Microeconomic Theory II

Consumer Choice

$\{coffee\}, \{tea\}, \{wine\}$

$\ell \quad \underline{1} \qquad 2 \qquad 3$

$$x = \begin{bmatrix} \underline{1} \\ 0 \\ 0 \end{bmatrix} \quad x = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad x = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Commodities: finite set of G&S, $\ell = 1, 2, \ldots, L$.

commodity bundle: vector of commodities

$$\underset{L \times 1}{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_L \end{bmatrix}$$

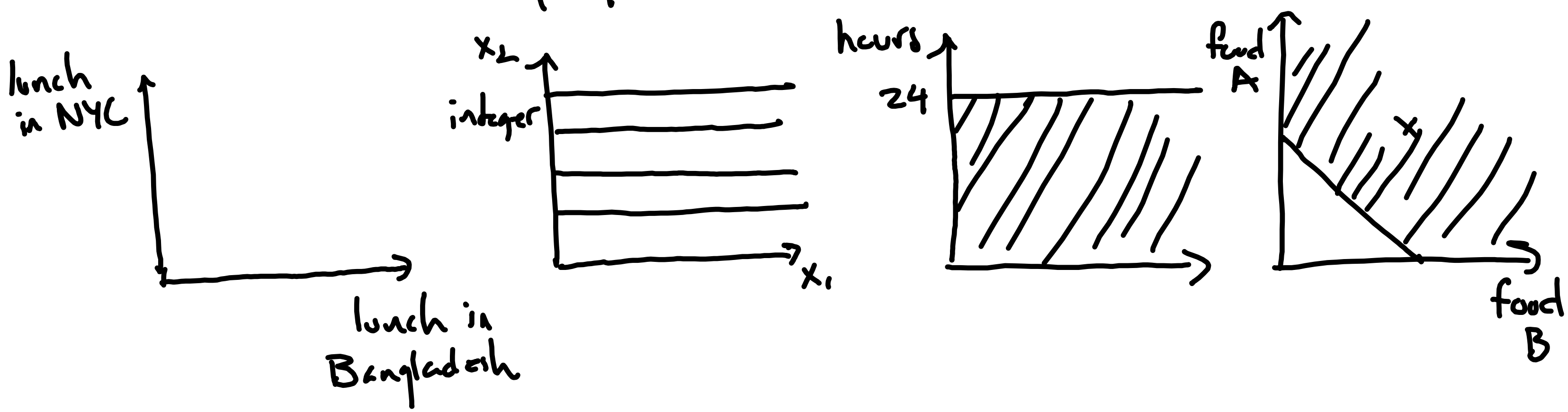
common assumptions

1. $x_i \geq 0$, or $x > 0$ (all elements non-negative, at least one positive)
2. indeces can be different times, states of the world, etc.
3. point in $\mathbb{R}^L$

Consumption Set

consumption set is a subset of commodity space $X \subset \mathbb{R}^L$ whose elements are (conceivably) available for consumers to consume

1. constraints: time (life span), indivisibility, necessity, geographic, institutional, physical



2. non-negative constraint (commonly)

$$X = \mathbb{R}^L_+ = \{x \in \mathbb{R}^L : x_\ell \geq 0 \ \forall \ell = 1, 2, \dots, L\}$$

this set is <u>convex</u>.

Competititive Budgets

1. Assumptions: completeness principle or universality — publically quoted prices

$$\underset{L\times 1}{p} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_L \end{bmatrix} \in \mathbb{R}^L$$

2. We can let $p_\ell \leq \emptyset$, but generally $p_\ell > \emptyset$ or $p >> \emptyset$. (all elements positive)

7. We generally assume price-taking.

A bundle is affordable if $\underbrace{\underset{1\times L}{p} \cdot \underset{L\times 1}{x}}_{} \leq \underset{1\times 1}{w}$

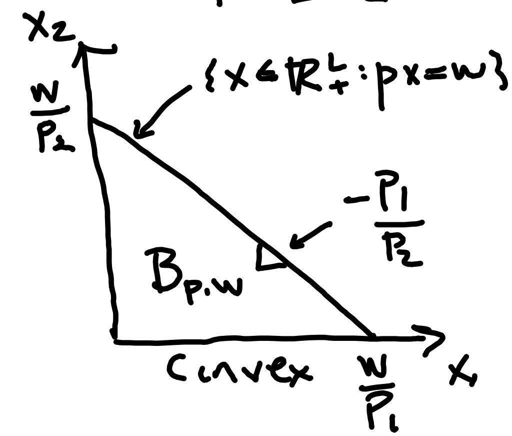
x' or element-by-element multiplication

$$p_1 x_1 + p_2 x_2 + \ldots + p_L \cdot x_L \leq w.$$

Walrasian Budget set is $B_{p,w} = \{x \in \mathbb{R}^L_+ : p \cdot x \leq w\}$, the set of all feasible consumption bundles given p, w.

The budget hyperplane is the set $\{x \in \mathbb{R}^L_+ : p \cdot x = w\}$ is the upper boundary of $B_{p,w}$.

for $L = 2$



Convex

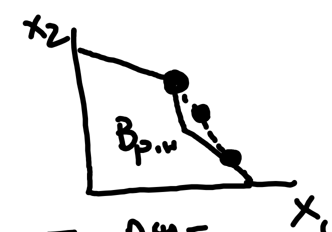
$B_{p,w}$ is a <u>convex</u> set b/c if x and x' are in $B_{p,w}$, then $x'' = \alpha x + (1-\alpha) x'$ for $\alpha \in [0, 1]$ is also.

Proof. 1. x and x' are non-negative $\Rightarrow x'' \in \mathbb{R}^L_+$

2. $px \leq w$ and $px' \leq w$

$\Rightarrow px'' = \alpha x + (1-\alpha) p x' \leq w$ for $\alpha \in [0, 1]$

$\Rightarrow x'' \in B_{p,w}$



non-convex and non-<u>Walrasian</u>

Demand Functions & Comparative Statics

Walrasian Demand Correspondence, $x(p,w)$, assigns a set of chosen consumption bundles for each (p,w). If single-valued, it is a demand fn.
$L\times 1$ 1×1 (if only one choice for any particular p,w)

Assumptions for (p,w) — later implications of theory

1. $x(p,w)$ is homogeneous of degree zero (HDZ)

$$x(p,w) = x(\alpha p, \alpha w) \text{ for } \alpha > 0$$

$\Rightarrow B_{p,w} = B_{\alpha p, \alpha w}$; a proportional change in price or wealth does not change the budget set.

2. $x(p,w)$ satisfies Walras' Law i.e. if for every $p \gg 0$ and $w > 0$ we have $px = w$; that is wealth is always spent over the consumer's lifetime.

Assuming that $x(p,w)$ is single-valued (and often continuous & differentiable) we can also write

$$x(p,w) = \begin{bmatrix} x_1(p,w) \\ x_2(p,w) \\ \vdots \\ x_L(p,w) \end{bmatrix}$$

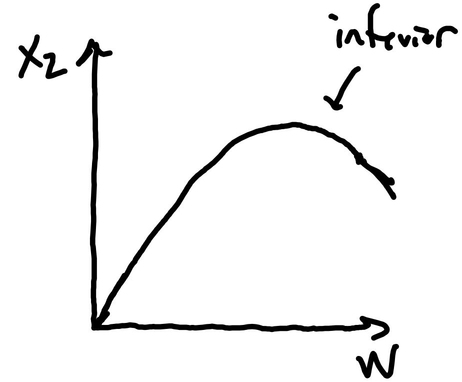
so that the Walrasian demand vector is a vector of commodity-specific demands.

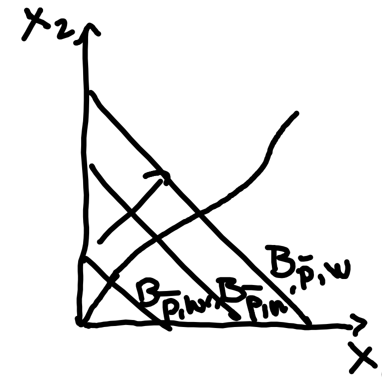
Comparative Statics

Wealth effects

For fixed $\bar{p}$, the fn. of wealth $x(\bar{p}, w)$ is the consumer's Engel fn.
It's image in $\mathbb{R}^2_+$, $E_p = \{x(\bar{p}, w) : w > 0\}$ is the wealth expansion path.

For $L = 2$.





In matrix notation, wealth effects are

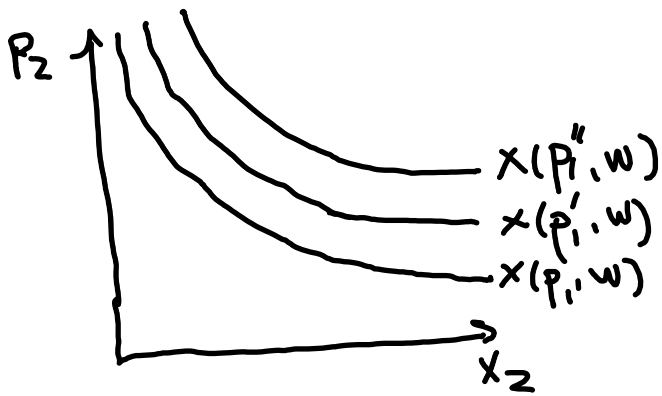
$$D_w x(p, w) = \begin{bmatrix} \dfrac{\partial x_1(p,w)}{\partial w} \\ \dfrac{\partial x_2(p,w)}{\partial w} \\ \vdots \\ \dfrac{\partial x_L(p,w)}{\partial w} \end{bmatrix}$$

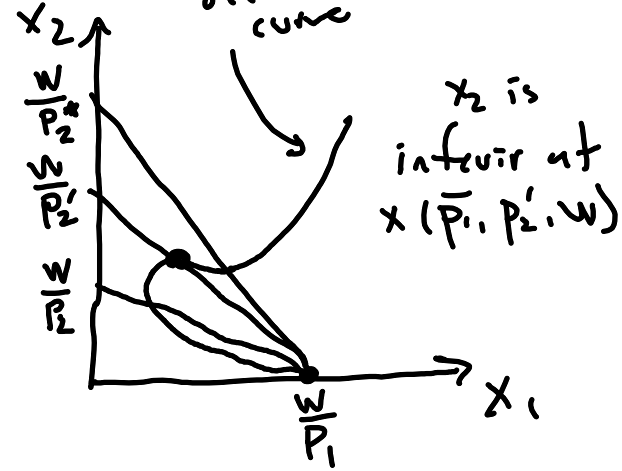
At any (p, w) $\partial x_\ell(p,w)/\partial w$ is the wealth effect on the ℓ^{th} good; if ≥ 0 the good is normal at (p, w); if < 0 inferior; if a commodity is normal at all (p, w), we say demand for that commodity is normal.

Price effects

If we hold wealth and the prices of all other goods constant, then we have the demand curve for commodity ℓ. The locus of points demanded in $\mathbb{R}^2$ over all possible values of p_ℓ is the offer curve.

In $L=2$.





$\partial x_\ell(p,w)/\partial p_k$ is price effect of p_k on demand for good ℓ

Prices effects in matrix form:

$$D_p x(p,w) = \begin{bmatrix} \frac{\partial x_1(p,w)}{\partial p_1} & \frac{\partial x_1(p,w)}{\partial p_2} & \cdots & \frac{\partial x_1(p,w)}{\partial p_L} \\ \frac{\partial x_2(p,w)}{\partial p_1} & \frac{\partial x_2(p,w)}{\partial p_2} & & \frac{\partial x_2(p,w)}{\partial p_L} \\ \vdots & & \ddots & \\ \frac{\partial x_L(p,w)}{\partial p_1} & \frac{\partial x_L(p,w)}{\partial p_2} & & \frac{\partial x_L(p,w)}{\partial p_L} \end{bmatrix}$$

For own price, generally $\frac{\partial x_\ell(p,w)}{\partial p_\ell} \leq 0$. If >0, we have "upward-sloping" demand $\Rightarrow$ Giffen good.

For cross-price if $\frac{\partial x_\ell(p,w)}{\partial p_k} < 0$ we've gross complements at (p,w), >0 gross substitutes, $=0$ unrelated.

What are implications of comparative statics & our assumptions on $x(p,w)$?

1. B/c HD0, $x(\alpha p, \alpha w) - x(p,w) = 0$. So, now we differentiate w.r.t. α and evaluate at $\alpha = 1$.

Prop. If Walrasian $x(p,w)$ is HD0, then $\forall p,w$

$$D_p x(p,w)\, p + D_w x(p,w)\, w = 0$$

or in sum-notation

$$\sum_{l=1}^{L} \frac{\partial x_l(p,w)}{\partial p_k} p_k + \frac{\partial x_l(p,w)}{\partial w} w = 0 \text{ for } l = 1, \dots, L.$$

Define elasticities of

$$\varepsilon_{l,k}(p,w) = \frac{\partial x_l(p,w)}{\partial p_k} \cdot \frac{p_k}{x_l(p,w)} \qquad \varepsilon_{l,w}(p,w) = \frac{\partial x_l(p,w)}{\partial w} \cdot \frac{w}{x_l(p,w)}$$

cross-price

wealth

these are % changes in $x(p,w)$ resulting from % changes in p & w.

By substitution,

$$\underbrace{\sum_{k=1}^{L} \epsilon_{l,k}(p,w)}_{\text{sum of cross-price elasticities}} + \epsilon_{l,w}(p,w) = 0 \quad \text{for } l = 1, \dots, L$$

* An equal % change in all prices & wealth leads to no change in demand.

2. By Walras' law, $p \cdot x(p,w) = w \;\forall p,w$. Diff. w.r.t.

prices

Prop- If Walrasian $x(p,w)$ satisfies WL, then for all p,w

$$\underbrace{\sum_{l=1}^{L} p_l \frac{\partial x_l(p,w)}{\partial p_k} + x_k(p,w)}_{\text{change in total exp}} = 0 \quad \text{for } k = 1, \dots, L$$

matrix notation

$$p D_p x(p,w) + x(p,w)^T = 0^T$$

Cournot Aggregation:

Total expenditure does not change in response to a change in prices (only).

... wealth

Prop. If Walrasian $x(p,w)$ satisfies WL, for all p,w

matrix notation

$$\underbrace{\sum_{l=1}^{L} p_l \frac{\partial x(p,w)}{\partial w} = 1}$$

$$p D_w x(p,w) = 1$$

Engel Aggregation: total expenditure changes by an amount equal to any wealth change.

What constraint does consistent choice put on demand?

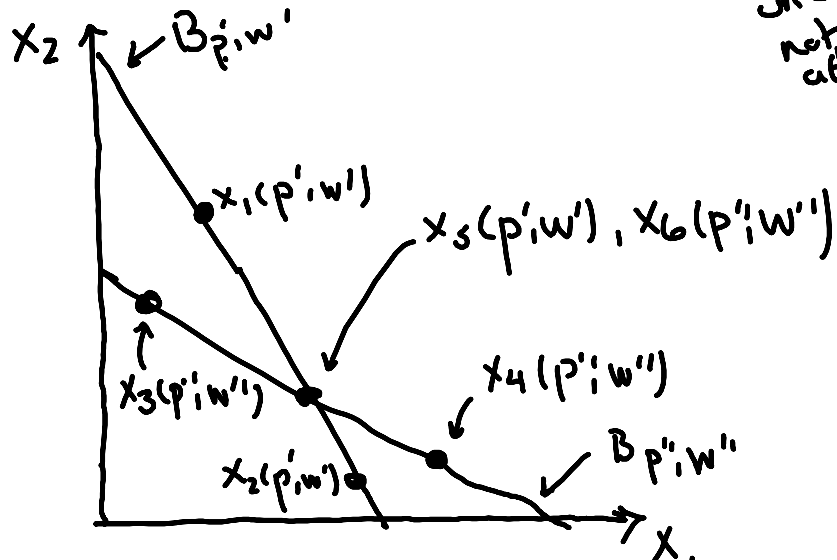
Assume $x(p,w)$ is single-valued, HD0, & WL.

$x'(p',w')$ is the bundle chosen under p', w'

(A) Def. $x(p,w)$ satisfies WARP if $\underbrace{p \cdot x(p',w') \leq w}_{\text{x is chosen when a bundle x' was affordable}}$ ✓ and $x(p',w') \neq x(p,w)$

implies $\underbrace{p' \cdot x(p,w) > w'}_{\text{x is not affordable when x' is chosen}}$ for any two (p,w) and (p',w').

L=2 Examples



at least one bundle not affordable

1) x_2 and x_4 satisfies WARP
2) x_1 and x_4 satisfies WARP
3) x_5 and x_4 satisfies WARP
4) x_2 and x_3 does not satisfy WARP
5) x_3 and x_5 does not satisfy WARP (if $x(p,w)$ single-valued)
6) x_5 and x_6 satisfies WARP

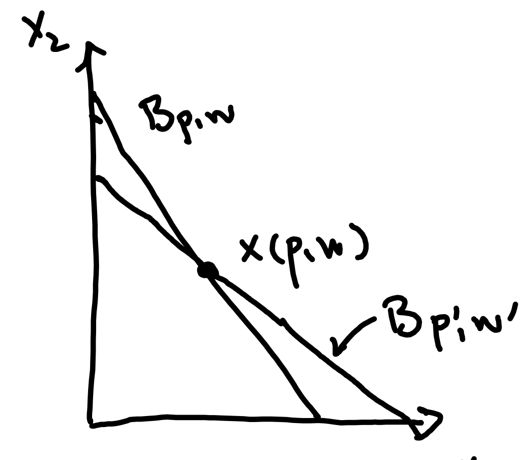
$x_2 = x$ $x_3 = x'$ is x_2 affordable when x_3 is chosen
no? WARP satisfied yes? WARP not satisfied

Price Changes and WARP

A change in price alters (i) the relative cost of commodities and (ii) consumer's real wealth.

To study WARP, we need to isolate the effects of price changes

Step-by-step in $\mathbb{R}^2$ we need to is change wealth to w' so that $x(p,w)$ is just affordable at w'.



$w' = p'x(p,w)$

1. start at $x(p,w)$
2. new prices at p'
3. set w' such that $p'x(p,w) = w'$
4. the wealth adjustment is

$$\Delta w = \Delta p \cdot x(p,w) \text{ where } \Delta p = (p' - p)$$

Δw is the Slutsky wealth compensation

Δp is the Slutsky compensated price changes

(*) Prop. Suppose $x(p,w)$ is HD0 and satisfies WL. Then $x(p,w)$ satisfies WARP iff

For any compensated price change from (p,w) to $(p',w') = (p', p'\cdot x(p,w))$ we have $(p'-p)\left[x(p',w') - x(p,w)\right] \leq 0$, w/ strict inequality whenever $x(p,w) \neq x(p',w')$.

$\Delta p \cdot \Delta x \leq 0$
compensated demand is downward-sloping

Proof. Because the proposition states iff, we must prove WARP $\Rightarrow \Delta p \cdot \Delta x \leq 0$ and <u>also</u> $\Delta p \cdot \Delta x \leq 0 \Rightarrow$ WARP.

I will use the shorthand $x \equiv x(p,w)$ and $x' \equiv x(p',w')$.

1. WARP $\Rightarrow \Delta p \cdot \Delta x \leq 0$

i.) if $x = x' \Rightarrow (p'-p)(x'-x) = 0$

ii) if $x \neq x'$, then

$$(p'-p)(x'-x) = p'(x'-x) - p(x'-x)$$
$$= \underbrace{p'x' - p'x}_{(a)} \overbrace{- px' + px}^{(b)}$$

$(a) = 0$ since $p'x' = w'$ by WL and $p'x = w'$ by compensated price change

So, for $\overbrace{px}^{= w \text{ by WL}} - px' \leq 0$, term $(b) \leq 0$.

WARP says x affordable under p', w' (by compensated price change) $\Rightarrow$ x' not affordable under p, w so $px' > w$, which implies the above.

2. $\Delta p \cdot \Delta x \leq 0 \Rightarrow$ WARP by contradiction

Suppose $\neg$ WARP, then $\exists$ a compensated price change such that $x \neq x'$, $px' = w$ and $p'x \leq w'$.

Then since $px = w$ and $p'x' = w'$ by WL,

$$p(x' - x) = 0 \text{ and } p'(x' - x) \geq 0$$

hence

$$(p' - p)(x' - x) \geq 0$$

but then $\Delta p \cdot \Delta x \geq 0$ which contradicts the inequality holding for all price changes.

Remember, this only holds for compensated price changes — in general, WARP does not mean demand for uncompensated changes is down-ward sloping.