Financial Economics

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Department of Economics

Prof: Erick W. Rengifo

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**Exercise set Nº 1**

**Part I: Time value of money, the Net Present Value and the Internal Rate of Return**

1. Use excel to answer this question. If you have the following financial transactions:

(10000, -6000,-5500)

(10000, -3200,-8960)

(10000, -6800,-4320)

1. What is the interest rate for each of the cash flows?
2. If your IRR is 15%, which one would you select? Why?
3. If your IRR is 12%, which one would you select? Why?
4. Generalize this analysis and present a graph that shows the regions in which one financial transaction is preferred to the other. Comment your results.
5. Assuming that the average annual return of the three-month T-bills is 3% and the average annual return for the SP500 is 9%. If you have the following investment transactions:

(-10000, 11000)

(-1000, 1200)

* 1. Which project would you execute if the future cash flow is a sure thing?
  2. Discuss and present a possible solution: If the future cash flows are simply expected ones and you consider that the risk of these investments can be comparable to an investment in stock markets, would your decision of part a. change? Why?

**Part II: Time value of money, the interest rate and the investment opportunity line**

The following question needs to be solved **graphically,** explaining the economic and financial intuition of these graphs.

Assume that you have two cash flows, one for time t0 and the other for time t1:

F0=10000

F1=15000

Assume that the interest rate (your opportunity cost of capital) is 10%

1. If you decide to smooth your consumption borrowing money based on a 20% decrease of future consumption. What should be the **total** money available for consumption at t0?
2. If the interest rate is 14% or 6%, how these different rates would impact your smoothing process? Which of the three scenarios would be more advantageous for you?
3. Assume that you have the opportunity to invest in real markets and that you have the following investment opportunities with their respective returns:

Investment opportunity 1 (I1): 500, 18%

Investment opportunity 2 (I2): 500, 10%

Investment opportunity 3 (I3): 500, 6%

Assume that your initial endowment (at t0) is 2000.

* 1. Assuming that the interest rate of the financial markets is 10% and that the return of each of the projects is a sure thing. Which project(s) should you do? Why? What is the technical criterion?
  2. Using your answer of part a., what should be your inter temporal cash flow for consumption?
  3. If you want to use the financial markets to save 30% of the money actually allocated for consumption (after the investment in real assets), what should be the total amount of money that you are going to have in the next period?
  4. Assuming that ignoring the technical rule you decide to do the three projects. Using a graph explain why this is not optimal. Can you confirm this intuition using numbers and the results of previous parts?

1. Explain, using the substitution and income effect, the effect of an increase of the capital market’s interest rate. (tip: assume 2 type of agents: net borrowers and net lenders).
   1. According to the substitution effect, what happens with each type of agents’ savings?
   2. According to the income effect, what happens with each type of agents’ savings?
   3. What is the global (aggregate) result in terms of savings?

**Part III: Preferences and utility functions**

1. Explain the intuition behind the idea that the indifference curves must not cross if the agent is a rational one.
2. Explain the intuition behind the idea that the marginal rate of substitution should equal the ratio of marginal utilities. Use a graphical representation and mathematical derivation for this question.
3. Assume that an individual’s utility function is . Using the MRSy,x equation and knowing that at time zero X=25, Y=30. Assume that at time one, x increases by 0.1, what is the change on Y to keep the utility level constant? Estimate the time-one value of Y and verify that the utility values at time zero and one are effectively equal. What happens if x changes by 3 units? Repeat the previous exercise and write your findings in terms of utility values (at time zero and time 1).
4. Assume that the utility function of a given agent is given by: U(X,Y) = logX + 3logY. And that this agent faces the following vector of prices: (10, 8) and that his income is 100. What should be the optimal consumption of X and Y for this specific agent? (Assume perfect divisibility of goods)

**Part IV: General competitive equilibrium**

1. Assume that we have two period of time (t0 and t1) and three states of nature in period t1.
   1. Assume that you know that the contingent claims’ prices are q1=0.8; q2=0.4 and q3=0.1. Assume that we have three securities (i=1, 2, 3) with the following state contingent payoffs:

Estimate the price of each security (P1, P2 and P3).

* 1. Now, starting from a. assume that you know the price of each of the security, build the state contingent payoff table and from there estimate the contingent claims’ prices (q1, q2 and q3) and verify that these values are exactly the same as the ones you were presented in part a.

1. Assume an exchange economy with two goods and two agents. What would happen with the vector of prices if we have an excess demand of good 1 (horizontal line) and an excess supply of good 2 (vertical line), in order to achieve the equilibrium.
2. Why Hypothesis 3 (convexity of preferences) is important to find the general competitive equilibrium.
3. How the intuition of general equilibrium in a timeless economy should be adapted to consider time and uncertainty? What is a contingent commodity?
4. Present an example of a contingent claim (also known as Arrow-Debreu Security).
5. Assume that we have two states of nature (up and down). Assume that there is a 50%-50% probability of ending in any of the states; assume that the payments for each of these states are 1.1 and 0.9, respectively. If the state contingent prices are 2 and 2.5, respectively,
   1. What should be the value of this security?
   2. How should the state contingent prices change if the probability of the Up-state increase to 80% (as opposed to 50%)?
   3. Explain the reason why even when an agent has equal probabilities of occurrence and equal expected losses and gains, agent’s preferences being convex implies that their marginal utilities are different (use figures to illustrate your point).
6. Suppose that each consumer has the Cobb-Douglas utility function ui(x1i, x2i)=x1iα x2i1-α. In addition the endowments are w1=(1,2) and w2=(2,1). What should be the vector of prices (p1\*, p2\*) in order to achieve equilibrium (supply=demand). [Note use an increasing transformation of the utility functions given by α ln x1i + (1-α) ln x2i].
7. Assume that you have two agents with the following utility functions: u1(x,y)=2ln(x)+ln(y) and u2(x,y)=ln(x)+3ln(y). The endowments are w1=(5,4) and w2=(2,6). What should be the vector of prices (px\*, py\*) in order to achieve equilibrium (supply=demand). Assume that px\*=2.5, what is py\* and what should be the optimal quantities of x and y for each agent?

**Part V: Time value of money, perpetuities and annuities**

1. As a winner of a given competition, you can choose one of the following prices:
   1. $100,000 now
   2. $180,000 at the end of 5 years
   3. $11,400 a year forever
   4. $19,000 for each of 10 years
   5. $6500 next year and increasing thereafter by 3% forever.

If the interest rate is 10%, which is the most valuable prize?

**Part VI: Choices in risky situations**

1. Given the following investment opportunities, with θ1 and θ2 representing two states of nature in Θ and assuming that the states probabilities Π1 and Π2 are equal to 0.6 and 0.4 respectively:

|  |  |  |  |
| --- | --- | --- | --- |
| **Investment No.** | **Initial investment** | **θ1** | **θ2** |
| 1 | -1000 | 1250 | 1750 |
| 2 | -1000 | 1300 | 1500 |
| 3 | -1000 | 1300 | 1800 |

* 1. Does any of this investments state-by-state dominate the others?
  2. Does any of this investments mean-variance dominate the others?
  3. Is there any contradiction between these two dominance measures? Which one is stronger?

1. Represent, using a tree diagram, the following lottery: ((x,y,p1),(x,(x,y,p3),p2),Π). Can this lottery be written as (x, y, Πp1 + (1-Π)p2 + (1-Π)(1-p2)p3). What is the probability associated to y?
2. Regarding the timing of uncertainty resolution, given the following tree diagram and assuming:

W(P1,  E(U)) = EU1/2,

U1(P1, P2(θ)) = (P1 + P2(θ))1/2

Does this agent prefer early or late resolution of the uncertainty? Prove it mathematically using the above functions.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| |  | | --- | |  | |  | π |  |  |  |
|  |  | 200 |  | 150 |  |
|  |  | 1-π |  |  |  |
|  |  | 200 |  | 75 |  |
|  |  |  | π |  |  |
|  |  |  |  | 150 |  |
|  |  | 200 |  |  |  |
|  |  |  | 1-π | 75 |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

1. An individual has a utility function U(x)=ln(x). He faces a risky decision q=(2000,5000,0.5) and a risk-free alternative s=3500. Is this a risk averse individual? What is his certain equivalent? Answer graphically and mathematically.
2. Replicate question 4 and define the individual as risk-neutral or risk lover. U(x)=ex. Also estimate individuals certain equivalent.
3. Replicate question 4 and define the individual as risk-neutral or risk lover. U(x)=5x. Also estimate individuals certain equivalent.
4. Using the concept of relative risk aversion, show that U(x)=ln(x) represents a more risk averse investor than U(x)=(x)1/2. Is this measure of risk aversion invariant to affine (linear) transformations? Present and example using a linear transformation of U(x)=ln(x).
5. You have the following lotteries (500, 1200, 0.3). If the investor is risk averse and has a CRRA utility function, with γ = 4 (coefficient of risk aversion). What is his certain equivalent? Is he a risk averse individual? Why? (answer in terms of risk premium).
6. Using the concept of relative risk aversion, show that U(x)=ln(x) represents a more risk averse investor than U(x)=(x)1/2. Is this measure of risk aversion invariant to affine (linear) transformations? Present and example using a linear transformation of U(x)=ln(x).
7. Based on the Absolute Risk Aversion (RA) and the odds result:

Estimate the following probabilities π(y,h), assuming that y=100 (wealth):

* 1. U(y)=(y)1/2, h=1.
  2. U(y)=(y)1/2, h=10.
  3. What is the “h” that makes π(y,h)=1?

1. Based on the Relative Risk Aversion (RR) and the odds result:

Estimate the following probabilities π(y,θ), assuming that y=100 (wealth):

* 1. U(y)=(y)1/2, θ =0.01.
  2. U(y)=(y)1/2, θ =0.10.
  3. What is the “θ” that makes π(y, θ)=1?

1. Compare your results from questions 10 and 11. Are they similar?
2. Repeat question 10 if U(y)=ln(y).
3. Repeat question 11 if U(y)=ln(y).
4. Compare your results from questions 10 and 13. Which individuals are more risk averse (the ones with U(y)=(y)1/2 or the ones with U(y)=ln(y)).
5. Based on the Relative Risk Aversion (RR) and CRRA utility function (with ) and the odds result:
   1. What is the coefficient of risk aversion (γ) that makes π(y, θ)=1, when y=100 and θ =0.50?
   2. What is the coefficient of risk aversion (γ) that makes π(y, θ)=1, when y=100 and θ =0.25?
   3. Which individual is more risk averse based on the coefficient of risk aversion?
   4. What is the coefficient of risk aversion of a risk neutral individual?

**Part VII: Modern Portfolio theory**

1. What is the difference between Capital, Asset and Security Allocation?
2. Is the standard deviation of a risky portfolio equal to the linear combination of the standard deviations of the securities included in the portfolio? When this is true?
3. When you can get a perfect hedging portfolio? Derive its formula.
4. Explain, using the CAL and any other material presented in class, what is the expected effect of raising the FED rates for emerging markets? Does this depend on the size of the increase? Does it make sense for local governments (emerging markets’ central banks) to increase their local interest rates? Why?
5. What is the minimum variance portfolio? Derive its formula. Explain the efficient vs the non-efficient portions of the Portfolio Opportunity Set (POS).
6. If you have a risk-free financial instrument, is the minimum variance portfolio part of the best CAL? If not, explain the way to obtain the efficient one.
7. Replicate, using excel, the capital and security allocation model. Be sure you understand how to arrive to each of the values presented in class (including the use of solver). Moreover, be sure that you understand how the model works (formulas and so on) and how you can implement them. Be sure that you understand the different between capital allocation and asset allocation.
8. Select 6 securities, 5 years of monthly data. Compute the monthly average returns, standard deviations and correlations. Assume a risk-free rate of 0.5% annual (transform to monthly). Using Excel create the optimal portfolio allocation for an individual with risk aversion coefficient of 3. Provide the risky portfolio weights and the weights between risk free and risky portfolio.
9. Explain the “Two Fund Separation Theorem”. Who is more likely to perform each of them (the technical and final decision about the portfolio allocation)?
10. In the Markovitz model, what is the impact of restricting the portfolio weights to be all non-negative (no short sales).
11. Why is the CAPM a general equilibrium model? How does the pricing mechanism work in order to achieve the equilibrium market portfolio?
12. Why the market portfolio should have a weight of 1 in:
13. Based on the equation presented in question 12, what should be the impact on the market risk premium of:
    1. An increase in market volatility?
    2. An increase in negative market sentiment that increases the average marker risk aversion.
14. What is the relationship/difference between the Capital Allocation Line (CAL) and the Capital Market Line (CML).
15. Why what matters in portfolio allocation is the marginal contribution of a given asset to the overall risk of the portfolio and not the individual risk? How is this idea incorporated in the CAPM?
16. Show:

Where rm represents the market returns.

1. Derive the expected return-beta relationship.
2. How can you use the security market line (SML) and how you can use it to determine whether (under the assumptions of the model) an asset is over- or under-valued.
3. Show that indeed the beta of the market equals 1. Use with n representing all available financial securities in the market.
4. Assume that you have 4 securities and that each enter your portfolio with the following weights: w1 = 0.2, w2 = 0.4, w3 = 0.1, w4 = 0.3. The corresponding betas are: β1 = 0.75, β1 = 0.90, β1 = 1.25, β1 = 1.50. What is the beta of your portfolio?
5. Using the data provided in question 20, determine the portfolio weights that allow you to get a portfolio beta of 1.
6. Using Excel, test of the Capital Asset Pricing Model (CAPM) using a simple ordinary least squares. Explain your results in terms of the alpha (intercept) and beta. For this, select any stock (yahoo finance is a good source for data), at least 2 years of daily data.

**Part VIII: Options**

1. What is the lower value for the price of an European put option with the following characteristics:

Stock price = US$ 12

Strike price = US$ 15

Duration = 1 month

Dividends = No

Rf = 6% annual

1. A 1-month European put on a stock that pays no dividends is sold in the market at US$ 2.5. The actual price of the stock is 47, the strike price is US$ 50 and rf is 6% annual. Is there any arbitrage opportunity? If there is one, how can you profit from it?
2. A 4-month European call on a stock that will pay dividends equal to US$ 0.5 at the end of the second month is sold in the market at US$ 2.8. The actual price of the stock is 28, the strike price is US$ 25 and rf is 8% annual. Is there any arbitrage opportunity? If there is one, how can you profit from it?
3. Compute the value of a 9-month European put on a stock with current price of US$ 25 and strike price of US$ 27, where the value of an European call on the same stock and with the same strike price is US$ 2.5 and the rf equals 10% annual.

If the market price of the put is US$ 2.35, identify the existence of an arbitrage opportunity and explain how you can profit from it.

1. Derive the price formula of the European put on a stock that pays no dividends. Use the portfolio replica as in the European call presented in class.
2. Using the binomial model compute the price of an American call on a stock that pays no dividends. The information that you have is the following:

Actual value of the stock = US$ 100

Strike price = US$ 95

Duration = 1 period

Rf = 10% per period

u = 1.25

d = 0.8

Do you expect any difference between this price and the one of a European call option on a stock that pays no dividends? Explain.

1. Re-do exercise six, but now assume that the duration is two periods.
2. Using the same data as in question six, what is the price of an American put? Can an American put worth more than a European one? Explain.
3. Re-do exercise eight, but now assume that the duration is two periods.
4. Assuming the following data, what should be the price of an European call? Actual value of the stock = US$ 60

Strike price = US$ 55

Duration = 2 periods

Rf = 6% per period

u1 = 1.1 u2 = 1.15

d1 = 0.9 d2 = 0.8

1. Assuming the following data, what should be the price of an European call if the stock will pay a dividend proportional to the actual price of the asset, established at 0.1, payable at t1?

Actual value of the stock = US$ 60

Strike price = US$ 55

Duration = 2 periods

Rf = 6% per period

u = 1.15

d = 0.8

1. Assuming the following data, what should be the price of an European put if the stock will pay a constant dividend of US$ 0.5 payable at t1?

Actual value of the stock = US$ 70

Strike price = US$ 72

Duration = 2 periods

Rf = 5% per period

u = 1.2

d = 0.9