

# MIDTERM EXAM

## MICROECONOMIC THEORY II

### SPRING 2018

#### Instructions

This exam has 10 questions worth 10 points each. You have no more than two hours to complete this exam.

Partial credit will be awarded preferentially for *incomplete* answers, as opposed to incorrect answers. Each question that you leave blank will earn you 2 points; completely incorrect answers will earn you 0 points.

To help me assign partial credit, please

- i. work neatly,
- ii. define your notation,
- iii. write your assumptions, and
- iv. describe what you are attempting to do.

Make your answers concise and answer only what is asked.

#### Directions

- ☐ A sheet will be circulated on which you will record your exam code next to your name. Do *not* write your name anywhere on your answers!
- ☐ Write your answers on the *front side* of the paper only.
- ☐ Label the *bottom of each page* of your answers with your *exam code* and the *page number*; i.e. if your exam code is “C”, put “C3” at the bottom of page 3 of your answers.
- ☐ When turning in your exam, place your pages *facing up in order* and *bind them* with the supplied paper-clip.

#### Revisions

Revisions are due before the beginning of class on **Tuesday, March 20<sup>th</sup>, 2018**.

Revisions must be submitted electronically on Blackboard as a \*.pdf file. Although I recommend that you learn how to typeset mathematics, scans of handwritten revisions are acceptable, provided that they are legible. You may scan your handwritten work for free using the department’s copier. As with your exam, label the *bottom of each page* of your revisions with your *exam code* and the *page number*; i.e. if your exam code is “C”, put “C3” at the bottom of page 3 of your revisions.

For each point lost, you can recover up to \_\_\_/10 a point. For each question missed, you must: (1) provide the correct answer to the question *with full work*, and (2) if applicable, an explanation of why your original answer was incorrect. Questions for which explanations are omitted or revisions containing errors will be awarded partial or no credit.

You may use your textbook and notes to complete your revisions. You may not consult your peers or other sources (e.g. me, the Internet) *except* for clarifications about the mathematical tools or materials in general. So, for example, asking a peer how to determine if a function is homothetic is permitted, while asking a peer to allow you to copy their answer to question 8 is prohibited.

<b>Grade:</b> before revisions: ____/100    +    for perfect revisions: ____    =    projected revised grade: ____/100    (____)
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Please see the instructions and directions on the previous page. As a reminder, to help me assign partial credit, please: (i.) work neatly, (ii.) define your notation, (iii.) write your assumptions, and (iv.) describe what you are attempting to do. Make your answers concise and answer only what is asked.

I. Answer questions 1 and 2 below.

1. Chose **one**—and only one—of the following two questions to answer.
  - i. Define local non-satiation. Use a graph under the assumption  $x = \mathbb{R}_+^2$  to explain why Walras' Law may not hold if preferences are not locally non-satiated.
  - ii. Assume  $x = \mathbb{R}_+^2$ . Lexicographic preferences are then defined as  $x \succsim y$  if either " $x > y$ " or " $x = y$  and  $x_2 \geq y_2$ ". Prove that lexicographic preferences violate the continuity axiom.
2. Draw a figure illustrating the relationships between the utility maximization and expenditure minimization problems. On the figure, identify where you would use Roy's identity, Shephard's lemma, and the Slutsky equations.

II. Consider an agent who chooses over three goods bundles  $x, x', x''$  at prices  $p, p', p''$  respectively, where

$$p = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \quad x = \begin{pmatrix} 5 \\ 19 \\ 9 \end{pmatrix},$$

$$p' = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad x' = \begin{pmatrix} 12 \\ 12 \\ 12 \end{pmatrix},$$

$$p'' = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \quad x'' = \begin{pmatrix} 27 \\ 11 \\ 1 \end{pmatrix}.$$

3. Consider all pairwise possible comparisons of the bundles and show that these data satisfy the Weak Axiom of Revealed Preference.
  4. Show that these data represent intransitive choices.
  5. Does there exist a utility function that represents the preferences of this agent? Explain.
- II. Suppose all agents in an economy have preferences over two goods that can be represented by Cobb-Douglas utility,

$$u_i(x_1, x_2) = x_1^{\alpha_i} x_2^{1-\alpha_i}$$

where  $\alpha_i \in (0, 1) \forall i$ , and  $x \in \mathbb{R}_+^2$ . The associated Walrasian and Hicksian demand functions are respectively

$$x_1(p, w_i) = \frac{\alpha_i}{p_1} w_i, \quad x_2(p, w_i) = \frac{1-\alpha_i}{p_2} w_i,$$

$$h_1(p, u_i) = \left( \frac{\alpha_i}{1-\alpha_i} \cdot \frac{p_2}{p_1} \right)^{1-\alpha_i} u_i, \quad h_2(p, u_i) = \left( \frac{1-\alpha_i}{\alpha_i} \cdot \frac{p_1}{p_2} \right)^{\alpha_i} u_i.$$

6. Derive  $x_1(p, w_i)$  from the agent's utility maximization problem. You may assume an interior solution.
7.
  - i. Should  $x_1(p, w_i)$  be homogenous of degree 0 or degree 1 in  $(p, w_i)$ ?
  - ii. Does this demand function satisfy the homogeneity requirement? Show your work.
8. Does aggregate demand depend only on aggregate wealth? Answer this by responding to i. and ii. below.
  - i. Derive the indirect utility function and explain how it addresses this question.
  - ii. What property does this agent's preferences satisfy that addresses this question? Show that it holds.
9. Does aggregate demand satisfy the uncompensated law of demand? Answer this by responding to i. and ii. below.
  - i. Show that  $D_p(p, w_i)$  is negative definite. Do this by demonstrating that  $d_{11} < 0$  and  $d_{11}d_{22} - d_{12}d_{21} > 0$ , where the  $d_{\ell k}$  denotes the  $(\ell, k)$  entry of  $D_p(p, w_i)$ .
  - ii. What can we infer from this about whether aggregate demand follows the uncompensated law of demand? Are additional assumptions required?
10. The Cobb-Douglas utility function has a history of wide use in empirical work. Based on its properties, in what ways is this justified? In what ways is this problematic?