# Analysis-01

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## 1 Exercise

Suppose $\sim$ is an equivalence relation on $S$.

We define two sets: $\langle a \rangle := \{x \in S | x \sim a\}$ and $S/\sim := \{\langle a \rangle | a \in S\}$. And a *partition* $P$ on $S$:

1. $P$ is a collection of some subsets of $S$: $P \subseteq P(S)$.
2. $\forall X, Y \in P$ if $X \neq Y$, then $X \cap Y = \emptyset$.
3. The union of all elements in $P$ equals to $S$.

**What you have to do:**

1. Prove that
$$\forall X, Y \in S, X = Y \text{ or } X \cap Y = \emptyset.$$
2. Prove that
$$\bigcup_{X \in S/\sim} X = S.$$
With 1 and 2 we can conclude that an equivalence relation defines a *partition* on $S$.
3. Show that if we have a *partition* of $S$, then there is a unique equivalence relation $\sim$ on $S$ corresponding to that partition. (**Hint:** consider the relation $R$: $xRy \Leftrightarrow \exists X \in P, \{x, y\} \subseteq X$)

## 2 Exercise

Prove that:

1. $\forall$ injective $f : A \to B$, $\exists$ surjective $h : B \to A$, such that:
$$\forall x \in A, h(f(x)) = x.$$
2. $\forall$ surjective $f : A \to B$, $\exists$ injective $h : B \to A$, such that:
$$\forall y \in B, f(h(x)) = y.$$