ADMM-Based Coordination of Electric Vehicles in Constrained Distribution Networks Considering Fast Charging and Degradation

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***Abstract*— Acting as a key to future environmentally friendly transportation systems, electric vehicles (EVs) have attached importance to develop fast charging technologies to accomplish the requirement of vehicle users. However, fast charging behaviors would cause degradations in EVs' batteries, as well as negative effects like new demand peak and feeder overloads to the connected distribution network, especially when plugging in large scale EVs. Decentralized coordination is encouraged and our goal is to achieve an optimal strategy profile for EVs in a decentralized way considering both the need of fast charging and reducing degradations in batteries and the distribution network. In this article, we innovatively model the EV fast charging problem as an optimization coordination problem subject to the coupled feeder capacity constraints in the distribution network. The need of fast charging is expressed by the total charging time, and the relative tendency to fully charge within the desired time period. We introduce a ℓ_0-norm of the charging strategy which is non-convex to represent the total charging time, and apply the ℓ_1-norm minimization to approximate the sparse solution of ℓ_0-norm minimization. The shorter the charging horizon is the stronger willing of fast charging the user has. The objective of the optimization problem tradeoffs the EVs' battery degradation cost, the load regulation in the distribution network, the satisfaction of charging and the total charging time, which is non-separable among individual charging behaviors. Even though alternating direction method of multipliers (ADMM) has been widely applied in distributed optimization with separable objective and coupled constraints, its decentralized scheme cannot be applied directly to the underlying non-separable EV charging coordination problem. Hence, a hierarchical algorithm based on ADMM is proposed such that the convergence to the optimal strategies is guaranteed under certain step-size parameter. Furthermore, a receding horizon based algorithm is proposed considering the forecast errors on the base demand and the EV arrival distribution. The results are demonstrated via some simulation results.**

***Index Terms*— Electric vehicles, constrained distribution network, fast charging, non-separable objective, alternating direction method of multipliers, gradient projection, hierarchical algorithm, convergence.**

Manuscript received December 23, 2019; revised June 21, 2020; accepted July 30, 2020. Date of publication August 21, 2020; date of current version December 24, 2020. This work was supported in part by the National Natural Science Foundation (NNSF) of China under Grant 61873303 and in part by the Beijing Institute of Technology Research Fund Program for Young Scholars. The Associate Editor for this article was Z. Hu. *(Corresponding author: Zhongjing Ma.)*

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Digital Object Identifier 10.1109/TITS.2020.3015122

I. Introduction

ELECTRIC vehicles (EVs) have emerged to lead to future green and intelligent transportation systems in the field of both things and energy [1]–[3]. All EVs mentioned in this article refer to the battery electric vehicles. With the development of fast charging and extreme fast charging (XFC) technologies, the number of EVs is increasing rapidly, but a series of new challenges are also introduced not only on the battery life but also on the stability of distribution networks. Due to the high power of EV fast charging and the unbalanced distribution of charging time, the feeder overloads of the distribution network can easily happen, especially when EVs are charged intensively during a certain period of peak load [4]–[6]. Consequently, compared with simply increasing corresponding facilities to expand the capacity of network feeders, coordinating EV fast charging behaviors provides a potentially more economical and effective solution, and how to efficiently and effectively coordinate charging is significant from different view points consisting of valley filling, frequency regulation and load balancing [7]–[10].

As stated in the literature [11], fast charging requires specific facilities and basically takes place in commercial charging stations to finish charging within a period from 15 minutes to 2 hours. Even though the driving range has been improved a lot while cutting down the charging time, EVs still need to be charged frequently and extreme fast charging is not always the need, e.g. when the vehicle is restricted to move in big cities. Moreover, under general scenarios, EVs rarely need to be charged immediately. Instead, EV users determine the desired charging time and fast charging stations in advance according to the using habit and the battery's state of charge (SOC), and then drive there according to the real-time traffic information which can be estimated by applications such as Baidu Map. Sometimes, when the user is not in a hurry, for protecting the batteries' life and decreasing the energy payment, they may choose a relatively slow charging. If in a hurry, they would like to charge their vehicles as fast as possible, imposing the desire of fast charging. Usually, fast charging means the short time period and the high charging rate.

In this work, we express the desire of fast charging of users by individual total charging time. The shorter the charging horizon is the stronger willing of fast charging the user has. Based on the desired charging horizon, we are motivated to establish an EV coordination problem to determine the proper charging time and rate, i.e., the charging strategy to satisfy the need of fast charging, and mitigate the degradation of batteries and avoid the overload of the distribution network. We apply a quadratic form of the degradation cost model with respect to the charging power derived in [8]. Consequently, the EV coordination problem minimizes the tradeoff among the EV users' satisfaction, the load regulation, the battery degradation cost and the total charging time, which models a non-separable objective function, i.e., it cannot be written as a summation form of individual objectives. Moreover, capacity limits of feeder lines draw coupling spatial constraints of charging strategies among individual EVs, and inter-temporal constraints are also addressed such that the total charging power of each EV has an upper limit associated with initial SOC and battery size. All these bring challenges to solve the optimization problem in a decentralized way.

Researches in decentralized coordination of EV charging are abundant, see [10], [12]–[16] and the reference therein. In [12], it developed a decentralized regularized primal-dual projection scheme to solve the multiuser problems with non-separable objective and constraints, while this method might introduce certain errors to the optimal solution. These works in [10], [13]–[15] adopted certain deviation penalty costs to substitute the feeder overload constraints, but the feeder capacity might be violated by applying the Lagrangian multipliers with slack parameters which could result in suboptimal solutions. The gradient projection-based method in [16] could handle the charging control problem of EVs with hard constraints of the feeder capacity, but the implemented solution might not be the system optimal solution.

The alternating direction method of multipliers (ADMM), which is commonly used in many fields [17]–[19], could implement the optimal solution to the underlying coordination problem in a centralized way. The decentralized scheme has difficulties in solving the optimization problem with a non-separable objective function which includes a nonlinear term of the total charging power of EVs in this problem. Some variations on the ADMM method, e.g. [20], [21], have been explored to solve the problems with non-separable functions in decentralized ways. However, in these algorithms, the charging strategies of individuals are updated successively, which may be time-consuming for large-scale systems. The proposed ADMM method for the sharing problem in [19], [22] could be applied to handle the non-separable EV charging problem in parallel, but each individual private information i.e., the coupled local cost function with respect to the total charging power of EV populations is required to send to the central system operator for the update of the auxiliary variable.

In this article, we design a hierarchical algorithm based on ADMM to solve the underlying EV coordination problem such that each individual EV could simultaneously update its own charging strategy. More specifically, we first propose an ADMM method for the constrained problem where the charging strategies of EVs, auxiliary and dual variables are updated in centralized ways, which is usually impractical for large-scale systems. Then for handling the non-separable load regulation term in the ADMM method, we further propose a decentralized algorithm based on the gradient projection method to parallel update the coordination behavior of each individual involved at each iteration step in the ADMM method, which can not be solved by the decomposition methods in the literature [8], [23] due to the temporal coupled inequality constraints. Next, based on the latest charging coordination behaviors collected from all individuals, the auxiliary and dual variables are updated in the ADMM method. This process repeats until the charging strategy does not update any longer. The optimality and convergence of the designed algorithm are guaranteed in case the step-size parameter of the gradient projection lies in a certain region.

Furthermore, besides the proposed hierarchical algorithm based on ADMM in this work, many papers have also proposed other hierarchical algorithms based on game approaches [24]–[26], ADMM [27], heuristic methods [28], [29] and other approaches [30], [31], to solve the problem related to the EV charging coordination. Reference [24] developed a new tri-level game theoretical approach for energy management of EVs and EV charging stations. In [27], a distributed manner by applying ADMM method was used to deal with the hierarchical distributed EV charging scheduling developed as the exchange problem with coupling constraints. Reference [30] considered the market-based control and price-based control into the hierarchical EVs' management system. Reference [31] proposed a search-swapping algorithm in the hierarchical coordinated framework to achieve power dispatch with near-optimal solutions guaranteed. However, these works didn't consider the need of EV fast charging in the modeling process.

Only a few papers studied the coordination of EV fast charging. Reference [29] proposed a hierarchical navigation strategy based on dynamic traffic/temperature data to decrease the peak-load and user's time and energy cost where the upper layer was the fast charging time selection and the underlayer was the route selection to fast charging stations (FCS). Reference [25] proposed a real-time charging navigation of EVs to FCS based on the non-cooperative game. In [26], the aggregation of EVs and FCSs was modeled as a leader-followers game which was reformulated as a bi-level optimization problem to provide regulation reserves for power systems. However, many works such as [25], [30] didn't consider the EV temporal coupled inequality constraints and [26] didn't consider the network spatial coupled constraints. In [24], [27], [31] the objective functions were separable, which was different from our work. References [28], [29] didn't prove the optimality and convergence of the proposed methods theoretically. Consequently, these algorithms illustrated above are all not suitable to deal with our proposed fast charging problem.

In summary, our work can promote the practical application in the XFC technology for EV applications by the economical coordination and advanced guidance to EV fast charging behaviours. The contributions of this article are as follows:

1) We innovatively model the EV fast charging problem as a constrained optimization problem related to the economic scheduling, which not only considers the users' economic cost and the need of fast charging, but also the reduction

TABLE I
LIST OF KEY SYMBOLS

$\mathcal{N}$	EV populations
$\mathcal{T}$	EV charging intervals
d_t	Total base demand at t
u_{nt}	Charging rate of EV n at instant t
γ_n	Maximum charging rate of EV n
Γ_n	Total charging power required to be fully charged of EV n
$\boldsymbol{u}$	Charging coordination trajectory of EVs
$\mathcal{U}$	Set of admissible charging strategies of EVs
$\mathcal{L}$	Set of feeders, $\mathcal{L} = \{l \mid l = 1, ..., L\}$
$\mathcal{M}$	Set of nodes, $\mathcal{M} = \{m \mid m = 1, ..., M\}$
$\mathcal{N}_l$	Collection of EVs whose power are supplied via feeder line l
$\mathcal{N}_m$	Collection of EVs charged at node m
β_l	Power capacity of feeder line $l \in \mathcal{L}$
V_{mt}	Total charging power of EVs plugged in node m at t
U_{lt}	Total charging power of EVs supplied via feeder line l at t
D_{lt}	Total base demand supplied via feeder line l at t

of the damage on the distribution network from the EV overcharging.

2) The need of fast charging is expressed by the total charging time, and the relative tendency to fully charge within the desired time period. And we introduce a ℓ_0-norm of the charging strategy to represent the total charging time.
3) A novel hierarchical algorithm based on ADMM is designed to solve the non-separable optimization problem with temporally spatially coupled inequality constraints, and its optimality and convergence are proved theoretically, while other methods in the literature [10], [12]–[16] may result in suboptimal solutions.

The rest of the paper is structured as follows. In Section II, we propose the coordination problem of EVs under capacity constraints of feeder lines in distribution networks. In Section III, we propose an ADMM method to solve the underlying problem, and further design a hierarchical algorithm by applying the gradient projection method to support the decentralized control of EV charging. We then verify the convergence and optimality of the proposed algorithm. A receding horizon based algorithm is proposed considering the forecast errors in Section IV. Numerical examples are shown in Section V to demonstrate the results developed in the paper. Finally, conclusions and future research directions are given in Section VI. A summary of the notations used throughout the paper is provided in Table I.

II. EV Charging Control in Constrained Distribution Networks

In this section, we introduce the models of individual EVs and the distribution network, and further the EV coordination problems considering the tradeoff among fast charging, battery degradation and distribution network effects.

A. Model of EV Charging & Distribution Networks

Consider the coordination of a population of EVs, denoted by $\mathcal{N} \equiv \{1, \cdots, N\}$, over a multi-time period $\mathcal{T} \equiv \{1, \cdots, T\}$. Let $\mathcal{T}_n \in \mathcal{T}$ denote the desired charging period of EV $n \in \mathcal{N}$ which is short if the user is in an emergency. The state of charge (SOC) of EV n at time t is given by soc_{nt}, and the dynamics of the SOC can be represented by the simplified model

$$soc_{n,t+1} = soc_{nt} + \frac{u_{nt}}{E_n}, \quad \forall t \in \mathcal{T}, \tag{1}$$

where u_{nt} (with units of kW) denotes the charging rate at $t \in \mathcal{T}$, and E_n denotes the battery capacity of EV n. At each time slot, the charging rate of each EV remains a constant.

Denote by Γ_n the total charging power required to be fully charged of EV n. Due to the limited capacity of feeder lines and the waiting time of queuing, EVs may not charge the full power during the period $\mathcal{T}_n$. Hence, the charging strategy of EV n, $\boldsymbol{u}_n \triangleq (u_{nt}; t \in \mathcal{T})$, satisfies the following:

$$u_{nt} \begin{cases} \in [0, \gamma_n], & t \in \mathcal{T}_n \\ = 0, & t \in \mathcal{T} \setminus \mathcal{T}_n \end{cases}, \quad \text{and} \quad \sum_{t=1}^{T} u_{nt} \le \Gamma_n, \tag{2}$$

where γ_n is an upper bound of the charging rate.

Remark: In need of fast charging, EV users prefer to charge at the maximum charging rate γ_n to obtain the shortest time for full charging $\lceil \frac{\Gamma_n}{\gamma_n} \rceil$ where $\lceil x \rceil$ represents the minimal integer value larger than or equal to x. The individual charging period could be set as $|\mathcal{T}_n| \ge \lceil \frac{\Gamma_n}{\gamma_n} \rceil$. If $|\mathcal{T}_n|$ is less than the shortest time, the EV could not be fully charged.

Let $\boldsymbol{u} \triangleq (\boldsymbol{u}_n; n \in \mathcal{N})$ represent the charging strategies of the EV populations. The set of admissible charging strategies of EV n is denoted by $\mathcal{U}_n$, and the set of admissible charging strategies for all the EVs, denoted by $\mathcal{U}$, is

$$\mathcal{U} \triangleq \mathcal{U}_1 \times \cdots \times \mathcal{U}_N. \tag{3}$$

For the distribution network, denote by $\mathcal{L} \equiv \{1, \ldots, L\}$ and $\mathcal{M} \equiv \{1, \ldots, M\}$ respectively the set of feeders and nodes (excluding the root node). Let $\mathcal{N}_l$ with $l \in \mathcal{L}$ denote a collection of EVs each of which is supplied with the electricity via feeder line l, and $\mathcal{N}_m$ with $m \in \mathcal{M}$ denote a collection of EVs each of which is charged at node m. Then we could introduce a matrix $\mathbb{A} \equiv [a_{lm}]_{L\times M}$ to represent the topology of the distribution network, such that $a_{lm} = 1$ in case $\mathcal{N}_m \in \mathcal{N}_l$ and $a_{lm} = 0$ otherwise. More specifically, $a_{lm} = 1$ means that the electricity at node m is supplied by feeder line l. For example, considering the distribution network shown in Fig. 1, we have $\mathbb{A} = \begin{bmatrix} 1&1&1&1&1 \\ 0&1&0&1&1 \\ 0&0&1&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \end{bmatrix}$.

For simplicity, we consider $V_{mt}(\boldsymbol{u}) \equiv \sum_{n\in\mathcal{N}_m} u_{nt}$, and $U_{lt}(\boldsymbol{u}) \equiv \sum_{n\in\mathcal{N}_l} u_{nt}$ which represent respectively the aggregated charging power of EVs plugged in node m, and the total power supplied via feeder line l at time t under charging behavior $\boldsymbol{u}$. Define the matrices $\mathbb{U}(\boldsymbol{u}) \equiv [U_{lt}]_{L\times T}$ and $\mathbb{V}(\boldsymbol{u}) \equiv [V_{mt}]_{M\times T}$. Then we can verify that $\mathbb{U} = \mathbb{A}\mathbb{V}$.

Denote the base demand connected at Node m by $\boldsymbol{d}_m = (d_{mt}, t \in \mathcal{T})$ and the aggregated base demand fed via feeder l by $D_{lt} \equiv \sum_{m\in\mathcal{M}_l} d_{mt}$ where $\mathcal{M}_l$ represents a set of nodes whose power is distributed via feeder l. The total base demand in distribution networks is denoted by $\boldsymbol{d} = (d_t, t \in \mathcal{T})$.

Denote by β_l the power capacity of feeder line $l \in \mathcal{L}$, then the power capacities of all the feeders in the distribution network at charging periods can be represented as

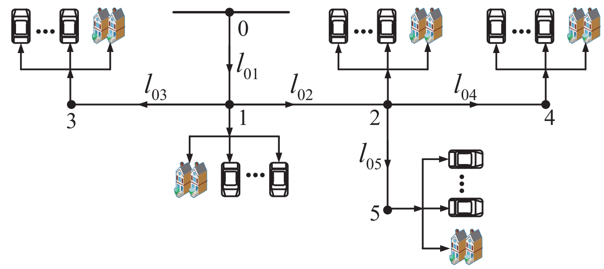


Fig. 1. Distribution network with 5 feeder lines.

$\mathbb{B} = [\boldsymbol{\beta}, \cdots, \boldsymbol{\beta}]_{L\times T}$ with $\boldsymbol{\beta} \equiv [\beta_l]_{L\times 1}$. For avoiding the localized overloading, the charging strategies of EVs should satisfy the feeder capacity constraints, such that

$$\mathcal{C} \triangleq \{\boldsymbol{u}, \text{ s.t. } U_{lt}(\boldsymbol{u}) + D_{lt} \le \beta_l, \ \forall (l, t) \in \mathcal{L} \times \mathcal{T}\}. \tag{4}$$

Furthermore, we introduce the normalized charging strategies denoted by $\boldsymbol{\xi}(\boldsymbol{u}) \equiv [\xi_{lt}]_{L\times T}$ with

$$\xi_{lt}(\boldsymbol{u}) \triangleq \frac{1}{\beta_l}(U_{lt}(\boldsymbol{u}) + D_{lt}), \quad \forall (l, t) \in \mathcal{L} \times \mathcal{T}. \tag{5}$$

Then we have the charging strategies $\boldsymbol{u}$ satisfying the feeder capacity constraints over the whole charging period $\mathcal{T}$, in case $\xi_{lt}(\boldsymbol{u}) \le 1$, for all $(l, t) \in \mathcal{L} \times \mathcal{T}$, which is adopted for demonstration in later simulation results.

B. EV Charging Coordination Problem

When the EV users have the need of fast charging, they prefer a high charging rate and try to finish charging within the desired time period as quickly as possible. The fast charging behaviors would accelerate the battery degradation [32]. Meanwhile, uncoordinated charging behaviors may create a new demand peak and bring the power loss in the distribution network. Thus it is necessary to regulate the EV load by the way of flatting the aggregate load which is widely adopted in the literature.

We give the centralized coordination problem of EV charging as below. The objective function is to minimize the total system cost in terms of the EVs and distribution network systems:

$$J_0(\boldsymbol{u}) \triangleq \sum_{n\in\mathcal{N}} \Bigg\{ \sum_{t\in\mathcal{T}} \Big\{ f_n(u_{nt}) + \omega_n h\big(d_t + \sum_{n\in\mathcal{N}} u_{nt}\big)\Big\} - \varpi_n g_n(\sum_{t\in\mathcal{T}} u_{nt}) + \eta_n \|\boldsymbol{u}_n\|_0 \Bigg\}, \tag{6}$$

where each term is specifically explained as below.

1) The first term $f_n(u_{nt})$: It represents the battery degradation cost of the nth EV. It is shown in Fig. 7 of [32] that the growth of battery resistance, hence the fade of battery energy capacity, is generally increasing and convex related to the charging rate. $f_n(u_{nt})$ is modeled as a quadratic form in [8], [33] derived by the charging current and voltage:

$$f_n(u_{nt}) = a_n u_{nt}^2 + b_n u_{nt} + c_n. \tag{7}$$

The model measures the energy capacity loss of the battery with parameters a_n, b_n and c_n.

Furthermore, more heat is generated at the battery side due to the fast charging behaviors with larger power, which may cause the high temperature environment. And the battery capacity fading can be accelerated exponentially as the temperature rises. Then an exponential form is derived in [34] to estimate the battery degradation cost:

$$f_n(u_{nt}) = A_n e^{-\frac{\theta_n}{\sigma_n u_{nt} + \varphi_n}}, \tag{8}$$

which is proved to be a convex function with positive coefficients A_n, θ_n, σ_n and φ_n.

2) The second term $h(\cdot)$: It denotes the load regulation which also represents the power loss in the distribution network with respect to the total power demand $d_t + \sum_{n\in\mathcal{N}} u_{nt}$, and $\omega_n \in (0, \omega_{max}]$ is the weighing parameter which reflects the relative tendency to reduce the adverse effects of fast charging behaviors on the power grid. It can be captured by minimizing the variance of the aggregate demand. Hence, a square form of $h(\cdot)$ has been widely adopted in the EV charging coordination problems [10], [23], [35], which is expressed as below:

$$h = \big(d_t + \sum_{n\in\mathcal{N}} u_{nt}\big)^2. \tag{9}$$

3) The third term $g_n(\cdot)$: It is the satisfaction function with respect to the total energy delivered over the charging periods. It can be seen from (2) that EV $n \in \mathcal{N}$ does not charge over time $t \in \mathcal{T} \setminus \mathcal{T}_n$, then we have $\sum_{t\in\mathcal{T}_n} u_{nt} = \sum_{t\in\mathcal{T}} u_{nt}$. EV users prefer to charge more power provided by the distribution network if possible until they reach their required energy Γ_n during their desired period $\mathcal{T}_n$. Then we propose the following form to represent the users' satisfaction:

$$g_n(\sum_{t\in\mathcal{T}} u_{nt}) = -(\sum_{t\in\mathcal{T}} u_{nt} - \Gamma_n)^2. \tag{10}$$

The weighing parameter $\varpi_n \in (0, \varpi_{max}]$ of EV n reflects the relative importance of fully charging over $\mathcal{T}_n$.

4) The last term $\|\boldsymbol{u}_n\|_0$: It is the cardinality of the charging strategy $\boldsymbol{u}_n$, i.e. the total number of non-zero elements in $\boldsymbol{u}_n$. The minimization of ℓ_0-norm represents the minimization of the total charging time. For the users in hurry, they prefer to finish a task as quickly as possible. It is unacceptable to wait too longer to complete the task. The weighing parameter $\eta_n > 0$ represents the relative tendency to fast charge within the desired time period $\mathcal{T}_n$.

However, it's challenging to handle the minimization of $\|\boldsymbol{u}_n\|_0$ even over the constraints given in (3) and (4) since $\|\boldsymbol{u}_n\|_0$ is a non-convex function [36]. To alleviate the computational difficulty, the convex function $\|\boldsymbol{u}_n\|_1$ is applied to approximate the cardinality minimization $\|\boldsymbol{u}_n\|_0$. Actually, the ℓ_1-norm minimization can exactly recover the sparse solution of ℓ_0-norm minimization, which has been widely adopted in many situations [36], [37]. In the field of power systems, [38] employed the ℓ_1-norm to handle the security-constrained optimal power flow problems. The authors in [39] adopted the sparse load shifting strategy to coordinate the smart appliances for the customers' satisfaction in the demand side management.

Thus we remodel the problem in (6) as the following charging control problem by applying the ℓ_1-norm convex approximation:

$$J(\boldsymbol{u}) \triangleq \sum_{n\in\mathcal{N}}\Big\{\sum_{t\in\mathcal{T}}\Big\{f_n(u_{nt})+\omega_n h\big(d_t+\sum_{n\in\mathcal{N}}u_{nt}\big)\Big\} - \varpi_n g_n(\sum_{t\in\mathcal{T}}u_{nt})+\eta_n\|\boldsymbol{u}_n\|_1\Big\}. \quad (11)$$

Remark: For those users who want to charge more energy as quickly as possible, they may choose to shorten desired charging duration $\mathcal{T}_n$ and at the same time to increase the value of weighing parameter ϖ_n and η_n. In case the feeder lines are not overloaded, the maximum charging rate is the upper bound Γ_n.

The following assumptions will apply throughout the paper:

(A1) $h(\cdot)$ is increasing, strictly convex and differentiable, and $h''(x)\le\varphi$, for all x.

(A2) $f_n(\cdot)$ is increasing, strictly convex and differentiable, and $f_n''(x)\le\phi$, for all $n\in\mathcal{N}$ and all x.

(A3) $g_n(\cdot)$ is non-decreasing, concave and differentiable, and $g_n''(x)\ge\psi$, for all $n\in\mathcal{N}$ and all x. ■

Remark: The above assumptions establish sufficient conditions for the convergence of the decentralized algorithm. The similar assumptions are widely adopted in the literature [16], [40].

Centralized EV fast charging coordination is formulated as the following optimization problem:

Problem 1:

$$\min_{\mathbf{u}\in\mathcal{U}\times\mathcal{C}} J(\boldsymbol{u}). \quad (12)$$

The objective is to implement a socially optimal charging strategies for all EVs, denoted by $\boldsymbol{u}^*$, with local constraints $\mathcal{U}$ and distribution network constraints $\mathcal{C}$ satisfied. ■

III. Decentralized Coordination Algorithm Based on ADMM

In this section, we propose a hierarchical charging coordination method to solve Problem 1. More specifically, an ADMM method is introduced in Section III-A to deal with the underlying problem, but the charging behaviors of EVs are coordinated in a centralized way with respect to the auxiliary and dual variables. Thus in Section III-B, we further design a decentralized iterative algorithm based on the gradient projection method to implement charging strategies of EVs related to each iteration step of the ADMM method. Then the latest charging strategies of all EVs are used to re-update the auxiliary and dual variables in the ADMM method. Consequently, this forms a hierarchical structure.

We also verify that the system can converge to the optimal solution by applying the hierarchical method, in case the step-size parameter satisfies certain conditions. The convergence rate of the decentralized algorithm is also analyzed and the developed results are given in later theorems.

A. Charging Coordination via ADMM Method

To solve the underlying problem via the ADMM method, we first introduce an indicator function of $\mathcal{C}$, that is,

$$I_{\mathcal{C}}(\boldsymbol{u}) = \begin{cases} 0, & \text{if } \boldsymbol{u}\in\mathcal{C},\\ +\infty, & \text{otherwise.}\end{cases}$$

Problem 1 is equivalently reformulated as follows by introducing an auxiliary variable $\boldsymbol{z}=\boldsymbol{u}$:

$$\begin{aligned}\min_{\boldsymbol{u}}\ & \sum_{n\in\mathcal{N}}\Big\{\sum_{t\in\mathcal{T}}\Big\{f_n(u_{nt})+\omega_n h\big(d_t+\sum_{n\in\mathcal{N}}u_{nt}\big)\Big\} - \varpi_n g_n(\sum_{t\in\mathcal{T}}u_{nt})+\eta_n\|\boldsymbol{u}_n\|_1\Big\}+I_{\mathcal{C}}(\boldsymbol{z})\\ \text{s.t.}\ & \boldsymbol{u}_n\in\mathcal{U}_n,\quad \forall n\in\mathcal{N}\\ & \boldsymbol{z}=\boldsymbol{u}.\end{aligned} \quad (13)$$

Give the following definitions:

$$\begin{aligned} f(\boldsymbol{u}) &\triangleq \sum_{n\in\mathcal{N}}\Big\{\sum_{t\in\mathcal{T}}\big\{f_n(u_{nt})+\omega_n h\big(d_t+\sum_{n\in\mathcal{N}}u_{nt}\big)\big\} - \varpi_n g_n(\sum_{t\in\mathcal{T}}u_{nt})+\eta_n\|\boldsymbol{u}_n\|_1\Big\},\\ g(\boldsymbol{z}) &\triangleq I_{\mathcal{C}}(\boldsymbol{z}).\end{aligned}$$

The optimal value of (13) could be noted by p^*. With regard to the problem, the Lagrangian is as below

$$L_0(\boldsymbol{u},\boldsymbol{z},\boldsymbol{y}) = f(\boldsymbol{u})+g(\boldsymbol{z})+\boldsymbol{y}^\top(\boldsymbol{u}-\boldsymbol{z}), \quad (14)$$

where $\boldsymbol{y}$ denotes the dual variable which is the Lagrange multiplier, and $\boldsymbol{y}^\top$ represents the transpose of vector $\boldsymbol{y}$. As in the method of multipliers, we have the augmented Lagrangian

$$L_\rho(\boldsymbol{u},\boldsymbol{z},\boldsymbol{y}) = f(\boldsymbol{u})+g(\boldsymbol{z})+\boldsymbol{y}^\top(\boldsymbol{u}-\boldsymbol{z})+\frac{\rho}{2}\|\boldsymbol{u}-\boldsymbol{z}\|_2^2$$

where $\rho>0$ is a penalty factor.

ADMM consists of the iterations [19]:

$$\boldsymbol{u}^{(k+1)} := \arg\min_{\boldsymbol{u}\in\mathcal{U}} L_\rho(\boldsymbol{u},\boldsymbol{z}^{(k)},\boldsymbol{y}^{(k)}) \quad (15)$$

$$\boldsymbol{z}^{(k+1)} := \arg\min_{\boldsymbol{z}\in\mathbb{R}} L_\rho(\boldsymbol{u}^{(k+1)},\boldsymbol{z},\boldsymbol{y}^{(k)}) \quad (16)$$

$$\boldsymbol{y}^{(k+1)} := \boldsymbol{y}^{(k)}+\rho(\boldsymbol{u}^{(k+1)}-\boldsymbol{z}^{(k+1)}), \quad (17)$$

where the superscript k is the iteration counter.

We refer to $\boldsymbol{r}^{(k+1)}=\boldsymbol{u}^{(k+1)}-\boldsymbol{z}^{(k+1)}$ as the primal residual and to $\boldsymbol{s}^{(k+1)}=-\rho(\boldsymbol{z}^{(k+1)}-\boldsymbol{z}^{(k)})$ as the dual residual at iteration $k+1$. A reasonable termination criterion is that the primal and dual residuals must be small, i.e.,

$$\|\boldsymbol{r}^{(k)}\|_2\le\epsilon^{pri} \text{ and } \|\boldsymbol{s}^{(k)}\|_2\le\epsilon^{dual}, \quad (18)$$

where $\epsilon^{pri}>0$ and $\epsilon^{dual}>0$ are feasibility tolerances.

Theorem 2: By applying the ADMM method in (15)-(17), the system converges to the optimal value p^* as iteration step $k\to\infty$.

Proof: $g(\boldsymbol{z})$ is closed, proper, and convex since the sets of constraints in the optimization problem are all closed nonempty convex sets [18]. Furthermore, by Assumptions (A1)-(A3), we can know that $f(\boldsymbol{u})$ is also closed, proper, and convex. Therefore, we have the strong duality holds for the original optimization problem (13). Then the optimal solution, denoted by $(\boldsymbol{u}^*, \boldsymbol{z}^*, \boldsymbol{y}^*)$, is the saddle point for the Lagrangian $L_0(\boldsymbol{u}, \boldsymbol{z}, \boldsymbol{y})$ in (14). Consequently, based on [19] we can get the objective convergence, $f(\boldsymbol{u}) + g(\boldsymbol{z}) \to p^*$ by the ADMM iterations in (15)-(17). ■

It is clearly observed from (15) that the charging strategies $\boldsymbol{u}^{(k+1)}$ of EVs are obtained in a centralized way since the objective function is non-separable. More specifically, $h(d_t + \sum_{n\in\mathcal{N}} u_{nt})$ with respect to the summation of base demand and the accumulated EVs charging power, can not be written as a summation form under Assumption (A1), i.e., $h(\boldsymbol{u}_n; n \in \mathcal{N}) \neq \sum_{n\in\mathcal{N}} h_n(\boldsymbol{u}_n)$. The charging behaviors among EV individuals are coupled in the objective function. Then each of the individual EVs can't update its own optimal strategy respectively.

In the centralized methods, the system operator needs to collect all the parameters featuring EV's characteristics, such as the desired charging intervals, initial and final SOC and the valuation functions and local constraints of EV populations. The complete information transmitted has huge dimensionality, and may create heavy communication signals. With the increasing of EV number, the centralized operation will bring high computational complexity since the dimensions of the optimization problem increase. Furthermore, the EV users may not be willing to share their private information with others.

Alternatively, we will novelly propose an iterative method to implement $\boldsymbol{u}^{(k+1)}$, where each individual has its own objective function. By the repeated update of individual charging strategy with respect to the previous strategies of EV populations, we can get the global solution $\boldsymbol{u}^{(k+1)}(\boldsymbol{u}_n; n \in \mathcal{N})$ in a decentralized way.

Remark: The proposed optimization problem can also be solved in a decentralized way by classifying the coupled load regulation term $\omega_n h\big(d_t + \sum_{n\in\mathcal{N}} u_{nt}\big)$ into the update part of the auxiliary variable z. However, if we apply the handling technique, each individual private information $\omega_n h(\cdot)$ is required to send to the central system operator for the update of variable z. It is usually impractical due to the autonomy and privacy of individual EVs.

B. Implementation of Optimal Charging Behavior in (15) With a Decentralized Method

In this section, we design a decentralized iteration procedure given in Algorithm 1 to implement $\boldsymbol{u}^{(k+1)}$ by applying a gradient projection method.

For convenience, we introduce a vector $\boldsymbol{v} = (1/\rho)\boldsymbol{y}$ known as the scaled dual variable. Based on (15), we define a function denoted by $H(\boldsymbol{u}; \boldsymbol{z}^{(k)}, \boldsymbol{v}^{(k)})$ subject to $\boldsymbol{u}$ with respect to $(\boldsymbol{z}^{(k)}, \boldsymbol{v}^{(k)})$, such that

$$\begin{aligned} H(\boldsymbol{u}) &\equiv H(\boldsymbol{u}; \boldsymbol{z}^{(k)}, \boldsymbol{v}^{(k)}) \\ &\triangleq \sum_{n\in\mathcal{N}} \Big\{ \sum_{t\in\mathcal{T}} \Big\{ f_n(u_{nt}) + \omega_n h\big(d_t + \sum_{n\in\mathcal{N}} u_{nt}\big) \Big\} \\ &\quad - \varpi_n g_n\big(\sum_{t\in\mathcal{T}} u_{nt}\big) + \eta_n \|\boldsymbol{u}_n\|_1 \Big\} + \frac{\rho}{2} \|\boldsymbol{u} - \boldsymbol{z}^{(k)} + \boldsymbol{v}^{(k)}\|_2^2. \end{aligned} \quad (19)$$

Then (15) can be written as below:

$$\boldsymbol{u}^{(k+1)} := \arg\min_{\boldsymbol{u}\in\mathcal{U}} H(\boldsymbol{u}). \quad (20)$$

Since we have $u_{nt} \geq 0$ for all $n \in \mathcal{N}, t \in \mathcal{T}$ in the local constraints $\mathcal{U}$, $\|\boldsymbol{u}_n\|_1$ can be viewed as the sum of all u_{nt}. Thus the optimization problem (20) is differentiable convex.

Gradient projection method [41] is commonly used for a variety of constrained optimization problems due to the computational superiority and fast convergence rates in conjunction with other methods. Hence, we propose an iterative algorithm based on the gradient projection method to obtain the optimal value $\boldsymbol{u}^{(k+1)}$ in (20). We first introduce some terms below.

We define the updated strategy of EVs denoted by $\widehat{\boldsymbol{u}}(\boldsymbol{u}) \equiv \big(\widehat{\boldsymbol{u}}_n(\boldsymbol{u}), n \in \mathcal{N}\big)$ with respect to a given charging strategy $\boldsymbol{u}$ by adopting the gradient projection method such that

$$\widehat{\boldsymbol{u}}(\boldsymbol{u}) = [\boldsymbol{u} - \alpha \nabla H(\boldsymbol{u})]^+_{\mathcal{U}}, \quad (21)$$

where $\nabla H(\boldsymbol{u})$ is the gradient of the function $H(\boldsymbol{u})$ in (19), $\alpha > 0$ is the step size, and $[\boldsymbol{x}]^+_{\mathcal{U}}$ represents the projection of $\boldsymbol{x}$ on $\mathcal{U}$.

It's still a centralized form in (21) to solve the optimization problem (20). Nevertheless, due to the decoupling property of the admissible set defined in (3), say $\mathcal{U} \triangleq \mathcal{U}_1 \times \cdots \times \mathcal{U}_N$, it can be proven that the updated strategy in (21) can be implemented by the following decentralized form:

$$\widehat{\boldsymbol{u}}_n(\boldsymbol{u}) = [\boldsymbol{u}_n - \alpha[\nabla H(\boldsymbol{u})]_n]^+_{\mathcal{U}_n}, \quad \forall n \in \mathcal{N}, \quad (22)$$

where $[\nabla H(\boldsymbol{u})]_n$ is the n-th row of $\nabla H(\boldsymbol{u})$.

The iteration procedure for decentralized operation of (20) is given by **Algorithm 1**. The superscript j denotes the iteration number. The convergence and optimality of Algorithm 1 will be analyzed in Theorem 3 below.

Consequently, we can get the optimal solution $\boldsymbol{u}^{(k+1)}$ in parallel by applying Algorithm 1. And all the individual EVs could update their charging strategies locally and simultaneously such that the centralized computation and communication burdens can be largely reduced.

Remark: In the EV decentralized coordination, users care only about their individual benefit and don't want to share the private information. To some extent, the load regulation term can be viewed as the incentive that the system gives to users since the behaviours of individuals can affect the interests of the system, which is illustrated in [23], [33]. Consequently, in the individual objective function, considering the incentive, battery degradation and charging demand and time, EV users will adjust their charging strategies in a way that benefits themselves. Meanwhile, the system can also be benefited.

Theorem 3 (Convergence and optimality of Algorithm 1): Consider Assumptions (A1)-(A3) and any initial charging strategy $\boldsymbol{u}^{(0)}$, Algorithm 1 can converge to the optimal solution $\boldsymbol{u}^\dagger = \boldsymbol{u}^{(k+1)}$ given in (20) in case $0 < \alpha < 2\big(\phi + \rho + N\varphi\omega_{max} - T\psi\varpi_{max}\big)^{-1}$.

Furthermore, if we attach another condition $\rho > N\varphi\omega_{max} - T\psi\varpi_{max}$, the convergence rate of Algorithm 1 is at least as fast as the geometric series of ratio $\kappa \equiv S_{max} + \alpha(N\varphi\omega_{max} - T\psi\varpi_{max})$ with

$$S_{max} = max\big\{|1 - \alpha\rho|, |1 - \alpha(\phi + \rho)|\big\}.$$

Algorithm 1 Implementation of $\boldsymbol{u}^{(k+1)}$ in (20) With a Decentralized Method

Require:
Set the initial strategy of EVs $\boldsymbol{u}^{(0)}$;
Set the step size $\alpha > 0$ and feasibility tolerance ϵ;
Set $j = 0$, $\zeta > \epsilon$;
Ensure:
Charging strategy of EVs;
while $\zeta > \epsilon$ **do**
  Update $\boldsymbol{u}_n^{(j+1)}(\boldsymbol{u}^{(j)}) := \widehat{\boldsymbol{u}}_n(\boldsymbol{u}^{(j)})$ by (22), $\forall n \in \mathcal{N}$;
  Update $\zeta = \|\boldsymbol{u}^{(j+1)} - \boldsymbol{u}^{(j)}\|$;
  Update $j = j + 1$;
end while

Then it can be derived that, Algorithm 1 converges for any $\epsilon > 0$, such that $\|\boldsymbol{u}^{(j+1)} - \boldsymbol{u}^{(j)}\| \leq \epsilon$, in $j^{\dagger}(\epsilon)$ iterations, with

$$j^{\dagger}(\epsilon) = \left\lceil \frac{1}{2\ln(\kappa)}\Big(2\ln(\epsilon) - \ln(T) - \ln(\sum_{n\in\mathcal{N}} \gamma_n^2)\Big) \right\rceil. \quad (23)$$

Proof: Please refer to Appendix. ∎

Remark: In practice, the iteration steps for convergence to the desired tolerance ϵ may be far less than the upper bound established in (23). The further demonstration is shown in simulation results in Section V.

In summary, based on the above analysis, the proposed decentralized method based on ADMM for Problem 1 can be viewed as a hierarchical iteration method in Algorithm 2. The charging strategies $\boldsymbol{u}^{(k+1)}$ in (15) can be implemented by Algorithm 1, in which the individual EVs can update their own charging strategies in parallel. And the auxiliary variable $\boldsymbol{z}$ is updated by solving a convex optimization problem in (24) when keeping $\boldsymbol{u}^{(k+1)}$ and $\boldsymbol{v}^{(k)}$ fixed. It is a gradient descent of the augmented Lagrangian multiplier in (25).

As the iterations in Algorithm 2 proceed, the primal and dual residuals converge to zero, and we can get the system optimal solution illustrated in Corollary 4.

Corollary 4: By applying Algorithm 2, the system converges to the optimal solution as iteration step $k \to \infty$.

Proof: Under Theorem 3, Algorithm 1 converges to $\boldsymbol{u}^{(k+1)}$. By this together with Theorem 2, we can get the optimal solution of Problem 1. ∎

Algorithm 2 Implementation of Charging Coordination via the Hierarchical Method

Require:
Set the initial strategy state $\boldsymbol{u}^0, \boldsymbol{z}^0, \boldsymbol{v}^0$;
Set the feasibility tolerances $\epsilon^{pri}, \epsilon^{dual}$;
Set $k = 0$, $r > \epsilon^{pri}$ and $s > \epsilon^{dual}$;
Ensure:
Charging strategy of EVs;
while $r > \epsilon^{pri}$ or $s > \epsilon^{dual}$ **do**
  Update $\boldsymbol{u}_n^{(k+1)}(\boldsymbol{z}^{(k)}, \boldsymbol{v}^{(k)})$ by **Algorithm 1**, $\forall n \in \mathcal{N}$;
  Determine the auxiliary variable by (16)

$$\boldsymbol{z}^{(k+1)} := \arg\min_{\boldsymbol{z}\in\mathbb{R}} \Big(I_{\mathcal{C}}(\boldsymbol{z}) + \frac{\rho}{2}\|\boldsymbol{u}^{(k+1)} - \boldsymbol{z} + \boldsymbol{v}^{(k)}\|_2^2\Big); \quad (24)$$

  Determine the dual variable by (17)

$$\boldsymbol{v}^{(k+1)} := \boldsymbol{v}^{(k)} + (\boldsymbol{u}^{(k+1)} - \boldsymbol{z}^{(k+1)}); \quad (25)$$

  Update $r = ||\boldsymbol{u}^{(k+1)} - \boldsymbol{z}^{(k+1)}||_2$;
  Update $s = || - \rho(\boldsymbol{z}^{(k+1)} - \boldsymbol{z}^{(k)})||_2$;
  Update $k := k + 1$;
end while

IV. Receding Horizon Implementation

In the fast charging model illustrated above, the desired charging intervals and charging station are chose in advance by each individual based on the EV users' using habit and the battery's SOC. By applying Algorithm 2, based on the users' choice the more efficient EV charging periods are obtained considering the capacities of feeder lines, users' time cost and economic cost. In the static optimization problem, we assume that the base demand and the desired charging intervals are known with certainty prior to the operation periods. Consequently, Algorithm 2 could be operated off-line ahead of actual operation in a decentralized way.

However, in reality it's pretty challenging even impossible to accurately predict the base demand. And the desired charging intervals and charging station may change over time. Consequently, the implemented charging strategies based on the inaccurate and dynamic information may not be the practical optimal solution. Hence, for enhancing the performance of the proposed algorithms, we assume that the predicted information and users' choice will be updated as the operation time goes, and in this section we will apply the receding horizon approach based on Algorithm 2 to mitigate the effects of uncertainties.

In the receding horizon control, at any time $t \in \mathcal{T}$, the EV coordination problem is regarded as a finite horizon problem over a window of T instants, denoted by $\mathcal{T}_{|t} \triangleq \{t, \cdots, t+T-1\}$. Under this control mechanism, at t, the system predicts the base demand trajectory $\boldsymbol{d}_{|t} \triangleq \{d_{i|t}, i \in \mathcal{T}_{|t}\}$ over T window and collects the information of EV populations $\mathcal{N}_{|t}$ charging at all nodes. Then based on $\boldsymbol{d}_{|t}$ and $\mathcal{N}_{|t}$, each individual EV could obtain the optimal strategy denoted by $\boldsymbol{u}^*_{n,\cdot|t}(\boldsymbol{d}_{|t}, \mathcal{N}_{|t}) \triangleq \{\boldsymbol{u}^*_{ni|t}, i \in \mathcal{T}_{|t}\}$ by applying Algorithm 2 to solve Problem 1.

However, the receding horizon approach only adopt the control strategy $\boldsymbol{u}^*_{nt|t}$ for the current time t, since the future predicted information are subjected to change. After solving the optimization problem at the current time t, the predictive horizon window shifts to next time $t+1$, collects the forecasted information similarly, and solves the Problem 1 repeatedly for the whole time frame. This process continues until the time frame ends. Denote the charging coordination strategies considering forecast errors by $\boldsymbol{u}_n^{er} \triangleq \{u_{nt}^{er}, t \in \mathcal{T}\}$, which can be implemented by Algorithm 3 expressed below.

In general, the predictions are 15 minutes in advance in the EV coordination. The EV is assumed to arrive a charging node within 15 minutes when it wants to charge. The 15 minutes are not too long, and they are acceptable. EV users may choose a charging station close to them. Then based on the obtained charging intervals calculated by our proposed method, users

Algorithm 3 Receding Horizon Based Algorithm

Require:
Initialize $t = 1$;
Ensure:
Charging strategy $\boldsymbol{u}_n^{er}$ of EVs;
while true **do**
Update the predicted base demand $\boldsymbol{d}_{|t}$ and EV populations $\mathcal{N}_{|t}$ at t;
Solve Problem 1 by applying Algorithm 2 and obtain the optimal strategy $\boldsymbol{u}_{n,\cdot|t}^{*}(\boldsymbol{d}_{|t}, \mathcal{N}_{|t})$ for all $n \in \mathcal{N}_{|t}$;
Determine strategy $u_{nt}^{er} = u_{nt|t}^{*}$;
Update EVs' SOC by (1);
Update $t := t + 1$;
end while

only need to choose the proper route to the desired charging station according to the real-time traffic information which can be estimated by applications such as Baidu Map.

V. Numerical Results

In Section V-A, we demonstrate the performance of the proposed hierarchical method via a 5-feeder test system illustrated in Fig. 1 and compare it with the centralized method. In Section V-B, we focus on the effects brought by the heterogeneity in EV populations. We then consider a more complex system, a 12-feeder test system, in Section V-C for further demonstration of the hierarchical method. Section V-D illustrates the performance of the receding horizon based algorithm. All simulations are conducted in MATLAB R2018b on a desktop with 3.60-GHz Intel Core i9.

A. Case Study I

Consider the 5-feeder system given in Fig. 1 and suppose that the capacities of the feeder lines for EVs are set to be $\boldsymbol{\beta} = [9300, 4800, 1200, 880, 720]$ (kW). The time horizon $\mathcal{T}$ for the EV charging coordination is 12:00 to 18:00 on one day in this simulation, by considering the willing of users to charge a certain amount of power before getting off work. We assume that the EV population size is $N = 350$ and consider that the length of the charging periods is 15 minutes. Here simply suppose that $\mathcal{T}_n = \mathcal{T}$, and all EVs are Tesla Model S with the identical battery capacity size 60 kWh, common minimum and maximum SOC limits which are equal to 30% and 90%, respectively, and share a common initial SOC of 30% and uniform maximum charging rate 18 kWh.

Suppose that the battery degradation cost is as formulated in [33], such that $f_n(u_{nt}) = 0.004u_{nt}^2 + 0.075u_{nt} + 0.003$. The stopping criterions ϵ^{pri}, ϵ^{dual} and ϵ are all set as 10^{-3}, and the penalty factor is selected to be constant, i.e., $\rho = 1$. We adopt the models $h(\cdot)$ in (9) and $g_n(\cdot)$ in (10) to respectively represent the load regulation and the satisfaction level of users with weighing parameters $\omega_n = 0.04/N$, $\varpi_n = 1$ and $\eta_n = 0.1$ for all $n \in \mathcal{N}$. The base load and distribution network data are collected from [42]–[44].

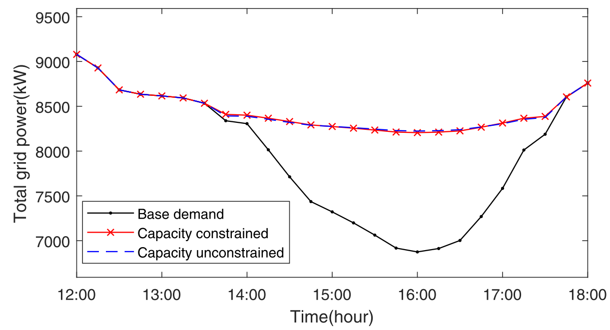


Fig. 2. Aggregated behaviors by the centralized method.

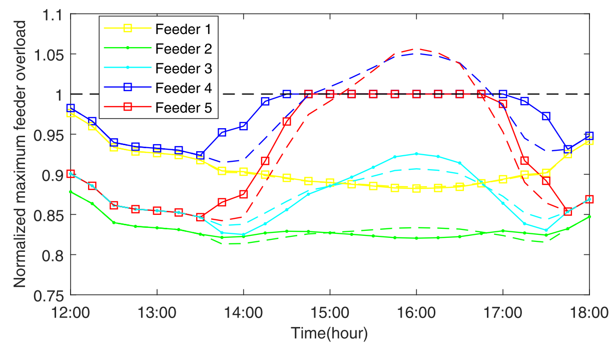


Fig. 3. The trajectory of $\boldsymbol{\xi}$ with or without considering the capacity constraints.

1) Centralized Control: We demonstrate the optimal coordination of EV charging by solving the optimization problem $\min_{\boldsymbol{u}\in\mathcal{U}} J(\boldsymbol{u})$ in the centralized method. Fig. 2 displays the converged update of the total demand, which is the summation of base demand and the accumulated EVs charging power. The dash lines represent the solution without capacity constraints $\mathcal{C}$ considered. However, under the strategy it can be seen from Fig. 3 that Feeder 4 and Feeder 5 represented by dash lines are overloaded. The solid lines in Fig. 2 represent the total demand which considers network constraints. Obviously, all capacity constraints are satisfied shown in Fig. 3. By contrast, we can observe that the charging power from 15:45 to 16:45 are reduced since the capacities of feeder lines are reached at these time, while charging power at other periods are increased.

2) Decentralized Control: We will apply the proposed Algorithm 2 to coordinate the charging behaviors of EV populations with the capacity constraints considered.

As stated in Theorem 3, the step size α should satisfy certain conditions for the system convergence, say $0 < \alpha < 0.039$ in this case. Then for demonstration we take $\alpha = 0.035$ which is consistent with the convergence condition.

We first initialize $(\boldsymbol{u}^0, \boldsymbol{z}^0, \boldsymbol{v}^0)$ both as 0. At the initial iteration step, say $k = 1$, we implement $\boldsymbol{u}_n^1$ by Algorithm 1 for all $n \in \mathcal{N}$, which converges in several iterations as illustrated in Fig. 4 (left). Then based on the collected charging strategies $\boldsymbol{u}^1 \equiv (\boldsymbol{u}_n^1; n \in \mathcal{N})$ and $\boldsymbol{v}^0$, we implement $\boldsymbol{z}^1$ by (24) in Algorithm 2. Similarly, $\boldsymbol{v}^1$ is updated by (25). The process is repeated until the termination criterions are satisfied, i.e., $r \le 10^{-3}$ and $s \le 10^{-3}$.

The optimal coordination results are shown in Fig. 5 with respect to the iteration step k. This converged profile

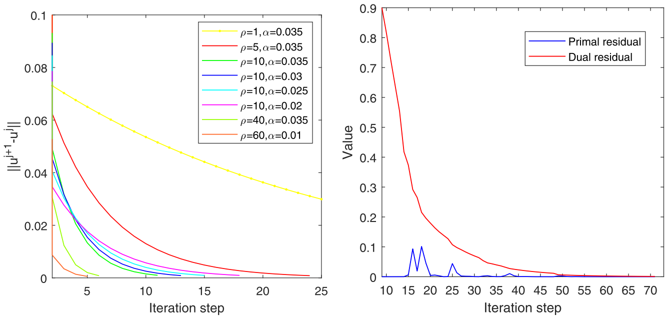


Fig. 4. (left) The convergence of Algorithm 1 under $k = 1$ with respect to different ρ and α. (right) The convergence of primal residual and dual residual of Algorithm 2.

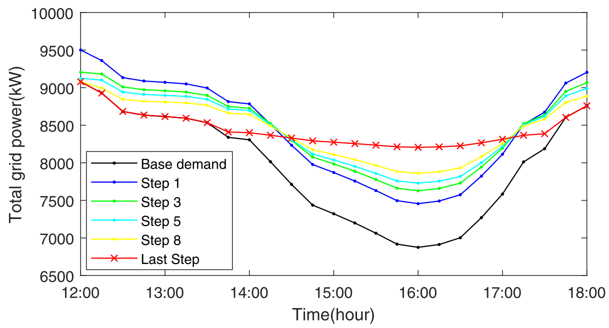


Fig. 5. Aggregated behaviors by applying Algorithm 2.

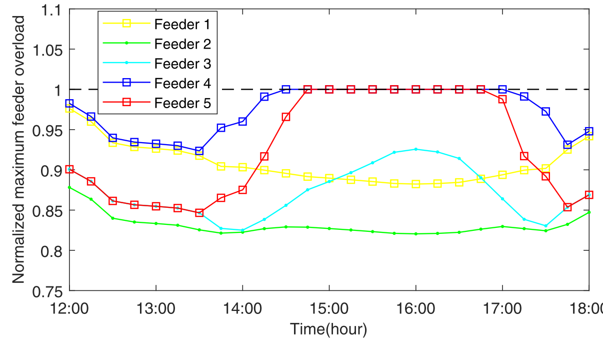


Fig. 6. The trajectory of $\boldsymbol{\xi}$ by applying Algorithm 2.

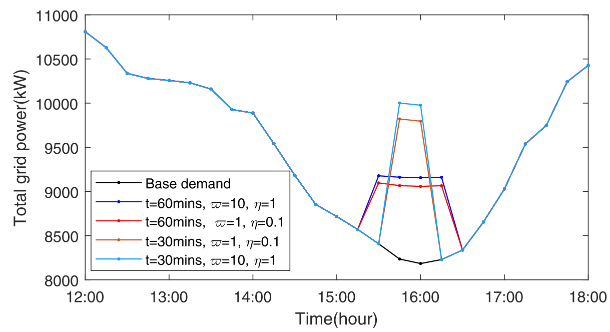


Fig. 7. Aggregated behaviors with respect to $\mathcal{T}_n$, ϖ_n and η_n.

essentially coincides with the one obtained by the centralized method. For demonstration, we only show step $k = 1, 3, 5, 8$ and the last step. From Fig. 6 we can clearly observe that all the capacity constraints are satisfied. The converged solution is valley-filling, and it illustrates the charging strategy obtained by applying Algorithm 2 could reduce the adverse impact on the power grid and the EV batteries. Fig. 4 (right) illustrates the optimal convergence behavior under Algorithm 2 in a few of iteration steps, which is consistent with Corollary 4. Furthermore, the average time of each outer iteration of decentralized Algorithm 2 is 1.1s and the total computation time is 78s in this case, which is much less than the total computation time 2057s of the centralized method, thus the centralized computation burdens can be largely reduced.

Fig. 4 (left) also displays the iteration steps of Algorithm 1 with respect to different values of penalty factor ρ and step size α under $k = 1$. As mentioned in Theorem 3, if $\rho > 50.08$ and $\alpha < 0.018$ in this case, it's going to converge faster. It implies that from (23) there exists an iteration step $j^{\dagger} = 124$ such that $\|\boldsymbol{u}^{(j+1)} - \boldsymbol{u}^{(j)}\|_1 \leq 10^{-3}$ with $\rho = 60$ and $\alpha = 0.01$. From Fig. 4 (left), we can observe that Algorithm 1 converges only in 5 iteration steps, which is less than the derived result 124 iteration steps. And the proper values of ρ and α can speed up the convergence of Algorithm 1. It's not necessarily the larger of these values the faster of the convergence. Moreover, a larger ρ may slow down the convergence of ADMM method, that is, the iteration steps k to converge will increase [19].

Furthermore, we consider the fast charging behaviours in a hurry. It means a shorted charging time and a higher charging rate by increasing the corresponding weighting parameters to charge more power. For demonstration, we assume the capacities of feeder lines are large enough not to overload. And there are 100 EVs to charge over the periods from 15:30 to 16:30 with $\varpi_n = 1, 10$ and $\eta_n = 0.1, 1$. Fig 7 shows the converged optimal strategies. To further speed up charging, we shorten the desired charging time slots to 30 minutes from 15:45 to 16:15. The charging behaviours are compared in Fig 7 under these different parameters. And Fig 8 displays the final SOC before leaving related to different weighting parameter ϖ_n. It can be seen that the lager ϖ_n is, the higher charging rate and the more total power have until EVs reach their maximum SOC. The EVs are almost fully charged when $\varpi_n = 10$.

When the desired charging period is very short such as 30 minutes and the user wants to charge his EV to a certain high level, the EV may choose to charge at the maximum charging rate γ_n. Thus in this case, the total charging time can not be reduced by increasing the weighting parameter ϖ_n and η_n due to the upper bound of charging rate. In general, EVs will experience fewer interruptions and less charging time slots via the sparse pattern compared to other demand-side management strategies in [45], [46], which is illustrated in [39].

B. Case Study II

For approaching realistic characteristics of EV populaions, we consider a heterogeneous case. For simplicity, we evenly divide the charging time from 12:00 to 18:00 into eight periods, the length of each of which is 45 minutes. We assume there are 200 EVs charging at the period from 15:45 to 16:30. And there are 100 EVs charging within each of the other periods. Also assume the initial SOC values of EVs that charge at each period approximately satisfy a Gaussian distribution [47], denoted by $N(\mu, \sigma^2)$ with $\mu = 0.5, \sigma = 0.1$.

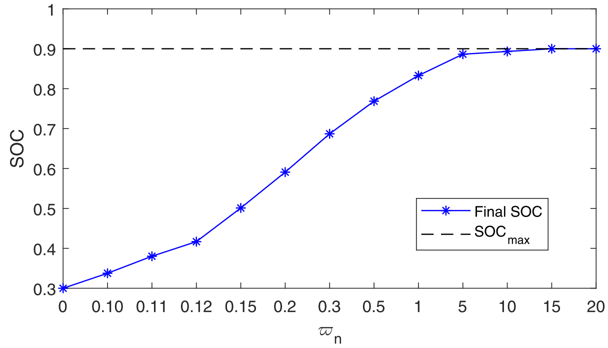


Fig. 8. Final SOC with respect to ϖ_n by charging from 15:45 to 16:15.

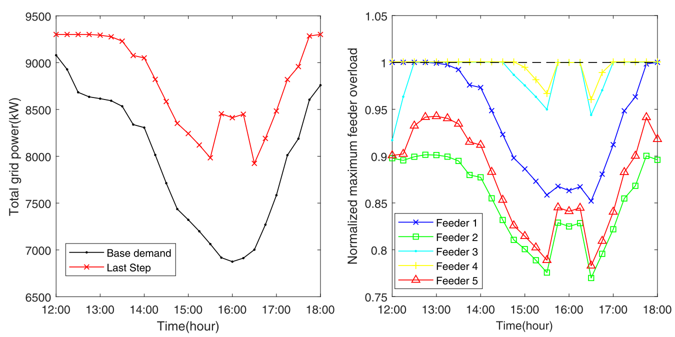


Fig. 9. Implemented coordination solution with heterogeneous EV populations by applying Algorithm 2.

TABLE II

CAPACITIES OF 12 FEEDER LINES

Feeder line	l_{01}	l_{02}	l_{03}	l_{04}	l_{05}	l_{06}
Capacity (kW)	20000	3000	4800	3200	820	9300
Feeder line	l_{07}	l_{08}	l_{09}	l_{10}	l_{11}	l_{12}
Capacity (kW)	3000	5200	3400	820	820	820

More specifically, Fig. 9 (left) illustrates the converged optimal fast charging behaviours with capacity constraints satisfied in Fig. 9 (right). During the limited periods from 15:45 to 16:30, the EVs are not fully charged since the system reaches the upper bound of the power.

C. Case Study III

In this subsection, we further study the charging problems in a distribution network with 12 feeder lines, as illustrated in Fig. 10, over the periods from 12:00 to 18:00. We suppose that there are 60 EVs connected to each of the feeders, and the capacities of feeder lines for EVs are shown in Table II.

Fig. 11 displays the updates of aggregated charging behaviors by using Algorithm 2. The EVs start charging in the valley and the system converges to the optimal solution within a few of iteration steps. From Fig. 12, we can know that the charging behaviors satisfy the capacity constraints of all feeder lines.

D. Case Study IV

In this subsection, we will illustrate the performance of the receding horizon based algorithm expressed in Algorithm 3.

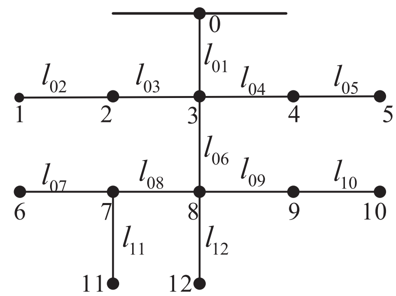


Fig. 10. Distribution network with 12 feeder lines.

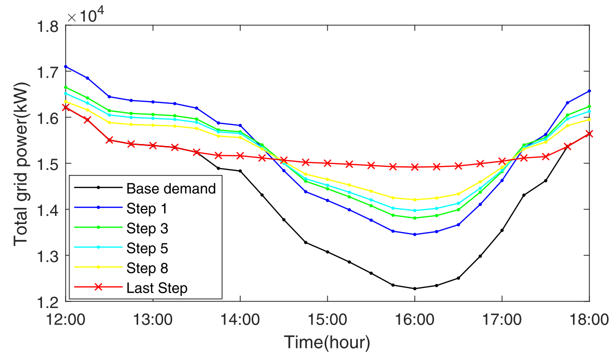


Fig. 11. Aggregated behaviors by applying Algorithm 2 for 12-feeder distribution networks.

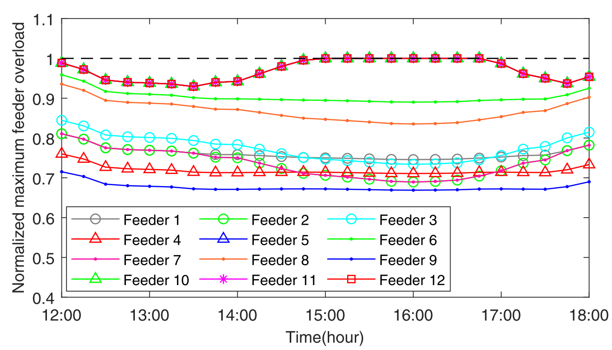
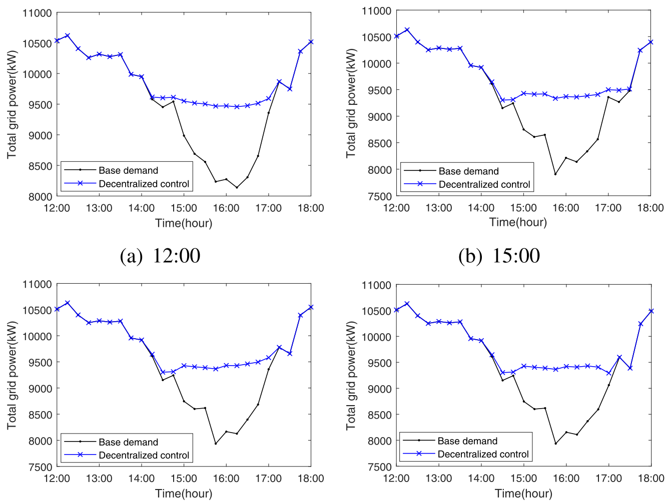


Fig. 12. The trajectory of ξ by applying Algorithm 2 for 12-feeder distribution networks.

The time horizon $\mathcal{T}$ for the EV coordination is 12:00 to 18:00, i.e., 6 hours. The size of the receding window T is 24 since each time instant is 15 minutes, then a total of 24 iterations ($t = 1, \cdots, 24$) are proceeded in Algorithm 3.

Fig. 13 displays the evolution of the receding process. For demonstration, we only show the iterations at time 12:00, 15:00, 16:00 and 17:00 among the whole 24 iterations. At the beginning of the operation periods, i.e. at 12:00, the information of base demand and the EV populations are predicted and collected, and by applying Algorithm 2, a valley-fill strategy which is called off-line strategy is obtained as given in Fig. 13(a). As time goes, the forecasted information are updated every 15 minutes, and the actual information can be obtained in the end of the operation period. The off-line strategy under the actual base demand is shown as the yellow line in Fig. 14, The red line in Fig. 14 represents the optimal



(a) 12:00 (b) 15:00

(c) 16:00 (d) 17:00

Fig. 13. EV coordination at different times considering both EVs and base demand forecast errors.

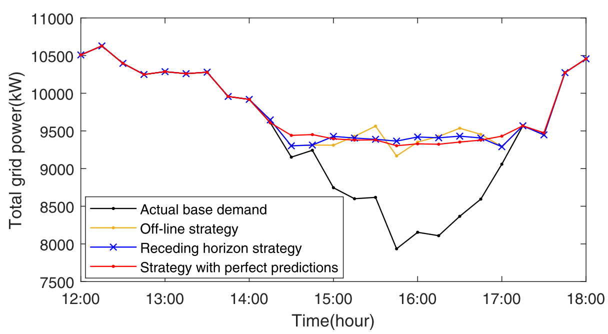


Fig. 14. EV strategies under Algorithm 2 and Algorithm 3 respectively.

strategy by Algorithm 2 for the actual demand, but it's impractical since the actual information can only be obtained in the end of the operation period. The coordination strategy by applying the receding Algorithm 3 is give as the blue line in Fig. 14. It can be observed that the proposed receding algorithm performs better than the off-line strategy.

VI. Conclusion

The fast charging behaviours could lead to the EV battery degradation and the capacity overloads of feeder lines in distribution networks. In this article, we have designed a decentralized hierarchical algorithm based on the ADMM method to solve the fast charging coordination problem of EV populations with a centralized non-separable objective function and coupled constraints, which considers the tradeoff among the EV users' satisfaction, the battery degradation cost, the load regulation and the total charging time. The desire of EV users to charge more power as quickly as possible is expressed by the weighing parameter of the satisfaction level and the individual total charging time which is represented by a non-convex ℓ_0-norm. And we apply the ℓ_1-norm minimization to approximate the sparse solution of ℓ_0-norm minimization. The optimality and convergence of the proposed method are guaranteed in case the step-size parameter is set in a certain region. Furthermore, a receding horizon based algorithm is designed to mitigate the effects of uncertainty over the base demand profiles and the EV populations. Simulations are developed to illustrate the results in this article. As ongoing researches, the proposed control framework will be extended by taking the renewable energy source into consideration.

Appendix
Proof of Theorem 3

Consider a pair of distinct charging behaviors $\boldsymbol{u}, \boldsymbol{x} \in \mathcal{U}$, and consider $U_t = \sum_{n\in\mathcal{N}} u_{nt}$ and $X_t = \sum_{n\in\mathcal{N}} x_{nt}$; then we have

$$\begin{aligned}\frac{\partial H(\boldsymbol{u})}{\partial u_{nt}} - \frac{\partial H(\boldsymbol{x})}{\partial x_{nt}} &\\ = \omega_n\big(h'(d_t+U_t) - h'(d_t+X_t)\big) + f_n'(u_{nt}) - f_n'(x_{nt}) &\\ - \varpi_n\big(g_n'(\sum_{t\in\mathcal{T}} u_{nt}) - g_n'(\sum_{t\in\mathcal{T}} x_{nt})\big) + \rho(u_{nt} - x_{nt}). & \quad (26)\end{aligned}$$

Since the first and second derivatives of $h(\cdot)$, $f_n(\cdot)$ and $g_n(\cdot)$ are continuous on $\mathbb{R}$ under Assumptions (A1)-(A3); by the mean value theorem, there exists some D_t between $d_t + U_t$ and $d_t + X_t$, w_{nt} between u_{nt} and x_{nt}, and e_n between $\sum_{t\in\mathcal{T}} u_{nt}$ and $\sum_{t\in\mathcal{T}} x_{nt}$ such that

$$h''(D_t) = \frac{h'(d_t+U_t) - h'(d_t+X_t)}{U_t - X_t}, \quad (27a)$$

$$f_n''(w_{nt}) = \frac{f_n'(u_{nt}) - f_n'(x_{nt})}{u_{nt} - x_{nt}} \quad (27b)$$

$$g_n''(e_n) = \frac{g_n'(\sum_{t\in\mathcal{T}} u_{nt}) - g_n'(\sum_{t\in\mathcal{T}} x_{nt})}{\sum_{t\in\mathcal{T}} u_{nt} - \sum_{t\in\mathcal{T}} x_{nt}}, \quad (27c)$$

by which together with (26), we have

$$\begin{aligned}\frac{\partial H(\boldsymbol{u})}{\partial u_{nt}} - \frac{\partial H(\boldsymbol{x})}{\partial x_{nt}} &\\ = \omega_n h''(D_t)(U_t - X_t) + \big(f_n''(w_{nt}) + \rho\big)(u_{nt} - x_{nt}) &\\ - \varpi_n g_n''(e_n)(\sum_{t\in\mathcal{T}} u_{nt} - \sum_{t\in\mathcal{T}} x_{nt}). & \quad (28)\end{aligned}$$

Furthermore, define a matrix $S \triangleq \mathrm{diag}\{S_n\}_{n=1,\cdots,N}$ in which $S_n \triangleq \mathrm{diag}\{S_{nt}\}_{t=1,\cdots,T}$ with $S_{nt} = f_n''(w_{nt}) + \rho$, and let $G(\boldsymbol{u}) = \sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}} \omega_n h\big(d_t + \sum_{n\in\mathcal{N}} u_{nt}\big)$ and $E(\boldsymbol{u}) = -\sum_{n\in\mathcal{N}} \varpi_n g_n(\sum_{t\in\mathcal{T}} u_{nt})$. It implies that

$$\begin{aligned}&\|\nabla H(\boldsymbol{u}) - \nabla H(\boldsymbol{x})\|\\ &= \big\|S(\boldsymbol{u}-\boldsymbol{x}) + \nabla G(\boldsymbol{u}) - \nabla G(\boldsymbol{x}) + \nabla E(\boldsymbol{u}) - \nabla E(\boldsymbol{x})\big\|\\ &\le \big\|S(\boldsymbol{u}-\boldsymbol{x})\big\| + \big\|\nabla G(\boldsymbol{u}) - \nabla G(\boldsymbol{x})\big\| + \big\|\nabla E(\boldsymbol{u}) - \nabla E(\boldsymbol{x})\big\|\end{aligned} \quad (29)$$

where the last inequality holds by the properties of Euclidean norm in [41].

Since $\omega_n \le \omega_{max}$, $\varpi_n \le \varpi_{max}$, $h''(x) \le \varphi$, $f_n''(x) \le \phi$, $g_n''(x) \ge \psi$ under Assumptions (A1)-(A3), it gives

$$\rho \le S_{nt} \le \phi + \rho. \quad (30)$$

And we have

$$\begin{aligned}\left\|\nabla G(\boldsymbol{u})-\nabla G(\boldsymbol{x})\right\|_2^2 &= \sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\big(\omega_n h''(D_t)(U_t-X_t)\big)^2\\ &\le N\varphi^2\omega_{max}^2\sum_{t\in\mathcal{T}}(U_t-X_t)^2\\ &\le N^2\varphi^2\omega_{max}^2\|\boldsymbol{u}-\boldsymbol{x}\|_2^2,\end{aligned}$$

where the last inequality holds by the Jensen's inequality $(U_t-X_t)^2 \le N\sum_{n\in\mathcal{N}}(u_{nt}-x_{nt})^2$ and $\|\boldsymbol{u}-\boldsymbol{x}\|_2^2 = \sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}(u_{nt}-x_{nt})^2$. Then it can be obtained that

$$\left\|\nabla G(\boldsymbol{u})-\nabla G(\boldsymbol{x})\right\| \le N\varphi\omega_{max}\|\boldsymbol{u}-\boldsymbol{x}\|. \tag{31}$$

Similarly, we have

$$\begin{aligned}&\left\|\nabla E(\boldsymbol{u})-\nabla E(\boldsymbol{x})\right\|_2^2\\ &= T\sum_{n\in\mathcal{N}}\Big(-\varpi_n g_n''(e_n)(\sum_{t\in\mathcal{T}}u_{nt}-\sum_{t\in\mathcal{T}}x_{nt})\Big)^2\\ &\le T\psi^2\varpi_{max}^2\sum_{n\in\mathcal{N}}(\sum_{t\in\mathcal{T}}u_{nt}-\sum_{t\in\mathcal{T}}x_{nt})^2\\ &\le T^2\psi^2\varpi_{max}^2\|\boldsymbol{u}-\boldsymbol{x}\|_2^2.\end{aligned}$$

Since $\psi < 0$ it gives that

$$\left\|\nabla E(\boldsymbol{u})-\nabla E(\boldsymbol{x})\right\| \le -T\psi\varpi_{max}\|\boldsymbol{u}-\boldsymbol{x}\|. \tag{32}$$

Hence, substituting (30)-(32) into (29) yields

$$\begin{aligned}&\|\nabla H(\boldsymbol{u})-\nabla H(\boldsymbol{x})\|\\ &\qquad\le (\phi+\rho+N\varphi\omega_{max}-T\psi\varpi_{max})\|\boldsymbol{u}-\boldsymbol{x}\|.\end{aligned} \tag{33}$$

Then the total cost $H(\boldsymbol{u})$ monotonically decreases on $\{\boldsymbol{u}^j\}$ in case $0<\alpha<2\big(\phi+\rho+N\varphi\omega_{max}-T\psi\varpi_{max}\big)^{-1}$, and

$$\begin{aligned}&H(\boldsymbol{u}^{(j+1)})-H(\boldsymbol{u}^{(j)})\\ &\le \Big(\frac{1}{2}(\phi+\rho+N\varphi\omega_{max}-T\psi\varpi_{max})-\frac{1}{\alpha}\Big)\|\boldsymbol{u}-\boldsymbol{x}\|_2.\end{aligned} \tag{34}$$

Therefore, based on the above analysis the sequence $\{\boldsymbol{u}^j\}$ generated by Algorithm 1 converges to an equilibrium, denoted by $\boldsymbol{u}^\dagger$ since $H(\boldsymbol{u})$ is bounded from below.

Based on Proposition 2.4 in [41], it gives that $\boldsymbol{u}^\dagger$ is the stationary point, that is, it satisfies

$$\nabla H(\boldsymbol{u}^\dagger)^\top(\boldsymbol{u}-\boldsymbol{u}^\dagger)\ge 0,\quad \forall\,\boldsymbol{u}\in\mathcal{U}. \tag{35}$$

We can know that $H(\boldsymbol{u})$ is convex by Assumptions (A1)-(A2). Hence, by Proposition 2.1.2 in [41], we have $\boldsymbol{u}^{(k+1)} = \boldsymbol{u}^\dagger$, that is, the system converges to the optimal solution $\boldsymbol{u}^{(k+1)}$ that minimizes $H(\boldsymbol{u})$ by applying Algorithm 1.

Furthermore, denote the respective updated profiles of charging behaviors $\boldsymbol{u},\boldsymbol{x}$ by $\widehat{\boldsymbol{u}},\widehat{\boldsymbol{x}}\in\mathcal{U}$ given in (21), we have

$$\begin{aligned}\|\widehat{\boldsymbol{u}}-\widehat{\boldsymbol{x}}\| &= \left\|[\boldsymbol{u}-\alpha\nabla H(\boldsymbol{u})]_{\mathcal{U}}^+-[\boldsymbol{x}-\alpha\nabla H(\boldsymbol{x}]_{\mathcal{U}}^+\right\|\\ &\le \left\|\big(\boldsymbol{u}-\alpha\nabla H(\boldsymbol{u})\big)-\big(\boldsymbol{x}-\alpha\nabla H(\boldsymbol{x})\big)\right\|\\ &= \left\|(\boldsymbol{u}-\boldsymbol{x})-\alpha\big(\nabla H(\boldsymbol{u})-\nabla H(\boldsymbol{x})\big)\right\|,\end{aligned} \tag{36}$$

where the inequality holds by Proposition 2.1.3 in [41].

By (36) together with the analysis in (29)-(32), it gives

$$\begin{aligned}&\|\widehat{\boldsymbol{u}}-\widehat{\boldsymbol{x}}\|\\ &\le \left\|(I-\alpha S)(\boldsymbol{u}-\boldsymbol{x})-\alpha\big(\nabla G(\boldsymbol{u})-\nabla G(\boldsymbol{x})+\nabla E(\boldsymbol{u})-\nabla E(\boldsymbol{x})\big)\right\|\\ &\le \left\|(I-\alpha S)(\boldsymbol{u}-\boldsymbol{x})\right\|+\alpha(N\varphi\omega_{max}-T\psi\varpi_{max})\|\boldsymbol{u}-\boldsymbol{x}\|.\end{aligned} \tag{37}$$

Since $\rho\le S_{nt}\le\phi+\rho$ given in (30), it implies that

$$\left\|(I-\alpha S)(\boldsymbol{u}-\boldsymbol{x})\right\|\le S_{max}\|\boldsymbol{u}-\boldsymbol{x}\|, \tag{38}$$

where $S_{max}=max\big\{|1-\alpha\rho|,|1-\alpha(\phi+\rho)|\big\}$. And S_{max} can be also represented as follows:

$$S_{max}=\begin{cases}1-\alpha\rho, & \text{if } \alpha\le\dfrac{2}{\phi+2\rho},\\ \alpha(\phi+\rho)-1, & \text{otherwise.}\end{cases}$$

Then substituting (38) into (37) gives

$$\|\widehat{\boldsymbol{u}}-\widehat{\boldsymbol{x}}\|\le\big(S_{max}+\alpha(N\varphi\omega_{max}-T\psi\varpi_{max})\big)\|\boldsymbol{u}-\boldsymbol{x}\|, \tag{39}$$

which illustrates that the update strategy given in (21) is a contraction map if $\kappa = S_{max}+\alpha(N\varphi\omega_{max}-T\psi\varpi_{max}) < 1$, and the sequence $\{\boldsymbol{u}^j\}$ can converge to $\boldsymbol{u}^\dagger$ based on the properties of contractive mapping.

We will establish the sufficient conditions of $\kappa<1$ by the backward induction:

(I) Assume $\alpha\le 2/(\phi+2\rho)$, then $\kappa<1$ gives

$$1-\alpha\rho+\alpha(N\varphi\omega_{max}-T\psi\varpi_{max})\big)<1,$$

which implies $\rho > N\varphi\omega_{max}-T\psi\varpi_{max}$ since $\alpha>0$.

(II) Assume $\alpha>2/(\phi+2\rho)$, then $\kappa<1$ gives

$$\alpha(\phi+\rho)-1+\alpha(N\varphi\omega_{max}-T\psi\varpi_{max})\big)<1$$

which implies $\alpha<2\big(\phi+\rho+N\varphi\omega_{max}-T\psi\varpi_{max}\big)^{-1}$. Only when $\rho > N\varphi\omega_{max}-T\psi\varpi_{max}$, it does make sense with $2/(\phi+2\rho)<\alpha<2\big(\phi+\rho+N\varphi\omega_{max}-T\psi\varpi_{max}\big)^{-1}$.

Therefore, based on the above analysis we can obtain that $\kappa<1$ in case $\rho>N\varphi\omega_{max}-T\psi\varpi_{max}$ under $\alpha<2\big(\phi+\rho+N\varphi\omega_{max}-T\psi\varpi_{max}\big)^{-1}$.

Furthermore, by (39) we have

$$\|\boldsymbol{u}^{(j+1)}-\boldsymbol{u}^{(j)}\|\le\kappa\|\boldsymbol{u}^{(j)}-\boldsymbol{u}^{(j-1)}\|, \tag{40}$$

which implies the following inequality:

$$\|\boldsymbol{u}^{(j+1)}-\boldsymbol{u}^{(j)}\|\le\kappa^j\|\boldsymbol{u}^{(1)}-\boldsymbol{u}^{(0)}\|. \tag{41}$$

With the maximum value of charging rate $\boldsymbol{u}$ specified in (2), it implies that

$$\|\boldsymbol{u}^{(j+1)}-\boldsymbol{u}^{(j)}\|\le\kappa^j(T\sum_{n\in\mathcal{N}}\gamma_n^2)^{1/2}, \tag{42}$$

which gives $\|\boldsymbol{u}^{(j+1)}-\boldsymbol{u}^{(j)}\|\le\epsilon$ for j satisfying (23).

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