Econ 6020: Macro I

Prob Set IV, Add'l Prob 2

6020 NK Prob
Additional Problem 2

NKPC $$\pi_t = \beta E_t \pi_{t+1} + \gamma x_t + \varepsilon_{\pi t} \quad (1)$$

NKIS $$x_t = E_t x_{t+1} - \theta\left[i_t - E_t \pi_{t+1} \right] + \varepsilon_{xt} \quad (2)$$

Agg M^d $$(m_t - p_t)^d = \mu + x_t - \alpha i_t \quad (3)$$

Target RMB $$(m_t - p_t)^s = \mu + \varepsilon_{mt} \quad (4)$$

~~[illegible]~~ $\varepsilon_{\pi t}$, ε_{xt}, ε_{mt} are white noise.

Solve for SODE in π_t

① Use (3), (4) and equic, $(m_t - p_t)^d = (m_t - p_t)^s$, to write

$$\mu + x_t - \alpha i_t = \mu + \varepsilon_{mt}$$

which gives $$-\alpha i_t = -x_t + \varepsilon_{mt} \text{ or}$$

$$i_t = \frac{1}{\alpha} x_t - \frac{1}{\alpha} \varepsilon_{mt} \quad (5)$$

② Subst (5) into (2) to get

$$x_t = E_t x_{t+1} - \theta\left[\frac{1}{\alpha} x_t - \frac{1}{\alpha} \varepsilon_{mt} - E_t \pi_{t+1} \right] + \varepsilon_{xt} \text{ or}$$

$$\left(1 + \frac{\theta}{\alpha}\right) x_t = E_t x_{t+1} + \theta E_t \pi_{t+1} + \frac{\theta}{\alpha} \varepsilon_{mt} + \varepsilon_{xt}$$

↓

$$\left(\frac{\alpha+\theta}{\alpha}\right)x_t = E_t x_{t+1} + \theta E_t \pi_{t+1} + \frac{\theta}{\alpha}\varepsilon_{mt} + \varepsilon_{xt} \quad \text{or}$$

$$x_t = \left(\frac{\alpha}{\alpha+\theta}\right)E_t x_{t+1} + \left(\frac{\alpha\theta}{\alpha+\theta}\right)E_t \pi_{t+1} + \left(\frac{\theta}{\alpha+\theta}\right)\varepsilon_{mt} + \left(\frac{\alpha}{\alpha+\theta}\right)\varepsilon_{\pi t} \qquad (6)$$

(3) SUBST (6) INTO (1)

$$\pi_t = \beta E_t \pi_{t+1} + \varepsilon_{\pi t} + \gamma\left[\left(\frac{\alpha}{\alpha+\theta}\right)E_t x_{t+1} + \left(\frac{\alpha\theta}{\alpha+\theta}\right)E_t \pi_{t+1} + \left(\frac{\theta}{\alpha+\theta}\right)\varepsilon_{mt} + \left(\frac{\alpha}{\alpha+\theta}\right)\varepsilon_{\pi t}\right]$$

OR

$$\pi_t = \left[\beta + \left(\frac{\alpha\theta\gamma}{\alpha+\theta}\right)\right]E_t \pi_{t+1} + \left(\frac{\alpha\gamma}{\alpha+\theta}\right)E_t x_{t+1} + \varepsilon_{\pi t} + \left(\frac{\theta\gamma}{\alpha+\theta}\right)\varepsilon_{mt} + \left(\frac{\alpha\gamma}{\alpha+\theta}\right)\varepsilon_{\pi t} \qquad (7)$$

(4) NOTE FROM (1) THAT

$$\gamma x_t = \pi_t - \beta E_t \pi_{t+1} - \varepsilon_{\pi t} \quad \text{SO}$$

$$\gamma x_{t+1} = \pi_{t+1} - \beta E_{t+1} \pi_{t+2} - \varepsilon_{\pi t+1} \quad \text{SO}$$

$$\gamma E_t x_{t+1} = E_t \pi_{t+1} - \beta E_t \pi_{t+2} - 0 \qquad (8)$$

(5) USE (8) IN (7) TO GET

$$\pi_t = \left[\beta + \left(\frac{\alpha\theta\gamma}{\alpha+\theta}\right)\right]E_t \pi_{t+1} + \left(\frac{\alpha}{\alpha+\theta}\right)\left(E_t \pi_{t+1} - \beta E_t \pi_{t+2}\right) + \varepsilon_{\pi t} + \left(\frac{\theta\gamma}{\alpha+\theta}\right)\varepsilon_{mt} + \left(\frac{\alpha\gamma}{\alpha+\theta}\right)\varepsilon_{xt}$$

↘

Collecting

$$\left(\frac{\alpha}{\alpha+\theta}\right)\beta E_t \pi_{t+2} - \left[\beta + \left(\frac{\alpha\theta\gamma}{\alpha+\theta}\right) + \left(\frac{\alpha}{\alpha+\theta}\right)\right] E_t \pi_{t+1} + \pi_t = \varepsilon_{\pi t} + \left(\frac{\theta\gamma}{\alpha+\theta}\right)\varepsilon_{mt} + \left(\frac{\alpha\gamma}{\alpha+\theta}\right)\varepsilon_{xt} \quad (9)$$

(6) or, multiplying through by $\left(\frac{\alpha+\theta}{\alpha\beta}\right)$,

$$E_t \pi_{t+2} - \left(\frac{\alpha+\theta}{\alpha\beta}\right)\left[\beta + \left(\frac{\alpha\theta\gamma}{\alpha+\theta}\right) + \left(\frac{\alpha}{\alpha+\theta}\right)\right] E_t \pi_{t+1} + \left(\frac{\alpha+\theta}{\alpha\beta}\right)\pi_t = \left(\frac{\alpha+\theta}{\alpha\beta}\right)\varepsilon_{\pi t} + \left(\frac{\alpha+\theta}{\alpha\beta}\right)\left(\frac{\theta\gamma}{\alpha+\theta}\right)\varepsilon_{mt} + \left(\frac{\alpha+\theta}{\alpha\beta}\right)\left(\frac{\alpha\gamma}{\alpha+\theta}\right)\varepsilon_{xt}$$

OR

$$E_t \pi_{t+2} - \left[\left(\frac{\alpha+\theta}{\alpha}\right) + \frac{\theta\gamma}{\beta} + \frac{1}{\beta}\right] E_t \pi_{t+1} + \left(\frac{\alpha+\theta}{\alpha\beta}\right)\pi_t = \left(\frac{\alpha+\theta}{\alpha\beta}\right)\varepsilon_{\pi t} + \left(\frac{\theta\gamma}{\alpha\beta}\right)\varepsilon_{mt} + \left(\frac{\gamma}{\beta}\right)\varepsilon_{xt}$$