

ECON 7010 Microeconomics 2 – Midterm Revision

Wei Ye

May 12, 2022

1. Revision to Question 6:

From (Mas-Colell et al. (1995)) Page 191: there are five cases for two bernoulli utility functions u_1 and u_2 , such that $u_2(\cdot)$ is more risk averse than $u_1(\cdot)$:

- 1.) $r_A(x, u_2) \geq r_A(x, u_1)$. Where $r_A(x)$ measures the Arrow-Pratt coefficient risk aversion at x .
- 2.) There exists an increasing concave function $\Phi(\cdot)$ such that $u_2(\cdot) = \Phi(u_1(x))$ at all x ; that is $u_2(\cdot)$ is a concave function of $u_1(\cdot)$, aka, $u_2(\cdot)$ is more concave than $u_1(\cdot)$
- 3.) $\pi(x, \epsilon, u_2) \geq \pi(x, \epsilon, u_1)$ for any x and ϵ . Where $\pi(\cdot)$ is probability premium.
- 4.) $c(F, u_2) \leq c(F, u_1)$ for any $F(\cdot)$.
- 5.) Whenever $u_2(\cdot)$ finds a lottery $F(\cdot)$ at least as good as a riskless outcome \bar{x} , then $u_1(\cdot)$ also finds $F(\cdot)$ at least as good as \bar{x} . That is, $\int u_2(x)dF(x) \geq u_2(\bar{x})$ implies $\int u_1(x)dF(x) \geq u_1(\bar{x})$ for any $F(\cdot)$ and \bar{x} .

Mistake analysis:

First, I didn't read the chapter 6 of the textbook carefully. Second, I believed that I can use First and second dominance theorem to depict the risk averse level. But obviously, it's a huge mistake, Third, I had a bad understanding of what are the conditions of risk-averse. Lastly, during the exam, I only knew if one utility function is more concave than the other, it should be more risk-averse. There are some conclusions associated with this: like $c(\cdot)$, and A-P risk coefficient and probability premium. That's what i did in the exam, however, i missed some conditions that are prerequisite to these.

2. Revision to Question 7:

- i.) Since $u(w) = -r_i w^2 + w$, we can derive first and second order derivative of u w.r.t w :

$$\begin{aligned} u'(w) &= -2r_i w + 1 \\ u''(w) &= -2r_i \end{aligned} \tag{1}$$

Because if the utility function is concave, the second order derivative should be less than 0. Thus, from (equation (1)), $-2r_i < 0 \rightarrow r_i > 0$.

- ii.) This part is easy to prove: Since $w < \frac{1}{2r_i}$, as $w \geq 0$. at least RHS is larger than 0, which means r_i is larger than 0 in which it satisfies the restriction indicated in part one.

The absolute risk aversion:

$$A(w) = -\frac{u''(w)}{u'(w)} = \frac{2r_i}{-2r_i w + 1} \quad (2)$$

Since $w < \frac{1}{2r_i} \rightarrow 1 - 2r_i w > 0 \rightarrow \text{equation}(2) > 0$.

$$\frac{\partial A(w)}{\partial w} = \frac{4r_i^2}{(-2r_i w + 1)^2} > 0 \quad (3)$$

Therefore, it cannot represent preferences that display decreasing absolute risk aversion. It's actually increasing absolute risk aversion.

- iii.) Quadratic utility function has been widely used in finance theory especially in financial economics¹ as the expected utility can be defined in the form of its means and variances².

Analysis: I made a stupid mistake during part 1 because of stupid algebra mistake. The reason why my original solutions to part 2 is wrong is that I just learnt the definition of coefficient of absolute prudence in macro class, SO when i saw absolute risk aversion, I thought you wanted us to compute absolute prudence. Meanwhile, I was assuming there is only relative risk aversion aka, arrow-Pratt **RELATIVE** risk aversion. For part c, I literally didn't know how to answer in the exam, it belongs to the classification I gave up. Now, after looking up some materials, all the questions and problems are clear. I know why you gave us this question on the exam. Because quadratic utility has been widely used in financial economics(You taught this class last semester.), and this question can be assumed as an illuminating question to pave the way to understanding financial economics in the future class. Very good question to appear in the exam, even though I lost points on the exam, it's a good way to learn.

3. Revision to question 5: **The study of two goods presumed to be gross complements.**

This is no need to correct. Like in the email you sent me a few weeks ago. I need to cross out the solutions i don't want next time in order to avoid some misunderstanding.

¹See the link: <https://www.d42.com/portfolio/analysis/quadratic-utility>

²Same reference source: see the above link and its footnote 9.

Bibliography

Mas-Colell, Andreu, Michael Dennis Whinston, Jerry R Green et al. (1995) *Microeconomic theory*, 1: Oxford university press New York.