HW10

Wei Ye[*]
ECON 5700

Due on August 25, 2021.

1   Question 1

Solution:

- $2 > 0$

- $2 - 1 = 1 > 0$

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$$-(-1)(-2+b) + 2(4-b^2) - (-1)(-2+b) = -2b^2 + 2b + 4$$

If $b \in (-1, 2)$, it's positive definite. if $b \in [-1, 2]$, it's positive semidefinite.

2   Question 2

Solution:

$$\begin{bmatrix} 4 & 12 & -16 \\ 12 & 37 & -43 \\ -16 & -43 & 98 \end{bmatrix} \xrightarrow[R_2 - 3R_1]{R_3 - (-4)R_1} \begin{bmatrix} 4 & 12 & -16 \\ 0 & 1 & 5 \\ 0 & 5 & 34 \end{bmatrix} \xrightarrow{R_3 - 5R_2} \begin{bmatrix} 4 & 12 & -16 \\ 0 & 1 & 5 \\ 0 & 0 & 9 \end{bmatrix} = U$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -4 & 5 & 1 \end{bmatrix}$$

Since from the conclusion we deduced in class: $S = L^T = \begin{bmatrix} 1 & 3 & -4 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$. ☺

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[*]I worked on my assignment sololy. Email: wye22@fordham.edu

3 Question 3

Solution:

$$A = LDL^T$$
$$= LddL^T$$
$$= R^T R$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -4 & 5 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 3 & -4 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 9 \\ 0 & 0 & 9 \end{bmatrix}$$

4 Question 4

Solution:

$$f(x, y) = x^2 + 3xy + y^2 - x + 3x$$
$$\text{s.t. } x + y - 1 = 0$$
$$\mathcal{L} = x^2 + 3xy + y^2 + 2x + \lambda(x + y - 1)$$
$$\frac{\partial \mathcal{L}}{\partial x} = 2x + 3y + 2 + \lambda = 0 \tag{1}$$
$$\frac{\partial \mathcal{L}}{\partial y} = 3x + 2y + \lambda = 0 \tag{2}$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = x + y - 1 = 0 \tag{3}$$

From these equations, we can deduce that $x^* = \frac{3}{2}, y^* = -\frac{1}{2}$.