Exercises #1

Instructions

Exercises #1 are due on Wednesday, January 26$^{\text{th}}$.

Exercises may be presented for credit as a hard copy at the end of the class meeting on the due date, or may be submitted electronically on Blackboard by the following Monday. If submitted on Blackboard, exercises should be attached as a Portable Document Format (`*.pdf`) file. It is possible to convert handwritten work to `*.pdf` using scanner or a camera-equipped device with Microsoft Office Lens (Android, iOS, or Windows), Google Drive (Android), or Apple Notes (iOS).

Exercises are "collaborative and open book" assignments. You are encouraged to make use of help from your peers, textbook, notes, and me, but you must submit your own answers. There is no penalty for incorrect answers; the expectation is simply for you to progress as far as you can on each question. Complete answers with explanations will be provided in recitation.

Questions

1.C.1 Consider the choice structure $(\mathscr{B}, C(\cdot))$ with $\mathscr{B} = \{\{x,y\},\{x,y,z\}\}$ and $C(\{x,y\}) = \{x\}$. Show that if $(\mathscr{B}, C(\cdot))$ satsifies the weak axiom, then we must have $C(\{x,y,z\}) = \{x\}$, $= \{z\}$, or $= \{x,z\}$.

1.C.2 Show that the weak axiom is equivalent to the following property holding:

> Suppose that $B, B' \in \mathscr{B}$, that $x, y \in B$, and that $x, y \in B'$. Then if $x \in C(B)$ and $y \in C(B')$, we must have $\{x,y\} \subset C(B)$ and $\{x,y\} \subset C(B')$.

1.C.3 Suppose that choice structure $(\mathscr{B}, C(\cdot))$ satisfies the weak axiom. Consider the revealed preferred relation $\succ^* \Leftrightarrow$ there is some $B \in \mathscr{B}$ such that $x, y \in B$, $x \in C(B)$, and $x \notin C(B)$, where x is the revealed at-least-as-good-as relation.

(b) Must $x \succ^* y$ be transitive?

(c) Show that if $\mathscr{B}$ includes all three-element subsets of X, then $x \succ^* y$ is transitive.

1.D.3 Let $x = \{x,y,z\}$, and consider the choice structure $(\mathscr{B}, C(\cdot))$ with

$$\mathscr{B} = \{\{x,y\},\{y,z\},\{x,z\},\{x,y,z\}\}$$

and $C(\{x,y\}) = \{x\}$, $C(\{y,z\}) = \{y\}$, and $C(\{x,z\}) = \{z\}$, as in Example 1.D.1. Show that $(\mathscr{B}, C(\cdot))$ must violate the weak axiom.