2 + \beta_4 x_1$$
2. $E(u|x_1,x_2)$ means error term u and corvariates are independ,i.e., $E(u|x_1,x_2) = 0$. For $E(u|x_1,x_2,x_2^2,x_1x_2)$, if we are give previous CE, then in this CE, x_2^2 and x_1x_2 are both redundent, because they can be expressed by x_1 and x_2.
3. Given part(b), $E(u|x_1,x_2)$ doesn't give any additional information about $Var(u|x_1,x_2)$. The only thing we can know for sure is this item is nonnegative. But for whetere it's constant or relies on other variables, aka, heterogeneity or homogenity, we have no information.

2 Question 2 – 2.2

Solution:

1. Since in this question, $\mu = E(x)$, thus:
$$\frac{\partial E(y|x)}{\partial x} = \delta_1 + 2\delta_2 x - 2\mu = \delta_1 + 2\delta_2(x-\mu)$$
2. Take expectation on both sides:
$$E(\frac{\partial E(y|x)}{\partial x}) = \delta_1 + 2\delta_2 E(x - E(x)) = \delta_1 + 0 = \delta_1$$

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3.

$$\begin{aligned}L(y|1,x) &= L(E(y)|1,x)\\&= L(\delta_0 + \delta_1(x_1-\mu) + \delta_2(x_1-\mu)^2|x)\\&= \delta_0 + \delta_1(x_1-\mu) + \delta_2 L((x_1-\mu)^2|1,x)\\&= \alpha_0 + \delta_1 x_1\end{aligned}$$

For the first equation above, I borrow the result from the Property LP.5 of Appendix 2.A.3 directly. And the last equation, I intend to negelect some extra proof, because this equation is based on what I guess. The last $L(.)$ would be a constant, so $\alpha_0 = \delta_0 - \delta_1\mu + L(.)$, and it's still a constant number, i.e., it doesn't rely on x.

3 Question 3 – 2.4

Solution:

Since the expectation of u given x v is the function of x and v, and u, v is uncorrelated with x, then, it's conveniently to kill x term in this conditional expectation. So the CE becomes: $E(u|x,v) = E(u|v)$. We assume that $E(u|v) = \rho_0 + \rho_1 v$, By LIE, $E(E(u|v)) = 0 = \rho_0 + \rho_1 E(v)$. From the question, $E(v) = 0$, then we can deduce that $\rho_0 = 0$. Thus, back to our previous equation, $E(u|v) = \rho_1 v$. ☺.

4 Question 4 – 2.7

Solution:[1]

Since $E(y|x,z) = g(x) + z\beta$, we can generalize y with corvariates x and z:

$$y = g(x) + z\beta + \epsilon \tag{1}$$

Take expectation on both sides of (1) on the condition of x:

$$E(y|x) = g(x) + \beta E(z|x) \tag{2}$$

(1)-(2):

$$y - E(y|x) = z\beta - \beta E(z|x) + \epsilon = \beta(z - E(z|x)) + \epsilon \tag{3}$$

Subsitute $\tilde{y}$ and $\tilde{z}$ into equation (3):

$$\tilde{y} = \beta\tilde{z} + \epsilon \tag{4}$$

Take expectation of equation of (4) on variable $\tilde{z}$:

$$E(\tilde{y}|\tilde{z}) = \beta\tilde{z} + 0 = \beta\tilde{z} \tag{5}$$

The reason why we get 0 from equation (5) is that the error term ϵ is uncorrelated with $\tilde{u}$, thus, $E(\epsilon|\tilde{z}) = 0$.

[1] I partly refer to the online resource about the beginning of this solution, because I forgot to set up a more flexible function of y, aka, euqation (1).