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Persistent and Transitory Shocks, Learning, and Investment Dynamics

This paper introduces a new approach to understanding investment. The distinctive feature of our approach is that shocks to the economic fundamentals have both persistent and transitory components, and that firms must disentangle the persistent from the transitory shocks. The model generates interesting dynamics. Simulations of the model show that the response of investment to changes in the interest rate can vary widely over time, that the current response of investment depends on the sequence of past shocks, that investment will respond less when the firm is confident about its beliefs and more when a change in economic fundamentals challenges the firm's beliefs, and that investment booms and crashes may occur without any change in the true state of the economy. Simulations of the model also show that it captures many "stylized facts" of investment dynamics documented in previous empirical studies.

THIS PAPER INTRODUCES a new approach to understanding investment. The novelty does not lie in the optimization problem, the equation for the evolution of capital, the technology faced by the firm, or in heterogeneity across firms. The distinctive features are that we model the shocks that affect investment as consisting of both transitory and persistent components and that we take seriously the problem that firms face in disentangling transitory from persistent shocks.

Looking back from today's vantage point, it is widely agreed that the real interest rate was moderately high in the United States in the 1960s and early 1970s, close to zero in the mid-to-late-1970s and then substantially higher in the 1980s. At the time, however, these persistent tendencies of the real interest rate were frequently obscured by short-run variations. Instead of ignoring this empirical tendency of the real interest

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rate to persistently gravitate to a certain level, we directly model it—and the ensuing consequences for investment behavior. Because of the substantial transitory fluctuations around these persistent interest rate levels, it also seems important to recognize that firms face the difficult problem of trying to determine whether or not the real interest rate has shifted from one persistent level to another. We therefore also model this learning process and explore its implications for investment behavior.

The resulting model generates interesting dynamics. For example, Caballero and Engel (1994) have argued that time variation in the response of investment to shocks is important in understanding investment. Simulations of our model show that the response of investment to changes in the interest rate can vary widely over time. This is partly because the model shares a feature of recent *S,s* models, namely, that the current response of investment depends on the sequence of past shocks.

The wide variability in the response of investment to a given change in the interest rate is closely linked to another interesting feature of investment dynamics in the model. A firm's beliefs matter, where "beliefs" refer to the firm's assessment of the probability that next period's economic fundamentals will be favorable.[1] Our simulations show that investment will respond less when the firm is quite *confident* about its beliefs.

Because of the importance of beliefs, there is an asymmetry in the effect of shocks. For example, if the firm believes that the real interest rate will be persistently high and a shock raises the interest rate, the change in the interest rate will have little or no effect on investment. We refer to such shocks as *confirming* shocks because they tend to confirm existing beliefs. On the other hand, if the firm believes that the real interest rate will be persistently high and a shock lowers the real interest rate, the change in the interest rate can have a large effect. We refer to these as *challenging* shocks.

For decades, economists have suggested that "animal spirits" play a role in investment and have tried to understand how and why this might be the case. The model in this paper provides a possible interpretation. If a sequence of transitory shocks leads firms to believe the state has changed, either an investment boom or a crash can occur without any change in the true state of economic fundamentals. These booms and crashes arise because of the important role played by beliefs and because of the asymmetric and variable effect of transitory shocks on beliefs.[2]

In addition to these interesting features, our model is able to capture some of the features of investment behavior documented in previous empirical research.[3] First,

1. Several authors have used the evolution of beliefs under learning to help explain economic dynamics. For example, Lewis (1989) and Arifovic (1996) use learning to explain exchange rate behavior, Timmermann (1996) shows that learning about dividends can account for the excess volatility and predictability of stock market returns, and Fuhrer and Hooker (1993) study how learning about monetary policy will affect inflation, unemployment, and output. Evans and Ramey (1992) show that monetary policy can have long-run real effects if it is costly for agents to revise their forecasts.

2. Howitt and McAfee (1992) show that booms and crashes can arise from Bayesian learning about an extraneous two-state Markov process. An aggregate externality in their model generates multiple equilibria and the extraneous random variable serves to coordinate beliefs. In our model, perfectly competitive firms attempt to learn the true state of an economic fundamental.

3. A variety of explanations have been proposed for the discrepancy between investment theory and empirical evidence, including irreversibility, finance constraints, measurement error in Q (for example,

empirical studies find that investment responds very sluggishly to shocks. Second, the estimated elasticity of the capital stock with respect to the relative price of capital is low. Third, several studies have found that the neoclassical theory of investment with adjustment costs (often referred to as the Q theory of investment) fails to explain a large portion of the variation in investment. Fourth, empirical studies find that Q investment equation residuals are serially correlated, suggesting an important misspecification. The simulations of our model reproduce all of these stylized facts.

The intuition for sluggish investment is straightforward. The expected sequence of future interest rates is relevant for a forward-looking firm. Suppose there are two states—high interest rate and low interest rate—both of which are persistent. Suppose further that the economy has been in the low-interest-rate state for some time and the firm is thus quite confident about the true state. A shift to the persistent high-interest-rate state implies that future interest rates will be high. Investment should fall. The firm, however, may not react immediately, because it must disentangle the regime shift from transitory shocks, and because its prior belief is that the low-interest-rate regime will persist. This causes substantial sluggishness.

The intuition for the small estimated elasticity of capital with respect to the interest rate is slightly different. Investment depends on the expected present value of a unit of capital. Transitory interest rate shocks have a small impact on investment because they only affect the current period.[4] Thus, firms will not respond to changes in the interest rate that they believe are transitory. The result, as we show, is a very low estimated elasticity.

The paper is organized as follows. Section 1 describes the model. Section 2 describes the simulation procedure and parameters. Section 3 illustrates some interesting (and realistic) aspects of investment dynamics in the model, such as variation through time in how the interest rate affects investment, and the role of beliefs. Section 4 presents simulation results on the speed of adjustment of the capital stock and provides a more detailed explanation of why the model yields sluggishness. Section 5 presents the estimated elasticity of the capital stock with respect to the interest rate and explains in more detail why the model yields an estimated elasticity close to that found in the actual data. Section 6 estimates the R^2 and Durbin-Watson statistics from simulations of the model and provides an explanation for the low R^2 and Durbin-Watson statistics in Q investment equations. Section 7 concludes.

because the stock market does not always reflect the expected present value of future dividends), aggregation (either over production units or heterogeneous capital goods), and others. See, for example, Bertola and Caballero (1994), Fazzari, Hubbard, and Petersen (1988), Blanchard, Rhee, and Summers (1993), and Hayashi and Inoue (1991), respectively. Chirinko (1993) and Caballero (1999) provide surveys and additional references.

4. Abel (1982) shows that a temporary increase in the investment tax credit (ITC) will increase investment at least as much as, and possibly more than, a permanent increase in the ITC. Our finding that transitory changes in the interest rate have a smaller effect than changes that are persistent does not contradict Abel's result. Rather, there is a difference between the effects of an ITC change and the effects of an interest rate change. In Abel's (1982) model, as in our model, a temporary change in the interest rate will have less of an effect than a permanent change.

1. THE MODEL

We begin by assuming a representative firm that minimizes the present value of its expected costs over an infinite horizon.[5] The firm discounts future profits using R_{t+1}, which denotes the gross rate of return on an alternative asset. We assume that R_{t+1} is stochastic, and is not observed by the firm until period $t+1$. The firm's objective is to minimize

$$E_t\left[\Phi(t)+\sum_{j=1}^{\infty}\left(\prod_{i=1}^{j}R_{t+i}^{-1}\right)\Phi(t+j)\right] \tag{1}$$

subject to

$$\Phi(t+j)=w_{t+j}L_{t+j}+p_{t+j}^{I}I_{t+j}+G(I_{t+j},K_{t+j}) \tag{2}$$

$$K_{t+j+1}=(1-\delta)K_{t+j}+I_{t+j} \tag{3}$$

and

$$F(\,K_t,L_t)=y\,. \tag{4}$$

Here $\Phi(t)$ denotes current costs measured in units of output. $F(K_t,L_t)$ and $G(I_t,K_t)$ denote the production and adjustment cost functions, respectively. The level of output, y, is given exogenously. Capital and labor inputs are K_t and L_t, gross investment is I_t. p_t^I denotes the price of the investment good and w_t the price of the labor input. The adjustment cost and the input prices are measured in units of output. To keep the model simple, we abstract from taxes.

We assume a constant-returns-to-scale,[6] Cobb-Douglas production function,

$$F(K_t,L_t)=A_tK_t^{\alpha}L_t^{(1-\alpha)},\qquad \alpha<1\,, \tag{5}$$

and an adjustment cost function of the form

5. We use cost minimization with fixed output rather than profit maximization, to ensure the stationarity of the problem. (Recall that the output level is indeterminate for a constant-returns-to-scale firm in a competitive output market.)

6. We assume CRS (and competitive markets) in order to focus the analysis on the effects of learning in a model which is otherwise completely standard. An alternative is to assume decreasing returns to scale:

$$F(K_t,L_t)=A_tK_t^{\alpha}L_t^{\beta}$$

$\alpha>0$, $\beta>0$, $0<\alpha+\beta<1$. In this case, (4) can be dropped. Results for this case are described in subsequent sections.

$$G(I_t, K_t) = \frac{\gamma_1}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t\,, \tag{6}$$

so that the first-order conditions are

$$\lambda_t(1-\alpha)A_t K_t^{\alpha} L_t^{-\alpha} = w_t\,, \tag{7}$$

$$\frac{I_t}{K_t} = \frac{1}{\gamma_1}(q_t - p_t^I) + \delta\,, \tag{8}$$

and

$$q_t = E_t R_{t+1}^{-1}\left[\lambda_{t+1}\alpha A_{t+1} K_{t+1}^{(\alpha-1)} L_{t+1}^{(1-\alpha)} - \frac{\delta^2\gamma_1}{2} + \frac{\gamma_1}{2}(I_{t+1}/K_{t+1})^2 + (1-\delta)q_{t+1}\right], \tag{9}$$

where q_t and λ_t denote the Lagrange multipliers on the constraints in (3) and (4), respectively. To abstract from the effect of variation in A_t, p_t^I, and w_t, we hold these variables constant at A, p^I, and w.

We use (4), (7), and (8) to eliminate L_{t+1}, λ_{t+1}, and (I_{t+1}/K_{t+1}), respectively, from the right-hand side of (9), and define $r_t \equiv \ln(R_t)$. The result is

$$q_t = E_t[\exp(-r_{t+1})]\left[\begin{array}{l}\left(\frac{w\alpha}{1-\alpha}\right)\left(\frac{y}{A}\right)^{\frac{1}{1-\alpha}} K_{t+1}^{\frac{-1}{1-\alpha}} \frac{\gamma_1\delta^2}{2} \\ \quad + \frac{\gamma_1}{2}\left(\frac{1}{\gamma_1}(q_{t+1} - p^I) + \delta\right)^2 + (1-\delta)q_{t+1}\end{array}\right]. \tag{10}$$

Next, use (8) to eliminate I_t from (3). The result is

$$K_{t+1} = \left[1 + \frac{1}{\gamma_1}(q_t - p^I)\right]K_t\,. \tag{11}$$

Thus, the firm will add to its capital stock when $q_t > p^I$ and reduce its capital stock when $q_t < p^I$.

Equations (10) and (11) form a system of nonlinear expectational difference equations in q_t and K_t. To solve these equations we take a first-order Taylor series expansion around the point (r, q_s, K_s). Here r denotes the unconditional mean of r_t and

(q_s,K_s) is the steady state of the nonstochastic system formed by (10) and (11) with r_{t+1} set equal to r for all t.

We show in the appendix that the linearization of (10) and (11) yields

$$q_t = q_s + \phi_1(K_t - K_s) + \phi_2 \sum_{i=0}^{\infty} b_2^{-i} E_t(r_{t+1+i} - r) \tag{12}$$

where ϕ_1, ϕ_2, and b_2, which are defined in the appendix, satisfy $\phi_1 < 0$, $\phi_2 < 0$, and $b_2 > \exp(r) > 1$. Intuitively, note that q_t in equation (12) gives the present value of the marginal unit of capital. Since $\phi_1 < 0$, the value of marginal capital will be decreasing in the current capital stock. Since $\phi_2 < 0$ and $b_2 > 0$, an increase in expected future interest rates will lower the present value of capital.

Agent Expectations and the Stochastic Process for r_t

To resolve the expectation on the right-hand side of (12) begin by considering the stochastic process for r_t. We assume that r_t follows

$$r_t = r_L + (r_H - r_L)S_t + \varepsilon_t \qquad \varepsilon_t \sim i.i.d\ N(0,\sigma^2). \tag{13}$$

Here $r_H > r_L$, S_t and ε_t are independent, and $S_t \in \{0,1\}$ follows a Markov-switching process with transition probability matrix[7]

$$P \equiv \begin{bmatrix} p_L & (1-p_H) \\ (1-p_L) & p_H \end{bmatrix} \tag{14}$$

where $p_L = \text{Prob}\{S_{t+1}=0|S_t=0\}$ and $p_H=\text{Prob}\{S_{t+1}=1|S_t=1\}$.

We now derive the firm's forecast of future interest rates and its measure of q_t. First consider the expectation of an agent who knows (13) and (14) and observes S_t. It is useful to define $\xi_t \equiv [(1-S_t), S_t]^T$. We can then write

$$\xi_{t+1} = P\xi_t + v_{t+1} \tag{15}$$

where $v_{t+1} \equiv \xi_{t+1} - E(\xi_{t+1}|\xi_t, \xi_{t-1}, \ldots)$. Since

$$P^j\xi_t = \begin{bmatrix} Prob(S_{t+j} = 0|S_t) \\ Prob(S_{t+j} = 1|S_t) \end{bmatrix} \tag{16}$$

7. Our convention for the off-diagonal elements of P follows Hamilton (1994); i.e., the element in row 2, column 1 is the probability that $S_t=0$ will be followed by $S_{t+1}=1$.

we have that

$$E(r_{t+j}-r|S_t,S_{t-1},S_{t-2},\ldots)=[(r_L-r)\ \ (r_H-r)]P^j\xi_t\,. \tag{17}$$

Using (17) in (12) we derive[8] that, if the current state is known, q_t will be

$$q_t^S=q_s+\phi_1\cdot(K_t-K_s)+\Psi_L+(\Psi_H-\Psi_L)\cdot Prob(S_{t+1}=1|S_t)\,. \tag{18}$$

where $\Psi_L>\Psi_H$.[9]

Next consider the expectation of an agent who knows the structure and parameters of (13) and (14) but does not observe S_t directly. Instead the agent must infer S_t from observed interest rates.[10] Let the firm's information set be $\Omega_t=\{r_t,r_{t-1},\ldots\}$ and let π_{t+1} denote the firm's assessment of $Prob\{S_{t+1}=1|\Omega_t\}$. Using (18) we have that, if the firm must learn the current state, q_t will be

$$q_t^L=q_s+\phi_1\cdot(K_t-K_s)+\Psi_L+(\Psi_H-\Psi_L)\pi_{t+1}\,. \tag{19}$$

Note from (18) that the agent is concerned with S_{t+1}, which affects r_{t+1} but does not affect $\{r_t,r_{t-1},\ldots\}$. The agent must therefore assess $Prob\{S_t=1|\Omega_t\}$, and then use the transition probabilities, p_L and p_H, to determine π_{t+1}. Begin at the end of period $t-1$ with Ω_{t-1} and π_t. The agent enters period t and observes r_t. The conditional probability density functions for r_t are

$$f(r_t|S_t=0)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left[\frac{-1}{2\sigma^2}(r_t-r_L)^2\right], \tag{20}$$

$$f(r_t|S_t=1)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left[\frac{-1}{2\sigma^2}(r_t-r_H)^2\right]. \tag{21}$$

Bayes' rule gives

8. The eigenvalues of $b_2^{-1}P$ are both positive and less than one in absolute value.

9. and where

$$\psi_L\equiv\frac{\varphi_2}{\Delta}[(r_L-r)(1-b_2^{-1}p_H)+(r_H-r)b_2^{-1}(1-p_L)];$$

$$\psi_H\equiv\frac{\varphi_2}{\Delta}[(r_L-r)b_2^{-1}(1-p_H)+(r_H-r)b_2^{-1}(1-b_2^{-1}p_L)];$$

$$\Delta\equiv[(1-b_2^{-1}p_L)(1-b_2^{-1}p_H)-b_2^{-2}(1-p_L)(1-p_H)].$$

10. For a model of investment in which learning plays a role, but the learning is about a fixed parameter, see Demers (1991)

$$Prob\{S_t = 1|\Omega_t\} = \frac{f(r_t|S_t = 1) \cdot \pi_t}{f(r_t|S_t = 0)(1-\pi_t) + f(r_t|S_t = 1) \cdot \pi_t}. \tag{22}$$

The transition probabilities, p_L and p_H, are used together with (22) to obtain[11] π_{t+1} from

$$\pi_{t+1} = p_H[Prob\{S_t = 1|\Omega_t\}] + (1-p_L) \cdot [1-Prob\{S_t = 1|\Omega_t\}]. \tag{23}$$

Using π_{t+1}, as given from (23), in (19) we have q_t^L, that is, q_t as perceived by the Bayesian firm which uses observed interest rates to learn the underlying Markov state.

2. SIMULATION PROCEDURES AND PARAMETERS

We simulate our model, calibrating the stochastic process for r_t to the actual behavior of real interest rates in the United States. Specifically, we base our choice of the parameters p_L, p_H, r_L, r_H, and σ on Garcia and Perron (1996), who use quarterly U.S. data to estimate Markov-switching models of the real interest rate. We focus on the specification reported in column two of Table 1 of Garcia and Perron (1996), which corresponds to the typical frequency of macroeconomic investment data (quarterly). To keep the theoretical model as simple as possible, we consider only two states (high and low interest rate), abstract from autoregressive terms (whose coefficients Garcia and Perron find to be small and not statistically significant), and, consistent with the model in section 1, assume that the variance of transitory shocks is the same in both states. For our baseline parameter setting, which assumes a quarterly frequency, we use $p_L = .970$, $p_H = .985$, $r_L = 0$, $r_H = .005$, and $\sigma = .00375$.

These values of r_L, r_H, and σ imply that the ratio $(r_H - r_L)/\sigma$ equals 4/3. This ratio is useful as a measure of how difficult it is for the Bayesian agent to distinguish persistent state changes from transitory shocks. If $(r_H - r_L)/\sigma$ is large then a state change causes a movement in the observed interest rate that is much larger than the typical transitory shock. This makes it relatively easy for the agent to infer the underlying state. If, on the other hand, $(r_H - r_L)/\sigma$ is small, it is more difficult for the agent to distinguish a state change from a transitory shock. It will therefore take longer for the agent to learn when there has been a change in the underlying state.

In addition, we normalize A, w, and p^I to unity and set α equal to .36. We fix y at 100 and set $\delta = .025$.[12] We have assumed convex adjustment costs so that the firm is

11. The application of Bayes' rule in equations (20) through (23) is the same as the likelihood- based procedure used in Hamilton (1989) to calculate the probability of a given state. Thus, by using (20) through (23) the representative agent follows Hamilton (1989) directly.

12. The choice of y affects the coefficients in the linearization in (12), so we checked the robustness of the results reported below by varying y from a low of 25 to a high of 400. Not surprisingly (since y is a scale parameter), variation in y had a negligible effect.

forward-looking. However, these adjustment costs are a potential source of sluggishness. To minimize this effect we set γ_1, which determines the convexity of adjustment costs, equal to 2. This value is well below empirical estimates of γ_1 from Q models in which adjustment costs are the only source of slow adjustment. [Hayashi (1982) estimates γ_1 at 23.6 and Summers (1981) offers a preferred estimate of 32.3.]

We draw a pair of initial values from the unconditional joint distribution[13] of π_t and S_t and then simulate the model for 163 periods. Our simulated time series will therefore have the same number of observations as our data set. With both simulated and actual data we use the first thirteen periods to set the lagged values in the regression equations described below, leaving a sample of 150 observations. For each parameter setting we repeat the procedure of generating the simulated data 10,000 times. We then examine the median value, over the 10,000 repetitions, of the important summary statistics. These statistics include the estimated mean lag, relative price elasticity of the capital stock, kurtosis of I/K, and the R^2 and Durbin-Watson statistics from Q investment regressions. To separate the effects of Markov switching from those of learning, we study the behavior of investment and capital under two alternative assumptions about the firm's information. First, in what we refer to as the "state-known" model, we assume that the firm observes S_t, so investment and the capital stock are determined by q_t^S as given in equation (18).[14] Alternatively, in the learning model, we assume that the firm must infer S_t from observed interest rates, so investment and the capital stock are determined by q_t^L as given in equation (19).

3. SOME INTERESTING ASPECTS OF INVESTMENT WITH LEARNING

Our model involves two distinctive features: (1) a stochastic process for the interest rate with both transitory and persistent shocks and (2) the introduction of learning about state changes (that is, persistent shocks). We begin by illustrating the behavior of investment when only the first feature is present, that is, when the interest rate state is observable to firms. Figure 1 presents a sample simulation. In period 25, the state changes from the high interest rate to the low interest rate. In the low-interest rate state, the firm anticipates that interest rates will be low for many periods, increasing the present value of future marginal products of capital. As a result, the desired capital stock is higher. Investment is higher than usual for several periods until the desired capital stock is achieved. As capital approaches the desired level, I/K re-

13. We obtain this joint distribution by simulating equations (13) through (15), for 10,000 periods using (20) through (23) to compute π_t for each period. Then to start each simulation of our model, the initial pair of values for π_t and S_t are obtained by a random (uniform) draw from this sample of 10,000.

14. In a few repetitions, S_t is constant for all 150 periods. This would make computation of the summary statistics for the state-known case impossible because, for example, I_t/K_t would be constant and calculating R^2 for the Q regressions in section 7 would require division by zero. To avoid this problem we add a very small Gaussian white noise shock (standard deviation .0001) to I_t/K_t as given from equation (8). Since, based on simulations, the variance of this shock is 0.121 percent of the variance of the state-known I_t/K_t, it has a negligible effect on our results. Nonetheless, we use identical shocks in the learning and state-known models to guarantee that our comparison of the two is in no way distorted.

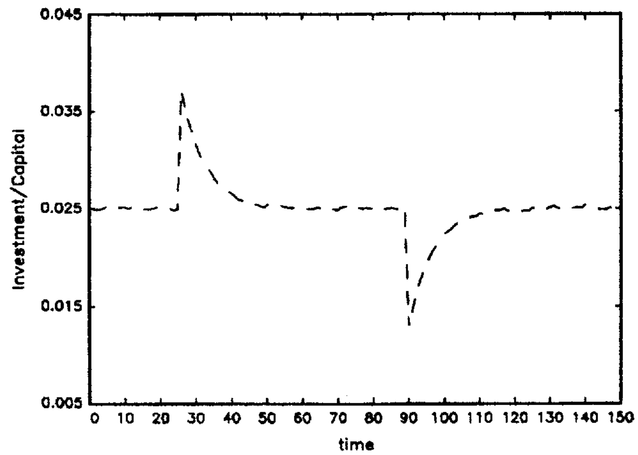

FIG. 1. Investment When the Interest Rate State Is Observable

turns to the replacement rate. In period 90, the state changes from the low interest rate to the high interest rate and investment is lower than usual for several periods.

In addition to the state changes, there are transitory shocks [the ε_t in (13), the equation which describes the stochastic process for the interest rate]. However, as the figure illustrates, if state changes are observable, then firms respond only to changes in the interest rate due to persistent shocks and not to those due to transitory shocks.

Once learning is introduced, the reaction of firms to changes in the interest rate is more complex. Consider the following example. Suppose the firm is quite sure that the economy is in the high-interest rate state (so π_{t+1} is close to 1). Now suppose there is a transitory shock which increases the interest rate by 10 percent. Since this shock confirms the firm's beliefs, π_{t+1} does not change and investment is unaffected by the change in the interest rate. Now consider a second example. Suppose that a sequence of changes in the interest rate leaves the firm somewhat uncertain about which state the economy is in, but the firm leans toward the view that interest rates are low (e.g., $\pi_{t+1} = .40$). Now suppose there is a transitory shock similar to the one in the previous example which increases the interest rate by 10 percent. Since this shock challenges the firm's beliefs, π may change substantially and investment will be affected by the shock.

Figure 2 illustrates how the sensitivity of investment to changes in the interest rate can vary in a model with learning.[15] In period 49, the firm is quite sure that the economy is in the high-interest rate state ($\pi_{t+1} = .976$). In period 50, a transitory shock pushes up the interest rate by 23 percent. From the point of view of the firm, this pushes up the probability that the economy is in the high-interest rate state. However, because the firm was already quite sure of the state and because the movement in the

15. Figure 2 shows a simulation with no state changes in which the economy is always in the high interest rate state.

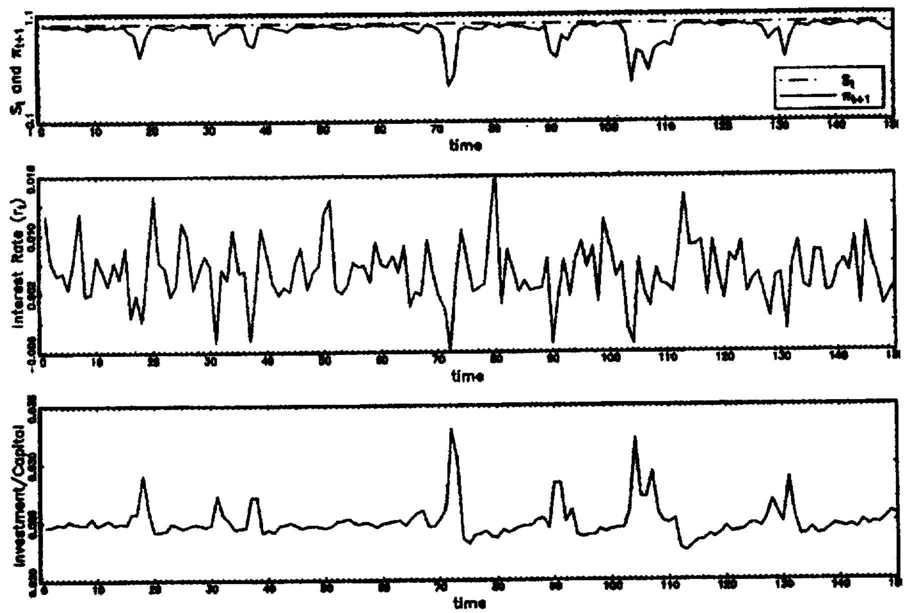

FIG. 2. Time Variability in the Responsiveness of Investment to Interest Rate Changes

interest rate tends to confirm the firm's beliefs, the effect is small: π_{t+1} rises to .984 and the percentage change in I/K is less than 1 percent.

The situation is quite different around period 112. A sequence of transitory shocks leaves the firm somewhat uncertain about the state; in period 111, $\pi_{t+1} = .73$. A transitory shock raises the interest rate by 21 percent in period 112. This pushes the firm's assessment of the likelihood of being in the high interest state up to .93 and leads to a percentage fall in I/K of about 10 percent.

An even more striking case occurs around period 72. Here a sequence of transitory shocks makes the firm somewhat unsure of the state, although it leans toward the view that the economy is in the high interest state; in period 71, $\pi_{t+1} = .88$. In period 72, a transitory shock reduces the interest rate by 23 percent. Because the firm was already somewhat unsure of the state and because the change in the interest rate challenges the firm's beliefs, the effect is dramatic: π_{t+1} falls to .29 and the percentage increase in I/K is 26 percent.

Thus a key distinctive feature of investment under learning is that the response of investment to changes in the interest rate will vary over time, depending on the sequence of past shocks as summarized by the firm's beliefs (that is, by π_{t+1}) and on whether a particular change in the interest rate confirms or challenges the firm's beliefs. This insight (that learning can make the response of a dependent variable to a change in an independent variable depend on the history of previous shocks) has the potential to help us better understand many economic phenomena, such as why stock prices, exchange rates, and long-term interest rates sometimes respond much more strongly to news than they do at other times, or why consumption sometimes moves dramatically with no obvious trigger (as it apparently did around the beginning of the 1990–91 recession).

In the particular case of investment, it has been argued that the variability in the response of investment to the interest rate is an important feature of investment behavior. For example, using postwar U.S. manufacturing data, Caballero and Engel (1994) find a substantial improvement in the performance of models with non-standard adjustment functions over traditional models based on convex adjustment costs. They state that "the main reason behind this gain . . . is a time-varying elasticity of aggregate investment with respect to shocks." As we have seen, this is a feature of our model with learning.

Learning can also lead to other interesting responses of investment to the interest rate. For example, since the firm has no way of telling whether shocks are transitory or persistent, an unusually big transitory shock or a sequence of transitory shocks of the same sign can make the firm think that the state has changed. Figure 2 illustrates this phenomenon. A sequence of low transitory shocks starting in period 102 leads to a substantial decrease in π_{t+1}. The firm is essentially fooled into thinking that a state change has occurred. There is a large increase in investment, followed by a drop as the firm realizes that the economy is still in the high-interest-rate state.

4. THE SLUGGISH RESPONSE OF INVESTMENT TO SHOCKS

The dynamics following a switch to the high-interest-rate regime are somewhat different in a model with learning than in the standard rational expectations model with convex adjustment costs and no learning. The firm doesn't know that the state has changed; it must disentangle the persistent state change from the transitory shocks. Furthermore, since $p_L = .97$ the firm's prior belief is that the low-interest rate regime will persist. As a result, π_{t+1}, the probability which the firm attaches to being in the high-interest-rate state next period takes several periods to respond. Thus the capital stock falls more gradually because the firm is not sure that the state has changed. Q tends to fall because of an increase in the probability of higher future interest rates. The typical paths of π_{t+1}, K, and q are illustrated in Figure 3, which shows the paths of the variables (averaged over 1,000 repetitions) for an example in which the state changes from low interest rate to high interest rate in period 50 and back to the low-interest-rate state in period 90.

The effect of learning depends on $(r_H - r_L)/\sigma$. When this ratio is low, it is harder for the firm to recognize a persistent shock and the effects of learning are more pronounced. This can be seen in Figure 3 which compares the path of capital without learning (that is, when the firm knows the current state) with the path of capital when the firm must learn the state. The latter is illustrated for several different values of $(r_H - r_L)/\sigma$. As the graph illustrates, the capital stock will take longer to move to its new desired level when $(r_H - r_L)/\sigma$ is low.

To see whether the model can account for the sluggish response of investment found in actual data, we measure the mean lag in the response of investment to a change in the value of capital. Let K_t^* denote the value of the capital stock that the firm would choose if there were no adjustment costs, $\Delta K_t^* \equiv K_t^* - K_{t-1}^*$, and $N_t \equiv I_t - \delta K_t$. We estimate the parameters of the regression

$$N_t = \zeta_0 + \zeta_1 \Delta K_t^* + \zeta_2 \Delta K_{t-1}^* + \zeta_3 N_{t-1} + \eta_t \, . \tag{24}$$

We use these estimated parameters to measure the mean lag in the response of investment to a change in K_t^*, that is, the number of periods on average that it will take for capital to adjust to the new K^*.[16] To assess the performance of the model, we compare statistics based on quarterly U.S. data over the period 1960:2–1997:4 (see Appendix B for data sources) with the corresponding statistics based on the simulated data over samples of the same length. To ensure stationarity of the actual data, we HP filter the variables in (24). In Table 1, we present the estimated mean lag from the HP filtered simulated data. For completeness, we also include statistics based on unfiltered simulated data. We compute K_t^* for the actual data assuming that output and relative prices other than the interest rate are fixed at the constant values that we use in the simulations.[17] In the U.S. data over the period 1960:2–1997:4, the mean lag is 6.3 quarters. The model produces a mean lag of approximately 5.0 quarters.

Our parameters for r_H, r_L and σ come from the empirical work of Garcia and Perron (1996), who estimate the stochastic process for the interest rate using quarterly U.S. data. To examine the robustness of the results, in Table 1 we consider a range of alternative values of $(r_H - r_L)$/& from one- half the empirical estimate of 4/3 to two times this value. As expected, the mean lag is a decreasing function of $(r_H - r_L)/\sigma$. Most of the values of $(r_H - r_L)/\sigma$ in Table 1 yield substantial sluggishness.

The other parameter which influences the mean lag is the adjustment cost parameter γ_1, which we have set equal to 2. In his widely cited paper, Summers' (1981) preferred estimate of this parameter is 32.3. This level of γ_1 (which has also been found in many other studies) has widely been regarded as a problem for investment models based solely on convex adjustment costs. One of the successes of the model presented here is that it accounts for the observed sluggishness without resort to extremely convex adjustment costs.

Our main results are based on constant returns to scale. As noted in footnote 6, an alternative is to assume decreasing returns to scale. Holding the other parameters constant but setting the sum of the exponents on K and L in the production function to 0.92, we obtain a mean lag of 5.8 quarters in the simulations. Variation in the degree of decreasing returns (where the sum of the exponents on K and L vary from 0.85 to 0.99) makes little difference to the mean lag in the simulations.

16. For details of the calculation of the mean lag, see Hall and Jorgenson (1967). We focus on this paper because it is a widely recognized classic in the investment literature and provides a specific procedure which we can replicate, but many other studies find substantial sluggishness. For additional references, see the Chirinko (1993) survey.

17. If instead we use the actual output and relative price data to compute K_t^*, the estimated mean lag is virtually the same.

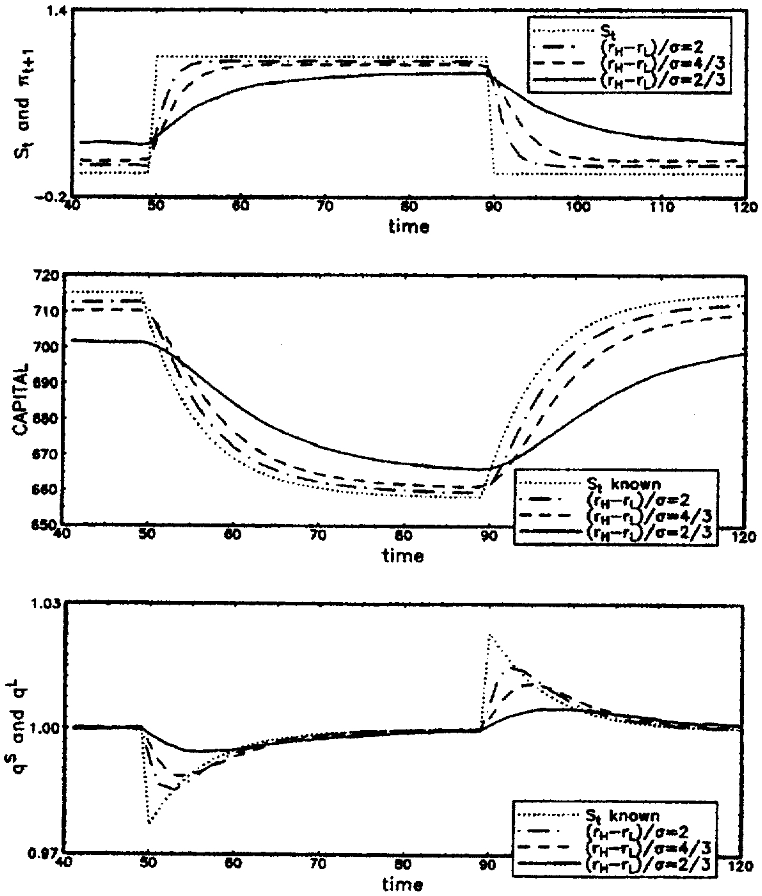

FIG. 3. Adjustment to Persistent Shocks with Learning (for different values of $(r_H - r_L)/\sigma$) and without Learning

TABLE 1

MEASURED SLUGGISHNESS OF INVESTMENT

$\frac{r_H - r_L}{\sigma}$	Mean Lag: Model Simulations (HP filtered)	Mean Lag: Model Simulations
0.67	8.69	18.18
1.00	6.94	14.30
1.33	4.96	10.10
1.67	3.91	8.08
2.00	3.40	6.88
2.33	3.07	6.21
2.67	2.84	5.66

5. THE RESPONSIVENESS OF INVESTMENT TO CHANGES IN THE INTEREST RATE

Much empirical work on investment has examined the responsiveness of investment to changes in the interest rate. The most common finding is an elasticity of the capital stock with respect to the interest rate substantially below the neoclassical benchmark of unity. [See Chirinko (1993) for a recent survey.]

As before we denote the cost of capital by c_t. We estimate the regression

$$\Delta \ln K_t = \gamma_{p0} + \sum_{i=1}^{7} \gamma_{pi} \Delta \ln\left(\frac{1}{c_{t-i}}\right) + \sum_{j=1}^{2} \omega_j^{EN} \Delta \ln K_{t-j} + e_t \,. \tag{25}$$

The estimate of the elasticity of the capital stock with respect to changes in the cost of capital is then given by

$$E_p = \frac{\sum_{i=1}^{7} \hat{\gamma}_{pi}}{1 - \sum_{j=1}^{2} \hat{\omega}_j^{EN}} \,. \tag{26}$$

In the quarterly U.S. data over the period 1960:2–1997:4, the estimated elasticity of the capital stock with respect to the cost of capital is 0.00.[18] The simulations yield an elasticity of 0.09.[19]

A natural question is how much these results depend on the parameters. Table 2 shows that the estimated elasticity is hardly affected by variation in $(r_H - r_L)/\sigma$ from half the empirical value of 4/3 to twice the empirical value.[20]

Why does our model yield estimated elasticities close to those in the empirical literature? The key idea is similar to the response of consumption to income changes in the Permanent Income Hypothesis, that is, the response depends on the persistence of shocks. Investment depends on the expected present value of future marginal products of capital. The more persistent the shock, the bigger the effect on the pres-

18. Using the delta method, the standard error is .01, so the estimated elasticity is insignificantly different from 0. For a description of the delta method, see Hendry (1995, pp. 741–42).

19. This replicates the specific procedure of Eisner and Nadiri (1968), who obtained estimated elasticities ranging from .04 to .33 (depending on details such as lag structures). As with Hall and Jorgenson (1967), this is a widely recognized classic paper. Many other papers find similar elasticities. For example, Shapiro (1986) reports an elasticity of .31; Bernstein and Nadiri (1989) report estimates clustering around .45; Morrison (1986) reports estimates of .18 (based on static expectations) and .05 (based on forward-looking expectations); and Meese (1980) reports statistically insignificant coefficients.

20. The estimated elasticity is affected by variation in $(r_H - r_L)/\sigma$ but not very much over this range. Also, variation of the adjustment cost parameter from half its base case value (of 2) to twice its base case value yields elasticity estimates which are all similarly low (ranging from .11 for the lowest value of γ_1 to .06 for the highest value of γ_1). If we keep the other parameters at their base case values but consider decreasing returns to scale (sum of the exponents on K and L equal to 0.92), the simulations yield an elasticity of 0.03.

TABLE 2
ELASTICITY OF CAPITAL STOCKS WITH RESPECT TO THE INTEREST RATE

$\frac{r_H - r_L}{\sigma}$	Elasticity: Model Simulations (HP filtered)	Elasticity: Model Simulations
0.67	0.081	0.151
1.00	0.083	0.153
1.33	0.086	0.152
1.67	0.088	0.155
2.0	0.092	0.157
2.33	0.096	0.161
2.67	0.099	0.164

ent value. If there is some cost of changing the capital stock, the firm must balance the benefits of investment (which are associated with moving closer to its frictionless optimal capital stock) against the costs. Purely transitory shocks will have a small effect on the present value and it will not be worthwhile to change the capital stock. Thus the elasticity of the capital stock with respect to the interest rate should depend on the persistence of changes in the interest rate.[21]

We can explore the effect of persistence in our model by changing the share of the variance of the interest rate due to persistent shocks. Let Θ denote the percent of the variance of r_t that is due to the variance of the persistent shock, S_t. From equation (13) in section 1, it follows that Θ equals $(r_H - r_L)^2 \operatorname{Var}(S_t)/ [(r_H - r_L)^2 \operatorname{Var}(S_t) + \sigma^2]$. A convenient way of changing Θ is to change the variance of transitory shocks while holding p_L, p_H, and $r_H - r_L$ constant. To focus purely on the effect of the persistence of changes to the interest rate, we shut down learning (by making the state observable to firms). As Figure 4 shows, there is a strong positive relationship between the estimated elasticity and the persistence of changes in the interest rate.

In comparing investment under learning to investment when the state is known, a key difference is that under learning π_{t+1} sometimes responds to changes in the interest rate due to transitory shocks (especially if they are shocks that challenge the firm's beliefs). This creates a mechanism through which learning makes investment more responsive to changes in the interest rate. The more difficult it is for the firm to disentangle persistent from transitory shocks, the more important this mechanism is. This is confirmed by Figure 5, which plots the elasticity under learning minus the elasticity without learning against $(r_H - r_L)/\sigma$. When $(r_H - r_L)/\sigma$ is low, there is a substantial difference in the elasticities. As $(r_H - r_L)/\sigma$ is increased, it becomes easier for firms to recognize persistent shocks and the difference in elasticities drops off to zero.

21. While this idea seems very intuitively appealing, we are not aware of a paper that works out the quantitative implications for the estimated elasticity. A paper that does discuss this issue, by Kiyotaki and West (1996), finds some empirical evidence supportive of the idea that estimated elasticities are linked to the persistence of shocks.

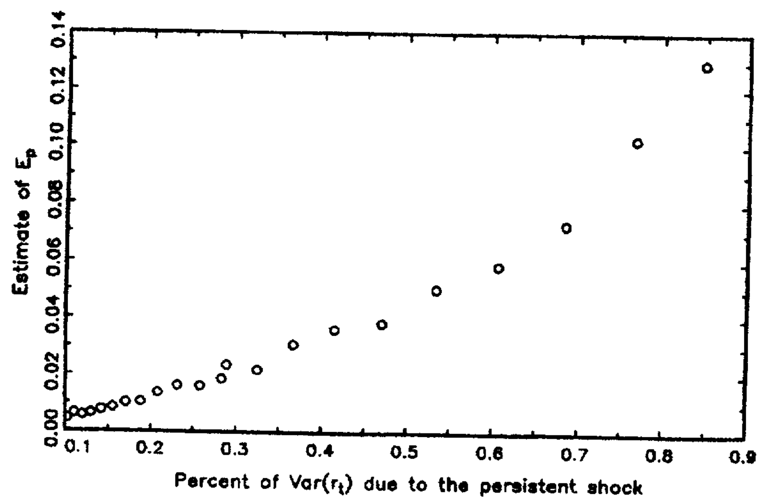

FIG. 4. Estimate of E_p versus Percent of Var(r_t) due to the Persistent Shock

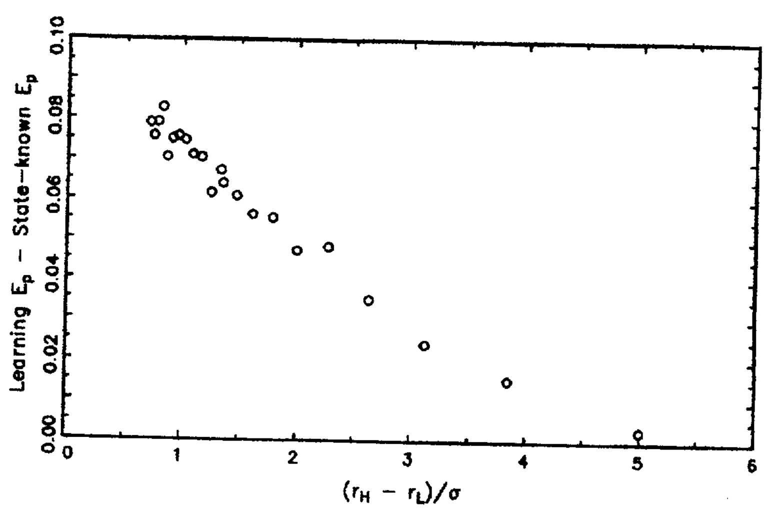

FIG. 5. The Difference between Estimates of E_p with and without Learning versus $(r_H - r_L)/\sigma$

6. R^2 AND DURBIN-WATSON

In his classic paper on investment and Q,[22] Hayashi (1982) estimates equations of the following form:

$$I_t/K_t = a_0 + b_0 Q_t + u_t^Q . \tag{27}$$

He finds that variation in Q leaves a large portion of the variation in investment unexplained, and that this unexplained portion is positively serially correlated. Specifically, he obtains an R^2 of .46 and a Durbin-Watson statistic of .43.[23]

The state of the art in constructing Q is provided by Abel and Blanchard (1986). To avoid the potential problems caused by a stock-market-based measure of Q, Abel and Blanchard construct Q using a linear forecast based on the current information set. Based on the assumptions of our model, we derive the best possible linear forecast that an econometrician could use to construct Q. It can be shown that $(r_t - r)$ follows the ARMA(1,1) process

$$(r_t - r) = \rho(r_{t-1} - r) + u_t - \theta u_{t-1} \tag{28}$$

where $\rho \equiv (p_H + p_L - 1)$, θ is a constant, and u_t is white noise.[24] It follows that the optimal linear forecast of $(r_{t+j} - r)$ is

$$E_t(r_{t+j} - r) = \rho^{j-1}\left[\frac{\rho - \theta}{1 - \theta L}\right](r_t - r) . \tag{29}$$

Using (29) in (12) we derive the econometrician's Q:

$$q_t^E = q_s + \phi_1 \cdot (K_t - K_s) + \left(\frac{\phi_2}{1 - b_2^{-1}\rho}\right)\left(\frac{\rho - \theta}{1 - \theta L}\right)(r_t - r) . \tag{30}$$

We endow the econometrician with knowledge of both the structure and the true parameters, ρ and θ, of the optimal-linear representation in (28). Since, according to (8), I_t/K_t depends on $q_t - p^I$, we simplify notation by defining $Q_t^E \equiv q_t^E - p^I$. In the quarterly U.S. data over the 1960:2–1997:4 period, the R^2 is .43 and the Durbin-

22. As Hayashi (1982) notes, Lucas and Prescott (1971) were the first to recognize the equivalence between neoclassical investment models with adjustment costs and the Q approach, and Abel (1977) showed that the optimal rate of investment is the one for which Q [$=(q - p^I)$] is equal to the marginal adjustment cost.

23. There is some evidence that similar issues may arise in models with irreversibility. For example, Bertola and Caballero (1994) introduce irreversibility and deal carefully with aggregation, but also find that their model leaves a large portion of the variation in investment expenditures unexplained and that the unexplained portion is serially correlated.

24. See Hamilton (1989), especially around page 362, and Hamilton (1994, pp. 102–05).

Watson statistic is .32.[25] In the simulated data, we obtain an R^2 of .55 and a Durbin-Watson statistic of .78. As Tables 3 and 4 show, the R^2 and Durbin- Watson statistics are qualitatively similar for a range of values of $(r_H - r_L)/\sigma$.[26]

In the next two subsections, we explain why our model yields results close to those obtained in empirical studies.

TABLE 3

THE R^2 FROM A REGRESSION OF INVESTMENT ON q

$\frac{r_H - r_L}{\sigma}$	R^2 Model Simulations (HP filtered)	R^2 Model Simulations
0.67	0.75	0.67
1.00	0.63	0.58
1.33	0.55	0.55
1.67	0.49	0.55
2.00	0.46	0.55
2.33	0.45	0.56
2.67	0.44	0.58

TABLE 4

THE DURBIN-WATSON STATISTIC FROM A REGRESSION OF INVESTMENT ON Q

$\frac{r_H - r_L}{\sigma}$	Durbin-Watson Statistic: Model Simulations (HP filtered)	Durbin-Watson Statistic: Model Simulations
0.67	0.71	0.30
1.00	0.74	0.36
1.33	0.78	0.42
1.67	0.82	0.48
2.00	0.86	0.54
2.33	0.90	0.60
2.67	0.94	0.66

25. Note that equation (30) contains a distributed lag of infinite order. With both the U.S. data and the simulated data we use the first thirteen of 163 observations to obtain lagged values of r_t in (30). As a check on this truncation procedure we also ran simulations that generated 250 observations and used the first 100 observations to obtain lagged values of r_t in (30). This increase in the initial lag length had no discernable effect on our results.

26. As γ_1 is varied from half its base case value of 2 to twice that value, the R^2 and Durbin-Watson statistics change very little. If we keep the other parameters at their base case values but consider decreasing returns to scale (sum of the exponents on K and L equal to 0.92), the simulations yield a R^2 of 0.57 and a Durbin-Watson statistic of 0.77.

A. R^2

In the absence of learning, the true econometric specification of the investment equation in our model is

$$I_t / K_t = \gamma_0 + \frac{1}{\gamma_1} Q_t^S + \omega_t \qquad (31)$$

where $Q_t^S \equiv q_t^S - p^I$, q^S is the q on which the firm bases its investment decisions [see equation (18)], and ω_t is the error term in the regression.

If the econometrician uses the optimal-linear forecasting rule, the estimated model will be

$$I_t / K_t = \gamma_0 + \frac{1}{\gamma_1} Q_t^E + \omega_t^E \qquad (32)$$

where

$$\omega_t^E = -\frac{1}{\gamma_1} \nu_t^S + \omega_t \qquad (33)$$

and

$$\nu_t^S \equiv Q_t^E - Q_t^S . \qquad (34)$$

Thus ν_t^S measures the gap between the Q used by the econometrician, Q^E, and the Q that determines the firm's choice of investment, Q^S. This gap will be included in the error term in (32). This has consequences for both the R^2 of the regression and the serial correlation of the residuals.

It is possible to show (a derivation is available from the authors) that

$$plim\, R^2 = \left(1 - \frac{\sigma_{\nu S}^2}{\sigma_{QS}^2 + \sigma_{\nu S}^2}\right) \frac{\sigma_{QE,(I/K)}}{\sigma_{(I/K)}^2} \frac{1}{\gamma_1}, \qquad (35)$$

Here $\sigma_{\nu S}^2$ is the variance of ν^S, σ_{QS}^2 is the variance of Q^S, $\sigma_{(I/K)}^2$ is the variance of I/K, and $\sigma_{QE,(I/K)}$ is the covariance of Q^E and I/K. Since $\sigma_{\nu S}^2 < \sigma_{\nu S}^2 + \sigma_{QS}^2$, the factor in parentheses will always be less than or equal to one. If there were no difference between Q^E and Q^S, ν_t^S and $\sigma_{\nu S}^2$ would be identically equal to zero. Ceteris paribus, the larger the ratio $\sigma_{\nu S}^2/\sigma_{QS}^2$, the smaller is R^2.

We begin by looking at what happens when there is no learning in the model. During a span of time when there are no state changes, $\sigma_{\nu S}^2/\sigma_{QS}^2$ will be very large, since transitory shocks cause Q^E to move with no change in Q^S. This is illustrated in Figure 6; for example, there is no state change from period 1 to 49, so Q^S is constant, but Q^E is affected by transitory shocks. However, when the state changes, both Q^E and Q^S move. Thus, state changes tend to reduce the ratio σ_{vS}/σ_{QS}. As Θ, the proportion of the variance of the interest rate due to persistent shocks, is increased, the ratio $\sigma^2_{vS}/\sigma^2_{QS}$ should become smaller. The relationship between σ_{vS}/σ_{QS} and the proportion of the variance of the interest rate due to persistent shocks is plotted in Figure 7; as suggested by the foregoing intuition, the relationship is negative.

Equation (35) establishes that R^2 is declining in the ratio σ_{vS}/σ_{QS}. By varying the proportion of the variance of the interest rate due to persistent shocks, it should therefore be possible to vary R^2. Using the same variation in Θ as in the previous figure, Figure 8 plots the relationship between R^2 and σ_{vS}/σ_{QS}. As indicated in (35), the relationship is negative.

Thus if the variance of the interest rate is largely accounted for by transitory shocks, $\sigma^2_{vS}/\sigma^2_{QS}$ will be high and R^2 will be low. In the state-known model (for the value of Θ based on the Garcia-Perron estimates of the stochastic process for the interest rate), R^2 is .09, which is lower than the R^2 of .43 based on U.S. data over the 1960:2–1997:4 period (even though we give the econometrician a big advantage by endowing her with knowledge of both the structure and the true parameters, ρ and θ, of the optimal-linear representation in (28)).

The second part of the story is the role of learning, which helps to increase R^2. The key idea is that under learning, investment reacts more (than without learning) to those changes in the interest rate that are due to transitory shocks. This means that, over a span of time in which there is no state change, Q^L (defined as $Q_t^L \equiv q_t^L - p^I$)

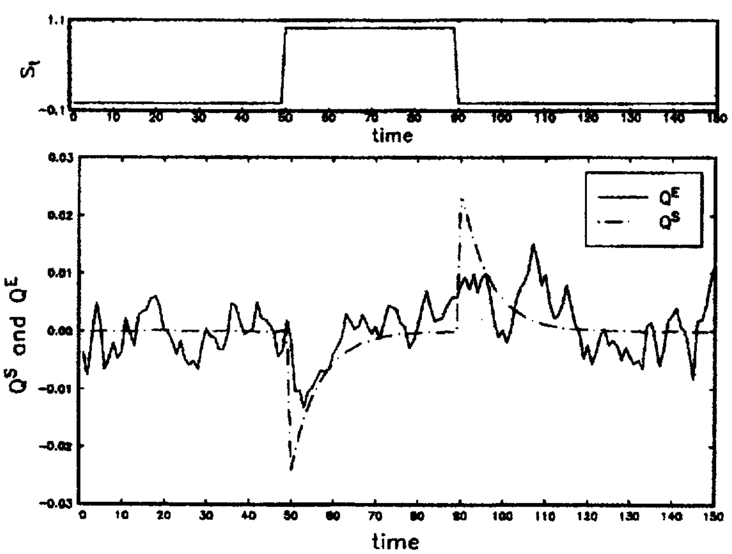

FIG. 6. Time Path of Q^S and Q^E

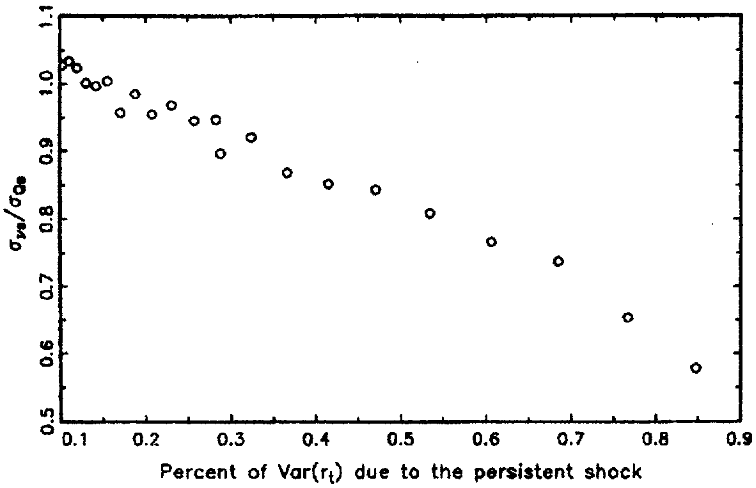

FIG. 7. The Relationship between σ_{vS}/σ_{QS} and the Percent of Var(r_t) due to the Persistent Shock

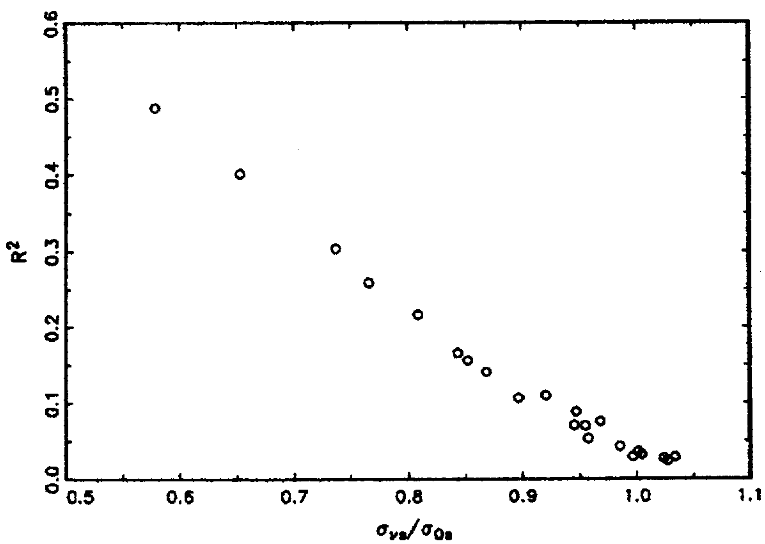

FIG. 8. The Relationship between R^2 and σ_{vS}/σ_{QS}

and Q^E move together more than Q^S and Q^E. This is illustrated in Figure 9. Consider, for example, periods 1–20 where there is substantial comovement of Q^L and Q^E. Since there is no state change in periods 1–20, there would be no comovement of Q^S and Q^E. The comovement of Q^L and Q^E due to transitory changes in the interest rate reduces the variance of v^L, where v^L is defined as $v^L = Q^E - Q^L$. This tends to reduce σ_{vL}/σ_{QL} and thus tends to increase the R^2 under learning relative to the R^2 in the state-known model. The more important learning is (that is, the harder it is for firms to distinguish transitory shocks from a state change), the more important this mechanism will be. Figure 10 shows how the difference between the R^2 under learning and the state-known R^2 varies as we vary $(r_H - r_L)/\sigma$. When $(r_H - r_L)/\sigma$ is large this difference is small. As $(r_H - r_L)/\sigma$ is reduced, the R^2 under learning increases relative to the state-known R^2. Thus modelling transitory and persistent shocks provides a qualitative explanation for the low R^2 found by empirical researchers. Introducing learning yields an R^2 that is quantitatively more realistic.

B. Durbin-Watson

We begin by looking at the model without learning. Economists frequently assume that rational expectations of future variables can be well captured by linear projections on the current information set. If the true stochastic process is nonlinear, sometimes this approach will not work well. Garcia and Perron (1996) show that the real interest rate follows a nonlinear stochastic process. As a result, a linear forecast of future interest rates (even the *optimal* linear forecast) does not fully capture the persistence of interest rate states. For example, when the economy is in the low interest state, the optimal linear forecast tends to predict higher interest rates than will occur.

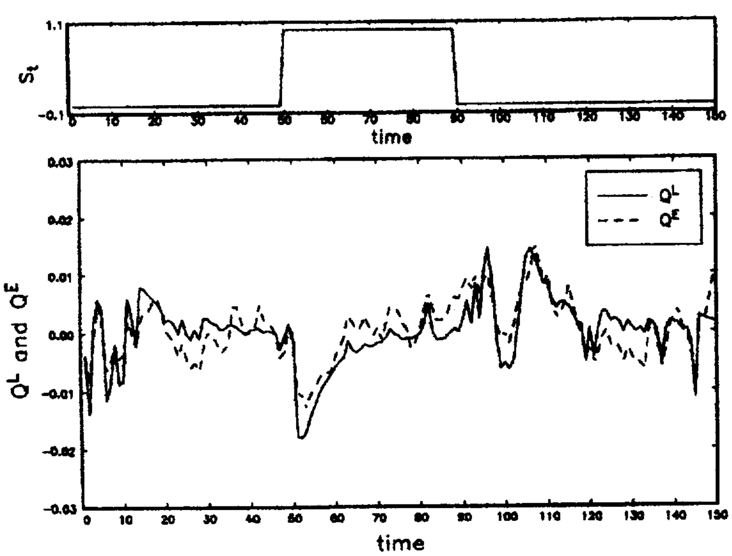

FIG. 9. Time Path of Q^L and Q^E

This leads to serial correlation of v^S within a given state. State changes reduce this serial correlation, because when the state changes, Q^S moves abruptly while the linear forecast moves gradually. The more important state changes are, the weaker the serial correlation of Q equation residuals and the larger the Durbin-Watson statistic. This is illustrated in Figure 11, which plots the Durbin-Watson statistic against Θ.

Once learning is introduced, the serial correlation of the Q equation residuals tends to be slightly higher, because when there is a state change Q^L moves less than Q^S, so the serial correlation of the residuals is not reduced as much.

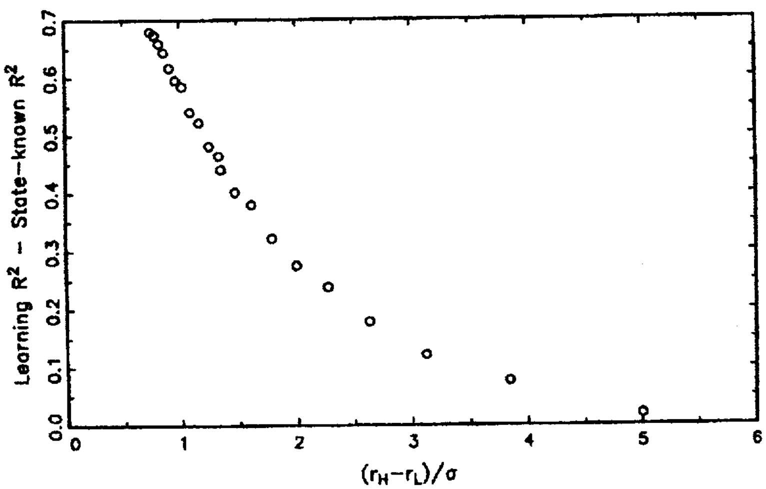

FIG. 10. The Difference between R^2 with Learning and R^2 without Learning versus $(r_H - r_L)/\sigma$

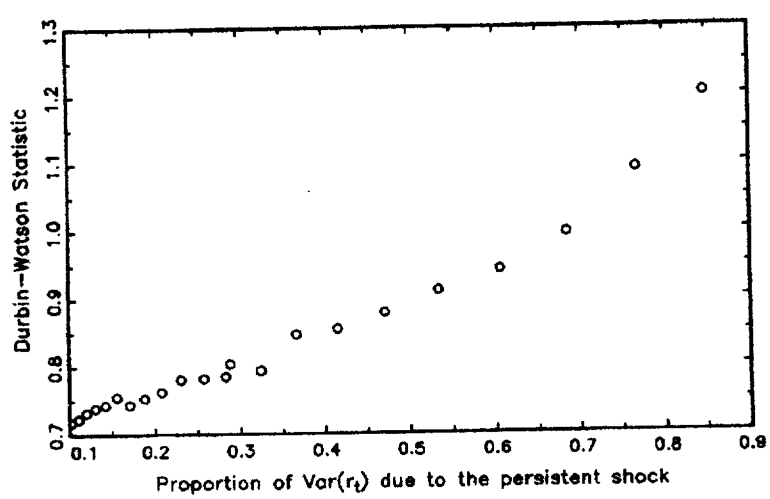

FIG. 11. The Relationship between the Durbin-Watson Statistic and the Proportion of Var(r_t) due to the Persistent Shock

7. CONCLUSION

In this paper, we present a relatively simple model which generates interesting dynamics and provides a unified explanation of several of the stylized facts in the investment literature. There are two distinctive aspects to the model. First, the model incorporates both persistent and transitory shocks. Our simulations suggest that linking the responsiveness of investment to the persistence of changes in the interest rate helps substantially in understanding important features of investment. Second, the model incorporates learning, in the specific sense that firms are not able to distinguish persistent changes in the interest rate from transitory changes. The introduction of learning into the model plays a particularly important role in explaining the sluggishness of investment in responding to changes in the interest rate. In addition, it leads to interesting (and realistic) investment dynamics.

APPENDIX A: LINEARIZATION

We linearize equations (10) and (11) around the point (r, q_s, K_s). Here, r denotes the unconditional mean of r_t and (q_s, K_s) is the steady state of the nonstochastic system formed by (10) and (11) with r_{t+1} set equal to r for all t. It follows from (11) with $\gamma_0 =$ that $K_{t+1} = K_t = K_s$ implies

$$q_s = p^I . \tag{A1}$$

Set $q_t = q_{t+1} = p^I$ and $r_{t+1} = r$ in (10) to obtain the steady state capital stock:

$$K_s = \left[\frac{\alpha w}{(1-\alpha)[e^r - 1 + \delta]p^I}\right]^{(1-\alpha)} \left[\frac{y}{A}\right]. \tag{A2}$$

Next, use (11) to eliminate K_{t+1} from (10). The result is

$$E_t[\exp(-r_{t+1})] \left[\begin{array}{l} \left(\frac{w\alpha}{1-\alpha}\right)\left(\frac{y}{A}\right)^{\frac{1}{1-\alpha}}\left((1-\delta)+\frac{1}{\gamma_1}(q_t - p^I)+\gamma_0\right)^{\frac{-1}{1-\alpha}} K_{t+1}^{\frac{-1}{1-\alpha}} \\ +(1-\delta)q_{t+1} \end{array}\right]$$

$$+ E_t[\exp(-r_{t+1})]\left[\frac{-\gamma_1\gamma_0^2}{2} + \frac{\gamma_1}{2}\left(\frac{1}{\gamma_1}(q_{t+1} - p^I)+\gamma_0\right)^2 + (1-\delta)q_{t+1}\right] - q_t = 0\,. \tag{A3}$$

The linear approximation to (A.3) is

$$E_t[f_K \cdot (K_t - K_s) + f_{q_t} \cdot (q_t - q_s) + f_{q_{t+1}} \cdot (q_{t+1} - q_s) + f_r \cdot (r_{t+1} - r)] = 0 \quad \text{(A4)}$$

where the partial derivatives, f_K, f_{qt}, f_{qt+1}, and f_r, are evaluated at (r, q_s, K_s). The linear approximation to (10) and (11) can then be written as

$$\begin{bmatrix} q_{t+1} - q_s \\ K_{t+1} - K_s \end{bmatrix} = \mathbf{A} \begin{bmatrix} q_{t+1} - q_s \\ K_{t+1} - K_s \end{bmatrix} + \mathbf{B}\, W_{t+1} \quad \text{(A5)}$$

where

$$\mathbf{A} = \begin{bmatrix} \dfrac{-f_{q_t}}{e^{-r}} & \dfrac{-f_K}{e^{-r}} \\ \dfrac{K_s}{\gamma_1} & 1 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} \dfrac{p^I}{e^{-r}} & 1 \\ 0 & 0 \end{bmatrix}, \qquad W_{t+1} = \begin{bmatrix} E_t(r_{t+1} - r) \\ q_{t+1} - E_t(q_{t+1}) \end{bmatrix}.$$

The roots of **A**, which we denote by b_1 and b_2, satisfy $0 < b_1 < 1 < e^r < b_2$. In (A.5) solve the unstable root, b_2, forward to obtain

$$q_t - q_s = \phi_1(K_t - K_s) + \phi_2 \sum_{i=0}^{\infty} b_2^{-i} E_t(r_{t+1+i} - r) \quad (15)$$

where

$$\phi_1 \equiv \frac{(-f_K / e^{-r})}{\left[b_1 + (f_{q_t} / e^{-r})\right]} < 0, \qquad \phi_2 \equiv \frac{-p^I}{e^{-r} b_2} < 0.$$

APPENDIX B: DATA

The following is a brief description of the sources and construction of the data. Investment is gross private domestic nonresidential fixed investment in equipment and software from the BEA National Accounts data. The price of investment goods is based on the corresponding price index, which we also use to adjust the nominal investment data for inflation. The capital stock is constructed using the perpetual inventory method with the same depreciation rate as in the paper. The initial value (that is, for 1959:1) of the capital stock is set equal to the 1958 end-of-period net fixed private nonresidential equipment capital stock, taken from the BEA Industry and Wealth data. Output is gross domestic product in current dollars, which we adjusted for inflation using the corresponding chain-type price index. Again, both series are taken from the BEA National Accounts data, which is available at www.bea.doc.gov. The interest rate is the three-month T-bill rate, which we adjusted for inflation using the consumer price index for all urban consumers.

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RONALD A. RATTI

On Optimal Contracts for Central Bankers and Inflation and Exchange-Rate-Targeting Regimes

This paper analyzes the issues of discretion and commitment in monetary policy under an exchange rate-targeting regime. Neither a linear state-contingent inflation contract for the central bank nor an explicit state-contingent inflation target combined with a weight-conservative central bank can now achieve the commitment equilibrium. It is shown that a state-contingent contract conditioned on the exchange rate and past output can implement the commitment equilibrium. Contracts conditioned on the exchange rate and inflation and on inflation and past output can also mimic the optimal rule under commitment.

FROM ITS INITIATION by Kydland and Prescott (1977), and through subsequent development by Barro and Gordon (1983a; 1983b), Backus and Driffill (1985a, b), Canzoneri (1985), Rogoff (1985), and others, the literature on dynamic inconsistency and monetary policy demonstrates that commitment to a policy rule might be systematically better than discretion.[1] The inflationary bias shown to arise under the discretionary equilibrium has led to arguments for greater central bank independence as a means to move closer to the commitment solution. In one of the most influential papers on the subject, Rogoff (1985) shows that delegation of operational independence to a central banker with larger (finite) weight on inflation in the loss function than society, that is, to a Rogoff weight-conservative central bank, would improve the discretionary equilibrium overall. However, the decision rule obtained would imply greater output variability and less inflation variability than would the optimal rule under commitment.

In analyses of the issues involved, Persson and Tabellini (1993) and Walsh (1995) suggest a principal-agent approach, in which costs are imposed on an instrument-independent central bank when inflation strays from target. Walsh (1995) was the first to propose optimal central bank contracts and demonstrates that a linear inflation

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1. Reviews of the literature on dynamic inconsistency and monetary policy can be found in Blanchard and Fischer (1989) and Fischer (1990).

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