第 1 章 绪论

内容提要

- ❑ 误差分析
- ❑ 有效数字
- ❑ 图表处理

此章涉及误差合成计算, 有效数字位数保留和实验数据处理等, 如果你在处理数据时感到迷茫和困惑, 不妨回过头来看看第一部分的内容.

1.1 误差

误差传递公式, 相对误差公式, 仪器误差计算, 误差来源分析.

提示

1. p11-(1-9), 表 1-1 下面的大括号, p12-例 1, 表 1-2, 表 1-3;
2. 误差的有效数字 1 2 位;
3. 科学计数法在误差表示结果中的使用;

1.2 有效数字

p27-1.5.2.

1.3 实验数据处理

列表和作图要写表名和图名, 多线并存时要写图注, 必要时可以使用计算机拟合 (相信你已经掌握了一部分 MATLAB 的知识).

1.4 绪论作业

作业 1.1 按有效数字运算规则计算下列各式

1. $(8.42 + 0.052 - 0.47)/2.001 = 4.00$;
2. $(100.25 - 100.23)/100.22 = 2 \times 10^{-4}$.

作业 1.2 单位变换

1. $m = 1.750 \pm 0.003$ kg, 写成以 g, mg, t 为单位.

   解

   (1). $m = (1.750 \pm 0.003) \times 10^3$ g;

   (2). $m = (1.750 \pm 0.003) \times 10^6$ mg;

   (3). $m = (1.750 \pm 0.003) \times 10^{-3}$ t.
2. $t = 1.8 \pm 0.1$ min, 写成以秒为单位.

   解 $t = 108 \pm 6$ s.

作业 1.3 计算下列数据的算数平均值, 标准偏差及平均值的标准偏差, 正确表达测量结果 (计算相对误差)

1. l_i (cm): 3.4298, 3.4256, 3.4278, 3.4190, 3.4262, 3.4234, 3.4263, 3.4242, 3.4272, 3.4216

   解

   (1). $\bar{l} = \dfrac{1}{10}\sum_{i=1}^{10} l_i = 3.4251$ (cm);

   (2). $s = \sqrt{\dfrac{1}{11}\sum_{i=1}^{12}(\Delta t_i)^2} = 0.003$ (cm);

   (3). $s_{\bar{l}} = \dfrac{s}{\sqrt{10}} = 0.001$ (cm);

   (4). $E = \dfrac{s_{\bar{l}}}{\bar{l}} \times 100\% = 0.029\%$.

2. t_i (s): 1.35, 1.26, 1.38, 1.33, 1.30, 1.29, 1.33, 1.32, 1.32, 1.34, 1.29, 1.36

   解

   (1). $\bar{t} = \dfrac{1}{12}\sum_{i=1}^{12} t_i = 1.32$ s;

   (2). $s = \sqrt{\dfrac{1}{11}\sum_{i=1}^{12}(\Delta t_i)^2} = 0.03$ s;

   (3). $s_{\bar{t}} = \dfrac{s}{\sqrt{12}} = 0.01$ s;

   (4). $E = \dfrac{s_{\bar{t}}}{\bar{t}} \times 100\% = 0.73\%$.

✍ **作业 1.4** 改写正确的误差传递公式 (算数合成法)

1. $E = \dfrac{4\rho l^3}{\lambda a b^3}$ $\dfrac{\Delta E}{E} = \dfrac{\Delta\rho}{\rho} + \dfrac{\Delta l}{l^3} - \dfrac{\Delta\lambda}{\lambda} - \dfrac{\Delta a}{a} - \dfrac{\Delta b}{b^2}$

   解

   $$\begin{aligned}\Delta E &= \left|\frac{\partial \ln E}{\partial \rho}\right|\Delta\rho + \left|\frac{\partial \ln E}{\partial l}\right|\Delta l + \left|\frac{\partial \ln E}{\partial \lambda}\right|\Delta\lambda + \left|\frac{\partial \ln E}{\partial a}\right|\Delta a + \left|\frac{\partial \ln E}{\partial b}\right|\Delta b \\ &= \frac{\Delta\rho}{\rho} + \frac{3\Delta l}{l} + \frac{\Delta\lambda}{\lambda} + \frac{\Delta a}{a} + \frac{3\Delta b}{b}\end{aligned}$$

2. $V = \dfrac{1}{6}\pi d^2$ $\dfrac{\Delta V}{V} = \dfrac{1}{2}\pi\dfrac{\Delta d}{d}$

   解 $\dfrac{\Delta V}{V} = \left|\dfrac{\mathrm{d}\ln V}{\mathrm{d}d}\right|\Delta d = \dfrac{2\Delta d}{d}$

3. $N = x^2 - 2xy + y^2$ $\Delta N = 2\Delta x + 2\Delta x\Delta y + 2\Delta y$

   解 $\Delta N = \left|\dfrac{\partial N}{\partial x}\right|\Delta x + \left|\dfrac{\partial N}{\partial y}\right|\Delta y = |2x - 2y|(\Delta x + \Delta y)$

4. $N = k\sin x$ $\Delta N = \Delta k + \tan x$

   解 $\Delta N = \left|\dfrac{\partial N}{\partial k}\right|\Delta k + \left|\dfrac{\partial N}{\partial x}\right|\Delta x = k|\cos x|\Delta x + |\sin x|\Delta k$

✍ **作业 1.5** 求出下列函数的算术合成法误差传递式 (等式右端未经说明者均为直接测得量, 绝对误差或相对误差任写一种)

1. $N = x + y - 2z$

   解 $\Delta N = \left|\dfrac{\partial N}{\partial x}\right|\Delta x + \left|\dfrac{\partial N}{\partial y}\right|\Delta y + \left|\dfrac{\partial N}{\partial z}\right|\Delta z = \Delta x + \Delta y + 2\Delta z.$

2. $Q=\dfrac{k}{2}\left(A^2+B^2\right),k$ 为常量

   解 $\Delta Q=\left|\dfrac{\partial Q}{\partial A}\right|\Delta A+\left|\dfrac{\partial Q}{\partial B}\right|\Delta B=kA\cdot\Delta A+kB\cdot\Delta B.$

3. $f=\dfrac{A^2-B^2}{4A}$

   解 $\Delta f=\left|\dfrac{\partial f}{\partial A}\right|\Delta A+\left|\dfrac{\partial f}{\partial B}\right|\Delta B=\dfrac{A^2+B^2}{4A^2}\cdot\Delta A+\dfrac{B}{2A}\cdot\Delta B.$

4. $n=\dfrac{\sin i}{\sin r}$

   解 $\Delta n=\left|\dfrac{\partial n}{\partial i}\right|\Delta i+\left|\dfrac{\partial n}{\partial r}\right|\Delta r=\left|\dfrac{\cos i}{\sin r}\right|\Delta i+\left|\dfrac{\sin i\cos r}{\sin^2 r}\right|\Delta r$

作业 1.6 改正标准偏差传递式中的错误

1. $L=b+\dfrac{1}{2}a, s_L=\sqrt{s_b^2+\dfrac{1}{2}s_a^2}$

   解 $s_L=\sqrt{\left(\dfrac{\partial L}{\partial a}\right)^2s_a^2+\left(\dfrac{\partial L}{\partial b}\right)^2s_b^2}=\sqrt{\dfrac{1}{4}s_a^2+s_b^2}$

2. $\nu=\dfrac{1}{2L}\sqrt{\dfrac{mgl_0}{m_0}}$; g 为常量, $\dfrac{s_\nu}{\nu}=\sqrt{\left(\dfrac{s_L}{L}\right)^2+\dfrac{1}{2}\left(\dfrac{s_m}{m}\right)^2+\dfrac{1}{2}\left(\dfrac{s_{l_h}}{l_0}\right)^2+\dfrac{1}{2}\left(\dfrac{s_{m_0}}{m_0}\right)^2}$

   解

$$\begin{aligned}\frac{s_\nu}{\nu}&=\sqrt{\left(\frac{\partial\ln\nu}{\partial L}\right)^2s_L^2+\left(\frac{\partial\ln\nu}{\partial m}\right)^2s_m^2+\left(\frac{\partial\ln\nu}{\partial l_0}\right)^2s_{l_0}^2+\left(\frac{\partial\ln\nu}{\partial m_0}\right)^2s_{m_0}^2}\\&=\sqrt{\left(\frac{s_L}{L}\right)^2+\left(\frac{s_m}{2m}\right)^2+\left(\frac{s_{l_0}}{2l_0}\right)^2+\left(\frac{s_{m_0}}{2m_0}\right)^2}\end{aligned}$$

作业 1.7 计算下列各式的结果, 并用算术合成法估算误差

1. $N=A+B-\dfrac{1}{3}C,\ A=0.5768\pm0.0002\ \text{cm},\ B=85.07\pm0.02\ \text{cm},\ C=3.247\pm0.002\ \text{cm}$

   解

$$E_N=\frac{\Delta N}{N}\times100\%=0.02\%$$

$$N=84.56\pm0.02\ \text{cm}$$

$$N=A+B-\frac{1}{3}C=0.5768+85.07-\frac{1}{3}\times3.247=84.56\ \text{cm}$$

$$\Delta N=\left|\frac{\partial N}{\partial A}\right|\Delta A+\left|\frac{\partial N}{\partial B}\right|\Delta B+\left|\frac{\partial N}{\partial C}\right|\Delta C=0.0002+0.02+\frac{1}{3}\times0.002=0.02\ \text{cm}$$

2. $V=1000\pm1\ \text{cm}^3$, 求 $\dfrac{1}{V}=\ ?$

   解 令 $f=\dfrac{1}{V}$,

$$\Delta f=\left|\frac{\mathrm{d}f}{\mathrm{d}V}\right|\Delta V=1\times10^{-6}\ \text{cm}^{-3}$$

$$E_{\frac{1}{V}}=\frac{\Delta\left(\dfrac{1}{V}\right)}{\dfrac{1}{V}}\times100\%=0.1\%$$

3. $\nu = \dfrac{h_1}{h_1 - h_2}, h_1 = 45.51 \pm 0.02\ \text{cm}, h_2 = 12.20 \pm 0.02\ \text{cm}$

   解

$$\nu = \frac{45.5}{45.5 - 12.20} = 1.366\ \text{cm}$$

$$\Delta\nu = \left|\frac{\partial \nu}{\partial h_1}\right|\Delta h_1 + \left|\frac{\partial \nu}{\partial h_2}\right|\Delta h_2 = 2 \times 10^{-4}\ \text{cm} = 1.366 \pm 2 \times 10^{-4}\ \text{cm}$$

$$E_\nu = \frac{\Delta \nu}{\nu} \times 100\% = 0.01\%$$

$$\nu = 1.366 \pm 2 \times 10^{-4}\ \text{cm}$$

作业 1.8

1. 利用单摆测定重力加速 g, 当摆角很小的时候有 $T = 2\pi\sqrt{\dfrac{l}{g}}$ 的关系. 式中 T 为周期, l 为摆长, 它们的测量结果分别为 $T = 1.9842 \pm 0.0002\ \text{s}, l = 98.81 \pm 0.02\ \text{cm}$, 求重力加速度及其不确定度, 写出结果表达式.

   解

$$u_g = \sqrt{\left(\frac{8\pi^2 l}{T^3}\right)^2 \Delta T^2 + \left(\frac{4\pi^2}{l^2}\right)^2 \Delta l^2} = 0.003\text{m/s}^2$$

$$g = \frac{4\pi^2 l}{T^2}, l = 0.9881 \pm 0.0002\ \text{m}$$

$$u_r = \frac{u_g}{g} \times 100\% = \sqrt{\frac{4\Delta T^2}{T^2} + \frac{\Delta l^2}{l^2}} \times 100\% = 0.028\%$$

$$\Longrightarrow \begin{cases} g = 9.908^2 \pm 0.003\ \text{m/s}^2 \\ u_r = 0.028\% \end{cases}$$