Exercises #5

Instructions

Exercises #5 are due on Wednesday, February 23rd.

Exercises may be presented for credit as a hard copy at the end of the class meeting on the due date, or may be submitted electronically on Blackboard by the following Monday. If submitted on Blackboard, exercises should be attached as a Portable Document Format (`*.pdf`) file. It is possible to convert handwritten work to `*.pdf` using scanner or a camera-equipped device with Microsoft Office Lens (Android, iOS, or Windows), Google Drive (Android), or Apple Notes (iOS).

Exercises are "collaborative and open book" assignments. You are encouraged to make use of help from your peers, textbook, notes, and me, but you must submit your own answers. There is no penalty for incorrect answers; the expectation is simply for you to progress as far as you can on each question. Complete answers with explanations will be provided in recitation.

Questions

4.B.1 Prove the sufficiency part of Proposition 4.B.1.

> A necessary and sufficient condition for the set of consumers to exhibit parallel, straight wealth expansion paths at any price vector p is that preferences admit indirect utility functions of the Gorman form with the coefficients w_i the same for every consumer i. That is:
>
> $$v_i(p, w_i) = a_i(p) + b(p)w_i.$$

Show also that if preferences admit the Gorman-form indirect utility functions with the same $b(p)$, then preferences admit expenditure functions of the form $e_i = (p, u_i) = c(p, u_i) + d_i(p)$.

4.C.3 Give a graphical two-commodity example of a preference relation generating a Walrasian demand that does not satsify the ULD property. Interpret.

4.C.11 Suppose that there are two consumers, 1 and 2, with utility functions over two goods, 1 and 2, of $u_1(x_{11}, x_{21}) = x_{11} + 4\sqrt{x_{21}}$ and $u_2(x_{12}, x_{22}) = 4\sqrt{x_{12} + x_{22}}$. The two consumers have identical wealth levels, $w_1 = w_2 = w/2$.

(a) Calculate the individual demand functions and the aggregate demand functions.

(b) Compute the individual Slutsky matrices $S_i(p, w/2)$ (for $i = 1, 2$) and the aggregate Slutsky matrix $S(p, w)$. [*Hint*: Note that for this two-good example, only one element of each matrix must be computed to determine the entire matrix.] Show that $dp \cdot S(p, w)dp < 0$ for all $dp \neq 0$ not proportional to p. Conclude that aggregate demand satisfies WARP.