

Homework 7 Solution

Check if it is a linear transformation

$$T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x - y + z \\ 2x + y - 3z \end{bmatrix}, \quad T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ x + 2y \\ 3x - 4y \end{bmatrix}$$

Let $T_A : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the matrix transformation corre-

sponding to $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$. Find $T_A(\mathbf{u})$ and $T_A(\mathbf{v})$,

where $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$.

Determine whether the given matrices are linearly transformation. If not, give counterexamples.

$$T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x^2 \end{bmatrix}, \quad T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} xy \\ x + y \end{bmatrix}$$

Find the matrix $S^{\circ}T(a)$ by (a) direct substitution and (b) by matrix multiplication of $[S][T]$.

$$T \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 + 2x_2 \\ 2x_2 - x_3 \end{bmatrix}, S \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} y_1 - y_2 \\ y_1 + y_2 \\ -y_1 + y_2 \end{bmatrix}$$

Find the eigenvalues and eigenvectors of the follow matrices

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$