Exercises #3

Instructions

Exercises #3 are due on Wednesday, February 9^{th}.

Exercises may be presented for credit as a hard copy at the end of the class meeting on the due date, or may be submitted electronically on Blackboard by the following Monday. If submitted on Blackboard, exercises should be attached as a Portable Document Format (`*.pdf`) file. It is possible to convert handwritten work to `*.pdf` using scanner or a camera-equipped device with Microsoft Office Lens (Android, iOS, or Windows), Google Drive (Android), or Apple Notes (iOS).

Exercises are "collaborative and open book" assignments. You are encouraged to make use of help from your peers, textbook, notes, and me, but you must submit your own answers. There is no penalty for incorrect answers; the expectation is simply for you to progress as far as you can on each question. Complete answers with explanations will be provided in recitation.

Questions

3.B.3 Draw a convex preference relation that is locally nonsatiated but not monotone.

3.C.2 Show that if $u(\cdot)$ is a continuous utility function representing $\succsim$, then $\succsim$ is continuous.

3.C.6 Suppose that in a two-commodity world, the consumer's utility function takes the form $u(x) = [\alpha_1 x_1^\rho + \alpha_2 x_2^\rho]^{\frac{1}{\rho}}$. This utility function is known as the *constant elasticity of substitution* or *CES* utility function.

(a) Show that when $\rho = 1$, indifference curves become linear.

(b) Show that as $\rho \to 0$, this utility function comes to represent the same preferences as the (generalized) Cobb-Douglas utility function $u(x) = x_1^{\alpha_1} x_2^{\alpha_2}$.

(c) Show that as $\rho \to -\infty$, indifference curves become "right angles"; that is, the utility function has in the limit the same indifference map as the Leontief utility function $u(x_1, x_2) = \min\{x_1, x_2\}$.

3.D.5 Consider again the CES utility function of Exercise , and assume that $\alpha_1 = \alpha_2$. For simplicity, use the monotone transformation $\tilde{u}(x) = \rho u(x)^\rho = \rho\,[\alpha_1 x_1^\rho + \alpha_2 x_2^\rho]$.

(a) Compute the Walrasian demand and indirect utility functions for this utility function.

(b) Verify that these two functions satisfy all of the properties of Propositions 3.D.2 and 3.D.3.

(c) Derive the Walrasian demand correspondence and indirect utility functions for the case of linear utility and the case of Leontief utility. (See Exercise). Show that the CES Walrasian demand and indirect utility functions approach these as ρ approaches 1 and $-\infty$, respectively.

(d) The *elasticity of substitution* between two goods is defined as

$$\xi_{12}(p, w) = \frac{\partial x_1(p, w)/x_2(p, w)}{\partial p_1/p_2} \frac{p_1/p_2}{x_1(p, w)/x_2(p, w)}.$$

Show that for the CES utility function, $\xi_{12}(p, w) = 1/(1-\rho)$, thus justifying its name. What is $\xi_{12}(p, w)$ for the linear, Leontief, and Cobb-Douglas utility functions?

3.D.6 Consider the three-good setting in which the consumer has utility function $u(x) = (x_1 - b_1)^\alpha (x_2 - b_2)^\beta (x_3 - b_3)^\gamma$.

(a) Can we assume that $\alpha + \beta + \gamma = 1$ without loss of generality? Do so for the rest of the problem.

(b) Write down the first-order conditions for the UMP, and derive the consumers' Walrasian demand and indirect utility functions. This system of equations is known as the linear expenditure system is do to Stone (1954).