HW4

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ECON 5700

Due on August 15, 2020.

1 Question 1

Solution:

1. (X, d) is called **Metric Space** where X is the nonempty subset of $\mathcal{R}$, and d is a function mapping from cartesian subsets to $\mathcal{R}$.
2. $B(a; \delta)$ is an open ball with a as center, δ as radius.
3. $B'(a : \delta)$ is a closed ball, and the left is the same with 2.
4. $\overline{E}$ is a closed set.
5. E° is an open set.

2 Question 2

Solution:

1. For any open set S, $a \in S$, by the defination of open set, there is open neighborhood of a as $B_\epsilon(a)$, $s.t.$ $B_\epsilon(a) \subset S$. Since $S \subset \cup_n^\infty S_n \longrightarrow B_\epsilon(a) \subset \cup_n^\infty S_n$, Thus, the union of any collection of open subsets of X is open.
2. (a) If the intersection of the finite open sets is empty, then by the defination of empty set, it's open.
   (b) If the intersection of the finite open sets is not empty, then we can find the small radium with a point in the intersection, such that this open neighborhood is in the intersection. In mathematics language, $r = \min\{r_1, r_2, ..., r_n\}$, and $a \in \cap_i^n S_i$, $s.t.$ $B_r(a) \subset \cap_i^n S_i$. Since the point is what we randomly pick in the intersection, thus, a is an interior point in the point. Thus, the intersection of finite open set is still open.

*I worked on my assignment sololy. Email: wye22@fordham.edu

3 Question 3

Solution:

Define the intersection of any collection of closed subsets of X as $\cap_i^\infty X_i$, Since X_i is closed, thus, X_i^c is open. By the result of **Question 2**: The union of any collection of open subsets of X is open, i.e., $\cup_i^\infty X_i^c$ is open. Thus, $() \cup_i^\infty X_i^c)^c$ is closed. By **De Morgan's law**, $(\cup_i^\infty X_i^c)^c = \cap_i^\infty X_i$. This means the intersection of any collection of closed subsets of X is closed.

4 Question 4

Solution:

$$\int \frac{dx}{\sqrt{1+4x}} = \frac{1}{2}(1+4x)^{\frac{1}{2}} + c$$

5 Question 5

Solution:

Let $u = 1 + x^2$, then $du = 2xdx$, $\frac{1}{2}du = xdx$

$$\int \frac{x}{1+x^2}dx = \int \frac{1}{u}\frac{1}{2}du = \frac{1}{2}\ln u + c = \frac{1}{2}\ln(1+x^2) + c$$

6 Question 6

Solution:

$$\int 2^x e^x dx = 2^x e^x - \int e^x d2^x = 2^x e^x - \int e^x 2^x \ln 2 dx$$

$$(1+\ln 2)\int 2^x e^x dx = 2^x e^x$$

$$\int 2^x e^x dx = \frac{2^x e^x}{1+\ln 2} + c$$

7 Question 7

Solution:

Rearrange the intergral:

$$\int x e^{-x^2} dx = \frac{1}{2}\int e^{-x^2} dx^2$$

Let $u = x^2$:

$$\frac{1}{2}\int e^{-x^2}dx^2 = \frac{1}{2}\int e^{-u}du = \frac{-1}{2}e^{-u} + c$$

Replace u with x:

$$\int xe^{-x^2} = -\frac{1}{2}e^{-x^2} + c$$

8 Question 8

Solution:

$$\frac{\ln x}{x^2}dx = \int \ln x \ d\ln x^2$$

Let $\ln x = u$, substitute it into the equation:

$$\int \ln x \ d\ln x^2 = \int u \ du^2 = \int 2u^2 du = \frac{2}{3}u^3 + c$$

Replace u with $\ln x$:

$$\frac{\ln x}{x^2}dx = \frac{2}{3}(\ln x)^3 + c$$

9 Question 9

Solution:

$$\begin{aligned}
\int x^2e^x dx &= x^2e^x - \int e^x dx^2 \\
&= x^2e^x - 2\int xde^x \\
&= x^2e^x - 2(xe^x - \int e^x dx) \\
&= x^2e^x - 2(xe^x - e^x) + c \\
&= 2e^x + x^2e^x - 2xe^x + c
\end{aligned}$$

10 Question 10

$$g(x) = \int_0^{x^2} \sqrt{1+t^2}dt$$

Solution:

Check!

See the file 'ipynb' file in my github website. It's located in PhD-Course-2021Fall-Math-ECON5700-HW4.

This question is questionable and can't be solved by hand, so I rely on Python to derive it. As you can see in my ipynb file, the result is ridiculous and insane!.

11 Question 11

Solution:

$$g(x) = \frac{1}{3}(x^3 - x^{\frac{3}{2}}) - \frac{1}{2}(x^2 - x)$$

$$g'(x) = x^2 - \frac{1}{2}x^{\frac{1}{2}} - x + \frac{1}{2}$$

At x=1:

$$g'(1) = 1 - \frac{1}{2} - 1 + \frac{1}{2} = 0$$

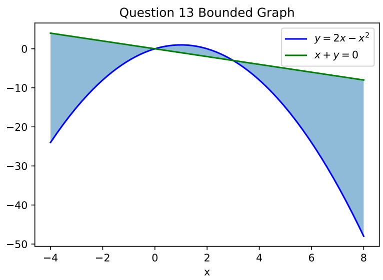
12 Question 12

Solution:

$$\int_0^2 (x^3 - x^2)dx = \frac{1}{4}x^4\Big|_0^2 - \frac{1}{3}x^3\Big|_0^2 = 4 - \frac{8}{3} = \frac{4}{3}$$

13 Question 13

Solution:



14 Question 14

Solution:

$$\int_{-2}^{2} \frac{dx}{x^3} = -\frac{1}{2}\frac{1}{x^2}\Big|_{-2}^{2} = -\frac{1}{8} + \frac{1}{8} = 0$$

Note, when the above equations exclude the point $x = 0$.