Micro Final

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Question 1:

a. max $U = q_1 q_2$ ①

s.t. $2q_1 + 5q_2 = 100$ ②

set up a Lagrangian Eqn:

$$\mathcal{L} = q_1 q_2 + \lambda(100 - 2q_1 - 5q_2)$$

Take the first order derivative of q_1, q_2 respectively:

$[q_1]$: $q_2 = 2\lambda$

$[q_2]$: $q_1 = 5\lambda$.

$$\Rightarrow \frac{q_2}{2} = \frac{q_1}{5} \quad \Rightarrow q_2 = \frac{2}{5} q_1. \quad ③$$

plug ③ into ②.

$$\Rightarrow 2q_1 + 5 \cdot \frac{2}{5} q_1 = 100$$

$$\Rightarrow 4q_1 = 100 \Rightarrow q_1 = 25.$$

b. The price elasticity:

$$ed_1 = \frac{\partial q_1}{\partial P_1} \cdot \frac{P_1}{q_1}$$

$$= 0$$

Because the price of q_1 is pre-determined, and it's constant, which means, we are in competitive market. Thus, the elasticity of demand of q_1 is 0.

2.

(a) $MC = \frac{\partial TC}{\partial Q} = 0.12Q^2 - 1.8Q + 10$

$\frac{\partial MC}{\partial Q} = 0.24Q - 1.8 = 0 \Rightarrow Q^* = 7.5$

$\frac{\partial^2 MC}{\partial Q^2} = 0.24 > 0$

As the second order derivative of MC with respect to Q is positive, so at Q^*, MC could get minimum.

(b) $AVC = \frac{TVC}{Q} = \frac{0.04Q^3 - 0.9Q^2 + 10Q}{Q}$

$= 0.04Q^2 - 0.9Q + 10$

$\frac{\partial AVC}{\partial Q} = 0.08Q - 0.9 = 0$

$\Rightarrow Q^* = 11.25$.

$\frac{\partial^2 AVC}{\partial Q^2} = 0.08 > 0$

Since second order derivative is positive, so at point Q^*, we can get the minimum AVC.

plug Q^* into AVC, AVC at Q^* =

$\Rightarrow 0.04 \times Q^{*2} - 0.9Q^* + 10$

$= \frac{81}{16} - \frac{81}{8} + 10$

AVC : $= \frac{79}{16}$

(c) when $MC = AVC$

$$\Rightarrow 0.12Q^2 - 1.8Q + 10 = 0.04Q^2 - 0.9Q + 10$$

$$0.08Q^2 - 0.9Q = 0$$

$$Q(0.08Q - 0.9) = 0$$

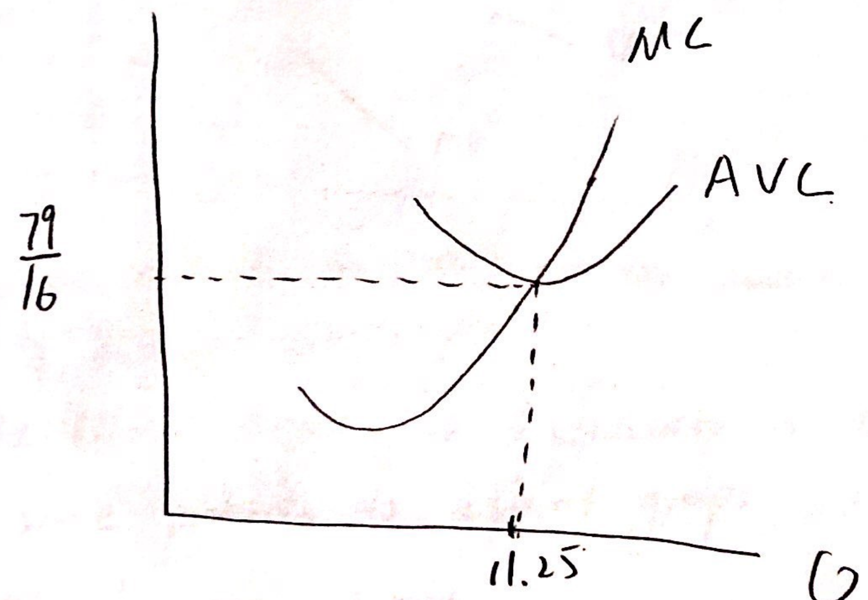
$$\Rightarrow Q_1 = 0$$

$$Q_2 = 11.25$$

we first eliminate $Q = 0$ as $Q > 0$

$\Rightarrow Q = 11.25 = Q^*$ at (b) where Q^* is the minimum point of AVC curve.

(d)

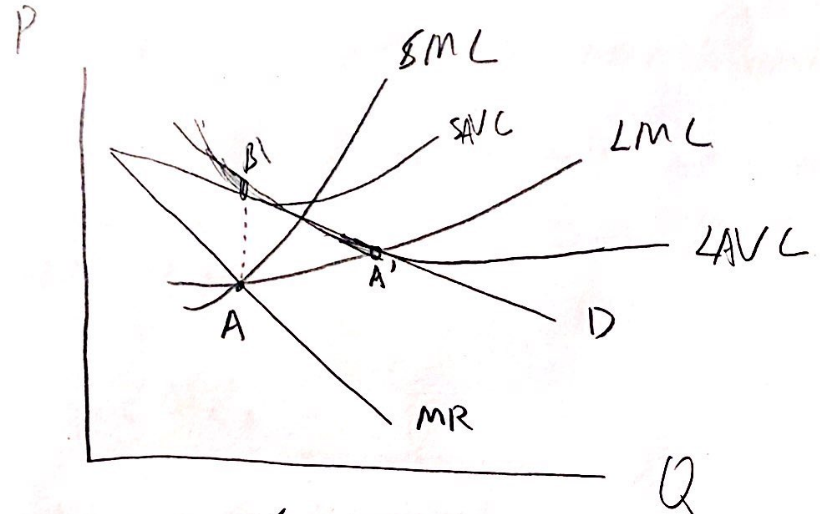


3.

(a) True

Because if in the inelastic curve, which's competitive market when P = MR = MC, the monopolistic producers will not get any profits and mark up (P-MR=0) if they enter this market. The only possible scenario for monopolistic producers to enter and produce is the mark-up [P > MR] is positive, which is on the elastic portion of demand curve.

(b)



So the point A' as seen in the above is located in the minimum point of long-run average point.

(c.) ① The third degree price discrimination is the producers sell the same products to different groups, and they charge different prices for each group.

② Because different groups of consumers have different elasticity of demand curve, the monopolists can get higher mark up if they charge different prices for each group. => higher total profits for the monopolists.

③ Conditons: 1) No collusion between differnt grops. [if differnt grop can exchange goods, the stratge will be fail]

2) The monopolist need to find a method to differentiate different groyps.

3) The podects are hetergeneous.

④ Example:

Theater: children have children price for the ticket
adults have adult price for the ticket.

Restaurant: Restaurants near Fordham University give fordham students some meal discounts.

d) $MR = P(1+\frac{1}{e})$

$MR_1 = MR_2$

$\Rightarrow P_1(1+\frac{1}{e_1}) = P_2(1+\frac{1}{e_2})$

$\Rightarrow 12(1-\frac{1}{3}) = P_2(1-\frac{1}{2})$

$\Rightarrow 8 = \frac{P_2}{2}$

$\Rightarrow P_2 = 16$

So need to charge \$16 at market 2.

4.

a. Suppose the price of product market is $P(Q)$, [As it's imperfect market] production function is $Q(V_1,V_2)$, as in the input market, it's perfect competition, so the price for each input factor is unchanged, The firm wants to maximize its profits

$$\Rightarrow \max_{Q} \pi: P(Q)Q(V_1,V_2) - \bar{P}_{V_1}V_1 - \bar{P}_{V_2}V_2.$$

Take the first order derivative of V_1, V_2.

$$[V_1]\quad MR\frac{\partial Q}{\partial V_1} = \bar{P}_{V_1}$$

$$\Rightarrow\quad MR\,MPPV_1 = \bar{PV_1} \qquad MR = \frac{\bar{PV_1}}{MPPV_1}.$$

$$\Rightarrow\quad MR\,MPPV_2 = \bar{PV_2}$$

$$[V_2]\quad MR\frac{\partial Q}{\partial V_2} = \bar{P}_{V_2}.$$

From these formulas, we can get the specific inputs V_1, V_2 for the firms to maximize its profits

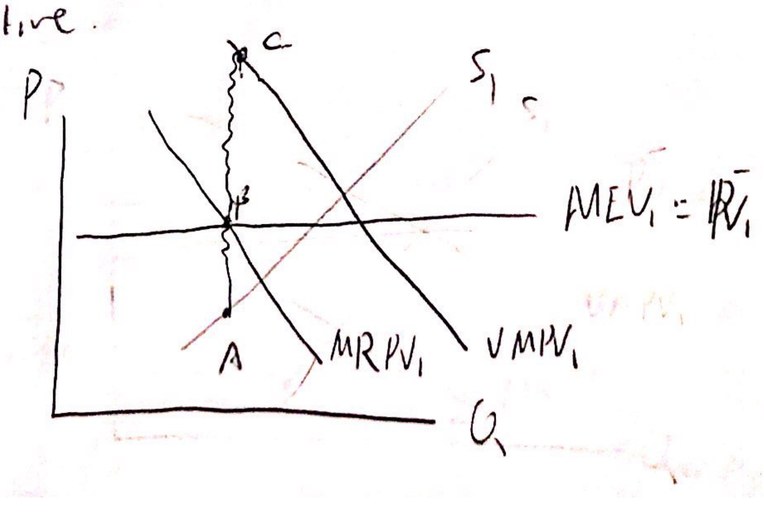
(b.) As $P > MR$ at imperfect market.

$$\Rightarrow P\cdot\frac{\partial Q}{\partial V_1} > MR\frac{\partial Q}{\partial V_1}$$

$$\Rightarrow VMPPV_1 > MRPV_1.$$

For $MEV_1 = \bar{PV_1}$ at this case b/c input market is competitive.

$\Rightarrow$



7. The figure is for V_1, but the same for V_2.

In the figure, MEV_1 intersects w/ $MRPV_1$ at point B, if we extend Q of B up and below to $VMPV_1$ and S_1, respectively, we can get A and C. $VMPV_1 > MRPV_1$ for each point b/c in imperfect market, $P > MR$.

$MRPV_1$ is negative w/ Q, so $\frac{\partial MRPV_1}{\partial Q} < 0 \Rightarrow \frac{\partial^2 \pi}{\partial V_1} < 0$, which means it's the point of max π.

BC in my figure represents the "monopolistic exploitation".