Homework 2

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ECON 7010- Microeconomics II

Due on Feb 2, 2022

1 Qeustion 1 – 2.E.1

Solution:

We apply $\mathcal{P} \cdot \mathcal{X}$: $\sum_i^3 p_i * x_i = \frac{w(\beta p_1 + p_2 + p_3)}{p_1 + p_2 + p_3}$

- When $\beta = 1$, $\sum_i^3 p_i * x_i = w$, it satisfies Walras' law. $x_1(\alpha p, \alpha w) = \frac{\alpha p_2}{\alpha(p_1+p_2+p_3)} \frac{\alpha w}{\alpha p_1} = x_1(p, w)$. Using the same method to derive other two, we get the same result, thus, it satisfies homogeneity of degree zero.

- When $\beta \in (0, 1)$, $\sum_i^3 p_i * x_i < w$, it violates Walras' law. For x_1 and x_2, they are same with previous, so we only watch for x_3, $x_3(\alpha p, \alpha w) = \frac{\alpha \beta p_1}{\alpha(p_1+p_2+p_3)} \frac{\alpha w}{\alpha p_3} = x_3(p, w)$, therefore, it satisfies homogeneity of degree zero.

2 Question 2 – 2.E.7

Solution:

By Walras' law, $p_1 x_1(p, w) + p_2 x_2(p, w) = w$, plugging $x_1 = \frac{\alpha w}{p_1}$ into Walras' law equation. We can obtain: $p_1 \cdot \frac{\alpha w}{p_1} + p_2 \cdot x_2(p, w) = w$, then deriving this equation, $x_2(p, w) = \frac{(1-\alpha)w}{p_2}$

To test homegeneity of degree zero:

$$x_1(\alpha p, \alpha w) = \frac{\alpha^2 w}{\alpha p_1} = x_1(p, w)$$

$$x_2(\alpha p, \alpha w) = \frac{\alpha(1-\alpha)w}{\alpha p_2} = x_2(p, w)$$

Thus, her demand function satisfies homogeneity of degree zero.

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3 Question 3 – 2.F.3

Solution:

(a) This question is a little tricky, now if we assume her behavior is consistent, i.e., satisfing WARP, it means $p_1x_1' + p_2x_2' \leq p_1x_1 + p_2x_2$ and $p_1'x_1 + p_2'x_2 > p_1'x_1' + p_2'x_2'$, however, if it violates WARP, we need to make induction condition have converse direction, which means $p_1'x_1 + p_2'x_2 \leq p_1'x_1' + p_2'x_2'$

Combine:

$$p_1x_1' + p_2x_2' \leq p_1x_1 + p_2x_2 \tag{1}$$

$$p_1'x_1 + p_2'x_2 \leq p_1'x_1' + p_2'x_2' \tag{2}$$

Plug $p_1 = 100$, $p_1' = 120$, $p_2 = 100$, $p_2' = 80$, $x_1 = 100$, $x_1' = 120$, $x_2 = 100$ into the above two equations.

$$100 * 120 + 100x_2' \leq 100 * 100 + 100 * 100$$

$$100 * 100 + 80 * 100 \leq 100 * 120 + 80x_2'$$

Thus, $x_2' \in [75, 80]$

(b) Since year 1's bundle is revealed preferred to the bundle in year 2.

$$p_1x_1' + p_2x_2' \leq p_1x_1 + p_2x_2 \tag{3}$$

$$p_1'x_1 + p_2'x_2 > p_1'x_1' + p_2'x_2' \tag{4}$$

Plug values into these equations:

$$100 * 120 + 100x_2' \leq 100 * 100 + 100 * 100$$

$$100 * 100 + 80 * 100 > 100 * 100 + 80x_2'$$

Therefore, $x_2' < 75$.

(c) As year 2's consumption bundle is revealed preferred to year 2's:

$$p_1'x_1 + p_2'x_2 \leq p_1'x_1' + p_2'x_2' \tag{5}$$

$$p_1x_1' + p_2x_2' > p_1x_1 + p_2x_2 \tag{6}$$

Plug values into these two equations:

$$100 * 100 + 80 * 100 \leq 100 * 120 + 80x_2'$$

$$100 * 120 > 100 * 100 + 100 * 100$$

Hence, $x_2' > 80$

4 Question 4 – 2.F.16

Solution:

(a)

$$x_1(\alpha p, \alpha w) = \frac{\alpha p_2}{\alpha p_3} = \frac{p_2}{p_3} = x_1(p, w)$$
$$x_2(\alpha p, \alpha w) = -\frac{\alpha p_1}{\alpha p_3} = -\frac{p_1}{p_3} = x_2(p, w)$$
$$x_3(\alpha p, \alpha w) = \frac{\alpha w}{\alpha p_3} = \frac{w}{p_3} = x_3(p, w)$$

Thus, $x(p, w)$ satisfies homogeneity of degree zero. Now, let's prove Walras' law:

$$\begin{aligned}\sum_i^3 p_i x_i &= p_1 \cdot \frac{p_2}{p_3} + p_2 \cdot \frac{-p_1}{p_3} + p_3 \cdot \frac{2}{p_3} \\ &= \frac{p_1 p_2 - p_1 p_2 + w p_3}{p_3} \\ &= w\end{aligned}$$

It satisfies Walras' law.

(b) If x satisfies WARP, it should $px' \leq w$, $p'x > w$, we prove by contradiction: Let $p = (1, 2, 1)$, w=1, thus, $x = (2, -1, 1)$, let $p' = (1, 1, 1)$, $w' = 2$,thus, $x' = (1, -1, 2)$.

$$p * x' = 2 > w$$

$$p' * x = 2 = w'$$

Therefore, it violates WARP.

5 Question 5 – 2.F.17

Solution:

(a)

$$\begin{aligned}x_k(\alpha x, \alpha w) &= \frac{\alpha w}{\alpha \sum_{l=1}^{L} p_l} \\ &= \frac{w}{\sum_{l=1}^{L} p_l} \\ &= x_k(x, w)\end{aligned}$$

Thus, it's homogeneous of degree zero.

(b)

$$\begin{aligned}
\sum_{k}^{L} x_k(p,w)p_k &= \sum_{k}^{L} p_k \cdot \frac{w}{\sum_{l=1}^{L} p_l} \\
&= \frac{w}{\sum_{l=1}^{L} p_l} \sum_{k}^{L} p_k \\
&= \frac{w}{\sum_{l=1}^{L} p_l} \cdot \sum_{l=1}^{L} p_l \\
&= w
\end{aligned}$$

Yes, it satisfies Walras' law.

(c) Suppose $px_k(p', w') \leq w$ and $p'x_k(p,w) \leq w'$. From the first inequality, $\sum_k^L p_k \frac{w'}{\sum_{l=1}^L p_l} \leq w$, which means $\frac{w'}{\sum_{l=1}^L p'_l} \leq \frac{w}{\sum_{l=1}^L p_l}$, from the second inequality, $\sum_k^L p'_k \frac{w}{\sum_{l=1}^L p_l} \leq w'$, thus, $\frac{w'}{\sum_{l=1}^L p'_l} \geq \frac{w}{\sum_{l=1}^L p_l}$, Hence, $x_k(p,w) = x_k(p', w')$, it satisfies WARP.