Exercises #7

Instructions

Exercises #7 are due on Wednesday, March 23$^{\text{rd}}$.

Exercises may be presented for credit as a hard copy at the end of the class meeting on the due date, or may be submitted electronically on Blackboard by the following Monday. If submitted on Blackboard, exercises should be attached as a Portable Document Format (`*.pdf`) file. It is possible to convert handwritten work to `*.pdf` using scanner or a camera-equipped device with Microsoft Office Lens (Android, iOS, or Windows), Google Drive (Android), or Apple Notes (iOS).

Exercises are "collaborative and open book" assignments. You are encouraged to make use of help from your peers, textbook, notes, and me, but you must submit your own answers. There is no penalty for incorrect answers; the expectation is simply for you to progress as far as you can on each question. Complete answers with explanations will be provided in recitation.

Questions

5.B.2 The Cobb-Douglas production function with two inputs is given by $f(z_1, z_2) = z_1^\alpha z_2^\beta$, where $\alpha \geq 0$ and $\beta \geq 0$. Verify that the marginal technical rate of substitution between the two inputs at $z = (z_1, z_2)$ is $MRTS_{12}(z) = \frac{\alpha z_1}{\beta z_2}$.

5.B.3 Show that for a single-output technology, Y is convex if and only if the production function $f(z)$ is concave.

5.C.1 Prove that, in general, if the production set Y exhibits non-decreasing returns to scale, then either $\pi(p) \leq 0$ or $\pi(p) = +\infty$.

5.C.8 Alpha Incorporated (AI) produces a single output q from two inputs z_1 and z_2. You are assigned to determine AI's technology. You are given 100 monthly observations. Two of these monthly observations are shown in the following table.

	Input prices		Input levels		Output price	Output level
Month	w_1	w_2	z_1	z_2	p	q
3	3	1	40	50	4	60
95	2	2	55	40	4	60

In light of these two monthly observations, what problem will you encounter in trying to accomplish your task?

5.C.10 Derive the cost function $c(w, q)$ and conditional factor demand functions (or correspondences) $z(w, q)$ for each of the following single-output constant return technologies with production functions with production functions given by

(a) $f(z) = z_1 + z_2$ (perfect substitutable inputs)

(b) $f(z) = \min z_1, z_2$ (Leontief technology)

(c) $f(z) = (z_1^\rho + z_2^\rho)^{\frac{1}{\rho}}$, $rho \leq 1$ (constant elasticity of substitution)

5.F.1 Give an example of a $y \in Y$ that is profit maximizing for some $p \geq 0$ with $p \neq 0$ but that is also inefficient.