HW6

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ECON 5700

Due on August 20, 2020.

1 Question 1

Solution:

Let $P = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. By deduction, we can get:

$$P^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$P^5 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Thus, if the remainder of k divide 4 is 0, then the format of the matrix is like P^4, if the remainder is 1, the matrix is like P^1. If the remainder is 2, the format is P^2. And if the remainder is 3, like P^3.☺

2 Question 2

Solution:

Since $(A+B)^2 = (A+B)(A+B) = A(A+B) + B(A+B) = A^2 + AB + BA + B^2$, if and only if $AB + BA = 2AB \longrightarrow BA = AB$, then the equation holds. Otherwise, we can't say the equation holds.

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3 Question 3

Solution:

It's dependent. let the coeffieient of the first matrix is a and the coefficient of the second matrix is b. We need to prove that:

$$a\begin{bmatrix}1 & 2\\ 4 & 3\end{bmatrix} + b\begin{bmatrix}2 & 1\\ -1 & 0\end{bmatrix} = \begin{bmatrix}-1 & -1\\ -1 & -1\end{bmatrix}$$

Solve this, we can get $a = -\frac{1}{3} \neq 0, b = -\frac{1}{3} \neq 0$. Thus, these matrices are dependent.

4 Question 4

Solution:

1. $$\left[\begin{array}{ccc|ccc}2 & 3 & 0 & 1 & 0 & 0\\ 1 & -2 & -1 & 0 & 1 & 0\\ 2 & 0 & -1 & 0 & 0 & 1\end{array}\right] = \left[\begin{array}{ccc|ccc}1 & 0 & 0 & 2 & 3 & -3\\ 0 & 1 & 0 & -1 & -2 & 2\\ 0 & 0 & 1 & 4 & 6 & 7\end{array}\right]$$

2. For this matrix the determinant is 0, thus, the inverse matrix doesn't exist.

5 Question 5

Solution:

1. $$\frac{1}{8-7}\begin{bmatrix}2 & -7\\ -1 & 4\end{bmatrix} = \begin{bmatrix}\frac{1}{4} & -\frac{7}{8}\\ -\frac{1}{8} & \frac{1}{2}\end{bmatrix}$$

2. The inverse matrix is:

$$\begin{bmatrix}0 & \frac{1}{2}\\ -\frac{1}{2} & 1\end{bmatrix}$$

3. Since this matrix $3 \cdot 4 - 4 \cdot 6 = 0$, this matrix doesn't have inverse matrix.

4. The inverse matrix is:

$$\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$$

6 Question 6

Solution:

Af ter matrix elimination, the matrix A would be:

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus, the $row(A) = span\{[1\ 1\ 0\ 1], [0\ 1\ -1\ 1], [0\ 0\ 0\ 1]\}$, To find the col(a), we need to transpose A first and make elimination, get:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

We get the $row(A^T) = span\{[1\ 0\ 0], [0\ 1\ 1], [0\ 0\ -2]\}$. Transpose this to get $col(A) = \{[1\ 0\ 0]^T, [0\ 1\ 1]^T, [0\ 0\ -2]^T\}$

To get null(A), we need to begin with RE(A): thus, $x_1 = -x_2 = -x_3, x_4 = 0$, let $x_3 = x_2 = t, x_1 = -t$.

$$\vec{x} = \begin{bmatrix} -t \\ t \\ t \end{bmatrix} = t \cdot \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

Thus, the null(A)=$t \cdot \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$, for $t \in \mathcal{R}$.

7 Question 7

Solution:

$$RE(A) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$$

$row(A) = span\{[1\ 0\ -1], [0\ 1\ 2]\}$ Assume the coefficient of the first matrix is a, b for the second, c for w. Solving this euqation for the matrix independence, $a = b = 0 = c$. Thus, it can't be expressed.

Follwing the method in Question 7, we can get the col(A)=$span\{[1\ 1]^T, [0\ 1]^T\}$

$$3 \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} + (-1) \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Thus, b in col(A).