Economics: ECON 6020, Macroeconomic Theory I, Fall 2021

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Office Hours: Mondays, 1 to 4 pm, and by appointment. Because of Covid, and consistent with department policy, I cannot meet with students in my office. Consequently, **I will conduct office-hour meetings via Zoom.**

Dear Students,

In this syllabus and in my first lecture I outline how I believe we can proceed this semester. We are in a period of transition to a normal, fully in-person, mode of instruction. But we are not quite there yet. I understand that this is still a difficult time and that you may face problems that I have not anticipated. Please contact me if you have a problem or a question. There may be reversals and I am learning as I go so how we conduct this course may evolve. We will do our best and, with an extra bit of patience, courtesy, and compassion, we will get through this time together.

I very much hope that you and your families are well.

Sincerely,
Bart Moore

Masks: University policy is that **all individuals must wear a mask indoors**, even if they are vaccinated. Consistent with this policy, I will wear a mask when I teach and **all students must wear a mask** when they are in my classroom.

Description: This course will introduce students to the models and mathematical technique essential to understanding macroeconomic theory. I will begin by using simple models of rational expectations equilibrium to introduce solution methods and to explain the Lucas critique. Next, I will develop the Ramsey-Cass-Koopmans (RCK) model to study dynamic general equilibrium and the fundamental concepts of optimal growth theory. I will then develop a real business cycle model from the dynamic stochastic general equilibrium model. I will use versions of this model to explain the permanent-income/life-cycle theory of consumption, the Ricardian equivalence hypothesis, and equilibrium asset pricing. Finally, we will examine the microeconomic foundations of sticky nominal prices and the New-Keynesian approach to monetary policy.

Grading: Your grade will be based on a midterm exam (40%) and a final exam (60%). We will discuss the format and timing of these exams in class.

Math Background: I assume that students are familiar with derivatives, partial derivatives, optimization, constrained optimization (the Lagrange method), and exponential and logarithmic functions. I also assume some familiarity with stochastic processes and linear algebra. There is a concise and useful review of mathematical techniques in Hamilton, James, **Time Series Analysis** (Princeton University Press, 1994), Appendix A, especially pp. 710-750.

Primary Text:

Wickens, Michael, **Macroeconomic Theory**, 2^{nd} ed, Princeton U. Press (Princeton, NJ: 2012).

Background Text:

Romer, David, **Advanced Macroeconomics**, third edition, McGraw-Hill (New York: 2006).
The analysis in this book is a bit out of date but it contains much useful background material. I place the chapter on consumption theory on A-res.

The primary text is available for purchase online or at the campus bookstore. Other readings (below) are available through JSTOR **(J),** Elsevier Science **(E)**, or will be placed on A-res **(R)**.

If you are not familiar with Macroeconomics at an undergraduate level, you may find it helpful to review an intermediate undergraduate text, such as Mankiw, N. Gregory, **Macroeconomics**, 10^{th} edition, Worth Publishers (New York: 2019). Read, especially, Chapters 3, 5, and 7-12. An advanced undergraduate text that is useful for background and intuition is Chugh, Sanjay K., **Modern Macroeconomics**, MIT Press, (2015). *See especially Chapters 1-8, 10, 13, 14, and 26.*

Course Outline and Readings

I. Simple models of rational expectations equilibrium.

Primary:

Blanchard, O. and Stanley Fischer, **Lectures on Macroeconomics**, (MIT Press, 1989).
Chapter 5, "Multiple Equilibria, Bubbles and Stability" (especially pp. 213- 224 and "Appendix to Chapter 5: A tool kit ..." pp. 264- 266). **(R)**

Sargent, Thomas J. **Macroeconomic Theory,** 2^{nd} Edition, (Academic Press, 1987), Chapter IX, "Difference Equations and Lag Operators" (especially pp. 176-85, 195-199). **(R)**

Hamilton, James, **Time Series Analysis** (Princeton University Press, 1994), Chapter 3, Stationary ARMA processes (especially pp. 43-61). **(R)**

Optional: Wickens, Mathematical Appendix, Section 17.8 (especially pp. 558-59).

Background:

DeJong, David, and Chetan Dave, **Structural Macroeconometrics,** 2^{nd} ed, Princeton U. Press (Princeton, NJ: 2011), Chapter 4, Linear Solution Techniques.

Lucas, Robert E., Jr. "Econometric Policy Evaluation: A Critique," in Brunner and Meltzer, eds. **The Phillips Curve and Labor Markets, Carnegie-Rochester Conference Series on Public Policy**, v.1, 1976, pp19-46.

II. The Dynamic General Equilibrium Model of Capital Accumulation.

Primary:

Wickens, Chapters 2, 3, and Chapter 17, Sections 17.1-17.6.

Barro, Robert J. and Xavier Sala-I-Martin, **Economic Growth**, second edition, MIT Press (Cambridge, MA: 2004), Chapter 1, (esp. pp. 23-37). **(R)**

Background:

Barro and Sala-I-Martin, Introduction, Appendix A.3, pp.604-618, and Chapter 2.

Romer, Chapter 2 Part A.

Romer, Paul, "Capital Accumulation in the Theory of Long-Run Growth", in Robert Barro, ed., **Modern Business Cycle Theory**, Harvard U. Press (Cambridge, MA: 1989) pp. 51-70.

King, Robert G. and Sergio Rebelo, "Transitional Dynamics and Economic Growth in the Neoclassical Model," **American Economic Review**, v. 83, n. 4 (September 1993) pp. 908-931.

Lucas, Robert E., Jr., "On the Mechanics of Economic Development," **Journal of Monetary Economics**, 22, 1988, pp. 3-42. **(E)**

III. The Decentralized Dynamic Stochastic Competitive General Equilibrium Model.

Primary:

Wickens, Chapters 4 and 16 (especially 16.1-16.4.2 and 16.8-16.9).

Romer, Chapters 4, 7 (especially pp. 346-65), and 11 (especially pp. 559-572).

Background:

Wickens, Chapter 5, Sections 5.1-5.3.

Prescott, Edward C. "Theory ahead of Business Cycle Measurement," **Carnegie-Rochester Conference Series on Public Policy,** 25 (Autumn 1986) pp. 11-44. **(E)**

King, R.G., C.I. Plosser, and S.I. Rebelo, "Production, Growth, and Business Cycles, I: The Basic Neoclassical Model," **The Journal of Monetary Economics,** 1988, v. 21, pp. 195-232. **(E)**

IV. Microeconomic Foundations of Sticky Prices.

Primary:

Wickens, Chapter 14 (especially pp. 402-426).

Background:

Blanchard, O. and Stanley Fischer, **Lectures on Macroeconomics**, (MIT Press, 1981). Chapter 10.3, "Aggregate Supply and Demand, Wage Indexation, and Supply Shocks", pp. 518-525 (also, pp. 555-565).

Romer, Chapter 6.

Mankiw, N. G. "Small Menu Costs and Large Business Cycles, …" **The Quarterly Journal of Economics**, v. 100, n. 2, (May 1985), pp. 529-538. **(J)**

Romer, David, "Keynesian Macroeconomics without the LM Curve", **The Journal of Economic Perspectives**, v. 14, no.2, Spring 2000. **(J)**

Gali, Jordi. **Monetary Policy, Inflation, and the Business Cycle.** Princeton U. Press (2008).

Recommended Problems.

I. Simple models of rational expectations equilibrium.

Problems:

1.) Assume that $|\beta|<1$. Prove the following three propositions.

(a) $1+\beta+\beta^2+\beta^3+... = \left(\frac{1}{1-\beta}\right)$.

(b) $1+\beta+\beta^2+\beta^3+...+\beta^N = \left(\frac{1-\beta^{N+1}}{1-\beta}\right)$.

(c) $\sum_{j=0}^{\infty}\beta^j j = \left[\frac{\beta}{(1-\beta)^2}\right]$.

2.) A standard intertemporal budget constraint is

$$a_{t+1} = (1+r)a_t + y_t - c_t \qquad (1)$$

where a_t, y_t, and c_t denote real assets, real income, and real consumption, respectively. Here $r>0$ denotes the real interest rate. Give a brief economic explanation of equation (1). Note that equation (1) is a first-order difference equation. Evaluate its root and solve it appropriately. What does your result say about the relationship of the present discounted value consumption to the present discounted value of income (both income and consumption being evaluated over an infinite horizon)?

3.) For this problem, consider the simple asset-pricing model derived in class and presented in Blanchard and Fisher, page 215. In class we solve the model using factorization, that is, "Sargent's method". On pages 218-19 Blanchard and Fisher develop a simpler (though more tedious) method known as repeated substitution which explicitly invokes the *law of iterated expectations*. Use repeated substitution to solve for the current REE price of the equity share as a function of expected future dividends. State clearly where and how you use the law of iterated expectations. How does your result compare to what we derived in lecture?

4.) Consider the equilibrium asset-pricing model discussed in lecture. Recall that equilibrium in this model satisfies

$$(1+r)P_t = E_t\left(P_{t+1} + D_{t+1}\right) \qquad (1)$$

where P_t is the price of an equity share, D_t is the dividend payment, and $r>0$ is the (net) real rate of interest. Let ε_t denote a white noise process. Find the rational expectations equilibrium solution for P_t when by each of the D_t is governed by each of the following stochastic processes.

(Problem continued next page.)

(a) MA(1): $D_t = \varepsilon_t + \theta\varepsilon_{t-1}$.

(b) General stationary AR(1): $D_t = \mu + \rho D_{t-1} + \varepsilon_t$, $\mu \neq 0, |\rho| < 1$. (*This one is a bit tedious, but instructive.*) Comment on how the solution in this case compares to the solutions to the two cases discussed in lecture.

(c) Two-state process: $D_{t+j} = D_1$ with probability ϕ and $D_{t+j} = D_0$ with probability $1-\phi$, where $\phi \in [0,1]$ and $D_1 > D_0$. How does an increase in ϕ affect P_t (give an ECONOMIC interpretation)?

5.) Consider the following model of REE:

$$m_t - p_t = \gamma - \alpha R_t, \quad \alpha > 0 \qquad (1)$$

$$R_t = r + E_t(p_{t+1} - p_t) \qquad (2)$$

Here, m_t denotes (log) nominal money supply, p_t the (log) price level, and R_t the nominal interest rate.

(a) Give an ECONOMIC explanation of equations (1) and (2).

(b) Suppose that the nominal money supply rule is $m_t = \mu + \varepsilon_t$, where ε_t is a white noise process. Solve for the REE values of p_t and R_t.

(c) Suppose instead that the nominal money supply rule is $m_t = \rho m_{t-1} + \varepsilon_t$, where $|\rho| < 1$ and ε_t is a white noise process. Once again, solve for the REE values of p_t and R_t.

II. DGE Capital Accumulation, Part A.

Wickens (1st Edition), Chapter 2: Problem 2.1. *(Note: the problems and solutions for the Wickens text are available at the website listed on page 14 of the text.)*

Additional Problem 1.) This problem is based on Romer, problem 2.7. Assume a Cobb-Douglas aggregate production function. Describe how each of the following affects the $\Delta c_{t+1} = 0$ and $\Delta k_{t+1} = 0$ loci in the phase diagram for optimal capital accumulation (e.g., Wickens, Figure 2.10).

a) A rise in θ.

b) A decline in an otherwise constant total factor productivity, $\Delta A < 0$.

c) An increase in the depreciation rate, $\Delta\delta > 0$.

Show how each affects the steady state equilibrium values of c_t and k_t. Also, describe briefly the dynamic behavior of c_t, k_t, and Y_t as they make the transition from their initial steady state to their new steady state values.

(Continued next page.)

Additional Problem 2. (This is in the book: Section 2.6, Labor in the Basic Model.) Suppose that the social planner seeks to maximize

$$\sum_{s=0}^{\infty} \beta^s U\left(c_{t+s}, l_{t+s}\right) \tag{1}$$

subject to

$$l_t + n_t = 1 \tag{2}$$

$$k_{t+1} = \left(1-\delta\right) k_t + y_t - c_t \tag{3}$$

$$y_t = F\left(k_t, n_t\right) \tag{4}$$

where $\beta = \frac{1}{1+\theta}$, $\theta > 0$, $\delta \in \left(0,1\right)$, $\alpha \in \left[0,1\right]$. The production function and the period utility function have standard properties.

A.) Set up the Lagrangian and derive the first-order conditions.

B.) Use the first-order conditions to derive the **intertemporal optimality condition** (which gives the relationship between this period's marginal utility of consumption and next period's marginal utility of consumption) and the **intratemporal optimality condition** (which gives the relationship between the current marginal utilty of consumption and the current marginal utility of leisure).

C.) How does the intertemporal optimality condition compare to the corresponding Euler equation in the model developed in class, i.e., the model that does not explicitly include labor?

Wickens (1st Edition), Chapter 2: Problem 2.4 (modified). *For this problem you may wish to consult my handwritten notes on A-Res in addition to the solution manual.*

(a) Instead of expressions for the long-run (steady-state) solutions for consumption, labor, and capital, instead derive expressions for the long-run (steady-state) values of $\frac{y}{k}$ and of $\frac{c}{l} = \frac{c}{1-n}$. Your answer should give the value of $\frac{y}{k}$ as a function of the model's parameters and $\frac{c}{1-n}$ as a function of $\frac{y}{n}$ and the model's parameters.

(b) What is the implied value of the long-run real wage as a function of $\frac{y}{n}$ and the model's parameters? What is the implied value of the real interest rate as a function of the model's parameters?

Omit part (c).

(d) Obtain an expressions for the long-run capital labor ratio, $\frac{k}{n}$, as a function of the long-run equilibrium value of the real wage and the real interest rate.

II. DGE Capital Accumulation. Part B.
Wickens (1st Edition), Chapter 3: Problems 3.1.

Additional Problems:

1.) This problem is based on Romer, problem 1.1. Answer the following three questions for both continuous time and discrete time. For discrete time derive the result using both the log approximation to percentages and again using a first-order Taylor series approximation. Assume that X_t grows at rate x and Z_t grows at rate at rate z.

(a) Show that their product, $X_t Z_t$, grows at rate x +z. (This will hold approximately for discrete time.)

(b) Show that the ratio, $\left(X_t / Z_t \right)$, grows at rate x – z. (Approximately for discrete time.)

(c) If $Z_t = X_t^{\alpha}$ then z = αx . (Approx. for discrete).

2.) This problem is based on Romer, problem 2.6. In class we considered the DGE model of capital accumulation when the growth rate of labor augmenting technical progress is ω. Using the modified capital accumulation equation and the modified intertemporal optimality condition sketch the phase diagram and locate the $\Delta c_{et+1} = 0$ and $\Delta k_{t+1} = 0$ loci and the saddle path. Note that here, $c_{et} = \dfrac{C_t}{N_{et}}$, $k_t = \dfrac{K_t}{N_{et}}$, and $N_{et} = (1+\omega)^t$.

Show how the $\Delta c_{et+1} = 0$ and $\Delta k_{t+1} = 0$ loci shift when there is an increase in ω and locate the new saddle path. Hint: $\dfrac{1}{1+\eta} \cong 1-\eta$ *and* $\dfrac{1-\delta}{1+\eta} \cong 1-\delta-\eta$.

III. The Decentralized DSCGE Model. *For the problem from Wickens, you may wish to consult my handwritten notes on A-Res in addition to the solution manual.*

Wickens (1st Edition), Chapter 4: Problem 4.1.

Additional Problem 1.) Consider a representative household that seeks to maximize

$$\sum_{s=0}^{\infty} \beta^s U(c_{t+s}) \qquad (1)$$

Where $\beta = \dfrac{1}{1+\theta}$, $0 < \theta < 1$,

$$U(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}, \ \gamma > 0, \ \gamma \neq 1. \qquad (2)$$

Let W_t denote the household's wealth in the initial period. Assume that the household's only source of income is interest on its wealth. Let $r(W_t)$ denote the real interest rate and note that here, the real interest rate is a (linear) declining function of W_t : specifically,

$$r(W_t) = \rho - \xi W_t, \ \ \rho > \theta, \ \xi > 0. \qquad (3)$$

(Problem continued next page.)

Household wealth therefore evolves according to

$$W_{t+s+1} = \left[1 + r\left(W_{t+s}\right)\right] W_{t+s} - c_{t+s} \qquad (4)$$

A.) Set up the Lagrangian and derive the intertemporal optimality condition.

B.) Derive and expression for the growth rate of consumption, $\frac{\Delta c_{t+s+1}}{c_{t+s}}$.

C.) Derive a condition that must hold if consumption is to be constant through time. (That is, derive the condition that must hold for the growth rate of consumption to be zero.)

D.) Construct a phase diagram (c_t on the vertical axis and W_t on the horizontal axis) illustrating the results of this problem. Derive and illustrate the dynamic behavior of W_t and c_t in all regions of the diagram. Locate the saddle path. Give the expressions for steady state wealth and steady state consumption, each in terms of the model's parameters.

E.) Use the phase diagram to illustrate how, beginning from a steady state equilibrium, an unanticipated increase in θ will affect the behavior of W_t and c_t through time. Can you offer an economic explanation for these effects?

Additional Problem 2.) This problem is based on Romer, problem 2.5. Consider a representative household that seeks to maximize

$$\sum_{s=0}^{\infty} \beta^s U(c_{t+s}) \qquad (1)$$

Where $\beta = \frac{1}{1+\theta}$, $\theta > 0$,

$$U\left(c_t\right) = \frac{c_t^{1-\gamma}}{1-\gamma}, \; \gamma > 0, \; \gamma \neq 1. \qquad (2)$$

Let W_t denote the household's wealth in the initial period. Assume that the household's only source of income is interest on its wealth. Let $r \geq 0$ denote the (constant) real interest rate. Household wealth therefore evolves according to

$$W_{t+s+1} = \left(1+r\right) W_{t+s} - c_{t+s} \qquad (3)$$

Derive the intertemporal optimality condition. Derive the growth rate of consumptions, $\frac{\Delta c_{t+s+1}}{c_{t+s}}$.

Is consumption increasing, decreasing, or constant through time?

Additional Problem 3.)
(Romer, Chapter 7: Problem 7.5, following Hansen and Singleton, 1983):
Suppose that the representative agent seeks to maximize

$$E_t \sum_{s=0}^{\infty} \beta^s U(c_{t+s}) \quad \text{where} \quad U(c_t) = \frac{c_t^{1-\theta}}{1-\theta}, \quad \theta > 0, \; \beta = \frac{1}{1+\rho}, \text{ and } \rho > 0,$$

subject to $A_{t+1} = (1+r)A_t + y_t - c_t$. Here y_t denotes exogenous random income. Assume that the real interest rate, r, is constant but not necessarily equal to ρ.

(a) Derive the Euler equation relating c_t to c_{t+1}.
(b) Suppose that the log of income is distributed normally, and that as a result the log of c_{t+1} is distributed normally; let σ^2 denote its variance conditional on information available at time t. Re-write the expression in part (a) in terms of $\ln c_t, E_t[\ln c_{t+1}], \sigma^2$ and the parameters r, $\theta, and\ \rho$.
(Hint: if a variable x is distributed normally with mean μ and variance V, $E[e^x] = e^{\mu}e^{V/2}$.)
(c) Show that the result in part (b) implies that the log of consumption follows a random walk with drift (i.e., $\ln c_{t+1} = a + \ln c_t + u_{t+1}$ where u_{t+1} is white noise).
(d) How do changes in r and σ^2 affect expected consumption growth, $E_t[\ln c_{t+1} - \ln c_t]$?

IV. Microeconomic Foundations of Sticky Prices.
Wickens Chapter 13 (1st Edition) : Problems 13.4 (omit part d, ii) and 13.5, parts a and c only (for part c assume that μ > 1).

Additional Problem 1: Consider the following simple New-Keynesian model:

$$x_t = -(R_t - E_t\pi_{t+1} - r) \qquad (1)$$
$$\pi_t = \beta E_t \pi_{t+1} + x_t + e_t, \qquad 0 < \beta < 1 \qquad (2)$$
$$R_t = r + \gamma \pi_t, \qquad \gamma > 1 \qquad (3)$$

where π_t denotes inflation, x_t denotes the GDP gap, R_t denotes the nominal federal funds rate, and r is the steady-state equilibrium real interest rate. Assume that e_t is $i.i.d.(0, \sigma^2)$. Also, note that $0 < \beta < 1 < \gamma$.

A.) Give a brief **ECONOMIC** explanation of each equation. What is the Taylor principle and does it hold in this model? What is the implied target inflation rate?
B.) Derive the rational expectations equilibrium values of $\pi_t, x_t,\ and\ R_t$, each as a function of e_t.
C.) Derive the policy that minimizes the Variance of π_t. Also, derive the policy that minimizes the Variance of x_t. (Note that $\gamma > 1$). Comment on the difference between these two policies and what it means for central-bank policy.