Homework 3

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ECON 7010- Microeconomics II

Due on Feb 9, 2022

1 Qeustion 1 – 3.B.1

Solution:

If we want to gain convex preference with locally nonsatiated but is not monotone, we need to make this two points one in lower preference point but higher combination of two goods if in $\mathcal{R}^2$[1] Question: What's the specific function form for this graph?

2 Question 2 – 3.C.2

Solution: This question I mainly refer to Definition 3.C.1. I also assume for any sequence of pairs $\{(x^n, y_n)_{n=1}^{\infty}\}$ with $x^n \succsim y^n$, and $\lim_{n\to\infty} x^n = x$, $\lim_{n\to\infty} y^n = y$. From the question, we know $u(\cdot)$ is a countinuous utility function with the relation $\succsim$, which means when $n \to \infty$, $u(x) > u(y)$, hence, $x \succsim y$, and $\succsim$ is continuous.

3 Question 3 – 3.C.6

Solution:

(a) When $\rho = 1$, the utility would be $u(x) = \alpha_1 x_1 + \alpha_2 x_2$, which is obviously a linear function.

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[1] See `https://felixmunozgarcia.files.wordpress.com/2017/08/recitation_1.pdf` In his graph, $y > x$ but for each direction x is better than y, which means it's not monotone.

(b) As $\rho \to 0$, we transfer $u(x)$ into $\ln u(x)$. $\ln u(x) = \frac{\ln(\alpha_1 x_1^\rho + \alpha_2 x_2^\rho)}{\rho}$ Use L' Hopital Rule:

$$\begin{aligned}\lim_{\rho\to 0} \ln u(x) &= \frac{\ln(\alpha_1 x_1^\rho + \alpha_2 x_2^\rho)}{\rho} \\ &= \frac{\alpha_1 x_1^\rho \ln x_1 + \alpha_2 x_2^\rho \ln x_2}{\alpha_1 x_1^\rho + \alpha_2 x_2^\rho} \\ &= \frac{\alpha_1 \ln x_1 + \alpha_2 \ln x_2}{\alpha_1 + \alpha_2}\end{aligned}$$

Multiply both sides by $\alpha_1 + \alpha_2$, we can obtain:

$$(\alpha_1 + \alpha_2) \ln u(x) = \ln(x_1^{\alpha_1} x_2^{\alpha_2})$$

Make operator of exp on both sides:

$$u(x)^{\alpha_1+\alpha_2} = x_1^{\alpha_1} x_2^{\alpha_2} == \text{Cobb-Douglas utility function}$$

(c) This question is tricky, I mainly use online solution[2], I write it in my way as below: Assume $x_1 < x_2$ in this case, so our problem becomes to prove $lim_{n\to\infty} u(x) = x_1$. When $x_1 \geq 0$ and $x_2 \geq 0$: as parameters α_1 and α_2 are both larger than 0. $\alpha_1 x_1^\rho \leq \alpha_1 x_1^\rho + \alpha_2 x_2^\rho$,thus:

$$\alpha_1^{\frac{1}{\rho}} x_1 \geq (\alpha_1 x_1^\rho + \alpha_2 x_2^\rho)^{\frac{1}{\rho}} \qquad 3.C.6(1)$$

As $x_1 \leq x_2$:

$$\alpha_1 x_1^\rho + \alpha_2 x_2^\rho \leq \alpha_1 x_1^\rho + \alpha_2 x_1^\rho = (\alpha_1 + \alpha_2)^\rho$$

Thus:

$$(\alpha_1 x_1^\rho + \alpha_2 x_2^\rho)^{\frac{1}{\rho}} \geq (x_1 + x_2)^{\frac{1}{\rho}} x_1 \qquad 3.C.6(2)$$

Combine (3.C.6(1)) and (3.C.6(2)), we can gain the relationship:

$$\alpha_1^{\frac{1}{\rho}} x_1 \geq (\alpha_1 x_1^\rho + \alpha_2 x_2^\rho)^{\frac{1}{\rho}} \geq (\alpha_1 + \alpha_2)^{\frac{1}{\rho}} x_1$$

By Squeeze Theorem, when $\rho \to -\infty$, this utility limits to $x_1 = \min\{x_1, x_2\}$ == Leontief Utility Function

4 Question 4 – 3.D.5

Solution:

When $\alpha_1 = \alpha_2 = 1$:

(a) $u(x) = (x_1^\rho + x_2^\rho)^{\frac{1}{\rho}}$ Make some monotone transformation of this utility function: $\hat{u(x)} = \rho u(x)^\rho = \rho(x_1^\rho + x_2^\rho)$ Take first order differential equaion: we can get the walrasian demand function: $x(p,w) = (\frac{w}{p_1^\delta + p_2^\delta})(p_1^{\delta-1}, p_2^{\delta-1})$, $\delta = \frac{\rho}{\rho-1}$. Thus, the indirect utility is $v(p,w) = \frac{w}{(p_1^\delta + p_2^\delta)^{\frac{1}{\delta}}}$

[2] See `https://fdocuments.in/document/mwg-solutions.html?page=4`

(b) first check homogeneity of degree zero:

$$x(\alpha p, \alpha w) = (\frac{\alpha w}{(\alpha p_1)^\delta + (\alpha p_2)^\delta})((\alpha p_1)^{\delta-1}, (\alpha p_2)^{\delta-1})$$
$$= \frac{w}{p_1^\delta + p_2^\delta}(p_1^{\delta-1}, p_2^{\delta-1})$$

Second: check Walras's law: $px(p,w) = (\frac{w}{p_1^delta + p_2^\delta})(p_1 p_1^{\delta-1}, p_2 p_2^{\delta-1}) = w$

For indirect utility function, $v(\alpha p, \alpha w) = (\frac{\alpha w}{(\alpha p_1)^\delta + (\alpha p_2)^\delta})$ For monotonity: $\frac{\partial v(p,w)}{\partial w} > 0$, and $\frac{\partial v(p,w)}{\partial p_l} < 0$

(c) For Leontief utility function:If $p_1 < p_2$, $x(p,w) = (\frac{w}{p}, 0)$, when $P_1 > p_2$, it's $(0, \frac{w}{p_2})$. Otherwise: $(\frac{w}{p_1})(\lambda, 1-\lambda)$. Indirect utility function is: $v(p,w) = \max(\frac{w}{p_1}, \frac{w}{p_2})$.

(d) $\frac{x(p,w)}{x_2(p,w)} = (\frac{p_1}{p_2})^{\delta-1}$, and $\frac{(\frac{x_1(p,w)}{x_2(p,w)})}{\frac{p_1}{p_2}} = (\frac{p_1}{p_2})^{\delta-2}$. For $\frac{d(\frac{x_1(p,w)}{x_2(p,w)})}{d(\frac{p_1}{p_2})} = (\delta-1)(\frac{p_1}{p_2})^{\delta-2}$ Thus, $\epsilon_{1,1}(p,w) = -(\delta-1) = \frac{1}{1-\rho}$ When it's linear function, $\epsilon_{1,2}(p,w) = \infty$; when it's leontif function, $\epsilon_{1,2}(p,w) = 0$. When it's for cobb-douglas function, it's 1.

5 Question 5 – 3.D.6

Solution:

(a) Transform the utility function into a new one as it's monotone continuous. $u(\hat{x}) = u(x)^{\frac{1}{\alpha+\beta+\gamma}}$, thus the sum of of the expotential terms are 1, therefore, we can use $\alpha + \beta + \gamma = 1$ WLOG.

(b) Check this later, how to derive it? $x(p,w) = (b_1, b_2, b_3) + (w - p \cdot b)(\frac{\alpha}{p_1}, \frac{\beta}{p_2}, \frac{\gamma}{p_3})$, and indirect utility function is: $v(p,w) = (w - p \cdot b)(\frac{\alpha}{p_1})^\alpha (\frac{\beta}{p_2})^\beta (\frac{\gamma}{p_3})^\gamma$