Microeconomic Theory II

Preference, Utility, & Choice

Is it possible to develop a <u>model</u> that explains observed choice?

Primitives: unobserved "preferences".

X is the choice set, or a list of alts.

(*) preference relation $x \succsim y \iff x$ at least as good as y

strict preference rel. $x \succ y \iff x \succsim y$ but not $y \succsim x$.

indifference rel. $x \sim y \iff x \succsim y$ and $y \succsim x$

($x \not\succ y$ and $y \not\succ x$)

ex.

$X = \{\{\text{coffee, tea}\}\}$

$X = \{\{\text{coffee, tea}\}, \{\text{coffee, tea, wine}\}\}$

$x = \{\text{coffee}\}$

$y = \{\text{tea}\}$

A $\succsim$ is rational if it is

1. complete: $\forall x, y \in X$ we have $x \succsim y$, $y \succsim x$, or both.
2. transitive: $\forall x, y, z \in X$, if $x \succsim y$ and $y \succsim z$, then $x \succsim z$.

If $\succsim$ is rational, then

i.) $\succ$ is irreflexive $\iff$ $x \succ x$ never holds
and transitive $\iff$ if $x \succ y$ and $y \succ z \Rightarrow x \succ z$

ii.) $\sim$ is reflexive $\iff$ $x \sim x$ always holds $(\forall x)$
is transitive $\iff$ if $x \sim y$ and $y \sim z \Rightarrow x \sim z$
is symmetric $\iff$ $x \sim y \iff y \sim x$

iii.) if $x \succ y$ and $y \succsim z$, then $x \succ z$

A **utility fn.** assigns a numerical value to each $x \in X$ ranking these elements of X consistently w/ preferences.

Def. A fn. $u: X \to \mathbb{R}$ is a utility fn. representing pref. rel. $\succsim$ if
$\forall x, y \in X$, $x \succsim y \iff u(x) \geq u(y)$

Prop. A $\succsim$ can be represented by a utility fn. **only** if it is rational.

Proof. Show that if $\exists$ a utility fn. that represents $\succsim$, then $\succsim$ must be (a) complete & (b) transitive.

(a) B/c $u(\cdot)$ is real-valued on X it must be $\forall x, y \in X$, either $u(x) \geq u(y)$ or $u(y) \geq u(x)$. By def. above $\Rightarrow$ either $x \succsim y$, $y \succsim x$, or both (completeness).

(b) WLOG, suppose $x \succsim y$ and $y \succsim z$. B/c $u(\cdot)$ represents $\succsim$ we must have $u(x) \geq u(y)$ and $u(y) \geq u(z) \Rightarrow u(x) \geq u(z)$. B/c $u(\cdot)$ represents $\succsim$, this $\Rightarrow x \succsim z$ (transitivity).

Choice

potential available choices

A choice structure $(\mathcal{B}, C(\cdot))$ consists of

i) $\mathcal{B}$ is a set of non-empty subsets of X.

Every element of $\mathcal{B}$ is a set $B \subset X$. For ex. $B \in \mathcal{B}$ could be a budget set.

ii) $C(\cdot)$ is a choice correspondence that assigns a non-empty set of chosen elements $C(B) \subset B$ for every set $B \in \mathcal{B}$. When single-valued, it is an individual's choice from B.

Review: X is the choice set
$\mathcal{B}$ is the potential available choices
B is a subset of $\mathcal{B}$
$C(B)$ yields a choice made from $\mathcal{B}$ when single-valued

What kind of restrictions might we need to say a choice structure is <u>reasonable</u> or 'exhibit basic consistency'.

- What does an unreasonable choice structure look like?

Ex. 1 Suppose $X = \{x, y, z\}$, $\mathcal{B} = \{\{x, y\}, \{x, y, z\}\}$

Then $C_1(\{x, y\}) = x$ and $C_1(\{x, y, z\}) = x$.

This seems fine.

$x \succsim^* y$ and

$x \succsim^* z$

satisfies WARP b/c x & y are never chosen, no violation is possible

a.

$y \succsim^* x$

(and $x \succsim^* y$

$x \succsim^* z$ and

$y \succsim^* z$)

b. $x \succ^* y$

WARP is violated

Ex. 2. Suppose $X = \{x, y, z\}$, $\mathcal{B} = \{\{x, y\}, \{x, y, z\}\}$.

Then $C_2(\{x, y\}) = x$ and $C_2(\{x, y, z\}) = \{x, y\}$.

b. a.

This is problematic b/c it seems like the availability of z (not chosen) affects the choice over x and y.

How can we rule out the type of inconsistency we're seeing in Ex. 2?

(A) <u>Def.</u> $\mathcal{B}, C(\cdot)$ satisfies the weak axiom of revealed preference (WARP) if the following holds

If, for some $B \in \mathcal{B}$ w/ $x, y \in B$, we have $x \in C(B)$, then for any $B' \in \mathcal{B}$ w/ $x, y \in B'$ and $y \in C(B')$, then we require $x \in C(B')$.

In words:

If x is chosen when y is available in one case, then there cannot be another case where both are available and y is chosen, but x is not.

Does this rule out Ex. #2?

If we observe choices, what can we can we infer their consistency?

Def. Given $\mathcal{B}, C(\cdot)$, the revealed preference rel. $\succsim^*$ is defined by

$$x \succsim^* y \iff \exists B \in \mathcal{B} \text{ such that } x, y \in B \text{ and } x \in C(B).$$

This allows us to restate WARP as follows:

"if x is revealed at as good as y, then y cannot be realed (strongly) prefered to x" (If $x \succsim^* y$, then $y \not\succ^* x$.)

Lets consider our two examples in revealed preference notation.

Can we go back and forth? Does rationality $\Rightarrow$ WARP? Does WARP $\Rightarrow$ rationality?

Yes.

Sometimes.

This is the important question to us in Micro?

A note on notation: Suppose rational $\succsim$ on X. If facing $B \subset X \Rightarrow$ $C^*(B, \succsim) = \{x \in B : x \succsim y \ \forall y \in B\}$ is the preference-maximizing choice correspondence. (This says preference-maxing behavior is to chose the most preferred alternatives.)

Does 'rationality $\Rightarrow$ WARP?

Prop. Suppose that $\succsim$ is rational. Then $(\mathcal{B}, C^*(\cdot, \succsim))$ satisfies WARP.

Proof. Suppose for that some $B \in \mathcal{B}$ we have $x, y \in B$ and $x \in C^*(B, \succsim)$. By def. of C^* this $\Rightarrow x \succsim y$. Now suppose that for $B' \in \mathcal{B}$ w/ $x, y \in B'$ we have $y \in C^*(B, \succsim)$. This implies by C^* $y \succsim z \; \forall z \in B'$.

But we know $x \succsim y \Rightarrow x \succsim z \; \forall z \in B' \Rightarrow x \in C^*(B', \succsim)$, but this is WARP by definition!

Does WARP $\Rightarrow$ rationality?

Def. Given $(\mathcal{B}, C(\cdot))$ we say rational $\succsim$ <u>rationalizes</u> $C(\cdot)$ relative to $\mathcal{B}$ if $C(B) = C^*(B, \succsim)$ $\forall B \in \mathcal{B}$; that is if $\succsim$ <u>generates</u> the choice structure $(\mathcal{B}, C(\cdot))$ when pref-maximizing choices are made.

Let's look at an example where WARP $\not\Rightarrow$ rationality (and see what might be wrong).

Ex. 3 Suppose $X = \{x, y, z\}$, $\mathcal{B} = \{\{x,y\}, \{y,z\}, \{x,z\}\}$, $C(\{x,y\}) = x$, $C(\{y,z\}) = y$, and $C(\{x,z\}) = z$.

This satisfies WARP (no opp. for contradictions.)

To have rational $\succsim$, we would need $x \succ y$ and $y \succ z \Rightarrow x \succ z$ by transitivity, but this is contradicted by the above.

<u>Prop.</u> Arrow (1959)

If $(\mathcal{B}, C(\cdot))$ is a choice structure such that

i) WARP is satisfied

ii) $\mathcal{B}$ includes <u>all</u> subsets of X up to three elements

then $\exists$ a rational $\succsim$ that rationalizes $C(\cdot)$ relative to $\mathcal{B}$; that is $C(B) = C^*(B, \succsim)$ $\forall$ $B \in \mathcal{B}$. Further, this rational $\succsim$ is <u>unique</u>!

Proof: Go through yourself.