Monetary Policy Regimes and Inflation in the New-Keynesian Model.

Bartholomew Moore
Department of Economics
Fordham University
441 East Fordham Road
Bronx, NY 10458
718-817-4049
bmoore@fordham.edu

July 31, 2013

JEL Classification codes: E52, E31, D84
Keywords: Monetary Policy, Markov Switching, Inflation Persistence, Expectations

Abstract

This paper shows that plausible modifications to the Taylor rule for monetary policy can help explain several empirical anomalies to the behavior of inflation in the new-Keynesian general equilibrium model. The key anomalies considered are (1) the persistence of inflation, both in reduced form and after conditioning on inflation's driving processes, (2) the positive correlation between the output gap and the change in the inflation rate, and (3) the apparent bias in survey measures of expected inflation.

The Taylor rule in this model includes the now standard assumption that the central bank smoothes changes to its target interest rate. It also includes Markov switching of a persistent inflation target between a low target rate and a high target rate. The model is calibrated to match Benati's (2008) result that, historically, changes in monetary policy lead to a statistically significant change in the persistence of inflation.

Matching Benati's result requires a reduction in an exogenous, hence structural, source of persistence. However, inflation in the model inherits additional, non-structural, persistence from the process that governs the inflation target. As a result, the model is able to replicate measures of inflation persistence, even after conditioning on inflation's driving processes. Agents with rational expectations and knowledge of the current inflation target will be aware of the possibility of a future target switch, causing their expectations to appear biased in small samples. Finally, with sticky nominal prices a discrete drop to the low-inflation target requires a loss of output while previously-set prices adjust.

I. Introduction.

The new-Keynesian Phillips curve, and in particular the Calvo (1983) aggregate supply curve, is very widely used in general equilibrium models of monetary phenomena. This is because it provides a description of sticky nominal prices that is both tractable and well-grounded in microeconomic theory. Unfortunately, there are a number of stylized empirical facts that are difficult to explain as the endogenous outcome of models that include the new-Keynesian Phillips curve.

Perhaps the most widely studied of these facts is the persistence of inflation. In a pure version of the Calvo aggregate supply curve, one derived solely from the profit maximizing behavior of monopolistically competitive firms, inflation is a forward-looking variable. That is, current inflation depends on expected future inflation and on current marginal costs, lagged inflation should have no explanatory power. Yet, empirical studies consistently show that inflation is persistent: lagged values of inflation have a statistically significant effect on current inflation. One possible explanation is that inflation inherits persistence from its driving variable, marginal costs, or from serially correlated “cost-push” shocks. Indeed, general equilibrium models designed to give a realistic representation of the data usually include features that impart persistence to inflation’s driving variables. For example, habit persistence causes optimal consumption to depend on lagged as well as expected-future consumption. Consequently, current output will depend on lagged and expected future output. Since marginal cost is proportional to output, marginal cost will be persistent and inflation may inherit that persistence.

The persistence puzzle has also therefore deepened by results in, for example, Fuhrer (2009), Kiley (2007), and especially Rudd and Whelan (2005, 2006) showing that lags of inflation retain their power to explain current inflation even after conditioning on variables such as unemployment, output, marginal cost, or the present discounted value of expected future marginal costs. In other words, the observed persistence of inflation does not appear to be inherited from output or from marginal costs.

Another common modification to sticky-price models that imparts persistence to inflation is to assume that those firms unable to reset their price optimally in a given period will instead update their price using a rule of thumb, for example in proportion to the CPI.[1] This brings lagged inflation directly in to the aggregate supply (AS) equation and renders inflation persistent, although it does so by assumption diminishing the appeal of the resulting AS equation by undermining its microeconomic foundation.

However, the persistence puzzle has again been deepened by results in Benati (2008) showing that historically a change in monetary policy often leads to a statistically significant change in the persistence of inflation. Benati concludes that "inflation persistence is not structural in the sense of Lucas (1976)", that is, it is not invariant with respect to changes in monetary policy. If the persistence properties of inflation change when monetary policy changes, inflation persistence cannot solely be caused by structural sources such as rule-of-thumb price setting, habit persistence, or, importantly, from serially correlated shocks.

Inflation persistence is not the only empirical fact anomalous to general equilibrium models based on the new-Keynesian Phillips curve. Another anomalous stylized fact is the acceleration phenomenon, the observed positive correlation between the change in the inflation rate and the GDP gap. Mankiw and Reiss (2002, p.1297) refer to the acceleration phenomenon as "the central finding from the empirical literature on

[1] See, for example, Christiano, Eichenbaum, and Evans (2005)

the Phillips curve …" and argue that the acceleration phenomenon is inconsistent with the standard sticky-price model.

The difficulties explaining the persistence of inflation, the persistence of inflation after conditioning, and the acceleration phenomenon as endogenous outcomes of a rational expectations sticky-price model has led several authors to suggest departures from rational expectations. Kiley (2007) argues that the assumption that some firms are rule-of-thumb price setters, discussed above, can be viewed as one such departure. Fuhrer (2009) and Rudd and Whelan (2006) both suggest that adaptive expectations or adaptive learning could explain the persistence of inflation. Milani (2007) shows that when agents learn the behavior of inflation, interest rates, and output using a constant-gain algorithm the need for other sources of persistence – for inherited persistence – is reduced. Mankiw and Reiss (2002) replace sticky prices with sticky expectations, that is, the assumption that agents form rational expectations but only update their information periodically, to explain a number of key stylized facts; including the persistence of inflation, the acceleration phenomenon, and the fact that disinflation causes recession.

Alternatives to rational expectations would seem to gain support from evidence of bias in survey measures of expected inflation. Survey measures of expected inflation appear to systematically underestimate inflation when it is rising and overestimate inflation when it is falling.[2] However, departures from rational expectations diminish the original appeal of the Calvo (1983) aggregate supply curve, because they undermine the discipline imposed by full rationality. Furthermore, the idea that the apparent bias in survey measures of expected inflation must represent a departure from rational

[2] Regarding the apparent bias in inflation forecasts see, for example, Evans and Wachtel (1993) or Thomas (1999).

expectations is not fully supported either by theory or by the data. Theoretically, deviations from rational expectations are by no means necessary to explain the appearance of bias. Evans and Wachtel (1993) argue that, if the inflation rate follows a two-state Markov switching process, expectations of inflation will appear biased even if they are formed rationally. Erceg and Levin (2003) develop a general equilibrium model in which the target inflation rate is governed by an unobserved near random walk and optimal forecasts are formed using the Kalman filter. In their model a large and persistent decline in the target inflation rate, such as occurred during the Volcker period, will lead to a sacrifice ratio near empirical estimates and to rational expectations that over predict a falling inflation rate. Andolfatto, Hendry, and Moran (2008), also show that the filtering problem that results when the inflation target is an unobserved random draw can, in the context of a general equilibrium model, explain the apparent bias of inflation expectations. Empirically, Ang, et al, (2007) show that survey measures of expected inflation are better at forecasting actual inflation than asset markets, time series models, or Phillips-curve regressions that predict inflation from measures of real economic activity.

In this paper, I argue that several of the stylized facts that appear anomalous to the new-Keynesian general equilibrium model can to a significant extent be explained by two highly plausible modifications to the central bank's monetary policy rule, interest-rate smoothing and inflation-target switching. Interest- rate smoothing is the now broadly accepted idea that the Fed adjusts the nominal federal funds rate towards its ultimate target gradually. This is modeled by assuming that, in the Taylor (1993) rule describing the nominal federal funds rate, the current fed funds rate is a weighted average of the lagged fed funds rate and the ultimate target rate; the weights summing to one. Inflation-target switching is here modeled by assuming that the target inflation rate is a two-state Markov switching process, so that the fed switches between a high-inflation-target policy and a low-inflation-target policy.

Using a simple version of the standard new-Keynesian model, I show that interest-rate smoothing and inflation-target switching will cause inflation in the model to exhibit persistence, even after conditioning on output or the present value of expected future marginal costs. Also, in the model output and the change in inflation are positively correlated thus replicating the acceleration phenomenon. I show that agents with fully rational expectations will appear to underestimate inflation when it is rising and overestimate inflation when it is falling. Finally, a switch from the high inflation target to the low inflation target leads to a loss of output so that disinflation causes recession.

The rest of the paper is organized as follows. Section two explains the model. Section three discusses the calibration. Section four presents and explains results from simulations of the model. Section five concludes. The model solution and more detailed explanation of the timing and information assumptions are given in Appendix B.

II. The Model.

I use a simple version of the standard new-Keynesian general equilibrium model. It consists of a Calvo (1983) aggregate supply curve, a linearized intertemporal Euler equation, and Taylor (1993) rule describing the behavior of the nominal federal funds rate.[3] The model is not intended to be a fully realistic description of the macro economy.

[3] Except for the inflation targets in the two-state version of the model all constants, including the steady state real interest rate, are normalized to zero.

Features essential to a complete description of the data are not included and exogenous sources of persistence are limited to serially correlated shocks. The model includes only those features necessary to show how interest-rate smoothing, inflation-target switching, and a moderate amount of exogenous persistence can interact to explain the stylized facts that are the subject of this paper.

I assume a continuum of monopolistically competitive firms, each producing a unique variety of output and facing a downward sloping demand curve. Each period a fraction of firms, $(1-\theta)$ where $0 \le \theta < 1$, is selected at random and allowed to reset their price. Thus, in every period, the fraction of firms that hold the nominal price of their output fixed is θ. A firm that changes its price recognizes that it will have to maintain its new price in each subsequent period with probability θ. Firms choose their nominal price to maximize the expected present discounted value of profits subject to this price-changing constraint. Assuming that capital is fixed and that marginal cost is proportional to the output gap, inflation will evolve according to

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + v_{st} . \tag{1}$$

Here π denotes inflation, x denotes the output gap (i.e., the deviation of log real GDP from its trend), β is the discount factor, $\kappa = (1-\theta)(1-\beta\theta)/\theta$,[4] and v_{st} is a stationary AR(1) shock:

$$v_{st} = \rho_s v_{st-1} + \varepsilon_{st} , \tag{2}$$

with $0 \le \rho_s < 1$ and $\varepsilon_{st} \sim$ i.i.d. $\mathrm{N}\left(0,\sigma_s^2\right)$. The shock, v_{st}, is sometimes referred to as a "cost-push" shock.

---

[4] This formula for κ assumes log utility for consumption and that labor supply is perfectly elastic. See Walsh (2003) pp. 232-39.

The representative household maximizes the expected present discounted value of its utility over an infinite horizon. Each period, utility is an increasing and separable function of a consumption aggregator and of real money balances. Utility is CES in the consumption aggregator with the coefficient of relative risk aversion denoted by σ. Assuming a closed economy, and a fixed capital stock (as mentioned above), and abstracting from the role of government expenditure, a log linearization of the intertemporal optimality condition yields

$$x_t = E_t x_{t+1} + \left(\frac{-1}{\sigma}\right) E_t \left(i_t - \pi_{t+1}\right) + v_{dt} \,. \tag{3}$$

Here i_t denotes the nominal interest rate, which in this model is the nominal federal funds rate, and v_{dt} is another a stationary AR(1) shock:

$$v_{dt} = \rho_d v_{dt-1} + \varepsilon_{dt} , \tag{4}$$

with $0 \le \rho_d < 1$ and $\varepsilon_{dt} \sim$ i.i.d. $\mathrm{N}\left(0, \sigma_d^2\right)$. Note that the placement of the expectations operator in the second term on the right hand side of equation (3) allows for the possibility that households and firms make their period t decisions before observing the current nominal federal funds rate.[5] Note also that equations (2) and (4) describe exogenously given persistent processes.[6]

The nominal federal funds rate is given by

$$i_t = \rho_F i_{t-1} + \left(1 - \rho_F\right) i_t^* + \varepsilon_{mt} , \tag{5}$$

where $0 \le \rho_F < 1$,

---

[5] This timing assumption is consistent with the recursiveness assumption often used in VARs to identify monetary policy shocks. See for example, Bernanke and Blinder (1992) or Bernanke and Mihov (1998).

[6] Equations (1) and (3) can be rigorously derived from the profit maximizing behavior of firms and the utility maximizing behavior of households. That derivation is not the focus of this paper and is relatively standard. The interested reader is referred to Gali (2008) or Walsh (2003, Ch. 5.4).

$$i_t^* = \pi^T(s_t) + \gamma_\pi \left[\pi_t - \pi^T(s_t)\right] + \gamma_x x_t , \tag{6}$$

and

$$\varepsilon_{mt} = \sigma_m(s_t)\varepsilon_t , \quad \varepsilon_t \sim i.i.d. N(0,1) . \tag{7}$$

In the above equations, i^* denotes the *eventual* target federal funds rate, ρ_F the degree of interest rate smoothing, and $\pi^T(s_t)$ the target inflation rate. The target inflation rate depends on the state variable, s_t. I assume that $\gamma_\pi > 1$, consistent with the Taylor principle, and that $\gamma_x \geq 0$. The Fed sets i_t^* to keep inflation near its target rate and if $\gamma_x > 0$ to keep the output gap near zero. From equation (6) we see that the Fed will increase i_t^* if π_t increases. If $\gamma_x > 0$ the Fed will also increase i_t^* in response to an increase in x_t. If $\rho_F = 0$ the Fed sets the fed funds rate at i_t^* in the current period. If on the other hand $\rho_F > 0$ the Fed moves towards its eventual target interest rate more gradually. Note from equation (5) that the realized fed funds rate is disturbed by the shock ε_{mt}, whose variance also depends on the state.

The random state variable, s_t, which governs $\pi^T(s_t)$ and $\sigma_m(s_t)$, takes the value 1 or 2, $s_t \in \{1,2\}$, and follows a Markov-switching process with transition probabilities $p_{ij} = \Pr\left[s_t = j \mid s_{t-1} = i\right]$. In what follows $s_t = 1$ will denote the low-inflation-target (and low-variance) state and $s_t = 2$ will denote the high-inflation-target (and high-variance) state. Thus, by assumption, $\pi^T(1) \leq \pi^T(2)$ and $\sigma_m(1) \leq \sigma_m(2)$.

I examine four versions of the monetary policy described by equations (5) through (7). These four models are summarized in Table 1 in order of increasing complexity. In each of these versions, all agents are assumed to know the structure and parameters of the model. In presenting my results I will emphasize the two versions of the model with Markov switching in the inflation target and in the variance of ε_{mt}. In the first of these, which I refer to as the *state-known* model, the monetary policy state, s_t, is observed by all agents in each period before market equilibrium is determined.[7] In the second of these, which I refer to as the *learning* model, households and firms do not observe s_t, but must infer its value from their observations of i_t.

In attempting to understand my results I also consider two versions of the model without policy switching, that is, with $\pi^T(1) = \pi^T(2) = \pi^T$ and $\sigma_m(1) = \sigma_m(2) = \sigma_m$. In one of these, which I call the *smoothing-only* model, $\rho_F > 0$. In the other, which I call the *simple* model, $\rho_F = 0$, so that the Fed does not smooth changes to the target interest rate.

III. Calibration.

I calibrate the model to match a key stylized fact, that the persistence of inflation is not structural in the sense of Lucas (1976). Rather, a change in monetary policy will lead to a significant change in Benati's (2008) reduced-form measure of inflation persistence.

The calibration of the model is summarized in Table 2. I calibrate to a quarterly frequency and, in the baseline calibration, choose values for β, σ, and θ (hence κ) that

---

[7] In simulations of the state-known model, it makes little difference whether s_t is observed before or after market equilibrium if determined. This is because the calibrated values of the transition probabilities are quite high, $p_{11} = .97$ and $p_{22} = .95$, so knowing last period's state allows agents to predict this period's state with a high degree of accuracy.

are standard in the literature. I set $\beta = 0.99$, implying a steady state real interest rate of approximately four percent annually, and set $\sigma = 1$ implying log utility. I assume that each period one third of firms reset their nominal price optimally so that $\theta = 2/3$ and $\kappa = 0.17$. Based on the estimates of Clarida, Gali, and Gertler[8] (2000), I set $\gamma_\pi = 1.7$, $\gamma_x = 0.5$, and $\rho_F = 0.75$. Using the estimates of Schorfheide (2005) I set $\pi^T(2) - \pi^T(1) = 5$, $p_{11} = 0.97$, and $p_{22} = 0.95$. I set $\pi^T(2)$ and $\pi^T(1)$ so that their unconditional mean is zero.[9] Again using the estimates of Schorfheide (2005) I set $\sigma_s = 0.46$, $\sigma_d = 0.09$, $\sigma_m(1) = 0.65$ and $\sigma_m(2) = 1.65$. For versions of the model without switching, I set the target inflation rate to zero and σ_m =1.025, which is the unconditional mean of $\sigma_m(1)$ and $\sigma_m(2)$.

The endogenously determined equilibrium value of π_t will inherit some or all of the persistence that is assumed for the exogenous shocks, v_{dt} and v_{st}. Indeed, if I calibrate the model based on Schorfheide's (2005) estimates and use $\rho_d = 0.8$ and $\rho_s = 0.98$ inflation exhibits substantial persistence in all four versions of the model by any of the several measures of persistence discussed below. Interestingly, in the model inflation inherits most of its persistence from v_{st}. For the values of $\beta, \theta, \kappa, \sigma_s,$ and σ_d considered here variation in ρ_d has only a small effect on the persistence of inflation. This is partly because κ is relatively small (though larger than many empirical estimates) and also because σ_s is five times σ_d, so that most of the persistent variation in π_t is due to v_{st}. In practice, my objective of calibrating the exogenous sources of persistence to match

[8] Schorfheide (2005) obtains similar estimates of γ_π and ρ_F, but holds $\gamma_x = 0$ by assumption.

[9] The unconditional probability that $s_t = 1$ is $(1 - p_{22})/(2 - p_{11} - p_{22})$. See Hamilton (1994, p 683).

Benati's (2008) result therefore means reducing ρ_s until the artificial data generated from simulations of the model replicates the Benati result.

Benati (2008), in his reduced-form test, considers the regression

$$\pi_t = a_0 + \sum_{j=1}^{n} a_j \pi_{t-j} + e_{at} \ . \qquad (8)$$

He examines several historical episodes in which monetary policy has changed, including several episodes in which a country has adopted a single inflation target well-known to the public. He then tests whether there was a statistically significant change in $\hat{A}(1) \equiv \sum_{j=1}^{n} \hat{a}_j$, the sum of the estimated coefficients on lagged inflation. For most historical episodes of a change in monetary policy he does find a significant change in $\hat{A}(1)$. He concludes that "inflation persistence is not structural in the sense of Lucas (1979)".

In the model considered here, the adoption of a single inflation target is represented by a change from the state-known version of the model to the version with a single inflation target and interest rate smoothing, the smoothing-only model. Such a change assumes only that the central bank replaces its two randomly chosen but highly persistent inflation targets with a single constant target. In modeling this change I assume that there is no change in the interest-rate smoothing parameter, ρ_F. It is important to note that in both of these two versions of the model, state-known and smoothing-only, the Fed's current inflation target is known to all agents.

I seek to calibrate ρ_s so that the model replicates Benati's result. To do this I examine, for different values of ρ_s, the probability that a change in policy from the state-known (i.e., two-target) policy to the smoothing-only (single-target) policy would cause a significant change in $\hat{A}(1)$. To calibrate ρ_s I therefore proceed as follows. I set all parameters other than ρ_s to the values specified in Table 2. I then choose a value for ρ_s. I begin from $\rho_s = 0.98$ and then reduce ρ_s along the grid of values .98, .95, .90, .85, … . For each value of ρ_s I simulate the state-known version of the model. Since below I will compare the model to the US sample from 1968:4 to 2009:3, in each repetition I simulate the model for 164 periods (quarters). I set the state variable in these simulations exogenously to match the pattern in the US data. Schorfheide (2005) estimates that the US economy was in the high-inflation-target regime from 1974:1 through 1982:3, but was otherwise in the low-target regime. I therefore set $s_t = 2$ for periods 22 through 56 and set $s_t = 1$ for all other periods.

In each repetition I use the artificial data to estimate Benati's reduced form regression, equation (8) above, with n = 4, and record the estimated value, $\hat{A}(1)$. I repeat this exercise 10,000 times and, using the 10,000 estimates of $A(1)$ find the cutoff value, call it $A_C(1)$, such that the estimate of $A(1)$ is greater than $A_C(1)$ in 95% of the repetitions. I then simulate the smoothing-only version of the model for the same parameter values (except, of course, I set $\pi^T(1) = \pi^T(2) = 0$ and $\sigma_m(1) = \sigma_m(2) = 1.025$). Again I simulate the model for 10,000 repetitions of 164 periods each and, again I use the artificial data from each repetition to estimate $A(1)$. Since the objective is to find a value of ρ_s that virtually guarantees that there will be a significant change in $\hat{A}(1)$, I then ask, what percent of the smoothing-only repetitions generate an estimate of $A(1)$ that is below the critical value, $A_C(1)$, obtained from the state-known version of the model?

The results of this exercise are reported in Table 3. Note from Table 3 that for $\rho_s = 0.98$ or 0.95 there is little difference in the estimates of $A(1)$ along this dimension. From the perspective of this test with $\rho_s = 0.98$ or 0.95 inflation persistence would appear to be structural, contradicting Benati's (2008) result. Note further that the percent of the smoothing-only estimates of $A(1)$ that are below $A_C(1)$ is monotonically decreasing in ρ_s. Since 99.13% of the estimates of $A(1)$ from the smoothing-only model are below $A_C(1)$ when $\rho_s = 0.70$ it is higly likely that the Benati test would show a significant change in inflation persistence as a consequence of the assumed change in monetary policy. I therefore use $\rho_s = 0.70$ in the simulations that follow.[10]

IV. Simulation Results.

In what follows I discuss several summary statistics obtained from the US data and the corresponding statistics obtained from simulations of the model. I simulate each of the four versions of the model. The parameters are the same in each version of the model except for the differences noted in Table 1. As with the Benati tests above I simulate the model for 10,000 repetitions of 164 periods each. I set $s_t = 2$ for periods 22 through 56, corresponding to 1974:1 to 1982:3 and set $s_t = 1$ for all other periods. For the summary statistics considered below I report the median value over these 10,000 repetitions.

[10] All of the results reported below are very similar if I set $\rho_s = 0.65$ or 0.75.

A. Autocorrelation of inflation and reduced-form persistence.

Fuhrer (2009) reviews several measures of inflation persistence and argues that one of the most useful reduced-form measures is the autocorrelation function. In Table 4, column 2, I list the autocorrelations of inflation in the sample at lags 1 through 6, 8, and 12. These reveal a high degree of persistence. The first autocorrelation is close to 0.9 and the autocorrelation of inflation declines slowly, remaining above 0.5 even after three years (twelve quarters).

The third column of Table 4 shows the median autocorrelation at each lag in the simple version of the model, the version without interest -rate smoothing or inflation-target switching. Note that the first autocorrelation is 0.686, slightly below the autocorrelation assumed for the exogenous shock v_s. Furthermore, the autocorrelations of inflation decline geometrically, roughly as powers of the first-order autocorrelation. Thus the reduced-form persistence of inflation in the simple model appears to be almost entirely inherited from the exogenous process for v_s.

The autocorrelations for the three other versions of the model, smoothing-only, state-known, and learning, are reported in columns (4) through (6) of Table 4, respectively. In the smoothing-only model and in the simple model, the moderate degree of exogenous persistence leaves the autocorrelations of inflation substantially below those from the data. This is especially true at the third lag and beyond. Somewhat counter intuitively, the simulations show that a policy of interest-rate smoothing leads to a reduction in inflation persistence.

However, inflation in the models with switching inherits persistence both from the moderately persistent cost-push shocks and from the highly persistent inflation targets. Consequently, inflation in the state-known model and, to a lesser extent, the learning model exhibits a relatively high degree of autocorrelation even at the longer lags. The median first-order autocorrelation of inflation in the state-known model is 0.770, only a bit below the corresponding value from the data and, as in the data, there is substantial autocorrelation of inflation into the twelfth lag.

Thus, in this calibration, a change in monetary policy would cause a significant decline in reduced-form measures of inflation persistence but the versions of the model with inflation-target switching, especially the version in which the inflation target is known, retain the ability to replicate the degree of inflation autocorrelation observed in the data.

B. Inflation regressions and persistence after conditioning.

It is well known[11] that it is possible to construct a general equilibrium model in which inflation exhibits reduced-form persistence because it inherits persistence from one or more of its driving processes. There are two objections to such an approach. The first is that such inherited persistence will be structural, contradicting Benati's (2008) empirical results.

A second objection to modeling inflation persistence as inherited from the driving process is that, if persistence is inherited in this way, then when inflation is appropriately conditioned on its driving variables the remaining persistence should be small. In other words, if persistence is largely inherited from some other variable, then inflation should

[11] See for example Fuhrer (2009) or Mankiw and Reiss (2002).

exhibit little persistence after appropriately conditioning on that variable. Empirically, this is not the case. Rudd and Whelan (2005, 2006) show that lagged inflation enters reduced-form inflation equations with large coefficients, even after conditioning on driving variables that may themselves be highly autocorrelated. Quoting Rudd and Whelan (2006, pp304-5), “The model predicts that inflation depends solely on current and expected future values of the output gap. Once we condition on this, no lagged variables – including lagged inflation – should have an effect on the current level of inflation.”

Repeated substitution for π_{t+j}, $j = 1, 2, 3,$ ... in equation (1) yields

$$\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k E_t x_{t+k} + v_{st} . \qquad (9)$$

Rudd and Whelan (2005, 2006) use a linear projection of the model’s endogenous variables to derive an estimate of the discounted forward sum on the right hand side of (9). Specifically, let z denote the vector of endogenous variables in the model, $z_t = \begin{bmatrix} x_t & \pi_t & i_t \end{bmatrix}'$. Then forecasts of future values of x_t can be formed as linear projections using an estimated VAR in z_t. Let Z_t denote the appropriately augmented vector (e.g., $Z_t = \begin{bmatrix} z_t & z_{t-1} & z_{t-2} & z_{t-3} \end{bmatrix}'$ for a fourth-order VAR). Then we can write the VAR in companion form as

$$Z_t = \Phi Z_{t-1} + \varepsilon_t , \qquad (10)$$

where ε_t is a vector of shocks.[12] Using the linear projection of future variables the discounted forward sum on the right hand side of (9) can be written as

[12] Variables here are measured as deviations from their long-run means.

$$\sum_{k=0}^{\infty} \beta^k \hat{E}_t x_{t+k} = e_1' \left(I - \beta \hat{\Phi} \right)^{-1} Z_t \,, \qquad (11)$$

where $\hat{\Phi}$ is the matrix of coefficient estimates and e_1' is a conformable row vector with a 1 as its first element and zeros everywhere else.

Denote the discounted forward sum in (11) by $\hat{DS}_t$. That is, let

$$\hat{DS}_t \equiv e_1' \left(I - \beta \hat{\Phi} \right)^{-1} Z_t \,. \qquad (12)$$

Rudd and Whelan (2005, 2006) reason that, if the persistence of inflation is inherited from its driving variables, then that persistence should largely be explained by $\hat{DS}_t$. Thus lagged values of inflation should have considerably reduced explanatory power in a reduced-form inflation equation that includes $\hat{DS}_t$. They compare the reduced form regression

$$\pi_t = b_0 + B(L)\pi_{t-1} + b_x x_t + e_{bt} \qquad (13)$$

to

$$\pi_t = d_0 + D(L)\pi_{t-1} + d_S \hat{DS}_t + e_{dt} \qquad (14)$$

where $B(L) = \sum_{j=1}^{n} b_j L^j$ and $D(L) = \sum_{j=1}^{n} d_j L^j$. In both regressions they use four lags, n=4. If inflation inherits its persistence from its driving processes then the estimate of $D(1)$ should be considerably smaller than the estimate of $B(1)$. Extending this argument the estimate of $D(1)$ should also be smaller than the estimate of $A(1)$ in equation (8). In this comparison, $A(1)$ is a reduced-form measure of inflation persistence, $B(1)$ and $D(1)$ measure reduced-form persistence after conditioning on the current output gap (or marginal cost) and on the forward sum $\hat{DS}_t$, respectively. Again quoting Rudd and Whelan (2006, p. 305),

"…the persistence problem stems from the fact that lagged inflation enters reduced-form inflation equations with large coefficients even after we have conditioned on driving variables (such as the current output gap) that are themselves highly correlated."

Column 2 of Table 5 reports the values of $A(1)$, $B(1)$, and $D(1)$ estimated using the US data with their 95% confidence intervals.[13] Columns 3 through 6 report the median estimates of $A(1)$, $B(1)$, and $D(1)$ from simulations of each of the four versions of the model and the 95% confidence intervals from the simulations. Note from column 2 that there is considerable inflation persistence in the US data as measured by $\hat{A}(1)$. Note further from the estimates $\hat{B}(1)$ and $\hat{D}(1)$ that in the data conditioning on the output gap does not reduce measured persistence and conditioning on the forward sum, $\hat{DS}_t$, results in only a small reduction in measured persistence.

The anomaly to the standard model that Rudd and Whelan (2005, 2006) highlight is apparent from the median estimates of $A(1)$, $B(1)$, and $D(1)$ obtained from the simple and smoothing-only versions of the model, reported in columns 3 and 4 of Table 5. While measures of inflation persistence such as $\hat{A}(1)$ or the autocorrelations reported in Table 4 will reflect the persistence that inflation inherits from other variables, in the standard model such inherited persistence will be substantially reduced by conditioning on current output or the discounted sum of expected future output. We see such a reduction in the median estimates of $B(1)$ and $D(1)$ in the simple and smoothing-only

[13] Constructed using the F distribution.

versions of the model. Note that this is true even though inflation in the model inherits most of its persistence from persistent cost-push shocks, which are less likely to be cleaned out by a low-order VAR and even though the exogenous persistence (via ρ_s) is set at the highest value that is likely to be consistent with Benati's (2008) result.

However, versions of the model with inflation-target switching do have the ability to match the degree of inflation persistence after conditioning. The state-known model in particular appears to replicate well the pattern observed in the data. The sum of the estimated coefficients $\hat{A}(1)$ is large, 0.823. Although this is not quite as large as in the data, it is substantially larger than the corresponding sum in either of the models without switching. More importantly, after conditioning on the expected present discounted value of the output gap, the sum of the coefficients on lagged inflation in this model remains high at 0.753. Thus, in the state-known model as in the data, including $\hat{DS}_t$ in the regression does little to reduce the sum of the estimated coefficients on lagged inflation. The results for the learning model are similar to those for the state-known model, but are less strong. Both $\hat{B}(1)$ and $\hat{D}(1)$ for the learning model are above the corresponding statistics in the models without switching, but below $\hat{B}(1)$ and $\hat{D}(1)$ in the state-known model.

In versions of the model with target switching, inflation inherits persistence from the persistent nonlinear switching process. This nonlinear source of persistence remains after conditioning on the linear projection from the VAR. Thus, versions of the model with switching in the target inflation rate are able to explain the measured persistence of inflation after conditioning. Versions of the model without switching are not.

C. The acceleration phenomenon, disinflation and recession.

When real GDP is below trend the inflation rate tends to fall and when real GDP is above trend the inflation rate tends to rise. Mankiw and Reiss (2002) call this *acceleration phenomenon* "the central finding from the empirical literature on the Phillips curve …" and argue that it is inconsistent with the Calvo aggregate supply curve under rational expectations. Indeed, this inconsistency is one of their main arguments in favor of their alternative model of sticky expectations. Interestingly, there is empirical evidence that the acceleration phenomenon, like inflation persistence, is not structural. Ball (2000) finds that prior to WWI when the gold standard was in effect output was correlated with the *level* of inflation, not its change.

Formally, the acceleration phenomenon is that $corr(\Delta\pi, x) > 0$. Table 6 reports this correlation for the one-year (four quarter) change in inflation, $\Delta\pi_{t+2,t-2} \equiv \pi_{t+2} - \pi_{t-2}$, and for the two-year change in inflation, $\Delta\pi_{t+4,t-4} \equiv \pi_{t+4} - \pi_{t-4}$. In Table 6, the output gap, x, is measured as the deviation of log GDP from its trend, where the trend is calculated using the Hodrick-Prescott filter (as in Mankiw and Reiss). Column 2 of Table 6 reports the correlation for the US data with the 95% confidence interval.[14] Columns 3 through 6 report the median correlations over 10,000 repetitions for each of the four versions of the model with the 95% confidence intervals from the simulations.

In the data the correlation is positive for both the one-year and the two-year change in the rate of inflation. In the simple model these correlations are zero to at least three decimal places. Thus analysis of the data confirms the presence of the acceleration

[14] Constructed using the Fisher z transformation.

phenomenon and simulations of the simple model confirm that, with respect to this version of the model, the acceleration phenomenon is anomalous.

Interestingly, from the simulations of the smoothing-only model it appears that interest-rate smoothing alone is sufficient to induce the acceleration phenomenon. For this version of the model the correlation of the output gap with the change in inflation is positive both for the one-year and for the two-year change in inflation, as reported in column 4 of Table 6 (although the confidence interval includes zero for the two-year change). Introducing switching in the target interest rate – either as the state-known model or the learning model - increases the correlation of the output gap with the change in inflation but only by a small amount. Based on this, it is fair to say that a policy with a single inflation target and interest-rate smoothing can generate the acceleration phenomenon, at least qualitatively. Since empirical studies strongly support the hypothesis that the Fed smoothes changes in the fed funds rate[15] the positive correlation of $\Delta\pi$ with the output gap should not be viewed as an anomaly the new-Keynesian Phillips curve under rational expectations. However, no version of the model considered here can replicate the increase in the correlation as we go from the one-year change in inflation to the two-year change in inflation.

To understand why interest-rate smoothing is sufficient to induce a positive correlation between the change in the one-year inflation rate and the output gap, consider Figure 1. Panels A and B of Figure 1 shows the behavior of key variables in the simple model. Panels C and D shows the same variables for the smoothing-only model. Since variation in v_{st} is the main source of variation in these versions of the model, Figure 1

[15] See, for example, Clarida, Gali, and Gertler (2000), Schorfheide (2005).

shows how a one-time one-standard deviation cost push shock, $v_{st} = \sigma_s$, affects inflation, the interest rate, the output gap, and the one-year change in inflation. To eliminate complicating detail, for these impulse response diagrams I set $\rho_s = 0$, so that there is no serial correlation in v_{st}, and set $\gamma_x = 0$, so that the fed funds rate does not respond to the output gap.

Consider first the responses in the simple model, where $\rho_F = 0$. Figure 1, Panel A, shows that the cost shock, which occurs in period 4, causes an increase in inflation and a decline in output. The Fed responds to the inflationary shock by increasing the fed funds rate. Since, in this version, the Fed does not smooth changes in its target rate, the contemporaneous increase in the fed funds rate is sufficient to restore π_t and x_t to their long-run equilibrium levels in the subsequent period. Figure 1, Panel B, compares the effect on the one-year (four-quarter) change in the inflation rate, $\Delta\pi_{t+2,t-2} \equiv \pi_{t+2} - \pi_{t-2}$, to the effect on the output gap. Note from Panel A that the quarterly inflation rate only changes in two periods: it increases in period 4 and decreases by an equal amount in period 5. Thus, $\Delta\pi_{t+2,t-2}$ will only be non-zero in two periods. It will be positive in period 2 because $\Delta\pi_{t+2,t-2}$ for t =2 includes the period 4 increase in the quarterly rate but not the equal period 5 decrease. And $\Delta\pi_{t+2,t-2}$ will be negative in period 6 because it will include the period 5 decline, but not the equal increase in period 4. The resulting pattern gives a positive and a negative realization of $\Delta\pi_{t+2,t-2}$ in periods 2 and 6, respectively, periods in which the output gap is at its long-run equilibrium level. Furthermore, because $\Delta\pi_{t+2,t-2}$ for t = 4 includes the offsetting period 4 increase and period 5 decrease in inflation, $\Delta\pi_{t+2,t-2} = 0$ in period 4, the only period in which the output gap deviates from its long run equilibrium. The resulting pattern in the simple model leaves no correlation between $\Delta\pi_{t+2,t-2}$ and x_t in response to the cost shock.

Consider next Panels C and D, which show the response to the same shock, $v_{st} = \sigma_s$, again with $\rho_s = 0$ and $\gamma_x = 0$, but now with interest-rate smoothing, $\rho_F = .75$. Here again the cost shock in period 4 causes an increase in π_t and a decrease in x_t. Note that the contemporaneous response of the interest rate in in Panel C is smaller than that in Panel A, but more persistent. In periods 5, 6, and 7, the fed funds rate remains above its long run equilibrium level, pushing inflation below its long run equilibrium level for several periods. In Panel D we see that $\Delta\pi_{t+2,t-2}$ is positive in period 2 and negative in period 3, periods in which x_t =0. However, for periods 4, 5, and especially 6, a negative one-year change in inflation correlates with values of x_t below the long-run equilibrium level. Thus, the gradual but persistent increase in interest rates causes a decline in the average inflation rate to coincide with a persistent drop in output. That is, it generates the acceleration phenomenon. In periods 7, 8, and 9 there is the opposite effect, one-year inflation recovering while the output gap is still negative, but this effect is qualitatively weaker than the positive correlation between $\Delta\pi_{t+2,t-2}$ and x_t that is established in period 4 through 6.

Although the response of output and inflation is more complicated and drawn out with $\rho_s > 0$ and $\gamma_x > 0$, the essential pattern is the same. Interest-rate smoothing causes an inflationary shock to be followed by a period of gradual adjustment in which output is below its long-run equilibrium and inflation is declining.

A stylized fact that is related to the acceleration phenomenon is the positive sacrifice ratio, which is to say, that disinflations cause a persistent loss of output. This, along with the acceleration phenomenon, is one of Mankiw and Reiss's (2002) main motivations for introducing sticky expectations. Ball (1994) has shown that for an aggregate supply relation of the form of equation (1) it is possible for a disinflation to cause an expansion. Still, the question is not whether some pattern of disinflation might in principle cause an expansion it is, rather, whether the new-Keynesian model predicts that disinflations like those that occur in the US economy (or other advanced economies) cause recessions. To answer this question for the present model I consider the behavior of the output gap following a switch from the high-inflation-target regime to the low-inflation-target regime, such as occurred in the US in 1982:4. To do this, I again simulate the state-known model for 10,000 repetitions of 168 periods each. As described above I set s_t =2 from period 22 (1974:1) through period 56 (1982:3), and s_t =1 for all other periods. To abstract from noise I then average the output gap in each period over the 10,000 repetitions. The average path of x_t, so constructed, is shown for periods 50 through 65 in Figure 2.

Before discussing the recessionary effect of the disinflation that begins in period 57, I must first digress on the implications of the Calvo aggregate supply curve for steady-state output. It is a widely noted[16] and unappealing feature of the Calvo aggregate supply relation that it is not consistent with the natural rate hypothesis. It is straightforward from equation (1) that when the model converges to a steady state the relationship between the steady-state inflation rate, π_s, and the steady-state output gap

[16] See, for example, McCallum and Nelson (1999)

will be $x_s = (1-\beta)\pi_s$. This being the case, in a model where there is a high-inflation target and a low-inflation target, the steady-state output gap will be higher when the inflation target is higher. This can be seen in the average behavior of the output gap shown in Figure 2. Prior to period 57, when the economy is in the high-inflation-target regime, the average output gap centers on its high-inflation-target steady state. Following the recession caused by the shift to the low-inflation-target regime, from, say, period 65 onward, the average output gap has converged to its low-inflation-target steady state value.

Thus, in evaluating the extent to which disinflation causes recession in the present model, the recession should be defined as the length of time during which output is below its low-inflation-target steady-state value. With this in mind, it is clear from Figure 2 that the disinflation brought on by a switch from the high to the low inflation target causes a persistent recession. In the period of the regime switch, period 57, the output gap falls 2.45 percentage points below its low-inflation-target steady-state value. As a consequence of the disinflation output stays below its steady state value for roughly five quarters. Thus, in the state-known model, the disinflation associated with a switch to the low-inflation target, causes a recession that is in some important respects similar to what we'd expect given historical experience.

D. The apparent bias in survey expectations.

It has been widely noted that survey measures of inflation expectations appear to be biased, especially in small samples.[17] Specifically, they appear to systematically

[17] See, for example, Thomas (1999), Andolfatto, et al (2008), or Evans and Wachtel (1993). .

under predict inflation when it is rising and over predict inflation when it is falling. Paradoxically, though widely documented to be biased, surveys do a better job forecasting inflation than many alternative methods. Ang, et al, (2007) compare survey forecasts of expected inflation to forecasts based on time series ARIMA models, to inflation forecasts implied by the term structure of interest rates, and to Phillips-curve regressions that use measures of real activity to forecast inflation. They find that the survey forecasts, especially the median forecasts, are better at predicting actual inflation than any of the three model-based alternatives. This supports Evans and Wachtel's (1993) interpretation that survey expectations are reasonably accurate forecasts and only appear to be biased. Specifically, Evans and Wachtel (1993) argue that inflation follows a two-state Markov switching process so that, during a period of high inflation, rational forward-looking agents aware of the possibility of a switch to a low-inflation regime will appear to have biased expectations ex post, if the switch does not occur.

Although this paper models a switch in the inflation target and not equilibrium inflation per se, it is easy to see how the argument of Evans and Wachtel would generalize. Indeed, both Andolfatto, et al (2008) and Erceg and Levin (2003) use changes in the Fed's inflation target as a means of explaining the apparent bias in survey measures of expected inflation. In contrast to the approach taken here, neither Andolfato et al (2008) nor Erceg and Levin (2003) model a switch between a low-inflation target regime and a high-inflation target regime. Andolfatto et al model the new inflation target as a random draw and Erceg and Levin model the target as a near random walk.[18] Both

[18] The persistent shock in Erceg and Levin's model of the target inflation rate is AR(1) with coefficient 0.999.

papers explain the apparent bias as arising from the signal extraction problem: it takes time for agents to infer the change in the unobserved target rate.

A common way of measuring the bias in survey expectations is to examine the implied ex post forecast error during a period of rising or falling inflation. Thomas (1999), for example, records the mean error in the one-year-ahead forecast of CPI inflation from the Livingston survey. He finds that, while the mean forecast error for his sample as a whole is small for the subsample when inflation is generally rising, the mean forecast error is 1.60%, and for the subsample when inflation is generally falling, the mean forecast error is -0.84%. Thus, it appears that survey measures of expected inflation systematically under predict inflation when it is rising and over predict inflation when it is falling.[19]

Applying the approach in Thomas (1999) to my data set, I calculate the mean error in the one-quarter-ahead forecast of inflation from the Survey of Professional Forecasters.[20] . Thomas determines periods of rising or falling inflation somewhat informally. As Schorfheide (2005) identifies 1982:4 as the period in which Fed policy switched to low inflation I take that to be the period in which inflation begins to decline. In Table 7, column 2, I report this mean for the full sample, 1968:4 to 2009:3, the period in my sample when inflation is rising, 1968:4 to 1982:3, and for the rest of the sample, from 1982:4 to 2009:3. My results confirm those of Thomas (1999). Although the mean forecast error is effectively zero over the full sample, the mean error is positive (inflation

[19] Erceg and Levin (2003) take a similar approach to examining the 1980:4 – 1985:4 period in the US and reach a similar conclusion. They note the same pattern in the U.K. and Canada.
[20] There is a great deal of coherence among different surveys. See Thomas (1999), Evans and Wachtel (1993), or Erceg and Levin (2003).

is higher than forecast) during the period of rising inflation, and the mean error is negative (inflation is lower than forecast) during the period of falling inflation.

To replicate this test in the context of the model, I simulate each version of the model and in each take the period of rising inflation to be periods 1 through 56, corresponding to 1968:4 to 1982:3, and take the period of falling inflation to be periods 57 to 164, corresponding to 1982:4 to 2009:3. In each simulation I calculate the mean forecast error in the full sample, and in both the period of rising inflation and the period of falling inflation. I repeat each simulation 10,000 times and in columns 3 through 6 of Table 7 report the median value across the 10,000 repetitions of this mean forecast error. Note that neither the simple model nor the smoothing-only model replicates the apparent bias in expectations: in the full sample and in both subsamples, the mean forecast error is effectively zero. This is not surprising and is, in fact, what we would expect from a rational expectations model with a constant target inflation rate.

However, both versions of the model with target switching do replicate, qualitatively, the apparent bias in one-step ahead forecasts. In both the learning and the state-known models the mean forecast error is positive in the period of rising inflation and negative in the period of falling inflation. This result is a bit stronger in the state-known model. This raises two questions. First, why does learning add so little to the bias? Both Erceg and Levin (2003) and Andolfatto, et al (2008) show that the bias can arise if agents must learn an unobservable target inflation rate. The second question raised by the strength of the state-known result is, what explains the apparent bias in one-step-ahead forecasts when the inflation target is known?

To answer the first question, note that in the learning model it is not very difficult for firms and households to infer Fed's inflation target. This is because, even in the high-inflation high-variance state, the difference in the inflation targets of five percentage points is large relative to the standard deviation of the interest rate shock. With γ_{π} = 1.7 and $\pi^{T}\left(2\right)-\pi^{T}\left(1\right)$ = 5 a switch from the high-inflation-target high-variance state to the low-inflation-target low-variance state will cause an increase in i_t^* that is twice large as $\sigma_m\left(2\right)$. From the low-inflation low-variance state, a switch to the high target causes an increase in i_t^* that is five times as large as $\sigma_m\left(1\right)$. After even a few periods, it is therefore unlikely that a random shock to the fed funds rate would look like a change in the inflation target. As a consequence the true state can be inferred with a reasonably high degree of accuracy and the learning model behaves much like the state-known model.

Why then, does there appear to be systematic forecast errors in the state-known model? To answer this question, consider the behavior of inflation and expected inflation as shown in Figure 3. The path of inflation shown in Figure 3, like the path of output in Figure 2, is obtained by averaging each observation over 10,000 repetitions. What is readily apparent from the figure is that on average $E_{t-1}\pi_t$ lies below π_t when the economy is in the high-inflation-target state, and $E_{t-1}\pi_t$ lies above π_t when the economy is in the low-inflation-target state. To understand why this is the case, note that when the economy is in the high-target state there is a 5% chance that next period the inflation target will switch to its low value (p_{11} = 0.95 implies p_{12} = 0.05). Thus, taking the weighted average of states, expected inflation is lower than current inflation so long as the high-inflation state persists. Given the magnitude of the gap between expected and realized inflation and the proportion of the 1968:4 to 1982:3 subsample for which the economy is in the high-inflation-target state, expected inflation is on average below actual inflation in that subsample. The same argument explains why $E_{t-1}\pi_t$ lies above π_t in the low-inflation state: there is a small probability that next period the economy will switch to the high-inflation state. Thus, expectations may appear biased in a given subsample and, because these errors will balance out on average over a long period, the forecast error will be smaller in large samples.

Note that there is a key difference between the argument here and that in Andolfatto, et al (2008) and Erceg and Levin (2003). In Andolfatto, et al and Erceg and Levin the apparent bias in expectations arises because there has been a shift in the *unobserved* inflation target. Expectations in these models appear biased because it takes time for households and firms to learn the new inflation target. These models imply that the central bank could enhance its credibility by making the unobserved target public.

In the model considered here, the state-known-model, the inflation target is observed directly by all agents. Nonetheless, expectations appear biased because there is the possibility that central bank policy will change in the near future. Here, an improvement in central bank credibility requires a reduction in the probability that there will be a switch in the observable inflation target.

A more formal approach to testing for biased expectations, one employed by both Thomas (1999) and Andolfatto, et al (2008), is to estimate the regression

$$\pi_t = \beta_0 + \beta_1 E_{t-1}(\pi_t) + e_t$$

and to test the null of no bias, that is, $H_0: \beta_0 = 0, \beta_1 = 1$. The results of this test using the one-quarter ahead inflation forecast from the SPF and the corresponding inflation rate from the US data are reported in column 2 of Table 8. Since the observations are non-overlapping I employ a standard F-test. As is typical in tests of this type, the null of no bias cannot be rejected in the full sample but is strongly rejected for the subsamples in which inflation is generally rising or generally falling.

In columns 3 through 6 of Table 8 I report the results of this bias test for simulations of each version of the model. Here, as in Table 7 and based on the results in Schorfeide (2005), I take the subsample in which inflation is generally rising to be periods 1 through 56, and the subsample in which inflation is generally falling to be from period 57 on. These columns report the per cent of 10,000 repetitions in which the null of $\beta_0 = 0, \beta_1 = 1$ is rejected at the 5% level. Not surprisingly, for versions of the model with a single target inflation rate (and rational expectations), the simple and smoothing-only models, the rate of rejection is uniformly close to 5%. These versions of the model cannot, therefore, explain the bias observed in the data. For versions of the model with switching in the inflation target, the state-known and learning models, there is little evidence of systematic bias either in the full sample or in the inflation-rising subsample. However, in the state-known model for the inflation-falling subsample the null of no bias is rejected in 45% of the repetitions. For this subsample then, the state-known model is capable of explaining the finding of bias, even though it is a model with fully rational expectations. The learning model also has some ability to replicate the finding of bias – the null of no bias is rejected in 25% of the repetitions. However, here as with the other anomalies, the results for the state-known model are closer to the data than those from the model with learning.

V. Conclusion.

This paper argues that two highly plausible modifications to the Taylor rule, the smoothing of changes to the fed funds rate and Markov switching of a persistent inflation target, can help to explain several stylized empirical facts that appear anomalous to the new-Keynesian Phillips curve. In so doing it strengthens the case for using the new-Keynesian Phillips curve in general equilibrium models, provided that the monetary policy rule is correctly specified. It also weakens, to some extent, the case for alternatives to rational expectations, at least in so far as phenomena such as inflation persistence or the apparent bias of survey measures of expected inflation are seen as requiring a departure from the rational expectations baseline.

The argument in this paper is in some respects similar to the argument in Erceg and Levin (2003) and to that in Andolfatto, et al (2008). It differs from those papers in three key respects. First, here the target inflation rate is constrained to take two values – high or low – whereas in those papers it evolves as the sum of two exogenous shocks, one persistent and the other transitory. Assuming a high and a low target seems more realistic. One can imagine that a change in the FOMC's philosophy is better described as a switch to a new target inflation rate, rather than as a purely exogenous mean-zero random disturbance. Second, I attempt to explain a broader range of empirical facts, taking up the persistence of inflation after conditioning and the acceleration phenomenon, in addition to the positive sacrifice ratio and the apparent bias of survey expectations.

Finally, and importantly, I find that the version of the model in which the inflation target is fully observed, does a better job explaining the stylized facts than the version of the model where agents must learn the inflation target using an optimal filter. This shifts the emphasis away from viewing credibility as a filtering problem (i.e., the central bank obscures or has trouble communicating its target) and towards the view that credibility is the problem of guaranteeing that the current inflation target will be maintained into the indefinite future.

The model in this paper is intentionally simple. Thus, several extensions that would make the model more realistic would be useful. One such extension would be to incorporate other plausible sources of persistence such as habit persistence in consumption or capital accumulation with adjustment costs. Another would be to see if the results generalize to other forms of policy switching, such as switching in the monetary policy reaction parameters.[21] Another interesting question is whether adaptive learning like that assumed in Milani (2007) or in Slobodyan and Wouters (2012) would endogenously generate a change in measured persistence when there is a change in monetary policy. Finally, if it were possible, it would be useful to estimate a model with switching in monetary policy across a change in policy regime, specifically across the types of policy change considered in Benati (2008). This would extend the estimation of Schorfheide (2005) by subjecting it to the constraint that a change in policy causes a significant change in the persistence of inflation.

[21] This form of policy switching is studied in Davig and Leeper (2007) and in Lubik and Schorfheide (2004).

References.

Andolfatto, David, Scott Hendry, and Paul Moran (2008). "Are Inflation Expectations Rational?" *Journal of Monetary Economics,* March, 55(2): pp. 406-22.

Ang, Andrew, Geert Bekaert, and Min Wei (2007). "Do Macro Variables, Asset Markets, or Surveys Forecast Inflation Better." *Journal of Monetary Economics,* May 54(4): pp. 1163-1212.

Ball, Laurence (1994) . "Credible Disinflation with Staggered Price Setting." *American Economic Review* 84(1): pp. 282-89.

Ball, Laurence (2000). "Near Rationality and Inflation in Two Monetary Regimes, NBER working paper 7988.

Benati, Luca (2008) "Investigating Monetary Persistence Across Monetary Regimes." *Quarterly Journal of Economics*, August, pp. 1005-1060.

Bernanke, Ben and Ilian Mihov (1998) "Measuring Monetary Policy", *Quarterly Journal of Economics,* August, 113 (3) pp. 869-902.

Bernanke, Ben and Alan Blinder (1992) "The Federal Funds Rate and the Channels of Monetary Transmission," *American Economic Review,* September, 82 (4) pp. 901-21.

Calvo, Guillermo A. (1983) "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics*, September, 12(3), pp. 383-98.

Christiano, Lawrence J., Martin Eichenbaum, and Charles Evans (2005) "Nominal Rigidities and the Dynamic Effects of a shock to Monetary Policy." *Journal of Political Economy*, 113(1) pp. 1-45.

Clarida, Richard, Jordi Gali, and Mark Gertler (2000) "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory", *Quarterly Journal of Economics*, February, 115(1) pp. 147-180.

Davig, Troy, and Eric Leeper (2007) "Generalizing the Taylor Principle", *The American Economic Review*, June v.97 (3), pp. 607-635.

Erceg, Christopher J. and Andrew T. Levin, (2003) "Imperfect Credibility and Inflation Persistence," *Journal of Monetary Economics*, May, 50(4), pp. 915-944.

Evans, Martin and Paul Wachtel (1993) "Inflation Regimes and the Sources of Inflation Uncertainty." *Journal of Money, Credit, and Banking,* 25(3), pp. 475-511.

Fuhrer, Jeffrey C. (2009). "Inflation Persistence" Federal Reserve Bank of Boston, working paper # 09-14.

Gali, Jordi. (2008) *Monetary Policy, Inflation, and the Business Cycle*. Princeton U. Press.

Hamilton, James D. (1994) *Time Series Analysis.* Princeton University Press.

Kiley, Michael T. (2007). “A Quantitative Comparison of Sticky-Price and Sticky-Information Models of Price Setting.” *Journal of Money, Credit, and Banking,* S39(1), pp. 101-124.

Lubik, Thomas A., and Frank Schorfheide, (2004). “Testing for Indeterminacy: An Application to U.S. Monetary Policy”, *The American Economic Review,* 94(1), pp190-217.

Lucas, Robert E., Jr. (1976). “Econometric Policy Evaluation: A Critique.” *Carnegie-Rochester Conference Series on Public Policy,* v.1, pp19-46.

Mankiw, N. Gregory and Ricardo Reiss (2002). “Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve.” *Quarterly Journal of Economics*, November, 117 (4) pp. 1295-1328.

McCallum, Bennett T. and Edward Nelson (1999). “Performance of Operational Policy Rules in an Estimated Semiclassical Structural Model,” in John B. Taylor, ed., *Monetary Policy Rules*, University of Chicago Press, pp. 15-45.

Milani, Fabio. 2007. “Expectations, Learning, and Macroeconomic Persistence.” *Journal of Monetary Economics*, October, 54(7), pp. 2065-2082.

Rudd, Jeremy and Karl Whelan (2005) “Does Labor’s Share Drive Inflation *?” Journal of Money, Credit, and Banking*, 37(2), pp. 297-312.

Rudd, Jeremy and Karl Whelan (2006) “Can Rational Expectations Sticky-Price Models Explain Inflation Dynamics?” *American Economic Review*, 96(1), pp. 303-20.

Schorfheide, Frank (2005) “Learning and Monetary Policy Shifts.” *Review of Economic Dynamics*, April, 8(2) pp.392-419.

Slobodyan, Sergey, and Raf Wouters, (2012). “Learning in A Medium-Scale DSGE Model with Expectations Based on Small Forecasting Models”, *American Economic Journal: Macroeconomics,* April, 4(2) pp. 65-101.

Taylor, John B. (1993). “Discretion versus Policy Rules in Practice.” *Carnegie-Rochester Conference Series on Public Policy,* 39, pp195-214.

Thomas, Lloyd B., Jr. (1999). "Survey Measures of Expected U.S. Inflation." Journal of Economic Perspectives, v.13 (4), Fall, pp. 125-144.

Walsh, Carl E. (2003) *Monetary Theory and Policy*. MIT Press, 2nd Edition.

Table 1: Four Versions of the Monetary Policy Rule.

The nominal federal funds rate is given by

$$i_t = \rho_F i_{t-1} + (1-\rho_F) i_t^* + \varepsilon_{mt}, \quad 0 \le \rho_F < 1, \qquad (5)$$

where

$$i_t^* = \pi^T(s_t) + \gamma_\pi \left[\pi_t - \pi^T(s_t)\right] + \gamma_x x_t \qquad (6)$$

and

$$\varepsilon_{mt} = \sigma_m(s_t)\varepsilon_t, \quad \varepsilon_t \sim i.i.d.N(0,1). \qquad (7)$$

(1) *Simple* model:
$\pi^T(1) = \pi^T(2) = 0$, $\sigma_m(1) = \sigma_m(2) = \sigma_m$, and $\rho_F = 0$.

(2) *Smoothing-only* model:
$\pi^T(1) = \pi^T(2) = 0$, $\sigma_m(1) = \sigma_m(2) = \sigma_m$, and $\rho_F > 0$.

(3) *State-known* model:
$\pi^T(1) < \pi^T(2)$, $\sigma_m(1) < \sigma_m(2)$, and $\rho_F > 0$.
s_t realized and observed before market equilibrium is determined.
ε_{mt}, hence i_t, realized and observed after market equilibrium is determined.

(4) *Learning* model:
$\pi^T(1) < \pi^T(2)$, $\sigma_m(1) < \sigma_m(2)$, and $\rho_F > 0$.
i_t, realized and observed after market equilibrium is determined.
s_t and ε_{mt} not observed. s_t inferred from observations of i_t.

Table 2: Calibration of Structural Parameters

Parameter	Value	Comments/Source
β	0.99	4% annual real rate
σ	1	Log utility
θ	2/3	1/3 of firms reset price each period
κ	0.17	$\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}$
ρ_F	0.75	Clarida, Gali, and Gertler (2000) and Schorfheide (2005)
γ_π	1.7	
γ_x	0.5	Clarida, Gali, and Gertler (2000)
$\pi^T(2)-\pi^T(1)$	5.0	Schorfheide (2005)
p_{11}	0.97	
p_{22}	0.95	
$\sigma_m(1)$	0.65	
$\sigma_m(2)$	1.65	
σ_d	0.09	
ρ_d	0.80	
σ_s	0.46	
ρ_s	0.70	Calibrated to match Benati's (2008) result as described in Section III.

Table 3: Percent of Smoothing-only Repetitions in which the Estimate of $A(1)$ is below $A_C(1)$.

ρ_s	$A_C(1)$ Derived from *state-known* model	% of *smoothing-only* repetitions with $\hat{A}(1) < A_C(1)$.
0.98	0.848	5.24
0.95	0.801	5.93
0.90	0.735	8.82
0.85	0.706	22.35
0.80	0.704	55.32
0.75	0.716	88.53
0.70	0.740	99.13
0.65	0.764	99.98
0.60	0.784	100.00
0.55	0.805	100.00

Table 4: Autocorrelations of Inflation.

(1) Lag	(2) Data	(3) Simple model	(4) Smoothing-only model	(5) State-known model	(6) Learning model
1	0.887	0.685	0.557	0.772	0.671
2	0.843	0.465	0.319	0.640	0.488
3	0.808	0.310	0.184	0.556	0.381
4	0.773	0.202	0.107	0.499	0.316
5	0.717	0.129	0.058	0.457	0.272
6	0.679	0.078	0.029	0.422	0.241
8	0.600	0.020	-0.002	0.370	0.197
12	0.508	-0.025	-0.019	0.283	0.139

Column (2) reports the autocorrelations of quarterly inflation in the US data. Columns (3) through (6) report the median autocorrelation of π_t over 10,000 repetitions of the model.

Table 5: Sum of Estimated Coefficients on Lagged Inflation.

(1)	(2) Data	(3) Simple model	(4) Smoothing-only model	(5) State-known model	(6) Learning model
$\hat{A}(1)$	0.938 *[.861, 1.015]*	0.678 *[.497, .797]*	0.565 *[.355, .717]*	0.823 *[.723, .886]*	0.723 *[.523, .837]*
$\hat{B}(1)$	0.963 *[.888, 1.038]*	0.126 *[.006, .397]*	0.550 *[.316, .733]*	0.829 *[.718, .901]*	0.750 *[.544, .873]*
$\hat{D}(1)$	0.828 *[.682, .974]*	0.346 *[-.084, .759]*	0.302 *[-.240, .620]*	0.753 *[.401, .878]*	0.657 *[.179, .813]*

Columns (3) through (6) report the median value of the relevant statistic over 10,000 repetitions of the model. The numbers in square brackets give the 95% confidence intervals.

Table 6: The Acceleration Phenomenon.

(1)	(2) Data	(3) Simple model	(4) Smoothing-only model	(5) State-known model	(6) Learning model
$corr(\Delta\pi_{t+2,t-2}, x_t)$	0.420 *[.351, .485]*	0 *[-.033, .032]*	0.244 *[.049, .416]*	0.300 *[.099, .471]*	0.305 *[.107, .474]*
$corr(\Delta\pi_{t+4,t-4}, x_t)$	0.534 *[.474, .589]*	0 *[-.042, .042]*	0.143 *[-.068, .339]*	0.172 *[-.053, .379]*	0.183 *[-.365, .392]*

Columns (3) through (6) report the median value of the relevant statistic over 10,000 repetitions of the model. The numbers in square brackets give the 95% confidence intervals.

Table 7: Mean forecast errors, $\left(\pi_t - E_{t-1}\pi_t\right)$.

	(2) Data	(3) Simple model	(4) Smoothing-only model	(5) State-known model	(6) Learning model
Full Sample 68:4 to 09:3	-0.02	0	-0.001	-0.046	-0.046
Inflation Rising 68:4 to 82:3	0.37	0.001	0	0.128	0.105
Inflation Falling 82:4 to 09:3	-0.22	0	-0.001	-0.133	-0.121

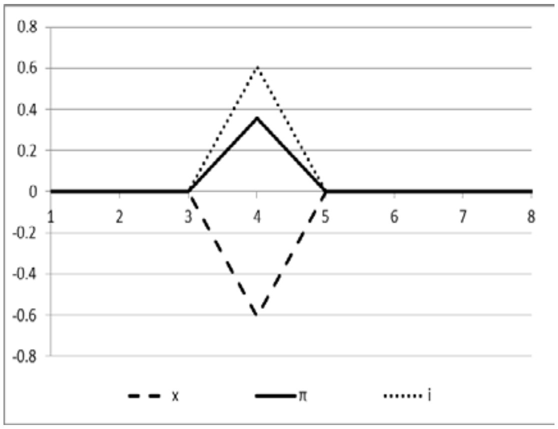
Table 8. Tests of H_0: $\beta_0 = 0, \beta_1 = 1$ in Regression $\pi_t = \beta_0 + \beta_1 E_{t-1}\left(\pi_t\right) + e_t$.

(1)	(2)	Per cent of repetitions in which H_0 is rejected			
	SPF and US Data	(3) Simple Model	(4) Smoothing-Only Model	(5) State-Known Model	(6) Learning Model
Full Sample 68:4 to 09:3	Fail to reject H_0 p-value = 0.909	5.27	5.43	8.24	6.45
Inflation Rising 68:4 to 82:3	Reject H_0 p-value = 0.005	5.72	5.52	8.51	5.39
Inflation Falling 82:4 to 09:3	Reject H_0 p-value = 0.000	5.08	5.20	45.17	24.43

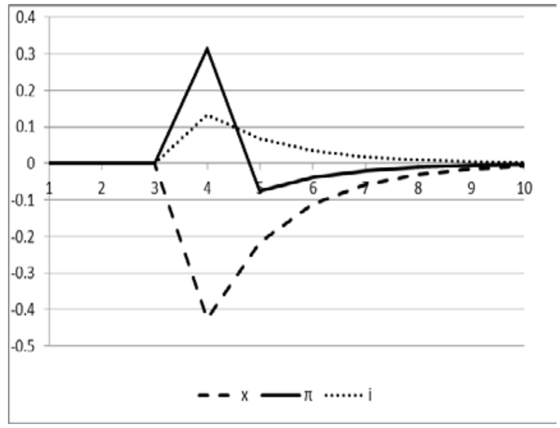
Column 2 tests the null using the SPF for expected inflation and the corresponding US data for actual inflation. Columns 3-6 report the percent of 10,000 repetitions that the null is rejected in simulations of each version of the model. As the observations are non-overlapping, the null is evaluated using a standard F-distribution.

Figure 1: Effect of $\rho_F > 0$ on the Response to an i.i.d. Cost-push Shock

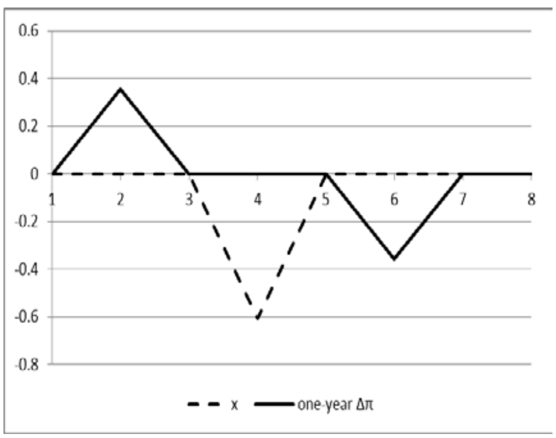
Panel A: x_t**,** π_t**, and** i_t **with** $\rho_F = 0$**.**



Panel C: x_t**,** π_t**, and** i_t **with** $\rho_F = .75$**.**



Panel B: x_t **and** $\Delta\pi_{t-2,t+2}$ **with** $\rho_F = 0$**.**



Panel D: x_t **and** $\Delta\pi_{t-2,t+2}$ **with** $\rho_F = .75$**.**

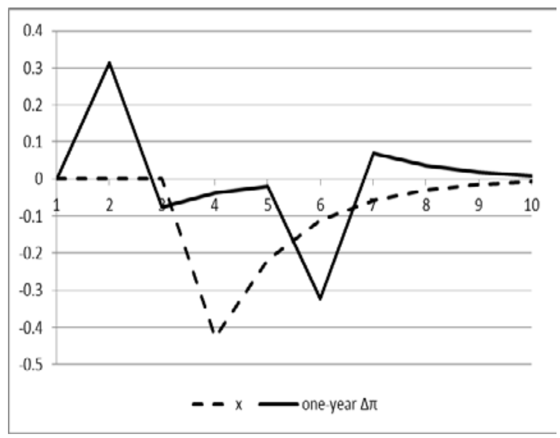


Figure 2: The Output Gap Following a Switch to the Low-inflation Target.

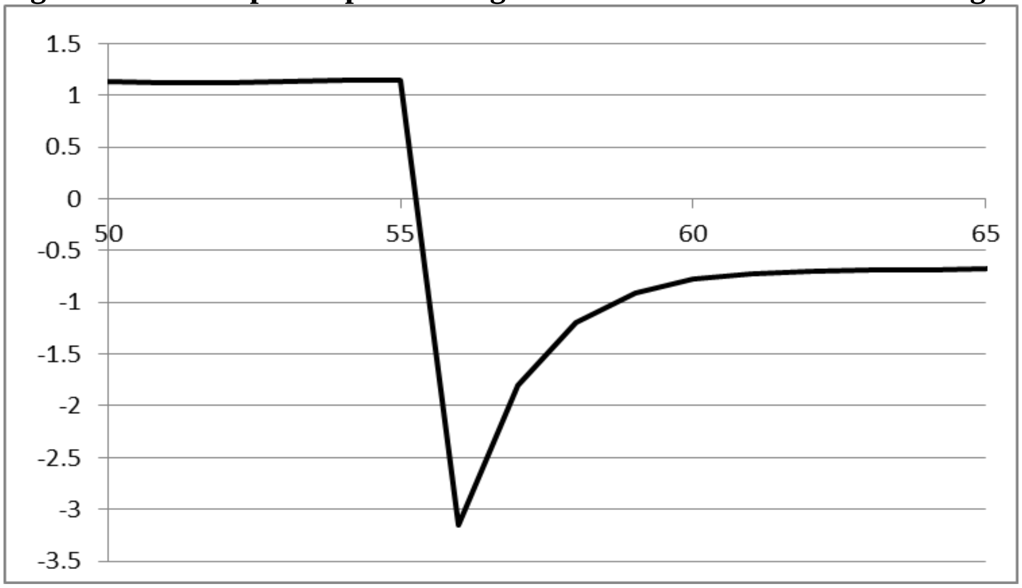
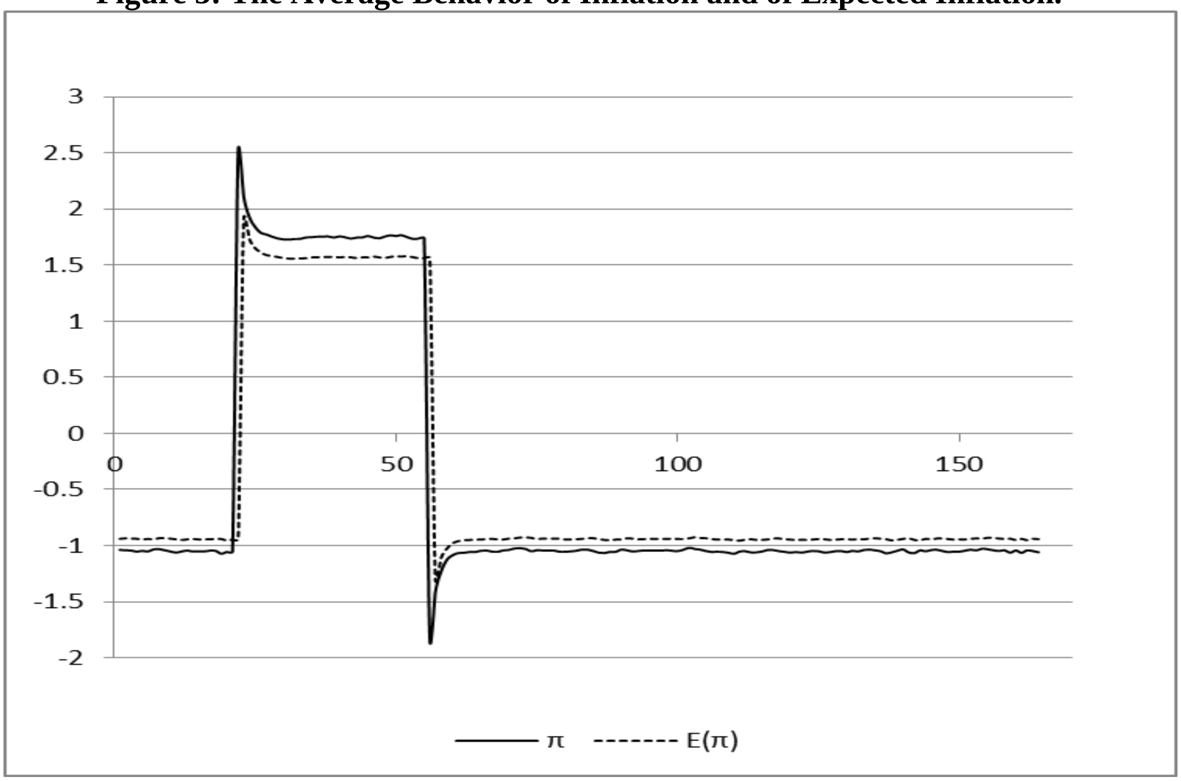


Figure 3: The Average Behavior of Inflation and of Expected Inflation.



Appendix A: Data and Sources

This paper uses quarterly data from 1968:4 to 2009:3. The data on expected inflation is the mean one-quarter-ahead inflation forecast from the Survey of Professional Forecasters (SPF), available at the Federal Reserve Bank of Philadelphia's website. The SPF collected data on forecasts of the GNP deflator from 1968:4 thorough 1991:4 and the GDP deflator thereafter. For expected inflation I use the SPF's "dpgdp2" series which is the mean forecast of the change in the GDP (or GNP) deflator.

Macroeconomic data on output, inflation, and the federal funds rate are from the Federal Reserve Bank of Saint Louis's FRED II database. For inflation I match the variable being forecast by the SPF and so use the change in the log of GNPCTPI (the GNP deflator) for 1968:4 through 1991:4 and the change in the log of GDPCTPI (the GDP deflator) from 1992 forward. (The two inflation series are very highly correlated, especially from 1968 through 1992.) The output gap is measured as the deviation of log real GDP (GDPC96) from its quadratic time trend. The exception is that, as noted in the paper, the output gap used to analyze the acceleration phenomenon is the deviation of log real GDP from its HP trend. The quarterly federal funds rate is the simple average of the monthly federal funds rate, (FEDFUNDS).

APPENDIX B: MODEL SOLUTION
(Not for publication.)

I. Solution for versions of the model in which ρ_F > 0.

Begin from the model as given in the paper.

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + v_{st}, \tag{1}$$

$$v_{st} = \rho_s v_{st-1} + \varepsilon_{st}, \tag{2}$$

with $0 \le \rho_s < 1$ and $\varepsilon_{st} \sim$ i.i.d. $\mathrm{N}\left(0, \sigma_s^2\right)$.

$$x_t = E_t x_{t+1} + \left(\frac{-1}{\sigma}\right) E_t \left(i_t - \pi_{t+1}\right) + v_{dt}, \tag{3}$$

$$v_{dt} = \rho_d v_{dt-1} + \varepsilon_{dt}, \tag{4}$$

with $0 \le \rho_d < 1$, and $\varepsilon_{dt} \sim$ i.i.d. $\mathrm{N}\left(0, \sigma_d^2\right)$.

$$i_t = \rho_F i_{t-1} + \left(1 - \rho_F\right) i_t^* + \varepsilon_{mt}, \tag{5}$$

where $0 \le \rho_F < 1$,

$$i_t^* = \pi^T\left(s_t\right) + \gamma_\pi \left[\pi_t - \pi^T\left(s_t\right)\right] + \gamma_x x_t, \tag{6}$$

$$\varepsilon_{mt} = \sigma_m\left(s_t\right)\varepsilon_t, \text{ and } \varepsilon_t \sim i.i.d. N(0,1). \tag{7}$$

The state variable, $s_t \in \{1, 2\}$, follows a Markov-switching process with transition probabilities $p_{ij} = \Pr\left[s_t = j \mid s_{t-1} = i\right]$. The random variables s_t, ε_{st}, ε_{dt}, and ε_t are mutually independent.

The information assumptions for the different versions of the model are discussed below. Here it is important to note that in all four versions of the model households and firms observe v_{st} and v_{dt} before equilibrium is determined. In other words, v_{st} and v_{dt} are in the period t information set. It follows from this assumption and from equations (1) and (3) that $E_t(\pi_t) = \pi_t$ and $E_t(x_t) = x_t$, so π_t and x_t are also in the period t information set. However, ε_{mt} is not observed before equilibrium is determined and is, therefore, not in the period t information set. It follows that $E_t\left(\varepsilon_{mt}\right)$=0 and, from (5), that $E_t(i_t) \neq i_t$.

Substitute (6) into (5) to get

$$i_t = \rho_F i_{t-1} + \left(1-\rho_F\right)\gamma_\pi \pi_t + \left(1-\rho_F\right)\gamma_x x_t + \gamma_0(s_t) + \varepsilon_{mt} \tag{B.1}$$

where

$$\gamma_0(s_t) = \left(1-\rho_F\right)\left(1-\gamma_\pi\right)\pi^T(s_t) \text{ for } s_t = 1, 2. \tag{B.2}$$

Define

$$\xi_{t\,(2\times1)} = \begin{cases} \begin{bmatrix}1 & 0\end{bmatrix}' & \text{if } s_t = 1 \\ \begin{bmatrix}0 & 1\end{bmatrix}' & \text{if } s_t = 2 \end{cases}. \tag{B.3}$$

Then

$$\gamma_0(s_t) = \left[\gamma_0(1) \quad \gamma_0(2)\right]\xi_t \tag{B.4}$$

And we can rewrite (B.1) as

$$i_t = \rho_F i_{t-1} + \left(1-\rho_F\right)\gamma_\pi \pi_t + \left(1-\rho_F\right)\gamma_x x_t + \left[\gamma_0(1) \quad \gamma_0(2)\right]\xi_t + \varepsilon_{mt} \tag{B.5}$$

Take expectations to get

$$E_t\left(i_t\right) = \rho_F i_{t-1} + \left(1-\rho_F\right)\gamma_\pi \pi_t + \left(1-\rho_F\right)\gamma_x x_t + \left[\gamma_0(1) \quad \gamma_0(2)\right]E_t\left(\xi_t\right). \tag{B.6}$$

Next, rewrite (1) as

$$E_t\left(\pi_{t+1}\right) = \frac{1}{\beta}\pi_t + \left(\frac{-\kappa}{\beta}\right)x_t + \left(\frac{-1}{\beta}\right)v_{st}. \tag{B.7}$$

Using (B.6) in (3) gives

$$x_t = E_t x_{t+1} + \left(\frac{1}{\sigma}\right)E_t\pi_{t+1} + v_{dt} + \left(\frac{-1}{\sigma}\right)\left\{\rho_F i_{t-1} + \left(1-\rho_F\right)\gamma_\pi \pi_t + \left(1-\rho_F\right)\gamma_x x_t + \left[\gamma_0(1) \quad \gamma_0(2)\right]E_t\left(\xi_t\right)\right\}$$

or

$$\left[1+\frac{(1-\rho_F)\gamma_x}{\sigma}\right]x_t = E_t x_{t+1} + \left(\frac{1}{\sigma}\right)E_t\pi_{t+1} + v_{dt} + \left(\frac{-\rho_F}{\sigma}\right)i_{t-1} + \left[\frac{-(1-\rho_F)\gamma_\pi}{\sigma}\right]\pi_t + \left(\frac{-1}{\sigma}\right)\left[\gamma_0(1) \quad \gamma_0(2)\right]E_t\left(\xi_t\right)$$

or, using (B.7),

$$E_t x_{t+1} = \left(\frac{\rho_F}{\sigma}\right)i_{t-1} + \left[\frac{(1-\rho_F)\gamma_\pi}{\sigma} - \frac{1}{\sigma\beta}\right]\pi_t + \left[1+\frac{(1-\rho_F)\gamma_x}{\sigma} + \frac{\kappa}{\sigma\beta}\right]x_t$$

$$+\left(\frac{1}{\sigma\beta}\right)v_{st} + (-1)v_{dt} + \left(\frac{1}{\sigma}\right)\left[\gamma_0(1) \quad \gamma_0(2)\right]E_t\left(\xi_t\right).$$

(B.8)

Collecting, equations (B.6), (B.7), and (B.8) can be written as

$$\begin{bmatrix} E_t\left(i_t\right) \\ E_t\left(\pi_{t+1}\right) \\ E_t\left(x_{t+1}\right) \end{bmatrix} = \mathbf{A}\begin{bmatrix} i_{t-1} \\ \pi_t \\ x_t \end{bmatrix} + \mathbf{B}\mathbf{v}_t + \mathbf{C}E_t\left(\xi_t\right), \tag{B.9}$$

where

$$\mathbf{A}_{(3\times3)} = \begin{bmatrix} \rho_F & (1-\rho_F)\gamma_\pi & (1-\rho_F)\gamma_x \\ 0 & \left(\frac{1}{\beta}\right) & \left(\frac{-\kappa}{\beta}\right) \\ \frac{\rho_F}{\sigma} & \left(\frac{(1-\rho_F)\gamma_\pi}{\sigma} - \frac{1}{\sigma\beta}\right) & \left(1+\frac{(1-\rho_F)\gamma_x}{\sigma} + \frac{\kappa}{\sigma\beta}\right) \end{bmatrix},$$

$$\mathbf{B}_{(3\times2)} = \begin{bmatrix} 0 & 0 \\ \frac{-1}{\beta} & 0 \\ \frac{1}{\sigma\beta} & -1 \end{bmatrix}, \ \mathbf{v}_t = \begin{bmatrix} v_{st} \\ v_{dt} \end{bmatrix}, \text{ and } \mathbf{C}_{(3\times2)} = \begin{bmatrix} \gamma_0(1) & \gamma_0(2) \\ 0 & 0 \\ \frac{\gamma_0(1)}{\sigma} & \frac{\gamma_0(2)}{\sigma} \end{bmatrix}.$$

Let $\mathbf{A} = \mathbf{Q}\Lambda\mathbf{Q}^{-1}$, where $\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$, the λ_i are the eigenvalues of $\mathbf{A}$,

$\mathbf{Q} = \begin{bmatrix} 1 & 1 & 1 \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix}$, and the columns of $\mathbf{Q}$ are the eigenvectors of $\mathbf{A}$. Denote the elements of $\mathbf{Q}^{-1}$ by

$$\mathbf{Q}^{-1} \equiv \begin{bmatrix} q^{11} & q^{12} & q^{13} \\ q^{21} & q^{22} & q^{23} \\ q^{31} & q^{32} & q^{33} \end{bmatrix}.$$

We can then write (B.9) as

$$\begin{bmatrix} E_t(i_t) \\ E_t(\pi_{t+1}) \\ E_t(x_{t+1}) \end{bmatrix} = \mathbf{Q\Lambda Q}^{-1} \begin{bmatrix} i_{t-1} \\ \pi_t \\ x_t \end{bmatrix} + \mathbf{B}\mathbf{v}_t + \mathbf{C}E_t(\xi_t). \tag{B.10}$$

Pre-multiply (B.10) by $\mathbf{Q}^{-1}$ and define

$$\mathbf{Q}^{-1} \begin{bmatrix} i_{t-1} \\ \pi_t \\ x_t \end{bmatrix} \equiv \begin{bmatrix} w_{1t} \\ w_{2t} \\ w_{3t} \end{bmatrix} \tag{B.11}$$

to get

$$\begin{bmatrix} E_t(w_{1t+1}) \\ E_t(w_{2t+2}) \\ E_t(w_{3t+3}) \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} w_{1t} \\ w_{2t} \\ w_{3t} \end{bmatrix} + \mathbf{D}\mathbf{v}_t + \mathbf{G}E_t(\xi_t) \tag{B.12}$$

where $\mathbf{D} = \mathbf{Q}^{-1}\mathbf{B}$ and $\mathbf{G} = \mathbf{Q}^{-1}\mathbf{C}$. Denote the elements of $\mathbf{D}$ and $\mathbf{G}$ by

$$\mathbf{D}_{(3\times 2)} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \\ d_{31} & d_{32} \end{bmatrix} \equiv \begin{bmatrix} \mathbf{d}_{1\,(1\times 2)} \\ \mathbf{d}_{2\,(1\times 2)} \\ \mathbf{d}_{3\,(1\times 2)} \end{bmatrix} \tag{B.13}$$

and

$$\mathbf{G}_{(3\times 2)} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \\ g_{31} & g_{32} \end{bmatrix} \equiv \begin{bmatrix} \mathbf{g}_{1\,(1\times 2)} \\ \mathbf{g}_{2\,(1\times 2)} \\ \mathbf{g}_{3\,(1\times 2)} \end{bmatrix}. \tag{B.14}$$

Now, (B.12) can be written as

$$E_t\left(w_{1t+1}\right) = \lambda_1 w_{1t} + \mathbf{d}_1\mathbf{v}_t + \mathbf{g}_1 E_t\left(\xi_t\right),$$

$$E_t\left(w_{2t+1}\right) = \lambda_2 w_{2t} + \mathbf{d}_2\mathbf{v}_t + \mathbf{g}_2 E_t\left(\xi_t\right),$$

$$E_t\left(w_{3t+1}\right) = \lambda_3 w_{3t} + \mathbf{d}_3\mathbf{v}_t + \mathbf{g}_3 E_t\left(\xi_t\right),$$

or, more simply, as

$$E_t\left(w_{jt+1}\right) = \lambda_j w_{jt} + \mathbf{d}_j\mathbf{v}_t + \mathbf{g}_j E_t\left(\xi_t\right) \qquad \text{for j = 1, 2, 3.} \tag{B.15}$$

Conjecture that λ_1 is inside the unit circle and that λ_2 and λ_3 are outside the unit circle. (These conjectures are verified numerically by the code.) Thus, for j = 2, 3 equation (B.15) must be solved forward. Using (B.15) write

$$\left(1-\lambda_j L\right) E_t\left(w_{jt+1}\right) = \mathbf{d}_j\mathbf{v}_t + \mathbf{g}_j E_t\left(\xi_t\right) \qquad \text{for j = 2, 3.}$$

Then

$$E_t\left(w_{jt+1}\right) = \left[\frac{1}{\left(1-\lambda_j L\right)}\right]\left[\mathbf{d}_j\mathbf{v}_t + \mathbf{g}_j E_t\left(\xi_t\right)\right] = \left[\frac{-\lambda_j^{-1}L^{-1}}{\left(1-\lambda_j^{-1}L^{-1}\right)}\right]\left[\mathbf{d}_j\mathbf{v}_t + \mathbf{g}_j E_t\left(\xi_t\right)\right] \tag{B.16}$$

Multiplying the right and left-hand sides of (B.16) by the lag operator, L, gives

$$w_{jt} = -\lambda_j^{-1}\sum_{k=0}^{\infty}\left(\lambda_j^{-1}L^{-1}\right)^k\left[\mathbf{d}_j\mathbf{v}_t + \mathbf{g}_j E_t\left(\xi_t\right)\right] \text{ or}$$

$$w_{jt} = \left(-\lambda_j^{-1}\right) E_t\left[\sum_{k=0}^{\infty}\lambda_j^{-k}\mathbf{d}_j\mathbf{v}_{t+k}\right] + \left(-\lambda_j^{-1}\right)\left[\sum_{k=0}^{\infty}\lambda_j^{-k}\mathbf{g}_j E_t\left(\xi_{t+k}\right)\right] \quad \text{for j = 2, 3.} \tag{B.17}$$

Using the definitions of $\mathbf{d}_{2,3}$, $\mathbf{v}_t$, and equations (2) and (4)

$$E_t\left[\sum_{k=0}^{\infty}\lambda_j^{-k}\mathbf{d}_j\mathbf{v}_{t+k}\right]=\sum_{k=0}^{\infty}\lambda_j^{-k}E_t\left[d_{j1}v_{st+k}+d_{j2}v_{dt+k}\right]=\sum_{k=0}^{\infty}d_{j1}\lambda_j^{-k}\rho_s^k v_{st}+\sum_{k=0}^{\infty}d_{j2}\lambda_j^{-k}\rho_d^k v_{dt}$$

And thus $E_t\left[\sum_{k=0}^{\infty}\lambda_j^{-k}\mathbf{d}_j\mathbf{v}_{t+k}\right]=\left[\frac{d_{j1}}{1-\lambda_j^{-1}\rho_s}\right]v_{st}+\left[\frac{d_{j2}}{1-\lambda_j^{-1}\rho_d}\right]v_{dt}$. Note that, since

$\left(-\lambda_j^{-1}\right)\left[\frac{d_{j1}}{1-\lambda_j^{-1}\rho_s}\right]=\left[\frac{d_{j1}}{\rho_s-\lambda_j}\right]$, etc., we have that

$$\left(-\lambda_j^{-1}\right)E_t\left[\sum_{k=0}^{\infty}\lambda_j^{-k}\mathbf{d}_j\mathbf{v}_{t+k}\right]=\psi_{js}v_{st}+\psi_{jd}v_{dt} \text{ for j= 2, 3,} \qquad \text{(B.18)}$$

where

$$\psi_{js}=\left[\frac{d_{j1}}{\rho_s-\lambda_j}\right] \text{ and } \psi_{jd}=\left[\frac{d_{j2}}{\rho_d-\lambda_j}\right] \qquad \text{(B.19)}$$

Smoothing-only model.

Consider $\sum_{k=0}^{\infty}\lambda_j^{-k}\mathbf{g}_jE_t\left(\xi_t\right)$. For the smoothing-only model $\pi^T\left(1\right)=\pi^T\left(2\right)=0$, which using (B.2) gives $\gamma_0\left(1\right)=\gamma_0\left(2\right)=0$. It follows that $\mathbf{C}=\mathbf{0}_{(3\times2)}$, $\mathbf{G}=\mathbf{0}_{(3\times2)}$, and $\mathbf{g}_j=\mathbf{0}_{(1\times2)}$ for j = 1, 2, 3. Thus, for the smoothing-only model

$$\sum_{k=0}^{\infty}\lambda_j^{-k}\mathbf{g}_jE_t\left(\xi_{t+k}\right)=0. \qquad \text{(B.20)}$$

Using (B.18) and (B.20) in (B.17) we have that, for the smoothing-only model,

$$w_{jt}=\psi_{js}v_{st}+\psi_{jd}v_{dt}, \qquad \text{for j = 2,3.} \qquad \text{(B.21)}$$

State-Known and Learning models.

Once again, consider $\sum_{k=0}^{\infty}\lambda_j^{-k}\mathbf{g}_j E_t\left(\xi_t\right)$. Let $\mathbf{P}=\begin{bmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{bmatrix}$, where $p_{ij}=\text{Prob}\left\{s_t = j \mid s_{t-1}=i\right\}$. [The elements of $\mathbf{P}$ are ordered as in Hamilton (1994, Chapter 22).] From the definition of ξ_t and the stochastic process governing s_t it follows that $E_t\left(\xi_{t+1}\right)=\mathbf{P}E_t\left(\xi_t\right)$ and, more generally, that

$$E_t\left(\xi_{t+k}\right)=\mathbf{P}^k E_t\left(\xi_t\right) \quad \text{for k = 0, 1, 2, … .} \tag{B.22}$$

Thus, for the state-known and learning models,

$$\sum_{k=0}^{\infty}\lambda_j^{-k}\mathbf{g}_j E_t\left(\xi_{t+k}\right)=\sum_{k=0}^{\infty}\lambda_j^{-k}\mathbf{g}_j\mathbf{P}^k E_t\left(\xi_t\right)=\mathbf{g}_j\left(\sum_{k=0}^{\infty}\lambda_j^{-k}\mathbf{P}^k\right)E_t\left(\xi_t\right)=\mathbf{g}_j\left(\mathbf{I}-\lambda_j^{-k}\mathbf{P}\right)^{-1}E_t\left(\xi_t\right).$$

Collecting,

$$\sum_{k=0}^{\infty}\lambda_j^{-k}\mathbf{g}_j E_t\left(\xi_{t+k}\right)=\mathbf{g}_j\left(\mathbf{I}-\lambda_j^{-1}\mathbf{P}\right)^{-1}E_t\left(\xi_t\right) \text{ for j = 2,3.} \tag{B.23}$$

Using (B.23) we can therefore write

$$\left(-\lambda_j^{-1}\right)\sum_{k=0}^{\infty}\lambda_j^{-k}\mathbf{g}_j E_t\left(\xi_{t+k}\right)=\Psi_{j\xi}E_t\left(\xi_t\right) \tag{B.24}$$

where

$$\Psi_{j\xi\,(1\times 2)}=\left(-\lambda_j^{-1}\right)\mathbf{g}_j\left(\mathbf{I}-\lambda_j^{-1}\mathbf{P}\right)^{-1} \text{ for j = 2, 3.} \tag{B.25}$$

Using (B.24) and (B.18) in (B.17) we have that, for the state-known and learning models,

$$w_{jt}=\psi_{js}v_{st}+\psi_{jd}v_{dt}+\Psi_{j\xi}E_t\left(\xi_t\right) \text{ for j = 2, 3} \tag{B.26}$$

where ψ_{js}, ψ_{jd}, and $\Psi_{j\xi}$ are given by equations (B.19) and (B.25).

For the state-known model $E_t\left(\xi_t\right)=\xi_t$, where ξ_t is defined in (B.4). [I also examined the case in which the state is known but is revealed after the households and firms observe i_t, the current fed funds rate. In this case $E_t\left(\xi_t\right)=\mathbf{P}\xi_{t-1}$. See footnote 7 in the text.] Thus, for the state-known model w_{2t} and w_{3t} are given by (B.26) with $E_t\left(\xi_t\right)=\xi_t$.

For the learning model we can write

$$E_t\left(\xi_t\right)=\begin{bmatrix}\phi_{t|t-1}\\ 1-\phi_{t|t-1}\end{bmatrix} \tag{B.27}$$

where $\phi_{t|t-1}=\text{Prob}\left\{s_t=1\,|\,I_t\right\}$ and where I_t denotes the households' and firms' current information set. Thus, for the model with learning, w_{2t} and w_{3t} are given by (B.26) where $E_t\left(\xi_t\right)$ is given from (B.27) and where $\phi_{t|t-1}$ is determined as described below.

Recall that s_t and ε_{mt} are mutually independent and independent from v_{st} and v_{dt}. Also, recall that i_t is observed after equilibrium π_t and x_t are determined. It follows that, in the learning model, where the state is not observed, the only information relevant to forming $\phi_{t|t-1}=\text{Prob}\left\{s_t=1\,|\,I_t\right\}$ is $\left\{i_{t-1},i_{t-2},i_{t-3},\,...\,\right\}\subset I_t$. Thus $\text{Prob}\left\{s_t=1\,|\,I_t\right\}$ = $\text{Prob}\left\{s_t=1\,|\,i_{t-1},i_{t-2},i_{t-3},\,...\right\}$ which explains my choice of subscript for $\phi_{t|t-1}$, conditioning on t-1. Let

$$i_t\left(s_t\right)\equiv\rho_F i_{t-1}+\left(1-\rho_F\right)\gamma_\pi\pi_t+\left(1-\rho_F\right)\gamma_x x_t+\gamma_0(s_t) \tag{B.28}$$

It follows that

$$i_t=i_t\left(s_t\right)+\varepsilon_{mt}. \tag{B.29}$$

Since $\varepsilon_{mt}\sim\text{i.i.d. }N\left[0,\sigma_m^2\left(s_t\right)\right]$,

$$f\left(\varepsilon_{mt} \mid s_t\right) = \frac{1}{\sqrt{2\pi}\,\sigma_m\left(s_t\right)} \exp\left\{\frac{-1}{2\sigma_m^2\left(s_t\right)}\left[i_t - i\left(s_t\right)\right]^2\right\} \tag{B.30}$$

for $s_t = 1, 2$.

Consider now how households and firms update their probability assessments, $\phi_{t|t-1}$.

(i) They enter period t with $\phi_{t|t-1}$.

(ii) They observe v_{st} and v_{dt} or, equivalently, π_t and x_t. From this they can infer $i_t\left(1\right)$ and $i_t\left(2\right)$.

(iii) They observe i_t and, using Bayes's rule, form

$$\phi_{t|t} \;=\; \text{Prob}\left\{s_t = 1 \mid i_t, i_{t-1}, i_{t-2}, \ldots\right\} \;=\; \frac{f\left(\varepsilon_{mt} \mid s_t = 1\right)\cdot\phi_{t|t-1}}{f\left(\varepsilon_{mt} \mid s_t = 1\right)\cdot\phi_{t|t-1} + f\left(\varepsilon_{mt} \mid s_t = 2\right)\cdot\left(1-\phi_{t|t-1}\right)}.$$

(B.31)

Note that $\left(\varepsilon_{mt} \mid s_t = 1\right) \;=\; i_t - i_t\left(1\right)$ and $\left(\varepsilon_{mt} \mid s_t = 2\right) \;=\; i_t - i_t\left(2\right)$.

(iv) Given $\phi_{t|t}$ from (B.31), households and firms form $\phi_{t+1|t}$ using

$$\phi_{t+1|t} = \phi_{t|t}\cdot p_{11} + \left(1-\phi_{t|t}\right)\cdot p_{21}. \tag{B.32}$$

Recovering π_t***,*** x_t***, and*** $E_t\pi_{t+1}$***.***

For each of the versions of the model considered to this point – smoothing-only, state-known, and learning – partition $\mathbf{Q}^{-1}$ in (B.11) appropriately, to write

$$\begin{bmatrix} q^{11}_{(1\times 1)} & \mathbf{Q}^{12}_{(1\times 2)} \\ \mathbf{Q}^{21}_{(2\times 1)} & \mathbf{Q}^{22}_{(2\times 2)} \end{bmatrix} \begin{bmatrix} i_{t-1} \\ \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} w_{1t} \\ w_{2t} \\ w_{3t} \end{bmatrix}. \tag{B.33}$$

The lower two rows of (B.33) are

$$\mathbf{Q}^{21} i_{t-1} + \mathbf{Q}^{22} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} w_{2t} \\ w_{3t} \end{bmatrix}$$

which gives

$$\begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \left(\mathbf{Q}^{22}\right)^{-1} \begin{bmatrix} w_{2t} \\ w_{3t} \end{bmatrix} - \left(\mathbf{Q}^{22}\right)^{-1} \mathbf{Q}^{21} i_{t-1}. \tag{B.34}$$

Thus, given w_{2t}, w_{3t}, and i_{t-1}, π_t and x_t can be obtained from (B.34). Furthermore, given π_t, x_t, i_{t-1}, and the realized values of s_t and ε_{mt}, i_t can be obtained from (B.1). For the smoothing-only, state-known, and learning versions of the model $E_t \pi_{t+1}$ can then be obtained from (B.9). Recall that for the smoothing-only model $\mathbf{C} = \mathbf{0}_{(3\times 2)}$, for the state-known model $E_t\left(\xi_t\right) = \xi_t$, and for the learning model $E_t\left(\xi_t\right)$ is given by (B.27).

II. Solution for the simple model.

In the simple model ρ_F = 0, $\pi^T\left(1\right) = \pi^T\left(2\right)$ = 0. It follows that in this version of the model $\gamma_0\left(1\right) = \gamma_0\left(2\right)$ = 0. This, together with ρ_F = 0, in (B.8) gives

$$E_t x_{t+1} = \left(\frac{\gamma_\pi}{\sigma} - \frac{1}{\sigma\beta}\right)\pi_t + \left(1 + \frac{\gamma_x}{\sigma} + \frac{\kappa}{\sigma\beta}\right) x_t + \frac{1}{\sigma\beta} v_{st} - v_{dt}. \tag{B.35}$$

Equations (B.7) and (B.35) can be written as

$$\begin{bmatrix} E_t \pi_{t+1} \\ E_t x_{t+1} \end{bmatrix} = \mathbf{A}_s \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} + \mathbf{B}_s \mathbf{v}_t \tag{B.36}$$

where $\mathbf{A}_s = \begin{bmatrix} \dfrac{1}{\beta} & \dfrac{-\kappa}{\beta} \\ \left(\dfrac{\gamma_\pi}{\sigma} - \dfrac{1}{\sigma\beta}\right) & \left(1 + \dfrac{\gamma_x}{\sigma} + \dfrac{\kappa}{\sigma\beta}\right) \end{bmatrix}$, $\mathbf{B}_s = \begin{bmatrix} \dfrac{-1}{\beta} & 0 \\ \dfrac{1}{\sigma\beta} & -1 \end{bmatrix}$, and $\mathbf{v}_t = \begin{bmatrix} v_{st} & v_{dt} \end{bmatrix}'$ as above.

Note that $\mathbf{A}_s = \mathbf{Q}_s \Lambda_s \mathbf{Q}_s^{-1}$ where $\Lambda_s = \begin{bmatrix} \lambda_1^s & 0 \\ 0 & \lambda_2^s \end{bmatrix}$, λ_1^s and λ_2^s are the eigenvalues of $\mathbf{A}_s$, $\mathbf{Q}_s = \begin{bmatrix} 1 & 1 \\ q_{21}^s & q_{22}^s \end{bmatrix}$, and where the columns of $\mathbf{Q}_s$ are the eigenvectors of $\mathbf{A}_s$. Pre-multiplying both sides of (B.36) by $\mathbf{Q}_s^{-1}$ then gives

$$\mathbf{Q}_s^{-1} \begin{bmatrix} E_t \pi_{t+1} \\ E_t x_{t+1} \end{bmatrix} = \Lambda_s \mathbf{Q}_s^{-1} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} + \mathbf{D}_s \mathbf{v}_t \tag{B.37}$$

where $\mathbf{D}_s = \mathbf{Q}_s^{-1} \mathbf{B}_s$. Let $\mathbf{D}_s = \begin{bmatrix} d_{11}^s & d_{12}^s \\ d_{21}^s & d_{22}^s \end{bmatrix}$. Then, defining

$$\begin{bmatrix} w_{1t}^s \\ w_{2t}^s \end{bmatrix} = \mathbf{Q}_s^{-1} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} \tag{B.38}$$

Equation (B.37) can be written as

$$E_t w_{1t+1}^s = \lambda_1^s w_{1t}^s + d_{11}^s v_{st} + d_{12}^s v_{dt}$$

$$E_t w_{2t+1}^s = \lambda_2^s w_{2t}^s + d_{21}^s v_{st} + d_{22}^s v_{dt}$$

or, simply,

$$E_t w_{jt+1}^s = \lambda_j^s w_{jt}^s + d_{j1}^s v_{st} + d_{j2}^s v_{dt} \qquad \text{for j = 1,2.} \tag{B.39}$$

Conjecture that λ_1^s and λ_2^s lie outside the unit circle so that (B.39) must ne solved forward for j=1, 2. (This conjecture is verified numerically by the code.) Thus, proceeding as in (B.16) –(B.19) above

$$w_{jt}^{s}=\left[\frac{-\left(\lambda_{j}^{s}\right)^{-1}}{1-\left(\lambda_{j}^{s}\right)^{-1}L^{-1}}\right]\left[d_{j1}v_{st}+d_{j2}v_{dt}\right]$$

$$=-\left(\lambda_{j}^{s}\right)^{-1}E_{t}\left[d_{j1}\sum_{k=0}^{\infty}\left(\lambda_{j}^{s}\right)^{-k}v_{st+k}+d_{j2}\sum_{k=0}^{\infty}\left(\lambda_{j}^{s}\right)^{-k}v_{dt+k}\right]$$

$$=\left(\frac{d_{j1}}{\rho_{s}-\lambda_{j}^{s}}\right)v_{st}+\left(\frac{d_{j2}}{\rho_{d}-\lambda_{j}^{s}}\right)v_{dt}$$

Or, simply,

$$w_{jt}^{s}=\psi_{js}^{s}v_{st}+\psi_{jd}^{s}v_{dt} \qquad \text{for j = 1, 2} \qquad \text{(B.40)}$$

where

$\psi_{js}^{s}=\left(\frac{d_{j1}}{\rho_{s}-\lambda_{j}^{s}}\right)$, and $\psi_{jd}^{s}=\left(\frac{d_{j2}}{\rho_{d}-\lambda_{j}^{s}}\right)$.

Inverting (B.38) we can obtain π_t and x_t from

$$\begin{bmatrix}\pi_{t}\\ x_{t}\end{bmatrix}=\mathbf{Q}_{s}\begin{bmatrix}w_{1t}^{s}\\ w_{2t}^{s}\end{bmatrix}. \qquad \text{(B.41)}$$

Given π_t and x_t, we can then obtain $E_t\pi_{t+1}$ for the simple model from (B.36).