Homework Solution-Selected Questions

Wei Ye*
Macroeconomics 1

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Macroeconomics 1 at Fordham University is not hard, and so for homework. Professor Moore gave us a bunch of questions and there was no need to submit. He also gave us solutions in scan version(bad), which is not a good reference for future students. I will select some questions from the questions sheet and do them by myself. All errors are my own. My principle is for questions with same logistics, I only do one.

1 Simple models of rational expectations equilibrium

1. (a) Since this geometric series has infinite terms, thus:

$$1 + \beta + \beta^2 + \beta^3 + ... = \frac{1}{1-\beta}$$

(b) This geometric series has **finite** terms, thus:

$$1 + \beta + \beta^2 + \beta^3 + ... + \beta^N = \frac{1-\beta^{N+1}}{1-\beta}$$

(c) This question is a little tricky, but it still can be solvable:

$$\begin{aligned}
\sum_{j=0}^{\infty} \beta^j j &= 0 + \beta + \beta^2 + 3\beta^3 + ... \\
&= \beta(1 + 2\beta + 3\beta^2 + 4\beta^3 + ...) \\
&= \beta((1 + \beta + \beta^2 + \beta^3 + ...) + (\beta + \beta^2 + \beta^3 + \beta^4) + (\beta^2 + \beta^3 + ...) + ...) \\
&= \beta((1 + \beta + \beta^2 + \beta^3 + ...) + \beta(1 + \beta + \beta^2 + ...) + \beta^2(1 + \beta + \beta^2 + \beta^3 + ...) + ...) \\
&= \beta((1 + \beta + \beta^2 + ...)(1 + \beta + \beta^2 + ...)) \\
&= \beta(\frac{1}{1-\beta} \cdot \frac{1}{1-\beta}) \\
&= \frac{\beta}{(1-\beta)^2}
\end{aligned}$$

*1st year PhD student in Economics Department at Fordham University. Email: wye22@fordham.edu

The crucial part of this question is to eliminate j and turn geometric series into what we are familar with like (a) or (b).

2. This question is backward question[1], I will solve it in a different method with what we learnt in class. In my way, you don't memorize any formulas derived in class, just make some simple computations. Let's go!

   (a) The equation means next period real assets is positively related to this periods assets plut its associated real interests, and current income, kicking off current period of consumption. From cash flow perspective, it's total cash inflow minus total cash outflow.

   (b) Derive the equation directly:

$$(1-(1+r)L)a_{t+1} = y_t - c_t$$

Thus:

$$a_{t+1} = (\frac{1}{1-(1+r)L})(y_t - c_t)$$

From the question, $r > 0$ is given, which means $1 + r > 1$. From the lecture, if the parameter before L in the denominator is bigger than 1, then we need to convert this problem to **Forward Solution Method**! Following what we derive above, and rearrange first:

$$a_{t+1} = (\frac{(1+r)^{-1}L^{-1}}{1-(1+r)^{-1}L^{-1}})(c_t - y_t)$$

The reason behind this is we need to apply the formula of sum of geometric series, if the denominator is $1 - x$, it's what we learnt in high school. If it's $x - 1$, we need to convet it to what we're familar with. Since it's forward method, LHS should be our current time instead of future time.

$$\begin{aligned}
a_t &= (\frac{1+r}{1-(1+r)^{-1}L^{-1}})(c_t - y_t)\\
&= \frac{1}{1+r}(\frac{1}{1-(1+r)^{-1}L^{-1}})(c_t - y_t)\\
&= \frac{1}{1+r}((c_t + (1+r)E_t c_{t+1} + (1+r)^2 E_t c_{t+2} + \ldots) - (y_t + (1+r)E_t y_{t+1} + (1+r)^2 E_t y_{t+2} + \ldots))\\
&= \frac{1}{1+r}(\sum_{j=0}^{\infty}(1+r)^j E_t(c_{t+j} - y_{t+j}))
\end{aligned}$$

[1]Actually, it's not. I assumed it's a backward question, but when I solved this quesiton, it violates the condition of backward question. So it should be classified as forward question. Yes, we are exploring from wrong assumption, then correct it. Good! One more, I forgot E_t when I first tried this question. We are not prophet, we don't have exact information about furture, if so, we would be billionaire. So, we use Expecation to express we would happen at current period given the information we have had. In mathematics, the information is called filtration.

It's obvious that the present discounted value consumption and the present discounted value of income are *positively* correlated. In economics, it means you only consume what you earn. If not, there will be ponzi scheme.[2]

3. Use law of iterated expectations (LIE) to solve REE Price model in class[3]. Before we solve this problem, let me reiterate the formula we will use in this quesiton: $P_t(1+r) = E_t(P_{t+1} + D_{t+1})$ (The logistics is simple, because the price today plus the potential interest are equal to future price and dividends. This is also assumed no arbitrage.)

   My Solution:

   Since $P_t(1+r) = E_t(P_{t+1} + D_{t+1})$, rearrange this equation would $P_t = \frac{1}{1+r}E_t(P_{t+1} + D_{t+1})$.

   Do some simple algebras by forwarding the equation:

$$P_{t+1} = \frac{1}{1+r}E_{t+1}(P_{t+2} + D_{t+2}) \tag{1}$$

$$P_{t+2} = \frac{1}{1+r}E_{t+2}(P_{t+3} + D_{t+3}) \tag{2}$$

   There are infinite equations, but I only write down two. Put the infinite equations back to the function of P_t.

$$\begin{aligned} P_t &= (\frac{1}{1+r})^n E_{t+n} ... E_t(P_{t+n}) + \sum_{i=0}^{n}(\frac{1}{1+n})^i E_t(\sum_{i=0}^{n} D_{t+i}) \\ &= (\frac{1}{1+n})^n E_t P_{t+n} + \sum_{i=0}^{n}(\frac{1}{1+n})^i E_t(\sum_{i=0}^{n} D_{t+i}) \\ &= 0 + \sum_{i=0}^{n}(\frac{1}{1+n})^i D_{t+i} \end{aligned}$$

   The second equation is by LIE, and the third equation is due to $E_t P_{t+n}$ is a constant number, if we discounted a constatn number from infinite future to today, it approximates to 0.

4. As before, the price equation is $(1+r)P_t = E_t(P_{t+1} + D_{t+1})$.

   (a) If there is MA(1) process: $D_t = \epsilon_t + \theta\epsilon_{t-1}$. Pluging this into our previous equation, it becomes:

$$(1+r)P_t = E_t(P_{t+1} + \epsilon_{t+1} + \theta\epsilon_t) \tag{3}$$

[2]See Dirk Krueger's lecture notes Chapter2, p19, version: 2012. `https://www.ssc.wisc.edu/~aseshadr/econ714/MacroTheory.pdf`

[3]Somehow I didn't get what the professor asked us to do. After reading his solution manual, I was still confused.

Rearrange this again:

$$P_t = (\frac{1}{1+r})E_t(P_{t+1} + \epsilon_{t+1} + \theta\epsilon_t) \tag{4}$$

The logic is the same with Question 3, then:

$$\begin{aligned} P_t &= (\frac{1}{1+r})^n E_t(P_{t+n}) + E_t(\sum_{i=1}^{n} \epsilon_{t+i} + \theta\epsilon_t) \\ &= 0 + 0 + \theta\epsilon_t \\ &= \theta\epsilon_t \end{aligned}$$

For the second equation, the second is 0 because $E_t(\epsilon_{t+i}) = 0 \ for \ \forall i \neq 0$. The third term is $\theta\epsilon_t$ because at time t, the information of ϵ_t is known, so it can be assumed as a number.

(b) Now the dividends function becomes AR(1) instead of MA(1): $D_t = \mu + \rho D_{t-1} + \epsilon_t$. $P_t = E_t(P_{t+1} + \mu + \rho D_{t-1} + \epsilon_t)$ Leave it intendedly, so tedious. Need to figure out later.

(c) If ϕ increases, the possibility of higher amount of dividends distributed increases as well. Because we know higher dividends implies less prices, so the future price will be affected negatively.

5. (a) For equation (1), we rearranging this equation to a new one, $\log(\frac{M_t}{P_t}) = \gamma - \alpha R_t$. Now it's clear! The log ratio of money supply over price level is negatively related with nominal interest rate, which means if If money supply increases, the corresponding nominal interest rate will decrease[4].

For equation (2), it's Fisher Equation. $E_t(p_{t+1} - p_t) = E_t(\log(\frac{P_{t+1}}{P_t}))$ to refect the expected inflation rate. So it's obvious that norminal interest rate is equationt to real interest rate plus expected inflation rate.

(b) In the quesiton, we have been given $m_t = \mu + \epsilon$. Rearranging our two equations to become:

$$\mu + \epsilon_t - p_t = \gamma - \alpha r - \alpha E_t(p_{t+1} - p_t) \tag{5}$$

Thus:

$$(1 - \frac{1+\alpha}{\alpha}L)E_t p_{t+1} = \frac{1}{\alpha}\gamma - \frac{1}{\alpha}\mu - r - \frac{1}{\alpha}\epsilon_t \tag{6}$$

Since as question 2, the parameter before L is $\frac{1+\alpha}{\alpha} > 1$ where $\alpha > 0$. We can't use backward method, instead, we should rely on foreward method to

[4] This conclusion is somehow confusing. However, let me explain it more intuitively. Money supply is for capital money supply, like saving amount in capital market. If saving amount is much higher, which means there is more redundent money in the market than actual needed, so interest rate will decrease to refect this relationship.

solve. Before doing computation, let's first derive : $\frac{1}{1-\frac{1+\alpha}{\alpha}L} = \frac{(\frac{1+\alpha}{\alpha})^{-1}L^{-1}}{1-(\frac{1+\alpha}{\alpha})^{-1}L^{-1}}$.
From equation (6):

$$E_t p_{t+1} = \frac{1}{\alpha}(\frac{1}{1-\frac{1+\alpha}{\alpha}L})\gamma - \frac{1}{\alpha}(\frac{1}{1-\frac{1+\alpha}{\alpha}L})\mu - \frac{1}{1-\frac{1+\alpha}{\alpha}L}r - \frac{1}{\alpha}(\frac{1}{1-\frac{1+\alpha}{\alpha}L})\epsilon_t \tag{7}$$

Thus, from equation (7):

$$E_t p_{t+1} = \frac{1}{\alpha}(\frac{(\frac{1+\alpha}{\alpha})^{-1}L^{-1}}{1-(\frac{1+\alpha}{\alpha})^{-1}L^{-1}})\gamma - \frac{1}{\alpha}(\frac{(\frac{1+\alpha}{\alpha})^{-1}L^{-1}}{1-(\frac{1+\alpha}{\alpha})^{-1}L^{-1}})\mu - \frac{(\frac{1+\alpha}{\alpha})^{-1}L^{-1}}{1-(\frac{1+\alpha}{\alpha})^{-1}L^{-1}}r - \frac{1}{\alpha}(\frac{(\frac{1+\alpha}{\alpha})^{-1}L^{-1}}{1-(\frac{1+\alpha}{\alpha})^{-1}L^{-1}})\epsilon_t \tag{8}$$

Multiply L on both sides of equation(**??**):

$$\begin{aligned} p_t &= \frac{1}{1+\alpha}(\frac{1}{1-(\frac{1+\alpha}{\alpha})^{-1}L^{-1}})\gamma - \frac{1}{1+\alpha}(\frac{1}{1-(\frac{1+\alpha}{\alpha})^{-1}L^{-1}})\mu - \frac{\alpha}{1+\alpha}(\frac{1}{1-(\frac{1+\alpha}{\alpha})^{-1}L^{-1}})r - \frac{1}{1+\alpha}(\\ &= \frac{1}{1+\alpha}(\frac{1}{1-(\frac{1+\alpha}{\alpha})^{-1}L^{-1}})(\gamma-\mu) - \frac{\alpha}{1+\alpha}(\frac{1}{1-(\frac{1+\alpha}{\alpha})^{-1}L^{-1}})r - \frac{1}{1+\alpha}(\frac{1}{1-(\frac{1+\alpha}{\alpha})^{-1}L^{-1}})\epsilon_t \\ &= \gamma - \mu - \alpha r - \frac{1}{1+\alpha}(\epsilon_t + (\frac{1+\alpha}{\alpha})^{-1}L^{-1}\epsilon_t + \ldots) \\ &= \gamma - \mu - \alpha r - (-\frac{1}{1+\alpha}\epsilon_t) + \sum_{i=1}^{\infty}(\frac{1+\alpha}{\alpha})^{-i}E_t(\epsilon_{t+i}) \\ &= \gamma - \mu - \alpha r + \frac{1}{1+\alpha}\epsilon_t \end{aligned} \tag{9}$$

The reason behind equation (9) is that ϵ_t is white noise, thus,$E_t(\epsilon_{t+i}) = 0$, where $i = 1, 2, \cdots$
Plug equation (9) into $R_t = r + E_t(p_{t+1} - p_t)$, so we can obtain:

$$R_t = r + \gamma - \mu - \alpha r - \gamma + \mu + \alpha - \frac{1}{1+\alpha}\epsilon_t = r - \frac{1}{1+\alpha}\epsilon_t$$

(c) Same with (b), but the expression of p_t is related to the monely supply of m_t instead of a constant like (b).

2 DGE Capital Accumulation, Part A

1. Sorry, I don't have textbook, so can't do this question.

2. (Additional Probelm 1) **Solution:**