Exercises #4

Instructions

Exercises #4 are due on Wednesday, February 16th.

Exercises may be presented for credit as a hard copy at the end of the class meeting on the due date, or may be submitted electronically on Blackboard by the following Monday. If submitted on Blackboard, exercises should be attached as a Portable Document Format (`*.pdf`) file. It is possible to convert handwritten work to `*.pdf` using scanner or a camera-equipped device with Microsoft Office Lens (Android, iOS, or Windows), Google Drive (Android), or Apple Notes (iOS).

Exercises are "collaborative and open book" assignments. You are encouraged to make use of help from your peers, textbook, notes, and me, but you must submit your own answers. There is no penalty for incorrect answers; the expectation is simply for you to progress as far as you can on each question. Complete answers with explanations will be provided in recitation.

Questions

3.E.8 For the Cobb-Douglas utility function, verify that the realtionships in

$$e(p, v(p, w)) = w \qquad \text{and} \qquad v(p, e(p, u) = u \tag{3.E.1}$$

and

$$h(p, u) = x(p, e(p, u)) \qquad \text{and} \qquad x(p, w) = h(p, v(p, w)) \tag{3.E.4}$$

hold. Note that the expenditure function can be derived by simply inverting the indirect utility function, and vice versa.

3.G.3 Consider again the three-good setting of Exercise 3.D.6 in which the consumer has utility function $u(x) = (x_1 - b_1)^\alpha (x_2 - b_2)^\beta (x_3 - b_3)^\gamma$. Assume that $\alpha + \beta + \gamma = 1$ and that $b_1 \geq 0$, $b_2 \geq 0$, and $b_3 \geq 0$.

(a) Derive the Hicksian demand and expenditure functions. Check that $e(p, u)$ is (i) homogenous of degree one in p, (ii) strictly increasing in u and nondecreasing in p_ℓ for any ℓ; and that $h(p, u)$ is homogeneous of degree zero in p, and (ii) provides no excess utility.

(b) Show that the derivates of the expenditure function are the Hicksian demand function you derived in (a).

(c) Verify that the Slutsky equation holds.

(d) Verify that the own-substitution terms are negative and that the compensated cross-price effects are symmetric.

3.G.15 Consider the utility function

$$u = 2x_1^{\frac{1}{2}} + 4x_2^{\frac{1}{2}}.$$

(a) Find the demand functions for goods 1 and 2 as they depend on prices and wealth.

(b) Find the compensated demand function $h(\cdot)$.

(c) Find the expenditure function, and verify that $h(p, u) = \nabla_p e(p, u)$.

(d) Find the indirect utility function and verify Roy's identity.