# HW3

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ECON 7910 Econometrics

Due on Oct 7, 2021

## 1 Question – 4.1

**Solution:**

1. Because $\log(wage) = \beta_0 + \beta_1 married + \beta_2 edu + z\gamma + u$ as our equation, rearrange this to obtain:

$$\begin{aligned} wage &= \exp(\beta_0 + \beta_1 married + \beta_2 edu + z\gamma + u) \\ &= \exp(u)\exp(\beta_0 + \beta_1 married + \beta_2 edu + z\gamma) \end{aligned}$$

The reason why we plug $\exp(u)$ out the above equation is that $u$ is independent with our covariate variabes, i.e., $E(u|married, edu, \gamma) = E(u) = constant$. Thus, we can derive an equation:

$$\begin{aligned} \hat{\theta} &= 100\left(\frac{\exp(u)\exp(\beta_0 + \beta_1 + \beta_2 edu + z\gamma) - \exp(u)\exp(\beta_0 + \beta_2 edu + z\gamma)}{\exp u \exp(\beta_0 + \beta_2 edu + z\gamma)}\right) \\ &= 100(\exp(\hat{\beta}_1) - 1) \end{aligned}$$

2. Since from (1), we already have $\theta_1 = 100(\exp(\beta_1) - 1)$. By delta method:

$$\sqrt{N}(\theta_1 - 100(\exp(\beta_1) - 1)) \xrightarrow{a} \mathcal{N}(0, (100\exp(\beta_1))avar(\beta_1)(100\exp(\beta_1))')$$

Thus, the standard error of $\hat{\theta}_1$ is $100\exp(\hat{\beta}_1)se(\hat{\beta}_1)$

One thing is unclear is how to assume $N = 1$, because, if $N \neq 1$, then the asymptotic standard error will be with N in its final result.

3. While estimating the effect of change educ to wage, we need to fix other variables.

$$\begin{aligned} \theta_2 &= \frac{\beta_2\exp(edu_1) - \exp(\beta_2 edu_0)}{\exp(\beta_2 edu_0)} \\ &= \exp(\beta_2(edu_1 - edu_0)) - 1 \end{aligned}$$

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Thus, with same logistics with (1): $\hat{\theta}_2 = 100\exp(\hat{\beta}_2(edu_1 - edu_0))$

About estimating asymptotic se of $\hat{(}\theta_2)$, the step is the same, but for the convenience of computation, we assume $edu_1 - edu_0 = \Delta edu$.

$$\sqrt{N}(\hat{\theta}_2 - 100\exp(\hat{\beta}_2\Delta edu)) \xrightarrow{a} \mathcal{N}(0, (100\Delta edu\exp(\hat{\beta}_2\Delta edu))Avar(\hat{\beta}_2)(100\Delta edu\exp(\hat{\beta}_2\Delta edu))')$$

Thus, the se of $\hat{\theta}_1$ is $100\Delta edu\exp(\hat{\beta}_2\Delta edu)\cdot se(\hat{\beta}_2)$.

4. For the first regression, see tha table 1: The $\hat{\theta}_1 = 0.065$, and $se(\hat{\theta}_1) = 0.006250$.

   The $\theta_2 = 0.0654$, and $se = 0.063$ (I actually don't know how to write R codes for $\Delta educ = 4$, because there is only code example on google like $\Delta = 1$.) For specific R codes, as below:

```
library(tidyverse)
library(stargazer)
library(margins)
#Question1-4.1(d)
wage_1<- read_csv('nls80.csv')
head(wage_1,n=5)
lwage_reg<-lm(lwage~married+educ+(exper+tenure+south+urban+black)
summary(lwage_reg)
stargazer(lwage_reg)
#For estimating the margin effect of educ with 4, need to dig out
summary(margins(lwage_reg, variables='educ'))
```

## 2 Question – 4.2

**Solution:**

1.

$$\begin{aligned}
E(\hat{\beta}|X) &= E((X'X)^{-1}X'Y) \\
&= E(\beta + (X'X)^{-1}X'u|X) \\
&= \beta + (X'X)^{-1}X'E(u|X) \\
&= \beta + 0 \\
&= \beta
\end{aligned}$$

2.

$$\begin{aligned}
Var(\hat{\beta}^2|X) &= E(((X'X)^{-1}X'Y)^2|X) - \beta^2 \\
&= \beta^2 - \beta^2 + E(((X'X)^{-1}X'U)^2|X) \\
&= (X'X)^{-1}X'X(X'X)^{-1}E(U^2|X) \\
&= (X'X)^{-1}\sigma^2
\end{aligned}$$

## 3 Question – 4.3

**Solution:**

1. Yes, the reason is as below:

$$\begin{aligned} E(u^2|x) &= Var(u|x) + E(u|x)^2 \\ &= Var(u|x) + 0 \\ &= \sigma^2 \end{aligned}$$

2. It's homoskedasticity, if it doesn't hold, there would some error terms in our regression models.

## 4 Question – 4.5

**Solution:**

1. First, since $\hat{\beta}$ is the coefficient of regression regarding to $x$ and $z$, and it's consistent with $\beta$. We can derive $\sqrt{N}(\hat{\beta}-\beta) \xrightarrow{a} \mathcal{N}(0, Avar(\beta))$ by CLT. And also for $\sqrt{N}(\tilde{\beta}-\beta) \xrightarrow{a} \mathcal{N}(0, Avar(\beta))$. But since $\tilde{\beta}$ is only from the regression of y w/ respect to x, however, $\hat{\beta}$ is for the integration of $x$ and $z$. From mathematical estimation perspective, The CLT of latter is weakly higher than the former, which means the substraction is positively semidefinite.

   Feel free to deduct points, because I can't convince myself in some sense, either.

2. I think it's better to use. Because in two-dimension, we can't only measure partial effect, but measurement errors. But in one dimensional space, it can't.

3. Why not in question 2.3? But in this question, it's better to not use, because there is an unobservable variable $z$, which viotates the assumption of uncorrelated covariates in question 2.3.

## 5 Appendix

Table 1

| | *Dependent variable:* |
|---|---|
| | lwage |
| married | 0.199*** |
| | (0.039) |
| educ | 0.065*** |
| | (0.006) |
| exper | 0.014*** |
| | (0.003) |
| tenure | 0.012*** |
| | (0.002) |
| south | −0.091*** |
| | (0.026) |
| urban | 0.184*** |
| | (0.027) |
| black | −0.188*** |
| | (0.038) |
| Constant | 5.395*** |
| | (0.113) |
| Observations | 935 |
| $R^2$ | 0.253 |
| Adjusted $R^2$ | 0.247 |
| Residual Std. Error | 0.365 (df = 927) |
| F Statistic | 44.747*** (df = 7; 927) |
| *Note:* | *p<0.1; **p<0.05; ***p<0.01 |