Lecture 4

Absolute convergence & conditional Convergence

$\boxed{AC}$: $\sum a_n$ is AC if $\sum |a_n|$ converges

- $\sum a_n$ is AC $\Rightarrow$ $\sum a_n$ is convergent

$\boxed{CC}$ $\sum a_n$ is conditionally convergent if $\sum a_n$ convergent but not absolutely convergent.

$\sum a_n$ ① $1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \frac{1}{7})$ > 1

② $(1 + \frac{1}{2}) - (\frac{1}{3} - \frac{1}{4}) - (\quad)$ $< \frac{3}{2}$

$\frac{3}{2}$ $\quad$ $\frac{1}{12}$

Thus. $1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} \cdots$ converges.

but $\sum |a_n|$ $\Rightarrow$ harmonic series $\Rightarrow$ dive

Ex 2. $\sum_{n=1}^{\infty} (-\frac{1}{n^2})^{n+2}$

Since $\sum_{n=1}^{\infty} (-\frac{1}{n^2})^{n+2}$ convergent

$\sum_{n=1}^{\infty} |-\frac{1}{n^2}|^{n+2}$ convergent $\Rightarrow$ Absolut convergent.

4) Power series.

$\sum a_n x^n = a_0 + a_1 x + \ldots + a_n x^n$

$\sum a_n (x - x_0)^n = a_0 + a(x - x_0) + \cdots + a(x - x_0)^n$

Domain of power series $f(x) = \sum a_n (x - x_0)^n$ is the set of x for which $\sum f(x)$ converges.

Interval of convergence.

5) Geometric Series.

$q_n = \frac{a_{n+1}}{a_n}$ $\quad a_{n+1} = q \cdot a_n \quad a_n = a_1 \cdot q^{n-1}$

$S_n = a_1 + a_2 + \dots + a_n = a_1 \frac{1-q^n}{1-q}$

$S_n = \sum_{n=1}^{\infty} a_n = a_1 \sum_{n=1}^{\infty} q^n$

when $n \to \infty$ $\quad S_n \to \frac{a_1}{1-q} \quad (0<|q|<1)$

$$\begin{cases} \text{converges to } \frac{a_1}{1-q} & \text{if } |q|<1 \\ \text{divergent} & \text{if } |q|>1 \end{cases}$$

Ex: $1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \dots$

$q = \frac{1}{x}$

$a_1 = 1, \quad a_2 = a_1 q, \quad a_3 = a_1 q^2 \dots$

$S_n = \frac{a_1}{1-q} = \frac{1}{1-\frac{1}{x}} = \frac{1}{\frac{x-1}{x}} = \frac{x}{x-1}$

$$\begin{cases} \text{converges to } \frac{x}{x-1} & |\frac{1}{x}|<1 \\ \text{divergent.} & |\frac{1}{x}|>1 \end{cases}$$