Coordination Optimization Algorithm Based on Improved ADMM of EV Considering Orderly Charging

IEEE Publication Technology Department

***Abstract*—With the rapid proliferation of electric vehicles (EVs), the challenges of integrating their charging demands into the power grid have intensified. This paper addresses the imperative of orderly EV charginga multi-objective optimization problem that balances grid stability, cost-efficiency, and user convenience. We propose a dual-pronged solution that encompasses the development of a sparse mathematical model-based distributed framework for EV management and an improved Alternating Direction Method of Multipliers (ADMM) algorithm. The framework facilitates the decoupling of complex charging coordination problems into multiple sub-problems, enhancing system interpretability and computational efficiency. The refined ADMM algorithm, with its multiple block Jacobi forms and the inclusion of a proximal term, not only allows for parallel computation but also achieves superior convergence rates, improving from ~~$O(1/k)$ to $o(1/k)$~~$O(1/k)$ to $o(1/k)$. Simulation results confirm the efficacy of our model in managing large-scale EV charging coordination, ensuring grid stability, and offering a scalable, privacy-preserving solution for EV users and operators. This study contributes to the advancement of smart grid technology by providing a robust mechanism for accommodating the burgeoning EV charging demand while maintaining grid integrity and optimizing user experience.**

***Index Terms*—Electric Vehicle Charging, Orderly Charging, Power Grid Stability, Multi-Objective Optimization, ADMM, Distributed Framework, Convergence Rate, Smart Grid.**

I. Introduction

As the number of Electric Vehicles (EVs) continues to climb, the concept of orderly charging is garnering increasing attention within the realms of scientific research and smart grid technologies. The surge in EV adoption is a direct result of swift advancements in EV technology coupled with rising gasoline prices. This trend underscores the vast potential and significant societal impact that EV development is poised to deliver. However, the fluctuating and unstable nature of integrating EVs into the power grid is expected to challenge the grid's optimal operation and dispatch, presenting a noteworthy hurdle [1], [2]. The cornerstone for surmounting this challenge lies in the deployment of coordinated charging scheduling method for electric vehicles. This method serves to facilitate seamless information exchange among grid operators, aggregator entities, and electric vehicle users, thereby ensuring the secure operation of the grid and reducing economic costs.

The orderly charging issue for EVs is inherently a multi-objective optimization problem that requires a delicate balance between various competing factors. First and foremost, the stability of the power grid is a critical factor as the burgeoning number of electric vehicles contributes to substantial and erratic demand. Without judicious coordination and control, this surge in consumption could precipitate fluctuations in grid performance, potentially triggering widespread power disruptions or necessitating costly enhancements to existing electrical infrastructure. Secondly, the cost of charging for EV users should also be taken into account. This factor is pivotal, as it significantly influences the uptake of electric vehicles within the constraint of a fixed power supply and shapes consumer readiness to participate in intelligent charging processes. Lastly, the convenience of the charging process must not be overlooked. Through judicious coordination, it is feasible to ensure that every EV user can recharge their vehicle without excessive waiting times, thus streamlining the user experience. *Unfortunately*, the optimization variables involved in addressing this problem are highly interconnected, which adds a layer of complexity to finding efficient solutions.

The approach for solving the numerous EV coordinated charging models can be broadly classified into two distinct categories: centralized [3], [4] and distributed [5], [6], [7], [8], [9], [10], [11], [12] approaches. The former involves the collection of all relevant parameters, entrusted to a single entity for centralized computation. The latter, on the other hand, enables parallel computation through multiple agents [13], requiring limited information exchange for collaborative computation. Consequently, distributed methods exhibit higher computational efficiency and scalability. They can coordinate larger groups of electric vehicles across the entire distribution network while providing more choices for electric vehicle customers during the decision-making process. Indeed, there is a wealth of research on the decentralized coordination of electric vehicle charging [6], [7], [8], [9], [10], [11], [12]. However, it is crucial to note that the methods mentioned above, including [6], [7], [8], [9], [10], [11], [12], require a central coordinator (or aggregator), leading to partial distributed optimization. Specifically, these methods necessitate a communication network where all information is transmitted through the aggregator to the electric vehicles, resulting in substantial communication overhead.

In this paper, we address the coordinated charging problem

Manuscript created October, 2020; This work was developed by the IEEE Publication Technology Department.

by employing the Alternating Direction Method of Multipliers (ADMM) as the foundational approach. As a fully distributed method, the ADMM has gained significant attention over an extended period. Since the ordered charging problem is a multi-objective and strongly coupled complex optimization issue, utilizing the traditional ADMM method for its resolution is deemed inefficient and cumbersome. Indeed, the ordered charging problem encompasses numerous optimization objectives, leading to the development of various models catering to distinct requirements (such as fast charging models, adjustable electricity pricing models, etc.). This study considers a mathematical model that balances the interests of charging users (e.g., electricity cost and charging satisfaction) with power grid stability and safety (e.g., peak-shaving of power grid). Achieving a balance among these diverse optimization objectives is facilitated by adjusting the charging power of each electric vehicle at different time intervals. It is noteworthy that the optimization objectives for power grid stability are intricately coupled with the interests of charging users. This intricacy poses a challenge for parallel computation in distributed ADMM algorithms. Conventional decoupling methods often compromise the inherent convergence speed of the ADMM algorithm. Consequently, the challenge is to propose an effective decoupling algorithm that simultaneously preserves convergence speed without significant degradation.

Over an extended period, scholars have endeavored to optimize and enhance the ADMM to address diverse requirements. It is noted that the original ADMM decomposes the objective function into two subproblems, employing an alternating optimization approach to concurrently achieve the overall optimization objective. In response to more intricate scenarios, scholars have naturally sought to directly extend the ADMM algorithm to tackle optimization problems involving three or more separable operators [14], [15]. However, the convergence of directly extended ADMM exhibits varying outcomes for different problems, as supported by the literature. Scholars have introduced the Gaussian alternating direction method to address this, incorporating correction strategies after the prediction step. Nevertheless, the additional correction steps result in a substantial increase in computational overhead.

The primary objective of this paper is to present a decoupling algorithm for ADMM that enables parallel computation without significantly compromising convergence speed. This paper introduces refinements to the original ADMM method, addressing the increasing demand for parallel computation and presenting a comprehensive solution to challenges associated with large-scale electric vehicle (EV) coordinated charging. To achieve this, we ingeniously decouple the initially intertwined multi-objective optimization problem into N+1 sub-problems, where N represents the number of electric vehicles involved. This strategic decoupling allows for the deployment of distributed parallel computation, significantly enhancing computational efficiency. The solutions to these sub-problems are meticulously aggregated and harmonized through Lagrangian multipliers, providing an effective coordination mechanism. This iterative process continues until reaching an acceptable error threshold, signifying the completion of iterations. To bolster the convergence speed, a proximity term is introduced, ensuring a more rapid and robust convergence. Notably, our approach successfully addresses challenges related to large-scale electric vehicle coordinated charging, including privacy concerns for EV owners and difficulties in accurately describing constraints. This accomplishment is pivotal in achieving a mutually beneficial outcome for the power grid, charging stations, and users, effectively balancing the interests of all stakeholders. The carefully orchestrated parallelized sub-problems, coupled with the coordination mechanism, not only enhance computational speed but also contribute to a scalable and adaptable solution. In short, our refined ADMM method not only improves computational efficiency but also navigates the complexities of large-scale electric vehicle coordination, ensuring privacy preservation and overcoming constraint description challenges. This achievement is a significant stride toward a win-win scenario for the power grid, charging stations, and users.

Additionally, the inclusion of proximity term in this paper makes a significant contribution to convergence. In fact, many methods struggle to converge to the optimal solution. In the field of distributed optimization, despite extensive research as evidenced by the cited literature such as [16], [17]-[18],[19] achieving a global optimal solution remains a formidable task. The study presented in [16] explores non-Euclidean stochastic strongly convex optimization methods. While this strategy demonstrates effectiveness, it entails some nodes waiting for others to complete computations, resulting in shortcomings in the speed and efficiency of the algorithm. Similarly, in [17]-[18], researchers propose a gradient tracking approach to solve optimization problems with linear objectives in strongly convex optimization. However, this method typically yields only local optimal solutions in general distributed optimization, hindering the attainment of a globally optimal solution. In [19], a primal-dual method is introduced to address large-scale distributed convex optimization and data analysis. Despite its long history and capability to handle constraints, it is important to note that this method is unable to solve nonsmooth problems and exhibits relatively slow convergence. Therefore, careful consideration of the trade-offs and limitations associated with the current methods is essential in the realm of distributed coordination. While existing research provides valuable insights, there is a noticeable gap that requires further exploration and refinement to establish more robust and optimal solutions in the field of distributed optimization.

The contributions of this article are outlined as follows:

1) Proposing a Sparse Mathematical Model-Based Distributed Framework for Electric Vehicle (EV) Management: We introduce a distributed framework for electric vehicle management based on a sparse mathematical model. By employing a method that decouples the originally tightly coupled issues into solutions for multiple sub-problems, the efficiency in handling system complexity is significantly improved. This approach enhances the interpretability of electric vehicle scheduling problems.
2) Enhancing Convergence Speed with an Improved ADMM Algorithm: To address the slow convergence of sparse models, this paper introduces an enhanced Alternating

Direction Method of Multipliers (ADMM) algorithm. We extend the classical ADMM method into multiple block Jacobi forms and incorporate a proximal term. This ensures not only parallel solvability but also higher convergence compared to the classical ADMM method. The convergence rate improves from ~~$O(1/k)$ to $o(1/k)$~~ $O(1/k)$ to $o(1/k)$.

The rest of the paper is structured as follows. In Section ~~II~~II, we propose the mathematical models for the distribution network and EV orderly charging scheduling. In Section ~~III~~III, we propose an improved ADMM method to solve the underlying problem~~, We then~~. subsequently, we verify the convergence and optimality of the proposed algorithm. Numerical examples are shown in Section ~~V~~ IV to demonstrate the results developed in the paper. Finally, conclusions and future research directions are given in Section ~~VI~~V.

II. EV MODEL CONSTRUCT

A. EV distributed optimization framework

In this ~~model,~~ paper, we assume that EV users upload their charging demand information to the aggregator when they connect their EVs to charging piles. Charging plan of each EV will then be sent back to EV users. The charging demand information include arrival time ~~t_i^c~~t_i^c, predefined departure time ~~t_i^d~~t_i^d, the SOC when connecting to charging piles ~~SOC_i^c~~ SOC_i^c and the expected SOC called ~~SOC_i^d~~ SOC_i^d of each EV. ~~The coordinated charging scheduling process is shown in Fig. 1.~~ The coordinated charging scheduling process is shown in Fig. 1.

In the scheduling process, the scheduling plan is usually executed by period of time for improving execution efficiency. The scheduling time is divided into several time periods. In the proposed EV charging scheduling model, a day is discretized into 96 time slots and the length of a time slot is Δt = 15 ~~min~~minutes. Then, the arrival time and departure time of each EV can be are expressed as:

$$I_i^c = \frac{t_i^c}{\Delta T}\lceil\frac{t_i^c}{\Delta t}\rceil, i = 1, 2, ..., N. \tag{1}$$

$$I_i^d = \frac{t_i^d}{\Delta T}\lceil\frac{t_i^d}{\Delta t}\rceil, i = 1, 2, ..., N. \tag{2}$$

Where ~~I_i^c~~ I_i^c represents the time slot when the ~~$i-th$~~ $i-th$ electric vehicle is connected to the charging station, while ~~I_i^d~~ I_i^d represents the time slot when the ~~$i-th$~~ $i-th$ electric vehicle departs. ~~t_i^c and t_i^d~~ t_i^c and t_i^d represent the arrival time and scheduled departure time of the ~~$EV i$~~ ith EV, respective. ~~$\lceil\frac{t_i^c}{\Delta t}\rceil$~~ $\lceil\frac{t_i^c}{\Delta t}\rceil$ is the smallest positive integer greater than or equal to ~~$\frac{t_i^c}{\Delta t}$, while $\lceil\frac{t_i^d}{\Delta t}\rceil$~~ $\frac{t_i^c}{\Delta t}$, while $\lceil\frac{t_i^d}{\Delta t}\rceil$ is the smallest positive integer greater than or equal to ~~$\frac{t_i^d}{\Delta t}$~~ $\frac{t_i^d}{\Delta t}$.

TABLE I
~~LIST OF KEY SYMBOLS~~ LIST OF KEY SYMBOLS

~~$\mathcal{N}$~~ $\mathcal{N}$	EV population
~~$\mathcal{T}$~~ $\mathcal{T}$	EV charging intervals
~~$p_{n,t}$~~ EV_n	n-th EV
$p_{n,t}$	Chargingng rate of EV n at instant t
~~Γ_n~~ Γ_n	Total charging power required to be fully charged of EV n

B. Charging Scheduling Model Constraints

Based on the piecewise linear model, the battery dynamics can be described as follows

$$SOC_{n,t+1} = SOC_{n,t} + \frac{q_n^+ p_{n,t}}{C_n}\Delta t - \frac{\omega_{n,t}}{q_n^- C_n}\Delta\frac{q_n^+ p_{n,t}}{C_n}\Delta t \tag{3}$$

$$SOC_n^{min} \leq SOC_{n,t} \leq SOC_n^{max} \tag{4}$$

where ~~$q_n^+, q_n^- \in (0,1]$ represent the~~ SOC denotes the state of charge, $q_n^+ \in (0,1]$ represents the energy conversion efficiency for charging~~and discharging of electric vehicles, respectively~~. To extend battery life, it is recommended to set ~~soc_n^{min} and soc_n^{max} at 15% and 90% respectively, $p_{n,t}$(kW~~ SOC_n^{min} and SOC_n^{max} at 15% and 90% respectively, $p_{n,t}$ (kW) represents the charging rate of ~~$EV n$~~EV_n at time slot ~~$t \in \mathcal{T}$, and C_n denotes $EV n$ In~~ $t \in \mathcal{T}$, and C_n denotes the charging rate of EV_n in each time slot, the charging rate for each electric vehicle remains constant.

Let ~~$\boldsymbol{p} \triangleq (\boldsymbol{p}_n; n \in \mathcal{N})$~~ $\boldsymbol{p} \triangleq (\boldsymbol{p}_n?; n \in \mathcal{N})$ represent the charging strategy for the population of ~~EV~~EV, The set of permissible charging strategies for ~~$EV n$~~ EV_n is denoted as ~~$\mathcal{P}_n$~~$\mathcal{P}_n$, and the set of permissible charging strategies for all electric vehicles is represented by ~~$\mathcal{P}$~~$\mathcal{P}$.

$$\mathcal{P} \triangleq \mathcal{P}_1 \times \cdots \times \mathcal{P}_N \tag{5}$$

The charging strategy is the most significant factor influencing grid investment. To ensure grid safety, the following inequality holds:

$$p_n^{min} \leq p_{n,t} \leq p_n^{max} \tag{6}$$

where ~~p_n^{min} and p_n^{max}~~ p_n^{min} and p_n^{max} represent the minimum and maximum charging power of ~~$EV n$~~EV_n, respectively. By determining whether to charge electric vehicles in each time slot, coordinated charging scheduling of electric vehicles is achieved. The charging status of ~~$EV n$~~ EV_n is as follows

$$x_{n,t} = \begin{cases} 1 & \text{if the } \cancel{EV}\ EV \text{ is charging} \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

The constraints satisfy the following form:

$$(1 - x_{n,t})p_{n,t} = 0 \tag{8}$$

Electric vehicles with urgent charging needs may not be able to obtain sufficient charge due to their short connection time. Therefore, the state of charge ~~Therefore, the state of charge (SOC)~~at the end of their charging period, denoted as $\cancel{SOC_n^{end}}$ SOC_n^{end}, may differ from the desired ~~SOC~~ SOC, represented as $\cancel{SOC_n^d}$ SOC_n^d

$$SOC_n^{end} = min\{SOC_i^C + \cancel{\frac{p_{n,t} \cdot \Delta t \times \eta_n}{C_n}} \frac{p_{n,t} \cdot \Delta t \times \eta_n}{C_n}, SOC_i^d\} \tag{9}$$

$$SOC_n^{min} \leq SOC_{\cancel{n,t}n}^{end} \leq SOC_N^{max} \tag{10}$$

where $\cancel{\eta}$ η represents the charging energy conversion efficiency of $\cancel{EV n}$ EV_n.

C. Objective Function Optimization

In this section, we delve into the intricacies of electric vehicle (EV) charging, placing a significant emphasis on addressing challenges related to minimizing costs, enhancing charging satisfaction, peak shaving, and formulating effective charging strategies. Our research focuses on the development of multi-objective optimization models for electric vehicles, aiming to strike a balance between minimizing charging costs and optimizing user satisfaction. Simultaneously, we implement strategies for peak shaving to establish an efficient charging scheduling policy for sustainable electric mobility.

To reduce the cost of electric vehicle charging, we have designed corresponding electricity price models based on different time periods throughout the day ~~fig.1,~~ (see Fig 4 for detial). The approach aims to mitigate the impact of a large number of electric vehicles randomly connecting to the grid. We aim to enhance the stability of the power system when accommodating a significant number of electric vehicles, with the overarching objective of minimizing the overall cost of electric vehicle charging, as follows:

$$G_1 = \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} E_t p_{n,t} \Delta t \tag{11}$$

where $\cancel{E_T}$ E_t represents the electricity price at time ~~t, $p_{n,t}$~~ t, $p_{n,t}$ denotes the charging rate of ~~$EV n$ at time t, and Δt~~ EV_n at time t, and Δt represents the length of a time slot.

EV users typically take into account the satisfaction function for the total energy delivered during the charging period. Given the opportunity, EV users would prefer to charge their vehicles with more electricity provided by the grid until they attain the required energy within the expected time ~~$t \in \mathcal{T}$,~~ $t \in \mathcal{T}$. They maintain this desire constant within the expected time frame until the required energy is achieved. The goal of electric vehicle owners is to maximize charging satisfaction, which is

$$G_2 = -\sum_{n \in \mathcal{N}} (\sum_{t \in \mathcal{T}} p_{nt} - \Gamma_n)^2. \tag{12}$$

To enable peak shaving and valley filling in the aggregated power of electric vehicle charging, we utilize the variance of the total power to reflect its stability. A smaller variance indicates a more stable grid. The total power encompasses the overall power from electric vehicle charging and residential electricity consumption (denoted as $\cancel{B_T}$ B_t), as illustrated below:

$$G_3 = \frac{1}{T} \sum_{t \in \mathcal{T}} [(\sum_{n \in \mathcal{N}} p_{nt} + B_t) - \overline{p+B}]^2, \tag{13}$$

where $\bar{B} = \frac{1}{T} \sum_{t \in \mathcal{T}} B_t$, $\bar{p} = \frac{1}{T} \sum_{t \in \mathcal{T}} \sum_{n \in \mathcal{N}} p_{nt}$.

Charging behavior significantly impacts critical battery characteristics, such as health status, cycle life, and the growth of resistance impedance. Intermittent charging tends to shorten the battery's lifespan. Therefore, it is crucial to minimize the number of charging cycles to maintain battery health. On the other hand, prolonged waiting times for completing charging tasks and/or frequent interruptions during the charging process are deemed unacceptable. These factors can cause discomfort for EV owners. Recent research emphasizes that the solution for charging time must be sufficiently sparse, as a sparse schedule implies a lower charging frequency.

$$G_4 = \|\boldsymbol{p}_n\|_0 \tag{14}$$

Where $\cancel{\|\boldsymbol{p}_n\|_0}$ $\|\boldsymbol{p}_n\|_0$ is the cardinality of the charging strategy, i.e. the number of non-zero elements in $\cancel{|\boldsymbol{p}_n|}$ $|\boldsymbol{p}_n|$. For EV owners with an urgent need for their vehicles, prioritizing charging is crucial, even though it comes with higher charging costs. On the other hand, owners who are not in a hurry to use their electric vehicles can shift the electric vehicle load to other time periods.

As the process of electrification in the automotive industry progresses, initially, attempts were made to model the charging strategy problem as a minimum $\cancel{l_2}$ l_2 norm problem for solving. However, the $\cancel{l_2}$ l_2 norm does not capture the sparsity required by the charging strategy of electric vehicles. In contrast, the $\cancel{l_0}$ l_0 norm counts the number of non-zero values in a vector. Therefore, the charging strategy is modeled as a minimum $\cancel{l_0}$ l_0 norm problem to achieve shorter charging time, improved battery lifespan, and increased user satisfaction. However, the minimum $\cancel{l_0}$ l_0 norm problem is NP-hard and cannot be solved using conventional methods. Under certain conditions, the minimum $\cancel{l_1}$ l_1 problem and the minimum $\cancel{l_0}$ l_0 norm problem are equivalent, meaning that solving the minimum $\cancel{l_1}$ l_1 problem can yield the solution to the minimum $\cancel{l_0}$ l_0 problem.

Therefore, we approximate the $\cancel{l_0}$ l_0 norm with the $\cancel{l_1}$ l_1 norm. We rephrase (14) into the following sparse-promoting charging control model as follows:

$$\tilde{G}_4 = \|p\boldsymbol{p}_n\|_1 \tag{15}$$

The above issue involves a multi-objective optimization model that integrates multiple objectives with distributed optimization algorithms. The equation achieves optimal scheduling goals for minimizing EV charging costs, maximizing user satisfaction, minimizing grid load variance, and optimizing the electric vehicle charging strategy by linearly weighting the terms in equations (11), (12),(13), and (15). Thus, the objective function is formulated as follows:

$$min \quad \omega_1 g_1 + \omega_2 g_2 + \omega_3 g_3 + \omega_4 \tilde{g}_4. \tag{16}$$

Due to the diverse dimensions of the four metrics EV charging cost, user satisfaction, grid load variance, and optimized electric vehicle charging strategy data normalization is required. In (16), ~~g_1, g_2, g_3, and g_4~~ g_1, g_2, g_3, and g_4 represent the dimensionless values of the four objective functions after normalization. ~~ω_1, ω_2, ω_3, and ω_4~~ ω_1, ω_2, ω_3, and ω_4 respectively denote the weight values assigned to each objective.

Since the objective function shown in (16) is minimized, for ~~$G_1, i = 1, 3$~~ $G_1, i = 1, 3$, the smaller the better, that is:

$$g_i = \frac{G_j - minG_j}{maxG_j - minG_j} \tag{17}$$

for ~~$\tilde{G}_2$~~ $\tilde{G}_2$, the smaller the better, that is:

$$g_4 = \frac{\tilde{G}_4 - min\tilde{G}_4}{max\tilde{G}_4 - min\tilde{G}_4} \tag{18}$$

~~for G_2~~ for G_2, the bigger the better 这里G2已经取负了, that is:

$$g_2 = \frac{maxG_2 - G_2}{maxG_2 - minG_2} \tag{19}$$

After dimensionless transformation of each objective function, (16) can be further expressed as follows:

$$\begin{aligned} min \quad & \tau_1 G_1 + \tau_2 G_2 + \tau_3 G_3 + \tau_4 \tilde{G}_4 \\ & = \tau_1 \sum_{n \in \mathcal{N}} \{ \sum_{t \in \mathcal{T}} E_t p_{n,t} \Delta t - \tau_2 (\sum_{t \in \mathcal{T}} p_{nt} - \Gamma_n)^2 \\ & + \tau_3 \frac{1}{T} \sum_{t \in \mathcal{T}} [(\sum_{n \in \mathcal{N}} p_{nt} + B_t) - \overline{p + B}]^2 + \tau_4 \|\boldsymbol{p}_n\|. \end{aligned} \tag{20}$$

Note:

The following assumptions apply to the entire article:

1) ~~G_1~~ G_1 is Lipschitz continuous, for all ~~x~~ x,
2) ~~G_3~~ G_3 is non-decreasing, concave, and Lipschitz continuous, for all ~~$n \in \mathcal{N}$ and all x~~ $n \in \mathcal{N}$ and all x.
3) ~~G_2~~ G_2 is non-decreasing, strictly convex and differentiable.

III. EV Charging Framework Based on Improved ADMM

******summarise the chapter 高度概括

A. Problem reformulation based on improved ADMM

The ADMM method has emerged as a potent technique for addressing large-scale structured optimization problems. Notably, ADMM excels in untangling spatial coupling constraints and breaking down the primary problem into numerous manageable subproblems, each amenable to straightforward resolution. The global objective is subsequently optimized. Nevertheless, in the context of this paper, where more than two variables are at play, we employ a modified ADMM algorithm. This adaptation extends ADMM to tackle optimization challenges characterized by three or more separable operators. Specifically, the augmented Lagrangian (21) is broken down into alternating and parallel steps. In each iteration, this method employs a new decomposition pattern to solve multiple smaller and easily solvable variables ~~x_j~~ x_j. Sub-problem variables ~~x_j~~ x_j can be computed in parallel. This approach enhances efficiency by allowing simultaneous processing of multiple variables, thereby accelerating the overall computation process.

For an optimisation problem with linear constraints, the objective function comprises the aggregate of ~~M~~ M independent functions devoid of interdependent variables. The corresponding mathematical model can be expressed as follows

$$\begin{aligned} min \quad & \sum_{i=1}^{M} \theta_i(x_i) \\ s.\ t. \quad & \sum_{i=1}^{\cancel{M} L} A_i x_i = c, \end{aligned} \tag{21}$$

where ~~θ_i~~ θ_i is a non-strictly convex function, ~~$M \geq 3$. L~~ $M \geq 3$. L is the number of equation constraints~~, $b \in \boldsymbol{R}^L$.~~ $b \in \boldsymbol{R}^L$.

B. Optimization strategy based on improved ADMM algorithm

~~(??) constitutes a~~
The objective function (20) contains coupled multi-objective ~~optimization model, Combining~~ functions. In the following, these will be decoupled into $N+1$ independent multi-objective ~~models with distributed optimisation algorithms~~ functions and combined with the distributed optimization algorithm.

~~Note that~~ Firstly, by equations (11) and (12), G_1, G_2 can be equivalently rewritten in the following form.

$$G_1 = \sum_{n \in \mathcal{N}} \underline{f_n(p_{nt})} = \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} E_t p \tag{22}$$

$$\underline{\sum_{n \in \mathcal{N}} g_n(\sum_{t \in \mathcal{N}} p_{nt})} G_2 = \underline{-\sum_{n \in \mathcal{N}} (\sum_{t \in \mathcal{T}} p_{nt} - \Gamma_n)^2} \underline{\equiv} - \sum_{n \in \mathcal{N}} (\|\boldsymbol{p}_n\|_1 - \Gamma_n)^2 \sum_{n \in \mathcal{N}} \tag{23}$$

where ~~$\boldsymbol{E} = (E_1, E_2, \cdots, E_T)^T$~~ $\boldsymbol{E} = (E_1, E_2, \cdots, E_T)^T$.

~~Moreover, by introducting the notation~~
Then, by introducing the notation

$$p_{N+1,t} = \sum_{n \in \mathcal{N}} p_{nt}, \boldsymbol{p}_{N+1} = (p_{N+1,1}, \cdots, p_{N+1,T})^T$$

~~We have~~ and applying (13), we arrive

$$\underline{h(\sum_{n \in \mathcal{N}} p_{nt})} G_3 = \underline{\frac{1}{T} \sum_{t \in \mathcal{T}} [(\sum_{n \in \mathcal{N}} p_{nt} + B_t) - \overline{p + B}]^2} \underline{\equiv} D(\boldsymbol{p}_{N+1} + B_t), \tag{24}$$

~~Here, $D(\cdot)$~~ where $D(\cdot)$ represents the variance.

~~Therefore, we~~ Combining equations (22)-(24) and (14), we ultimately obtain an equivalent form of equation ~~(??) :~~

minmize

20) as follows.

$$\min_{\boldsymbol{p} \in \mathcal{P}} \quad \theta_1(\underline{\mathbf{p}} \boldsymbol{p}_1) + \cdots + \theta_N(\underline{\mathbf{p}} \boldsymbol{p}_N) + \theta_{N+1}(\underline{\mathbf{p}} \boldsymbol{p}_{N+1}), \tag{25}$$

$$\underline{\text{subject to}}\ \text{s.t} \quad \underline{\mathbf{p}} \boldsymbol{p}_1 + \cdots + \underline{\mathbf{p}} \boldsymbol{p}_N - \underline{\mathbf{p}} \boldsymbol{p}_{N+1} = \mathbf{0}, \tag{26}$$

Algorithm 1 Jacobi-Proximal ADMM

Initialize: $\cancel{\boldsymbol{p}_i^0 (i=1,2,\cdots,M) \text{and } \lambda^0}$ $\boldsymbol{p}_i^0 (i=1,2,\cdots,M)$ and λ^0

for k=0,1,... **do**

Update $\cancel{\mathbf{p}_i}$ $\boldsymbol{p}_i$ for $\cancel{i=1,...,M}$ $i=1,...,M$ in parallel by:

$$\cancel{\mathbf{p}_i^{k+1} = \text{argmin}\,\{\theta_i(\mathbf{p}_i) + \tfrac{\rho}{2}\|A_i\mathbf{p}_i + \sum_{j\neq i} A_j\mathbf{p}_j^k - c - \tfrac{\Lambda^k}{\rho}\|_2^2 + \tfrac{1}{2}\|\mathbf{p}_i - \mathbf{p}_i^k\|_\Psi^2\}}$$

$$\boldsymbol{p}_i^{k+1} = \text{argmin}\,\{\theta_i(\boldsymbol{p}_i) + \tfrac{\rho}{2}\|A_i\boldsymbol{p}_i + \sum_{j\neq i} A_j\boldsymbol{p}_j^k - c - \tfrac{\Lambda^k}{\rho}\|_2^2 + \tfrac{1}{2}\|\boldsymbol{p}_i - \boldsymbol{p}_i^k\|_\Psi^2\}$$

Update $\cancel{\Lambda^{k+1} = \Lambda^k - \gamma\rho(\sum_{i=1}^M A_n\mathbf{p}_n^{k+1} - c)}$ $\Lambda^{k+1} = \Lambda^k - \gamma\rho(\sum_{i=1}^M A_n\boldsymbol{p}_n^{k+1} - c)$

end for

C. ~~*Optimization strategy based on improved ADMM algorithm*~~

Algorithm 2 Implementation of Charging Coordination via the Parallel Method

Input: Set the initial strategy state $\cancel{\boldsymbol{p}^0}$ $\boldsymbol{p}^0$,

Input: Set the feasibility tollerance $\cancel{\varepsilon^{pri}, \varepsilon^{dual}}$ $\varepsilon^{pri}, \varepsilon^{dual}$,

Input: Set $\cancel{k=0, r>\varepsilon^{pri} \text{ and } s>\varepsilon^{dual}}$ $k=0, r>\varepsilon^{pri}$ and $s>\varepsilon^{dual}$

Output: $\cancel{\mathbf{p}_n^k (n=1,...,N+1)}$ $\boldsymbol{p}_n^k (n=1,...,N+1)$

1: **for** k=0,1,... **do**

2: Update $\cancel{\mathbf{p}_n}$ $\boldsymbol{p}_n$ for $\cancel{n=1,...,N+1}$ $n=1,...,N+1$ in parallel by:

3: $\cancel{\mathbf{p}_n^{k+1} = \text{argmin}\,\{\Theta_n(\mathbf{p}_n) + \tfrac{\rho}{2}\|A_n\mathbf{p}_n + \sum_{j\neq n} A_j\mathbf{p}_j^k - c - \tfrac{\Lambda^k}{\rho}\|_2^2 + \tfrac{1}{2}\|\mathbf{p}_n - \mathbf{p}_n^k\|_\Psi^2\}}$ $\boldsymbol{p}_n^{k+1} = \text{argmin}\,\{\Theta_n(\boldsymbol{p}_n) + \tfrac{\rho}{2}\|A_n\boldsymbol{p}_n + \sum_{j\neq}$

4: Update the dual variable $\cancel{\Lambda}$ Λ by:

5: $\cancel{\Lambda^{k+1} = \Lambda^k - \gamma\rho(\sum_{n=1}^{N+1} A_n\mathbf{p}_n^{k+1} - c)}$ $\Lambda^{k+1} = \Lambda^k - \gamma\rho(\sum_{n=1}^{N+1} A_n\boldsymbol{p}_n^{k+1} - c)$

6: **if** $\cancel{\sum_{n=1}^{N+1}\|\mathbf{p}_n^{k+1} - \mathbf{p}_n^k\|_F \le \varepsilon^{pri}}$ $\sum_{n=1}^{N+1}\|\boldsymbol{p}_n^{k+1} - \boldsymbol{p}_n^k\|_F \le \varepsilon^{pri}$ **then**

7: Break.

8: **end if**

9: **end for**

where

$$\cancel{\mathcal{J}_1() = \sum_{n=1}^{N+1}}\theta_n(\boldsymbol{p}_n), \cancel{\theta_n(n)} = \Delta t \boldsymbol{E}\boldsymbol{p}_n - (\|\boldsymbol{p}_n\|_1 - \Gamma_n)^2 + \|\boldsymbol{p}_n\|_1,$$

$$\theta_{N+1}(\boldsymbol{p}_{N+1}) = D(\boldsymbol{p}_{N+1} + B_t).$$

To tackle the core problem using the ADMM method, we initially introduce an indicator function of $\cancel{\mathcal{C}}\mathcal{P}_n$, denoted as:

$$\mathcal{I}_{\cancel{\mathcal{C}}\mathcal{P}_n}(\boldsymbol{p}_n) = \begin{cases} 0 & \text{if } \boldsymbol{p}_n \in \mathcal{P}_n \\ \infty & \text{otherwise.} \end{cases}$$

and let

$$\Theta_n(\boldsymbol{p}_n) := \theta_n(\boldsymbol{p}_n) + \mathcal{I}_{\mathcal{P}_n}(\boldsymbol{p}_n), \forall 1 \le n \le N, \quad (27)$$

$$\Theta_{N+1}(\boldsymbol{p}_{N+1}) := \theta_{N+1}(\boldsymbol{p}_{N+1}) \quad (28)$$

Thus, problem (20) can be equivalently ~~written~~ rewritten in the following form:

$$\cancel{min}\,\text{min} \quad \sum_{n=1}^{N+1}\Theta_n(\boldsymbol{p}_n), \quad (29)$$

$$\text{s.t} \quad \sum_{n=1}^{N+1}\boldsymbol{p}_n = \cancel{0}\,\mathbf{0}.$$

~~To utilize Amproved ADMM, we define~~

$$\cancel{=_{1,\cdots,N,N+1}{}^T \in \mathcal{R}^{(N+1)\times T}}$$

The augmented lagrangian ~~:=~~ ~~where $\rho > 0$~~ of (29) is:

$$\mathcal{L}_\rho(\boldsymbol{p}_1,\cdots,\boldsymbol{p}_{N+1},\boldsymbol{u}) = \sum_{n=1}^{N+1}\Theta_i(\boldsymbol{p}_i) + \frac{\rho}{2}\|\sum_{i=1}^{N}\boldsymbol{p}_i - \boldsymbol{p}_{N+1} + u\|_2^2, \quad (30)$$

where $\rho > 0$ is the standard penalty parameter, $\cancel{u}$ u is referred to as the scaling dual variable or scaling Lagrange multiplier. The variables $\cancel{p}$ $\boldsymbol{p}_i$ and $\cancel{u}$ u are updated simultaneously by adding additional approximation terms. The specific update procedures are as follows.

$$\boldsymbol{p}_n^{k+1} := \arg\min_{\boldsymbol{p}\in\mathcal{P}}\Theta_n(\boldsymbol{p}_n) + \frac{\rho}{2}\|\boldsymbol{p}_n^k + \sum_{n\neq j}p_j^k - \boldsymbol{u}^k\|_2^2 + \frac{1}{2}\|\boldsymbol{p}_n - \boldsymbol{p}_n^k\|_\Psi^2,$$

$$\cancel{\Lambda^{k+1}}\Lambda^{k+1} = \Lambda^k + \gamma\rho(\sum_{n=1}^{N+1}A_n\cancel{\mathbf{p}}\boldsymbol{p}_n^{k+1}), \quad (31)$$

~~where $\Phi, \Psi, \gamma > 0$~~ where $\Phi, \Psi, \gamma > 0$ satisfy

$$\Phi \succ \rho(\frac{1}{\mu_1} - 1)I, \quad \Psi \succ \rho(\frac{1}{\mu_2} < 2 - \gamma)I,$$
$$\mu_1 + \mu_2 < 2 - \gamma, \tag{32}$$

~~for $\mu_1 > 0, \mu_2 > 0$.~~ for $\mu_1 > 0, \mu_2 > 0$. Here, (32) is ~~suffucuent~~ sufficient for the convergence of the improved ADMM.

~~Several~~ There are several viable selections of ~~Φ, Ψ, and γ exist~~ Φ, Ψ, and γ that satisfy (32). For this purpose, a straightforward choice applied here involves setting ~~$\Phi = \phi I$, $\Psi = \psi I$~~ $\Phi = \phi I$, and $\Psi = \psi I$, thereby fulfilling

$$\phi \succ \rho(\frac{1}{\mu_1} - 1), \psi \succ \rho(\frac{1}{\mu_2} < 2 - \gamma)$$
$$\mu_1 + \mu_2 < 2 - \gamma \tag{33}$$

Remark 1：It is important to note that the Jacobian ADMM mentioned in [20] is parallel and can be directly applied to solve the model. However, it requires all electric vehicles to share information, while our preference here is for each intelligent agent to be connected only to a few neighbors.

Remark2: We propose that ~~$Amproved-ADMM$~~ $Amproved-ADMM$ offers several advantages over the conventional ADMM. Firstly, in the ADMM variant presented in [13], the variables ~~(P, X, u)~~ (P, X, u)? must be updated sequentially, similar to the classical centralized ADMM. This means that in iteration ~~k + 1, updating P~~ $k+1$, updating P first and using it to update ~~X~~X, and then updating ~~u~~ u are predetermined steps, and variable updates cannot occur simultaneously. In contrast, our proposed ~~$Amproved-ADMM$~~ $Amproved-ADMM$ algorithm significantly enhances the algorithm presented in [Classical] by allowing simultaneous updates of ~~P, X, u~~ P, X, u in any order. This is particularly crucial for accelerating computation time in parallel and distributed computing, especially when the number of agents is large. Secondly, the ability to update variables in parallel accelerates the convergence speed of our proposed DCJ-ADMM algorithm to ~~$o(1/k)$~~ $o(1/k)$, which is noticeably faster than the ~~$O(1/k)$~~ $O(1/k)$ convergence speed in the algorithm presented in ADMM

C. Algorithms

If the number of iterations reaches a given threshold ~~$iter_{max}$~~ $iter_{max}$ or satisfies the following conditions simultaneously [21], then ADMM meets the following criteria:

$$\boldsymbol{r}^{k+1} = \|\boldsymbol{P}^{k+1} - \boldsymbol{Y}^{k+1}\|_F$$

where $\boldsymbol{P} := [\boldsymbol{p}_1, \ldots, \boldsymbol{p}_N, \boldsymbol{p}_{N+1}]' \in \mathcal{R}^{(N+1)\times T}$.

$$\boldsymbol{s}^{k+1} = \|\rho(\boldsymbol{Y}^{k+1} - \boldsymbol{Y}^k)\|_F$$

this shows that when the residual ~~$\boldsymbol{r}^k$ and $\boldsymbol{s}^k$~~ $\boldsymbol{r}^k$ and $\boldsymbol{s}^k$ are small as follow

$$\|\boldsymbol{r}^k\|_2 \le \varepsilon^{pri} \qquad \|\boldsymbol{s}^k\|_2 \le \varepsilon^{dual}$$

where ~~$\varepsilon^{pri} > 0$ and $\varepsilon^{dual} > 0$~~ $\varepsilon^{pri} > 0$ and $\varepsilon^{dual} > 0$ are feasibility tolerances for the primal and dual feasibity conditions () and (). These tolerances can be chosen using an absolute and relative criterion, such as

$$\varepsilon^{pri} = \sqrt{N}\varepsilon^{abs} + \varepsilon^{rel} max\{\|p\|_2, \|-x\|_2\},$$
$$\varepsilon^{dual} = \sqrt{N}\varepsilon^{abs} + \varepsilon^{rel}\|\mu^k\|_2$$

where ~~$\varepsilon^{abs} > 0$~~ $\varepsilon^{abs} > 0$ is an absolute tolerance and ~~$\varepsilon^{rel} > 0$~~ $\varepsilon^{rel} > 0$ is a relative tolerance. A reasonable value for the relative stopping criterion might be ~~10^{-3}~~ 10^{-3} or ~~10^{-4}~~ 10^{-4}. A pseudo-code for the proposed Improved -ADMM is then give in Algorithm 2. Here,

$$A_n = 1, \forall 1 \le n \le N; \quad A_{N+1} = -1; \quad c = 0.$$

下面的定理需要证明或者引用参考文献？

Theorem 1. *For ~~$k \ge 1$~~ $k \ge 1$, we have*

$$\|u^k - u^*\|_G^2 - \|u^{k+1} - u^*\|_G^2 \ge \|u^k - u^{k+1}\|_Q^2$$

where ~~$\|u^k - u^{k+1}\|_Q^2 := \|x^k - x^{k+1}\|_{G_x}^2 + \frac{2-\gamma}{\rho\gamma^2}\|\lambda^k - \lambda^{k+1}\|^2 + \frac{2}{\gamma}(\lambda^k -$~~
$\|u^k - u^{k+1}\|_Q^2 := \|x^k - x^{k+1}\|_{G_x}^2 + \frac{2-\gamma}{\rho\gamma^2}\|\lambda^k - \lambda^{k+1}\|^2 + \frac{2}{\gamma}(\lambda^k - \lambda^{k+1})$

Theorem 2. *If* ~~*$Q \succ 0$ and $M_x \succeq 0$, then $\|u^k - u^{k+1}\|_M^2 = o(1/k)$, however $\|x^k - x^{k+1}\|_{M_x}^2 = o(1/k)$ and $\|\lambda^k - \lambda^{k+1}\|^2 = o(1/k)$.*~~ *$Q \succ 0$ and $M_x \succeq 0$, then $\|u^k - u^{k+1}\|_M^2 = o(1/k)$, however $\|x^k - x^{k+1}\|_{M_x}^2 = o(1/k)$ and $\|\lambda^k - \lambda^{k+1}\|^2 = o(1/k)$.*

IV. Results and Discussion

In this section, we will present and discuss the experimental results obtained, highlighting a comparative analysis of efficiency between the traditional ADMM algorithm and our proposed approach. The results will shed light on the advantages and potential improvements offered by our approach in terms of optimization results and computational efficiency.

A. Assumptions and Parameter Settings

charging rate
p_n^{min} and p_n^{max}
η

For simplicity, we assumed all EVs to be BYD Seagull 305 with the same battery capacity of 30 kWh.

1) EV depart and return: The times for electric vehicles to depart from and return to the grid within a day followed normal distributions of ~~$N(18, 3.3^2)$~~ $N(18, 3.3^2)$ (see Fig. 1) and ~~$N(8, 3.24^2)$~~ $N(8, 3.24^2)$ (see Fig. 2), respectively [22].

2) fundamental load data: In this paper, we consider the fundamental load data following the electricity usage pattern [23], and the specific data is illustrated in Fig. 3. The typical profile of the basic load curve within a daily-cycle for a regional microgrid can be observed. Peak loads are observed to occur approximately at 6 hours, 10 hours, and 18 hours. The highest observed load is approximately 710.62 kW, while the lowest load recorded is 445.69 kW.

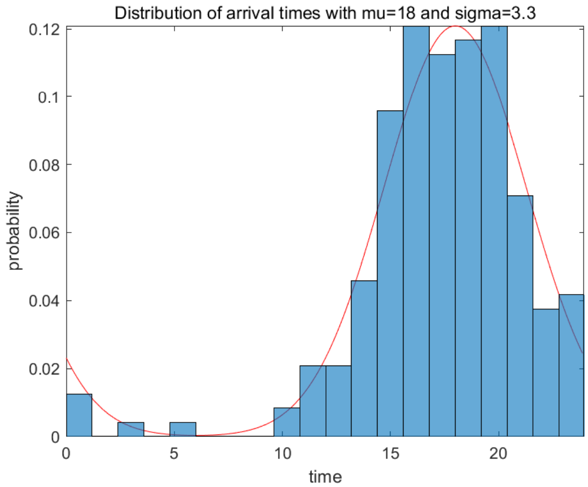


Fig. 1. arrive

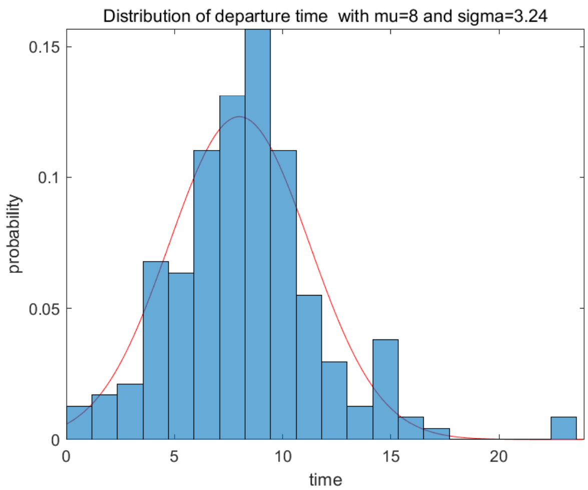


Fig. 2. depart

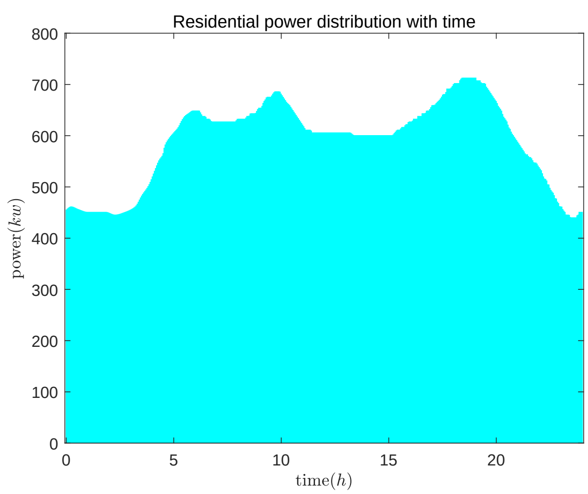


Fig. 3. Typical basic load profile in one-day cycle.

3) electric price: We have set the electricity prices within a day as shown in the Fig. 4. Two high-priced time periods can be observed, from 9:00 am to 1:00 pm and from 6:00 pm to

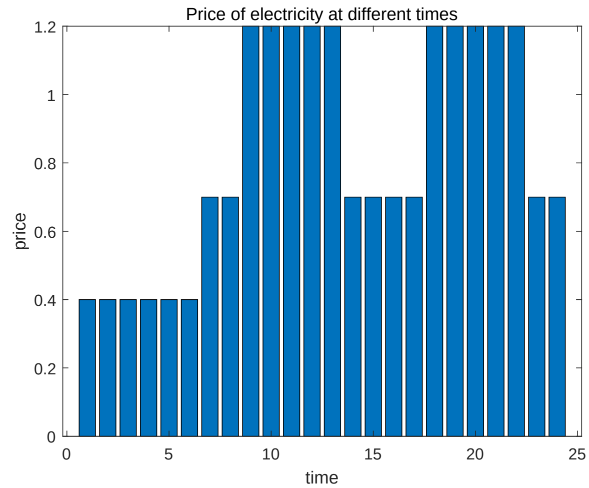


Fig. 4. TOU electricity price

10:00 pm. During peak electricity consumption periods, prices are generally higher, which in turn may suppress the desire of electric vehicle users to charge their vehicles. The pricing setup in this paper is similar to that assumed in the literature [?].

4) assume about SOC: We assume that the initial state of charge (~~SOC_c~~ SOC_c) of electric vehicles is uniformly and continuously distributed between 0.1 and 0.3, while the expected charging completion state (~~SOC_d~~ SOC_d) is uniformly and continuously distributed between 0.7 and 0.9.

B. Effect of Varying ~~ρ~~ ρ on Convergence Speed

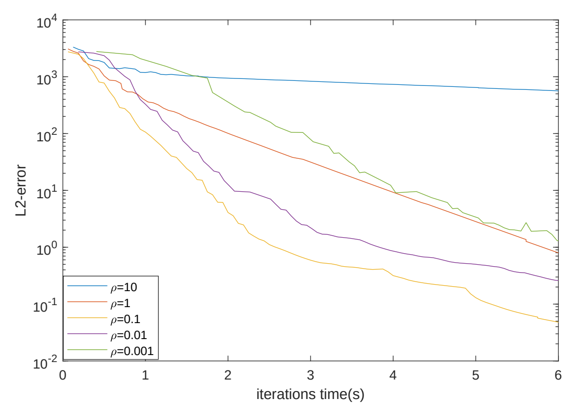


Fig. 5. The convergence of Algorithm 2 under k=1 with respect to different ~~ρ~~ ρ

As depicted in Fig. 5, the convergence behavior of the improved ADMM algorithm under different ~~ρ~~ ρ conditions is examined. It can be observed from the graph that the convergence is least favorable when ~~$\rho = 10$~~ $\rho = 10$, while it attains optimal convergence when ~~$\rho = 0.1$~~ $\rho = 0.1$. It is important to note that the convergence does not exhibit a linear improvement with increasing values of ~~ρ~~ ρ, nor does it necessarily improve with decreasing values. Instead, an optimal convergence is achieved within a moderate range of ~~p~~ ρ values.

C. Effect of Peaking shaving and valley filling

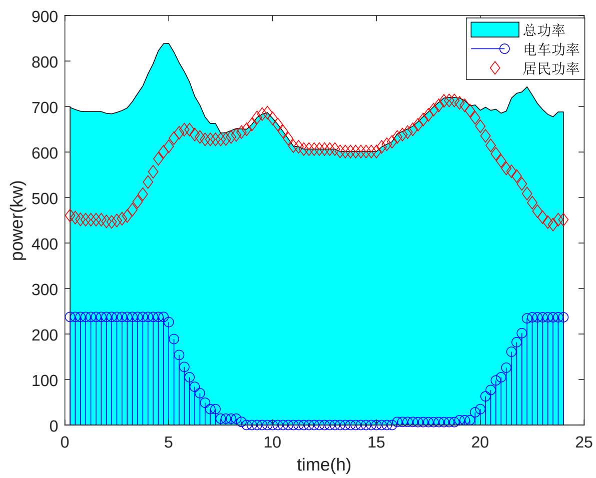


Fig. 6. Rusult of no Peaking shaving and valley filling

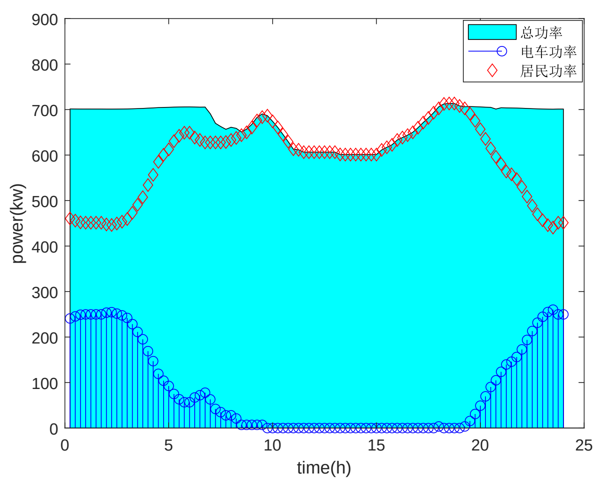


Fig. 7. Rusult of Peaking shaving and valley filling

Total power is the sum of the power consumed by electric vehicles and residential electricity usage. Our objective is to minimize the difference between the maximum (peak power) and minimum (low power) values of the total power. Considering that residential electricity usage typically peaks during the day and decreases at night, which differs from the charging pattern of electric vehicles, peak shaving and valley filling of the grid's total power can be achieved through proper optimization control. In this case study, we selected 100 electric vehicles for investigation. The weight coefficients for each optimization objective were chosen as (1, 1, 0, 1) and (1, 1, 100, 1) to test the differences in the effectiveness of power control. Specifically, Fig. 6 illustrates the power situation without power stabilization, while Fig. 7 shows the power profile after peak shaving and valley filling optimization. It is evident that the power peaks at 5 AM and 11 PM have been eliminated, resulting in a significant reduction in the absolute power difference from ~~250kw~~ 250 kw to 100 kw.

D. Computational efficiency compared to traditional ADMM

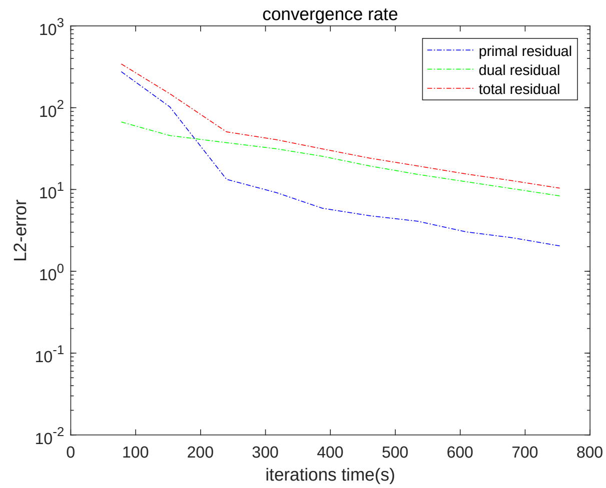


Fig. 8. Result of covergence by ADMM

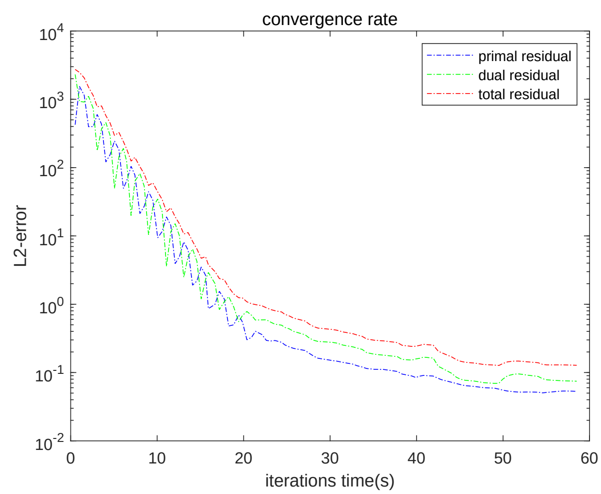
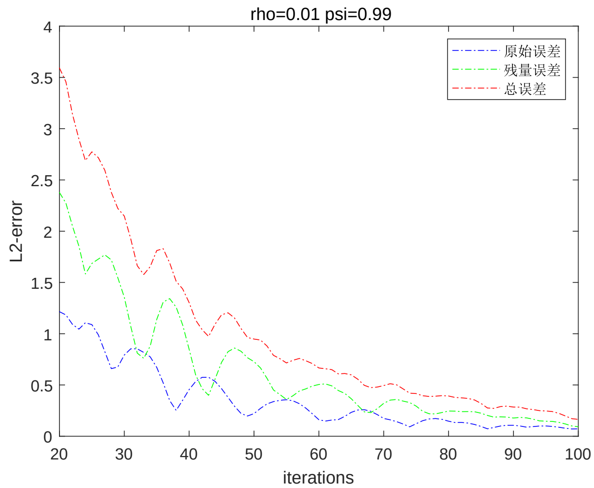
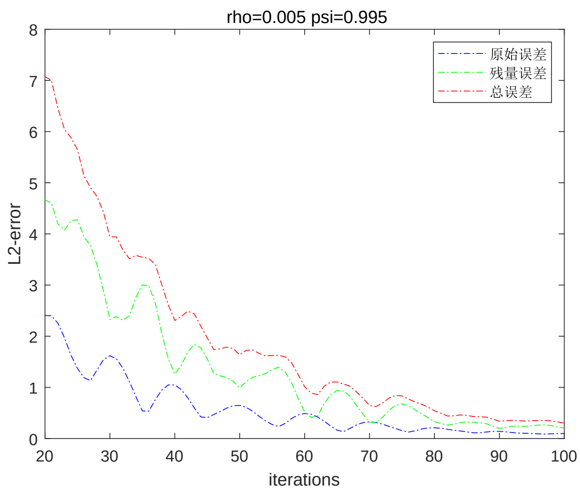


Fig. 9. Result of covergence by Amproved ADMM

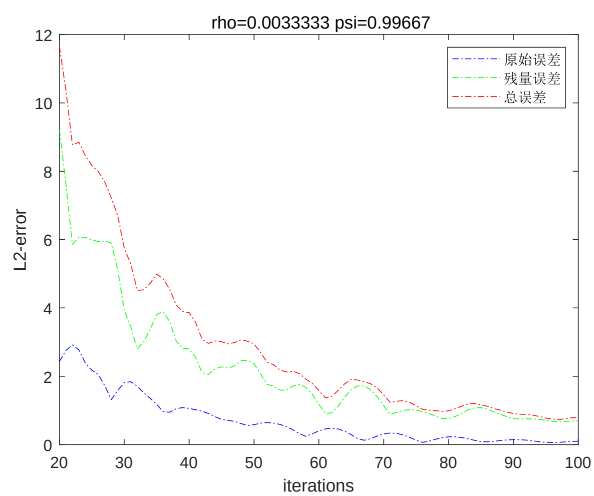
We conducted separate calculations of the decay rates for the primal and dual errors throughout the iterative process. In order to facilitate the observation of error decay trends, we defined the total error as the sum of these two errors. The convergence results of the classical ADMM method are depicted in Fig. 9, while the improved Jacobi-proximal ADMM method's convergence results are presented in Fig. 9. A comparison between the two methods reveals that the improved ADMM method demonstrates accelerated convergence when addressing the optimization problems investigated in this study. Within a CPU time of 60 seconds, it achieves a reduction in error to ~~10^{-2}~~ 10^{-2}. Consequently, the implementation of parallel algorithms significantly enhances computational efficiency.



(a)100EV



(a)200EV



(a)300EV

Fig. 10. Visual comparisons of original models.

E. Numerical performance of large vehicles

To further investigate the performance of our proposed model in handling large-scale data, we conducted experiments for different numbers of electric vehicles (EVs), ranging from 100 to 300. The results were compared by analyzing the error decay trends of the algorithm, as depicted in Fig. 10. It is evident that increasing the number of EVs does not significantly impede the convergence of the error. This highlights the robustness and scalability of our proposed algorithm for addressing large-scale optimization problems. In addition, the algorithm's promising performance in handling big data implies its potential applications in real-world scenarios, where the amount of data is typically large and diverse. Therefore, our proposed model could be instrumental in providing efficient and effective solutions to various complex optimization tasks across a broad range of domains.

V. Conclusion

References

[1] M. Li, M. Lenzen, F. Keck, B. McBain, O. Rey-Lescure, B. Li, and C. Jiang, "Gis-based probabilistic modeling of bev charging load for australia," *IEEE Transactions on Smart Grid*, vol. 10, no. 4, pp. 3525–3534, 2018.

[2] P. Wang, D. Wang, C. Zhu, Y. Yang, H. M. Abdullah, and M. A. Mohamed, "Stochastic management of hybrid ac/dc microgrids considering electric vehicles charging demands," *Energy Reports*, vol. 6, pp. 1338–1352, 2020.

[3] N. I. Nimalsiri, E. L. Ratnam, C. P. Mediwaththe, D. B. Smith, and S. K. Halgamuge, "Coordinated charging and discharging control of electric vehicles to manage supply voltages in distribution networks: Assessing the customer benefit," *Applied Energy*, vol. 291, p. 116857, 2021.

[4] J. De Hoog, T. Alpcan, M. Brazil, D. A. Thomas, and I. Mareels, "Optimal charging of electric vehicles taking distribution network constraints into account," *IEEE Transactions on Power Systems*, vol. 30, no. 1, pp. 365–375, 2014.

[5] F. Kennel, D. Görges, and S. Liu, "Energy management for smart grids with electric vehicles based on hierarchical mpc," *IEEE Transactions on industrial informatics*, vol. 9, no. 3, pp. 1528–1537, 2012.

[6] M. Liu, P. K. Phanivong, Y. Shi, and D. S. Callaway, "Decentralized charging control of electric vehicles in residential distribution networks," *IEEE Transactions on Control Systems Technology*, vol. 27, no. 1, pp. 266–281, 2017.

[7] L. Zhang, V. Kekatos, and G. B. Giannakis, "Scalable electric vehicle charging protocols," *IEEE Transactions on Power Systems*, vol. 32, no. 2, pp. 1451–1462, 2016.

[8] X. Zhou, S. Zou, P. Wang, and Z. Ma, "Voltage regulation in constrained distribution networks by coordinating electric vehicle charging based on hierarchical admm," *IET Generation, Transmission & Distribution*, vol. 14, no. 17, pp. 3444–3457, 2020.

[9] T. Rahman, Y. Xu, and Z. Qu, "Continuous-domain real-time distributed admm algorithm for aggregator scheduling and voltage stability in distribution network," *IEEE Transactions on Automation Science and Engineering*, vol. 19, no. 1, pp. 60–69, 2021.

[10] X. Zhou, P. Wang, and Z. Gao, "Admm-based decentralized charging control of plug-in electric vehicles with coupling constraints in distribution networks," in *2018 37th Chinese Control Conference (CCC)*. IEEE, 2018, pp. 2512–2517.

[11] S. Fahmy, R. Gupta, and M. Paolone, "Grid-aware distributed control of electric vehicle charging stations in active distribution grids," *Electric Power Systems Research*, vol. 189, p. 106697, 2020.

[12] X. Zhou, S. Zou, P. Wang, and Z. Ma, "Admm-based coordination of electric vehicles in constrained distribution networks considering fast charging and degradation," *IEEE Transactions on Intelligent Transportation Systems*, vol. 22, no. 1, pp. 565–578, 2020.

[13] N. I. Nimalsiri, C. P. Mediwaththe, E. L. Ratnam, M. Shaw, D. B. Smith, and S. K. Halgamuge, "A survey of algorithms for distributed charging control of electric vehicles in smart grid," *IEEE Transactions on Intelligent Transportation Systems*, vol. 21, no. 11, pp. 4497–4515, 2019.

[14] M. Kraning, E. Chu, J. Lavaei, S. Boyd *et al.*, "Dynamic network energy management via proximal message passing," *Foundations and Trends® in Optimization*, vol. 1, no. 2, pp. 73–126, 2013.

[15] Q. Liu, X. Shen, and Y. Gu, "Linearized admm for nonconvex nonsmooth optimization with convergence analysis," *IEEE access*, vol. 7, pp. 76 131–76 144, 2019.

[16] D. Yuan, Y. Hong, D. W. Ho, and G. Jiang, "Optimal distributed stochastic mirror descent for strongly convex optimization," *Automatica*, vol. 90, pp. 196–203, 2018.

[17] S. Cheng, S. Liang, Y. Fan, and Y. Hong, "Distributed gradient tracking for unbalanced optimization with different constraint sets," *IEEE Transactions on Automatic Control*, 2022.

[18] A. Nedic, A. Olshevsky, and W. Shi, "Achieving geometric convergence for distributed optimization over time-varying graphs," *SIAM Journal on Optimization*, vol. 27, no. 4, pp. 2597–2633, 2017.

[19] D. Jakovetić, D. Bajović, J. Xavier, and J. M. Moura, "Primal–dual methods for large-scale and distributed convex optimization and data analytics," *Proceedings of the IEEE*, vol. 108, no. 11, pp. 1923–1938, 2020.

[20] W. Deng, M.-J. Lai, Z. Peng, and W. Yin, "Parallel multi-block admm with o (1/k) convergence," *Journal of Scientific Computing*, vol. 71, pp. 712–736, 2017.

[21] S. Boyd, N. Parikh, E. Chu, B. Peleato, J. Eckstein *et al.*, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Foundations and Trends® in Machine learning*, vol. 3, no. 1, pp. 1–122, 2011.

[22] L. Liu and K. Zhou, "Electric vehicle charging scheduling considering urgent demand under different charging modes," *Energy*, vol. 249, p. 123714, 2022.

[23] Y. Zheng, Y. Shang, Z. Shao, and L. Jian, "A novel real-time scheduling strategy with near-linear complexity for integrating large-scale electric vehicles into smart grid," *Applied Energy*, vol. 217, pp. 1–13, 2018.

[24] B. He, M. Tao, and X. Yuan, "Alternating direction method with gaussian back substitution for separable convex programming," *SIAM Journal on Optimization*, vol. 22, no. 2, pp. 313–340, 2012.

[25] B. Shakerighadi, A. Anvari-Moghaddam, E. Ebrahimzadeh, F. Blaabjerg, and C. L. Bak, "A hierarchical game theoretical approach for energy management of electric vehicles and charging stations in smart grids," *Ieee Access*, vol. 6, pp. 67 223–67 234, 2018.

[26] J. Tan and L. Wang, "Real-time charging navigation of electric vehicles to fast charging stations: A hierarchical game approach," *IEEE transactions on smart grid*, vol. 8, no. 2, pp. 846–856, 2015.

[27] T. Zhao, Y. Li, X. Pan, P. Wang, and J. Zhang, "Real-time optimal energy and reserve management of electric vehicle fast charging station: Hierarchical game approach," *IEEE Transactions on Smart Grid*, vol. 9, no. 5, pp. 5357–5370, 2017.

[28] B. Khaki, C. Chu, and R. Gadh, "Hierarchical distributed framework for ev charging scheduling using exchange problem," *Applied energy*, vol. 241, pp. 461–471, 2019.

[29] W. Qi, Z. Xu, Z.-J. M. Shen, Z. Hu, and Y. Song, "Hierarchical coordinated control of plug-in electric vehicles charging in multifamily dwellings," *IEEE Transactions on Smart Grid*, vol. 5, no. 3, pp. 1465–1474, 2014.

[30] F. Xia, H. Chen, L. Chen, and X. Qin, "A hierarchical navigation strategy of ev fast charging based on dynamic scene," *IEEE Access*, vol. 7, pp. 29 173–29 184, 2019.

[31] J. Hu, C. Si, M. Lind, and R. Yu, "Preventing distribution grid congestion by integrating indirect control in a hierarchical electric vehicles' management system," *IEEE Transactions on Transportation Electrification*, vol. 2, no. 3, pp. 290–299, 2016.

[32] S. Yang, S. Zhang, and J. Ye, "A novel online scheduling algorithm and hierarchical protocol for large-scale ev charging coordination," *IEEE Access*, vol. 7, pp. 101 376–101 387, 2019.

[33] Z. Ma, S. Zou, and X. Liu, "A distributed charging coordination for large-scale plug-in electric vehicles considering battery degradation cost," *IEEE Transactions on Control Systems Technology*, vol. 23, no. 5, pp. 2044–2052, 2015.

[34] A. Ghavami, K. Kar, and A. Gupta, "Decentralized charging of plug-in electric vehicles with distribution feeder overload control," *IEEE Transactions on Automatic Control*, vol. 61, no. 11, pp. 3527–3532, 2016.

[35] J. Koshal, A. Nedić, and U. V. Shanbhag, "Multiuser optimization: Distributed algorithms and error analysis," *SIAM Journal on Optimization*, vol. 21, no. 3, pp. 1046–1081, 2011.

[36] M. R. Sarker, M. A. Ortega-Vazquez, and D. S. Kirschen, "Optimal coordination and scheduling of demand response via monetary incentives," *IEEE Transactions on Smart Grid*, vol. 6, no. 3, pp. 1341–1352, 2014.

[37] B. Yang, J. Li, Q. Han, T. He, C. Chen, and X. Guan, "Distributed control for charging multiple electric vehicles with overload limitation," *IEEE Transactions on Parallel and Distributed Systems*, vol. 27, no. 12, pp. 3441–3454, 2016.

[38] O. Ardakanian, C. Rosenberg, and S. Keshav, "Distributed control of electric vehicle charging," in *Proceedings of the fourth international conference on Future energy systems*, 2013, pp. 101–112.

[39] Z. Ma, N. Yang, S. Zou, and Y. Shao, "Charging coordination of plug-in electric vehicles in distribution networks with capacity constrained feeder lines," *IEEE Transactions on Control Systems Technology*, vol. 26, no. 5, pp. 1917–1924, 2017.

[40] N. I. Nimalsiri, E. L. Ratnam, C. P. Mediwaththe, D. B. Smith, and S. K. Halgamuge, "Coordinated charging and discharging control of electric vehicles to manage supply voltages in distribution networks: Assessing the customer benefit," *Applied Energy*, vol. 291, p. 116857, 2021.

[1] [2] [3] [4] [5] [6] [7] [8] [9] [10] [11] [12] [13] [14] [15] [24] [25] [26] [27] [28] [29] [30] [31] [32] [33] [34] [35] [36] [37] [38] [39] [20] [23] [22] [21] [40]