Math Camp
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Homework 8

Compute the determinant of $$A = \begin{bmatrix} 2 & -3 & 0 & 1 \\ 5 & 4 & 2 & 0 \\ 1 & -1 & 0 & 3 \\ -2 & 1 & 0 & 0 \end{bmatrix}$$
Compute the determinant of $$A = \begin{bmatrix} 2 & -3 & 1 & 0 & 4 \\ 0 & 3 & 2 & 5 & 7 \\ 0 & 0 & 1 & 6 & 0 \\ 0 & 0 & 0 & 5 & 2 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$
Compute the determinant of AB, where $A = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 1 \\ 2 & 1 \end{bmatrix}$
Compute the determinant of A^{-1}, where $A = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$
Determine whether A is diagonalizable, find the matrix P and so the diagonal format if so. $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

Find the determinants of the following matrices, assuming that

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 4$$

$$\begin{vmatrix} 2a & 2b & 2c \\ d & e & f \\ g & h & i \end{vmatrix} \quad \begin{vmatrix} 2a & b/3 & -c \\ 2d & e/3 & -f \\ 2g & h/3 & -i \end{vmatrix} \quad \begin{vmatrix} d & e & f \\ a & b & c \\ g & h & i \end{vmatrix} \quad \begin{vmatrix} a-c & b & c \\ d-f & e & f \\ g-i & h & i \end{vmatrix} \quad \begin{vmatrix} 2c & b & a \\ 2f & e & d \\ 2i & h & g \end{vmatrix} \quad \begin{vmatrix} a+2g & b+2h & c+2i \\ 3d+2g & 3e+2h & 3f+2i \\ g & h & i \end{vmatrix}$$

A is a 2×2 matrix with eigenvectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ corresponding to eigenvalues $\lambda_1 = \frac{1}{2}$ and $\lambda_2 = 2$, respectively, and $\mathbf{x} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$. Find $A^{10}\mathbf{x}$.

Compute

$$\begin{bmatrix} -1 & 6 \\ 1 & 0 \end{bmatrix}^{10}$$

Let us define the **algebraic multiplicity** of an eigenvalue to be its multiplicity as a root of the characteristic equation, i.e. the number of repeated times for a certain a certain eigenvalue. Let us define the **geometric multiplicity** of an eigenvalue be the dimension of its corresponding eigenspace, i.e. the number of independent eigenvectors for a certain eigenvalue. The **Diagonalization Theorem** says that the **algebraic multiplicity** of each eigenvalue equals its **geometric multiplicity**. find all (real) values of k for which A is diagonalizable

$$A = \begin{bmatrix} 1 & 0 & k \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$