Exercises #11

Instructions

Exercises #11 are due on Wednesday, April 27th.

Exercises may be presented for credit as a hard copy at the end of the class meeting on the due date, or may be submitted electronically on Blackboard by the following Monday. If submitted on Blackboard, exercises should be attached as a Portable Document Format (`*.pdf`) file. It is possible to convert handwritten work to `*.pdf` using scanner or a camera-equipped device with Microsoft Office Lens (Android, iOS, or Windows), Google Drive (Android), or Apple Notes (iOS).

Exercises are "collaborative and open book" assignments. You are encouraged to make use of help from your peers, textbook, notes, and me, but you must submit your own answers. There is no penalty for incorrect answers; the expectation is simply for you to progress as far as you can on each question. Complete answers with explanations will be provided in recitation.

Questions

12.D.2 Show that with J firms, repeated choice of any price $p \in (c, p^m]$ can be sustained as a stationary SPNE outcome path of the infinitely repeated Bertrand game using Nash reversion strategies if and only if $\delta \geq (J-1)/J$. What does this say about the effect of having more firms in the market on the difficulty of sustaining collusion?

12.D.3 Consider an infinitely repeated Cournot duopoly with discount factor $\delta < 1$, unit costs of $c > 0$, and inverse demand function $p(q) = a - bq$, with $a > c$ and $b > 0$.

(a) Under what conditions can the symmetric joint monopoly outputs $(q_1, q_2) = (q_1^m/2, q_2^m/2)$ be sustained with strategies that call for $(q_1^m/2, q_2^m/2)$ to be played if no one has yet deviated and for the single-period Cournot (Nash) equilibrium to be played otherwise?

(b) Derive the minimal level of δ such that the output levels $(q_1, q_2) = (q, q)$ with $q \in [(a-c)/2b, (a-c)/b]$ are sustainable through Nash reversion strategies. Show that this level of δ, $\delta(q)$, is an increasing, differentiable function of q.