HW8

Wei Ye*
ECON 5700

Due on August 22, 2020.

1 Question 1

Solution:

$$\begin{aligned}\det(A) &= (-1)^5 \cdot 2 \begin{bmatrix} 2 & -3 & 1 \\ 1 & -1 & 3 \\ -2 & 1 & 0 \end{bmatrix} \\ &= -2[(-1)^4(-2)(-9+1) + (-1)^5(6-1)] \\ &= -2(16-5) \\ &= -22\end{aligned}$$

2 Question 2

Solution:

Because A is upper triangle matrix, thus, its determinant is:

$$\det(A) = 2 \cdot 3 \cdot 1 \cdot 5 \cdot -1 = -30$$

3 Question 3

Solution:

$$AB = \begin{bmatrix} 12 & 3 \\ 16 & 5 \end{bmatrix}$$

$$\det(AB) = 12 \cdot 5 - 16 \cdot 3 = 12$$

*I worked on my assignment sololy. Email: wye22@fordham.edu

4 Question 4

Solution:

$$A^{-1} = \begin{bmatrix} \frac{3}{4} & -\frac{1}{4} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$
$$\det(A^{-1}) = \frac{3}{8} - \frac{1}{8} = \frac{1}{4}$$

5 Question 5

Solution:

$$\left[A - \lambda\right] = \begin{bmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & 1 \\ 1 & 1 & -\lambda \end{bmatrix}$$
$$= 0$$

Thus, $\lambda_1 = 2$, $\lambda_2 = -1$, and $\lambda_3 = 1$.
Thus, the matrix $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
From this, we can obtain the diagonal matrix is :

$$A = \mathcal{P}D\mathcal{P}^{-1}$$

$$[e_1\ e_2\ e_3]\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}[e_1\ e_2\ e_3]^{-1}$$

6 Question 6

Solution:

1. The first matrix =8
2. The second matrix is $4 \cdot 2 \cdot \frac{1}{3} \cdot (-1) = -\frac{3}{8}$
3. The result is -4
4. The result is 4
5. the result is $2 \cdot (-1) = -2$
6. The result is 4

Need to check as I get the lecture note 8. Forgot the specific rules.

7 Question 7

Solution:

$$
\begin{aligned}
A^{10} &= \mathcal{P}D^{10}P \\
&= [e_1\ e_2]\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 2 \end{bmatrix}[e_1\ e_2]^{-1} \\
&= [e_1\ e_2]\begin{bmatrix} \frac{1}{2^{10}} & 0 \\ 0 & 2^{10} \end{bmatrix}[e_1\ e_2]^{-1}
\end{aligned}
$$

8 Question 8

Solution: Let $A = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}$,

$$
\begin{aligned}
\begin{bmatrix} -1-\lambda & 6 \\ 1 & -\lambda \end{bmatrix} &= -\lambda(-1-\lambda) - 6 \\
&= 0
\end{aligned}
$$

We can obtain the eignvalues of the matrix, $\lambda_1 = -3, \lambda_2 = 2$. $D = \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}$

$$
A^{10} = \mathcal{P}D^{10}\mathcal{P}^{-1} = [e_1\ e_2]\begin{bmatrix} (-3)^{10} & 0 \\ 0 & 2^{10} \end{bmatrix}[e_1\ e_2]^{-1}
$$

9 Question 9

Solution: First, we need to calculate the the eigenvalues of the matrix,

$$
\begin{bmatrix} 1-\lambda & 0 & k \\ 0 & 1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix} = 0
$$

Thus $\lambda_1 = \lambda_2 = \lambda_3 = 1$. Thus, the algebraic multiplicity is 3. $A - \lambda I = \begin{bmatrix} 0 & 0 & k \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. If the matrix is diagonaize, it should be that $AM = GM \longrightarrow k = 0 \in \mathcal{R}$