ECON 6700: Mathematical Methods in Economics II, Fall 2021

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Office Hours: Mondays, 1 to 4 pm, and by appointment. Because of Covid, and consistent with department policy, I cannot meet with students in my office. Consequently, **I will conduct office-hour meetings via Zoom.**

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Dear Students,

In this syllabus and in my first lecture I outline how I believe we can proceed this semester. We are in a period of transition to a normal, fully in-person, mode of instruction. But we are not quite there yet. I understand that this is still a difficult time and that you may face problems that I have not anticipated. Please contact me if you have a problem or a question. There may be reversals and I am learning as I go so how we conduct this course may evolve. We will do our best and, with an extra bit of patience, courtesy, and compassion, we will get through this time together.

I very much hope that you and your families are well.

Sincerely,
Bart Moore

Masks: University policy is that **all individuals must wear a mask indoors**, even if they are vaccinated. Consistent with this policy, I will wear a mask when I teach and **all students must wear a mask** when they are in my classroom.

Course description: This course will introduce the mathematical tools necessary for advanced Economic analysis. Emphasis is given to the tools used to analyze equilibrium in a dynamic environment; difference equations, stochastic processes, and dynamic programming. I provide an introduction to sequences and convergence. I also introduce the Kuhn-Tucker approach to constrained optimization. I assume that all students are familiar with the topics covered in ECON 5700: Mathematical Methods in Economics I.

Requirements and grading: Your grade will be based on a midterm exam (40%) and a final exam (60%). I recommend problems on this syllabus. The TA will go over many of these problems during the lab session. The problems and their answers supplement and extend the material that I present in lecture so you should work them through carefully.

<u>Primary Texts:</u> The first two texts listed below are available for purchase online or in the campus bookstore. Readings designated with an **(R)** will be placed on A-res.

1.) Simon, Carl P., and Lawrence Blume, *Mathematics for Economists,* Norton & Co.,1994. (Hereafter, S&B).
2.) Hamilton, James, *Time Series Analysis* (Princeton University Press, 1994).
3.) Sargent, Thomas J. *Dynamic Macroeconomic Theory,* Harvard U. Press (Cambridge, MA, 1997), Chapter 1: Dynamic Programming (DMT). **(R)**

Course Outline and Readings

I. Eigenvalues, Eigenvectors, and Difference Equations.

A. Eigenvalues and Eigenvectors (1½ weeks).
S&B; Chapters 23 & 16.1-16.2.
Hamilton; Appendix, pp. 729-32.

B. Deterministic Difference Equations (1½ weeks).
S&B; Chapter 23 (continued) & Appendix A.3.
Hamilton; Appendix on complex numbers, pp. 704-11, 713-17.

C. Stochastic Difference Equations and Lag Opertaors (2 weeks).
Hamilton; Chapters 1 & 2.
Sargent, Thomas J. *Macroeconomic Theory,* 2nd Edition, (Academic Press, 1987), Chapter IX. **(R)**

II Stationary ARMA processes (1½ weeks).
Hamilton; Chapter 3 (esp. 3.1-3.5) and Chapter 4, (esp., 4.1 and 4.2).

<u>Background:</u>
Hamilton; Chapter 15, Models of Nonstationary Time Series.

III. Markov Chains (2 weeks).
Hamilton; Chapter 22, (esp., 22.1 and 22.2).
S&B, Chapter 23.6 (pp. 615-620) and pages 113-115.

<u>Background:</u>
Grinstead and Snell, *Introduction to Probability,* Chapter 11: Markov Chains. **(R)**

Mehra, R. and E. Prescott, The Equity Premium: a Puzzle, *Journal of Monetary Economics,* 1985, 15(2), pp.145-62.

IV. Expectational Difference Equations (1 week).

Blanchard, O. and Stanley Fischer, *Lectures on Macroeconomics,* (MIT Press, 1981). Chapter 5, "Multiple Equilibria, Bubbles and Stability" (especially pp. 213- 226 and "Appendix to Chapter 5: A tool kit ..." pp. 261- 274). **(R)**

Background:

Sargent, Thomas J. *Macroeconomic Theory,* 2nd Edition, (Academic Press, 1987), Chapter XIV.

Appendix B to Moore, B.J. "Monetary Policy Regimes and Inflation in the New Keynesian Model," *Journal of Macroeconomics*, June 2014, **(R)**

DeJong, D. and C. Dave, *Structural Macroeconometrics*, 2nd Edition, (Princeton U. Press, 2011), Chapter 4: Linear Solution techniques.

V. Optimization with Inequality Constraints (1 week).

Chiang, Alpha C. *Fundamental Methods of Mathematical Economics,* 3rd Edition, (McGraw-Hill, 1984), Chapter 21: Nonlinear Programming.

S&B, Chapters 18& 19 and Chapter 16.3 (pp. 386-393).

Background:

Mas-Colell, A., M. Whinston, and J. Green, *Microeconomic Theory*, (Oxford U. Press, 1995), Appendix, Section M.K: Constrained Maximization.

VI. Dynamic Programming (2 weeks).

Sargent, DMT, Chapter 1: Dynamic Programming. **(R)**

Background:

Sargent, T. J. and L. Lindquist, *Recursive Macroeconomic Theory*, 2nd Edition, (MIT Press, 2004).

King, Ian, "A Simple Introduction to Dynamic Programming in Macroeconomic Models," U. of Auckalnd w.p., April 2002. **(R)**

VII. Sequences and Convergence (1 week).

S&B, Ch.12 and Ch. 29 (especially pp.803-809).

Background:

Sargent DMT appendix.

Recommended Problems.

I.A. Eigenvalues and Eigenvectors.
S&B Exercises: 23.2 a-c; 23.5; 23.7 a, b; 23.13; 23.29, 23.48.
Prove S&B Theorem 23.16 for a (2X2) matrix (this is in the book).
S&B Ex.: 16.1 a-f. For these use Theorem 16.1, or Thm. 23.7, or Thm. 23.9

I.B. Deterministic Difference Equations.
The following problems are modified versions of problems in Chiang, A.C., and R. Wainwright, *Fundamental Methods of Mathematical Economics,* 4th Edition, McGraw Hill, 2004.

1.) For each of the difference equations below (i) find the stationary-state value of y_t, which you should denote by y_s, (ii) rewrite the difference equation in terms of $z_t \equiv (y_t - y_s)$, (iii) give the general solution and, using the given initial condition, the definite solution to the FODE, (iv) evaluate whether the DE converges or diverges and whether it is oscillatory or non-oscillatory. Sketch those labeled (a) and (c).

C&W 17.2, #4:
(a) $y_{t+1} + 3y_t = 4, \qquad y_0 = 4,$
(b) $2y_{t+1} - y_t = 6, \qquad y_0 = 7,$
(c) $y_{t+1} = 0.2y_t + 4, \qquad y_0 = 4,$

C&W 17.3, #3:
(a) $y_{t+1} - \frac{1}{3}y_t = 6, \qquad y_0 = 1,$
(b) $y_{t+1} + 2y_t = 9, \qquad y_0 = 4,$
(c) $y_{t+1} + \frac{1}{4}y_t = 5, \qquad y_0 = 2.$

2.) For each of the difference equations below (i) find the stationary-state value of y_t, which you should denote by y_s, (ii) rewrite the difference equation in terms of $z_t \equiv \begin{bmatrix} y_t - y_s \\ y_{t-1} - y_s \end{bmatrix}$, (iii) give the general solution and, using the given initial conditions, the definite solution to the SODE.

C&W 18.1, #4:
(a) $y_{t+1} + 3y_t - \frac{7}{4}y_{t-1} = 9, \qquad y_0 = 3, y_{-1} = 1,$
(b) $y_{t+1} - 2y_t + 2y_{t-1} = 1, \qquad y_0 = 4, y_{-1} = 3,$
(c) $y_{t+1} - y_t + \frac{1}{4}y_{t-1} = 2, \qquad y_0 = 7, y_1 = 4.$

3.) C&W 16.2, #6. Use the Euler relations to show the following:

(a) $e^{-i\pi} = 1$ (b) $e^{i(\pi/3)} = \frac{1}{2}\left(1+\sqrt{3}\cdot i\right)$ (c) $e^{i(\pi/4)} = \frac{\sqrt{2}}{2}(1+i)$ (d) $e^{-3i(\pi/4)} = -\frac{\sqrt{2}}{2}(1+i)$

4.) C&W 16.2, #7. Find the Cartesian form of each of the following. Graph (a) and (c).

(a) $2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$ (b) $4e^{i(\pi/3)}$ (c) $\sqrt{2}e^{i(\pi/4)}$

5.) C&W 16.2, #8. Find the exponential and polar form of each. Graph the polar form.

(a) $\frac{3}{2}+\frac{3\sqrt{3}}{2}i$ (b) $4\left(\sqrt{3}+i\right)$

I.C. Stochastic Difference Equations. (Additional) Problems:

1.) Consider the second-order difference equation

$$y_t = c+\phi_1 y_{t-1}+\phi_2 y_{t-2}+w_t \qquad (1)$$

where c, ϕ_1, and ϕ_2 are constants and w_t is a mean-zero random variable.

(a) Rewrite equation (1) as a first-order vector difference equation, that is, as

$$\xi_t = \mathbf{F}\xi_{t-1}+\mathbf{v}_t \qquad (2)$$

giving explicit definitions of ξ_t, $\mathbf{F}$, and $\mathbf{v}_t$.

(b) Show that the eigenvalues of $\mathbf{F}$ are the roots of the difference equation (1).

2.) Prove that the roots of $1-\phi_1 z-\phi_2 z^2 = 0$ are inverse of the roots of $\lambda^2-\phi_1\lambda-\phi_2 = 0$.

3.) A widely-studied version of the stochastic growth model can be written as the following system of two first-order difference equations:

$$-C_t = -C_{t+1}+a_1 K_{t+1}+a_2 e_{t+1} \qquad (1a)$$

$$K_{t+1} = b_1 K_t + b_2 w_{t+1}+b_3 C_t \qquad (1b)$$

where $a_1, a_2, b_1, b_2,$ and b_3 are constants and e_t and w_t are random variables.

(a) Combine (1a) and (1b) to obtain a single second-order difference equation in K_t.

Write the second-order system that you derive as first-order vector difference equation of the form

$$\xi_t = \mathbf{F}\xi_{t-1}+\mathbf{v}_t \qquad (2)$$

giving complete definitions of $\xi_t, \mathbf{F}$, and $\mathbf{v}_t$ in terms of the original specification in (1a) and (1b).

(b) Next define $\mathbf{x}_t \equiv \begin{bmatrix} C_t \\ K_t \end{bmatrix}$. Write the original system, equations (1a) and (1b) as a vector difference equation of the form

$$\mathbf{x}_{t+1} = \mathbf{A}\mathbf{x}_t + \mathbf{B}\mathbf{u}_{t+1} \qquad (3)$$

giving complete definitions of the matrices $\mathbf{A}$ and $\mathbf{B}$ and the vector $\mathbf{u}_{t+1}$ in terms of the original specification in (1a) and (1b).

(c) How do the eigenvalues of $\mathbf{F}$ compare to the eigenvalues of $\mathbf{A}$?

4.) The dynamic behavior of the capital stock, K_t, in a widely-studied version of the stochastic growth model is governed by the following second-order difference equation:

$$K_{t+2} - \left(1 + \tfrac{1}{\beta} + a_1 b_3\right) K_{t+1} + \tfrac{1}{\beta} K_t = u_{t+2}. \qquad (2)$$

Here, $0 < \beta < 1$ is the discount factor, $a_1 < 0$, $b_3 < 0$, and u_{t+2} is a random shock. Denote the roots of this second-order difference equation by λ_1 and λ_2.

Prove that $\quad 0 < \lambda_1 < 1 < \dfrac{1}{\beta} < \lambda_2$.

How do the roots, λ_1 and λ_2, in this problem compare to the roots of the system formed by equations (1a) and (1b) in Problem 3 of Section IC, above?

5.) Let $\{w_t\}_{t=-\infty}^{\infty}$ be a bounded sequence of random variables. Show that

$y_t = a_0 \phi^t + \sum_{i=0}^{\infty} \phi^i w_{t-i}$ is a solution to the difference equation $y_t = \phi y_{t-1} + w_t$, where $|\phi| < 1$.

If the sequence of solutions, $\{y_t\}_{t=-\infty}^{\infty}$, is bounded, what value must the constant a_0 take? Explain.

6.) Let ϕ and c denote constants. Prove the following two propositions:

(a) If $|\phi| < 1$ then $\left(\dfrac{1}{1-\phi L}\right) c = \left(\dfrac{1}{1-\phi}\right) c$.

(b) If $|\phi| > 1$ then $\left(\dfrac{-\phi^{-1} L^{-1}}{1-\phi^{-1} L^{-1}}\right) c = \left(\dfrac{1}{1-\phi}\right) c$.

II. Stationary ARMA Processes.

1.) Suppose that Y_t is covariance stationary. As in Hamilton let γ_j denote the *j*th autocovariance of Y_t and let ρ_j denote its *j*th autocorrelation.

(a) Derive γ_j and ρ_j, $j = 0, 1, 2$, for the case where Y_t is MA(1) and for the case where Y_t is MA(2).

(b) Suppose instead that Y_t is AR(1), that is, $y_t = \phi y_{t-1} + \varepsilon_t$ where ε_t is white noise. Derive γ_j and ρ_j, $j = 0, 1, 2$. What can you conclude (or conjecture) about the relationship between the autoregressive parameter, ϕ, and the autocorrelation, ρ_j.

2.) Suppose that Y_t is a stationary AR(2) process. That is

$$y_t = c + \phi_1 y_{t-1} - \phi_2 y_{t-2} + \varepsilon_t \qquad (1)$$

where c, ϕ_1, and ϕ_2 are constants and ε_t is white noise. Furthermore, suppose that $\phi_1 = (a+b)$ and $\phi_2 = ab$ where $|a|<1$ and $|b|<1$. Derive the $\mathrm{MA}(\infty)$ representation of Y_t and give explicit expressions for the MA coefficients in terms of the constants, a and b.

3.) Consider the following stationary ARMA(p,q) process:

$$y_t = c + \phi_1 y_{t-1} + \ldots + \phi_p y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} \qquad (1)$$

Here, c, ϕ_j, and θ_j are constants and $\varepsilon_t \;\; i.i.d.(0, \sigma^2)$

(a) Rewrite (1) as the first-order vector difference equation

$$\xi_t = \mathbf{F}\xi_{t-1} + \mathbf{G}\mathbf{v}_t \qquad (2)$$

giving complete definitions of $\xi_t, \mathbf{F}, \mathbf{G},$ and $\mathbf{v}_t$ in terms of the parameters of the original specification in (1).

4.) The present-value model of asset prices gives that the equilibrium price of an asset is the expected present discounted value of its dividend payments. Thus, if P_t denotes the current price of the asset and $r > 0$ denotes the known constant interest rate we have the equilibrium condition

$$P_t = E_t\left[\left(\frac{1}{1+r}\right)d_{t+1} + \left(\frac{1}{1+r}\right)^2 d_{t+2} + \left(\frac{1}{1+r}\right)^3 d_{t+3} + \ldots\right] \qquad (1).$$

Here $E_t[\bullet]$ denotes $E[\bullet | I_t]$ where I_t, the current information set, is

$$I_t = \{d_t, d_{t-1}, d_{t-2}, ...; P_{t-1}, P_{t-2}, P_{t-3}, ...; \Theta\}$$

where Θ denotes the structure of the relevant stochastic process including its parameters (and including r). In each case below, solve for P_t as a function of the elements of I_t. In each case you should assume that ε_t is white noise and that the (unconditional) mean of d_t is nonzero.

a.) d_t is MA(1): $d_t = \mu + \varepsilon_t + \theta\varepsilon_{t-1}$, $\mu \neq 0$.

b.) d_t is MA(2): $d_t = \mu + \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2}$, $\mu \neq 0$.

c.) d_t is AR(1): $d_t = c + \phi d_{t-1} + \varepsilon_t$, $c \neq 0$.

III. Markov Chains.

S&B Exercises: 23.31, 23.32, 23.33a,b.

Additional Problems.

1.) Let $\mathbf{P}$ be the transition probability matrix for an ergodic two-state Markov chain. Using Hamilton's notation, let $\mathbf{P} = \begin{bmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{bmatrix}$. Derive an expression for each of the eigenvalues of $\mathbf{P}$ as a function of p_{11} and p_{22}.

2.) Hamilton, Chapter 22, Exercise 22.1. Use your results from Additional Problem 1 to complete this exercise.

3.) You will need the (simple) result in (a) in your answer to (b).

a.) Suppose that $\mathbf{A}$ is an invertible matrix. Show that $(\mathbf{A}')^{-1} = (\mathbf{A}^{-1})'$.

b.) Suppose that $\mathbf{P}_{(N\times N)}$ is the transition probability matrix for an ergodic Markov chain and that $\mathbf{P} = \mathbf{T}\Lambda\mathbf{T}^{-1}$ where Λ is the diagonal matrix that has the eigenvalues of $\mathbf{P}$ along its diagonal ordered so that the largest eigenvalue is the (1,1) element of Λ. Let $\mathbf{y}'_{(1\times N)}$ denote the first row of $\mathbf{T}^{-1}$. Show that $\mathbf{y}_{(N\times 1)}$ is the eigenvector of $\mathbf{P}'$ that is associated with the eigenvalue of 1.

4.) Consider again the present-value model of asset prices. Denote the current price of an asset by A_t and the constant interest rate by $r > 0$. Thus,

$$A_t = E_t\left[\left(\frac{1}{1+r}\right)d_{t+1} + \left(\frac{1}{1+r}\right)^2 d_{t+2} + \left(\frac{1}{1+r}\right)^3 d_{t+3} + ...\right] \qquad (1)$$

where d_{t+j} is the dividend payment in period $t+j$. Suppose that the mean dividend is governed by an ergodic three-state Markov chain, specifically

$$d_{t+j} = \mu_{t+j} + \varepsilon_{t+j}, \qquad j = 0,1,2,... \tag{2}$$

where ε_{t+j} is white noise and where $\mu_{t+j} = \mu_1$ if $s_{t+j} = s_1$, $\mu_{t+j} = \mu_2$ if $s_{t+j} = s_2$, and $\mu_{t+j} = \mu_3$ if $s_{t+j} = s_3$.

a.) Use Hamilton's notation to describe how to use the vector ξ_{t+j} to summarize the state in $t+j$.

b.) Derive expressions for $E(d_{t+1} \mid \xi_t)$, $E(d_{t+2} \mid \xi_t)$, and in general for $E(d_{t+j} \mid \xi_t)$.

c.) Explain, given the assumptions of the problem, why

$$E_t\left[\left(\frac{1}{1+r}\right)d_{t+1} + \left(\frac{1}{1+r}\right)^2 d_{t+2} + \ldots\right] = \left\{E\left[\left(\frac{1}{1+r}\right)d_{t+1} + \left(\frac{1}{1+r}\right)^2 d_{t+2} + \left(\frac{1}{1+r}\right)^3 d_{t+3} + \ldots\right] \middle| \xi_t\right\}$$

d.) Solve for A_t as a function of r, $\mathbf{P}$, $[\mu_1, \mu_2, \mu_3]$, and ξ_t. (In formulating your answer you may want to review the section in Hamilton's appendix, page 732, entitled *Matrix Geometric Series*. Note especially the result in equation [A.4.33].)

5.) Five lily pads in a pond lie along a straight line, north to south. They are numbered, north to south; 1, 2, 3, 4, 5.
a.) If a frog sits on pad 2, 3, or 4, next period it will jump off the pad going north or south with equal probability. If it jumps onto pad 1 (northernmost) or pad 5 (southernmost) it will stay there. Write down the transition probability matrix for this Markov chain. Is it ergodic? Is it regular? Explain.
b.) Now suppose that the frog is biased. If it is on pad 2, 3, or 4, next period it will jump off the pad but it is twice as likely to jump south as it is to jump north. However, if it jumps onto pad 5 (southernmost) next period it will jump north to pad 4. As before, if the frog jumps on to pad 1 (northernmost) it will stay there. Once again, write down the transition probability matrix for this Markov chain. Is it ergodic? Is it regular? Explain.

IV. A. Expectational Difference Equations.
Sargent, *Macroeconomic Theory,* Chapter IX, Exercise 6.

Additional Problems:
1.) Consider the following two-equation model of monetary equilibrium under rational expectations. The first equation, based on the Cagan form of the money demand function says that the natural logarithm of real money balances is inversely and linearly related to the nominal interest rate:

$$m_t - p_t = \gamma - \alpha R_t, \qquad \alpha > 0 \tag{1}$$

where m_t denotes (log) nominal money, p_t the (log) price level, and R_t the nominal interest rate. The second equation is Fisher's equation:

$$R_t = r + E_t(p_{t+1} - p_t). \tag{2}$$

This says that the nominal interest rate equals a constant real interest rate, r, plus the expected rate of inflation. Note that, via the log approximation to percentages $p_{t+1} - p_t$ is the inflation rate. (cf. Blanchard & Fischer page 216 and Sargent, *Macroeconomic Theory*, pp. 195-7.)

(a) Suppose that the nominal money supply rule is $m_t = \mu_0 + \mu_1 t + \varepsilon_t$, where t is the time index and where ε_t is a white noise process. Solve for the rational expectations equilibrium (REE) values of p_t and R_t.
(b) Suppose instead that the nominal money supply rule is $m_t = \rho m_{t-1} + \varepsilon_t$, where $|\rho| < 1$ and ε_t is a white noise process. Once again, solve for the REE values of p_t and R_t.

2.) Consider the following simple discrete-time version of the Dornbusch (1976) exchange-rate model:

Money market: $m_t - p_t = -\alpha R_t + \psi y_t, \quad \alpha > 0, \psi > 0,$ (1)

Interest parity: $R_t = R^* + E_t\left(\Delta e_{t+1}\right),$ (2)

Price adjustment: $\Delta p_{t+1} = \delta\left(e_t - p_t\right), \quad \delta > 0.$ (3)

In the above R_t denotes the domestic nominal interest rate and R^* denotes the (constant) foreign nominal interest rate. Also, m, p, y, and e denote the logarithms of the domestic money supply, price level, income, and exchange rate, respectively. Note that the exchange rate is the domestic-currency price of a unit of foreign currency so that an *increase* in e is a *depreciation*. Also note that $\Delta e_{t+1} = e_{t+1} - e_t$ and $\Delta p_{t+1} = p_{t+1} - p_t$.
The **economics** of the model, briefly, are as follows. Equation (1) is the Cagan-form of the money demand function, the demand for real money is inversely related to the nominal interest rate and positively related to income. Equation (2) is an arbitrage equation, it says that the domestic interest rate must adjust up above the foreign rate to compensate investors for any expected depreciation of the domestic currency. Equation (3), which can be re-written as $p_{t+1} = \delta e_t + \left(1-\delta\right) p_t$. Thus next period's (log) price level is set as a weighted average of this period's price level and, reflecting the effect of import prices, this period's exchange rate. Note that a *depreciation*, an *increase* in e_t will cause an increase in p_{t+1}.
Simplify the model further by setting $y = R^* = 0$ and assume that $m_t = \bar{m}$, that is the (log) money supply is constant. Assume perfect foresight so that $E_t\left(\Delta e_{t+1}\right) = \Delta e_{t+1}$.
(a) Derive expressions that give Δe_{t+1} and Δp_{t+1} each as a function of p_t and e_t. What are the stationary-state (steady-state) values of p and e?
(b) Construct the phase diagram for the system. (Put e on the horizontal axis.) Locate and draw the $\Delta e_{t+1} = 0$ and $\Delta p_{t+1} = 0$ loci. Locate and label the stationary state. Characterize (draw) the dynamic behavior of p and e in all four regions of the diagram. Locate and draw the saddle path.
(c) In this model p is assumed to be predetermined (it cannot "jump" discontinuously) and e is not predetermined (it can "jump" discontinuously). Is equilibrium in this model determinate? Explain.
(d) Begin in the stationary state with the log money supply constant at $\bar{m}$. Suppose that there is an unanticipated one-time increase in the log money supply to $\tilde{m} > \bar{m}$. Using the phase diagram and also separate plots of p and e versus time, describe the dynamic behavior of p and e, beginning from before the increase and continuing through the time when they have converged to their new stationary-state values.

V. Optimization with Inequality Constraints
Chiang: Exercise 21.2, Probs. 1, 4, and 5.
Exercise 21.6, Prob. 1.
S&B: Exercise 16.6, items a, c, and e. (Some of the matrices are a bit large so you may want to evaluate their determinants using your computer.)
S&B: Exercises 18.10 and 18.13

VI. Dynamic Programming.
Sargent, DMT: Exercise 1.1. Derive the form of the value function and an expression giving the coefficient on $\ln k$ (in the value function) as an explicit function of the model's parameters. You do not need to derive the constant term.

Sargent DMT: Additional Problem. Work through the Dynamic Portfolio problem that begins on the bottom of page 33 of Sargent, DMT. Assume log utility and that there are two assets. Derive the two equations in k_1 and k_2 that appear on page 35 of DMT.

Sargent, *Macroeconomic Theory*, Chapter IX, Exercise 11. Is the equilibrium in this exercise determinate? Explain.

Additional Problem. Consider the following stochastic optimal-investment problem. A firm seeks to maximize the present discounted value of its profits. It produces output, y_t, using capital inputs, k_t, according to a linear production function, $y_t = Ak_t$, where A is a positive constant. It sells each unit of output for the exogenous price p_t where p_t is $i.i.d.(\overline{p},\sigma^2)$. The only cost is a quadratic cost of investment. Denote investment (new capital) by I_t. Capital depreciates at rate δ where $0<\delta<1$. Thus, each period the firm's revenue is $R(I_t,k_t) = p_t Ak_t - \frac{\alpha}{2}I_t^2$ where $\alpha > 0$. The firm's problem is to maximize $E_t\sum_{i=0}^{\infty}\beta^i R\left(I_{t+i},k_{t+i}\right)$, subject to $k_{t+1+i} = \left(1-\delta\right)k_{t+i} + I_{t+i}$.

Appropriate side conditions apply but need not be discussed in your answer to this question.

Set up the dynamic programming problem. Derive the Euler equation using the method discussed in class. Be careful to write out Bellman's equation and explain the value function. Also, be explicit about how the Envelope Theorem is used in your derivation of the Euler equation.

VII. Sequences and Convergence.

1) S&B Ex 12.5
2) Use the result from S&B Ex 12.5 to prove Theorem 12.3 (Most of which is in the text)
3) S&B Ex 12.6
4) S&B Ex 12.10 - Proof of 2nd part of Theorem 12.4
5) S&B Ex 12.12
6) S&B Prove theorem 12.5 (This is in the text)
7) S&B Ex 12.15
8) S&B Ex 12.17
9) S&B Ex 12.21
10) S&B Ex 29.2
11) S&B Ex 29.4
12) S&B Prove Lemma 29.2 (This is in the text)
13) Prove the Following Proposition: $| |X| - |Y| | \leq |X - Y|$ when $|X| \leq |Y|$