HW1

Wei Ye*
ECON 5700

Due on August 13, 2020.

1 Question 1

Solution:

Since $|\sin(\frac{1}{x})| \leq 1 \to |x^2 \cdot \sin(\frac{1}{x})| \leq x^2$, in analytical language, it's bounded.

Because $\lim_{x\to 0} -x^2 \leq x^2 \cdot \sin(\frac{1}{x}) \leq \lim_{x\to 0} x^2$, the $LHS = 0$ and $RHS = 0$ as well. Then by **Squeeze Theorem:** we can prove $\lim_{x\to 0} x^2 \sin(\frac{1}{x}) = 0$ ■

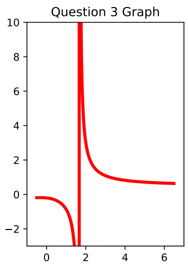
2 Question 2

Solution:

$$\lim_{x\to\infty} \left(\sqrt{x^2+1} - x\right) = \lim_{x\to\infty} \frac{\sqrt{x^2+1}-x}{1} \cdot \frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x} = \frac{x^2+1-x^2}{\sqrt{x^2+1}+x} = \lim_{x\to\infty} \frac{1}{\sqrt{x^2+1}+x} = 0 \ ■$$

3 Question 3

Solution:



4 Question 4

Solution:

By **L' Hopital Rule:** $\lim_{x\to\infty} \frac{6x-1}{10x+4} = \frac{6}{10} = \frac{3}{5}$

*I worked on my assignment sololy.Email: wye22@fordham.edu

5 Question 5

Solution:

$\lim_{x \to \infty} x^2 - x = \infty$

6 Question 6

Solution:

$\lim_{x \to \infty} x^3 = \infty$. $\lim_{x \to -\infty} x^3 = -\infty$

7 Question 7

Solution:

By **L' Hopital's Rule:** $\lim_{x \to \infty} \frac{x^2+x}{3-x} = \lim_{x \to \infty} 2x + 1 = \infty$

8 Question 8

Solution:

If $f(x) = \sqrt{x}$, then $f'(x) = \frac{1}{2\sqrt{x}}$. The domain of $f'(x)$ is $(0, \infty)$

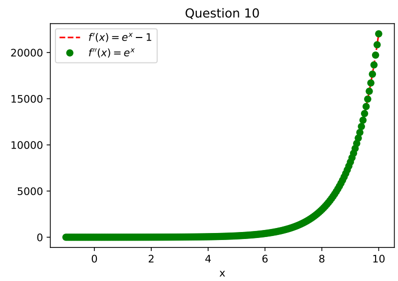
9 Question 9

Solution:

$$f'(x) = \frac{-3}{(2+x)^2}$$

10 Question10

Solution:



11 Question 11

Solution:

1. $$f'(x) = (1+x)e^x$$

2. $$f^{(n)}(x) = (n+x)e^x$$

12 Question 12

Solution:
$y' = \frac{e^x(1-x)^2}{(1+x^2)^2}$, if $x = 1$, then $y' = 0$, and the tangent line is $y = \frac{1}{2}e$

13 Question 13

Solution:

$$g'(t) = \frac{9(t-2)^8(2t+1)^9 - (t-2)^9 9(2t+1)^8 2}{(2t+1)^{18}} = \frac{45(t-2)^8}{(2t+1)^{10}}$$

14 Question 14

Solution:[1]
Step1: Assume a general function $F(x, y)$:

$$\frac{\partial F}{\partial x}\mathrm{d}x + \frac{\partial F}{\partial y}\mathrm{d}y = 0$$

Thus:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\partial F}{\partial x}}{-\frac{\partial F}{\partial y}}$$

Step2: Specify $F(\cdot)$ in our question: $F(x, y) = \sin(x+y) - y^2\cos(x)$:

$$y' = \frac{\cos(x+y) + y^2\sin(x)}{\cos(x+y) - 2y\cos(x)}$$

[1] Reference:https://math.stackexchange.com/questions/2485251/using-implicit-differentiation-find-y-prime-if-sinx-y-y2-cosx

15 Question $15^\star$

Solution:

Step1: Take logarithms on both sides:

$$\ln y = \ln(x^{\frac{3}{4}}(x^2+1)^{\frac{1}{2}}) - \ln(3x+2)^5 = \ln(x^{\frac{3}{4}}) + \ln((x^2+1)^{\frac{1}{2}}) - \ln(3x+2)^5$$

Step2: Take differentiation on both sides w.r.t x:

$$\frac{\mathrm{d}\ln y}{\mathrm{d}x} = \frac{3}{4x} + \sqrt{x^2+1}x + \frac{15}{3x+2}$$

In the end:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = y \cdot \left(\frac{3}{4x} + \sqrt{x^2+1}x + \frac{15}{3x+2}\right)$$

16 Question 16

Solution:

$$y = x^{x^{\frac{1}{2}}}$$

Step 1: Take log on both sides:

$$\log y = x^{\frac{1}{2}} \log x$$

Step 2: Take differentiation on both sides:

$$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}x^{-\frac{1}{2}} \log x + x^{-\frac{1}{2}}$$

In the end:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = y\left(\left(\frac{1}{2}\log x + 1\right)x^{-\frac{1}{2}}\right)$$

17 Question 17

Solution:

$$\lim_{x\to\infty} \frac{\ln x}{\sqrt[3]{x}} = \lim_{x\to\infty} \frac{x^{-1}}{\frac{1}{3}x^{-\frac{2}{3}}} = \lim_{x\to\infty} 3\frac{1}{\sqrt[3]{x}} = 0$$

18 Question 18

Solution:

$$\lim_{x\to0^+} x\ln x = \lim_{x\to0^+} \frac{\ln x}{\frac{1}{x}} = \lim_{x\to0^+} -x = 0$$

The second equation is by L' Hopital's Rule.

19 Quesition 19

Solution:

$$\lim_{x\to 0^+} x^x =^{2} \lim_{x\to 0^+} (e^{\ln x})^x = e^{\lim\limits_{x\to 0^+} x\ln x} = e^0 = 1$$

I directly use the result of Question18.

[2]Reference: https://www.youtube.com/watch?v=hjEwb-zfJFM