HW2

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ECON 7910 Econometrics

Due on Sep 30, 2021

1 Question – 3.1

Prove Lemma 3.1

Solution:

Since $N^0 x_N = x_N$, meanwhile, from the question, we know $x_N \xrightarrow{P} a$, where a is a constant. Thus $N^0 x_N$ is bounded. From these, we can get the conclusion: $x_N = O_p(1)$.

2 Question – 3.3

Prove under the assumption of lemma 3.4, we have $g(x_N) = O_p(1)$.

Solution:

Since from the contents of lemma, $x_N \xrightarrow{P} c$ and $g(x_N) \xrightarrow{P} g(c)$. Since $g(c)$ is bounded, which means $g(x_N)$ is bounded as well. $N^0 g(x_N)$ is bounded $\longrightarrow g(x_N) = O_p(1)$.

3 Question – 3.5

Solution:

1. The sample averge is: $\frac{\sigma^2}{N}$. Since $\sqrt{N}(\bar{y}_N - \mu)\ \mathcal{N}(0, \sigma^2)$. Thus, $Var(\sqrt{N}(\bar{y}_N - \mu)) = \sigma^2$.
2. The asymptotic variance is σ^2.
3. $AVar(\bar{y}_N) = \frac{\sigma^2}{N}$. $Var(\bar{y}) = \frac{\sigma^2}{N}$ from question (a). They are equal in this case.
4. The standard variance of $\bar{y}_N$ is $\frac{\sigma}{\sqrt{N}}$.
5. First, get the asymptotic variance of $\bar{y}_N$, then make square root of this variance to get asymptotic standard error.

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4 Question – 3.7

Solution:

1. From Slusky Theorem, $plim(\log(\hat{\theta})) = \log(plim(\hat{\theta}))$. Thus, $\hat{\gamma}$ is a consistent estimator of γ.

2. First, assume the asymptotic variance of $\sqrt{N}(\hat{\theta} - \theta)$ as $AVar(\sqrt{N}(\hat{\theta} - \theta))$. And, $\hat{\gamma} = \log(\hat{\theta})$ Thus, $AVar(\sqrt{N}(\hat{\gamma} - \gamma)) = AVar(\sqrt{N}(\log(\hat{\theta}) - \log(\theta)))$. By Delta Method:
$$Avar(\sqrt{N}(\hat{\gamma} - \gamma)) = \frac{1}{\theta} Avar(\sqrt{N}(\hat{\theta} - \theta)) \frac{1}{\theta} = (\frac{1}{\theta})^2 Avar(\sqrt{N}(\hat{\theta} - \theta))$$

3. $\hat{\theta} = 4$ and $se(\hat{\theta}) = 2$. The trick is assuming $Avar(\sqrt{N}(\hat{\theta} - \theta)) = \sigma^2$, Then, $se(\hat{\theta}) = (\frac{\sigma^2}{N})^{\frac{1}{2}} \longrightarrow \frac{\sigma^2}{N} = 4$. Thus, $\sigma^2 = 4N$. Go on our calculation $se(\hat{\gamma}) = (\frac{4N\frac{1}{\theta^2}}{N\theta^2}) = \frac{2}{\theta}$. Finally, plug $\theta = 4$ into $se(\hat{\gamma}) = \frac{2}{4} = \frac{1}{2}$.

4. For null hypothesis $H_0 : \theta = 1$, if we have been given some information from part(c), then $|t| = |\frac{1-4}{2}| = |\frac{-3}{2}|$. We need to compare the t value from computation with t-criterion table to determine in which confidence interval we reject our null hypothesis.

5. This question is a transformation one from one variable to another linear or non-linear variable, aka, Wald statistic. $H_0 : \gamma = 0$. $t = \frac{\log(4) - log(1)}{se(\hat{\gamma})} = \frac{\log(4)}{\frac{1}{2}} = 2\log(4)$. In the final step, we need to repeat the step we did in the last question, to compare t value with the value in the t-table given some confidence interval.

5 Question – 3.8

Solution:

1. Since $\hat{\theta}$ is the asymptotic normal estimation of θ, and $\hat{\gamma} = \frac{\hat{\theta_1}}{\hat{\theta_2}}$ is the estimation of $\gamma = \frac{\theta_1}{\theta_2}$. By the continuous function of θ.
$$plim\hat{\gamma} = plim\frac{\hat{\theta_1}}{\hat{\theta_2}} = \frac{\theta_1}{\theta_2} = \gamma$$

2. By delta method,
$$AVar(\hat{\gamma}) = (\frac{1}{\theta_2}, -\frac{\theta_1}{\theta_2^2}) Avar(\hat{\theta}) (\frac{1}{\theta_2}, -\frac{\theta_1}{\theta_2^2})'$$

3. Using the result of (b), we can derive:
$$se(\hat{\gamma}) = (\frac{Avar(\hat{\gamma})}{N})^{\frac{1}{2}} = \sqrt{71.2} \approx 8.438$$
Note: 1 sample size means $N = 1$.