ANS. TO PROB. SET I

ECON 6020: MACRO THEORY I

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PROBLEM 1 Let $|\beta| < 1$.

(a) Prove That $(1+\beta+\beta^2+\cdots) = \left(\frac{1}{1-\beta}\right)$.

$$1 = (1+\beta+\beta^2+\cdots) - (\beta+\beta^2+\beta^3+\cdots)$$

$$= (1-\beta)(1+\beta+\beta^2+\beta^3+\cdots)$$

Thus, Dividing Through by $(1-\beta)$ we have

$$\left(\frac{1}{1-\beta}\right) = 1+\beta+\beta^2+\beta^3+\cdots$$

QED (a)

(b) Prove That $(1+\beta+\beta^2+\cdots+\beta^N) = \left(\frac{1-\beta^{N+1}}{1-\beta}\right)$

$$1+\beta+\beta^2+\cdots+\beta^N = (1+\beta+\beta^2+\beta^3+\cdots) - (\beta^{N+1}+\beta^{N+2}+\beta^{N+3}+\cdots)$$

$$= (1+\beta+\beta^2+\beta^3+\cdots) - \beta^{N+1}(1+\beta+\beta^2+\beta^3+\cdots)$$

$$= \left(\frac{1}{1-\beta}\right) - \beta^{N+1}\left(\frac{1}{1-\beta}\right)$$

$$= \left(\frac{1-\beta^{N+1}}{1-\beta}\right)$$

QED (b)

(c) Prove That $\sum_{j=0}^{\infty} \beta^j \cdot j = \left[\frac{\beta}{(1-\beta)^2}\right]$.

$$\sum_{j=0}^{\infty} \beta^j \cdot j = \beta^0 \cdot 0 + \beta \cdot 1 + \beta^2 \cdot 2 + \beta^3 \cdot 3 + \cdots$$

$$= \beta + 2\beta^2 + 3\beta^3 + \cdots$$

$$= \beta(1 + 2\beta + 3\beta^2 + \cdots)$$

$$= \beta\left[(1 + \beta + \beta^2 + \beta^3 + \beta^4 + \cdots) + (\beta + \beta^2 + \beta^3 + \beta^4 + \cdots) + (\beta^2 + \beta^3 + \beta^4 + \cdots) + \cdots\right]$$

$$= \beta\left[(1 + \beta + \beta^2 + \beta^3 + \cdots) + \beta(1 + \beta + \beta^2 + \cdots) + \beta^2(1 + \beta + \beta^2 + \cdots) + \cdots\right]$$

$$= \beta\left[(1 + \beta + \beta^2 + \beta^3 + \cdots)(1 + \beta + \beta^2 + \beta^3 + \cdots)\right]$$

$$= \beta\left[\left(\frac{1}{1-\beta}\right)\left(\frac{1}{1-\beta}\right)\right]$$

$$= \frac{\beta}{(1-\beta)^2}$$

QED (c)

Problem 2: Consider eqn (1)

$$\boxed{a_{t+1} = (1+r)\,a_t + y_t - c_t \qquad (1)}$$

This equation says that your real assets next period, a_{t+1}, will be interest and principle on your real assets this period, $(1+r)a_t$, plus ~~your real~~ the real value of this period's savings. Note that savings is income minus consumption expenditure, $y_t - c_t$.

Let $R \equiv 1+r$ denote the gross real interest rate. Since $r>0$ then $R>1$. Use the log operator to write (1) as

$$a_{t+1} = R\,L\,a_{t+1} + (y_t - c_t) \qquad \text{or}$$

$$\boxed{(1-RL)\,a_{t+1} = (y_t - c_t) \qquad (2)}$$

From (2) it is clear that the root

of the first-order difference eqn (1) is R. Since $|R|>1$ we solve (2) forward. Thus

$$a_{t+1} = \left(\frac{1}{1-RL}\right)(y_t - c_t)$$

$$a_{t+1} = \left(\frac{-R^{-1}L^{-1}}{1-R^{-1}L^{-1}}\right)(y_t - c_t)$$

or, multiplying through by L

$$a_t = R^{-1}\left(\frac{1}{1-R^{-1}L^{-1}}\right)(c_t - y_t)$$

$$\text{or } a_t = \left(\frac{1}{1+r}\right)\sum_{j=0}^{\infty}\left[R^{-j}L^{-j}(c_t - y_t)\right] \qquad (3)$$

Allowing that $L^{-j}c_t = E_t\, c_{t+j}$, etc., Eqn (3) gives

$$a_t = \left(\frac{1}{1+r}\right)\sum_{j=0}^{\infty}\left[\left(\frac{1}{1+r}\right)^j E_t\left(c_{t+j} - y_{t+j}\right)\right]$$

$$\cancel{a_t = \left(\frac{1}{1+r}\right)\sum_{j=0}^{\infty} E_t\, c_{t+j} - \left(\frac{1}{1+r}\right)\sum_{j=0}^{\infty}}$$

$$a_t = \left(\frac{1}{1+r}\right)\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j \underset{t}{E}\, c_{t+j} - \left(\frac{1}{1+r}\right)\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j \underset{t}{E}\, y_{t+j}$$

OR

$$\boxed{\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j+1} \underset{t}{E}\, c_{t+j} = a_t + \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j+1} \underset{t}{E}\, y_{t+j}} \quad (4)$$

The left-hand side of (4) is the expected present discounted value of lifetime consumption. The right-hand side of (4) is current wealth (real assets) plus the expected present discounted value of lifetime income. So Eqn (4) says that the expected PDV of lifetime consumption must equal ~~your~~ the expected PDV of lifetime income plus ~~your~~ your current wealth

PROBLEM 3: Use Repeated Substitution and The Law of iterated Expectations to Solve for the current Price of the equity share as a function of Expected future dividends.

Begin from the Equilibrium condition as Derived in Class;

$$P_t(1+r) = \underset{t}{E}(P_{t+1} + D_{t+1}) \qquad (1)$$

Re-write (1) as

$$P_t = \left(\frac{1}{1+r}\right)\underset{t}{E}\left(P_{t+1} + D_{t+1}\right)$$

OR

$$\boxed{P_t = \left(\frac{1}{1+r}\right)\underset{t}{E} D_{t+1} + \left(\frac{1}{1+r}\right)\underset{t}{E} P_{t+1} \qquad (2)}$$

We Thus need to find an Expression for $\underset{t}{E} P_{t+1}$ as a function of Expected Future Dividends.

From (2) it follows That

$$P_{t+1} = \left(\frac{1}{1+r}\right)\underset{t+1}{E} D_{t+2} + \left(\frac{1}{1+r}\right)\underset{t+1}{E} P_{t+2} \quad (3)$$

Taking current expectations of (3) gives

$$E_t P_{t+1} = \left(\frac{1}{1+r}\right) E_t E_{t+1} D_{t+2} + \left(\frac{1}{1+r}\right) E_t E_{t+1} P_{t+2} \qquad (4)$$

But, the <u>Law of iterated Expectations</u> implies that $E_t E_{t+1} D_{t+2} = E_t D_{t+2}$ and $E_t E_{t+1} P_{t+2} = E_t P_{t+2}$

so that (4) becomes

$$\boxed{E_t P_{t+1} = \left(\frac{1}{1+r}\right) E_t D_{t+2} + \left(\frac{1}{1+r}\right) E_t P_{t+2} \qquad (5)}$$

Substitute from (5) into (2) to get

$$\boxed{P_t = \left(\frac{1}{1+r}\right) E_t D_{t+1} + \left(\frac{1}{1+r}\right)^2 E_t D_{t+2} + \left(\frac{1}{1+r}\right)^2 E_t P_{t+2} \qquad (6)}$$

Repeating the procedure above we have

$$P_{t+2} = \left(\frac{1}{1+r}\right) E_{t+2} D_{t+3} + \left(\frac{1}{1+r}\right) E_{t+2} P_{t+3} \qquad \text{so}$$

$$E_t P_{t+2} = \left(\frac{1}{1+r}\right) E_t E_{t+2} D_{t+3} + \left(\frac{1}{1+r}\right) E_t E_{t+2} P_{t+3} \qquad (7)$$

But, the <u>LAW of ITERATED EXPECTATIONS</u> implies

$$\underset{t}{E}\,\underset{t+2}{E}\,D_{t+3} = \underset{t}{E}\,D_{t+3} \quad \text{and} \quad \underset{t}{E}\,\underset{t+2}{E}\,P_{t+3} = \underset{t}{E}\,P_{t+3}$$

and thus

$$\underset{t}{E}\,P_{t+2} = \left(\frac{1}{1+r}\right)\underset{t}{E}\,D_{t+3} + \left(\frac{1}{1+r}\right)\underset{t}{E}\,P_{t+3} \qquad (8)$$

Use (8) in (6) to get

$$P_t = \left(\frac{1}{1+r}\right)\underset{t}{E}\,D_{t+1} + \left(\frac{1}{1+r}\right)^2\underset{t}{E}\,D_{t+2} + \left(\frac{1}{1+r}\right)^3\underset{t}{E}\,D_{t+3} + \left(\frac{1}{1+r}\right)^3\underset{t}{E}\,P_{t+3} \quad (9)$$

It is clear that repeated substitution, as above, will give

$$\boxed{P_t = \left(\frac{1}{1+r}\right)\underset{t}{E}\,D_{t+1} + \left(\frac{1}{1+r}\right)^2\underset{t}{E}\,D_{t+2} + \left(\frac{1}{1+r}\right)^3\underset{t}{E}\,D_{t+3} + \cdots} \quad (10)$$

or

$$\boxed{P_t = \sum_{j=1}^{\infty}\left(\frac{1}{1+r}\right)^j\underset{t}{E}\,D_{t+j}} \qquad (11)$$

Which is the <u>same result</u> as we obtained in lecture.

PROBLEM 4: Begin from the equilibrium condition

$$(1+r)\,P_t = E_t\left[P_{t+1} + D_{t+1}\right]. \qquad (1)$$

Let $R \equiv (1+r)$ and note that, since $r>0$ then $R>1$.

Rewrite (1) as

$$R\,P_t = E_t P_{t+1} + E_t D_{t+1} \qquad \text{or}$$

$$E_t P_{t+1} - R\,P_t = -E_t D_{t+1} \qquad \text{or}$$

$$(1-RL)\,E_t P_{t+1} = -E_t D_{t+1} \qquad (2)$$

Note that the root is $R>1$. So we solve (2) forward;

$$E_t P_{t+1} = \left(\frac{1}{1-RL}\right)\left(-E_t D_{t+1}\right) = \left[\frac{-R^{-1}L^{-1}}{1-R^{-1}L^{-1}}\right]\left(-E_t D_{t+1}\right)$$

or

$$P_t = R^{-1}\left[\frac{1}{1-R^{-1}L^{-1}}\right] E_t D_{t+1} \qquad \text{or}$$

$$P_t = \left(\frac{1}{1+r}\right)\sum_{j=0}^{\infty} R^{-j} L^{-j} E_t D_{t+1} \qquad \text{or}$$

$$\boxed{P_t = \sum_{j=1}^{\infty} \left(\frac{1}{1+r}\right)^j \underset{t}{E}\, D_{t+j} \qquad (2)}$$

We will use (2) to evaluate $\underset{t}{E} D_{t+j}$ and P_t for each of the stochastic processes in this problem.

(a) MA(1): $D_t = \varepsilon_t + \theta \varepsilon_{t-1}$

Since ε is iid mean zero we have that

$$\underset{t}{E} D_{t+1} = \underset{t}{E}\left[\varepsilon_{t+1} + \theta \varepsilon_t\right] = \underset{t}{E}\, \cancel{\varepsilon_{t+1}}^{\,0} + \theta \underset{t}{E}\, \cancel{\varepsilon_t}^{\,\varepsilon_t}$$

so $\boxed{\underset{t}{E} D_{t+1} = \theta \varepsilon_t \qquad (3.1)}$

Also, $\underset{t}{E} D_{t+2} = \underset{t}{E}\left[\varepsilon_{t+2} + \theta \varepsilon_{t+1}\right] = \underset{t}{E}\, \cancel{\varepsilon_{t+2}}^{\,0} + \theta \underset{t}{E}\, \cancel{\varepsilon_{t+1}}^{\,0}$

so $\underset{t}{E} D_{t+2} = 0$

Similarly, $\underset{t}{E} D_{t+3} = \underset{t}{E}\left[\varepsilon_{t+3} + \theta \varepsilon_{t+2}\right] = 0$ and

$$\boxed{\underset{t}{E} D_{t+j} = 0 \quad \text{for } j = 2, 3, 4, \ldots \qquad (3.2)}$$

Using (3.1) and (3.2) in (2) Gives

$$P_t = \left(\frac{1}{1+r}\right)\theta\,\varepsilon_t + \left(\frac{1}{1+r}\right)^2 \cdot 0 + \left(\frac{1}{1+r}\right)^3 \cdot 0 + \cdots$$

or

$$\boxed{P_t = \left(\frac{\theta}{1+r}\right)\varepsilon_t \qquad (4)}$$

(b) General stationary AR(1): $D_t = \mu + \rho D_{t-1} + \varepsilon_t$

Since ε is iid mean zero we have

$$\left.\begin{aligned}
\underset{t}{E}\, D_t &= D_t \\
\underset{t}{E}\, D_{t+1} &= \mu + \rho D_t \\
\underset{t}{E}\, D_{t+2} &= \mu + \rho\mu + \rho^2 D_t \\
\underset{t}{E}\, D_{t+3} &= \mu + \rho\mu + \rho^2\mu + \rho^3 D_t \\
\underset{t}{E}\, D_{t+4} &= \mu + \rho\mu + \rho^2\mu + \rho^3\mu + \rho^4 D_t
\end{aligned}\right] \quad (5)$$

Using (5) in (2) gives

$$P_t = R^{-1}(u+\rho D_t) + R^{-2}(u+\rho u+\rho^2 D_t) +$$
$$R^{-3}(u+\rho u+\rho^2 u+\rho^3 D_t) + R^{-4}(u+\rho u+\rho^2 u+\rho^3 u+\rho^4 D_t)+\cdots$$

or

$$P_t = R^{-1}u + R^{-2}(1+\rho)u + R^{-3}(1+\rho+\rho^2)u + R^{-4}(1+\rho+\rho^2+\rho^3)u+\cdots$$
$$+R^{-1}\rho[1+R^{-1}\rho+R^{-2}\rho^2+R^{-3}\rho^3+\cdots]D_t \qquad (6)$$

Note That

$$R^{-1}\rho[1+R^{-1}\rho+R^{-2}\rho^2+R^{-3}\rho^3+\cdots] = \left(\frac{\rho}{1+r}\right)\left(\frac{1}{1-R^{-1}\rho}\right)$$
$$= \left(\frac{\rho}{1+r}\right)\left[\frac{1}{1-\left(\frac{\rho}{1+r}\right)}\right] = \left(\frac{\rho}{1+r}\right)\left[\frac{1}{\frac{1+r}{1+r}-\frac{\rho}{1+r}}\right] = \left(\frac{\rho}{1+r}\right)\left(\frac{1+r}{1+r-\rho}\right)$$

so

$$R^{-1}\rho[1+R^{-1}\rho+R^{-2}\rho^2+\cdots] = \left(\frac{\rho}{1+r-\rho}\right) \qquad (7)$$

use (7) in (6) to get

$$P_t = R^{-1}u[1+R^{-1}(1+\rho)+R^{-2}(1+\rho+\rho^2)+R^{-3}(1+\rho+\rho^2+\rho^3)+\cdots]$$
$$+\left(\frac{\rho}{1+r-\rho}\right)D_t \qquad (8)$$

From Problem 1 part(b) we have

$$1+\rho = \frac{1-\rho^2}{1-\rho} \qquad (9.1)$$

$$1+\rho+\rho^2 = \frac{1-\rho^3}{1-\rho} \qquad (9.2)$$

$$1+\rho+\rho^2+\rho^3 = \frac{1-\rho^4}{1-\rho} \qquad (9.3)$$

etc.

Using $1 = \frac{1-\rho}{1-\rho}$ and (7.1) - (7.3) ~~on~~ in the First term on the Right-hand Side of (8) we have

$$R^{-1}u\left[1 + R^{-1}(1+\rho) + R^{-2}(1+\rho+\rho^2) + \cdots\right] =$$

$$R^{-1}u\left[\frac{1-\rho}{1-\rho} + R^{-1}\left(\frac{1-\rho^2}{1-\rho}\right) + R^{-2}\left(\frac{1-\rho^3}{1-\rho}\right) + \cdots\right] =$$

$$R^{-1}u\left[\left(\frac{1}{1-\rho}\right) + R^{-1}\left(\frac{1}{1-\rho}\right) + R^{-2}\left(\frac{1}{1-\rho}\right) + \cdots\right]$$

$$-R^{-1}u\left[\frac{\rho}{1-\rho} + R^{-1}\left(\frac{\rho^2}{1-\rho}\right) + R^{-2}\left(\frac{\rho^3}{1-\rho}\right) + \cdots\right]$$

$$= R^{-1}u\left(\frac{1}{1-\rho}\right)\left[\frac{1}{1-R^{-1}}\right] - R^{-1}u\left(\frac{\rho}{1-\rho}\right)\left[\frac{1}{1-R^{-1}\rho}\right]$$

$$= \left(\frac{\mu}{1+r}\right)\left(\frac{1}{1-\rho}\right)\left(\frac{1+r}{r}\right) - \left(\frac{\mu}{1+r}\right)\left(\frac{\rho}{1+\rho}\right)\left[\frac{1+r}{1+r-\rho}\right]$$

$$= \left(\frac{\mu}{1-\rho}\right)\left[\frac{1}{r} - \frac{\rho}{1+r-\rho}\right] = \left(\frac{\mu}{1-\rho}\right)\left[\frac{1+r-\rho-\rho r}{r(1+r-\rho)}\right]$$

$$= \left[\frac{(1+r)(1-\rho)}{r(1+r-\rho)}\right]\frac{\mu}{1-\rho} = \left[\frac{1+r}{(1+r-\rho)r}\right]\mu$$

So, Collecting

$$R^{-1}\mu\left[1+R^{-1}(1+\rho)+R^{-2}(1+\rho+\rho^2)+\cdots\right] = \left[\frac{1+r}{(1+r-\rho)r}\right]\mu \quad (10)$$

using (10) in (8)

$$\boxed{P_t = \left[\frac{1+r}{(1+r-\rho)r}\right]\mu + \left(\frac{\rho}{1+r-\rho}\right)D_t \qquad (11)}$$

In Lecture we discussed two cases

(i) $D_t = \mu + \varepsilon_t$. Relative to the case of the General AR(1), this is the specific case where $\mu \neq 0$ and $\rho = 0$. Using $\rho = 0$ in (11) gives $P_t = \frac{\mu}{r}$, which is what we derived in lecture.

(ii) $D_t = \rho D_{t-1} + \varepsilon_t$. Relative to the General AR(1) This is the case where $\mu = 0$. Using $\mu = 0$ in (11) gives $P_t = \left(\frac{\rho}{1+r-\rho}\right) D_t$, which is what we derived in lecture.

Thus, each of the cases examined in lecture is a ~~Specials~~ Special Case of the General AR(1).

(C) Two-State Process:

$D_{t+j} = D_1$ w/ prob ϕ, $D_{t+j} = D_0$ w/ prob $1-\phi$

where $\phi \in [0, 1]$ and $D_1 > D_0$

Here

$$\underset{t}{E} D_{t+1} = \phi D_1 + (1-\phi) D_0$$

$$\underset{t}{E} D_{t+2} = \phi D_1 + (1-\phi) D_0$$

and, in general,

$$\boxed{\underset{t}{E} D_{t+j} = \phi D_1 + (1-\phi) D_0 \quad j = 1, 2, 3, \ldots} \tag{12}$$

Using (12) in (2) Gives

$$P_t = \sum_{j=1}^{\infty}\left(\frac{1}{1+r}\right)^j\left[\phi D_1 + (1-\phi) D_0\right] \quad \text{or}$$

$$\cancel{P_t = \left(\frac{1}{1+r}\right)\sum_{j=0}^{\infty}\left[D_0 + \phi(D_1 - D_0)\right]}$$

$$P_t = \left(\frac{1}{1+r}\right)\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j\left[D_0 + \phi(D_1 - D_0)\right]$$

$$P_t = \left(\frac{1}{1+r}\right)\left[\frac{1}{1-\left(\frac{1}{1+r}\right)}\right]\left[D_0 + \phi(D_1 - D_0)\right]$$

$$\boxed{P_t = \frac{1}{r}\left[D_0 + \phi(D_1 - D_0)\right] \qquad (13)}$$

Eqn (12) gives the REE value of P_t for this case.

$$\text{Note that} \quad \frac{\partial P_t}{\partial \phi} = \frac{1}{r}(D_1 - D_0) > 0 \qquad (14)$$

So an increase in ϕ causes an increase in the Equilibrium price of the Equity Share.

Economically: If ϕ goes up there is an increase in ~~the~~ expected future dividends. This leads to an increase in the expected present discounted value of dividend payments and, hence, an increase in the equilibrium price.

Problem 5:

$$m_t - p_t = \gamma - \alpha R_t, \quad \alpha > 0 \qquad (1)$$

$$R_t = r + E_t(p_{t+1} - p_t) \qquad (2)$$

(a) Economically, eqn (1) is a money-market equilibrium condition. The LHS of (1) is the (log) supply of real money balances and the RHS of (1) is the (log) demand for real money balances. Note that the demand for real money is negatively related to the nominal interest rate. This is b/c the nominal ~~inte~~ interest rate is the opportunity cost of holding money. (Since (1) gives the log of real money demand as a linear function of R_t it is a version of the Cagan money demand function.)

Eqn (2) is Fisher's Eqn. It says that the nominal ~~interest rate~~ interest rate will be the (constant) real interest rate plus the expected rate of inflation.

To answer parts (b) and (c) it is useful first to solve for p_t as a function of expected future values of m_t.

Begin by using (2) to eliminate R_t from (1),

$$m_t - p_t = \gamma - \alpha\left[r + E_t\, p_{t+1} - p_t\right]. \qquad (3)$$

Let $\psi \equiv \gamma - \alpha r$ and (3) becomes

$$m_t - p_t = \psi - \alpha E_t\, p_{t+1} + \alpha p_t \quad \text{or}$$

$$\alpha E_t\, p_{t+1} - (1+\alpha) p_t = \psi - m_t \quad \text{or}$$

$$E_t\, p_{t+1} - \left(\frac{1+\alpha}{\alpha}\right) p_t = \left(\frac{\psi}{\alpha}\right) - \frac{1}{\alpha} m_t \quad \text{or}$$

$$\left[1 - \left(\frac{1+\alpha}{\alpha}\right) L\right] E_t\, p_{t+1} = \left(\frac{\psi}{\alpha}\right) - \frac{1}{\alpha} m_t \qquad (4)$$

Since the root of this DE is $\left(\frac{1+\alpha}{\alpha}\right) > 1$ we solve eqn (4) forward ↘

$$\underset{t}{E}\, p_{t+1} = \left[\frac{1}{1-\left(\frac{1+\alpha}{\alpha}\right)L}\right]\left[\frac{\psi}{\alpha} - \frac{1}{\alpha} m_t\right] \qquad \text{or}$$

$$\underset{t}{E}\, p_{t+1} = \left[\frac{-\left(\frac{\alpha}{1+\alpha}\right)L^{-1}}{1-\left(\frac{\alpha}{1+\alpha}\right)L^{-1}}\right]\left(\frac{\psi}{\alpha} - \frac{1}{\alpha} m_t\right)$$

or, multiplying through by L

$$p_t = \left(\frac{1}{1+\alpha}\right)\left[\frac{1}{1-\frac{\alpha}{1+\alpha}L^{-1}}\right](m_t - \psi)$$

or

$$p_t = \left(\frac{1}{1+\alpha}\right)\left[\frac{1}{1-\left(\frac{\alpha}{1+\alpha}\right)}\right](-\psi) + \left(\frac{1}{1+\alpha}\right)\left[\frac{1}{1-\left(\frac{\alpha}{1+\alpha}\right)L^{-1}}\right] m_t$$

or

$$\boxed{p_t = -\psi + \left(\frac{1}{1+\alpha}\right)\sum_{j=0}^{\infty}\left(\frac{\alpha}{1+\alpha}\right)^{j} \underset{t}{E}\, m_{t+j} \qquad (5)}$$

We will use (5) to answer parts (b) and (c), below.

(b) $\boxed{m_t = \mu + \varepsilon_t, \quad \varepsilon_t \sim iid(0, \sigma^2) \qquad (6)}$

Solve for p_t

From (6) $$E_t\, m_t = m_t \qquad (7.1)$$

$$E_t\, m_{t+1} = \mu$$

$$E_t\, m_{t+2} = \mu$$

$$E_t\, m_{t+j} = \mu \qquad j = 1, 2, 3, \ldots \quad (7.2)$$

Subst from (7.1) and (7.2) into (6) to get

$$p_t = -\psi + \left(\frac{1}{1+\alpha}\right) m_t + \left(\frac{1}{1+\alpha}\right)\left(\frac{\alpha}{1+\alpha}\right) \sum_{j=0}^{\infty} \left(\frac{\alpha}{1+\alpha}\right)^j \mu \quad (8)$$

Using $\psi \equiv \gamma - \alpha r$, and ~~(7.1)~~ (6) in (8) gives

$$p_t = (\alpha r - \gamma) + \left(\frac{1}{1+\alpha}\right)\mu + \left(\frac{1}{1+\alpha}\right)\left(\frac{\alpha}{1+\alpha}\right) \sum_{j=0}^{\infty} \left(\frac{\alpha}{1+\alpha}\right)^j \mu$$

$$+ \left(\frac{1}{1+\alpha}\right)\varepsilon_t \qquad \text{or}$$

$$p_t = (\alpha r - \gamma) + \left(\frac{1}{1+\alpha}\right) \sum_{j=0}^{\infty} \left(\frac{\alpha}{1+\alpha}\right)^j \mu + \frac{1}{1+\alpha}\varepsilon_t$$

$$p_t = (\alpha r - \gamma) + \left(\frac{1}{1+\alpha}\right)\left[\frac{1}{1 - \left(\frac{\alpha}{1+\alpha}\right)}\right]\mu + \frac{1}{1+\alpha}\varepsilon_t$$

$$\boxed{p_t = (\alpha\Gamma - \gamma) + \mu + \left(\frac{1}{1+\alpha}\right)\varepsilon_t \qquad (9)}$$

Eqn (9) is the REE Solution for p_t

Solve for R_t

From (2)

$$R_t = \Gamma + \underset{t}{E}\, p_{t+1} - p_t \qquad (2)$$

From (9) we have

$$p_{t+1} = (\alpha\Gamma - \gamma) + \mu + \left(\frac{1}{1+\alpha}\right)\varepsilon_{t+1} \qquad (10)$$

Thus, since $\underset{t}{E}\,\varepsilon_{t+1} = 0$,

$$\boxed{\underset{t}{E}\, p_{t+1} = (\alpha\Gamma - \gamma) + \mu \qquad (11)}$$

Use (10) and (11) in (2) to get

$$R_t = \Gamma + (\alpha\Gamma - \gamma) + \mu - \left[(\alpha\Gamma - \gamma) + \mu + \left(\frac{1}{1+\alpha}\right)\varepsilon_t\right]$$

or

$$\boxed{R_t = \Gamma + \left(\frac{-1}{1+\alpha}\right)\varepsilon_t \qquad (12)}$$

Eqn (12) is the REE Solution for R_t.

(c) Now we suppose instead that

$$\boxed{m_t = \rho m_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim iid(0, \sigma^2) \quad (13)}$$

Solve for p_t

From (13)

$$\underset{t}{E}\, m_t = m_t \qquad (14.1)$$

$$\underset{t}{E}\, m_{t+1} = \rho m_t$$

$$\underset{t}{E}\, m_{t+2} = \rho^2 m_t$$

$$\vdots$$

$$\underset{t}{E}\, m_{t+j} = \rho^j m_t \quad j = 1, 2, 3, \ldots \qquad (14.2)$$

Using (14.1) and (14.2) in (5) gives

$$p_t = -\psi + \left(\frac{1}{1+\alpha}\right) \sum_{j=0}^{\infty} \left(\frac{\alpha}{1+\alpha}\right)^j \rho^j m_t \quad \text{or}$$

$$p_t = -\psi + \left(\frac{1}{1+\alpha}\right) \left[\frac{1}{1 - \frac{\alpha\rho}{(1+\alpha)}}\right] m_t \quad \text{or}$$

$$\boxed{p_t = -\psi + \left[\frac{1}{1+\alpha-\alpha\rho}\right] m_t \qquad (15)}$$

Eqn (15) is the REE Solution for p_t

Note: $-\psi = (\alpha\Gamma - \gamma)$

Solve for R_t

From (15)

$$p_{t+1} = -\psi + \left[\frac{1}{1+\alpha-\alpha\rho}\right] m_{t+1} \qquad (16)$$

Thus, using (14.2)

$$E_t p_{t+1} = -\psi + \left[\frac{1}{1+\alpha-\alpha\rho}\right] \rho m_t \qquad (17)$$

Using (15) and (17) in (2)

$$R_t = r + \left[-\psi + \left(\frac{\rho}{1+\alpha-\alpha\rho}\right) m_t\right] - \left[-\psi + \left(\frac{1}{1+\alpha-\alpha\rho}\right) m_t\right]$$

So that

$$\boxed{R_t = r + \left[\frac{\rho-1}{1+\alpha-\alpha\rho}\right] m_t \qquad (18)}$$

Eqn (18) is the REE Solution for R_t