ECON 7920
Econometrics II
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Problem Set 6
Due Date: May 3, 2022

Chapter 14 Problems:
14.1, 14.2

Problem 1

Consider the population model: $gr6085_i = \theta_{01} + \theta_{02} gdp60_i + \theta_{03} gpop6085_i + \theta_{04} sec60_i + \theta_{05} corruption_i + u_i$.[1]

a. Under what assumptions is the structual model identified? Be specific.

b. Estimate the population mode via two-step efficient GMM using each variable as an instrument for itself. What are the moment conditions implied by this assumption?

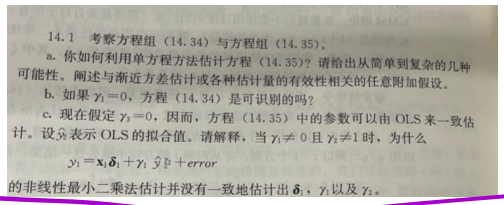
c. Test the null hypothesis that each of the population coefficients are equal to zero? What do you conclude?

d. Now use *ptrade* as an IV for *corruption*. State the moment conditions implied by using *ptrade* as an IV for *corruption*. What conditions are required so that *ptrade* is a valid and relevant IV for corruption?

e. Compute an F-stat to test whether *ptrade* is sufficiently strong. What do you conclude?

f. Now use *ptrade* and elf as IVs for corruption. Do you reject the validity assumption on your IVs? Are the IVs sufficiently strong?

[1] For this problem you will need the script files gfunction.R and qderivfungmm.R. You will also be using the data set Mauro1995.csv.



14.1 考察方程组（14.34）与方程组（14.35）。

a. 你如何利用单方程方法估计方程（14.35）？请给出从简单到复杂的几种可能性。阐述与渐近方差估计或各种估计量的有效性相关的任意附加假设。

b. 如果 $\gamma_1=0$，方程（14.34）是可识别的吗？

c. 现在假定 $\gamma_3=0$，因而，方程（14.35）中的参数可以由 OLS 来一致估计。设 $\hat{y}_2$ 表示 OLS 的拟合值。请解释，当 $\gamma_1\neq 0$ 且 $\gamma_2\neq 1$ 时，为什么

$$y_1=\mathbf{x}_1\boldsymbol{\delta}_1+\gamma_1\hat{y}_2^{\gamma_2}+error$$

的非线性最小二乘法估计并没有一致地估计出 $\boldsymbol{\delta}_1$，γ_1 以及 γ_2。

(a) 14.34: $y_1=x_1\delta_1+\gamma_1 y_2^{\gamma_2}+u_1$ 14.35: $y_2=x_2\delta_2+\gamma_3 y_1+u_2$

if we want to estimate 14.33. we can use 2SLS in which. (x_1, x_2) as instruments. if $\gamma_2\neq 1$, we can use linear method to impove the efficiency. if its heteroskedasticity, then heteroskedasticity of unkown is used.

(b) if $\gamma_1=0 \Rightarrow y_1=x_1\delta_1+u_1 \Rightarrow \delta_2$ disppers. $\Rightarrow$ Not identified.

(c) if $\gamma_3=0 \quad y_2=x_2\delta_2+u_2$.

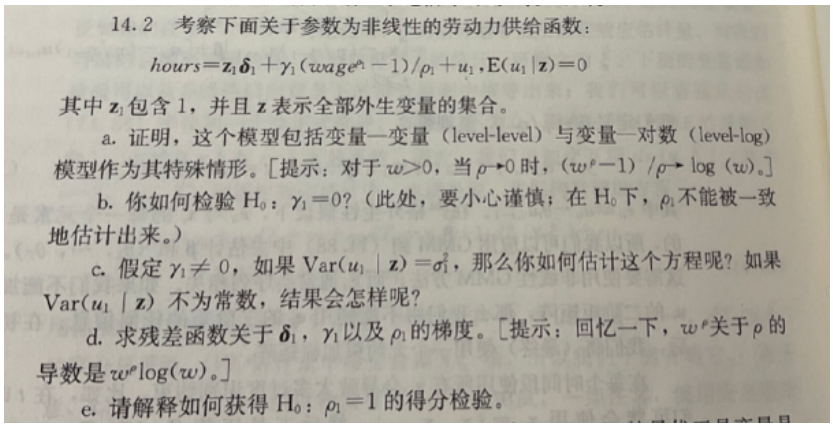
$$E(y_1|x)=x_1\delta_1+\gamma_1 E(y_2^{\gamma_2}|x)+\underbrace{E(u_1|x)}_{0 \text{ by assumption}}$$

$$=x_1\delta_1+\gamma_1 E(y_2^{\gamma_2}|x)$$

when $\gamma_2\neq 1 \Rightarrow E(y_2^{\gamma_2}|x)\neq E(y_2|x)^{\gamma_2}$

$$\therefore E(y_1|x)\neq x_1\delta_1+\gamma_1 E(y_2|x)^{\gamma_2}$$

Although we can estimate δ_2 consistently, the two-step NLS estimater of y_{i1} on x_{i1}, $(x_{i1}\hat{\delta}_2)^{\gamma_2}$ can't be consistent for δ_1, γ_2.



14.2 考察下面关于参数为非线性的劳动力供给函数：

$$hours=\mathbf{z}_1\boldsymbol{\delta}_1+\gamma_1(wage^{\rho_1}-1)/\rho_1+u_1,\ \mathrm{E}(u_1\mid\mathbf{z})=0$$

其中 $\mathbf{z}_1$ 包含 1，并且 $\mathbf{z}$ 表示全部外生变量的集合。

a. 证明，这个模型包括变量—变量（level-level）与变量—对数（level-log）模型作为其特殊情形。[提示：对于 $w>0$，当 $\rho\to 0$ 时，$(w^{\rho}-1)/\rho\to\log(w)$。]

b. 你如何检验 H_0：$\gamma_1=0$？（此处，要小心谨慎；在 H_0 下，ρ_1 不能被一致地估计出来。）

c. 假定 $\gamma_1\neq 0$，如果 $\mathrm{Var}(u_1\mid\mathbf{z})=\sigma_1^2$，那么你如何估计这个方程呢？如果 $\mathrm{Var}(u_1\mid\mathbf{z})$ 不为常数，结果会怎样呢？

d. 求残差函数关于 $\boldsymbol{\delta}_1$，γ_1 以及 ρ_1 的梯度。[提示：回忆一下，w^{ρ} 关于 ρ 的导数是 $w^{\rho}\log(w)$。]

e. 请解释如何获得 H_0：$\rho_1=1$ 的得分检验。

a. if in $wage^{\rho_1}$, $\rho_1=1$ ⇒ it becomes level-level.

when $\rho_1\to 0$

$$hours=z_1\delta_1+\gamma_1\frac{[wage^{\rho_1}-1]}{\rho_1}+u_1$$

$$\to z_1\delta_1+\gamma_1\log(w)+u_1$$

⇒ level-log

b $H_0: \gamma_1=0$

Since under H_0, it can't be estimated consistently

we fix ρ_1 and use t-test to make some estimation.

c. if $\gamma_1\neq 0$ And $\mathrm{Var}(u_1|z)=\sigma_1^2$

we use GMM method to estimate.

d $r(\theta)=hours-z_1\delta_1-\gamma_1\frac{(wage^{\rho_1}-1)}{\rho_1}$

where $\theta=(\delta_1,\gamma_1,\rho_1)$

∴ Acordiy to stochastic gradient

$$\nabla_\theta r(\theta)=\left(-z_1,-\frac{(wage^{\rho_1}-1)}{\rho_1},-\gamma_1\log w\right)$$

e. Goal: to gain some test of H_0

everything is the same w/ d, in GMM, but should w/ estimation.