HW2

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ECON 5700

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1 Question 1

Solution:

1. $z = 3y^2 - 2x^2 + x, (2, -1, 3)$
   Let $f(x, y) = 3y^2 - 2x^2 + x$, then $f_x = -4x + 1$ and $f_y = 6y$, thus, the targent place would be:
   $$z = (-4 \cdot 2 + 1)(x - 2) + 6 \cdot (-1) \cdot (y + 1) + 3 = -7x - 6y + 11$$
2. $z = \sqrt{xy}, (1, 1, 1)$
   Let $f(x, y) = \sqrt{xy}$, then $f_x = \frac{1}{2}x^{-\frac{1}{2}}y^{\frac{1}{2}}$, and $f_y = \frac{1}{2}x^{\frac{1}{2}}y^{-\frac{1}{2}}$.
   $$z = \frac{1}{2}(x - 1) + \frac{1}{2}(y - 1) + 1 = \frac{1}{2}(x + y)$$
3. $z = x\sin(x + y), (-1, 1, 0)$
   Let $f(x, y) = x\sin(x+y)$, then $f_x = \sin(x+y)+x\cos(x+y)$, and $f_y = x\cos(x+y)$
   $$z = -1 \cdot (x + 1) + (-1) \cdot (y - 1) + 0 = -(x + y)$$

2 Question 2

$f(x, y) = 1 + x\ln(xy - 5)$ at point $(2, 3)$ Why exisitence and find the linearization at that point.

Solution:

$f(2, 3) = 1+2\ln(6-5) = 1$, and $f_x = \ln(xy-5)+\frac{xy}{xy-5}$, $f_y = \frac{x^2}{xy-5}$. At the point$(2, 3)$, $f_x = 6,\ f_y = 4$ whereas the existence of both f_x and f_y, thus, the differentiation of this function exists. And the linearization funciton is as below:

$$f(x, y) = f_x(x - 2) + f_y(y - 3) + f(2, 3) = 6x + 4y - 23$$

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3 Question 3-Differential

$$z = e^{-2x} \cos 2\pi t$$
$$m = p^5 q^3$$
$$R = \alpha\beta^2 \cos\gamma$$

Solution:

1. Let $f(x,t) = z$, then

$$\mathrm{d}z = f_x(x,t)\mathrm{d}x + f_t(x,t)\mathrm{d}t = -2e^{-2x}\cos\pi t\ \mathrm{d}x + 2\pi e^{-2x}\sin 2\pi t\ \mathrm{d}t$$

2. Let $f(p,q) = m$, then:

$$dm = f_p \cdot \mathrm{d}p + f_q \cdot \mathrm{d}q = 5p^4q^3\ \mathrm{d}p + 3p^5q^2\ \mathrm{d}q$$

3. let $f(\alpha,\beta,\gamma) = R$, then:

$$\mathrm{d}R = f_\alpha\ \mathrm{d}\alpha + f_\beta\ \mathrm{d}\beta + f_\gamma\ \mathrm{d}\gamma = \beta^2\cos\gamma\ \mathrm{d}\alpha + 2\alpha\beta\cos\gamma\ \mathrm{d}\beta + \alpha\beta^2\sin\gamma\ \mathrm{d}\gamma$$

4 Question 4

Solution:

$$t\frac{\partial g}{\partial s} + s\frac{\partial g}{\partial t} = t\left[f_s \cdot 2s + f_t \cdot (-2s)\right] + s\left[f_s(-2t) + f_t(2t)\right] = 2stf_s - 2stf_t - 2stf_s + 2stf_t = 0$$

5 Question 5

Solution:

1. $f(x,y) = \sin(2x+3y)$ at $P(-6,4)$, $u = \frac{1}{2}(\sqrt{3}i - j)$

   (a) $f_x = 2\cos(2x+3y)$, and $f_y = 3\cos(2x+3y)$. Thus, the gradient is :

   $$\nabla f(x,y) = (2\cos(2x+3y), 3\cos(2x+3y))$$

   (b) At point P, the gradient would be:

   $$\nabla f(2,3) = (2\cos(0), 3\cos 0) = (2,3)$$

   (c) at the vector u, the gradient relative to point p is:

   $$\nabla f(2,3) = 2\cdot\frac{1}{2}(\sqrt{3}i - j) + 3\cdot\frac{1}{2}(\sqrt{3}i - j) = \frac{5}{2}\sqrt{3}i - \frac{5}{2}j$$

2. $f(x,y,z) = x^2yz - xyz^3$ at $P(2,-1,1)$, $u =< 0, \frac{4}{5}, -\frac{3}{5} >$

   (a) $f_x = 2xyz - yz^3$, $f_y = x^2z - xz^3$, and $f_z = x^2y - 3xyz^2$. Thus, the gradient is:
   $$\nabla f(x,y,z) = (2xyz - yz^3, x^2z - xz^3, x^2y - 3xyz^2)$$

   (b) At point P, the gradient would be:
   $$\nabla f(2,-1,-1) = (3,-2,2)$$

   (c) At the vector u, the gradient relative to the point p is:
   $$\nabla f(2,-1,-1) = 3 \cdot 0 + -2 \cdot \frac{4}{5} + 2 \cdot -\frac{3}{5} = -\frac{14}{5}$$

6 Question 6

Solution:
$f_x = \frac{1}{2}x^{-\frac{1}{2}}y^{\frac{1}{2}}$,$f_y = \frac{1}{2}x^{\frac{1}{2}}y^{-\frac{1}{2}}$. At the point $(2,8)$, $f_x = 4$, $f_y = 1$, thus, the final result w.r.t the vector would be:
$$4 \cdot 5 + 1 \cdot 4 = 24$$

7 Question 7

Solution:
$f_x = y\cos(xy)$, and $f_y = x\cos(xy)$. At the point $(1,0)$, $f_x = 0$, $f_y = 1$.Thus, the maximum change of rate is $\sqrt{0+1^2} = 1$ And occur at the direction of change $(0,1)$.

8 Question 8

Solution: $f_x = 20xy - 10x - 4x^3 = 0$, and $f_y = 10x^2 - 8y - 8y^3 = 0$, thus, one critical point would be $(0,0)$.

$f_{xx} = 20y - 10 - 12x^2$, and $f_{yy} = 8 - 24y^2$, $f_{xy} = 20x$. Thus, $D = -80 < 0$, which is a saddle point. Must be kidding me Can't solve even I use the online solver.Check later.

9 Question 9

Solution:

1. $f_x = 2x + y$, $f_{xx} = 2$, $f_y = x + 2y + 1$, $f_{yy} = 2$. $f_{xy} = 1$
   $$D = f_{xx}f_{yy} - (f_{xy})^2 = 2 \cdot 2 - 1 = 3 > 0$$
   In the meanwhile, $f_{xy} > 0$. Thus, it's min value.

2. $f_x = 1 - 2xy + y^2$, $f_{xx} = -2y$, $f_{xy} = -2x + 2y$, $f_y = -x^2 - 1 + 2xy$, $f_{yy} = 2x$. Let $f_x = f_y = 0 \to x^2 = y^2$. Thus, 4 senarios exist.

$$D = f_{xx}f_{yy} - [f_{xy}]^2 = -4(x-y)^2 + 4xy$$

At $(0,0)$ where D=0, the point is uncertain. If $D > 0$, $xy > (x-y)^2$, then it makes no sense. This question is questionable.

Make no sense. NEED TO check FURTHER!

3. $f_x = 6xy - 12x$, $f_y = 3y^2 + 3x^2 - 12y$, $f_{xx} = 6y - 12$, $f_{yy} = 6y - 12$. Let $f_x = 0$ and $f_y = 0$.

   (a) When $x = 0, y = 4$, $D = 144 > 0$, and $f_{xx} = 12 > 0$, it's min value point.
   (b) When $x = 0, y = 0$, $D = 144 > 0$, and $f_{xx} = -12 < 0$, it's max value point.
   (c) When $x = 2, y = 2$, $D = -144 < 0$, it's a saddle point.
   (d) When $x = -2, y = 2$, $D = -144 < 0$, it's a saddle point.

4. $f_x = 3x^2 - 12y$, $f_{xx} = 6x$, $f_y = -12x + 24y^2$, $f_{yy} = 48y$, and $f_{xy} = -12$. Let $f_x = f_y = 0$. Two scenarios exist:

   (a) When $x = 0, y = 0$, $D = -144 < 0$, it's a saddle point.
   (b) When $x = 2, y = 1$, $D = 432 > 0$, and $f_{xx} = 12 > 0$, it's min value point.

10 Question 10

Solution:

(a) $\frac{\partial y}{\partial x_1} = 6x_1^2 - 22x_1x_2$, $\frac{\partial y}{\partial x_2} = 11x_1^2 + 6x_2$.
(b) $\frac{\partial y}{\partial x_1} = 7 + 6x_2^2$, $\frac{\partial y}{\partial x_2} = 12x_1x_2 - 27x_2^2$.
(c) $\frac{\partial y}{\partial x_1} = 2x_2 - 4$, $\frac{\partial y}{\partial x_2} = 2x_1 + 3$
(d) $\frac{\partial y}{\partial x_1} = \frac{5}{x_2-2}$, $\frac{\partial y}{\partial x_2} = \frac{5x_1+3}{(x-2)^2}$

11 Question 11

Solution:

(a)

$$|\mathbb{J}| = \begin{vmatrix} 6x_1 & 1 \\ 36x_1^3 + 12x_1x_2 + 48x_1 & 6x_1^2 + 2x_2 + 8 \end{vmatrix} = 0$$

Thus, it's functionally dependent.

(b)

$$|\mathbb{J}| = \begin{vmatrix} 6x_1 & 4x_2 \\ 5 & 0 \end{vmatrix} = -20x_2$$

Thus, if $x_2 = 0$, it's functionally dependent, otherwise it's not.