

Computer Vision

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We firstly sketch some of the basic feature extraction methods, then we introduce those infamous Deep Neural Networks. Next we proceed with feature extraction and matching based on histograms of gradients - they build the basis of many tasks such as object instance detection and image retrieval. We then introduce object detection based on the sliding window technique, i.e. suitable for face and pedestrian detection. It follows a treatment of image processing techniques - segmentation and morphological processing - and of shape recognition techniques. We overview the essential tracking methods - for regions and moving objects. We close with a survey of video surveillance, in-vehicle vision system and remote sensing. The methods are explored in Matlab and sufficient code snippets are provided to immediately explore all those concepts; Python code is provided partially. It is possible to follow the notes without any particular prerequisites, but basic knowledge of linear algebra, signal processing and pattern recognition is of great advantage.

Prerequisites basic programming skills; enthusiasm to write a lot of code

Recommended basic statistical pattern recognition, basic linear algebra, basic signal processing

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1 Introduction

Computer Vision is the field of interpreting image content. It is concerned with the classification of the entire image, such as in a system classifying photos uploaded to the internet (Facebook, Instagram). Or Computer Vision is concerned with the recognition of objects in an image, such as detecting faces or car license plates (Facebook, GoogleStreetView). Or it is concerned with the detection of aspects of an image, such as cancer detection in biomedical images.

Origin Computer Vision was originally founded as a sub-discipline of the field of Artificial Intelligence in the 1970s. The founding goal was to create a system that has the same perceptual capabilities as the human visual system has - your eyes and most of your brain. The human visual system can easily interpret any scene with little effort: it perfectly discriminates between thousands of categories, and it can find objects in scenes within a time span of several hundred milliseconds only; it easily switches between several types of recognition processes with a *flexibility* and *swiftness*, whose complexity and dynamics have not been well understood yet. It quickly turned out, that that goal was rather ambitious.

Instead, Computer Vision has focused on a set of specific recognition challenges, to be introduced in Section 1.2. Those challenges can be often implemented in different ways, with each implementation having advantages and disadvantages. Throughout the decades, many applications have been created (Section 1.3), and some of those implemented tasks begin now to outperform a human observer - such as face identification, letter recognition, or the ability to maneuver through traffic (autonomous vehicles). And that itself is astounding, even though the original goal of an omni-vision system has not been achieved yet. Today, Computer Vision is considered its own field.

Frontier Computer Vision is still considered a frontier, despite its evolution over almost 50 years. The success of modern Computer Vision is less the result of truly novel algorithms, but rather a result of increasing computer speed and memory. In particular shape recognition is - despite its simple sounding task - still not properly understood. And even though Google has a system that can recognize thousands of classes, the system occasionally fails so bluntly, that one may wonder what other algorithms need to be invented in order to achieve a flawless recognition process. If those algorithms will not be invented, then household robots may always make some nerve-stretching errors, such as mistaking the laundry basket for the trash bin, confusing the microwave with the glass cabinet, etc. Thus, despite all the progress that has been made, it still requires innovative algorithms.

In particular in the past several years, Computer Vision has received new impetus by the use of so-called Deep Learning algorithms, with which one can classify decently large image collections. And that is the reason why we treat that topic relatively early (Section 5), after a quick warm up with the classical methods. We then proceed with a method that had been popular just before the arrival of Deep Learning algorithms, namely feature extraction and matching (Sections 6 and 7). Later on, we continue with traditional techniques (Section 9), and we also mention approaches to the most enigmatic challenge of Computer Vision, namely shape recognition (Section 11).

1.1 Related Fields

Several fields are related to Computer Vision, two of which are closely related, namely *Image Processing* and *Machine Vision*; in fact, those two fields overlap with Computer Vision to such an extent, that their names are sometimes used synonymously. Here is an attempt to discriminate between them, although there exist no agreed definitions and distinctions:

Image Processing is concerned with the transformation or other manipulation of the image with the goal to emphasize certain image aspects, e.g. contrast enhancement, or extraction of low-level features such as edges, blobs, etc; in comparison, Computer Vision is rather concerned with higher-level feature extraction and their interpretation for recognition purposes.

Machine Vision is concerned with applying a range of technologies and methods to provide imaging-based automatic inspection, process control and robot guidance in industrial applications. A machine-vision system has typically 3 characteristics:

- 1) objects are seen against an uniform background, which represents a 'controlled situation'.
- 2) objects possess limited structural variability, sometimes only one object needs to be identified.
- 3) the exact orientation in 3D is of interest.

An example is car license plate detection and reading at toll gates, which is a relatively controlled situation. In comparison, computer vision systems often deal with objects of larger variability and objects that are situated in varying backgrounds. Car license plate detection in GoogleStreetView is an example of an object with limited variability but varying context.

There are two other fields that overlap with Computer Vision:

Pattern Recognition (Machine Learning) is the art of classification (or categorization). To build a good computer vision system, it requires substantial knowledge of classification methodology. Sometimes it is even the more significant part of the computer-vision system, as in case of image classification, for which so-called Deep Neural Networks have produced the best classification accuracy so far (Section 5). Clearly, we cannot treat classification in depth in this course and we will merely point out how to use some of the classifiers (Section ??; Appendix E).

Computer Graphics is sometimes considered as part of Computer Vision. The objective in Computer Graphics is to represent objects and scenes as compactly and efficiently as possible; however there is no recognition of any kind involved.

1.2 Recognition - An Overview

We firstly explain the three principal recognition processes and their challenges. Then we mention other recognition objectives.

Classification (Categorization): an object or scene is assigned to a class (category), such as 'car', 'apple', 'beach scene', etc. In praxis, the discrimination between a few classes is manageable, however the larger the number of classes we desire to discriminate, the more challenging it is to deal with the *intra-class variability*. It is difficult to express the *structural variability* between instances within the same class - think of how differently chairs can look like.

Identification: an individual instance of an object is recognized. In principle, this process is a specific type of classification (as introduced just above), whereby the challenge is to discriminate between endlessly subtle structural variability. Examples: face identification, fingerprint identification, identification of a specific vehicle.

Detection (Localization): the image is searched for either a specific object class, or for an object instance or it is tested for a specific condition; the number of object occurrences is counted. The challenge is to create an efficient search that can find the object irrespective of its size: does the object cover the entire image? Or is it small and therefore difficult to detect? Examples: face detection, vehicle detection in an automatic road toll system, detection of possible abnormal cells or tissues in medical images.

In the literature, the term object recognition often implies some combination of those processes - occasionally it stands for only one of those three processes.

Here are some other frequent recognition tasks:

Motion Analysis: the movement of an object is investigated. One can be interested in merely detecting the movement, which is called *tracking*. Or one can identify specific movements, in which case it is an identification task.

Retrieval: here we sort images according to certain criteria. For instance we pass an image to the system, e.g. of a white rose, and the system returns us the 20 most similar images, e.g. rose, tulip,

sunflower etc. The order is determined based on some sort of 'comparison', similar to the comparison made during classification or identification processes.

Pose Estimation: determines the exact position or orientation of a specific object relative to the camera.

Example: assisting a robot arm in retrieving objects from a conveyor belt in an assembly line or picking parts from a bin.

1.3 Areas of Application (Examples)

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The following list of areas merely gives an overview of where computer vision techniques have been applied so far; the list also contains applications of image processing and machine vision, as those fields are related:

Medical imaging: registering pre-operative and intra-operative imagery; performing long-term studies of people's brain morphology as they age; tumor detection, measurement of size and shape of internal organs; chromosome analysis; blood cell count.

Automotive safety: traffic sign recognition, detecting unexpected obstacles such as pedestrians on the street, under conditions where active vision techniques such as radar or lidar do not work well.

Surveillance: monitoring for intruders, analyzing highway traffic, monitoring pools for drowning victims.

Gesture recognition: identifying hand postures of sign level speech, identifying gestures for human-computer interaction or teleconferencing.

Fingerprint recognition and biometrics: automatic access authentication as well as forensic applications.

Visual authentication: automatically logging family members onto your home computer as they sit down in front of the webcam.

Robotics: recognition and interpretation of objects in a scene, motion control and execution through visual feedback.

Cartography: map making from photographs, synthesis of weather maps.

Radar imaging: target detection and identification, guidance of helicopters and aircraft in landing, guidance of remotely piloted vehicles (RPV), missiles and satellites from visual cues.

Remote sensing: multispectral image analysis, weather prediction, classification and monitoring of urban, agricultural, and marine environments from satellite images.

Machine inspection: defect and fault inspection of parts: rapid parts inspection for quality assurance using stereo vision with specialized illumination to measure tolerances on aircraft wings or auto body parts; or looking for defects in steel castings using X-ray vision; parts identification on assembly lines.

The following are specific tasks which can be often solved with image processing techniques and pattern recognition methods and that is why they are often marginally treated only in computer vision textbooks - if at all:

Optical character recognition (OCR): identifying characters in images of printed or handwritten text, usually with a view to encoding the text in a format more amenable to editing or indexing (e.g. ASCII).

Examples: mail sorting (reading handwritten postal codes on letters), automatic number plate recognition (ANPR), label reading, supermarket-product billing, bank-check processing.

2D Code reading reading of 2D codes such as data matrix and QR codes.

1.4 Organization of a Computer Vision System

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The organization of a computer vision system depends strongly on its application and consequently there does not exist a general recognition scheme. The following list introduces terminology that is used to describe the stages (phases) in a recognition process; they do not strictly appear in that order and they can not always be clearly distinguished. The first two stages clearly belong to the field of image processing; the following three stages (3-5) represent the 'meat' of computer vision; the final stage (6) corresponds essentially to pattern recognition.

1) Image acquisition: is the processes of measuring the outer signal by some sensor and generating a corresponding map of values, an image, that represents that signal. There exist different types of cameras and measurements (Appendix A).

- 2) Image processing:** is an early manipulation of the raw image toward the type of features extracted later. This can be for example contrast enhancement to assure that relevant information can be detected; generation of a scale-space representation to enhance image structures at locally appropriate scales.
- 3) Feature extraction:** is the process of extracting specific types of information (features) from the image in order to facilitate later classification processes. Typical examples of such features are lines, edges, blobs, corners, etc.; more complex features may be related to texture, shape or motion.
- 4) Detection/segmentation:** is the process of deciding which image points or regions of the image are relevant for further processing. Examples are: selection of a specific set of interest points; segmentation of one or multiple image regions which contain a specific object of interest.
- 5) High-level processing:** At this step the input is typically a small set of data, for example a set of points or an image region which is assumed to contain a specific object. The remaining processing deals with, for example:
- Verification that the data satisfy model-based and application specific assumptions.
 - Estimation of application specific parameters, such as object pose or object size.
 - Image recognition: classifying a detected object into different categories.
 - Image registration: comparing and combining two different views of the same object.
- 6) Decision making:** Making the final decision required for the application, for example:
- Pass/fail on automatic inspection applications
 - Match / no-match in recognition applications
 - Flag for further human review in medical, military, security and recognition applications

1.5 Historical Note

In the early years of computer vision, the paradigm for recognition was formulated as a process, which gradually and meticulously reconstructs the spatial 3D layout of the scene from the 2D image. This 3D reconstruction process was often divided into low-level, mid-level and high-level vision process, a division partly reflected in the above list of stages (Section 1.4). It was inspired by the fact that we humans perceive the world as a 3D space. Over the years, it has become clear that this paradigm is too elaborate and too complicated. Presently, the focus lies on solving recognition tasks with 'brute-force' approaches, e.g. the use of Deep Neural Networks or approaches based on extensive matching of image patches, for which classical techniques such as edge detection or image segmentation hardly play a role. Some of the classical techniques have therefore moved a bit into the background. This is also reflected in recent text books. For instance, Forsyth and Ponce's book follows the structure of the classical paradigm (low/mid/high-level vision), but the treatment of edge detection and image segmentation is rather marginal; Szeliski's book organization is centered around the recent feature-matching approaches, but still contains substantial material on image segmentation for instance. But no book contains the latest, breath-taking developments, namely the use of Deep Neural Networks for image classification. We therefore will start with that topic first (Section 5).

1.6 From Development to Implementation

Usually one develops a system first in a higher-level language, such as Matlab, Python, GNU Octave, R, Scilab, etc. Once this testing phase is completed, then one would 'translate' the system into a lower-level language such as Cython, C++ or even C for instance, to make the application run in real-time if that is necessary.

Matlab (<http://www.mathworks.com/>) is extremely convenient for prototyping (research) because its 'formulation' is very compact and because it probably has the largest set of functions and commands. It offers an image processing toolbox that is very rich in functionality, but one can manage without the toolbox - we give plenty of code examples to do so. Since a few years, Matlab also features a computer vision toolbox that is continuously growing in scope. Use `doc` or `help` to read about the functions and commands it provides. It is useful to familiarize yourself with the image processing toolbox by starting with `doc images`.

Octave, R: (<https://www.gnu.org/software/octave/>, <https://www.r-project.org/>) For training purposes one can certainly also use software packages such as R and Octave, in which most functions have the same name as in Matlab.

Python (<https://www.python.org/>) is perhaps the most popular language by now. Coding in Python is slightly more elaborate than in Matlab and does not offer the flexibility in image display that Matlab has. Python's advantage is, that it can be relatively easily interfaced to other programming languages that are suitable for mobile app development for instance, whereas for Matlab this is very difficult. In Python, the initialization process is a bit more explicit and the handling of data-types (integer, float, etc.) is also a bit more elaborate, issues that make the Python code a bit lengthier than in Matlab.

What is a bit confusing about Python is that there are different distributions providing different editing features. If one looks for a straightforward transition from Matlab to Python, then the Anaconda/Spyder distribution is perhaps the easiest one: it provides many of the functions that we introduce in this script.

I recommend in particular to install Python version 3.5.1, a version that includes also the libraries (modules) called *tensorflow* and *keras* as well as the module *skimage*. Tensorflow is the most trending software package for training DeepNeuralNetworks (coming up in Section 5). Keras is the corresponding high-level interface. The newer Python version 3.6 does not include Tensorflow yet (August 2017).

If one switches between any of those high-level languages, then the following summary can be of use: <http://mathesaurus.sourceforge.net/matlab-python-xref.pdf>.

In the following we mention programming languages that are considered rather lower-level and that require more care in initializing and maintaining variables. If you process videos, then you probably need to implement your time-consuming routines into one of those languages.

Cython: Is essentially the same code as Python, but offers to specify certain variables and procedures in more detail with a notation similar to C (C++). That additional notation can speed up the code by several factors. Cython is included in the Anaconda distribution. (Not to be confused with *C*Python, which is the canonical Python implementation).

C++, C: For implementation into C or into one of its variants (i.e. C++), we merely point out that there exist C libraries on the web with implemented computer vision routines. The most prominent one is called *wiki OpenCV*, see <https://opencv.org/>. Many of the routines offered by those libraries can also be easily accessed through Python by importing them. Typically, you need to install those libraries separately.

1.7 Reading

Sonka, M., Hlavac, V., and Boyle, R. (2008). *Image Processing, Analysis, and Machine Vision*. Thomson, Toronto, CA. Introductions to topics are broad yet the method sections are concise. Contains many, precisely formulated algorithms. Exhaustive on texture representation. Oriented a bit towards classical methods, thus, not all newer methods can be found. Written by three authors, but reads like if authored by one person only.

Szeliski, R. (2011). *Computer Vision: Algorithms and Applications*. Springer. Meticulous and visually beautiful exposure of many topics, including on graphics and image processing; Strong at explaining feature-based recognition and alignment, as well as complex image segmentation methods with the essential equations only. Compact yet still understandable appendices explaining matrix manipulations and optimization methods.

Forsyth, D. and Ponce, J. (2010). *Computer Vision - A Modern Approach*. Pearson, 2nd edition. Exhaustive on topics about object, image and texture classification and retrieval, with many practical tips in dealing with classifiers. Equally exhaustive on tracking. Strong at explaining object detection and simpler image segmentation methods. Slightly more praxis oriented than Szeliski. Only book to explain image retrieval and image classification with feature methods.

Davies, E. R. (2012). *Computer and Machine Vision*. Elsevier Academic Press, Oxford. Rather machine vision oriented (than computer vision oriented). Contains extensive summaries explaining advantages and disadvantages of each method. Summarizes the different interest points detectors better than any other book. Treats video surveillance and automotive vision very thoroughly. Only book to contain automotive vision.

Prince, S. (2012). *Computer Vision: Models, Learning, and Inference*. Computer Vision: Models, Learning, and Inference. Cambridge University Press. Also a beautiful exposure of some computer vision topics; very statistically oriented, starting like a pattern recognition book. Contains up-to-date reviews of some topics.

Wikipedia Always good for looking up definitions, formulations and different viewpoints. Even textbooks sometimes point out wikipedia pages. But wikipedia's 'variety' - originating from the contribution of different authors - is also its shortcoming: it is hard to comprehend the topic as a whole from the individual articles (websites). Wikipedia is what it was designed for after all: an encyclopedia. Hence, textbooks remain irreplaceable.

Furthermore, because different authors are at work at wikipedia, it can happen that an intuitive and clear illustration by one author is being replaced by less a intuitive one by another author. I therefore recommend to copy/paste a well illustrated problem into a word editor (e.g. winword) in order to keep it.

1.8 Exercises

The following two study problems should help sharpen our understanding of the recognition processes as introduced in Section 1.2.

1. Google provides with the stand-alone program 'Google Goggles' an illustration of object recognition. Which exact recognition processes does it emulate?
2. There exist systems that perform object recognition for automated checkout lanes in retail. What do you think how the individual recognition processes are addressed in such a system?
3. What type of processes would an autonomous vehicle - such as the Tesla car - use in particular?

2 Simple Image Manipulations

To get acquainted with some of the basics, we perform a few simple image manipulations in this section. Firstly, we learn about the image format and some basic operations such as thresholding and data type conversions (Section 2.1). Then we introduce a simple detector for face part localization (Section 2.2).

2.1 Image Format, Thresholding, Conversion

A typical digital image, for instance a jpeg image, comes as a three-dimensional array. The first two dimensions correspond to the spatial axes x and y . The third dimension holds color information in three 'channels', namely red, green and blue (RGB). The color values generally range from 0 to 255 and are stored as unsigned integers, specifically as `uint8`, the number 8 standing for 8 bits, a data-type designed to hold exactly that range ($255 = 2^8 - 1$). For each pixel then, there exist 24 bits (3×8 bits), which allows to store $256 \times 256 \times 256 \approx 16.7$ million colors. Despite that rich color information, it is often more convenient to computer only with a gray-scale version of that image, thus reducing the information back to 8 bits per pixel. To convert a RGB image into gray-scale image, one converts the color values to a gray value L by adding the three components according to a specific ratio, for instance:

$$L = 0.2989 \cdot R + 0.5870 \cdot G + 0.1140 \cdot B \quad (1)$$

In Matlab one can load an image with the command `imread`. To convert it into a gray-level image there exists the function `rgb2gray`. To display the image we use the function `imagesc` (image scale) and for that purpose we initialize a figure with the function `figure`. The function `clf` clears the figure. With the command `subplot` we can pack several images into the same figure. The following code shows how to use those commands, its output is shown in Figure 1.

```
clear; % clear memory
Irgb = imread('yellowlily.jpg'); % load jpg image
Igry = rgb2gray(Irgb); % convert it to gray-scale
IrgbCen = Irgb(400:1200,300:900,:); % zoom into center
Igreen = Irgb(:,:,2); % green channel only
BWflw = Igry>100; % thresholded (black-white image)
Iblur = conv2(single(Igry),ones(25,25)); % blurring the image

%% ----- Plotting -----
figure(1); clf; [nr nc] = deal(3,2);
subplot(nr,nc,1); imagesc(Irgb); title('Original');
subplot(nr,nc,2); imagesc(Igry); colormap(gray); title('Gray-Scale');
subplot(nr,nc,3); imagesc(IrgbCen); title('Sub-Selection (Zoom)');
subplot(nr,nc,4); imhist(Igreen); title('Histogram of Green Channel');
subplot(nr,nc,5); imagesc(BWflw); title('Black-White (Logical) Image');
subplot(nr,nc,6); imagesc(Iblur); title('Blurred Image');
```

To select a part of the image - see comment 'zoom into center' -, we specify the row numbers first - vertical axis first -, followed by specifying the column numbers - horizontal axis. That is, one specifies the indices as in matrices in mathematics.

Black-White Image We can threshold an image by applying a relational operator, see line `BWflw = Igry>100`, in which case the image is automatically converted into a *logical* data-type, that is true or false, namely one bit (value one and zero respectively). An image of that data-type is also called a *black-white* image sometimes, hence the variable's name `BW`. In the code example above we attempted to separate the flower from its background, a foreground/background segregation as it is also called. We have chosen the threshold somewhat arbitrarily and of course it would make sense to choose a threshold based on a histogram, a histogram of intensity values for instance, as shown in the figure. We elaborate on that in Section 9. After we have segmented, one often manipulates the black-white image by so-called *morphological* operations, to be introduced in Section 10.

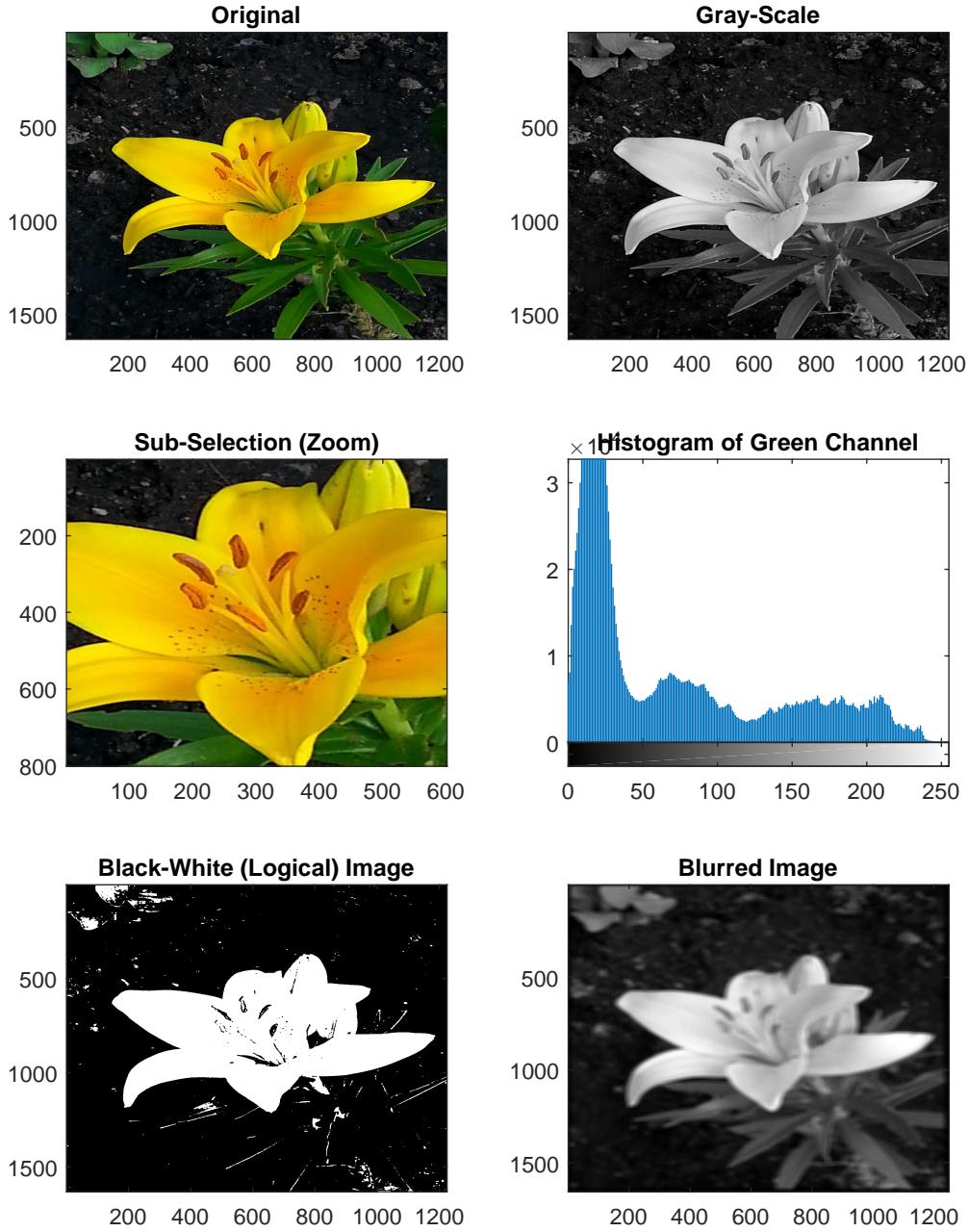


Figure 1: Some simple image manipulations.

Upper Left: original RGB image; each pixel is represented by 24 bits (3 chromatic channels \times 8 bits).

Upper Right: gray-level: RGB values combined to a single luminance value according to Eq. 1; each pixel is now represented by 8 bits only (values from 0 to 255).

Center Left: zoomed image; selection occurs with matrix-style indexing (rows/columns).

Center Right: image histogram of the green channel: the x -axis represents the chromatic value, ranging from 0 to 255; the y -axis represents the pixel-count.

Lower Left: result of thresholding the gray-level image at value equal 100: white represents on-pixel (true), black off-pixel (false).

Lower Right: gray-level image blurred with an average filter: at each pixel the average over its 25x25 pixel-neighborhood is taken.

Blurred Image Sometimes it is useful to blur an image because a blurred image helps analyzing the 'coarse' structures of an image, which otherwise are difficult to detect in the original image with all its details. We can blur an image by averaging over a local neighborhood at each pixel in the image, in Matlab done with the function `conv2`. In our example we take a 25x25 pixel neighborhood, generated with `ones(25,25)` and merely sum up its 625 pixel values: this summation operation is done for each pixel in the image. We will come back to that in Section 3.

Data-Type Conversion The function `imread` returns a jpeg image as data-type `uint8`, which is not very practical for certain computations. For many image-processing functions we need to convert the image into a floating-number data-type. In Matlab that would be for instance `single` or `double`, lower and higher precision respectively. In the above code, we did that for the function `conv2`, but we could also write a separate line if desired, `Irgb = single(Irgb)`.

Many functions are flexible and will produce an output of the same data-type; others expect a specific data-type as input; some functions produce a specific data-type as output such as the thresholding operation. It is best to be always aware of what data type an image is - or any variable -, and what type the functions expect and produce.

In Python, the code looks very similar, but we need to 'import' those functions from modules. In particular the module `skimage` holds a lot of functions for computer vision and image processing, see also Appendix G:

```
from numpy import arange, ones, histogram
from skimage.io import imread
from skimage.color import rgb2gray
from scipy.signal import convolve2d

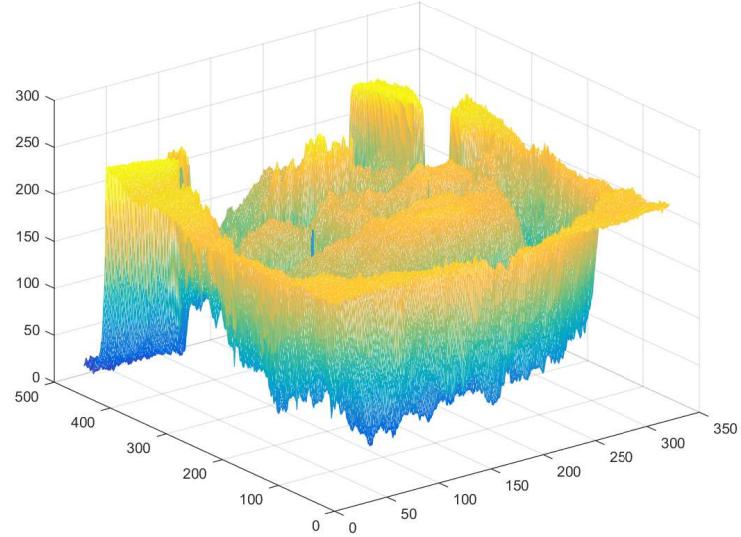
Irgb = imread('KlausIohannis.jpg') # loading the image
Igry = rgb2gray(Irgb) # convert to gray-scale
IrgbCen = Irgb[100:500,400:700,:]
Igreen = Irgb[:, :, 2]
BWflw = Igry>0.3
Iblur = convolve2d(Igry,ones((25,25))) # blurring the image

# %% ---- Plotting
from matplotlib.pyplot import figure, subplot, imshow, plot, hist, title, cm
figure(figsize=(10,6)); (nr,nc) = (3,2)
subplot(nr,nc,1); imshow(Irgb); title('Original')
subplot(nr,nc,2); imshow(Igry, cmap=cm.gray); title('Gray-Scale')
subplot(nr,nc,3); imshow(IrgbCen); title('Sub-Selection (Zoom)')
subplot(nr,nc,4); hist(Igreen.flatten(),bins=arange(256)); title('Histogram of Green Channel')
subplot(nr,nc,5); imshow(BWflw); title('Black-White (Logical) Image')
subplot(nr,nc,6); imshow(Iblur, cmap=cm.gray); title('Blurred Image')
```

Note that in Matlab the function `rgb2gray` returns a map of data-type `uint8` - if the input was of type `uint8` - , whereas in Python the function `skimage.color.rgb2gray` immediately down-scales the intensity values to an interval $\in [0 \text{--} 1.0]$ and assigns them to a floating data-type.

Topology To obtain an impression what type of stimulus an image represents, we recommend to observe the image with the function `mesh`, as given in the last line in the Matlab code block. This illustrates better that the image array holds an 'intensity landscape' (Fig. 2). For that reason computer vision scientists often borrow terminology from the field of topology to describe the operation of their algorithms, such as the term 'watershed' in case of a segmentation algorithm.

Figure 2: An image as observed from a three-dimensional perspective (command `mesh` in Matlab). The image appears as a landscape whose elevation - the vertical axis - represents intensity with values ranging typically from zero to 255 (for images coded with 8 bits). It is not easy for a human to understand the semantic content of a scene from this perspective, because the human visual system is trained to interpret frontal views of images. But this perspective illustrates better what the computer vision system receives as input.



2.2 Face Part Detection

With some simple image manipulations we can sometimes gain a lot of information. For instance to detect facial features in a portrait, it can suffice in many cases to look at the sum of pixel intensity values along each dimension, see Fig. 3. We generate two such *profiles*, one by integrating the pixel values along the horizontal axis (dimension), and one by integrating along the vertical axis. In Matlab this is easily done as follows:

```
Pver = sum(Ig,1); % vertical intensity profile
Phor = sum(Ig,2)'; % horizontal intensity profile
```

Those profiles are shown in black in the Fig. 3. In such profiles it is - in principle - relatively easy to locate facial features by observing local maxima, minima, etc. The 'raw' profile sums are a bit 'noisy' however - like most raw signals: they contain too many 'erratic' extrema that make detection of the facial features difficult. We therefore smoothen the profiles a bit by low-pass filtering them. The smoothed versions are shown in magenta in the Figure; now it is easier to locate facial features. Smoothing can be done by averaging over a local neighborhood in the signal at each pixel in the profile. We have done this above already to obtain a blurred image in two dimensions with the function `conv2`. Here we do it in one dimension only. And we do so with a so-called Gaussian filter, a function whose shape is a 'bump': it is an elegant way to filter a signal, see Appendix C.2 for its exact shape. For the blurring of the image above we had used a flat filter, which is a rather crude way.

The size of the filter matters: a small size does not smoothen very much, a large size flattens the signal to much. We therefore make the filter size dependent on the image size by choosing a fraction of it. The Gaussian values are generated with function `pdf`.

```
nPf = round(w*0.05); % # points: fraction of image width
LowFlt = pdf('norm', -nPf:nPf, 0, round(nPf/2)); % generate a Gaussian
Pverf = conv(Pver, LowFlt, 'valid'); % filter vertical profile
Phorf = conv(Phor, LowFlt, 'valid'); % filter horizontal profile
```

If you are not familiar with the convolution process you should consult Appendix B - for the moment the Appendix B.1 on one-dimensional convolution is sufficient. Those concepts are part of the field of signal processing and we cannot give lengthy introductions to those. But with Matlab you can conveniently explore them and obtain a quick understanding of those operations, that is why we give a lot of code examples in the Appendix.

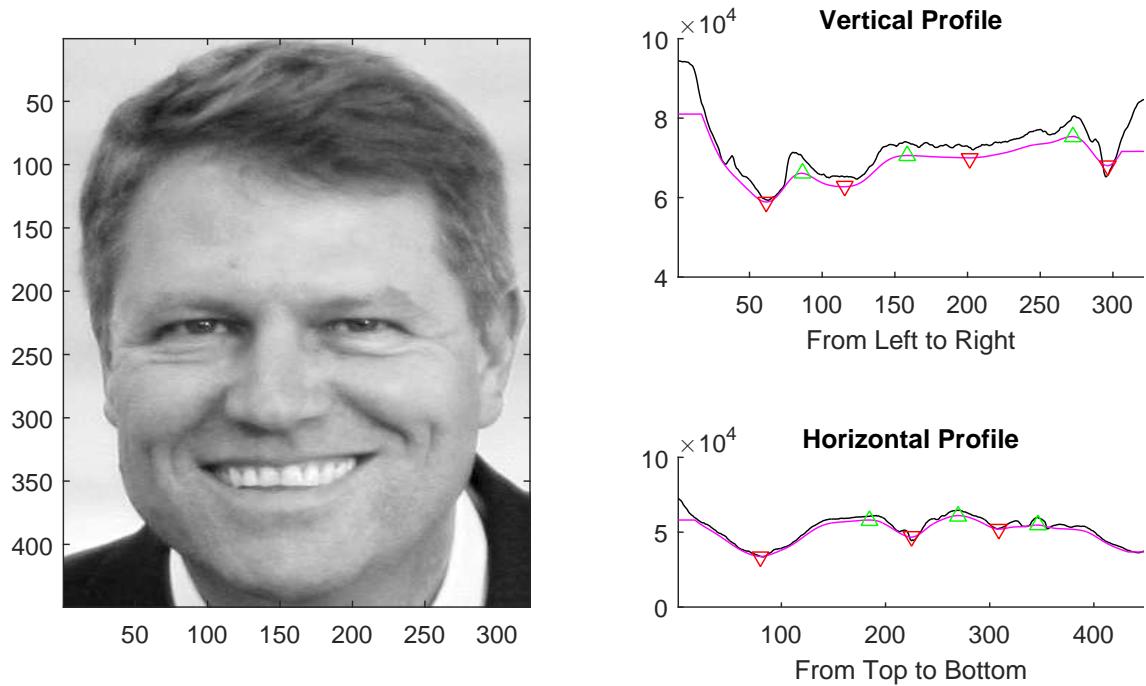


Figure 3: Intensity profiles for a face image. The vertical profile is generated by summing the pixel intensity values along the y-axis (vertical; column-wise); the horizontal profile is generated by summing along the x-axis (horizontal; row-wise). To what face parts do the extrema in the profiles correspond to? Code in Appendix I.1.

For extrema detection we can use the function `findpeaks`, which returns the local maxima as well as their location in the signal. In order to find the local minima we invert the distribution. The code in Appendix I.1 shows how to apply those functions.

This type of face part localization is somewhat simple, but its temporal complexity is low and that is why this approach is often used as a first phase in a more elaborate facial feature tracking system. Many applied computer vision systems consist of cascades of sub-systems that progressively carry out a task with increasing precision.

2.3 Exercises

2.3.1 Displaying an Image, Coordinate System

1. Start with the code block given above. Add the function `colorbar` after the plotting function `imagesc`: this allows you to conveniently observe the image range. Divide the intensity values of the image by 2, the image is now 'darker' an effect which might not be immediately obvious, because the function `imagesc` will scale the intensity range. Use then `imagesc(Igry, [0 255])` to observe the effect. Add the command `hold on`, which allows you to continue plotting on the image - which we do in the next exercise.
2. Understand the coordinate system in the displayed figure: plot something, e.g. an asterisk at `plot(20,40,'*')`. Observe that the image is displayed in matrix mode, that is on the y-axis we have rows, on the x-axis we have columns. If you prefer to see your plots on top of the image in Cartesian coordinates you need to switch axes, either by using the `axis` command (e.g. `axis xy`) or by swapping the x and y dimensions of your plotting coordinates.

2.3.2 Localizing Face Parts

Download an image database containing faces, where the face is placed approximately in the image center: search the Internet for 'face database'; a few tens of images suffice. Use the code in Appendix I.1 as a starting point. Manually construct some rules that identify facial features. There are certain appearances of users, which make identification of certain facial features difficult. Which ones are they?

3 Image Processing I: Scale Space, Pyramid and Gradient Image

For many computer vision tasks, it is useful to blur the image and to analyze those different blurs separately. We have introduced the idea of a blurred image already in the previous section, but here we generate this blur repeatedly with increasing filter sizes arriving at a so-called *scale space* as shown in Fig. 4. This space will be introduced in the subsequent Section 3.1. Then we introduce the image pyramid, Section 3.2, which is - coarsely speaking - a reduction of the scale space.

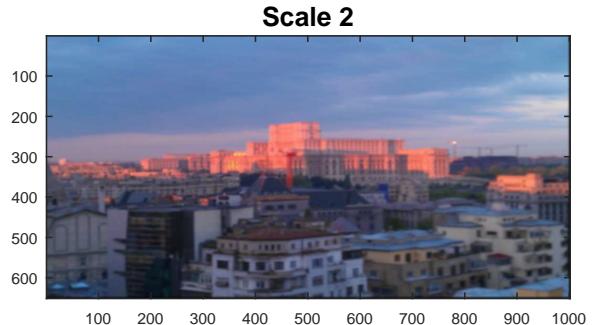
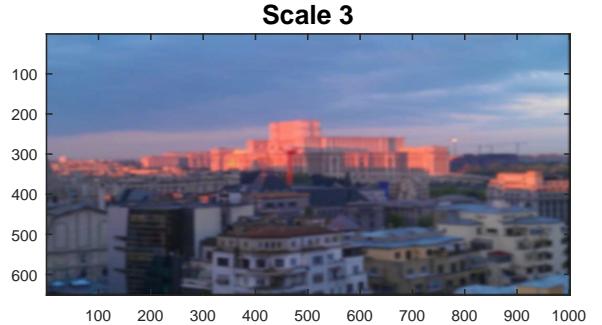


Figure 4: Scale space of an image. Observe the increasing blur from bottom to top.

Original: unfiltered image I_o .

Scale 1: I_o was smoothed with a Gaussian filter of sigma equal one, $\sigma = 1$.

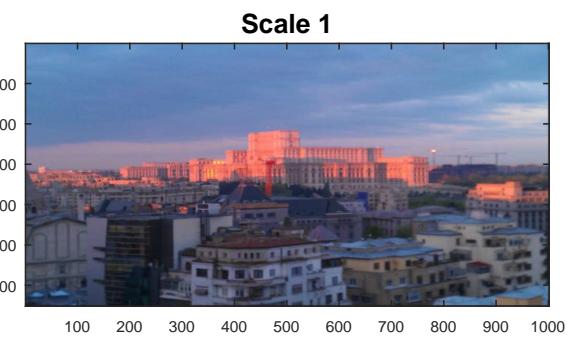
Scale 2: I_o was smoothed with $\sigma = 2$.

Scale 3: I_o was smoothed with $\sigma = 3$.

It is called a space because one can regard it as a three-dimensional space, with the third dimension corresponding to a fine-to-coarse axis with variable σ .

At coarser scales (larger values of σ) it is easier to find contours and regions as a whole, but structures are sometimes smeared with other different structures. Coarser scales are often down-sampled to obtain a more compact representation of the scale space, which so forms a pyramid, see Fig. 5. Code in Appendix I.2.

(In this case, the filtering was performed for each color channel separately, once for the red, once for green and once for blue image.)



For many computations it is also useful to know how the intensity landscape is 'oriented' at each pixel. Specifically, we would like to know the 'surface slope' at each image pixel for a small pixel neighborhood. This is expressed with the gradient image, to be explained in Section 3.3.

3.1 Scale Space

wiki Scale_space

Sze p127, s3.5, p144

FoPo p164, s4.7, p134

SHB p106, s4.3

An image is blurred by convolving it with a two-dimensional filter that averages across a small neighborhood. In our introductory example of the previous Section we merely used a summation function, but typically image blurring is done with a Gaussian filter as we did for filtering the face profiles (Section 2.2). Here the Gaussian filter is a two-dimensional function and looks like shown in the first four patches of Fig. 9. It is expressed as $g(x, y, \sigma)$, where x and y are the image axes and where σ is the standard deviation regulating the amount of blur - also called the smoothing parameter. In the language of signal processing one expresses the blurring process as a convolution, indicated by the asterisk *. One says, the image $I_o(x, y)$ is convolved by the filter $g(x, y, \sigma)$

$$I_c(x, y) = I_o(x, y) * g(x, y, \sigma), \quad (2)$$

resulting in a coarser image I_c . If you are unfamiliar with the 2D-convolution process, then consult now Appendix B.2 to familiarize yourself with it.

If this blurring is done repeatedly, then the image becomes increasingly coarser, the corresponding intensity landscape becomes smoother, illustrated in Fig. 4. Practically, there are different ways to arrive at the scale space. The most straightforward implementation is to low-pass filter the original image repeatedly with 2D Gaussians of increasing size, that is first with $\sigma = 1$, then with $\sigma = 2$, etc. That approach is however a bit slow and there exist fast implementation, for which we refer to the Appendix B.2.

The resulting stack of images, $\{I_o, I_1, I_2, \dots\}$, is called a *scale space*. In a typical illustration of the scale space, the bottom image corresponds to the original image; subsequent, higher images correspond to the coarser images. In other words, the images are aligned vertically from bottom to top and one can regard that alignment as a *fine-to-coarse* axis. That axis is now labeled σ . The resulting space is then expressed as

$$S(x, y, \sigma). \quad (3)$$

The axis σ can be understood as a smoothing variable: a σ -value equal zero corresponds to the original image, that is, there is no smoothing; a σ -value equal one corresponds to the first coarse image, etc. In applications, sigma values typically range from one to five, $\sigma = 1, 2, \dots, 5$.

Application The scale space is used for the following feature detection processes in particular:

1. Region finding: we can easily determine brighter and darker regions by subtracting the scales from each other, a technique we will return to in Section 4.1.
2. Verification of structures across the scale (axis): for instance, if a specific feature can be found at different scales, then it is less likely to be accidental, a technique to be used in feature detection in general (Section 6).
3. Finding more 'coherence': for instance, contours appear more continuous at coarser scales (Section 4.2).

In Matlab we can create a Gaussian filter with the function `fspecial` and convolve the image with the command `conv2`:

```
Fsc1 = fspecial('gaussian', [3 3], 1); % 2D gaussian with sigma=1
Fsc2 = fspecial('gaussian', [5 5], 2); % 2D " " sigma=2
Isc1 = conv2(Io, Fsc1, 'same'); % filtering at scale=1
```

The image processing toolbox also offers commands such as `imgaussfilt` and `imfilter` to generate blurs of images and are probably the preferred functions, because those are optimized for speed.

In Python the submodule `scipy.ndimage` contains the function `gaussian_filter` to blur images:

```
scipy.ndimage.gaussian_filter(I, sigma=1.0)
```

Generating such a scale space gives us more information that we intend to exploit to improve our features extraction. This gain in information comes however at the price of more memory usage, as we now have multiple images. And multiple images means more computation when searching for features. But we can reduce this scale space, without much loss of information by 'compressing' the coarse scales, which leads to the *pyramid* treated in the next section.

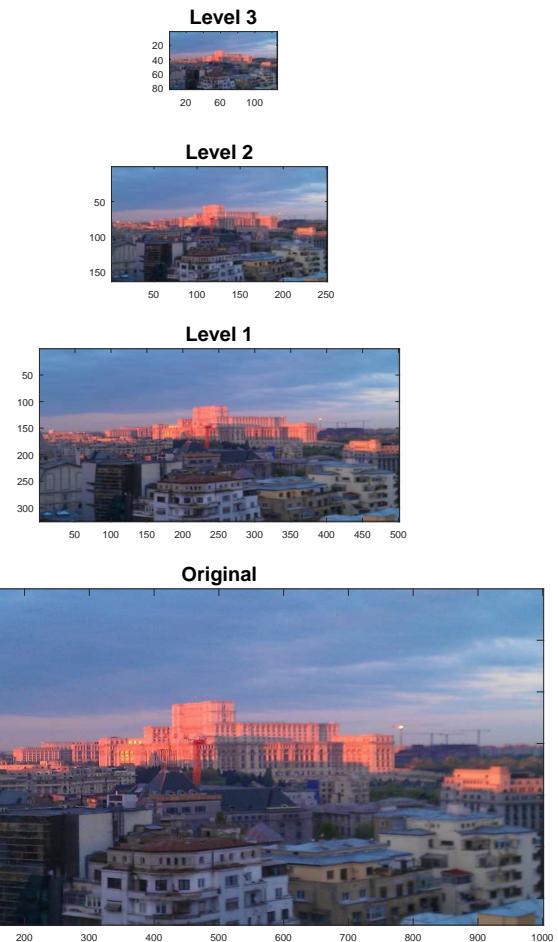


Figure 5: Multi-resolution pyramid of an image. The images of the scale space (Figure 4) are down-sampled by every second pixel: the resolution is halved with each scale along the fine-to-coarse axis.

Original: $P_0 = \text{original image } I_o$

Level 1: $P_1 = \text{sub-sampled of } I_{c1} (I_{c1} = I_o * g(1))$

Level 2: $P_2 = \text{sub-sampled of } I_{c2} (I_{c2} = I_o * g(2))$

Level 3: $P_3 = \text{sub-sampled of } I_{c3} (I_{c3} = I_o * g(3))$.

It is called a pyramid, because one could align the images such that they form a pyramid - for simplicity we show the images frontally.

3.2 Pyramid

Operating with the entire scale space S is computationally expensive, as it is rather large and to some extent redundant. Because coarser scales do not possess any fine structure anymore, it makes sense to subsample them: for each new, coarser scale I_c , only every second pixel is taken, once along the horizontal axis and once along the vertical axis. We so arrive at the (octave) pyramid with levels P_0 to P_3 for instance, see Fig. 5. In higher-level languages we can down-sample 'manually' by selecting every second row and column, i.e. sub-sampling rows with:

```
Idwn = Isc1(1:2:end,:); % sub-sampling rows
```

followed by sub-sampling columns by

```
Idwn = Idwn(:,1:2:end); % sub-sampling columns
```

wiki [Pyramid_\(image_processing\)](#)

In Matlab there exists also the function `downsample`, which however works for rows only - if the input is a matrix; we then need to flip the matrix to down-sample it along the columns. Matlab also offers the function `impyramid`, which carries out both the Gaussian filtering as well as the down-sampling. But we can also operate the other direction and up-sample:

```
I1 = impyramid(I0,'reduce'); % filter and downsampling every 2nd
I0 = impyramid(I1,'expand'); % upsampling every 2nd
```

If other down- or up-sampling steps - than halving and doubling- are required, then the function `imresize` may be more convenient.

Python `skimage.transform.pyramid_reduce` and `.pyramid_expand`

Application In many matching tasks, it is more efficient to search for a pattern starting with the top level of the pyramid, the smallest resolution, and then to work downward toward the bottom levels, a strategy also called coarse-to-fine search (or matching). More specifically, only after a potential detection was made at the coarse level, then one starts to verify by moving towards finer levels, where the search is more time consuming due to the higher resolution.

3.3 Gradient Image

wiki Image-gradient

The gradient image describes the steepness at each point in the intensity landscape, more specifically how the local 'surface' of the landscape is inclined. At each pixel, two measures are determined: the direction of the slope, also called the gradient; and the magnitude - the steepness - of the slope. Thus the gradient image consists of two maps of values, the direction and the magnitude. That information is most conveniently illustrated with arrows, namely as a vector field, see Figure 6: the direction of the arrow corresponds to the gradient; the length of the arrow corresponds to the magnitude.

To determine the gradient, one takes the derivative in both dimensions, that is the difference between neighboring pixels along both axes, $\frac{\partial I}{\partial x}$ and $\frac{\partial I}{\partial y}$ respectively. This operation is typically expressed with the nabla sign ∇ , the gradient operator:

$$\nabla I = \left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right)^T. \quad (4)$$

whereby the gradient information is expressed as a vector. We give examples: a point in a plane has no inclination, hence no magnitude and an irrelevant direction - the gradient is zero; a point in a slope has a certain direction - an angle value out of a range of $[0, 2\pi]$ - and a certain magnitude representing the steepness. The direction is computed with the arctangent function using two arguments (`atan2`), returning a value $\in [-\pi, \pi]$ in most software implementations. The magnitude, $\| \nabla I \|$, is computed using the Pythagorean formula.

Determining the gradient field directly on the original image does not return 'useful' results typically, because the original image is often too noisy (irregular). Hence the image is typically firstly low-pass filtered with a small Gaussian function, e.g. $\sigma = 1$, before determining its gradient field.

In Matlab there exists the routine `imggradient` which returns the magnitude and direction immediately (Image Processing Toolbox); if one needs also the derivatives later again in the code, then one can use `imggradientxy` to obtain two matrices corresponding to the individual gradients. If we lack the toolbox, we can use the following piece of code, whereby the function `gradient` returns two matrices - of the size of the image -, which represent the gradients along the two dimensions:

```
Fg      = fspecial('gaussian',[3 3],1); % 2D gaussian of size 3x3 with sigma=1
Isc1   = conv2(I, Fg, 'same');
[Dx Dy] = gradient(single(Isc1));        % gradient along both dimensions
Dir    = atan2(-Dy, Dx) + pi;            % [-pi,pi]
bk0    = Dir<0;                         % negative values
Dir(bk0) = Dir(bk0)+2*pi;                % [0,2*pi]
Mag    = sqrt(Dx.^2+Dy.^2);              % gradient magnitude
```

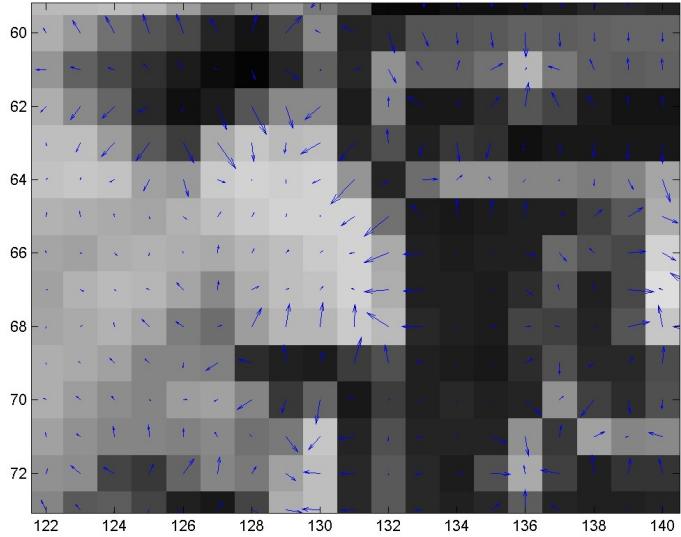


Figure 6: Gradient image. The arrows point into the direction of the gradient - the local slope of the neighborhood; the length of the arrows corresponds to the magnitude of the gradient. The gradient direction points toward high values (white=255; black=0).

(What does the picture depict? Hint: generate a coarse scale by squinting your eyes.)

In the example code, the angle values are shifted into the positive range $[0, 2\pi]$. One can plot the gradient field using the function `quiver`:

```
% --- Plotting:
X = 1:ISize(2); Y = 1:ISize(1);
figure(1); clf; colormap(gray);
imagesc(I, [0 255]); hold on;
quiver(X, Y, Dx, Dy);
```

In Python we can use the function `gradient` of the `numpy` module:

```
Dx, Dy = numpy.gradient(I)
```

Application The gradient image is used for a variety of tasks such as edge detection, feature detection and feature description (coming up).

3.4 Exercises

We are going to write a few functions, but instead of placing the code immediately into separate function scripts, we firstly develop our code in a *testing* script, which we call `t_NameOfScript` (t for test), because debugging is easier in a script than in a function script. Once we have developed our code sufficiently well in a script, then we copy the commands into a *function* script called `f_NameOfFunction` (f for function). Do not erase the code in your testing script - leave it there! You then apply the function `f_NameOfFunction` at the end of your testing script to verify its output, e.g. by taking the differences of the output in the testing script and the output of the function script.

1. Write a script `t_Pyramid`, which generates a pyramid for an image (see Appendix I.2 for an example). Store the levels in a `cell` or `struct`. Display the pyramid with `subplot`.
 - a. Understand the filters first: plot them and observe their maximum value at the center. Use `colorbar` to plot a scale for each. Apply the command `imagesc` with limits `[0 0.15]`.
 - b. Observe the filtered images carefully. What (intensity) range do the different levels have? Use `colorbar` again to illustrate. A value of 0 is what color? What is the maximal value?
 - c. Observe the image borders carefully. At the borders one can not apply the filters properly: what is done instead? What options does one have?

- d. Finally, place the code into a function `f_Pyramid`.
- 2. Write a script `t_GradImg` which generates a gradient image, see Section 3.3 for hints.
 - a. Display the maps for directions and magnitudes in separate images (`imagesc`). Use `colorbar` to verify the ranges. Where are the magnitude values the largest?
 - b. Now display the gradients - direction and magnitude - with the command `quiver` and place the output on top of the image (don't forget `hold on`). For large images it may take a long time - several seconds - to display all the arrows. Zoom into the results to verify the output. In what direction do the arrows point? Upward, downward in the intensity image? To what measure corresponds the length of the arrows?
 - c. Finally, write the function `f_GradImg`, which takes as input the image `I` and returns two maps, one with the direction values and one with the magnitudes. Return the maps in a structure `GM: GM.Dir = Dir, GM.Mag = Mag`, it is handier for later use. Now verify the function at the end of your testing script with `any(GM.Dir(:)-Dir(:))`: if 0 then it is ok, if not, then we may have an error - though there may be differences in rounding; verify with `any(sum(abs(GM.Dir(:)-Dir(:)))>0.1)`.

4 Feature Extraction I: Regions, Contours & Texture

Now we make a first step toward reading out the structure in images. We will try to find edges in the intensity landscape, dots, regions, repeating elements called texture, etc. This feature extraction is useful for locating objects, finding their outlines and perhaps even classifying them.

Regions can be relatively easily obtained by subtracting the images of the scale space from each other. This is an operation called band-pass filtering and will be introduced in Section 4.1. Edges can be obtained by observing where there is a steep drop in the intensity landscape, to be treated in Section 4.2. And for texture detection, the techniques are essentially a mixture of region and edge detection to be highlighted in Section 4.3.

4.1 Regions (Blobs, Dots) with Bandpass Filtering

Many regions are often either darker or brighter in comparison to their surround. One way to identify them would be by applying a threshold to the intensity values. That would be a technique that works well if the image consist of a homogenous background and foreground objects that stand out clearly from the background; we elaborate on that a bit later (Section 10). Here we use a technique that is slightly more flexible, but also more complicated. Specifically, we intend to exploit the scale space that we have developed previously. We subtract the images of the scale space S (Equation 3). That is for two images of S , one fine I_{fine} (e.g. $\sigma = 1$) and one coarse I_{coarse} (e.g. $\sigma = 2$), we take their difference

$$I_{\text{band}} = I_{\text{fine}} - I_{\text{coarse}} \quad (5)$$

in which case positive values correspond to brighter regions, and negative values correspond to darker regions. Figure 7 shows an example where those differences are taken between levels of the scale space.

As the fine and coarse image correspond to low-pass filtered images, their subtraction corresponds to the process of *band-pass filtering*. During band-pass filtering a certain 'center' range ('band') of values is selected, as opposed to the low-pass filtering for which the lower range of values is preferred. See also Appendix C for more explanations.

If the low-pass filtering occurred with Gaussian functions, that is g in Equation 2 is a Gaussian filter, then the corresponding band-pass filter is a difference-of-Gaussian filter, a DOG filter. It is perhaps the most common band-pass filter in image processing.

In Matlab we can generate this 'band space' by applying merely the difference operator to the scale space `SS`, in which case however the signs for brighter and darker regions are switched due to the implementation of `diff`:

```
%> ===== DOG =====
DOG      = diff(SS,1,3);      % brighter is negative, darker is positive
% --- set borders to 0:
for i=1:size(DOG,3), p=i*3;
    DOG(:,:,i) = padarray(DOG(p:end-p+1,p:end-p+1,i),[p p]-1);
end
%> ===== BRIGHTER/DARKER =====
[BGT DRK] = deal(DOG);    clear DOG; % clear DOG to save memory
BGT(BGT>0) = 0;           % all positive values to 0
DRK(DRK<0) = 0;           % all negative values to 0
BGT      = -BGT;          % switch sign to make brighter positive
```

The band-pass space `DOG` contains one level less than the scale space. In this piece of code we also set the border values to zero for reason of simplicity. The variables `BGT` and `DRK` hold then only positive and negative values (left and right column in Figure 7). Of course one could also take other threshold values than zero to select perhaps only very bright or very dark regions.

Python provides functions to perform blob detection, that can be found in submodule `skimage.feature`, for example function `blob_dog`.

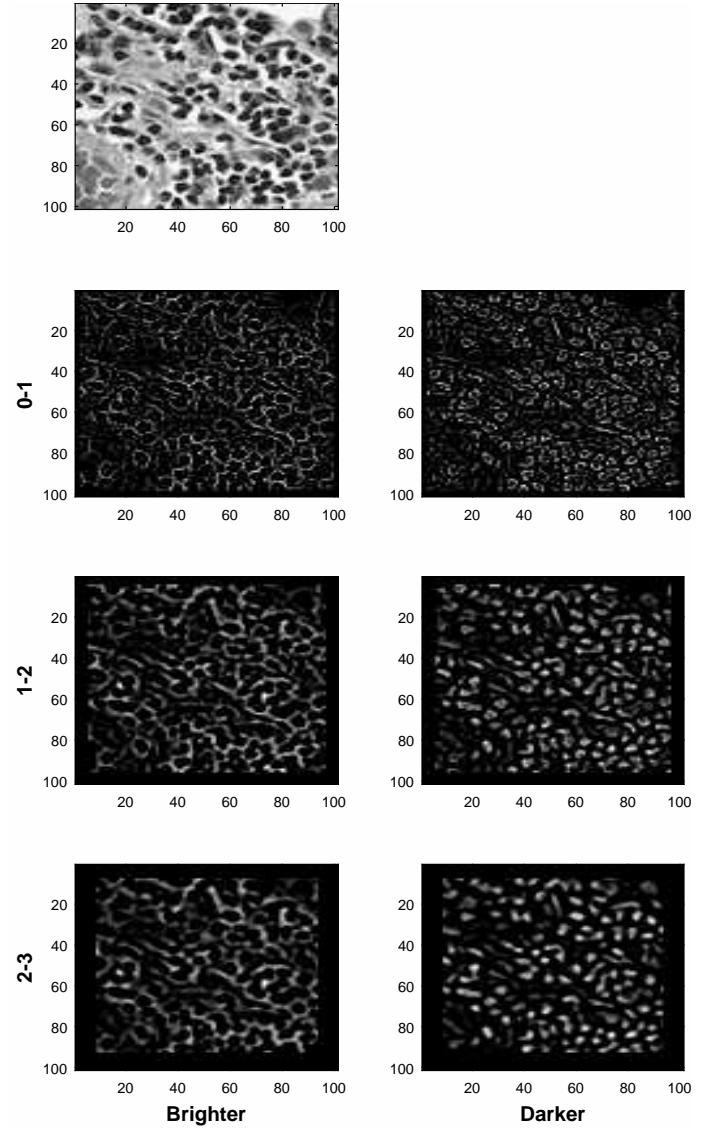
Figure 7: Example of a band-pass space, in this case a Difference-of-Gaussian (DOG) space: it is suitable for detecting regions.

Left column represents brighter regions (`BGT` in code): negative values were set to zero.

Right column represents darker regions (`DRK` in code): positive values were set to zero and the absolute value of the negative values is displayed.

The second row labeled **0-1** is the subtraction of the input image and its low-pass filtered version. The third row labeled **1-2** is the subtraction of the corresponding two adjacent levels in the scale space; etc.

If one intended to select only the nuclei in this image of cell tissue, then the lower right image (2-3, darker) might be a good starting point and one would continue with morphological processing as will be presented in Section 10.



Now that we have regions, we are interested in manipulating them. For example we wish to eliminate small regions and would like to know the approximate shape of the remaining larger regions. We continue with that in Section 10.

Generating and dealing with the entire band-pass space is time consuming. If we develop a very specific task it might suffice to chose a few 'dedicated' scales. For instance to select nuclei from histological images that have image sizes of several thousands pixel (for either row or column), then generating the entire space might be too expensive. Rather one would design a bandpass filter whose size matches approximately the size of a typical nucleus. The original image would then be low-pass filtered with the corresponding two suitable sigmas, the resulting two filtered images subtracted from each other, and then a threshold is applied to the difference image.

4.2 Contour Extraction

There are different types of contours: edge contours, ridge contours, river contours and iso-contours. Edge contours are understood as the lines along steep drops in the intensity image, a line of large gradient values (see Figs. 2 and 6 again). Ridge contours describe the ridges in the intensity topology. River contours describe the bottom of the valleys. Iso-contours correspond to the line of equal height in a topological map:

they have the same intensity value. In most scenarios, when one speaks of contours, one refers to edge contours only. The other three types have been rarely exploited so far. We therefore treat edge contours mostly.

Contours are extracted in two phases. The first phase is called *edge detection* and determines at each pixel, whether it qualifies as a candidate edge. Toward that goal, the image is filtered with a local neighborhood filter that has the shape of an edge (Section 4.2.1). The output of an edge detector is typically only a logical (binary) map with those pixels set to one (or true), where edges are present, see right graph in Fig. 8. In the second step, called *edge following*, the neighboring edge candidates are linked together to arrive at an actual contour, also called tracing sometimes (Section 4.2.2).

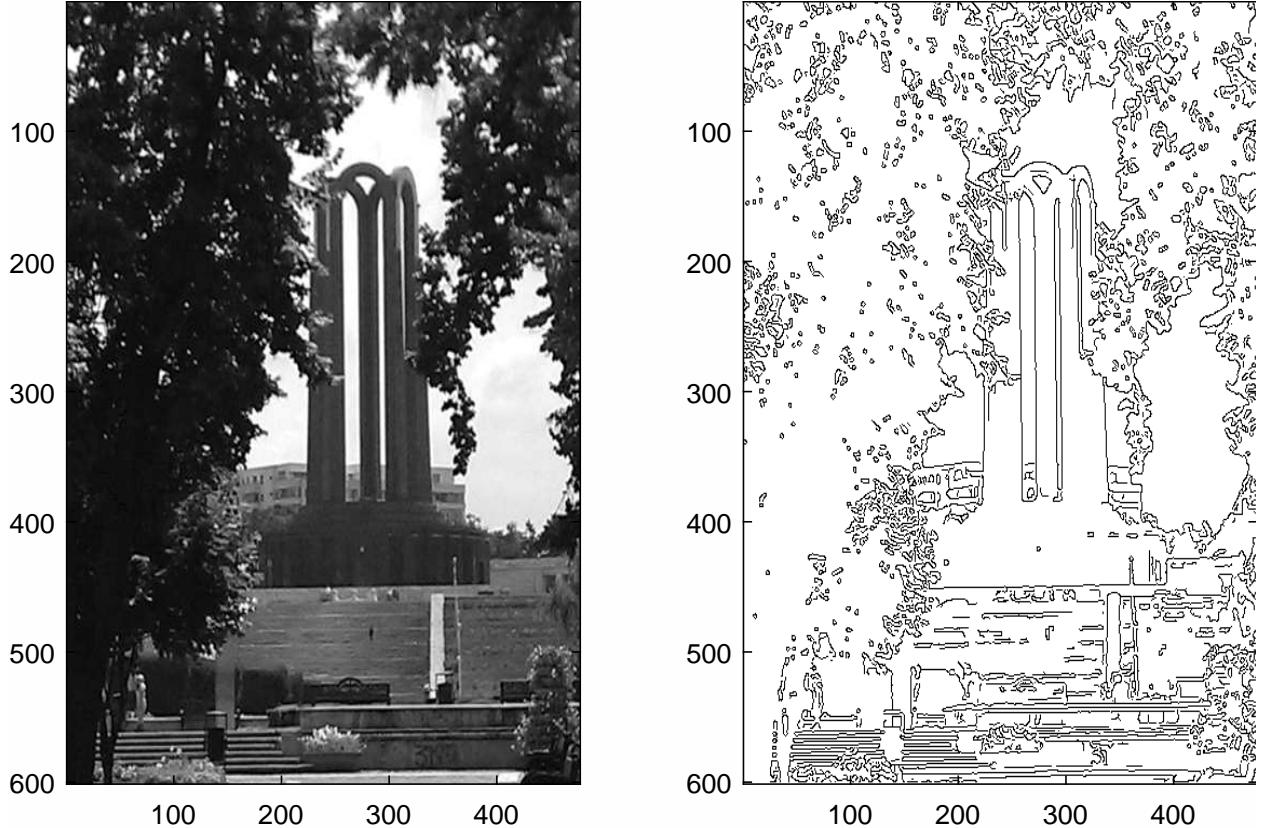


Figure 8: **Left:** input image. **Right:** binary map with on-pixels representing edges as detected by an edge-detection algorithm. There are different algorithms that can do that, the most elegant one is the Canny algorithm which is based on the gradient image as introduced in Section 3. An edge following algorithm traces the contours in such a binary map.

4.2.1 Edge Detection

An edge is an abrupt change in contrast with a certain orientation or direction; it is observed locally only, that is in a neighborhood of limited size. Edges can be detected using different techniques. The principal technique is to convolve the image with two-dimensional filters that emphasize such edges in the intensity landscape. The filter masks of such oriented filters therefore exhibit an 'orientation' in their spatial alignment. Here are tow primitive examples for a vertical and diagonal orientation filter respectively.

$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix}. \quad (6)$$

Thus, the image is convolved multiple times with different orientation masks to detect all edge orientations resulting thus in different output maps, which are then combined to a single output map by a mere logical AND operation.

Matlab provides the function `edge` to perform this process and it provides different techniques. Python offers pretty much the same set of techniques, but in different modules, `skimage.feature` and `skimage.filters`. Those functions return as output a binary map where on-pixels correspond to locations of pixels. The code example in Appendix I.3.2 displays the output of different edge detection techniques.

- Robert detector: this detector is rather primitive and is outdated by now, but it is still used in some industrial applications: its advantage is its low processing duration, its downside is that it does not detect all edges.
- Prewitt and Sobel detector: those detectors find more edges, but at the price of more computation.
- Cannv detector: this is most elaborate detection technique.

```
Medg = edge(I, 'canny', [], 1);
```

whereby the empty brackets [] stands for no particular choice of threshold parameters (and thus the threshold is determined automatically) and 1 is the scale value. Typically, scale values up to 5 are specified. Hence, with this technique one can easily perform low-pass-filtering and edge detection using one function only.

When we study the binary output map of that function carefully, then we may observe that some contours are broader than one pixel, in particular when the contour runs along the diagonal axes. This can be problematic when doing edge following, because it is easier to trace contours that show a width of a single pixel only. To avoid such thickened contours, one can employ the operations of *morphological processing*, which allow to conveniently modify a binary . With the so-called *thinning* operation we can 'slim' those thick contour locations. Or perhaps we wish to remove isolated pixels immediately by using an operation *clean*:

```
Medg = edge(I, 'canny', [], 1);
Medg = bwmorph(Medg,'clean'); % removes isolated pixels
Medg = bwmorph(Medg,'thin'); % turns 'thick' contours into 1-pixel-wide contours
```

We will elaborate a bit more on morphological processing in Section 10.

Matlab	<code>edge</code> (one function for all edge detectors)
Python	<code>skimage.feature.canny, skimage.filters.roberts/sobel/prewitt</code>
Code Examps	Appendix I.3.2

4.2.2 Edge Following

Edge following is also called *contour* or *boundary tracing* sometimes, although the term boundary stands strictly speaking for the silhouette of a region.

Matlab There are several ways to do this: two involve the use of Matlab functions, one way would be to write our own routine, see also example in Section I.3.3.

1. `bwboudaries`: this routine is useful for finding the boundaries of regions. In case of a contour, the routine will trace around the contour and half the contour pixels match with the opposite pixels: if one preferred the pixel coordinates only, then one had to eliminate the duplicate coordinates.
2. `bwttraceboundary`: here one specifies a starting point and the function will then trace until it coincides with its starting point. In contrast to the function `bwboudaries` above, it does not treat a contour as a region and pixels are 'recorded' only once. Hence, we write a loop which detects starting points and trace contours individually. We need to take care of when tracing should stop for an individual contour.
3. Own routine: the easiest approach would be trace contours using their end- and branchpoints. The endpoints can be found with the command `Mept = bwmorph(Medg, 'endpoints')`; the branchpoints can be found with the option '`branchpoints`'.

Python There exists no routine designed specifically for that task. Python however offers a routine to measure iso-contours, called `skimage.measure.find_contours`. One specifies a height value at which the map is thresholded and then the corresponding region boundaries are taken. This can be exploited to emulate boundary detection in binary images. If one specifies a value of 0.8 (between 0.5 and 1.0), then the contours lie closer to the region pixels; if one specifies a value of 0.2 (between zero and 0.5), then the contours lie closer to the adjacent exteriors pixels of the region. The example in Section I.3.3 demonstrates that.

4.2.3 Other Contours

The other contour types, the ridge and river contours as well as the iso-contours, have been used in specific tasks only so far. For example, ridge contours were exploited in finger print identification; iso-contours in nuclei detection in histological images.

4.3 Texture

Dav p209, ch8
FoPo p194, pdf 164

Texture is observed in the structural patterns of object surfaces such as wood, grain, sand, grass, and cloth. But even scenes, that consists of many objects, can be regarded as texture.

The term texture generally refers to the repetition of basic texture elements called *textons* (or *texels* in older literature). Natural textures are generally random, whereas artificial textures are often deterministic or periodic. Texture may be coarse, fine, smooth, granulated, rippled, regular, irregular, or linear.

One can divide texture-analysis methods into three broad categories:

Statistical: these methods are based on describing the statistics of individual pixel intensity values. We only marginally mention these methods (Section 4.3.1), as they have been outperformed by methods of spectral analysis (Section 4.3.2).

Structural: in structural analysis, primitives are identified first, such as circles or squares, which then are grouped into more 'global' symmetries. We also do not treat these methods as no dominant method has evolved so far (see SHB pch 15 for discussion).

Spectral: in those methods, the image is firstly filtered with a variety of filters such as blob and orientation filters, followed by a statistical description of the filtered output. Caution: these methods are sometimes also called 'structural'.

4.3.1 Statistical

ThKo p412

One can distinguish between first-order statistics and second-order statistics. First-order statistics take merely measures from the distribution of gray-scale values. In second-order statistics, one attempts to express also spatial relationships between pixel values and coordinates.

First-Order Statistics Let v be the random variable representing the gray levels in the region of interest. The first-order histogram $P(v)$ is defined as

$$P(v) = \frac{n_v}{n_{tot}} \quad (7)$$

with n_v the number of pixels with gray-level v and n_{tot} the total number of pixels in the region (`imhist` in Matlab for an entire image). Based on the histogram (equ. 7), quantities such as moments, entropy, etc. are defined. Matlab: `imhist`, `rangefilt`, `stdfilt`, `entropyfilt`

ThKo p414

Dav p213, s8.3
SHB p723, s15.1.2

Second-Order Statistics The features resulting from the first-order statistics provide information related to the gray-level distribution of the image, but they do not give any information about the relative positions of the various gray levels within the image. Second-order methods (cor)relate these values. There exist different schemes to do that, the most used one is the gray-level cooccurrence matrix (`graycomatrix` in Matlab), see Dav p213, s8.3 SHB p723, s15.1.2 for more.

The use of the cooccurrence matrix is memory intensive but useful for categories with low intensity variability and limited structural variability - or textural variability in this case. For 'larger' applications, the use of a spectral approach may return better performance.

4.3.2 Spectral [Structural]

FoPo p202
Pnc p277, s.13.1.5

In the spectral approach the texture is analyzed at different scales and described as if it represented a spectrum, hence the name. One possibility to perform such a systematic analysis is the use of wavelets ([wiki Wavelet](#)), which has found great use in image compression with the jpeg format. There are two reasons why wavelets are not optimal for texture representations. First, they are based on a so-called mother wavelet only, that is they are based on a single filter function. Second, they are useful for compression, but less so for classification. To represent texture, it is therefore more meaningful to generate more complex filters.

In the following we introduce a bank of filters that has been successful for texture description, see Figure 9 (see Appendix I.3.4 for code). It is one specific bank of filters, namely the one by authors Leung and Malik. But other spectral filter banks look very similar. All these filters are essentially a mixture of the filtering processes we have introduced before. There are four principally different types of filters in that bank:

Filter no. 1-4: those filters are merely Gaussian functions for different sigmas as we used them to generate the scale space (Section 3.1).

Filter no. 5-12: those eight filters are bandpass filters as we mentioned them in Section 4.1. In this case the blob filter is much larger and is a Laplacian-of-Gaussian (LoG) filter.

Filter no. 13-30 (rows 3, 4 and 5): these are oriented filters that respond well to a step or edge in an image - it corresponds to edge detection as introduced for contour extraction in Section 4.2. In this case, the first derivative of the Gaussian function is used.

Filter n. 31-48 (bottom three rows): those filters correspond to a bar filter. It is generated with the second derivative of the Gaussian.

To apply this filter bank, an image is convolved with each filter separately, each one returning a corresponding response map. In order to detect textons in this large output, one applies a quantization scheme as discussed in Section 7. Algorithm 1 is a summary of the filtering procedure.

1. In a first step, each filter is applied to obtain a response map R_i ($i = 1..n$). R_i can have positive and negative values.
2. In order to find all extrema, we rectify the maps obtaining so $2n$ maps R_j ($j = 1..2n$).
3. We try to find the 'hot spots' (maxima) in the image, that potentially correspond to a texton, by aggregating large responses. We can do this for example by convolving R_j with a larger Gaussian; or by applying a max operation to local neighborhoods.
4. Finally, we locate the maxima using an 'inhibit-on-of-return' mechanism that starts by selecting the global maximum followed by selecting (the remaining) local maxima, whereby we ensure that we do not return to the previous maxima, by inhibiting (suppressing) the neighborhood of the detected maximum.

Thus, for each image we find a number of 'hot spots', each one described by a vector x_l . With those we then build a dictionary as will be described in Section 7.

4.4 Special Features

4.4.1 Straight Line Segments, Circles

If an application requires the extraction of straight lines only, then there exist some dedicated techniques to find them. The most popular one is the Hough transform, which places straight line equations on each point of the diagonal toward all directions. The transform returns a matrix whose axes correspond to two variables: one is the distance ρ of the straight line from the image origin; and the other is the angle θ . The matrix is a 2D histogram, called an *accumulator* here, with the maximum corresponding to the longest,

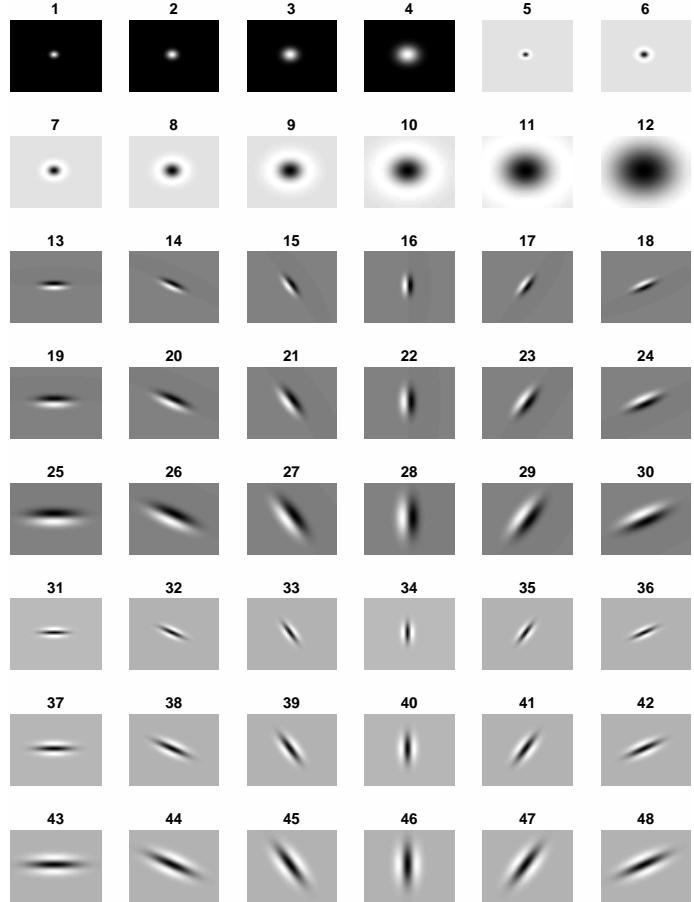


Figure 9: A bank of filters for texture analysis (the Leung-Malik filter bank): blob and orientation filters for different scales. Filter patches are generated for a size of 49×49 pixels.

Filters 1-4: Gaussian function (low-pass filter) at four different scales (sigmas).

Filters 5-12: a blob filter - a LoG in this case - at eight different scales.

Filters 13-30 (rows 3, 4 and 5): an edge filter - first derivative of the Gaussian in this case - for six different orientations and three different scales.

Filter 31-48 (bottom three rows): a bar filter - generated analogously to the edge filter.

straightest line in the image. For both variables, the bin size needs to be specified, which requires some tuning by the user. Example: the image contains a single line running from point $(x = 0, y = 1)$ to point $(x = 1, y = 0)$, then the transform has a single point at $(\rho = \sqrt{2}/2, \theta = 45^\circ)$.

Matlab: see the example under [houghlines](#)

Straight lines are used to find vanishing points in road scenes for instance (Figure 28). Or are used for calibration of internal and external camera parameters.

The Hough transform can be modified to search also for circles, see [imfindcircles](#).

Algorithm 1 Local filtering for texture.

Input : image I , set of n filters F_i (see Figure 9) ($i = 1..10$)

Parameters : radius for suppression in maximum search

1) Apply each filter F_i to the image to obtain a response map $R_i = I * F_i$

2) Rectification: for each R_i compute $\max(0, R_i)$ and $\min(0, -R_i)$, resulting in $2n$ maps R_j ($j = 1..2n$)

3) For each R_j , compute local summaries R_j^s by either

- convolving with a Gaussian of scale approximately twice the scale of the base filter
- taking the maximum value over a certain radius r .

4) Locate the maxima (in each map?) using an inhibition-of-return mechanism: $l = 1..n_{max}$ with coordinates $[x_l, y_l]$

Output : set of 'hot spot' vectors $x_l(j)$ whose components j correspond to the filter response R_j at location (x_l, y_l)

4.5 Exercises

4.5.1 Region Detection

The example in Appendix I.3.1 shows how we can filter for specific structures, in this case nuclei. Two types of band-pass filters are used: the difference-of-Gaussians (DoG) and the Laplacian-of-Gaussian (LoG). The filter sizes and threshold were chosen somewhat arbitrarily.

1. Is the filtering optimal? What is the size of a typical nucleus? What filter size would you suggest?
2. Why is the intensity range different between the DoG and the LoG output?
3. Would the use of color information improve segmentation-by-filtering? Try it out.

4.5.2 Edge Detection & Following

1. The code in Appendix I.3.3 should help clarify the differences between the different edge/boundary detection functions. Observe carefully how many contours are extracted with each method. Why are three contours extracted with `bwboundaries`?
2. Contour Extraction: Write a script `t_ContExtr` (contour extraction), which performs edge detection with `edge` and contour tracing using `bwboundaries`. The output of `bwboundaries` is a list of contours, whose coordinates we simply draw with the command `plot`, see `doc bwboundaries` for further examples. Plot the contours in two different images: once on top of the edge map `EM` and once on top of the gray-scale image `I`. Use color yellow, it contrasts well to the gray-scale image. Mark the starting point of a contour with a star marker (**).
3. Perform edge detection in the scale space, meaning for each scale separately. Display all levels including contours into a single figure. Observe how the edges change across scale: how do they differ across scale?

5 Classification with Deep Neural Networks

We now turn toward image classification, the process of assigning a scene or an object label to an entire image. One would think that this can be done with the feature extraction methods introduced in the previous section, and indeed there are innumerable attempts to do so. That approach is also called *feature engineering*. It has however lost its appeal - at least for the moment - with the arrival of the approach of *Deep Learning*, also called *feature learning*. With Deep Learning we let a so-called Deep Neural Network (DNN) find out the features that are necessary for getting the image classes discriminated. Not only are DNNs (or Deep Nets) more convenient - as we do not need to write elaborate feature extraction algorithms - but they also perform better, sometimes much better.

Deep Neural Networks are elaborations of traditional Artificial Neural Networks (ANNs), a methodology that exists already since decades. For a primer on ANNs we refer to Appendix (D.1). In this section we immediately start with Deep Neural Networks. Its most popular type is the so-called Convolutional Neural Network (CNN), which we introduce in Section 5.1. For this, we will switch to the programming language Python, as it has become the dominant language to explore neural network architectures. The big tech companies provide libraries to run such networks. Those libraries use similar terminology, but have slightly different ways to set up, initialize and run networks. One term that is often used is *tensor*, which is a function manipulating vectorial functions. In the context of Deep Nets, a tensor is often understood as an object representing batches of images, which makes the tensor three-dimensional or four-dimensional: two dimensions for the spatial dimension x and y , one dimension for the number of images in the batch, and another dimension if the image contains color information (typically 3: red, green, blue). A tensor can also be two- or even one-dimensional, in which case they are sometimes called simple tensors. Practically we do not need to learn novel algebra with this term, it is merely a different label for certain processes.

The three most popular Deep Net packages are:

PyTorch: this package is provided by Facebook. It is perhaps the most trending package. In particular *transfer learning* can be conveniently done with those libraries, coming up in Section 5.2.

Tensorflow: this package is provided by Google. It is considered somewhat intricate to understand and to apply, and that is why a simpler API was developed for it, called **Keras**. We use Keras for introducing MLPs (Appendix D.1) and for introducing a simple CNN (Section 5.1 next).

FastAI: a packed that promises to deliver state-of-the-art classification accuracies. It uses in particular ensemble of Deep Nets, which provide better results than individual Deep Nets. It is based on PyTorch.

For other packages we refer to the wiki pages: [wiki Comparison_of_deep_learning_software](#).

Data Augmentation One challenging issue of training an image classification system is the collection of labeled image material - we need to instruct the system what the categories (classes) are. The collection is a laborious process and sometimes one has few images per class only, think of medical images where the number of images for affected patients is rarer than for healthy patients. In order to maximally exploit the available image material, one often enlarges the material by introducing slight manipulations of the images, a process called *data augmentation*. Data augmentation aims to increase the (class) variability as to mimick the variability present in real-world classes - think of how many types of cars or chairs there exist. Those manipulations consist of randomly stretching, cropping, rotating the images. Some of those manipulations we will introduce in Section 14.1.

Learning Duration A fundamental downside of Deep Learning is that it takes very long to train a Deep Net. One solution to the problem is to apply massive computing power, available however mostly to big tech companies and universities. Another way to mitigate the downside is to exploit the so-called CUDA cores of the graphic cards in a PC. Software packages (PyTorch, Tensorflow) provide routines to exploit the available CUDA cores on a computer. A third way to counteract the problem is to use the trick of *transfer learning*. The latter two tricks will be addressed in particular in Section 5.2.

5.1 A Convolutional Neural Network (CNN)

wiki Convolutional_neural_network

Roughly speaking, a Convolutional Neural Network (CNN) can be regarded as an elaboration of a MLP (Section D.1). It is elaborated by the following two principal tricks in particular, see also Figure 10:

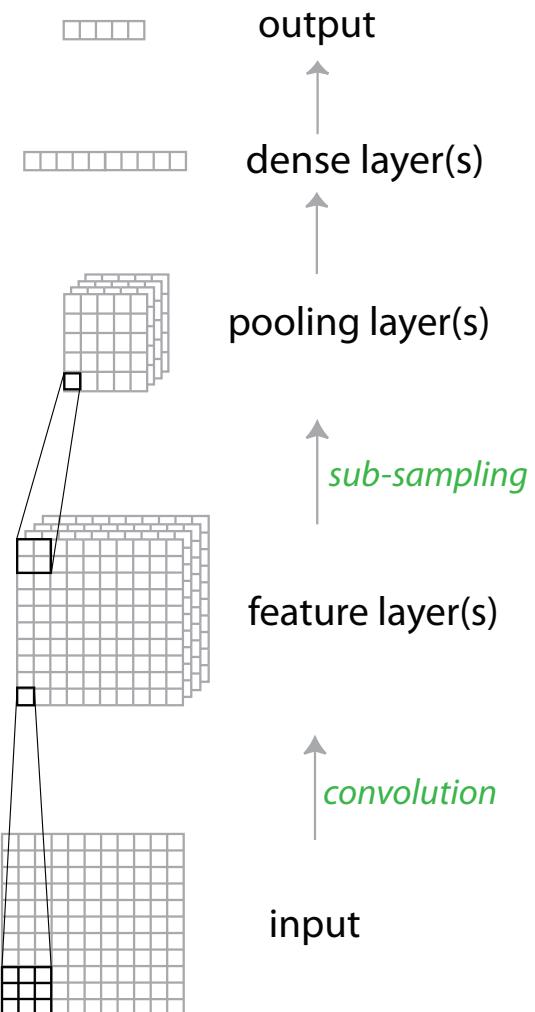
1. CNNs make actual use of the two-dimensionality of images, namely by applying quasi-convolutions to find discriminative, local features, similar to the feature extraction process discussed in Section 4 (Section 4.3.2 in particular). The difference is that in a 'traditional' convolution there is only a single kernel mask (K in equation 32), but in a CNN there are many individually learned kernel masks K_i , one for each local neighborhood i . The corresponding layers are also called *feature layers* or *feature maps*.
2. CNNs subsample the feature layers to so-called *pooling layers* akin to building a spatial pyramid discussed in Section 3.1.

Figure 10: The typical (simplified) architecture of a Convolutional Neural Network (CNN) used for classifying images. It is essentially an MLP elaborated by convolution and subsampling processes as introduced in the previous sections. The architecture has in particular several alternating feature and pooling layers. The term 'hidden' layer is not really used anymore in a CNN, as there are so many different types of hidden layers, that more specific names are needed.

The **feature layer(s)** is sometimes also called *feature map(s)*: it is the result of a convolution of the image, however not one with a single, fixed kernel, but one with many individual, learned kernels: each unit observes a local neighborhood in the input image and learns the appropriate weights.

The **pooling layer** is merely a lower resolution of the feature layer and is obtained by the process of sub-sampling. This sub-sampling helps to arrive at a more global 'percept'.

In a typical CNN, there are several alternations between feature and pooling layer. The learning process tries to find optimal convolution kernels K_i for each neighborhood i that help to separate the image classes.



Between the last pooling layer and the output layer, there lies typically a dense layer: it is a flat (linear) layer, all-to-all connected with its previous pooling layer and its subsequent output layer, again as a complete bipartite graph. This is sufficient introduction for the moment and we now start looking at some code as written in Keras. The code is merely an extension of the MLP code example of Appendix D.1, extended by convolution and pooling layers:

```

# https://github.com/fchollet/keras/blob/master/examples/mnist_cnn.py
# Trains a simple convnet on the MNIST dataset.
# Achieves 99.25% test accuracy after 12 epochs
from __future__ import print_function
import keras
from keras.models import Sequential
from keras.layers import Dense, Dropout, Flatten
from keras.layers import Conv2D, MaxPooling2D
from LoadMNIST import LoadMNIST
batchSz    = 128
nEpoch     = 12

#%%% ----- Load Database & Labels -----
Isz        = (28, 28, 1)
TREN, TEST, Lbl = LoadMNIST()      # load the digits
# extending by one dimension
TREN      = TREN.reshape(60000, Isz[0], Isz[1], 1) # [60000 28 28 1]
TEST      = TEST.reshape(10000, Isz[0], Isz[1], 1) # [10000 28 28 1]

#%%% ===== Build Network =====
N         = Sequential()
N.add( Conv2D(32, kernel_size = (3,3), activation = 'relu', input_shape = Isz))
N.add( Conv2D(64, (3,3), activation = 'relu') )
N.add( MaxPooling2D( pool_size = (2,2) ) )
N.add( Dropout(0.25) )
N.add( Flatten() )
N.add( Dense(128, activation = 'relu') )
N.add( Dropout(0.5) )
N.add( Dense(10, activation = 'softmax') )
N.summary()
N.compile( loss      = keras.losses.categorical_crossentropy,
            optimizer = keras.optimizers.Adadelta(),
            metrics   = ['accuracy'])

#%%% ===== Learning =====
N.fit(TREN, Lbl.TrenMx,
      batch_size = batchSz,
      epochs     = nEpoch,
      verbose    = 1,
      validation_data = (TEST, Lbl.TestMx) )

#%%% ===== Evaluation =====
score = N.evaluate(TEST, Lbl.TestMx, verbose=0)
print('Test loss: ', score[0])
print('Test accuracy:', score[1])

```

A typical CNN has many alternations between feature and pooling layers: for small tasks CNNs frequently have 3 or more such alternations, resulting in a 8-layer network or larger networks; for large tasks, CNNs can consist of several hundreds of layers.

5.2 Transfer Learning (PyTorch)

So far we have played with an image collection, whose image sizes are merely 28x28 pixels. If one intends to classify images of larger size, then we consequently convolve a larger area and that takes time as mentioned previously already. Fortunately, the big tech companies provide trained networks, which can be exploited to train novel image classes on them. That transfer of knowledge, namely from a *pre-trained* network to a modified network learning novel classes, is called *transfer learning*.

A pre-trained network is typically trained with thousands or even millions of images for hundreds of different classes, mostly general categories such as dogs, cars, lamps, flowers, etc.; an often-used collection is the *ImageNet* data set. Those pre-trained nets can then be used as feature extractors for other classification challenges that contain other categories. One would then adapt to the new task by merely making an adjustment to the top weight layer. That would be the quickest and simplest case and will be introduced in Section 5.2.1. A bit more elaborate is to adjust the entire network to the task, also called *fine-tuning* a

network (Section 5.2.2).

The libraries provided by PyTorch are particularly convenient to use transfer learning. A full example is given in Appendix I.5.3. Here we explain excerpts of it.

One specifies a desired pre-trained model with a single line, in this case we select the ‘resnet18’ model:

```
from torchvision import models
MOD      = models.resnet18(pretrained=True)
```

There exist other models, e.g. alexnet, densenet, inception. Often they come as a range of versions learned on different resolutions. For instance ‘resnet34’ provides higher resolution, but it also takes longer to train on it.

PyTorch is also convenient because it provides functions to easily augment the dataset. This is done with the module `transforms`. The various types of transforms can be lumped together to a single object, called `AUGNtrain` in the following code snippet:

```
from torchvision import transforms
AUGNtrain = transforms.Compose([
    transforms.RandomResizedCrop(szImgTarg),
    transforms.RandomHorizontalFlip(),
    transforms.ToTensor(),
    transforms.Normalize(ImgMean, ImgStdv) ])
```

Furthermore, PyTorch provides a routine `ImageFolder` in module `torchvision.datasets` to extract images and class labels from your collected data set automatically. You need to pass only the path of the folder that contains your classes, labeled accordingly, ie. all dog images placed into a folder called ‘dog’. The images can be of varying size. A routine called `DataLoader` in module `torch.utils.data` will automatically prepare the data set.

```
FOLDStrain = torchvision.datasets.ImageFolder(dirImagesTrain, AUGNtrain)
LOADERtrain = torch.utils.data.DataLoader(FOLDStrain, batch_size=szBtch, shuffle=True)
```

5.2.1 Fixed Feature Extraction

As introduced already, the fastest way to arrive at some results is to use the pre-trained network as is - without making further adjustments to it. To do that we proceed with two steps. One is to freeze the layers by preventing it to calculate gradients during the learning process: `requires_grad = False`. The second step is to replace the top layer with our problem set, for instance for a 5-class problem we need to insert a final weight layer that connects from the last dense layer to the final 5-unit output layer; function `nn.Linear` does that (`nClss` is the number of classes):

```
# freezing gradient calculation
for param in MOD.parameters():
    param.requires_grad = False
# replaces last fully-connected layer with new random weights:
MOD.fc = nn.Linear(MOD.fc.in_features, nClss)
```

Now we initialize the training parameters. With the function `SGD` we select an optimization procedure called *stochastic gradients*. Note that we specify the training only for the final layer by selecting `.fc`.

By calling `lr_scheduler.StepLR` we specify more learning parameters. Then we choose a loss function, a function that calculates the classification accuracy.

```
# only parameters of final layer are being optimized
optim = SGD(MOD.fc.parameters(), lr=lernRate, momentum=momFact)

# Decrease learning rate every szStep epochs by a factor of gamma
sched = lr_scheduler.StepLR(optim, step_size=szStep, gamma=gam)

crit = nn.CrossEntropyLoss()
```

5.2.2 Fine Tuning

To adjust the weights of the entire networks, we modify the steps of the previous section in two ways. One is, we refrain from freezing the parameters, so we drop the `for param` loop. Another step, is to omit the attribute `.fc`, which in turn instructs that the weights of the entire net are being re-learned:

```
# .fc is omitted - as opposed to above:  
optim = SGD(MOD.parameters(), lr=lernRate, momentum=momFact)
```

Adjusting the parameters of the entire net will take more time, but we gain classification accuracy. Other than that we can use the same code example as in I.5.3.

5.3 Summary and Notes

Deep Neural Networks (DNNs) are the most successful image classifiers to date - they are able to discriminate any set of classes decently. The CNN (Convolutional NN) is the most popular image classifier, but also the DBN (Deep Belief Network) is used.

DNNs have two disadvantages. One is that the design of the appropriate architecture is a bit of a heuristic endeavor, as well as the tuning of the learning parameters; the DBN is in that respect more elegant. Another reason is that learning itself is a very slow process. But the learning duration of CNNs can be accelerated with certain tricks and there is research going on trying to reduce the learning duration of such systems.

Another disadvantage is that they perform only classification. If one wanted to determine the exact position of the object - or its outline - one had to return to the classical feature extraction methods.

Practically, one hardly trains such a system by oneself, and rather uses transfer learning to achieve decent results.

6 Feature Extraction II: Patches

Sze p181, ch4, p205

Dav p149, ch6

We turn toward a feature that is sometimes generally referred to as *patch*. A patch is typically a small, square-sized pixel array of the image. From an image one selects a set of patches from specific locations and their collection represents the image. The extracted patch is then transformed; the most frequent transformation is the generation of a histogram of the local gradients of the patch. Such transformations make the patch relatively unique and distinct and thus suitable for matching between images; they are therefore also called 'features'. Patches are taken at points where there appears to be a corner or other 'interesting' structure (Figure 11), hence they are also called 'corners', 'interest points' or 'keypoint features'. But we remain with the term patch or feature for simplicity.

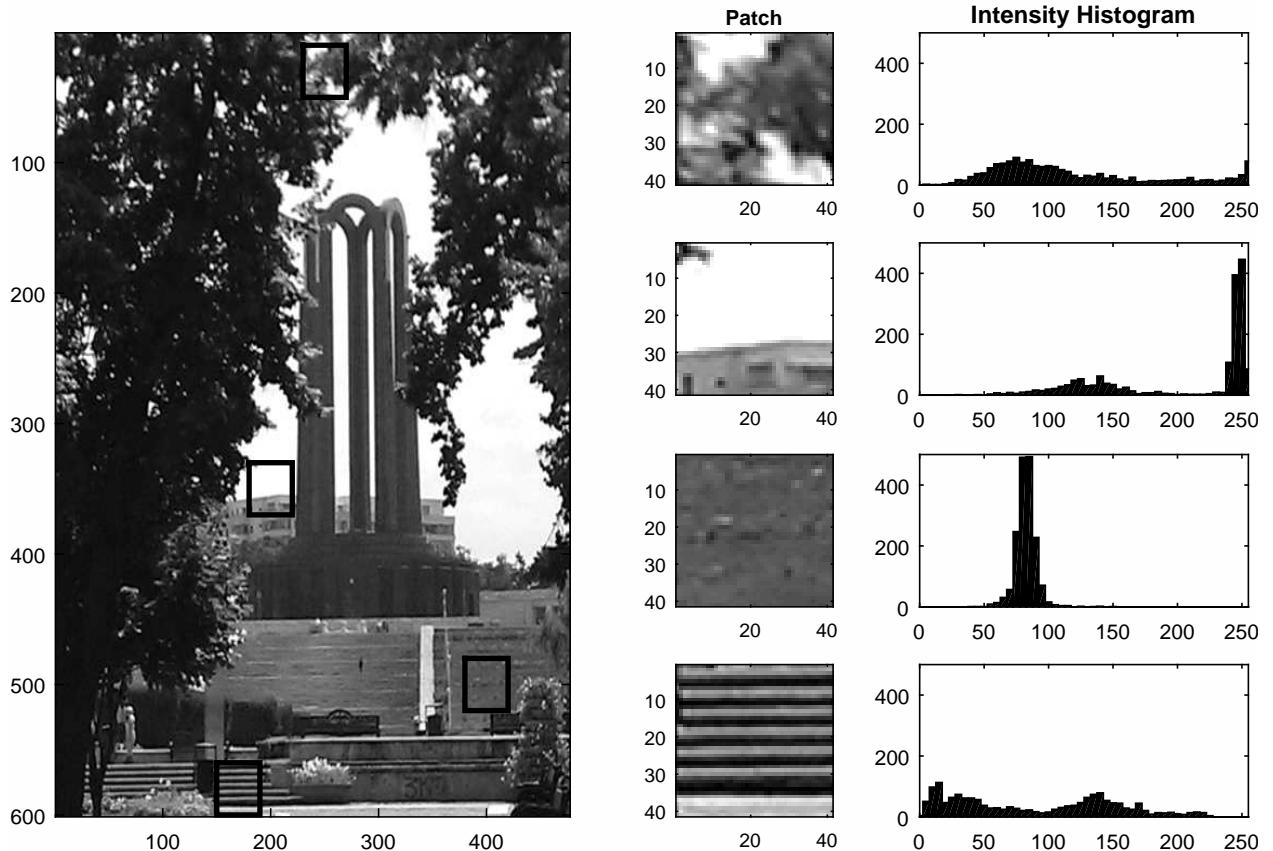


Figure 11: Examples of image patches and their corresponding intensity histogram.

Center column: four patches extracted from the picture.

Right column: histogram of intensity values for each patch.

We will discuss where in the image such patches can be taken (some studies take them at random locations!); and what kind of statistics can be taken from such patches - intensity is the simplest type of information, but one can also work with gradients as introduced in Section 3.3.

Patches are applied in feature-based correspondence techniques such as stereo matching, image stitching, fully automated 3D modeling, object instance detection as well as video stabilization. A key advantage of using matching with sets of keypoints is that it permits finding correspondences even in the presence of clutter (occlusion) and large scale and orientation changes. Patches used to be employed also for image classification, but were surpassed by Deep Neural Networks.

The process of finding and matching keypoints consists of three stages:

1. Feature detection: search for unique patch locations, that are likely to match well in other images.

2. Feature description: conversion of the patch into a more compact and stable (invariant) descriptor that can be matched against other descriptors.
3. Feature matching: weighting of feature descriptors and matching with descriptors of other images.

6.1 Detection

One way to find corners is to find edges, and then walk the edges looking for a corner. This approach can work poorly, because edge detectors often fail at corners. Also, at very sharp corners or unfortunately oriented corners, gradient estimates are poor, because the smoothing region covers the corner. At a 'regular' corner, we expect two important effects. First, there should be large gradients. Second, in a small neighborhood, the gradient orientation should swing sharply. We can identify corners by looking at variations in orientation within a window, which can be done by autocorrelating the gradients:

$$\mathcal{H} = \sum_{\text{window}} \{(\nabla I)(\nabla I)^T\} \quad (8)$$

$$\approx \sum_{\text{window}} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \quad (9)$$

whereby $I_x = I_o * \frac{\partial g}{\partial x}$ and $I_y = I_o * \frac{\partial g}{\partial y}$ (g is a Gaussian). In a window of constant gray level (that is without any strong gradient), both eigenvalues of this matrix are small because all the terms are small. In a window containing an edge, we expect to see one large eigenvalue associated with gradients at the edge and one small eigenvalue because few gradients run in other directions. But in a window containing a corner, both eigenvalues should be large. The Harris corner detector looks for local maxima of

$$C = \det(\mathcal{H}) - k \left(\frac{\text{trace}(\mathcal{H})}{2} \right)^2 \quad (10)$$

where k is some constant, typically set between 0.04 and 0.06. The detector tests whether the product of the eigenvalues (which is $\det(\mathcal{H})$) is larger than the square of the average (which is $(\text{trace}(\mathcal{H})/2)^2$). Large, locally maximal values of this test function imply the eigenvalues are both big, which is what we want. These local maxima are then tested against a threshold. This detector is unaffected by translation and rotation.

Algorithm 2 Feature detection.

Sze p185
FoPo p179, pdf 149
Dav p158, s6.5 s6.7
SHB p156, s5.3.10
Pnc p281, s13.2.2

-
1. Compute the horizontal and vertical derivatives I_x and I_y of the original image by convolving it with derivatives of Gaussians.
 2. Compute the three images corresponding to the outer products of these gradients.
 3. Convolve each of these images with a larger Gaussian.
 4. Compute a scalar interest measure using for instance equation 10.
 5. Find local maxima above a certain threshold and report them as detected feature point locations.
-

Matlab's image processing toolbox provides the command `corner`; this is how we would code it ourselves:

```
%% ----- Step 1
gx      = repmat([-1 0 1], 3, 1);           % derivative of Gaussian (approximation)
gy      = gx';
Ix      = conv2(I, gx, 'same');
Iy      = conv2(I, gy, 'same');
%% ----- Step 2 & 3
Glrg   = fspecial('gaussian', max(1, fix(6*sigma)), sigma); % Gaussian Filter
Ix2    = conv2(Ix.^2, Glrg, 'same');
Iy2    = conv2(Iy.^2, Glrg, 'same');
IxIy   = conv2(Ix.*Iy, Glrg, 'same');
%% ----- Step 4
k       = 0.04;
HRS    = (Ix2.*Iy2 - IxIy.^2) - k*(Ix2 + Iy2).^2;
```

As noted above already, we have presented the working principles of the Harris corner detector. There are many types of feature detectors but they are all based on some manipulation of the gradient image (see for instance [Dav p177, s6.7.6](#)). See also website links in [FoPo p190, s5.5](#) for code examples.

To now select corners in the 'corner image' ([HRS](#)) (step 5), we select maxima and suppress their neighborhood to avoid the selection of very near-by values. Here's a very primitive selection mechanism:

```
%% ----- Step 5
% Extract local maxima by performing a grey scale morphological
% dilation and then finding points in the corner strength image that
% match the dilated image and are also greater than the threshold.

sze = 2*radius+1; % size of dilation mask.
Hmx = ordfilt2(HRS,sze^2,ones(sze)); % grayscale dilate.

% Make mask to exclude points within radius of the image boundary.
bordermask = zeros(size(HRS));
bordermask(radius+1:end-radius, radius+1:end-radius) = 1;

% Find maxima, threshold, and apply bordermask
bHRS = (HRS==Hmx) & (HRS>thresh) & bordermask;

[r,c] = find(bHRS); % find row,col coords.
PtsIts = [r c]; % list of interest points [nPts 2]
```

Feature Tracking: Most features found at coarse levels of smoothing are associated with large, high-contrast image events because for a feature to be marked at a coarse scale, a large pool of pixels need to agree that it is there. Typically, finding coarse-scale phenomena misestimates both the size and location of a feature. At fine scales, there are many features, some of which are associated with smaller, low-contrast events. One strategy for improving a set of features obtained at a fine scale is to track features across scales to a coarser scale and accept only the fine-scale features that have identifiable parents at a coarser scale. This strategy, known as feature tracking in principle, can suppress features resulting from textured regions (often referred to as noise) and features resulting from real noise.

6.2 Description

The most popular descriptor is the scale invariant feature transform (SIFT), which is formed as follows:

- take the gradient (0-360 deg) at each pixel (from ∇I) in a 16×16 window around the detected keypoint (Section 3.3), using the appropriate level of the Gaussian pyramid at which the keypoint was detected.
- the gradient magnitudes are downweighted by a Gaussian fall-off function (shown as a circle in Figure 12) in order to reduce the influence of gradients far from the center, as these are more affected by small misregistrations.
- in each 4×4 quadrant, a gradient orientation histogram is formed by (conceptually) adding the weighted gradient value to one of 8 orientation histogram bins. To reduce the effects of location and dominant orientation misestimation, each of the original 256 weighted gradient magnitudes is softly added to $2 \times 2 \times 2$ histogram bins using trilinear interpolation. (Softly distributing values to adjacent histogram bins is generally a good idea in any application where histograms are being computed).
- form an $4 \cdot 4 \cdot 8$ component vector \mathbf{v} by concatenating the histograms: the resulting 128 non-negative values form a raw version of the SIFT descriptor vector. To reduce the effects of contrast or gain (additive variations are already removed by the gradient), the 128-D vector is normalized to unit length: $\mathbf{u} = \mathbf{v} / \sqrt{\mathbf{v} \cdot \mathbf{v}}$.
- to further make the descriptor robust to other photometric variations, values are clipped to $t = 0.2$: form \mathbf{w} whose i 'th element w_i is $\min(u_i, t)$. The resulting vector is once again renormalized to unit length: $\mathbf{d} = \mathbf{w} / \sqrt{\mathbf{w} \cdot \mathbf{w}}$.

FoPo p187, s5.4.1, pdf157

Dav p173, s6.7.3

Pnc p284, s13.3.2

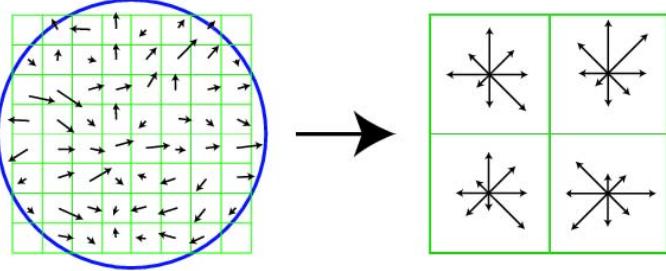


Figure 12: Forming SIFT features. Left: Gaussian weighting (for a 8x8 field in this illustration). Right: formation of histograms demonstrated on 2x2 quadrants. [Source: Szeliski 2011; Fig 4.18]

The following code fragments give an idea of how to implement steps a-c:

```

EdGrad = linspace(0,2*pi,8); % edges to create 8 bins
[yo xo] = deal(pt(1),pt(2)); % coordinates of an interest point
Pdir = Gbv.Dir(yo-7:yo+8,xo-7:xo+8); % 16 x 16 array from the dir map
Pmag = Gbv.Mag(yo-7:yo+8,xo-7:xo+8); % 16 x 16 array from the mag map
Pw = Pmag .* fspecial('gaussian',16, 4); % weighting center
BLKmag = im2col(Pw, [4 4], 'distinct'); % quadrants columnwise for magnitude
BLKdir = im2col(Pdir, [4 4], 'distinct'); % quadrants columnwise for direction
Gh = [];
for k = 1:16
    [HDir Bin] = histc(BLKdir(:,k), EdGrad);
    HDw = accumarray(Bin, round(BLKmag(:,k)*10000), [8 1]);
    Gh = [Gh; HDw];
end

```

6.3 Matching

To compare two 2 descriptor lists (originating from two different images for instance), d_i and d_j ($i = 1,..k, j = 1,..l$), we take the pairwise distances and form a $k \times l$ distance matrix D_{ij} . Then we take the minimum in relation to one descriptor list, e.g. $\min_i D_{ij}$, and obtain the closest descriptor from the other descriptor list. That would be a simple correspondence and may suffice if we compare a list of image descriptors and a list of category descriptors, as we will do for image classification (section 7). If we intend to establish correspondences for registration (section 14), we want to find the mutual matches.

```

L1, L2: the 2 descriptor lists, [nD1 x nDim] and [nD2 x nDim]
% ----- Compact version:
DM = pdist2(L1,L2);
% ----- Explicit version: (building DM ourselves)
DM = zeros(nD1,nD2);
for i = 1 : nD1
    iL1rep = repmat(L1(i,:), nD2, 1); % replicate individual vector of L1 to size of L2
    Di = sqrt(sum((iL1rep - L2).^2,2)); % Euclidean distance
    DM(i,:) = Di; % assign to distance matrix
end
% ----- Correspondence with respect to one list
[Mx2 Ix1] = min(DM,[],1); % [1 x nD2]
[Mx1 Ix2] = min(DM,[],2); % [nD1 x 1]
% ----- Correspondence mutual
IxP1to2 = [(1:nD1)' Ix2]; % [nD1 x 2] pairs with indices of 1st list and minima of 2nd list
IxP2to1 = [(1:nD2)' Ix1']; % [nD2 x 2] pairs with indices of 2nd list and minima of 1st list
bMut1 = ismember(IxP1to2, IxP2to1(:,[2 1]), 'rows'); % binary array of mutual matches in list 1
IxMut1 = find(bMut1); % mutually corresponding pairs with indexing to list 1
IxPMut1 = IxP1to2(IxMut1,:); % [nMut1 x 2] mutual pairs of list 1

```

One may also want to use a for-loop for the maximum operation, that is to take the maximum row-wise (to avoid costly memory allocation).

Note For large databases with thousands of vectors, these "explicit but precise" distance measurements are too slow anyway. Instead, faster but slightly inaccurate methods are used, as for instance hashing functions or kd-trees.

6.4 Notes

The advantage of the previously introduced histograms of gradients are their versatility, that is they can be used in many computer vision tasks. Their downside is that they cannot be used find actual object outlines (nor are they considered a credible approach to a more capable recognition system).

C implementations with Matlab interface can be found here for instance:

```
http://www.vlfeat.org/
http://www.aishack.in/2010/05/sift-scale-invariant-feature-transform/
```

6.5 Exercises

1. Write a script `t_CornerDet`, which detects interest points in an image following the algorithm 2, see code hints in Section 6.1. Display the corner map `HRS`: what is its range? How do you find the maxima? To find maxima you could apply different methods, for instance using morphological operations (`bwmorph`), or the statistical solution as given under step 5 in Section 6.1. Name the variable for detected points `PtsInt` (no. points x 2 [row/column]). Display the detected points on the gray-scale image - be aware of axes differences (Cartesian and matrix axes). Use the plotting option 'marker-size' to draw markers whose size corresponds to the degree of interest, the corresponding value in the corner map `HRS`.

Exclude interest points, that lie too close to the image bounds by either applying another bordermask (see step 5 in 6.1), or by operating on the list of interest points:

```
[nRow nCol] = size(Image); % no. of rows and columns of the images
ms = 8; % margin size
b_NearHorz = PtsInt(:,1)<ms+1 & PtsInt(:,1)>nRow-ms; % near horizontal borders
b_NearVert = PtsInt(:,2)<ms+1 & PtsInt(:,2)>nCol-ms; % near vertical borders
b_Near = b_NearHorz | b_NearVert; % near any border
PtsInt = PtsInt(~b_Near,:); % keeping inside border
```

Finally copy the code into a function `f_CornerDet`, whose input is the image `I` and the parameters `k`, `radius` and `thres`, and whose output is a list of interest points `PtsInt`. Play with the parameters and observe the changes, e.g. process the image with three different values for radius and plot the output in different images. Do the same for `k` and `thres`.

2. Write a script `t_DescGradHist`, which generates the gradient descriptors as treated under Section 6.2. For each interest point of `PtsInt`, we take its 16×16 pixel neighborhood and generate the 1×128 -dimensional gradient histogram. Display the histograms using the command `imagesc`. What is the maximum amplitude of all histograms? If all looks good, create the corresponding function `f_DescGradHist`, whose input are the image and the list of interest points, and the output is the list (matrix) of vectors `DSC`; choose the format number of points (rows) times number of dimensions (columns).
3. Write a script `t_DescMatch`, which matches the descriptor lists of a pair of similar images, `DSC1` and `DSC2`. Use the `pdist2` command to find the pairwise distances. Caution! this step is memory consuming: make sure you obtain fewer than 2000 interest points per image, i.e. use `imresize` to scale the image. To save further on memory, use only single precision instead of the default double precision, because we do not really require double precision in our application. Plot the distance matrix, check its data range.

To find the mutual correspondences, it is useful to firstly study a mock example. Open a script `t_indexing` and take two short, random lists to clarify the assignments, e.g. `nD1=5; nD2=8; L1 = rand(nD1,3); L2 = rand(nD2,3);`, where dimensionality is only 3 to keep everything simple. Generate the distance matrix and determine the mutual correspondences as exemplified in Section 6.3). Study the method exactly. Then apply to your real-word example. Sort the minimum distances for the mutual matches in increasing order and plot them.

Verification I: To verify visually, plot some of the best matching, corresponding pairs. Place the two images into one figure, then plot the first best 30 matches and denote the corresponding point pairs in the two images with a text label using `text`. We expect that some will match and some won't due to variability. Pay attention to the axes (x/y versus row/column, see also note in Section 1.6).

Verification II: if descriptors do not match, then take the same image twice - that should return exact correspondence.

If some points match, copy the code into a function `f_DescMatch`: its input is the two list of descriptors `DSC1` and `DSC2`, the output are arrays with corresponding point indices for the best matches.

4. It is worthwhile documenting the above functions. Write for each one an explanation of what it does (1-3 sentences) into its header. Specify the dimensions of the input and output variables.

It is always useful to display some text output in the command window. Use `tic` and `toc` to measure how long the functions take to process an image. Display how many interest points are detected with `fprintf`. Determine and display the minimum and maximum distances between features. And so on.

5. Take two slightly different images, e.g. room scene photographed from 2 slightly different angles, for instance take a step sideways for the second shot (angle change of few degrees is sufficient). Plot the results in two different figures and observe again how the contour output differs amongst the two images. Notice the variability - it is one of the reasons that makes recognition in images so difficult.

7 Feature Quantization

FoPo p203, s6.2, pdf 164
Sze p612, s14.4.1, pdf 718

We now move towards representations for objects and scenes using the patches as obtained in the previous Section 6. For some time, that was the most successful method for scene classification system, but has lost its appeal since Deep Neural Networks took over. Nevertheless, it is useful to understand the method, because it has its advantages too: it does not need as many training samples as a DNN and it learns much, much quicker.

A generic way to exploit such patches is to collect a large number of patches for a category and to find clusters within them, that are representative for that category. To illustrate the idea, we look at an example from image compression, specifically color encoding:

Example Quantization An image is stored with 24 bits/pixel and can have up to 16 million colors. Assume we have a color screen with 8 bits/pixel that can display only 256 colors. We want to find the best 256 colors among all 16 million colors such that the image using only the 256 colors in the palette looks as close as possible to the original image. This is color quantization where we map from high to lower resolution. In the general case, the aim is to map from a continuous space to a discrete space; this process is called vector quantization. Of course we can always quantize uniformly, but this wastes the colormap by assigning entries to colors not existing in the image, or would not assign extra entries to colors frequently used in the image. For example, if the image is a seascape, we expect to see many shades of blue and maybe no red. So the distribution of the colormap entries should reflect the original density as close as possible placing many entries in high-density regions, discarding regions where there is no data. Color quantization is typically done with the k-Means clustering technique.

The Principal Applied to Features In our case, we aim to find clusters amongst our features that represent typical 'parts' of objects, or typical 'objects' of scenes. In the domain of image classification and object recognition, these clusters are sometimes also called (visual) 'words', as their presence or absences in an image, corresponds to the presence or absence of words in a document; in texture recognition they are also called 'textons'. The list of words represents a 'pooled' representation or a 'dictionary' (aka 'bag of words'), with which we attempt to recognize objects and scenes. Thus, in order to apply this principal, there are two phases: one is building a dictionary, and the other is applying it; which would correspond to training and testing in machine learning terminology. Figure 13 summarizes the approach. We will merely point out how to use the machine learning techniques and omit lengthy explanations in order to progress with the concepts in computer vision.

7.1 Building a Dictionary

We quantize features as follows:

1. Collect patches $\mathbf{x}_i(d)$ from images (or image patches for objects) of the same category, e.g. n_p in total ($i = 1..n_p$). These patches can be represented in various ways, e.g. only by pixel values, in which case the dimensionality n_d corresponds to the number of pixels ($d = 1..n_d$); or by a SIFT histogram, in which case the dimensionality corresponds to the histogram length ($n_d = 128$ as for original SIFT features). Normalization can sometimes improve performance - try different normalization schemes.
2. This step is optional. You may want to try a principal component analysis (PCA) to reduce the dimensionality of your features. This maybe in particularly useful if you use pixel intensities as dimensions only, but can also be tried for SIFT features. We now denote the reduced dimensionality with n_r and the reduced vectors as $\mathbf{x}_i^r(d)$ with $d = 1..n_r$.
3. Quantize vectors with a clustering technique and obtain a list of clusters $c_j(d)$ with $j = 1..n_c$. In the simplest case, a cluster is represented by the mean of its members. Determine a threshold that decides when a tested feature is close enough to the center.

Algorithm 3 Building a dictionary for a category with $\mathbf{x}_i \in \mathcal{D}^L$. Compare with upper half of Figure 13.

- 1) Collect many training patches $\mathbf{x}_i(d)$ $(i = 1..n_p; d = 1..n_d)$
 - 2) Optional: apply the PCA: $\mathbf{x}_i(d) \rightarrow \mathbf{x}_i^r(d)$ $(d = 1..n_r)$
 - 3) Find k (n_c) cluster centers $\mathbf{c}_j(d)$ $(j = 1..n_c; d = 1..n_r)$
-

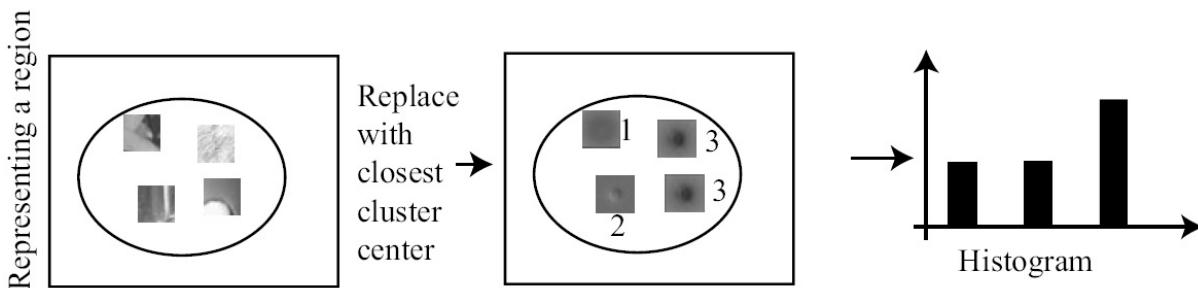
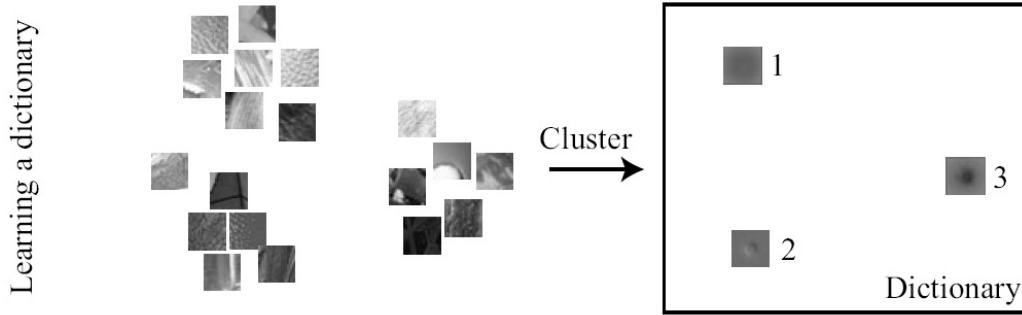


Figure 13: Using image patches for classification.

Upper part Learning a dictionary: 3 clusters were found in the training set (using some clustering technique), whose average values are shown as 'Dictionary'.

Lower part Representing a region: describing an object or scene. For a given image, replace detected texture patches with the closest cluster label and create a histogram. [Source: Forsyth/Ponce 2010; Fig 6.8]

7.1.1 Vector-Quantization using the k-Means Algorithm

The most frequent clustering technique used for vector quantization is the k-Means algorithm, a widely used machine learning technique. For this technique we need to provide the number of expected cluster k , hence the name k-Means, which we here however denote as n_c . In other words, we have to estimate the number of words of our dictionary prior to its usage. This is a bit odd as we rather prefer an algorithm that chooses the optimal number of clusters automatically, but unfortunately we have to determine that number ourselves by systematic testing.

K-means is a 'quick-and-dirty' method to cluster - in comparison to computationally more expensive and slower algorithms. It is an iterative procedure in which the cluster centers and sizes are gradually evolved by comparing the individual data points (vectors) sequentially. The procedure starts by randomly selecting n_c data points (from n_t total data points), which are taken as initial cluster centers. Then, the remaining data points are assigned to the nearest cluster centers and the new cluster centers are determined. Because the new cluster centers will be in slightly different locations, a new assignment labeling is carried out and the new centers determined, etc. The most beautiful illustration for this process is in Pnc p291, s.13.4.4.

To apply this technique in Matlab we organize our patches (features) in a $nF \times nD$ matrix **FTS**. We then cluster with the command **kmeans**: $\mathbf{Ixs} = \text{kmeans}(\mathbf{FTS}, k)$, whereby **Ixs** is a vector of length **nF** containing the cluster indices ($\in 1..k[n_c]$). See also Appendix E for more details. There are different ways how to

calculate the clusters during evolvement, see the options of `kmeans`.

7.2 Applying the Dictionary to an Image

One collects patches from a testing image, vector quantizes them by identifying the index of the closest cluster center, then computes a histogram with bins corresponding to the cluster indices that occur within the image. A bit more elaborate and step by step:

1. For a given testing image (or object), find interest points and describe the patches around them: $\mathbf{v}_m(d)$ with $m = 1..n_q$. Apply the PCA (if it was used before): $\rightarrow \mathbf{v}_m^r(d)$ with $d = 1..n_r$.
2. For each patch \mathbf{v}_m (or \mathbf{v}_m^r) find the nearest cluster center \mathbf{c}_j ($j = 1..n_c$) - if there is one (thresholding!).
3. Create a histogram $H(j)$, which counts the occurrences of (quantized) features, that is the cluster centers. The total histogram count is less equals n_q as some features may not have exceeded the threshold (ideally none for the features of a different category).

Histograms can now be used for classification or retrieval.

Algorithm 4 Applying the dictionary (for one category), $\mathbf{x}_i \in \mathcal{D}^T$. Compare with lower half of Figure 13.

- 1) For each relevant pixel m in the image: compute the vector representation \mathbf{v}_m around that pixel
 - 2) Obtain j , the index of the cluster center \mathbf{c}_j closest to that feature
 - 3) Add a value of one to the corresponding bin in the histogram $H(j)$.
-

7.3 Classification

A classifier is trained (or learned) with a so-called *training* dataset. To estimate its prediction performance it is applied to a so-called *testing* (or *sampling*) set. It requires a training and a testing set: the classifier is learned on the training set and then its performance is verified on the testing set. See also Appendix E for implementation details. When working with a dictionary, we need to partition the training set as well: one partition is used for building the dictionary, the other partition is used for generating histograms as training 'material' for the classifier.

Example: We have 30 images per category. For each category we use 25 instances for training and 5 for testing. Of the 25 training instances, we use 5 for building the dictionary (algorithm 3), the other 20 are used for generating histograms (algorithm 4). The actual classifier is then trained with those 20 histogram vectors and tested on the 5 training histograms, which were also generated with algorithm 4. We do this for 3-5 folds (see appendix).

As you may have noticed, there are many parameters that influence performance. The optimization of such a system is equally challenging (if not even more) as developing just the system - hence enthusiasm to deal with much code is of benefit. For the moment, we attempt to get the classification system going with a moderate performance and leave fine tuning to experts in classification. We mention here only that applying the principal component analysis may result in the largest performance improvement as well as the tuning of the feature thresholds.

7.4 Principal Component Analysis (PCA)

The PCA transform can provide us with a more compact representation, that is we transform the data such that we can omit seemingly irrelevant dimensions. In some cases, we have to apply the PCA, otherwise the classifier will not work at all. But using the PCA, we possibly loose somewhat of the discriminatory power of the higher dimensional space. See Appendix for implementation details.

To build a good classifier, the PCA is applied to the training set only. The resulting coefficients are then used to transform the testing set.

7.5 Exercises

Let us apply a dictionary ('bag of words') to image/object classification. Download a database with at least 5 classes and at least 25 testing images per class (with fewer training images it is difficult to train a classifier). Examples:

- The urban and natural scene collection: <http://cvcl.mit.edu/database.htm>. 60 percent can be easily reached with 25 training images and 5 testing images (6-fold crossvalidation).
- Satellite images: <http://vision.ucmerced.edu/datasets/landuse.html>. 70 percent can be easily reached using all 100 images per category. Try with 25 training and 5 testing images per category (6-fold crossvalidation).
- The Caltech 101 collection: http://en.wikipedia.org/wiki/Caltech_101. Choose a subset of categories if you prefer. (This one's hard even though the categories are objects only and are centered in the image).
- Or you may even photograph your own collection, which is more exciting but also time-consuming.

To preprocess, match and classify the images we break down the recognition sequence into several scripts:

1. Write a script `img_desc`, which extracts the interest points and corresponding descriptors for each image. Loop over the images and save the descriptors as `Desc` with the command `save`. Create a directory 'Dat-Desc' where you place the individual files. If you need to save on memory, then convert the descriptors into single format `Desc = single(Desc)`.

2. Write a script `img_concat`, which loads the individual descriptor lists and concatenates them into a single array of size: total no. descriptors for collection x no. dimensions. Speed up this procedure by initializing with `DESC = zeros(nDscTotEst,nDim,'single');` where `nDscTotEst` is an overestimation of the total number of descriptors for the collection. You trim the list `DESC` to the correct size at the end of the loop. Make sure you save as data type `single`.

Along with concatenating all image descriptor lists, also create an array of indices `IXD` of size 'total no. descriptors for collection x 3', where the first column corresponds to the image index, the 2nd column to the feature index and the 3rd is the category label.

3. Write a script `desc_cluster`, which loads the entire descriptor list `DESC` and perform clustering on it, see Appendix E. Use `kmeans` to find clusters. Generate three clusterings for k equal 25, 50 and 100 as the number of clusters, which correspond to three different sets of words. Create an average vector for each cluster (word).

4. Write a script `img_match`, which matches the average cluster vectors (words) against the descriptor list of each image. For each image determine the word count.

5. Write a script `img_classif`, which classifies the images using the word vectors. Try first without principal component analysis.

8 Object Detection

FoPo p549, ch 17

Object detection is the localization of an object category in a scene. For example, we would like to determine how many faces there are in a group photo; or how many pedestrians there are in a street scene. The principal technique is to train a classifier that discriminates between the desired object and any other object. This training is done on manually selected images. With the trained classifier we then search the image for potential matches, whereby the search is done on a subset of pixel locations, because it would be too costly to apply the classifier at each pixel: one uses 'windows' that typically overlap and are taken from a grid - it is a coarse scanning of the image. This is also called the sliding window technique sometimes.

For the training phase, we collect two datasets of image windows, each window of the same size $n \times m$. One set contains windows of the object of relatively fixed size and reasonably centered in the image. The other set contains 'distractor' images that constitute the non-object information. We then train a classifier to discriminate between these two sets of windows (classes) - a Deep Neural Network for instance. In the testing phase, we pass $n \times m$ windows of a new image to the classifier; the window is moved by a step size of few pixels to speed up the search (e.g. Δx and $\Delta y=3$ pixels). There are three challenges with this technique:

- 1) Size invariance: the detection system should be invariant to object size. This can be achieved by a search over scale, meaning by using the pyramid (Section 3.1): to find large objects, we search on coarser scales (layers), to find small objects we search on a finer scales. Put differently, we apply the $n \times m$ window in each layer of the pyramid.
- 2) Avoiding multiple counts: the very same object instance in an image should not be counted multiple times, which may happen due to the sliding search: the smaller the step sizes, the higher the chance for repeated detection. To avoid multiple counts, the neighboring windows are suppressed, when a local maximum was detected, also called nonmaximum suppression.
- 3) Accelerating spatial search: searching for a match in the highest image resolution is time consuming and it is more efficient to search for a match in the top pyramidal layers first and then to verify on lower layers (finer scales), that means by working top-down through the pyramid, e.g. first P_3 , then P_2 , etc. This strategy is also known as coarse-to-fine matching.

The technique in summary:

Algorithm 5 Sliding window technique for object detection.

==== TRAINING: Train a (binary) classifier on $n \times m$ image windows with positive (object) examples and windows with negative (non-object) examples.

==== TESTING:

Parameters detection threshold t , step sizes Δx and Δy

- 1) Construct an image pyramid.
- 2) For each level of the pyramid:
 - apply the classifier to each $n \times m$ window (moving by Δx and Δy) and obtain strength c .
 - if $c > t$, then add window to a list \mathcal{L} including response value c .
- 3) Rank list \mathcal{L} in decreasing order of c values $\rightarrow \mathcal{L}^{seq}$.
- 4) For each window \mathcal{W} in sequence \mathcal{L}^{seq} (starting with maximal c):
 - remove all windows $\mathcal{U} \neq \mathcal{W}$ that overlap \mathcal{W} significantly where the overlap is computed in the original image by expanding windows in coarser scales $\rightarrow \mathcal{L}^{red}$.

\mathcal{L}^{red} is the list of detected objects.

There are obviously tradeoffs between search parameters (e.g. step sizes) and system performance (e.g. detection and localization accuracy). For example, if we work with training windows that tightly surround the object, then we might be able to improve object/distractor discrimination, but we will have to use smaller step sizes for an actual performance improvement. Vice versa, if we use windows that surround the object only loosely, then we can use smaller steps sizes but our discrimination and localization performance suffers.

Matlab: see 'Neighborhood and Block Operations' under the image processing toolbox.

8.1 Face Detection

A typical face detection system uses the following learning tricks to improve performance:

- a) non-face images are collected from aerial images or vegetation for instance (Figure 14b).
- b) the set of collected face images is augmented by artificially mirroring, rotating, scaling, and translating the images by small amounts to make the face detectors less sensitive to such effects (Figure 14a).
- c) after an initial set of training images has been collected, some optional pre-processing can be performed, such as subtracting an average gradient (linear function) from the image to compensate for global shading effects and using histogram equalization to compensate for varying camera contrast (figure 14c).

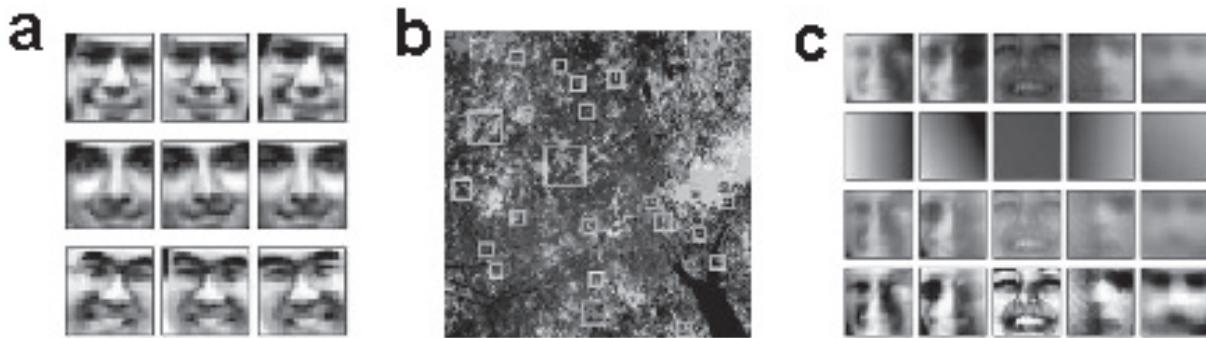
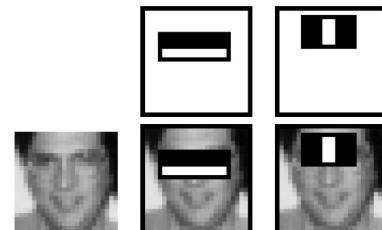


Figure 14: Pre-processing stages for face detector training (Rowley, Baluja, and Kanade 1998a): **a)** artificially mirroring, rotating, scaling, and translating training images to generate a training set with including larger variability; **b)** using images without faces (looking up at a tree) to generate non-face examples; **c)** pre-processing the patches by subtracting a best fit linear function (constant gradient) and histogram equalizing. [Source: Szeliski 2011; Fig 14.3]

Viola-Jones Algorithm The most frequently used face detection algorithm is probably the one by Viola and Jones. It uses features consisting of 2-4 rectangular patches of different polarity, see upper row of figure 15. The pixels inside the white rectangles are subtracted from the pixels inside the black pixels; rectangles are computed with the integral image (Section 8.1.1). To find out which combinations of rectangles (orientations and size) are representative for a category, it is necessary to try out all combinations, which is a very time-intensive procedure (despite the rapid computation of rectangles). This feature selection can be done with a 'boosting' classifier. The most significant 2 features are shown in Figure 15; there exist also a number of other less significant features. Testing an image occurs very rapidly by searching for the most significant features first; if they are present, the search continues; if they are not present, the search is stopped.

The primary advantage of this detection system is that it is extremely fast and runs in real time. The downside of the system is that it detects only vertically oriented faces (the majority of faces is vertically oriented anyway), and the long learning duration as just mentioned.

Figure 15: Face detection with groupings of rectangular patches. Top row: the 2 most significant rectangle-based combinations (in isolation). The horizontally oriented feature represents the eyes and the cheekbones; the vertically oriented ones represent the region covering left eye-nose bridge-right eye. [Source: Szeliski 2011; Fig 14.6]



Applications Face detectors are built into most of today's digital cameras to enhance auto-focus and into video conferencing systems to center on the speaker. They are also used in consumer-level photo organization packages, such as iPhoto, Picasa, and Windows Live Photo Gallery. Finding faces and allowing users to tag them makes it easier to find photos of selected people at a later date or to automatically share them with friends. In fact, the ability to tag friends in photos is one of the more popular features on Facebook.

8.1.1 Rectangles

Rectangular regions can be detected rapidly by use of the integral image, aka summed area table, which is computed as the running sum of all the pixel values from the origin:

$$I_s(i, j) = \sum_{k=0}^i \sum_{l=0}^j I_o(k, l). \quad (11)$$

To find now the summed area (integral) inside a rectangle $[i_0, i_1] \times [j_0, j_1]$, we simply combine four samples from the summed area table:

$$R_s(i_0..i_1, j_0..j_1) = I_s(i_1, j_1) - I_s(i_1, j_0) - I_s(i_0, j_1) + I_s(i_0, j_0) \quad (12)$$

Matlab

```
Is = cumsum(cumsum(Io,1),2); % integral image
Rp = Is(i1,j1)-Is(i1,j0)-Is(i0,j1)+Is(i0,j0); % a rectangular pixel patch
```

Python

```
skimage.transform.integral_image
```

8.2 Pedestrian Detection

According to Dalal and Triggs, one can typify the structure of pedestrians into 'standing' and 'walking':

- standing pedestrians look like lollipops (wider upper body and narrower legs).

- walking pedestrians have a quite characteristic scissors appearance.

Dalal and Triggs use histograms of gradients (HOG) descriptors, taken from a regular grid of overlapping windows (figure 16). Windows accumulate magnitude-weighted votes for gradients at particular orientations, just as in the SIFT descriptors (see previous section). Unlike SIFT, however, which is only evaluated at interest point locations, HOGs are taken from a regular grid and their descriptor magnitudes are normalized using an even coarser grid; they are only computed at a single scale and a fixed orientation. In order to capture the subtle variations in orientation around a person's outline, a large number of orientation bins is used and no smoothing is performed in the central difference gradient computation.

Figure 16 left shows a sample input image, while Figure 16 center left shows the associated HOG descriptors. Once the descriptors have been computed, a support vector machine (SVM) is trained on the resulting high-dimensional continuous descriptor vectors. Figures 16 center right and right show the corresponding weighted HOG responses. As you can see, there are a fair number of positive responses around the head, torso, and feet of the person, and relatively few negative responses (mainly around the middle and the neck of the sweater).

Matlab

```
extractHOGfeatures
```

Python

```
skimage.feature.hog
```

Applications Needless to say, that pedestrian detectors can be used in automotive safety applications. Matlab even has a code example for pedestrian detection.

Sze p106, pdf 120

Pnc p275

Dav p175

SHB p101, alg 4.2

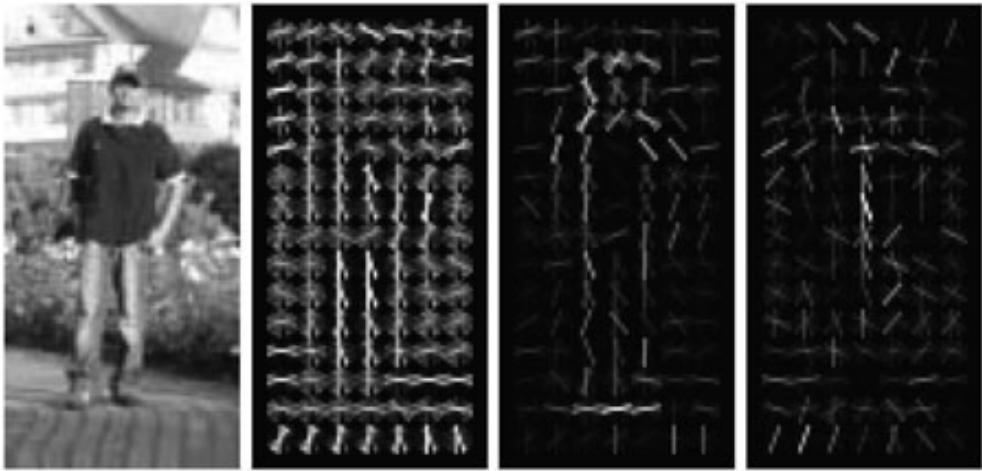


Figure 16: **Left:** typical pedestrian window.

Center left: HOG descriptor. Each of the orientation buckets in each window is a feature, and so has a corresponding weight in the linear SVM.

Center right: HOG descriptor weighted by positive weights, then visualized (so that an important feature is light). Notice how the head and shoulders curve and the lollipop shape gets strong positive weights.

Right: HOG descriptor weighted by the absolute value of negative weights, which means a feature that strongly suggests a person is not present is light. Notice how a strong vertical line in the center of the window is deprecated (because it suggests the window is not centered on a person).

[Source: Forsyth/Ponce 2010; Fig 17.7]

8.3 Improvement by Knowing Context

The sliding window technique is obviously a bit simple. The technique works with objects that exhibit limited variability in appearance (gradients) and structure, meaning that do not deform too much. Some improvement could be made if we knew more about the scene. Let's take pedestrian detection as an example. Pedestrians (and like most objects) appear in a typical *context*: pedestrians are all about the same absolute size, have their feet on or close to the ground, and are usually seen outdoors, where the ground is a plane. Thus, if we knew the horizon of the ground plane and the height of the camera above that ground plane, we could exclude many windows immediately. For instance, windows whose base is above the horizon would be suspect because they would imply pedestrians in the air; windows whose base is closer to the horizon should be smaller (otherwise, we would be dealing with gigantic pedestrians). The height of the camera above the ground plane matters because in this problem there is an absolute scale, given by the average height of a pedestrian. Assume the horizon is in the center of the image. Then, for cameras that are higher above the ground plane, legitimate pedestrian windows get smaller more quickly as their base approaches the horizon. There are two strong sources of information about the horizon and the camera height. First, the textures of the ground, buildings, and sky are all different, and these can be used to make a rough decomposition of the image that suggests the horizon. Second, observing some reliable detection responses should give us clues to where the horizon lies, and how high the focal point is above the ground plane. Hoiem et al. (2008) show that these global geometric cues can be used to improve the behavior of pedestrian and car detectors (see also Hoiem et al. (2006)).

8.4 Notes

Of course, one could also try to build object representations with SIFT features (or other histograms at interest point detectors), e.g. by determining the spatial alignment (configuration) between interest points but then our system grows very large and becomes slower. There exist however matching optimization methods.

9 Segmentation (Image Processing II)

SHB p176, ch6
FoPo p285, ch9, pdf255
Sze p235, ch5.pdf267

Image segmentation is the task of delineating objects or meaningful regions. For a scene, image segmentation aims at partitioning the scene into its constituent objects and regions, often with the aim at performing a foreground/background segregation. For an object - in face of some background -, image segmentation aims at finding the exact silhouette and possibly the object's parts. For other applications, the exact goal may differ as well. Functionally speaking, segmentation is the search for groups of pixels of a certain 'coherence'.

Historical note. Image segmentation is one of the oldest and most widely studied problems in computer vision. It was once thought to be an essential, early step in a systematic reconstruction of the semantic image content (see introduction again). But due to the difficulties of obtaining consistent segmentation results across different image types (scenes, objects, texture,...), which would correspond to human interpretation -, it has lost its significance for recognition. Nowadays much recognition is performed without any segmentation - see Deep Neural Networks. But segmentation algorithms are heavily used for example in medical image analysis (e.g. in X-rays) and in consumer applications, where users initiate segmentation by pointing out which regions are to be segmented.

If our task does not require precise segmentation, but merely a coarse localization of objects, then segmentation by thresholding can be sufficient. We have given an example in Section 2 already, in the upcoming Section 9.1 we elaborate on that. If our tasks requires reasonably precise outlines, then region growing as introduced in Section 9.2 could be more suitable. If we desire a precise segregation between a large foreground object and its background, then a statistical classifier is probably the best choice (Section 9.3).

9.1 Thresholding: Global, Local, Band and Multi-Level

SHB p177, s6.1
Dav p82, ch4, pdf119

Choosing an appropriate threshold is often done by looking at the intensity distribution of the image: if the objects and background are of distinct intensity or colors, then the distribution is bimodal and we chose the minimum between the two modes as the threshold, see for instance the histogram in the upper right of Fig. 17 (see also SHB p24, s.2.3.2). There exists a variety of methods on how to calculate the optimal threshold. One of the early methods was developed by Otsu, see graph center left in Fig. 17.

Global, Local, Band Thresholding If one applies a single threshold to the entire image, then that is also called *global* thresholding. It can however be more suitable sometimes, to apply a threshold that depends on its neighborhood, that is an image window around the pixel to be thresholded; that would be called a local threshold - because neighborhoods are local. We can also specify a range for which pixel values remain unmodified, but the values outside that range would be set to zero: that would be called a *band* threshold.

Multi-Level Thresholding For more complex scenes, thresholding is of limited use, but can be exploited for locating objects (or regions), that possess a relatively distinct gray-level. For instance, thresholding has been applied for road segmentation and for vehicle location in in-vehicle vision systems. The intensity distribution for such images is often multi-modal and methods to identify the correct threshold or range of values are sometimes called *multi-level thresholding*. One possible step toward that goal would be the smoothening of the distribution and extrema detection as we did for face part detection (exercise 2.3.2).

In Matlab one can look at the intensity histogram with the function `imhist`; the function `graythresh` finds an optimal threshold between two peaks; the function `im2bw` thresholds the image using the level as specified by `graythres`. The function `multithresh` is based on the method used in function `graythres`.

Caution `graythresh/im2bw`. The function `graythresh` always generates the level as a scalar between 0 (black) and 1 (white) irrespective of the image's class type ('uint8', 'single', etc.). Correspondingly, the function `im2bw` expects a level specified between 0 and 1 and the original image class type. If you have converted the original image to class 'single' already - as recommended in previous exercises - then this may produce wrong results. Here it is better to work with the original image class type.

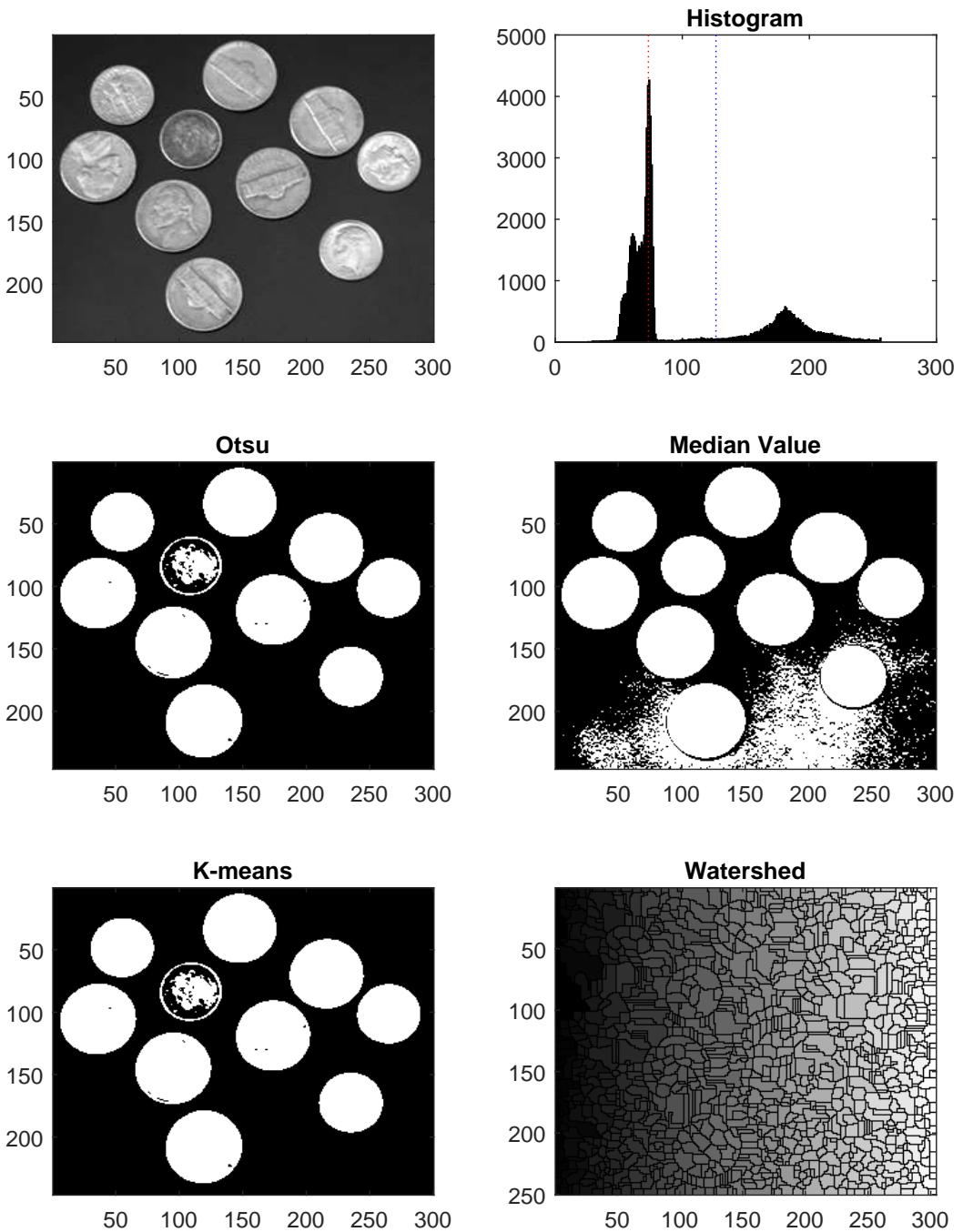


Figure 17: Segmentation methods in comparison.

Upper Left: image 'coins' from Matlab, `imread('coins.png')`.

Upper Right: intensity histogram distribution; the vertical stippled line is the threshold value of Otsu's method (in Matlab `graythresh`).

Center Left: binary map as thresholded with Otsu's method.

Center Right: output as produced with a median threshold (shown for comparison).

Lower Left: clustering result with K-Means algorithm (`kmeans`).

Lower Right: output of the watershed algorithm. The watershed algorithm typically over-segments. In this case, the outline of the coins can be recognized nevertheless (with some effort).

In Python those functions can be found in particular in sub-module `skimage.filters`, e.g. `threshold_otsu`. Python offers more methods to determine thresholds than Matlab does.

Application Thresholding is particularly appealing for tasks that require fast segmentation, as in video processing for example.

9.2 Region Growing: The Watershed Algorithm

SHB p233, s6.3.4

Sze p251, s5.2.1, pdf283

In region growing one starts with several, selected pixels and then keeps expanding from those pixels until some stopping condition is fulfilled. The most popular algorithm is the watershed algorithm and as its name implies, one floods the intensity landscape until the rising water level meets the watersheds. To imagine that, we observe the image as an intensity landscape, see Fig. 2 again. The algorithm determines the landscape's (local) minima and from those one grows outward until the flood front encounter another growing flood front. The points of encounter form lines that correspond to watersheds. The resulting regions can be regarded as catchment basis where rain would flow into the same lake.

Watershed segmentation is usually applied to a smoothed version of the gradient magnitude image ($\|\nabla I\|$, Section 3.3), thus finding smooth regions separated by visible (higher gradient) boundaries.

Watershed segmentation often leads to over-segmentation (see lower right in Figure 17), that is a segmentation into too many regions. Watershed segmentation is therefore often used as part of an interactive system, where the user first marks seed locations (with a click or a short stroke) that correspond to the centers of different desired components.

Matlab `watershed`

Python `skimage.segmentation.watershed`

Advantages no specification of the number of clusters necessary or any other parameter.

Disadvantages over-segmentation, slow

9.3 Clustering: Statistical Methods

Segmentation can also be carried out with statistical methods, in particular with *clustering* methods. Clustering is sometimes considered as part of the topic of machine learning, known there as *unsupervised classification* methods (because we have no actual labels/supervision). In principle, any clustering method can be used for image segmentation, but practically we prefer the ones that segment an image in reasonable time. One can roughly distinguish between the fast and the slow methods. Among the fast methods are the ubiquitous *K-Means* method (Section 9.3.1), a method already used several times in this script; another fast method is the *Mean-Shift* and its variants (Section 9.3.2). A slower method is the *Normalized-Cut* method, which is often used for foreground-background segregation (Section 9.3.3).

All those statistical methods treat the image as a matrix, namely number of image pixels times number of 'features'. In the simplest case, the features are the three chromatic channels RGB, thus a $n_{pixels} \times 3$ matrix, meaning we try to find clusters in three dimensions. We can also add the pixel coordinates as x and y values, thus generating a $n_{pixels} \times 5$ matrix; that would then be clustering in five dimensions. We can also add local texture measurements, as discussed in texture features (Section 4.3), in which case we move toward a high-dimensional space. In short, we select the features we think that are suitable to obtain our preferred segmentation results. The more features we take however, the slower becomes the clustering process as the dimensionality grows correspondingly.

9.3.1 K-Means

We used the K-Means procedure already for feature quantization (Section 7.1.1), namely on a 128-dimensional problem. Here we use it to segment much lower-dimensional spaces, for instance in Fig. 17 we used merely one dimension, namely gray-scale intensity. If we have a color image, then it would be obvious to start exploring the three RGB dimensions.

The advantage of using the K-Means algorithm is that it works relatively fast; its downside is that one needs to specify a number of clusters, meaning one needs to know how many different objects one expects.

Suppose we have a color image with of a flower with a homogeneous background (leaves of other plants). Then perhaps we could start the segmentation with $k=3$: one cluster for the flower leaves, one cluster for the background, and one for the rest of the flower. Doing this only on the color channels we would code:

```
szI      = size(Irgb);                      % [rows columns 3]
IrgbCol = reshape(Irgb,[szI(1)*szI(2) 3]); % turn image into data matrix for clustering [nPix 3]
L       = kmeans(IrgbCol,3);                 % labels with values E [1,2,3]
```

Matlab `kmeans`

Advantages relatively fast: faster than region growing, but slower than thresholding

Disadvantages specification of k : number of expected clusters needs to be specified beforehand

9.3.2 Mean-Shift, Quick-Shift

The *Mean-Shift* procedure is similar to the K-Means in its use, but here one specifies a bandwidth value h and not an expected number of clusters. That has the obvious advantage that we do not need to provide a particular k , yet if we do not specify an adequate bandwidth value, then the results can be unsatisfying. Thus the challenge here is to estimate a reasonable bandwidth, but in some cases that is better than providing a fixed k . The mean-shift has the down-side that it is slower than the K-Means algorithm and for that reasons a fast variant was developed named *Quick-Shift*.

Another down-side is that the algorithm works only for limited dimensionality, up to 6 or 7 dimensions in practice. The cause for that limitation is that the algorithm does not use distance measurements between 'pixels', but it computes gradients.

This type of segmentation is often used for tracking moving objects. An object in a scene often has a characteristic color histogram 'signature' that is distinct from its context and that is exploited with this type of algorithm.

9.3.3 Normalized Cut

The normalized-cut algorithm is a clustering algorithm that is used in particular for generating two clusters. It is often used for a segregation between foreground and background, for example in a graphics programs to extract an object from its background. To initiate the procedure it requires two samples, one from each cluster, which practically are often provided by the user, for instance manually by mouse-clicks, one pixel from the background and one from the foreground.

The advantage of this clustering procedure is that it can arrive at better results than the other clustering algorithms. Its downside is that it takes considerably much more time to arrive at the segmentation result and that it requires two samples as initial input. It is therefore rather used for individual images. The algorithm starts typically by relating the features at each pixel with all its neighboring pixels, which results in much more computation than the above k-Means and mean-shift procedures.

An example of how to apply normalized cuts is given in Section 9.4 of the SciKit-Learn documentation.

9.4 Notes

Color In many classification problems, color information is hardly of use as objects can appear in many different colors. In segmentation tasks however, color information is more likely to be of actual benefit. It may therefore be worth observing the images also in a different color space, other than the typical RGB space. For instance, segmentation of natural objects (leafs, fruits, landscapes) is sometimes done in the Hue-Saturation-Value (HSV) space, which can be obtained by converting with the command `rgb2hsv`. There exist many other color spaces, see Appendix F for an overview. But be advised that even when research articles state that a certain transformation helps improving segmentation, it is not necessarily what everyone agrees on. For instance for the task of skin detection, there exists many studies advertising their transformations. But other studies say that there is no real significance advantage by transforming the original RGB space. Thus, perhaps trying to deal with the original RGB space could be sufficient, at least in a first step.

Moving on Many segmentation algorithms return a binary (logical) map, also called *black-white* image as introduced in Section 2. That map is often manipulated with operations known as morphological processing, coming up in the next Section 10.

9.5 Exercises

Study the example given in Appendix I.6 - it produces Figure 17.

1. The use of the median threshold is motivated by the assumption that the majority of background values is clustered around a frequent value. This is indeed the case as we can observe in the histogram, but the problem is that the background intensity values are rather widely distributed and that inevitably will assign many background pixels to the object. A better threshold is the algorithm by Otsu, which takes the spread of fore- and background pixels into account.
2. If you run the script multiple times, then you observe that the K-Means algorithm changes its foreground/background assignment. Why?
3. On which aspect was the K-Means algorithm tested? You could try a 3-component vector per pixel (intensity and x/y coordinates). Create the x and y coordinates with `repmat`:

```
[m n] = size(I); % size of image
Xco = repmat(1:n,m,1);
Yco = repmat((1:m)',1,n);
FET = [I(:) Xco(:) Yco(:)];
Ix = kmeans(FET,2);
LbMx = reshape(Ix,[m n]);
```

Display the label matrix `LbMx` with `imagesc`. Explain the odd results. Increase the number of clusters (`k`) one by one. What happens now? Obviously, coordinates are of little use in this example. When then could they be of benefit?

4. Take some other images and compare the method.
5. Include color information (RGB values).
6. Take several images of the same type, e.g. one type of flower images; or street scenes of certain perspectives. Make a comparison between the instances of the same class.

10 Morphology and Regions (Image Processing III)

Sze p112, s3.3.2, p127

Dav p185, ch7

Morphological processing is the local manipulation of the structure in an image - its morphology - toward a desired goal. Often the manipulations aim at facilitating the measurement of features such as regions, contours, shapes, etc. Morphological processing can also carry out filtering processes.

We have already given two use cases for morphological processing. In one example, morphological processing 'polished' the map of edge pixels that one has obtained with some edge-detection algorithm (Section 4.2). And we have mentioned several times that after application of a segmentation algorithm one would like to continue modifying the black-white image toward a specific goal. In both cases one would apply so-called *binary* morphology, coming up in Section 10.1. But one can also apply such manipulations to gray-scale images, called *gray-scale* morphology, which we will quickly introduce in Section 10.2.

After we have completed (segmentation and) morphological processing, we often want to count the objects or regions in the black-white image. Or if we know their count already, then perhaps we intend to localize the regions and describe them by a few parameters. This is called region finding or labeling. This is a fairly straightforward process and will be explained in Section 10.3.

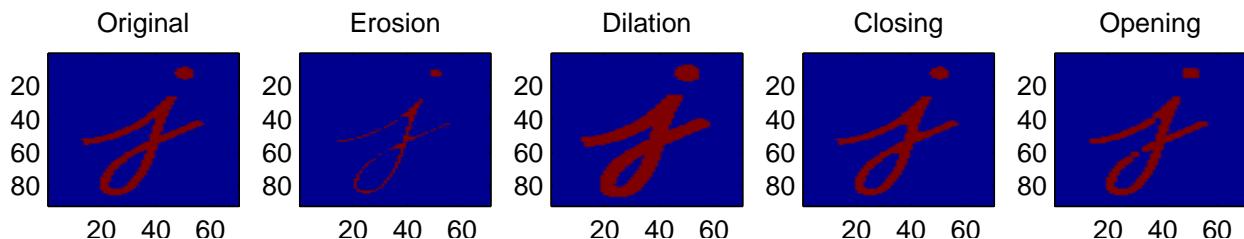


Figure 18: Binary morphological processing on a hand-written letter 'j'. From left to right:

Original: the object to be manipulated.

Erosion: a slimmed version of the original object: note that the resulting object is now ruptured.

Dilation: a thickened version of the original.

Closing: this process consists of two operations: first dilation of the original, followed by erosion. (the effects of any actual closing are not really viewable in this case)

Opening: first erosion of the original, followed by dilation. Note that some of the ruptures are now more evident.

10.1 Binary Morphology

Binary morphology manipulates only binary (black-white) images whereby the result is again a binary image. There exist two basic operations, the erosion and the dilation operation, which - as the names imply - make the *object(s)* shrink or grow in some specified way. Those two basic operations are then combined to form more complex operations. We introduce those operations with Figure 18:

Erosion: the object loses one or several 'layers' of pixels along its boundary.

Dilation: the object fattens by one or several layers of pixels along its boundary.

If one applies the above two operations in sequence, then that causes the following modifications:

Closing: is the dilation followed by an erosion, then that fuses narrow breaks and fills small holes and gaps.

Opening: is the erosion followed by a dilation, then that eliminates small objects and sharpens peaks in an object.

The last two sequential operations can be summarized as follows: they tend to leave large regions and smooth boundaries unaffected, while removing small objects or holes and smoothing boundaries.

Other combinations of those basic operations are possible, leading to relatively complex filtering operations such as the **tophat** and **bothat** operations, which however can also become increasingly time-

consuming. When developing an application, it is difficult to foresee which morphological operations are optimal for the task: one simply has to try out a lot of combinations of such operations and observe carefully the output.

More algorithmically speaking, the image is modified by use of a *structuring element*, which is moved through the image - very much like in the convolution process. The structuring element can be any shape in principle: it can be a simple 3×3 box filter as in the examples above; or it can be a more complicated structure, for instance some simple shape that one attempts to find in the image, leading so essentially to a filter process.

If one deals with large images, then one of the first steps toward object characterization or region finding might be to eliminate very small regions, which in software programs we can do with a single function. This is of benefit because morphological operations are carried out much faster after one has eliminated any unlikely object candidates.

Matlab Binary manipulations are carried out with commands starting with the letters `bw` standing for black-white. Here are examples of some essential manipulations, whereby most manipulations can be found in function `bwmorph`:

```
BW      = imclearborder(BW);      % deletes regions touching image border
BW      = bwareaopen(BW,4);      % deletes regions < 4 pixels (4-pix remain!)
areaObjs = barea(BW);          % summed areas of all objects

BW      = bwmorph(BW,'dilate',1);    % dilates by one
BW      = bwmorph(BW,'erode',2);    % erodes by two
BWper  = bwperim(BW);           % returns boundary pixels of objects
BWrem  = bwmorph(BW,'remove');    % removes interior of pixels
BWthn  = bwmorph(BW,'thin',inf);  % thinned
```

The operation ‘thin’ creates skeleton-like structures. It is particularly useful for contour tracing: one would apply first a thinning operation before trying to trace contours in a map.

More sophisticated manipulations can be achieved using a structural element defined with `strel`. There also exist functions such as `bwareafilt` and `bwpropfilt` to carry out complex filtering operations.

Python The functions are found in different sub-modules, in `scipy.ndimage.binary_xxx` as well as in `skimage.morphology`. In examples:

```
from skimage.morphology import remove_small_objects, remove_small_holes, \
                                binary_erosion, binary_dilation, thin, disk
BWlrg  = remove_small_objects(BW, 4)
Bwnoho = remove_small_holes(BW, 4)
Bwdil  = binary_dilation(BW)          # dilation by one (=disk(1)=5-pix cross)
Bwer   = binary_erosion(BW,disk(2))  # erosion by two
BWthin = thin(BW)                   # thins automatically to inf
```

10.2 Grayscale Morphology

The above introduced operations also exist in gray-scale morphology, but here the operation is not the change of a bit value, yet the selection of an extrema value in the neighborhood under investigation. In gray-scale morphology one talks of the *structural function*: in the simplest case, the function takes the maximum or minimum.

Matlab The operations are found under the initial letters `im`, for instance `imdilate`, `imerode`, `imopen` and `imclose`.

Python The operations can be found in `scipy.ndimage.grey_xxx` or some of them in `skimage.morphology`. For instance `skimage.morphology.dilation` carries out dilation for a gray-scale image and can also be applied to a black-white image, but for the latter the above functions starting with `binary_` are faster.

10.3 Region Finding, Description (Properties) and Boundary Detection

After segmentation and morphological processing, we need algorithms that search those regions that consist of connected pixels. In that context, one talks of *connected components*, which are defined as regions of adjacent pixels that have the same input value (or label). Connected components can be your objects of interest (value true or other labels), whereby the background may consist of one or more regions (value false); or the connected components may be some other 'targets'.

Label Matrix To find connected components, one runs an algorithm that labels the regions increasingly, 1, 2, 3, ... n_{regions} . The labeled regions remain in the map whereby the value 0 signifies background. Labeling can occur with two principal different types of *connectivity*: the 'conservative' type uses only a so-called *4-connectivity* or 4-connected neighborhood and uses only the 4 neighbors along the vertical and horizontal axes. The 'liberal' type uses the so-called *8-connectivity* or 8-connected neighborhood and uses all 8 neighbors; it typically results in fewer and larger regions than the 4-connectivity.

Region Properties After we have located the regions, we may want to describe them. Software packages typically provide a function that describes regions with a number of simple measures based on geometry or statistics, e.g. various measures of the region's extent in the image, statistical moments, etc.

Region Boundaries If we require the region boundaries, then we can obtain them by applying the edge-detection algorithms as introduced already in Section 4.2.1.

Matlab We can find and characterize connected components in different ways:

1. `bwlabel`: returns a map with connected pixels set to a certain integer value for a given object. The objects are numbered 1,... n_{objects} . We then write a loop to find the objects' indices using `find`.
2. `bwconncomp`: returns a structure with fields corresponding to the objects' indices, information that the function `bwlabel` does not provide. If one desires a labeled matrix, as it is created with `bwlabel`, then we apply the function `labelmatrix`.

The advantage of the use of `bwconncomp` is, that it requires less memory than `bwlabel`.

Matlab's function `regionprops` can be applied in a variety of ways. Here are explained three ways. We can apply the function `regionprops` directly to the black-white image `BW`, in which case we have skipped the function `bwconncomp` - it is carried out by `regionprops` in that case. Or we apply the function `bwconncomp` first and then feed its output to `regionprops`. Or we use the function `bwboundaries`, which is useful if we intend to describe the silhouette of shapes, for example with a radial description as introduced in the next section on the topic of shape.

```

I      = imread('cameraman.tif');
BW    = im2bw(I,128/255);

%% ===== RegionProps directly from BW =====
RPbw = regionprops('table',BW,I,'maxintensity','area');

%% ===== RegionProps via bwconncomp =====
CC   = bwconncomp(BW);
RPcon = regionprops('table',CC,I,'maxintensity','area');

%% ===== RegionProps via bwboundaries =====
[aBon MLbon] = bwboundaries(BW,8,'noholes');
RPbon = regionprops('table',MLbon,I,'maxintensity','area');

%% ----- Verification -----
assert(all(RPbw.Area==RPcon.Area)); % compare direct and bwconncomp
% via bwboundaries: only same for bwboundaries(BW,8,'noholes');
assert(all(RPcon.Area==RPbon.Area)); % compare bwconncomp and bwboundaries

```

The function `regionprops` provides only simple region descriptions. For more complex descriptions, one inevitably moves toward the topic shape description and that will be introduced in the upcoming section.

Python Python offers less flexibility than Matlab. The sub-module `skimage.measure` provides the functions `label` and `regionprops`. The `label` function requires type `int` as input, and the function `regionprops` takes only a label matrix as input:

```

from skimage.measure import label, regionprops
...
LB = label(BW)      # we call 'label' first, because...
RG = regionprops(LB) # ...input to 'regionprops' must be of type int! (not logical)
nShp = len(RG)
print('# Shapes', nShp)

```

To obtain the list values as a table - as a single array - , we write a loop as follows, a formulation that is called *list comprehension* in Python:

```
Ara = asarray([r.area for r in RG])      # obtaining all area values
```

10.4 Exercises

The output of the segmentation methods in Fig. 17 appear reasonable at first sight, but they are not sufficient, if we intend for instance to count the number coins: one coin was detected only partially and would therefore be counted multiple times. We thus perform some morphological operations to achieve this goal.

1. Use the following lines to arrive at the correct coin count starting with the logical map as obtained by the Otsu-thresholding method. The opening and closing calls will solidify somewhat the coin with lower intensity.

```

LbOts = bwlabel(BWots);
fprintf('# objects after segm: %d\n', max(LbOts(:)));
BWmod = bwmorph(BWots,'open');
BWmod = bwmorph(BWmod,'close');
Lbmod = bwlabel(BWmod);
fprintf('# objects after morph: %d\n', max(Lbmod(:)));
figure(3);clf;imagesc(BWmod);

```

2. Alternatively, use the following erosion operation on the median-threshold image.

```
BWmod = bwmorph(BWmed, 'erode', 7);
```

This is certainly the more elegant solution, as we have only one type of morphological operation to arrive at the correct coin count - instead of two as above.

Both examples are merely improvised solutions. Whether they would be useful also for other images with coins would have to be demonstrated of course. But it is clear that there is not necessarily one solution. That the output of the median filter can be used to obtain the correct coin count is not immediately clear from the segmentation output. The application of morphological processing requires experience.

Try both processing - segmentation and morphology - on Matlab's 'rice' image (`imread('rice.png')`).

11 Shape

Shape means the geometry of an object or its form; it is a 'structure' consisting of a few segments, typically without texture. Shape description techniques are used in the following applications for example:

- Medical imaging: to detect shape changes related to illness (tumor detection) or to aid surgical planning
- Archeology: to find similar objects or missing parts
- Architecture: to identify objects that spatially fit into a specific structure
- Computer-aided design, computer-aided manufacturing: to process and to compare designs of mechanical parts or design objects.
- Entertainment industry (movies, games): to construct and process geometric models or animations

In many applications, the task to be solved is the process of *retrieval*, namely the ordering (sorting) of shapes, and less so the process of classification, see Section 1.2 again to understand the difference. In a retrieval process, one shape is compared to all other shapes and that is computationally much more intensive than just classification. Thus, one major concern in retrieval is the speed of the entire matching process.

The number of shape matching techniques is almost innumerable. Each technique has its advantages and disadvantages and works often for a specific task and merely under certain conditions. Textbooks are typically shy of elaborating on this topic, because there is no dominating method and it is somewhat unsatisfactory and endless to present all techniques. Perhaps the two most important aspects in choosing a shape matching technique are its matching duration and its robustness to shape variability.

Shape Variability Depending on the task or the collection, shapes can vary in size or in spatial orientation; they can be at different positions in the image; they can appear mirrored; their parts may be aligned slightly differently amongst class instances; their context may differ. These different types of 'condition' are also called *variability*. Ideally, a shape matching technique would be invariant to all those variabilities. The table below shows the terminology used with respect to those desired shape matching properties:

Variability	Invariance
size	scaling
orientation	rotation
position	translation
laterality (mirroring)	reflection
alignment of parts	articulation
blur, cracks, noise	deformation
presence of clutter	occlusion

Practically, it is impossible to account for all these invariances and therefore one needs to observe what type of variability is present in the shape database and make a choice of the most suitable technique. This choice is also important for classification techniques.

The first three sections introduce shape descriptions of increasing complexity. Section 11.1 introduces simple shape descriptions suitable for rapid retrieval. Section 11.2 introduces techniques based on point comparisons: those techniques have a longer matching duration but show better retrieval accuracy. Section 11.3 introduces part-based descriptions: they can be even more accurate, but the matching techniques are rather complicated. Thus, the choice of a matching technique is also a matter of dealing with a speed-accuracy tradeoff. In the final section 11.4, we have a word on shape classification systems.

11.1 Compact Description

In compact descriptions, the shape is expressed by a number of parameters and those are used to form a feature vector. By using vectors one can conveniently apply traditional classification methodology. In Section 11.1.1, we mention simple measures based on the boundary or interior of the shape. In Section

11.1.2 we introduce the radial description, which is probably the most efficient description that uses feature vectors.

11.1.1 Simple Measures

It is not difficult to come up with a few simple boundary and region measurements. One can also apply the statistics of *moments* to the shape region (interior). Here are examples of measures and their suggested definitions:

Area	size of (shape) region (<code>bwarea</code>)
Circularity	e.g. area shape / area circle (circle with equal diameter)
Principle Axis Ratio	(also called eccentricity or aspect ratio) ratio between axis of elongation and its orthogonal axis (or length of major axis / length of minor axis)
Euler number	$= S - N$: # contiguous parts - # holes. Example shape '3': 1=1-0; shape 'B': -1=1-2; shape '9': 0=1-1. Matlab: (<code>bweuler</code>)
Bending Energy	degree of curvature ($\sum \kappa(s)$)

Matlab `regionprops`, introduced in Section 10.3 already
Python `regionprops` in submodule `skimage.measure`

Advantages compact, useful if very large number of shapes are to be matched, can serve as a triage
Disadvantages not very discriminative

11.1.2 Radial Description

If the shape is a continuous curve, that is a single, closed curve, then we can extract many more useful parameters by determining its radial signature, see Figure 19. The radial signature is the sequence of distances $R(s)$ from the shape's center point to each silhouette (curve) point s . To obtain the center point, one simply averages the curve points.

For a circle, the radial signature would be a constant value. For an ellipse, the signature would be undulating with two 'mounds'. For a triangle, there would be three sharp peaks. For a square, there would be four peaks. And so on. For a complex shape, such as the the pigeon shape in the figure, the signature is relatively complex.

There are two relatively straight-forward analyses we can do with the radial signature. One is a Fourier analysis, meaning we express the signature as a spectrum of frequencies. This is an enormously powerful analysis, which merits its own lectures, but is rather the topic of a signal processing course. The other analysis is an investigation of the extrema present in the signature. The Fourier analysis is more discriminative than the extrema analysis, but combining both can yield even better results.

Fourier Analysis The Fourier analysis transforms a signal into a spectrum, which in digital implementation is a sequence of so-called Fourier descriptors (FD). We apply this discrete Fourier transform to the (unmodified) radial signature ($R(s)$) and normalize it by its first value:

```
FDabs = abs(fft(Rad)); % fast Fourier
FDn = FDabs(2:end)/FDabs(1); % normalization by 1st FD
```

In the lower right of Fig. 19, the first 50 Fourier descriptors are shown. But typically, the first 5 to 10 Fourier descriptors are sufficient for discriminating the shapes.

Derivative Analysis Finding extrema in the signature is easier if we first low-pass filter the signature - very much as in the analysis of facial profiles, see introductory exercise. The number of extrema corresponds to the number of corners in a shape, whereby here corner means a curvature higher than its context. Two corners would correspond to an ellipse or bicorn shape, three corners to a triangle or trident shape, etc.

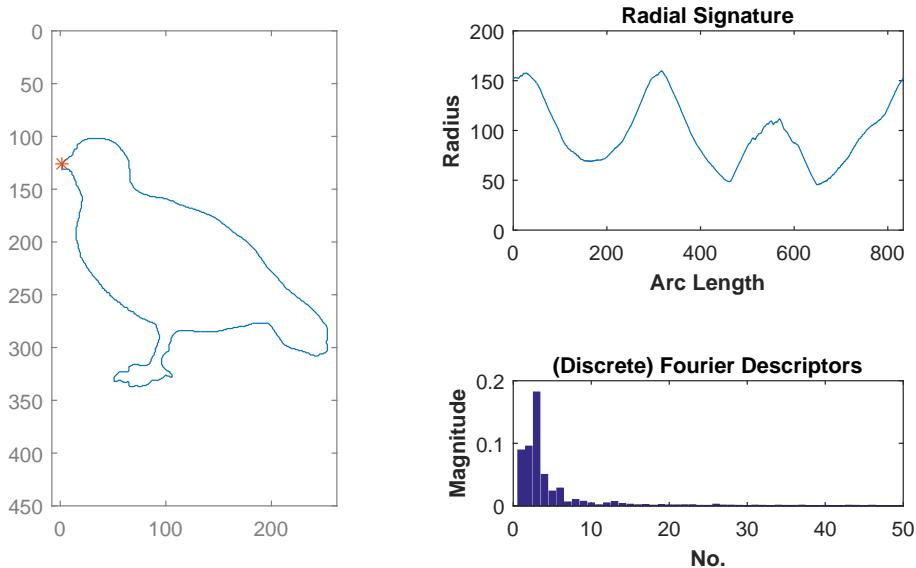


Figure 19: **Left:** a pigeon shape; the asterisk marks the beginning of the signature. **Upper Right:** radial signature: the distances between pole (shape center) and the individual shape points. **Lower Right:** discrete Fourier descriptors of the radial signature.

The presented descriptions can not discriminate large sets of shapes, but their use may serve as a triage for a more complex description and matching.

11.2 Point-Wise

There are two cases of point-wise formats one can distinguish. In one format, a shape is expressed as a single boundary, a sequence of points, which typically corresponds to its silhouette (Section 11.2.1). In the other format, a shape is considered as a set of points (Section 11.2.2). Because the shape comparison with such formats is relatively time-consuming, one prefers to know the approximate alignment between the two shapes before an accurate similarity is determined. This is also known as the *correspondence problem*, meaning which points in one shape correspond to which others in another shape, at least in an approximate sense.

11.2.1 Boundaries

In this case, the list of points of one shape, are compared to the point list of another shape, by somehow determining a distance (or similarity) measure between the two lists. The simplest way would be to take the pairwise distances between the two lists of points and to sum the corresponding minima to arrive at a measure of similarity. The pairwise point matching is computationally costly and the computational complexity is said to be square, expressed also as $O(N^2)$, where N is the number of pixels and O is the symbol for complexity. And to solve the correspondence problem one could simply shift the two shapes against each other to find a minimum for the correspondence. This would increase the complexity to $O(N^3)$, that is it is cubic now and thus rather impractical.

Of course, the complexity were greatly reduced if one used so-called *landmarks* or *key-points* only, namely points on the shape that are at locations of high curvature in a shape. The problem is that such key-points are difficult to determine consistently. The search for corners in the radial description (Section 11.1.2) may serve as key-points to find the approximate correspondence.

The most efficient boundary matching technique is based on observing the local orientations along the boundary and including them in the matching process. The detailed steps are as follows:

1. Sample an equal number of points $i = 1, \dots, N$ from each shape, equally spaced along the boundary.
2. Determine the local orientation ω at each point, for instance the angle of the segment spanning several pixels on both sides of the center pixel. Thus the shape is described by a list of N points with three values per point: x- and y-coordinate, as well as orientation ω .
3. Determine the farthest point using the radial description. This will serve as a correspondence.

When matching two shapes, one would take the point-wise distances (including orientation ω) using the farthest point as reference. The point-wise distances are also taken in reverse order to account for asymmetric shapes. Thus, the matching complexity is $O(2N)$ only.

11.2.2 Sets of Points

Here again we can distinguish between two cases: 1) shapes consist of multiple boundaries - and not only of one as assumed above; 2) shapes have limited intra-class variability and consist of few points.

1) Multiple Boundaries The most successful approach is called *Shape Context* (Belongie, Malik & Puzicha, 2002) and is based on taking local radial histograms at selected points of the shape. The selection of such key-points may not be completely consistent, but that would be compensated by a flexible matching procedure. At each key-point, a circular neighborhood of points is selected and a one-dimensional histogram is generated counting the number of on-pixels as a function of radial distance.

2) Limited Variability; Few Points This case is rather useful for localization and less for retrieval (or classification). We assume that we know the shape's key-points and that its articulation is limited (see again property list given in the introduction). The goal is then to find the target shape in another image. We are then faced with two tasks: the correspondence problem and the transformation problem.

11.3 Toward Parts: Distance Transform & Skeleton

Because we humans interpret a shape as an alignment of segments or parts, it was assumed from the early days of computer vision, that a shape description should also consist of segments or parts somehow. Decades of trial-and-error has taught computer vision scientists, that such a description is difficult to achieve. What exactly is supposed to be a segment or a part and how those should be represented, are still unsolved problems. But if one intended to work toward that direction, then the distance transform could be a piece of the puzzle, because it appears a 'natural' step toward extracting parts. After we introduced that distance transform (Section 11.3.1), we mention the difficulties with obtaining segments and parts (Section 11.3.2).

11.3.1 Distance Transform

The distance transform is typically determined for a binary image. The transform calculates at each background (off) pixel the distance to the nearest object (on) pixel. This results in a scalar field called *distance map* $D(i, j)$. The distance map looks like a landscape observed in 3D, which is illustrated in Figure 20. The distance values inside a rectangular shape form a roof-like shape, a *chamfer* (shapes used in woodworking and industrial design); the interior of a circle looks like a cone. The distance transform is also sometimes known as the *grassfire transform* or symmetric-axis transform, since it can also be thought of a propagation process, namely a fire front that marches forward until it is canceled out by an oncoming fire front. Wherever such fronts meet, that is where they form symmetric points, which correspond to the ridges in the distance map: it is a roof-like skeleton for the rectangle and a single symmetric point for a circle - the peak of the cone.

Matlab
Python

`bwdist`
`scipy.ndimage.distance_transform_edt`

Sze p113, s 3.3.3, pdf 129

Dav p240, s 9.5

SHB p19, s 2.3.1

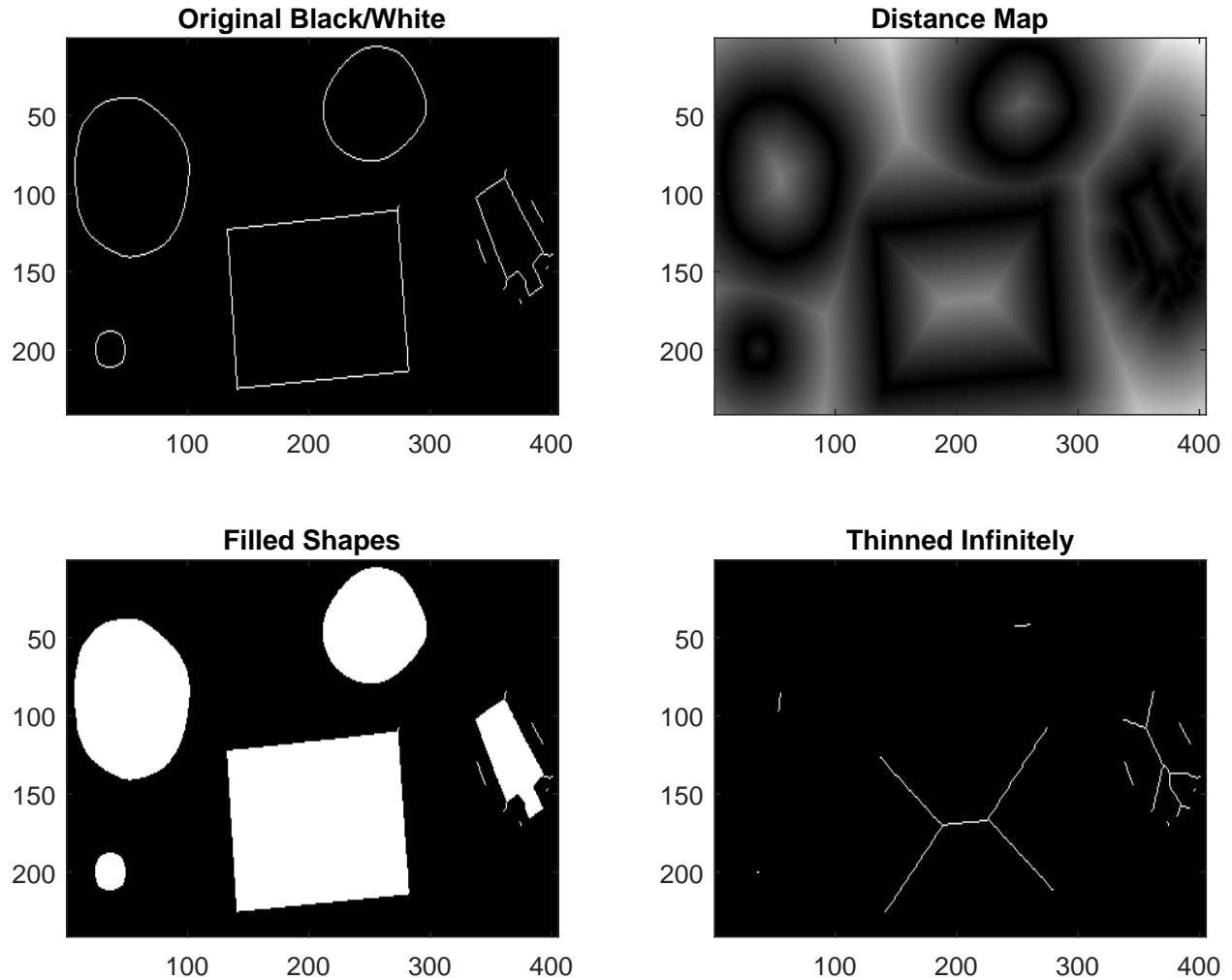


Figure 20: Distance map and skeleton.

Upper Left: input image.

Upper Right: its distance map obtained with `bwdist`.

Lower Left: Filled shapes of the original image.

Lower Right: quasi skeletons of the shapes, obtained using the 'thin' option of `bwmorph`.

Applications: fast chamfer matching (binary image alignment), feathering in image stitching and blending, nearest point alignment for range data merging, level sets

11.3.2 Symmetric Axes, Skeleton

The symmetric axes are the ridges in the distance map, the contours that look like veins (upper right in Figure 20). Extracting those is relatively difficult - at least no one has succeeded so far. Instead, they can be approximated by various other algorithms. One is a thinning algorithm, similar to the erosion operation for morphological processing (Section 10.1), which when carried out infinitely results in a skeleton resembling the sym-axes, see lower right in Figure 20 whereas the starting point are the filled shapes (lower left in figure). Such skeletons are then fragmented and a shape is expressed as a structural description, a description by parts in a certain alignment. No powerful method exists so far.

11.4 Classification

When it comes to classification, the most successful systems are not computer vision algorithms, but Deep Neural Networks as introduced in Section 5. There are two disadvantages with DNNs for shape recognition. One is, that they require the shape to be fairly well centered in the image; thus, a search algorithm is necessary that finds the exact shape center. The other disadvantage is, that they require fairly long to learn the features, as pointed out already in Section 5.

The power of those networks comes from their robustness to local changes: small changes in the boundary do have little consequences in networks. In contrast, for any of the shape description techniques introduced above, such small changes can result in relatively different features and thus tendentially more wrong classifications than with DNNs. Take for instance the two rectangular shapes in Figure 20: the corresponding skeletons in the lower right graph, show sufficient differences that make a robust comparison difficult.

11.5 Exercises

First we create our own set of stimuli, see Appendix I.7.1. In our subsequent testing scripts, we load that image, threshold it to obtain a logical map, then extract the shapes and finally compare them.

Appendix I.7.2 shows an example of how to extract and describe shapes, conveniently done with the Matlab function `regionprops`. In that example, we create a three-dimensional vector for each shape using the shape measurements 'area', 'eccentricity' and 'equivalent diameter'. Then we compare each shape with each other one using the Euclidean distance measure, resulting in the distance matrix `DM`. This matrix is then sorted and we observe the first few most similar shapes, row-wise.

1. The sorting does not look very impressive. Hint: there is no normalization. By what aspect is the sorting dominated?
2. Add normalization:

```
MxV      = max(Vec,[],1);  
Vec      = bsxfun(@ldivide,Vec,MxV);
```

3. To what modifications is the shape description robust? See the properties again given in the introduction.
4. The distance measure in the example is not normalized. Normalize and observe whether there is improvement.

Appendix I.7.3 shows how to do the same with a Fourier-transformed radial signature.

1. The shape was already normalized. Exclude normalization and observe the sorting. What invariance has been lost?

12 Image Search & Retrieval

FoPo p657, ch 21, pdf 627

An image retrieval system is a computer system for browsing, searching and retrieving images from a large database of digital images. Browsing, searching and retrieving are search processes of increasing specificity:

- browsing: the user looks through a set of images to see what is interesting.
- searching: the user describes what he wants, called a *query*, and then receives a set of images in response.
- retrieval: the user uploads an image and in return obtains the most similar images, ranked by some similarity measure.

Most traditional and common methods of image retrieval utilize some method of adding metadata such as captioning, keywords, or (textual) descriptions to the images. Retrieval is then carried out only with those annotated words, that is, computer vision techniques are not really involved. Because this manual image annotation is time-consuming, laborious and expensive, a large amount of research has been carried out on automatic image annotation. Additionally, the increase in social web applications and the semantic web have inspired the development of several web-based image annotation tools.

To search for images, a user may provide query terms such as keyword, image file/link, or click on some image or image features, and the system will return images "similar" to the query. The similarity used for search criteria could be meta tags, color distribution in images, region/shape attributes, descriptors as above (see scene classification in above subsection), etc.

Content-based image retrieval (CBIR) is the application of computer vision to the domain of image retrieval. CBIR aims at avoiding the use of textual descriptions and instead retrieves images based on similarities in their contents (textures, colors, shapes etc.).

12.0.1 Applications

Finding Near Duplicates

1) Trademark registration: A trademark needs to be unique, and a user who is trying to register a trademark can search for other similar trademarks that are already registered (see Figure 21 below).

2) Copyright protection.

				query	1: 0.086	2: 0.108	3: 0.109
				query	1: 0.046	2: 0.107	3: 0.114

Figure 21: A trademark identifies a brand; customers should find it unique and special. This means that, when one registers a trademark, it is a good idea to know what other similar trademarks exist. The appropriate notion of similarity is a near duplicate. Here we show results from Belongie et al. (2002), who used a shape-based similarity system to identify trademarks in a collection of 300 that were similar to a query (the system mentioned in Section 11.2.2). The figure shown below each response is a distance (i.e., smaller is more similar). *This figure was originally published as Figure 12 of Shape matching and object recognition using shape contexts, by S. Belongie, J. Malik, and J. Puzicha, IEEE Transactions on Pattern Analysis and Machine Intelligence, 2002, ©IEEE, 2002.*

Semantic Searches Other applications require more complex search criteria. For example, a *stock photo library* is a commercial library that survives by selling the rights to use particular images. An automatic method for conducting such searches will need quite a deep understanding of the query and of the images

in the collection. Internet image search shows one can build useful image searches without using deep object recognition methods (it is a safe bet that commercial service providers do not understand object recognition much better than the published literature). These systems seem to be useful, though it is hard to know how much or to whom.

Trends and Browsing In data mining, one uses simple statistical analyses on large datasets to spot trends. Such explorations can suggest genuinely useful or novel hypotheses that can be checked by domain experts. Good methods for exposing the contents of images to data mining methods would find many applications. For example, we might data mine satellite imagery of the earth to answer questions like: how far does urban sprawl extend?; what acreage is under crops?; how large will the maize crop be?; how much rain forest is left?; and so on. Similarly, we might data mine medical imagery to try and find visual cues to long-term treatment outcomes.

12.1 Retrieval Methods

Sze p604, s 14.3.2, pdf687
FoPo p662, s 21.2, pdf632

We now review techniques from document retrieval, that found their ways into image retrieval. A typical text information retrieval systems expects a set of query words. With those query words an initial set of putative matches is selected from an index (Section 12.1.1). From this list they chose documents with a large enough similarity measure between document and query (Section 12.1.2). These are ranked by a measure of significance, and returned (Section 12.1.3). For the purpose of image retrieval we can think of the words as being the 'visual words' as in developed in Section 7, and of documents as being the images. Some more comparison is given in the final section 12.1.4.

12.1.1 Indexing Documents

Much of text information retrieval is shaped by the fact that a few words are common, but most words are rare. The most common words - typically including 'the', 'and', 'but', 'it' - are sometimes called *stop words* and are ignored because they occur frequently in most documents. Other words tend to be rare, which means that their frequencies can be quite distinctive. Example: documents containing the words 'stereo', 'fundamental', 'trifocal' and 'match' are likely to be about 3D reconstruction.

Assume now that the total number of non-stop words is N_w ; and we work with N_d documents. For each document j we determine the frequency f with which a word t occurs. This will generate a $N_w \times N_d$ table $\mathcal{D}_{t,j}$ in which an entry represents the frequency f for a word t in a document j :

$$\{f_{t,j}\} = D(t, j) \quad (13)$$

The table is sparse as most words occur in few documents only. We could regard the table as an array of lists. There is one list for each word, and the list entries are the documents that contain that word. This object is referred to as an inverted index, and can be used to find all documents that contain a logical combination of some set of words. For example, to find all documents that contain any one of a set of words, we would take each word in the query, look up all documents containing that word in the inverted index, and take the union of the resulting sets of documents. Similarly, we could find documents containing all of the words by taking an intersection, and so on. This represents a coarse search only, as the measure f is used as a binary value only (and not as an actual frequency). A more refined measure would be the word count, or even better, a frequency-weighted word count. A popular method is the following:

tf-idf stands for 'term frequency-inverse document frequency' and consists of two mathematical terms, one for 'term frequency', the other for 'inverse document frequency'. With N_t as the number of documents that contain a particular term, the idf is

$$\text{idf} = \frac{N_d}{N_t} \cdot \frac{\text{total}}{\text{containing term}} \quad (14)$$

Practical tip: add a value of one to the denominator to avoid division by zero. With $n_t(j)$ for the number of times the term appears in document j and

$n_w(j)$ for the total number of words that appear in that document
the tf-idf measure for term t in document j is

$$f_{t,j} = \left(\frac{n_t(j)}{n_w(j)} \right) / \log\left(\frac{N_d}{N_t}\right). \quad (15)$$

We divide by the log of the inverse document frequency because we do not want very uncommon words to have excessive weight. The measure aims at giving most weight to terms that appear often in a particular document, but seldom in all other documents.

12.1.2 Comparing Documents

Assume we have a fixed set of terms that we will work with. We represent each document by a vector \mathbf{f} , with one entry for each term. Put differently, we select a subset of dimension t of the table. Two document vectors, e.g. $\mathbf{f}_1, \mathbf{f}_2$, can then be compared using the dot product and normalization by their respective lengths:

$$\text{sim} = \frac{\mathbf{f}_1 \cdot \mathbf{f}_2}{\|\mathbf{f}_1\| \|\mathbf{f}_2\|}. \quad (16)$$

The cosine similarity weights uncommon words that are shared more highly than common words that are shared. Expressed differently, two documents that both use an uncommon word are most likely more similar than two documents that both use a common word.

12.1.3 Ranking Documents

FoPo p535, s16.2.2, pdf 508

In response to a user query, a set of N_b documents is returned, where N_b needs to be specified, i.e. 100 items are returned to the user. To determine how fitting the selected documents are, one calculates two measures:

Recall: the percentage of relevant items that are actually recovered

$$R = \frac{n_r}{N_r}, \quad \frac{\text{retrieved}}{\text{total}} \quad (17)$$

where n_r is the number of recovered relevant items and N_r is the total number of relevant items for this query. Example: if the database has a total of 200 relevant items for the user query and the selection returned 60 relevant items, then the recall value is 60/200.

The larger we make our selection N_b , the larger will be R and if we return all documents then R will be one.

Precision: the percentage of recovered items that are actually relevant

$$P = \frac{n_r}{N_b}, \quad \frac{\text{retrieved}}{\text{query set}} \quad (18)$$

Example (continued): the precision value would be 60/100. The larger we make the selection N_b , the lower will be the precision value typically: if N_b is set to be 1000 and if this happen to return us all 200 relevant items, the precision is 200/1000.

F measure: To summarize the recall and precision values, different formulas can be used. A popular one is the F_1 -measure, which is a weighted harmonic mean of precision and recall:

$$F_1 = 2 \frac{PR}{P+R}. \quad (19)$$

Precision-Recall Curve To obtain a more comprehensive description of the system, one calculates the precision values for increasing recall by systematically increasing N_b . One plots recall on the x-axis against

precision on the y-axis and this curve is called the precision-recall curve .

Average Precision An important way to summarize a precision-recall curve is the average precision, which is computed for a ranking of the entire collection. This statistic averages the precision at which each new relevant document appears as we move down the list. $P(r)$ is the precision of the first r documents in the ranked list, whereby r corresponds to N_b in equation 18; N_r the total number of documents in the collection. Then, the average precision is given by

$$A = \frac{1}{N_r} \sum_{r=1}^{N_r} P(r). \quad (20)$$

which corresponds to the area under the curve.

Example In response to a query the (total of) 3 relevant items are found at positions 2, 13 and 36:

$$A = \frac{1}{3} \left(\frac{1}{2} + \frac{2}{13} + \frac{3}{36} \right) = 0.2457$$

Implementation Given an index vector with *sorted* retrieval positions, `Ix`, the measure is computed as follows:

```
Ixs    = sort(Ix);          % sort in increasing order
nItm  = length(Ixs);       % number of relevant items (N_r)
A      = sum( (1:nItm) ./ Ixs ) ./ nItm;   % average precision
```

12.1.4 Application to Image Retrieval

In order to apply the above measures to CBIR, we simply use 'visual words' - as in developed in Section 7 - instead of text words, and then use exactly the above equations. Or one can use both textual and visual information to create measures that weigh both types of information.

In the domain of texture recognition (Section 4.3.2), those words are also called textons, but in object recognition and image classification they tend to be called visual words. Because the histogram distributions for those words are generally different from those for gradients (as in SIFT), alternate distance measures such as the χ^2 -squared kernel may perform better.

It is tempting to believe that good systems should have high recall and high precision values, but obtaining high values can also be costly. The following examples illustrate that trade-offs are sometimes made:

- Patent searches: Patents can be invalidated by finding prior art (material that predates the patent and contains similar ideas). A lot of money can depend on the result of a prior art search. This means that it is usually much cheaper to pay someone to wade through irrelevant material than it is to miss relevant material, so very high recall is essential, even at the cost of low precision.
- Web and email filtering: Some companies worry that internal email containing sexually explicit pictures might create legal or public relations problems. One could have a program that searched email traffic for problem pictures and warned a manager if it found anything. Low recall is fine in an application like this; even if the program has only 10% recall, it will still be difficult to get more than a small number of pictures past it. High precision is very important, because people tend to ignore systems that generate large numbers of false alarms.

12.2 Exercise

For simplicity we work with an artificial retrieval distribution. It is an exponential distribution where relevant items are marked as one.

```

n      = 50;
Ix    = round(exprnd(1,[n 1])*100)+1;      % indices
B     = zeros(n,1);
B(Ix) = 1;                                % retrieval list

```

Write a loop that generates the precision-recall curve and calculate the average precision value. Note that in the artificial distribution the total number of relevant items N_r is not necessarily equal `n`, because some of the indices may be the same due to rounding. Thus, you need to calculate N_r first, i.e. with `nnz(B)`.

Place it into a function `f_PrecRecCrv`. Try out a random distribution, meaning a retrieval where relevant items are equally distributed across `B`.

13 Tracking

FoPo p356, ch 11, pdf 326

Tracking is the pursuit of one or multiple moving objects and possibly the interpretation of their dynamics. It has many applications:

Motion Capture: is the recording of the 3D configuration of a moving person using markers, e.g. white patches placed on the joints of a person dressed in a black suit. Such recordings are used for animation, e.g. rendering a cartoon character, thousands of virtual extras in a crowd scene, or a virtual stunt avatar.

Recognition from Motion: is the object identification by recognizing its (motion) dynamics. In some sense it is motion capture with discrimination.

Surveillance: is the monitoring of activities and the warning when a problem case is detected. Example airport traffic surveillance: different kinds of trucks should move in different, fixed patterns - if they do not, it is suspicious; similarly, there are combinations of places and patterns of motions that should never occur (e.g., no truck should ever stop on an active runway).

Targeting: A significant fraction of the tracking literature is oriented toward (a) decision what to shoot, and (b) hitting it. Typically, this literature describes tracking using radar or infrared signals (rather than vision), but the basic issues are the same: What do we infer about an object's future position from a sequence of measurements? Where should we aim?

There exist two simple but effective methods for tracking:

1. Tracking by detection: we have a strong model of the object, strong enough to identify it in each frame. We localize it, link up the instances, and that would be our track.
2. Tracking by matching: we have a model of how the object moves. We have a domain in the n th frame in which the object sits, and then use this model to search for a domain in the $(n + 1)$ 'th frame that matches it.

13.1 Tracking by Detection

FoPo p357, s 11.1.1

• **Case '1 object in each frame':** If there is only one object in each frame, we try to build a reliable detector (for that object) and observe its position in each frame (Figure 22a). If the detector is not reliable we treat the problem as if there were multiple objects, see next.

Example: Tracking a red ball on a green background: the detector merely needs to look for red pixels. In other cases, we might need to use a more sophisticated detector, e.g. tracking a frontal face.

• **Case 'Multiple objects (or unreliable detector)':** If objects enter or leave the frame (or the detector occasionally fails), then it is not enough to just report the object's position. We then must account for the fact that some frames have too many (or too few) objects in them. Hence, we maintain a *track*, which represents a timeline for a single object (Figure 22b).

Typically, the tracks from the previous frame are copied to the next frame, and then object detector responses are allocated to the tracks. How we allocate depends on the application (we give some examples below). Each track will get at most one detector response, and each detector response will get at most one track. However, some tracks may not receive a detector response, and some detector responses may not be allocated a track. Finally, we deal with tracks that have no response and with responses that have no track. For every detector response that is not allocated to a track, we create a new track (because a new object might have appeared). For every track that has not received a response for several frames, we prune that track (because the object might have disappeared). Finally, we may postprocess the set of tracks to insert links where justified by the application. Algorithm 6 breaks out this approach.

The main issue in allocation is the cost model, which will vary from application to application. We need a charge for allocating detects to tracks. For slow-moving objects, this charge could be the image distance between the detect in the current frame and the detect allocated to the track in the previous frame. For objects with slowly changing appearance, the cost could be an appearance distance (e.g., a χ^2 -squared distance between color histograms). How we use the distance again depends on the application. In cases where the detector is very reliable and the objects are few, well-spaced, and slow-moving, then a greedy

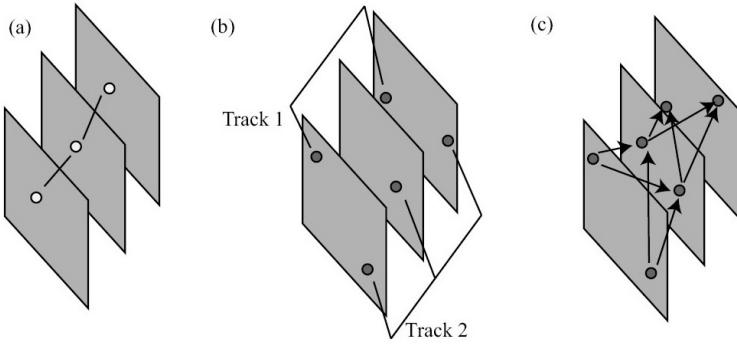


Figure 22: **a:** case one object only (with a distinctive appearance in each frame): the detector responses are linked to form a simple space-time path. **b:** if some instances drop out, we will need to link detector responses to abstract tracks: track 1 has measurements for frames n and $n + 2$, but not for frame $n + 1$. **c:** if there is more than one instance per frame, a cost function together with weighted bipartite matching could be enough to build the track. [Source: Forsyth/Ponce 2010; Fig 11.1]

Algorithm 6 Tracking multiple objects (or tracking with unreliable object detector). i =time; t =track.

Notation:

Write $x_k(i)$ for the k 'th response of the detector in the i th frame

Write $t(k, i)$ for the k 'th track in the i th frame

Write $*t(k, i)$ for the detector response attached to the k 'th track in the i th frame

(Think C pointer notation)

Assumptions: Detector is reasonably reliable; we know some distance d such that $d(*t(k, i - 1), *t(k, i))$ is always small.

First frame: Create a track for each detector response.

N'th frame:

Link tracks and detector responses by solving a bipartite matching problem.

Spawn a new track for each detector response not allocated to a track.

Reap any track that has not received a detector response for some number of frames.

Cleanup: We now have trajectories in space time. Link anywhere this is justified (perhaps by a more sophisticated dynamical or appearance model, derived from the candidates for linking).

algorithm (allocate the closest detect to each track) is sufficient. This algorithm might attach one detector response to two tracks; whether this is a problem or not depends on the application.

The more general algorithm solves a *bipartite matching problem*, meaning tracks on one side of the graph are assigned to the detector responses on the other side of the graph. The edges are weighted by matching costs, and we must solve a maximum weighted bipartite matching problem (Figure 22c), which could be solved exactly with the Hungarian algorithm, but the approximation of a greedy algorithm is often sufficient. In some cases, we know where objects can appear and disappear, so that tracks can be created only for detects that occur in some region, and tracks can be reaped only if the last detect occurs in a disappear region.

Background subtraction FoPo p291, s 9.2.1 is often a simple-but-sufficient detector in applications where the background is known and all trackable objects look different from the background. In such cases, the background-subtracted objects appear as blobs and those are taken as detector responses. It is the simplest form of foreground/background segmentation.

Example: People tracking on a fixed background, such as a corridor or a parking lot. If the application does not require a detailed report of the body configuration, and if we expect people to be reasonably

large in view, we can reason that large blobs produced by background subtraction are individual people. Weaknesses: if people stand still for a long time, they might disappear; it would require more work to split up the large blob of foreground pixels that occurs when two people are close together; and so on - many applications require only approximate reports of the traffic density, or alarms when a person appears in a particular view.

In many tracking tasks, nothing more complex is required. The trick of creating tracks promiscuously and then pruning any track that has not received a measurement for some time is quite general and extremely effective.

13.2 Tracking Translations by Matching

FoPo p360, s11.1.2 or s11.2
Sze p337, s8.1, pdf384

Example: Tracking soccer players on a television screen, with players of height 10-30 pixels. Detailed body-part dynamics can not be tracked due to low resolution and high frame rate (30Hz). Instead, we assume that the domain translates and we thus track the player as a box. We can model a player's motion with two components. The first is the absolute motion of a box fixed around the player and the second is the player's movement relative to that box. To do so, we need to track the box, a process known as *image stabilization*. As another example of how useful image stabilization is, one might stabilize a box around an aerial view of a moving vehicle; now the box contains all visual information about the vehicle's identity.

In each example, the box translates. If we have a rectangle in frame n , we can search for the rectangle of the same size in frame $n + 1$ that is most like the original, e.g. using the sum-of-squared differences (or SSD) of pixel values as a test for similarity and search for its minimum over a small neighborhood.

In many applications the distance the rectangle can move in an inter-frame interval is bounded because there are velocity constraints. If this distance is small enough, we could simply evaluate the sum of squared differences to every rectangle of the appropriate shape within that bound, or we might consider a search across the scale space (or even better the pyramid) for the matching rectangle.

Matching Principle The simplest way to establish an alignment between two images or image patches is to shift one image relative to the other. Given a *template* image $I_0(\mathbf{x})$ sampled at a set of discrete pixel locations $\{\mathbf{x}_i = (x_i; y_i)\}$, we wish to find where it is located in image $I_1(\mathbf{x})$. A least squares solution to this problem is to find the minimum of the sum of squared differences (SSD) function

$$E_{SSD}(\mathbf{u}) = \sum_i [I_1(\mathbf{x}_i + \mathbf{u}) - I_0(\mathbf{x}_i)]^2 = \sum_i e_i^2, \quad (21)$$

where $\mathbf{u} = (u; v)$ is the displacement and $e_i = I_1(\mathbf{x}_i + \mathbf{u}) - I_0(\mathbf{x}_i)$ is called the residual error (or the displaced frame difference in the video coding literature). (We ignore for the moment the possibility that parts of I_0 may lie outside the boundaries of I_1 or be otherwise not visible.) The assumption that corresponding pixel values remain the same in the two images is often called the *brightness constancy constraint*.

Tracking Initiation We can start tracks using an interest point operator. In frame 1, we find all interest points. We then find the location of each of these in the next frame, and check whether the patch matches the original one: if so, it belongs to a track; if not, the track has already ended. We now look for interest points that do not belong to tracks and create new tracks there. Again, we advance tracks to the next frame, check each against their original patch, reap tracks whose patch does not match well enough, and create tracks at new interest points.

13.3 Exercise

Understand the problems of motion detection by trying to detect, localize and track a motion in a short video. To start with, use the example in Appendix I.8.

1. Try other videos as well.
2. Can you find a procedure that automatically determines a suitable threshold for eliminating 'noise'?

3. Try with background subtraction, that is take the first frame as background and subtract it from the following frames.
4. Attempt to track an object, that is, select a suitable object patch that can be followed robustly.
5. Can you improve object tracking by using color, namely exploiting the 3 components red/green/blue?

14 Alignment / Registration

FoPo p397, ch 12, pdf 446

Sze p273, ch 6, pdf 311

Alignment is the challenge of determining how exactly an object or scene has changed its configuration from one image to another. Such information is employed for instance in medical support or cartography. Stated more explicitly, we are given an object and we try to detect it somewhere else and we would like to know how exactly it moved, that is how exactly it changed its orientation, shape and perspective - or mathematically speaking how it *transformed*.

In the simplest case, the object has moved straight from one location to another and otherwise it did not change - that would be a mere *translation*, see Figure 23. But the object may have also been rotated slightly during the *motion*, in which case this would correspond to a so-called *Euclidean* transformation. The object may have also shrunk or enlarged during motion, in which case the transformation is expressed with *similarity*. The task to reconstruct the transformation that the object has undergone during a motion is called alignment or registration. The object does not need to have changed its position but it is convenient to imagine the reconstruction as a 'motion estimation'. Put differently, alignment tries to find a transformation that takes one configuration of points to another configuration.

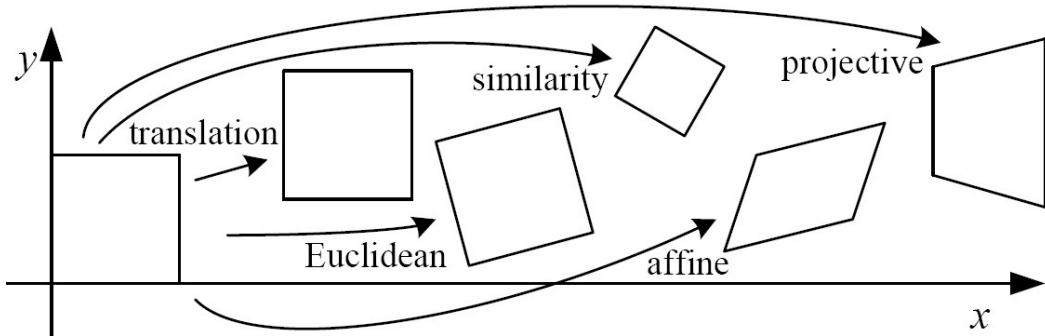


Figure 23: Basic set of 2D planar transformations. [Source: Szeliski 2011; Fig 3.45]

In the most straightforward form, the two datasets have the same dimensionality, for instance we are registering 2D data to 2D data or 3D data to 3D data, and the transformation is rotation, translation, and perhaps scale. Here are two examples:

Medical Support: We have an MRI image (which is a 3D dataset) of a patient's interior that we wish to superimpose on a view of the real patient to help guide a surgeon. In this case, we need to know the rotation, translation, and scale that will put one image on top of the other.

Cartography: We have a 2D image template of a building that we want to find in an overhead aerial image. Again, we need to know the rotation, translation, and scale that will put one on top of the other; we might also use a match quality score to tell whether we have found the right building.

We treat only 2D feature-based alignment for the beginning. First we explain how some of the 2D motions of Figure 23 are expressed mathematically (Section 14.1). Then we learn how to estimate them (Section 14.2). Finally, we learn how to robustly estimate them, that is even in the presence of noise and clutter (Section 14.4). The culminating exercise will be to estimate the motion of the two slightly rotated photographs taken in a previous exercise.

14.1 2D Transforms

Sze p33, s 2.1.2, pdf 36

Figure 23 showed the global parametric transformations for rigid 2D shapes. Here we introduce the formulas that express those transformations with matrix multiplications whereby two frequent notations are given in the table below: notation 1 is more explicit with respect to the individual motions; notation 2 concatenates the individual motions into a single matrix such that the entire motion can be expressed as a single matrix multiplication, which can be convenient sometimes.

Translation: simplest type of transform - merely a vector t is added to the points in x . To express this as a single matrix the identity matrix is used (unit matrix; square matrix with ones on the main diagonal and zeros elsewhere) and the translation vector is appended resulting in a 2×3 matrix, see also Figure 24.

2D Euclidean Transform: consist of a rotation and translation. The rotation is achieved with the a so-called orthonormal rotation matrix R whose values are calculated by specifying the rotation angle θ . In notation 2, the single matrix is concatenated as before and it remains a 2×3 matrix.

Scaled Rotation: merely a scale factor is included to the previous transform. The size of the single matrix in notation 2 does not change.

Affine: Here a certain degree of distortion is allowed, but we do not elaborate on this transformation any further here. The single matrix for notation 2 still remains of size 2×3 .

Transform	Notation 1	Notation 2	Comments
Translation	$x' = x + t$	$x' = [I \ t] x$	I is the 2×2 identity matrix
Rotation + Translation = 2D rigid body motion = 2D Euclidean transformation	$x' = Rx + t$	$x' = [R \ t] x$	$R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$ = orthonormal rotation matrix: $RR^T = I$ and $ R = 1$ Euclidean distances are preserved.
Scaled Rotation = Similarity Transform	$x' = sRx + t$	$x' = [sR \ t] x$ $x' = \begin{bmatrix} a & -b & t_x \\ b & a & t_y \end{bmatrix} x$	no longer requires $a^2 + b^2 = 1$
Affine	$x' = Ax$	$x' = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \end{bmatrix} x$	Parallel lines remain parallel

14.2 Motion Estimation with Linear-Least Squares

To estimate the motion between two objects we require two types of information. One is the correspondence between the two sets of points, which is not so trivial to establish in real-word scenes (see feature detection before), but for the moment we assume that the correspondence is known. The other type of information is the kind of transformation we expect. To address this we could simply work with complex transformations in order to be prepared for any type of transformation (i.e. affine transformation), which however do not return us the individual motion parameters explicitly; in scaled rotation in contrast we have explicit parameters for rotation, scale and translation.

Formulated mathematically, given a set of matched feature points $\{x_i, x'_i\}$ and a chosen planar transformation f with parameters p (e.g. t, R, \dots),

$$x' = f(x; p), \quad (22)$$

how can we reconstruct the best estimate of the motion parameters values? The usual way to do this is to use least squares, i.e., to minimize the sum of squared residuals

$$E_{LS} = \sum_i \|r_i\|^2 = \sum_i \|f(x_i; p) - x'_i\|^2, \quad (23)$$

where

$$r_i = x'_i - f(x_i; p) = \hat{x}'_i - \tilde{x}'_i \quad (24)$$

is the residual between the measured location \hat{x}'_i and its corresponding current predicted location $\tilde{x}'_i = f(x_i; p)$. See also Appendix for more explanation on linear least squares.

For simplicity we assume now a linear relationship between the amount of motion $\Delta x = x' - x$ and the unknown parameters p :

$$\Delta x = x' - x = J(x)p. \quad (25)$$

where $\mathbf{J} = \partial\mathbf{f}/\partial\mathbf{p}$ is the Jacobian of the transformation \mathbf{f} with respect to the motion parameters \mathbf{p} . \mathbf{J} is shown in figure 24 and has a particular form for each transformation. In this case, a simple *linear* regression (linear-least-squares problem) can be formulated as

$$E_{LLS} = \sum_i \|\mathbf{J}(\mathbf{x}_i)\mathbf{p} - \Delta\mathbf{x}_i\|^2, \quad (26)$$

which - after some reformulation - equates to

$$\mathbf{p}^T \mathbf{A}\mathbf{p} - 2\mathbf{p}^T \mathbf{b} + c. \quad (27)$$

The minimum can be found by solving the symmetric positive definite (SPD) system of normal equations:

$$\mathbf{A}\mathbf{p} = \mathbf{b}, \quad (28)$$

where

$$\mathbf{A} = \sum_i \mathbf{J}^T(\mathbf{x}_i) \mathbf{J}(\mathbf{x}_i) \quad (29)$$

is called the Hessian and

$$\mathbf{b} = \sum_i \mathbf{J}^T(\mathbf{x}_i) \Delta\mathbf{x}_i. \quad (30)$$

Transform	Matrix	Parameters p	Jacobian J
translation	$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix}$	(t_x, t_y)	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Euclidean	$\begin{bmatrix} c_\theta & -s_\theta & t_x \\ s_\theta & c_\theta & t_y \end{bmatrix}$	(t_x, t_y, θ)	$\begin{bmatrix} 1 & 0 & -s_\theta x - c_\theta y \\ 0 & 1 & c_\theta x - s_\theta y \end{bmatrix}$
similarity	$\begin{bmatrix} 1+a & -b & t_x \\ b & 1+a & t_y \end{bmatrix}$	(t_x, t_y, a, b)	$\begin{bmatrix} 1 & 0 & x & -y \\ 0 & 1 & y & x \end{bmatrix}$
affine	$\begin{bmatrix} 1+a_{00} & a_{01} & t_x \\ a_{10} & 1+a_{11} & t_y \end{bmatrix}$	$(t_x, t_y, a_{00}, a_{01}, a_{10}, a_{11})$	$\begin{bmatrix} 1 & 0 & x & y & 0 & 0 \\ 0 & 1 & 0 & 0 & x & y \end{bmatrix}$

Figure 24: Jacobians of the 2D coordinate transformations $\mathbf{x}' = \mathbf{f}(\mathbf{x}; \mathbf{p})$ (see table before), where we have re-parameterized the motions so that they are identity for $\mathbf{p} = 0$. [Source: Szeliski 2011; Tab 2.1]

14.3 Exercises

14.3.1 Preparing Transformations

Place the code in Appendix I.9 into a script called `t_2DTransforms`. In this script, a simple shape was specified with coordinates given as `Co`. It was then transformed in various ways, followed by estimating the corresponding motions - we expect a small error of course. The estimation procedure is done with `lsqlin` or `lsqnonneg`, whereby `A` and `b` are passed as arguments.

Now we are going to write separate functions for the individual transformations and motion estimations and we verify them at the end of the testing script `t_2DTransforms` by simply plotting their output into another figure for visual comparison.

1. Write a function `f_TransRot`, which carries out a transformation for a given list of coordinates, meaning the function input is `Co` and the corresponding parameters for rotation; the function output is the set of transformed coordinates. Then write a function `f_TransEuc`, which carries out a Euclidean transformation, etc. Verify by plotting the functions' output into a separate figure.
(Or more elegantly: write a single function `f_TransLin`, which performs a (linear) transformation according to the user's specification: the input is a list of coordinates and a string specifying the type of transformation. In the function we would use `if-then` or `switch` statements to carry out the desired transformation. Use `varargin` for variable input arguments.)
2. Write functions for motion estimation: one called `f_EstMotSim` for similarity, whose input are two lists of coordinates and whose output are the estimated parameters with estimation errors. Always verify the functions' output in your testing script. Then write another function for motion estimation with affinity `f_EstMotAff`, etc.
3. Take a more complex shape now, e.g. with 10 coordinates.

14.3.2 Motion Estimation

We now do motion estimation with 'noisy' input, once with artificial noise and once using a real application:

1. Add some random offset to the transformed coordinates, e.g. `CoTN = CoT + rand(nP, 2)*0.1`, where `CoT` are the transformed coordinates, `nP` are the number of points (coordinates) and `0.1` scales the amount of noise. Use `Co` and `CoTN` as input - and not `CoT`. Now we will have estimation errors. Increase the scale factor.
2. Try to estimate the motion between the two photographs you made in the previous exercise. Do this in another script called `t_Panoram`. To match the two photographs, what transformations could suffice? It actually depends how you moved your hand exactly between the two shots. Translation may suffice, but including rotation - which would account for slight rotations - may be beneficial.

14.4 Robust Alignment with RANSAC

You may have experienced some difficulties in the previous motion-estimation exercise. For instance, some points may not have had a corresponding point in the other image; other points were matched to the wrong points in the other image. One can summarize such 'misses' as outliers and noise, and they make motion estimation in real world applications difficult; in other words we have a *correspondence* problem. It therefore requires more robust estimation techniques, for instance an iterative process consisting of 2 steps:

- 1) selection of a random subset of points and motion estimation with them.
- 2) verification of the estimate with the remaining set of points.

Example: We are fitting a line to an elongated point cloud which contains about 50% outliers. If we draw pairs of points uniformly and at random, then about a quarter of these pairs will consist of 'inlier' data points. We can identify these inlier pairs by noticing that a large proportion of other points lie close to the fitted line. Of course, a better estimate of the line could then be obtained by fitting a line to the points that lie close to our current line.

Algorithm Fischler and Bolles (1981) formalized this approach into an algorithm called RANSAC, for RANDOM SAmple Consensus (algorithm 7). It starts by finding an initial set of inlier correspondences, i.e., points that are consistent with a dominant motion estimate: it selects (at random) a subset of n correspondences, which is then used to compute an initial estimate for \mathbf{p} . The *residuals* of the full set of correspondences are then computed as

$$\mathbf{r}_i = \tilde{\mathbf{x}}'_i(\mathbf{x}_i; \mathbf{p}) - \hat{\mathbf{x}}'_i, \quad (31)$$

where $\tilde{\mathbf{x}}'_i$ are the *estimated* (mapped) locations and $\hat{\mathbf{x}}'_i$ are the sensed (detected) feature point locations.

The RANSAC technique then counts the number of inliers that are within ϵ of their predicted location, i.e., whose $\|\mathbf{r}_i\| \leq \epsilon$. The ϵ value is application dependent but is often around 1-3 pixels. The random

FoPo p332, s 10.4.2, pdf 30
Sze p281, s 6.1.4, pdf 318
Pnc p342, s 15.6
SHB p461,s.10.2

Algorithm 7 RANSAC: Fitting structures using Random Sample Consensus. FoPo p332, pdf 305

Input : $\mathcal{D}, \mathcal{D}^*$

Parameters:

- n the smallest number of points required (e.g., for lines, $n = 2$, for circles, $n = 3$)
- k the number of iterations required
- t the threshold used to identify a point that fits well
- d the number of nearby points required to assert a model fits well

Until k iterations have occurred:

- Draw a sample of n points from the data \mathcal{D} uniformly and at random $\rightarrow \mathcal{D}^s; \mathcal{D}^c = \mathcal{D} \setminus \mathcal{D}^s$
- Fit to \mathcal{D}^s and obtain estimates \mathbf{p}
- **For** each data point $\mathbf{x} \in \mathcal{D}^c$: if point close,
that is smaller than a threshold: $\|\mathbf{r}_i\|^2 < t$, then $\mathbf{x}_i \rightarrow \mathcal{D}^{good}$
- end**
- If there are d or more points close to the structure ($|\mathcal{D}^{good}| \geq d$), then \mathbf{p} is kept as a good fit.
 - refit the structure using all these points.
 - add the result to a collection of good fits $\rightarrow \mathcal{P}^{good}$

end

Choose the best fit from \mathcal{P}^{good} , using the fitting error as a criterion

selection process is repeated k times and the sample set with the largest number of inliers is kept as the final solution of this fitting stage. Either the initial parameter guess \mathbf{p} or the full set of computed inliers is then passed on to the next data fitting stage.

Matlab's computer vision toolbox provides a function, but is only accessible in the newest version I think. In the appendix we give an example of an implementation in Matlab, to be explained in the exercise section. Python offers the function `ransac` in submodule `skimage.measure`.

14.5 Closing Notes

For range images, the most widely used 3D registration technique is the iterated closest point (ICP) algorithm (Sze p515, s12.2.1, pdf 588).

The most complex registration problem treats objects that can deform. In this case, the family of transformations that could register the two datasets is large, and the search for a particular transformation is correspondingly more difficult. Registering deformable objects is a core technology for medical image analysis, because human organs deform and because it is quite usual to image the same body component using different imaging modes.

14.6 Exercises

1) Appendix I.10 contains a function and a testing script. Copy the function code into a script called `f_RanSaC` and the testing code into a script called `t_Ransac`. Study the example; compare with previous exercise script `t_2DTransforms`. Play with the parameters to understand what is happening. Which parameter names in the script correspond to which parameter names in algorithm 7. With what type of transformation does the given function `f_RanSac` work? Note: the function script assumes 'corresponding' points already!

To verify the function script, you can also test it by appending the following lines to `t_2DTransforms`, where `Co` and `Caff` are from that testing script:

```
Opt.nMinPts      = 7;
Opt.nMaxIter     = 3;
Opt.thrNear      = 0.2;
Opt.nNear        = 5;
Opt.xlb          = '';
Opt.Match        = '';
b_plot.dist      = 1;
RSprm            = f_RanSaC(Co, Caff(:,[1 2]), Opt, b_plot);
RSprm'
```

We now optimize the function script for readability. First make a copy of the function and call it `f_RanSaCRaw`, which we use for verification. Now optimize the function script by using the transformation functions which you have written in the previous exercise, that is replace the inline function with those function scripts.

- 2) Now apply RanSac to motion estimation of your two images from the previous exercise. Now you should be able to determine the degree of motion more reliable than before, even under noisy conditions.
- 3) Try to create a panorama of your room, or any other viewpoint. Translation may suffice for minimization, that is a 'simpler' J can be used.

15 Systems

15.1 Video Surveillance

Dav p578, ch22

Surveillance is useful for monitoring road traffic, monitoring pedestrians, assisting riot control, monitoring-of crowds on football pitches, checking for overcrowding on underground stations, and generally is exploited in helping with safety as well as crime; and of course surveillance is used in military applications. Traditional video surveillance heavily relies on simple matching operations and on background subtraction, but recent systems, that use the principles of feature detection and matching (as introduced in section 6), have often outperform the traditional systems easily. It may thus be somewhat futile to dwell too long in old methods and rather apply the new methods, although a combination of old and new methods may always be worth trying. We already mentioned some aspects of surveillance (e.g. in sections 13 and 9), but we round the picture by adding some aspects and by giving some examples.

The Geometry The ideal camera position is above the pedestrian, at some height H_c , and the camera's optical axis has a declination (angle δ) from the horizontal axis (see Figure 25). This is simply the most suitable way to estimate the distance and height H_t of the pedestrian, thereby exploiting triangulation and the knowledge of where the position of the pedestrian's feet.

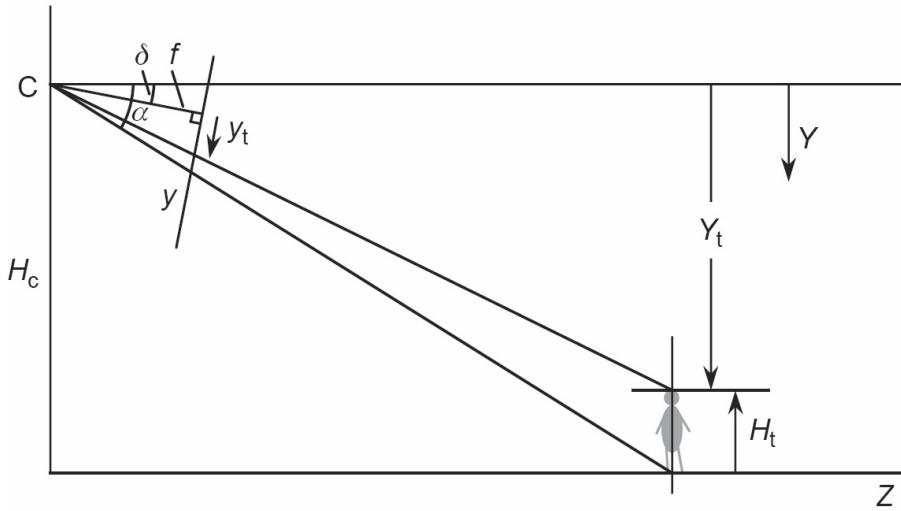


Figure 25: 3-D monitoring: camera tilted downwards. δ is the angle of declination of the camera optical axis.
[Source: Davies 2012; Fig 22.2]

Foreground/Background Separation

SHB p776, s 16.5.1

The idea of background subtraction is to eliminate the 'static' background to obtain only the (moving) foreground foreground objects. Although that appears to be a straightforward idea in principle, it is a challenging task because the background also changes continuously, e.g. the illumination changes throughout the day, vegetation flutters, shadows wander (of fixed objects and of clouds), etc. Furthermore, some of these background changes aggravate the challenge of detecting initial object motion. As mentioned before, using interest point features, much more stable motion detection and tracking can be provided (Section 13.2).

Vehicle License Plate Detection License plate recognition is a challenging task due to the diversity of plate formats and the nonuniform outdoor illumination conditions during image acquisition. Therefore, most approaches work only under restricted conditions such as fixed illumination, limited vehicle speed,

designated routes, and stationary backgrounds. Algorithms (in images or videos) are generally composed of the following three processing phases:

- 1) Extraction of a license plate region. An effective method is described in Figure 26.
- 2) Segmentation of the plate characters. Thresholding as described previously (Section 9.1)
- 3) Recognition of each character. A typical optical-character recognition system will do, see Pattern Recognition.

Scholar google "Anagnostopoulos" for a review.

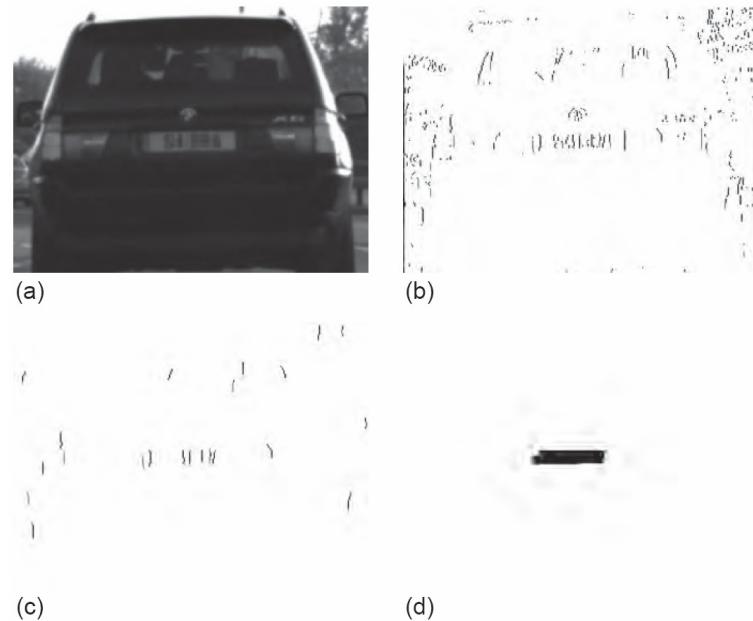


Figure 26: Simple procedure for locating license plates. (a) Original image with license plate pixelated to prevent identification. (b) Vertical edges of original image (e.g. with Sobel detector). (c) Vertical edges selected for length. (d) Region of license plate located by morphological operations, specifically horizontal closing followed by horizontal opening, each over a substantial distance (in this case 16 pixels).

[Source: Davies 2012; Fig 22.13]

Vehicle Recognition (Research) Depending on the purpose, vehicles are categorized into types (e.g., car, truck, van,...) or more specifically, their make and model is identified (e.g. Volkswagen, Golf). A type classification has been approached with an analysis of depth images. For make/model identification, many systems have focused on analyzing the car's front, which can be regarded as its face. A typical system locates the license plates first and then measures other front features in relation to it, such as head lights, radiator grill, etc.:



Figure 27: Make and model identification using the front features and their relations to the license plate.

So far, (exploratory) studies have used up to 100 auto model tested on still images. Scholar google 'Pearce and Pears' for a recent publication.

15.2 Autonomous Vehicles

Dav p636, ch23

Two decades ago, many experts would have considered it impossible that one day fully autonomous vehicles would drive on the road. Meanwhile most car companies develop a system that is capable of nearly or fully autonomous driving. Here we mention some of the principal issues addressed in such a system.

Such a system contains four interacting processes: environment perception, localization, planning and control. About two third of the system are involved in perception. Interestingly enough this proportion is about the same for the human brain.

Vision processing (environment perception) relies to a large degree on range cameras, because a range image can be easier segmented than an intensity image (see Kinect). A set of different range cameras is used covering different depth ranges with resolution down to 0.1 degree and depth up to 300m. Both radar and lidar is used (Appendix A).

The perception processes consist of the detection of pedestrians, bicyclists, cars, traffic signs, traffic lights, telephone poles, curbs, etc. The algorithms all appear to be based on techniques as we have introduced so far, meaning there is no particular magic involved; it rather requires an elaborate coordination of the different detection processes. One example was given already with car license plate recognition, we give more examples below. The recognition tasks are typically solved using multiple techniques complementing each other, as a single technique is often insufficient to reliably solve the problem. We here mention in particular techniques used in a 2D gray-level image.

Roadway/Roadmarker Location: is addressed for example with multilevel thresholding (Section 9.1); or with vanishing point detection using RANSAC on edge points (see Figure 28).

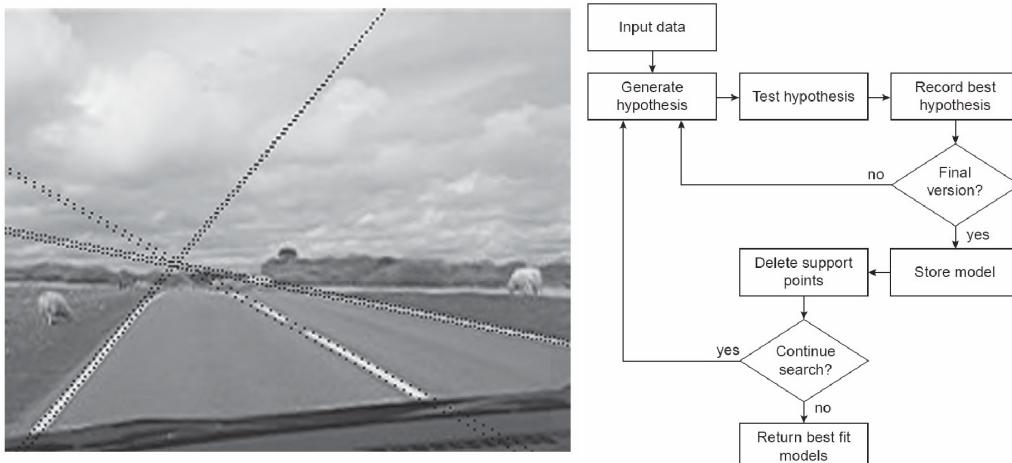


Figure 28: **Left:** Lane markings identification with RANSAC by placing straight lines through detected edge points. While the lane markings converge to approximately the right point on the horizon line. [Source: Davies 2012; Fig 23.2]. **Right:** Flow chart of a lane detector algorithm. [Source: Davies 2012; Fig 23.4]

Locating of Vehicles: there are two very simple tricks to detect cars (in 2D gray-scale images):

- shadow induced by vehicle: Importantly, the strongest shadows are those appearing beneath the vehicle, not least because these are present even when the sky is overcast and no other shadows are visible. Such shadows are again identified by the multilevel thresholding approach (Section 9.1).
- symmetry: The approach used is the 1-D Hough transform, taking the form of a histogram in which the bisector positions from pairs of edge points along horizontal lines through the image are accumulated. When applied to face detection, the technique is so sensitive that it will locate not only the centerlines of faces but also those of the eyes. Symmetry works also for plant leaves and possibly other symmetric shapes.

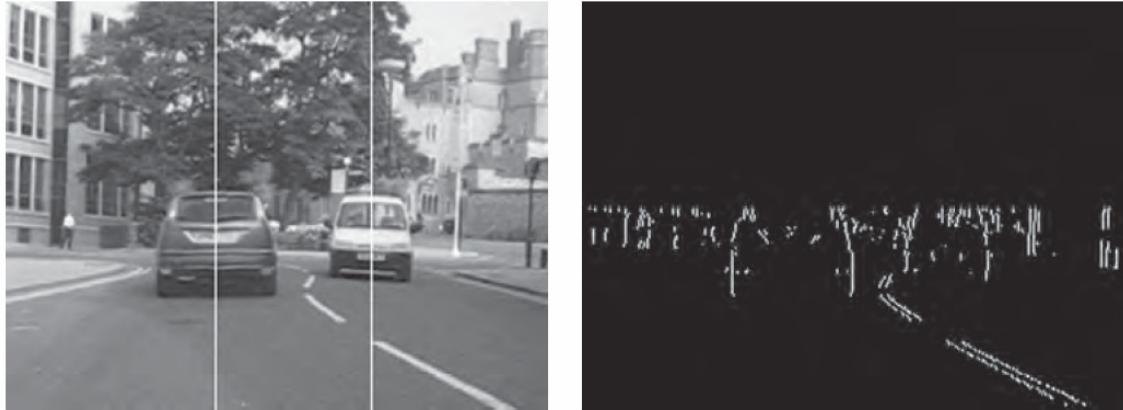


Figure 29: Detecting cars by exploiting the symmetry of vertical segments. [Source: Davies 2012; Fig 23.6]

Locating Pedestrians: We have introduced a pedestrian detection algorithm already in Section 8.2, but here we mention some other techniques that ensure a high recognition accuracy:

- a) detection of body parts, arms, head, legs. The region between legs often forms an (upside-down) V feature.
- b) Harris detector (Section 6.1) for feet localization.
- c) skin color (see also Section F.1).

A substantial effort in developing such perception software is spent in temporal optimization of the implemented algorithms. Meanwhile car-part supplier companies have also started to develop recognition software. Thus, in the future, car companies will probably buy components of the recognition process from suppliers.

15.3 Remote Sensing

wiki Remote_sensing

Dav p738, ch23

Remote sensing is the analysis of images taken by satellites or aircrafts ([wiki Satellite.imagery](#), [wiki Aerial.photography](#)). The images one obtains are huge and manipulating them therefore takes a lot of time - it is practically impossible to apply sophisticated image-processing methods on large areas; for instance scanning the ocean for floating debris is unfeasible at this point. From a methodological viewpoint, there is therefore nothing particular to explain here. We now merely give some background information on this topic, in particular on satellite imagery.

In satellite imagery one can distinguish between four types of resolution: spatial, spectral, temporal and radiometric. The more modern the satellite, the higher are those resolutions in general.

- Spatial Resolution: nowadays, the resolution can be approximately 30 centimeters per pixel.
- Spectral Resolution: satellite images can be simple RGB photographs, but also a broad range of electromagnetic waves is typically measured. Early satellite recorded so-called multi-spectral images, where at each pixel several bands of the electromagnetic spectrum were recorded, sometimes up to 15 bands [wiki Multispectral.image](#). Table 1 gives an impression of how some of those bands can be exploited. Meanwhile there exist satellites that record several tens or even hundreds of bands, generating so-called hyperimages, which permit detailed selections; the amount of storage required for such images is very large however.
- Temporal Resolution: if one intends to track changes over time, then the temporal resolution is of interest: it can be several days. It is also called revisiting frequency sometimes.

Table 1: Example of bands and their use. Given ranges are approximate - exact values depend on satellite.

Band Label	Range (nm)	Comments
Blue	450-520	atmosphere and deep water imaging; depths up to 150 feet (50 m) in clear water.
Green	515-600	vegetation and deep water; up to 90 feet (30 m) in clear water.
Red	600-690	man-made objects, in water up to 30 feet (9 m), soil, and vegetation.
Near-infrared (NIR)	750-900	primarily for imaging vegetation.
Mid-infrared (MIR)	1550-1750	imaging vegetation, soil moisture content, and some forest fires.
Far-infrared (FIR)	2080-2350	imaging soil, moisture, geological features, silicates, clays, and fires.
Thermal infrared	10400-12500	emitted instead of reflected radiation to image geological structures, thermal differences in water currents, and fires, and for night studies.
Radar & related tech		mapping terrain and for detecting various objects.

- Radiometric Resolution: concerns the 'range' of values and starts typically at 8 bits (256 values).

There exist software tools that preprocess the raw satellite images in order to transform them into a format that is more suitable for object detection and classification, see [wiki Remote_sensing_application](#). The larger the area under investigation, the more time consuming is this transformation.

To compare methods one can use the following Kaggle competition:

<https://www.kaggle.com/c/dstl-satellite-imagery-feature-detection/data>

A Image Acquisition

Digital image acquisition is the process of analog-to-digital conversion of the 'outer' signal to a number, carried out by one or several image sensors (cameras). The conversion can be very complex and involves the generation of an output that is 'visible' to the human eye. We here merely give an overview of the different types of sensors:

Light-sensitive cameras: measure from the visible part of the electromagnetic spectrum; the most common device in daily life.

Multi/hyper-spectral sensors: measure from a broader part of the electromagnetic spectrum (than the light-sensitive cameras) with individual sensors tuned to specific bands; originally developed for remote sensing (satellite imagery, Section 15.3), but now also employed in document and painting analysis.

Range sensors (rangefinders): is a device that measures the distance from the observer to a target, in a process called ranging or rangefinding. Methods include laser, radar, sonar, lidar and ultrasonic rangefinding. Applications are for example surveying, navigation, more specifically for example ballistics, virtual reality (to detect operator movements and locate objects) and forestry.

Tomography device: generates an image of a body by sections or sectioning, through the use of any kind of penetrating wave. The method is used in radiology, archeology, biology, atmospheric science, geophysics, oceanography, plasma physics, materials science, astrophysics, quantum information, and other areas of science.

Radar (Radio Detection And Ranging): is an object-detection system that uses radio waves to determine the range, angle, or velocity of objects. It can be used to detect aircraft, ships, spacecraft, guided missiles, motor vehicles, weather formations, and terrain.

Lidar (Light Detection And Ranging): is a surveying method that measures distance to a target by illuminating the target with a pulsed laser light, and measuring the reflected pulses with a sensor.

This 'raw' image may require some manipulation such as re-sampling in order to assure that the image coordinate system is correct; or noise reduction in order to assure that sensor noise does not introduce false information.

The pixel values typically correspond to light intensity in one or several spectral bands (gray images or colour images), but can also be related to various physical measures, such as depth, absorption or reflectance of sonic or electromagnetic waves, or nuclear magnetic resonance.

B Convolution [Signal Processing]

Expressed in the terminology of applied mathematics, convolution is the repeated multiplication of one function on the domain of another function producing a third function; it is considered an 'operation' and is similar to cross-correlation. In signal processing terms, the first function is the signal - in our case often an image -, and the second function is a so-called *kernel* and manipulates a local neighborhood of that image at each location. This is easier to understand in one dimension first (Section B.1), then we introduce this for two dimensions (Section B.2).

B.1 In One Dimension

Our signal is for instance the face profile as in Figure 3. We wish to determine the extrema of that signal but that is easier if we smoothen the signal first with a low-pass filter. In plain words, we average the signal over a small neighborhood at each pixel. Let us clarify the convolution operation on a very simple signal `S`: our signal is zero everywhere except for one value in center which holds the value 2. We convolve it with three different kernels, `Ka`, `Kb` and `Kg` using the function `conv` (Python: `scipy.signal.convolve`). Then we plot the three new functions.

```
clear;
S = [0 0 0 0 2 0 0 0 0]; % the signal
Ka = [1 1 1]/3; % averaging kernel
Kb = [.25 .5 .25]; % triangle function
Kg = pdf('norm', -2:2, 0, 1); % 5-pixel Gaussian

Sa = conv(S, Ka, 'same');
Sb = conv(S, Kb, 'same');
Sg = conv(S, Kg, 'same');

%% ----- Plotting
figure(1);clf;
Xax = 1:nPix;
plot(Xax, S, 'k'); hold on;
plot(Xax, Sa, 'r*');
plot(Xax, Sb, 'bt');
plot(Xax, Sg, 'g.');
```

So far, not much has happened. The new function looks like its kernel, but scaled in amplitude. It becomes more interesting if we make the signal more complicated: turn on another pixel in signal `S` and observe. To ensure that you understand the detailed convolution process, look at the following explicit example:

```
Sa2 = zeros(1,nPix);
for i = 2:nPix-1
    Nb = S(i-1:i+1); % the neighborhood
    Sa2(i) = sum(Nb .* Ka); % multiplication with kernel and summation
end
assert(nnz(Sa2-Sa)==0); % verify that it is same as 'conv'
```

The example is a simplified implementation of the convolution process, namely we do not observe the boundary values and it works only for kernel of length equal three pixels, with `i` running from 2 to `nPix-1`. But it contains the gist of the convolution operation.

When applying a convolution function you need to pay attention to what type of treatment you prefer for the boundary values. Matlab offers three options: full, valid and same; they return outputs of different sizes. We refer to the documentation for details. They do matter, so when you apply a convolution you need to think about the boundary values. If you prefer to set your own boundary values, then you compute only the central part of the convolution - in Matlab with option `valid` - and then use `padarray` to set the boundary values. We did that in the example of the face profiles, see Appendix I.1.

Mathematical Formulations In engineering equations the convolution can be written as:

$$(S * K)[k] = \sum_i^n S[i]K[k - i] \quad (32)$$

where $*$ is the convolution symbol, i is the signal's variable, n the number of pixels of the signal and k is the Kernel's variable. That was the formulation for the discrete convolution. The continuous convolution is written as:

$$(S * K)(k) = \int_{-\infty}^{\infty} S(i)K(k - i)\delta i \quad (33)$$

B.2 In Two Dimensions [Image Processing]

In two dimensions the convolution process integrates over a local two-dimensional neighborhood. In case of an image the neighborhood can be a 3x3 pixel neighborhood, or a 5x5 pixel neighborhood, etc. The mathematical formulation remains essentially the same as above; one can understand the kernel as of any dimension, but the signal and kernel need to be defined clearly from the beginning.

In Matlab we now use the function `conv2`, in Python we use `scipy.signal.convolve2d`:

```
clear;
% --- the image
I = zeros(10,10);
I(5,5) = 1;
% --- kernels
Ka = ones(3,3)/9; % averaging kernel
Kg = fspecial('gaussian',5,1); % 5x5 Gaussian kernel
% --- convolutions
Ia = conv2(I, Ka);
Ig = conv2(I, Kg, 'same');

%% ----- Plotting
figure(1);clf;[nr nc] = deal(2,2);
subplot(nr,nc,1); imagesc(I,[0 1]);
subplot(nr,nc,2); imagesc(Ia); colorbar;
subplot(nr,nc,3); imagesc(Ig); colorbar;
```

Speeding Up Because image convolution is a relatively time-consuming operation due to the repeated multiplication of two matrices, there exist methods to speed up image convolution. Those speed-ups work only if the kernel shows specific characteristics, in particular it needs to be symmetric. The Gaussian function for instance is suitable for speed up. In that case, an image convolution with a 2D Gaussian function can be separated into convolving the image twice with the 1D Gaussian function in two different orientations:

```
Kg1 = normpdf(-2:2); % a one-dim Gaussian
Iblr1 = conv2(I,Kg1'*Kg1,'same'); % with 2D function
Iblr2 = conv2(conv2(I,Kg1,'same'),Kg1','same'); % with 1D functions
Iblr3 = imgaussfilt(I,1); % special function
```

The product `Kg1'*Kg1` creates a 2D Gaussian - we could have also generated it using `fspecial('gaussian',[5],1)` for instance.

For small images, the durations for each of those three different versions are not much different - the duration differences become evident for larger images. Use `tic` and `toc` to measure time; or use `clock`.

C Filtering [Signal Processing]

An image can be filtered in different ways depending on the objective: one can measure emphasize or even search certain image characteristics. The term filter is defined differently depending on the specific topic (signal processing, computing, etc.). Here the term is understood as a function, which takes a small neighborhood and computes with its pixel values a certain value; that computation is done for each neighborhood of the entire image. That local function is sometimes called kernel.

For educational purposes, we differentiate here between five types of filtering. In the first one, the kernel calculates simple statistics with the neighborhood's pixel values (Section C.1). Then there are three basic techniques to emphasize a certain 'range' (band) of signal values: low-pass, band-pass and high-pass filtering (Sections C.2, C.3 and C.4, respectively). Finally, there can be complex filter kernels.

C.1 Measuring Statistics

Sometimes we intend to measure very simple statistics. Those in turn can be used for a variety of tasks, in particular texture description. One can determine the range of values within the neighborhood, their mean, their standard deviation etc. In Matlab those statistical measurements are provided with `rangefilt`, `stdfilt`, `medfilt2`, `ordfilt2`, etc.

There are various ways to apply your own specialized filters, for which it is easiest to arrange the image first into a matrix in which your neighborhoods are aligned column-wise using Matlab's `im2col`/`col2im` functions. That allows you to conveniently process the neighborhoods. Here is an example taking the average value for a neighborhood:

```
s = 5; % side length
I = reshape(linspace(0,1,s*s),[s s]); % [s s] stimulus
C = im2col(I,[3 3]); % extract [3 3] patches
Mn = mean(C,1); % take mean
Ims = col2im(Mn,[1 1],[s s]-2); % rearrange to matrix
Im = padarray(Ims,[1 1]); % pad to make it same size as I
```

C.2 Low-Pass Filtering

Because signals are often noisy, applying a low-pass filter to the image is often one of the first steps and that serves to squash the 'erratic' values. That is why images are typically filtered with at least a 3x3 Gaussian filter initially, no matter the exact task. The Gaussian filter is perhaps the most frequently used low-pass filter and it is worth looking at its one-dimensional shape, see Figure 30.

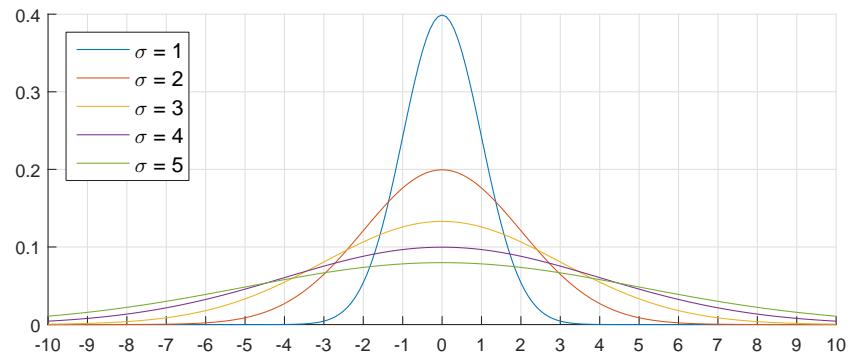


Figure 30: Gaussian function for five different values of sigma (1,...,5) placed at $x=0$. The parameter sigma determines the width of the bell-shaped curve.

In two dimensions, the Gaussian looks as depicted in Figure 9, namely in the first four patches (filter no. 1-4).

One can take also other functions for low-pass filtering, such as a simple average, see code block above or code in the Appendix on Convolution above, where `Ka = ones(3,3)/9` was used. The reason why the Gaussian function is so popular, is that it has certain mathematical advantages.

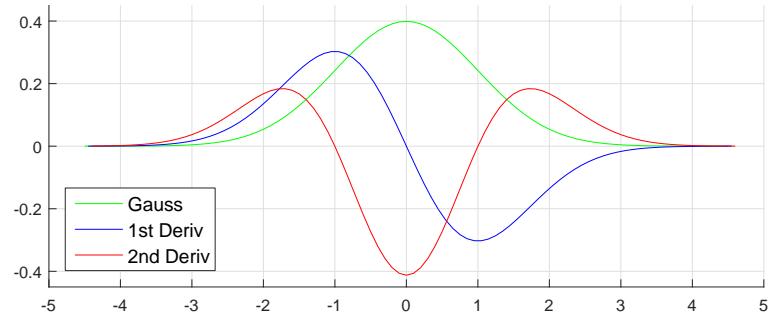
C.3 Band-Pass Filtering

One example of band-pass filtering is region detection with the Difference-of-Gaussian (DOG) as we introduced in Section (4.1). Another example are the Laplacian-of-Gaussian (LoG) filters used for texture detection (Figure 9).

C.4 High-Pass Filtering

Edge detection is an example for high-pass filtering. In some algorithms the first derivative of the Gaussian is used as high-pass filter, see also Figure 31.

Figure 31: The Gaussian function (green) and its first (blue) and second (red) derivative. The Gaussian function itself is often used as low-pass filter. The first derivative of the Gaussian is often used as high-pass filter, for example in edge detection. The second derivative is occasionally used as band-pass filter, for example as region detection.



C.5 Function Overview

The variety of filtering commands can be confusing sometimes, that is why we provide here a short summary of frequently used filtering operations:

Table 2: Overview of commonly used image filtering methods. `I` = image.

Command	Comments
<code>rangefilt(I,true(rad*2+1))</code>	local range. Neighborhood is logical matrix.
<code>stdfilt(I,true(rad*2+1))</code>	local std dev. Neighborhood is logical matrix.
<code>Flt = fspecial('gaussian',[3 3],0.5)</code>	creates a filter. To be used with <code>conv2, imfilter ...</code>
<code>conv2(I,Flt,'same')</code>	convolution
<code>convolve2d(I,Flt,mode='same')</code>	in <code>scipy.signal</code>
<code>im2col(I,[3 3])</code>	extracts blocks from image, <code>[nPix 9]</code> .
<code>col2im(B,[3 3])</code>	inverse of <code>im2col</code> .
<code>colfilt</code>	column-wise filtering, uses <code>im2col</code> and <code>col2img</code> .
<code>imfilter</code>	filter multi-dimensional images.
<code>nlfilt</code>	general non-linear sliding filter.
<code>filter2(Flt, I, 'same')</code>	filtering with a FIR filter.

Table 3: Filtering of one-dimensional signals. `S` = one-dimensional signal.

Command	Comments
<code>F1d = normpdf(-2:2) = pdf('norm',-2:2,0,1)</code>	generates a Gaussian (normal) filter for five pixels with $\mu=0$ (mean) and $\sigma=1$ (standard deviation)
<code>F1d = get_window('gaussian',1),5)</code>	in <code>scipy.signal</code>
<code>conv(S,F1d,'same')</code>	1D-convolution. Watch the orientation of vectors!
<code>convolve(S,F1d,'same')</code>	in <code>scipy.signal</code>

D Neural Networks

The element of a neural network (NN) is a so-called *Perceptron*. A perceptron essentially corresponds to a model of a linear classifier as in traditional Machine Learning. Formulated in the terminology of neural networks, a Perceptron consists of a ‘neural’ unit that receives input from other units. The inputs are weighted, then summed and then thresholded. For a two-class problem, there is essentially one integrating unit; for a multi-class problem there are several integrating units, whose count corresponds to the number of classes to be discriminated.

If such a Perceptron is stacked, then we talk of a *Multi-Layer Perceptron* (MLP). We show how to tune such a MLP in Keras, an API for Google’s tensorflow, coming up next in Section ???. There are also ‘richer’ ways to wire neural units; we introduce one such type of network in Section D.2.

D.1 A Multi-Layer Perceptron (MLP) - Warming up to Keras

[wiki Multilayer_perceptron](#)

An MLP is a neural network consisting of four or more layers (Figure 32): an input layer, receiving the image in our case; two or more hidden layers that combine information; and an output layer, that indicates the selected class for an input image. A layer consists of (*neural*) *units*. The input layer has a unit count that typically corresponds to the number of pixels of the input image. The hidden layers have a unit count that is often a multiple of the input layer count. The output layer has a unit count that corresponds to the number of classes to be distinguished.

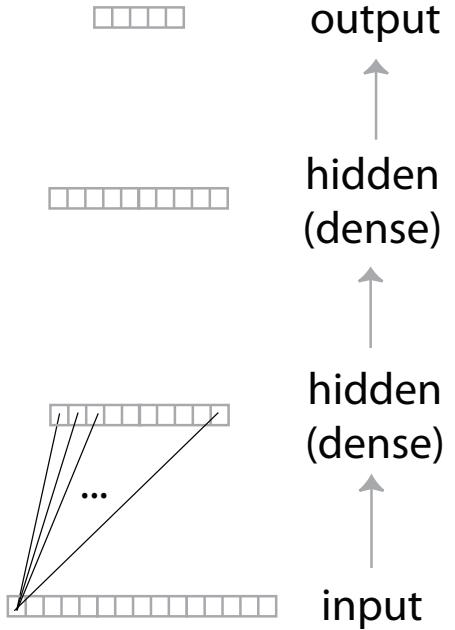


Figure 32: A Multi-Layer Perceptron (a general classifier in fact). The architecture has four layers: input layer, two hidden (dense) layers, and an output; classification flow occurs from bottom to top.

The **input layer** is linear and corresponds to the number of image pixels. The **(first) hidden layer** receives the values from the input layer with all-to-all connectivity (only the connections from one pixel to all hidden units are indicated). Each connection has a weight whose exact value will be learned during the learning process. The corresponding weight matrix has the size of number of input pixels times number of hidden units. The **(second) hidden layer** receives the output of the first hidden layer. The corresponding weight matrix has dimensions corresponding to the sizes of the two hidden layers. The **output layer** has the same number of units as the number of classes to be discriminated. For instance if we intend to discriminate the digits 0 to 9, then there would be 10 output units. The output layer receives the values of the previous hidden layer.

The units between two layers are typically all-to-all connected that is the connections build a *complete bipartite graph* ([wiki Bipartite_graph](#)). Each connection holds a weight value that will be learned during the training process.

A unit typically sums the values it receives - weighted by the corresponding connection weights - and then passes the summed value c to a so-called *activation* function that is adjustable by a parameter b :

$$a = f(c, b) \tag{34}$$

The output a of the activation function is then passed to the next layer. There are many types of activation functions ([wiki Activation_function](#)), a popular one in Neural Networks is the rectified linear unit (relu), where b is a thresholding parameter: values below b are set to zero. The way such a unit works is essentially like a linear classifier ([wiki Linear_classifier](#)).

An Example We demonstrate how such a network can be programmed in Keras, a Python library (module) designed to test network structures [wiki Keras](#). It is recommended to download Python version no. 3.5, which includes the modules for Tensorflow and Keras.

We use a database of handwritten digits (0-9), the so-called MNIST database, containing 60000 training images and 10000 testing images. How those images can be loaded is shown in Appendix I.4 and we import that function into our script with the line `from LoadMNIST import LoadMNIST`. Note that the function loads the images as two dimensions (28 x 28). The training images are therefore stored as a three-dimensional array 60000 x 28 x 28. A single digit image can be viewed by `imshow(TREN[1,:])` (not shown in code).

```
# https://github.com/fchollet/keras/blob/master/examples/mnist_mlp.py
# Trains a Multi-Layer Perceptron on the MNIST dataset.
# Achieves 98.20% test accuracy after 12 epochs
from __future__ import print_function
from keras.models import Sequential
from keras.layers import Dense, Dropout
from keras.optimizers import RMSprop
from LoadMNIST import LoadMNIST
batchSz = 128      # batch size (# images per learning step)
nEpoch = 12         # number of epochs = learning duration

# %% ----- Load Database & Labels -----
TREN, TEST, Lbl = LoadMNIST()           # load the digits
TREN = TREN.reshape(60000, 784)          # make image linear
TEST = TEST.reshape(10000, 784)

# %% ----- Build Network -----
N = Sequential()
N.add(Dense(512, activation='relu', input_shape=(784,))) # input - hidden1
N.add(Dropout(0.2))
N.add(Dense(512, activation='relu'))                  # hidden1 - hidden2
N.add(Dropout(0.2))
N.add(Dense(nClass, activation='softmax'))            # hidden2 - output
N.summary()
N.compile(loss = 'categorical_crossentropy',
           optimizer = RMSprop(),
           metrics = ['accuracy'])

# %% ----- Learning -----
N.fit(TREN, Lbl.TrenMx,
      batch_size = batchSz,
      epochs = nEpoch,
      verbose = 1,
      validation_data = (TEST, Lbl.TestMx))

# %% ----- Evaluation -----
score = N.evaluate(TEST, Lbl.TestMx, verbose=0)
print('Test loss: ', score[0])
print('Test accuracy:', score[1])
```

For the MLP network we do not even bother that the image has two dimensions but we 'linearize' the image by aligning all pixels in a single column: the input to the network is therefore a vector of length 784, `TREN.reshape(60000,784)`. We now discuss the three sections 'Build Network', 'Learning' and 'Evaluation'.

Build Network The network architecture is determined with five `N.add` commands. The first one determines the weights between the input layer and the first hidden layer, meaning there are $512 \times 784 = 401408$ weight parameters between those two layers. The second `N.add` determines a certain drop out rate, which helps the learning process by ignoring certain weights occasionally. The third `N.add` adds another hidden layer with 512 units. Thus we have $512 \times 512 = 262144$ weight values between the first and second hidden layer. The fourth `N.add` specifies again the drop out rate. The fifth `N.add` specifies the output layer, namely 10 units for 10 classes. With `N.summary()` the unit count is displayed.

There are many types of learning schemes and tricks that can be applied to learn a neural network. Here, those options are specified with the command `N.compile`.

Learning Learning occurs with the `N.fit` command. We learn on the training set `TREN` and the corresponding labels in `Lbl.TrenMx`. The parameter `batchSz` determines how many images are used per training step. There exists a rough optimum, too few or too many images per batch results in less efficient learning. The parameter `nEpoch` corresponds to the learning duration essentially.

Evaluation Evaluation takes place with `N.evaluate`. Here we employ the test set `TEST` and the corresponding labels `Lbl.TestMx` to estimate the prediction accuracy of the network.

The NNs as introduced so far are considered meanwhile traditional Neural Networks or *shallow* NNs, because they use few layers as opposed to the many layers used by a Deep Net.

D.2 Deep Belief Network (DBN)

wiki Deep_belief_network

A Deep Belief Network (DBN) is another type of Deep Neural Network. It is employed more rarely than the CNN, because it does not achieve quite the same performance as a CNN, but has the advantage that its overall architecture is simpler: often, with two layers you can achieve almost the same performance as with a CNN. Such a DBN has essentially a similar architecture to the Multi-Layer Perceptron, but the DBN contains also connections within the same layer. That particular characteristic makes it extremely powerful, but the downside is that learning is terribly slow.

A Belief Network is a network that operates with so-called *conditional dependencies*. A conditional dependence expresses the relation of variables more explicitly than just by combining them with a weighted sum - as is done in most other classifiers. However determining the full set of parameters for such a network is exceptionally difficult. Deep Belief Networks (DBNs) are specialized in approximating such networks of conditional dependencies in an efficient manner, that is at least partially and in reasonable time. Popular implementations of such DBNs consist of layers of so-called *Restricted Boltzmann Machines* (RBMs).

Architecture The principal architecture of a RBM is similar to the dense layer as used in an MLP or CNN. An RBM however contains an additional set of *bias* weights. Those additional weights make learning more difficult but also more capable - they help solving those conditional dependencies. The typical learning rule for a RBM is the so-called *contrast-divergence* algorithm.

The choice of an appropriate topology for the entire DBN is relatively simple. With two hidden layers made of RBMs, one can obtain already fantastic results. A third layer rarely helps in improving classification accuracy.

Learning Learning in a DBN occurs in two phases. In a first phase, the RBM layers are trained individually one at a time in an unsupervised manner (using the contrast-divergence algorithm): the RBMs perform quasi a clustering process. Then, in the second phase, the entire network is fine-tuned with a so-called back-propagation algorithm. As with CNNs, a Deep Belief Network takes much time to train. The downside of a DBN is, that there exists no method (yet) of speeding up the learning process, as is the case for CNNs.

Code Keras does not offer (yet?) methods to run a DBN, but tensorflow does. Tensorflow is however a bit trickier in specifying and running a network.

E Classification, Clustering [Machine Learning]

Given are the data `DAT` and - if present - some class (group) labels `Lbl`:

`DAT`: matrix of size `[nObs x nDim]` with rows corresponding to images (whose pixels are taken columnwise) or features (some attributes extracted from images); and columns corresponding to the dimensionality (no. pixels per image or no. of attributes, respectively).

`Lbl`: a one-dimensional array `[nObs x 1]` with entries corresponding to the class (group or category) membership. This label vector allows us to train classifiers; if this array does not exist, then we can apply only clustering algorithms.

E.1 Classification

To properly determine the classification performance, the data set `DAT` is divided into a training set and a testing set. With the training set the classifier model is learned, with the testing set the model's performance is determined. To generate those two sets the label vector `Lbl` is used. We generate indices for training and testing set with `crossvalind`, then loop through the folds using `classify` or other classification functions:

```
Ixs = crossvalind('Kfold', Lbl, 3); % indices for 3 folds
Pf = []; % initialize performance structure
for i = 1 : 3
    IxTst = Ixs==i; % i'th testing set
    IxTrn = Ixs~=i; % i'th training set
    LbOut = classify(DAT(IxTst,:), DAT(IxTrn,:), Lbl(IxTrn)); % classification
    nTst = nnz(IxTrn);
    Pf(i) = nnz(LbOut==Lbl(IxTst))/nTst*100; % in percent
end
fprintf('Pc correct: %4.2f\n', mean(Pf));
```

If the bioinfo toolbox is available, then the command `classperf` can be used to determine performance slightly more convenient.

Classification Errors? Chances are fairly good that you will not succeed with such a straightforward classification attempt. Matlab may complain with some error that the covariance matrix can not be properly estimated. In that case, there are several options:

1. You can always try a kNN classifier (`knnclassify` in the bioinfo toolbox): it is somewhat simple, but you always obtain some results.
2. Use the Principal Component Analysis to reduce the dimensionality, see next Section E.1.1.
3. Use a Support-Vector machine: `svmclassify` (bioinfo toolbox). This is a very powerful classifier but it discriminates between two classes only. To exploit this classifier you then need to learn c classifiers, each one distinguishing between one category and all others (c = number of classes).

E.1.1 Dimensionality Reduction with Principal Component Analysis

The Principal Component Analysis (PCA) finds a more compact data space for the original data: the number of dimensions `nDim` of the data is 'reduced', perhaps only 70% of its original dimensionality. In the following code `nObs` is the number of training samples:

```
coeff = pca(DAT); % [nDim, nDim] coefficients
nPco = round(min(size(DAT))*0.7); % suggested # of observations
PCO = coeff(:,1:nPco); % select the 1st nPco eigenvectors
DATRed = zeros(nObs,nPco); % allocate memory
for i = 1 : nObs,
    DATRed(i,:) = DAT(i,:) * PCO; % transform each sample
end
```

The reduced array `DATRed` is now being used for classification above.

E.2 Clustering

Given are the data `DAT` in the format as above, but there are no labels available: in some sense we try to find labels for the samples.

K-Means To apply this clustering technique we need to provide the number of assumed groups k (denoted as n_c called in our case):

```
Ixs = kmeans(DAT, nc);
```

`Ixs` is a one-dimensional array of length `nObs` that contains the numbers ($\in 1..n_c$) which represent the cluster labels. We then need to write a loop that finds the corresponding indices, for instance with the following function. In this function, the variable `Pts` is equal the variable `DAT` in our above notation.

```
% Cluster info.
% IN Cls      vector with labels as produced by a clustering algorithm
%     Pts      points (samples)
%     minSize   minimum cluster size
%     strTyp    info string
% OUT I .Cen   centers
%         .Ix    indices to points
%         .Sz    cluster size
%
function I = f_ClsInfo(Cls, Pts, minSize, strTyp)
nCls = max(Cls);
nDim = size(Pts,2);
H = hist(Cls, 1:nCls);
IxMinSz = find(H>=minSize);
I.n = length(IxMinSz);
I.Cen = zeros(I.n,nDim,'single');
I.Ix = cell(I.n,1);
I.Sz = zeros(I.n,1);
for i = 1:I.n
    bCls = Cls==IxMinSz(i);
    cen = mean(Pts(bCls,:),1);
    I.Cen(i,:) = cen;
    I.Ix{i} = single(find(bCls));
    I.Sz(i) = nnz(bCls);
end
nP = size(Pts,1);
I.notUsed = nP-sum(I.Sz);

%% ---- Display
fprintf('%d Cls %9s Sz %1d-%2d #PtsNotUsed %d oo %d\n', ...
    I.n, strTyp, min(I.Sz), max(I.Sz), I.notUsed, nP);

end % MAIN
```

F Color Spaces

wiki Color_space

There are certain color spaces in which segmentation of specific objects is sometimes easier, see table 4. To obtain these color spaces, one uses commands such as `rgb2hsv` for example; for complex spaces we need to call two commands:

```
LabForm = makecform('srgb2lab'); % prepares transformation structure
Icol    = applycform(Icol, LabForm); % applies above structure
```

Table 4: Commonly used color spaces. f : some complex function.

Space	Calculation	Comments
RGB wiki RGB_color_model	measured	- high correlation between channels; significant perceptual non-uniformity; mixing of chrominance and luminance data
normalized RGB wiki Rg_chromaticity	with $s = R + G + B$, then $r = R/s$ $g = G/s$ $b = B/s$	+ reduces shadow and shading effects (under certain assumptions) essentially a 2D space because the 3rd component is = 1-the other two Usage: computer vision <code>bsxfun(@rdivide,I,sum(I,3));</code>
HSI (HSV HSL) Hue Saturation Intensity (HS Value) HS Brightness) wiki HSL_and_HSV	$H = f(R, G, B)$ $S = 1 - 3 \frac{\min(R, G, B)}{R+G+B}$ $I = \frac{1}{3}(R + G + B)$	+ gives the user a good impression about the resulting color for a certain color value; describes color with intuitive values, based on the artists idea of tint, saturation and tone. - hue discontinuities; computation of I, V or L conflicts with properties of color vision. • Hue: dominant color such as red, green, purple and yellow. • Saturation: colorfulness in proportion to its brightness. • Intensity, Lightness or Value: related to luminance; Usage: design and editing (e.g. within graphics design tools) <code>rgb2hsv</code>
YC_bC_r wiki YCbCr	$Y = \text{eq. 1 above}$ $C_b = R - Y$ $C_r = B - Y$	Y also called <i>luma</i> here C_b, C_r also called <i>chroma</i> here Usage: digital video; image compression <code>rgb2ycbcr</code>
CIELAB or CIE Lab (related: CIELUV) wiki Lab_color_space	complex	+ perceptually uniform: small changes to a component do not change perception much - computationally intensive. • L: lightness • a: green-to-magenta • b: blue-to-yellow Usage: design and editing <code>makecform, applycform</code>

F.1 Skin Detection

Skin detection is the process of segmenting skin from its background. It is used for example for face detection in videos; or in medical applications, to distinguish between tumor and skin, in which case the skin is background and the tumor is foreground. There are many algorithms for skin detection, often geared towards a specific objective. For instance for face detection in videos, one requires a fast algorithm. Often, the algorithms involve a transformation from the RGB color space into another color space in which the red tones are (hopefully) isolated or emphasized. However, the advantage of those other color spaces has been occasionally questioned and one may therefore start developing an algorithm using the 'original' RGB space first, in order to see whether it is sufficient. For instance the following fast skin-detection algorithm is based on a set of hard rules in RGB space (`I` is a RGB color image):

```
% --- Minimum Range Condition
Mn      = min(I,[],3);
Mx      = max(I,[],3);
Brg     = (Mx-Mn)>15;

% --- Minimum Value Condition
```

```

R      = I(:,:,1);
G      = I(:,:,2);
B      = I(:,:,3);
Bmn   = R>95 & G>40 & B>20;

% --- Dominant Red Condition
Bc3   = abs(R-G)>15 & R>G & R>B;

% --- All Conditions
S      = Brg & Bmn & Bc3;

```

S is a binary image with on-pixels corresponding to skin.

For tumor/skin discrimination in medical images, the following color information has been used (Vezhnevets et al. 2003):

- The blue channel of the (original) RGB space
- The ratio between the red and the green channel of the RGB space
- The *a* channel of the Lab color space
- The *H* channel of the HSV space

G Python Modules and Functions

Python offers functions for computer vision and image processing in various modules. Some functions exist in multiple modules. Most functions can be found in module `skimage`, which also contains a number of graphics routines. Its documentation can be found at <http://scikit-image.org>. A number of functions can be found in submodule `numpy.ndimage`. For signal processing and clustering we employ `scipy.signal` and `scipy.cluster` respectively.

Table 5: Overview of the `skimage` module, <http://scikit-image.org>

Utility Functions	Comments
<code>img_as_float</code> , <code>img_as_XXX</code> <code>dtype_limits</code>	conversion returns minimum and maximum intensity
Submodule	Functions/Comments
<code>color</code>	all types of conversions, e.g. <code>rgb2gray</code>
<code>data</code>	example images such as <code>coins</code>
<code>feature</code>	detection of blobs: <code>blob_dog</code> , <code>blob_XXX</code> edges: <code>canny</code> corners: <code>corner_harris</code> , <code>corner_XXX</code> maxima: <code>peak_local_max</code>
<code>filters</code>	edge detection (Prewitt, Sobel, ...); threshold determination (<code>threshold_otsu</code>); complex filters
<code>io</code>	input/output, e.g. <code>imread</code>
<code>measure</code>	various functions, e.g. <code>find_contours</code> (iso-contours), <code>regionprops</code> , <code>label</code> , <code>points_in_poly</code> , <code>ransac</code>
<code>morphology</code>	black-white: <code>binary_closing</code> , <code>binary_XXX</code> gray-scale: <code>dilation</code> , <code>erosion</code> , ... extrema: <code>h_minima</code> , <code>h_maxima</code> , <code>local_minima</code> , <code>local_maxima</code> various: <code>label</code> , <code>watershed</code>
<code>segmentation</code>	<code>watershed</code> , <code>quickshift</code> , <code>clear_border</code>
<code>transform</code>	<code>pyramid_reduce</code> , <code>pyramid_expand</code> , <code>resize</code>

Table 6: Overview of the `scipy` module, <https://docs.scipy.org/>.

Submodule	Functions/Comments
<code>cluster</code>	<code>vq.kmeans</code>
<code>constants</code>	Physical and mathematical constants
<code>fftpack</code>	Fast Fourier Transform routines
<code>interpolate</code>	Interpolation and smoothing splines
<code>io</code>	Input and Output
<code>linalg</code>	Linear algebra
<code>ndimage</code>	N-dimensional image processing
<code>signal</code>	<code>convolve</code> , <code>convolve2d</code>
<code>spatial.distance</code>	<code>pdist</code> , <code>squareform</code>
<code>special</code>	Special functions
<code>stats</code>	Statistical distributions and functions

Table 7: Matlab-Python equivalents.

Matlab	Python
<code>padarray(A, [2 2])</code>	<code>numpy.pad(A, 2, 'constant', constant_values=0)</code>

H Coding

H.1 Programming Hints for Matlab

To write fast-running code in Matlab, one should exploit Matlab's matrix manipulating commands in order to avoid the slower `for` loops (see for instance `repmat` or `accumarray`; see also Matlab's documentation on 'vectorization'). Writing a kNN classifier can be conveniently done using the `repmat` command. However, when dealing with high dimensionality and large number of samples, exploiting this command can in fact slow down computation because the machine will spend a significant amount of time allocating the required memory for the large matrices. It may in fact be faster to maintain one for loop, and to use `repmat` only limitedly in that case. Thus there is a speed/memory tradeoff.

H.2 Organizing Your Code

Images and Data Paths Create separate folders for images, data obtained from preprocessing, data from matching images, etc. Create a global structure variable, which contains the path names to those folders, e.g.:

```
FOLD.ImgSatImg = 'C:/CollectionSatelliteImages/' ;
FOLD.ImgPanorama = 'D:/Photos/Panorama/' ;
FOLD.DatSatImg = 'D:/Data/ImgPreProc/SatImg/' ;
```

With the command `dir` you can obtain a list of all images, e.g. `ImgNames = dir(FOLD.ImgSatImg)` and then access the filenames `FileNames(1).name`. The first two entries will contain '' and '..', thus start with `FileNames(3)`. Use `fileparts` to separate the path into its components. Create a script called `init_img`, where you specify such paths and obtain the filenames. You will call that script at the beginning of each of your scripts.

Time-Consuming Preprocessing When you process hundreds of images - or even a few large ones - and you tune an algorithm that operates only on the final steps of the recognition sequence, then you may want to save some data from preprocessing first, and then load those data only, instead of carrying out the time-consuming preprocessing every time. Use `save` and `load` for saving and loading data.

Overviewing Scripts Create a file called `README.SatImg` (or some other appropriate ending), where you list all the essential scripts line by line, with a few keywords on the right-hand side. It will help you maintaining overview of your collection of scripts. From there you can easily access every script by moving the mouse cursor onto the script name and right-clicking your mouse.

I Code Examples

In the following you will find the Matlab code for the example tasks mentioned throughout the script (light green shade). I have started to provide corresponding Python code as well (light blue shade), but be aware that I made only one-on-one 'translations' and one or the other formulation in Python could be written more compact or more elegantly. For simplicity I often import the entire module with *. For Python I recommend to also browse the official tutorial <https://docs.python.org/3/tutorial/>.

I.1 Face Profiles

```
clear;
Iorg    = imread('KlausIohannisCrop.jpg'); % image is color [m n 3]
Ig      = rgb2gray(Iorg);                  % turn into graylevel image
Ig      = Ig(35:end-35,35:end-35);        % crop borders a bit more
[h w]   = size(Ig);                      % image height and width

%% ===== Raw Profiles =====
Pver    = sum(Ig,1);                      % vertical intensity profile
Phor    = sum(Ig,2)';                      % horizontal intensity profile

%% ===== Smoothen Profiles =====
nPf    = round(w*0.05);                   % # points: fraction of image width
LowFlt = pdf('norm', -nPf:nPf, 0, round(nPf/2)); % generate a Gaussian
Pverf   = conv(Pver, LowFlt, 'valid');       % filter vertical profile
Phorf   = conv(Phor, LowFlt, 'valid');        % filter horizontal profile
Pverf   = padarray(Pverf,[0 nPf],'replicate'); % extend to original size
Phorf   = padarray(Phorf,[0 nPf],'replicate'); % extend to original size

%% ===== Detect Extrema =====
[PksVer LocPksVer] = findpeaks(Pverf);      % peaks
[TrgVer LocTrgVer] = findpeaks(-Pverf);       % troughs (sinks)
[PksHor LocPksHor] = findpeaks(Phorf);         % peaks
[TrgHor LocTrgHor] = findpeaks(-Phorf);        % troughs

%% ----- Plotting -----
figure(1);clf;
subplot(1,2,1);
    imagesc(Ig); colormap(gray);
subplot(2,2,2); hold on;
    plot(Pver,'k');
    plot(Pverf,'m');
    plot(LocPksVer,PksVer,'g^');
    plot(LocTrgVer,abs(TrgVer),'rv');
    set(gca,'xlim',[1 w]);
    xlabel('From Left to Right');
    title('Vertical Profile');
subplot(3,2,6); hold on;
    plot(Phor,'k');
    plot(Phorf,'m');
    plot(LocPksHor,PksHor,'g^');
    plot(LocTrgHor,abs(TrgHor),'rv');
    set(gca,'xlim',[1 h]);
    xlabel('From Top to Bottom');
    title('Horizontal Profile');
```

This would be the corresponding Python code:

```
from numpy           import asarray, diff, nonzero, shape, sign, sum
from skimage.io     import imread
from skimage.color   import rgb2gray
from scipy.signal    import convolve, get_window, argrelmax, argrelmin

#%%% ---- Load Image ----
Irgb   = imread('c:/klab/do_lec/compvis/Matlab/KlausIohannisCrop.jpg') # image is color [m n 3]
```

```

Ig      = rgb2gray(Irgb)           # turn into graylevel image
Ig      = Ig[34:-36,34:-36]        # crop borders a bit more
(h,w)   = shape(Ig)              # image height and width

#%%% ===== Raw Profiles =====
Pver    = sum(Ig,axis=0)          # vertical intensity profile
Phor    = sum(Ig,axis=1)          # horizontal#%%% ---- Loading and Analysing

#%%% ===== Smoothen Profiles =====
sgm     = 10
nPf    = round(w*0.10);           # #points: fraction of image width
LowFlt = get_window('gaussian',sgm),nPf) # generate a Gaussian
LowFlt = LowFlt / sum(LowFlt)       # normalize the filter
Pverf  = convolve(Pver, LowFlt, 'same') # filter vertical profile
Phorf  = convolve(Phor, LowFlt, 'same') # filter horizontal profile

#%%% ===== Detect Extrema =====
LocMaxVer = argrelmax(Pverf)      # peaks
LocMinVer = argrelmin(Pverf)      # troughs (sinks)
LocMaxHor = argrelmax(Phorf)       # peaks
LocMinHor = argrelmin(Phorf)       # troughs (sinks)

#%%% ---- Plotting
from matplotlib.pyplot import figure, subplot, imshow, plot, title, xlim, ylim, cm
figure(figsize=(10,6))
subplot(1,2,1)
imshow(Ig, cmap=cm.gray)
subplot(2,2,2)
plot(Pver,'k')
plot(Pverf,'m')
plot(LocMaxVer[0],Pverf[LocMaxVer],'g^')
plot(LocMinVer[0],Pverf[LocMinVer],'rv')
xlim(1,w)
title('Vertical Profile')
subplot(2,2,4)
plot(Phor,'k')
plot(Phorf,'m')
plot(LocMaxHor[0],Phorf[LocMaxHor],'g^')
plot(LocMinHor[0],Phorf[LocMinHor],'rv')
ylim(1,h)
title('Horizontal Profile')

```

I.2 Image Processing I: Scale Space and Pyramid

```

clear;
Icol    = imread('autumn.tif');      % uint8 type; color
Ig      = single(rgb2gray(Icol));    % turn into single type

%% ----- Initialize
nLev    = 5;
[SS PY aFlt] = deal(cell(nLev,1));
SS{1}   = Ig;                      % scale space: make original image first level
PY{1}   = Ig;                      % pyramid:           "       "       "       "       "

%% ===== Scale Space and Pyramid
for i = 1:nLev-1
    Flt    = fspecial('gaussian', [2 2]+i*3, i);    % 2D Gaussian
    aFlt{i} = Flt;                                  % store for plotting
    Ilpf   = conv2(Ig, Flt, 'same');    % low-pass filtered image
    SS{i+1} = Ilpf;
    % --- Downsampling with stp
    stp    = 2^i;
    Idwn   = downsample(Ilpf,stp);    % first along rows
    PY{i+1} = downsample(Idwn',stp)';    % then along columns
end

%% ----- Plotting
figure(1);clf;
[nr nc] = deal(nLev,3);
for i = 1:nLev
    if i<nLev,
        subplot(nr,nc,i*nc-2);
        imagesc(aFlt{i});
    end
    subplot(nr,nc,i*nc-1);
    imagesc(SS{i});
    subplot(nr,nc,i*nc);
    imagesc(PY{i});
end

```

For the Python code, we also wrote a function `fspecialGauss`, which mimics Matlab's `fspecial` function, see separate code block below.

```

from numpy          import shape
from scipy.signal   import convolve2d
from skimage         import data
from skimage.color   import rgb2gray
from skimage.transform import resize
from fspecialGauss import fspecialGauss
#%%% ----- Load & Transform -----
Irgb   = data.chelsea()
Ig     = rgb2gray(Irgb)    # turn into single type
(m,n)  = shape(Ig)

#%%% ----- Initialize -----
nLev    = 5
SS = {}; PY={}; aFlt={}
SS[0]   = Ig;                      # scale space: make original image first level
PY[0]   = Ig;                      # pyramid:           "       "       "       "       "

#%%% ===== Scale Space and Pyramid =====
for i in range(1,nLev):
    Flt    = fspecialGauss(2+i*3, i)          # 2D Gaussian
    aFlt[i-1] = Flt;                          # store for plotting
    Ilpf   = convolve2d(Ig, Flt, mode='same') # low-pass filtered image
    SS[i]   = Ilpf
    # --- Downsampling with stp
    stp    = 2**i
    PY[i]   = resize(Ilpf,(m//stp,n//stp))

```

```

# %% ----- Plotting
from matplotlib.pyplot import *
figure()
(nr,nc) = (nLev,3)
for i in range(0,nLev):
    if i<nLev-1:
        subplot(nr,nc,i*nc+1);
        imshow(aFlt[i]);
        subplot(nr,nc,i*nc+2);
        imshow(SS[i]);
        subplot(nr,nc,i*nc+3);
        imshow(PY[i]);

```

The function `fspecialGauss`:

```

import numpy

def fspecialGauss(size,sigma):
    x, y = numpy.mgrid[-size//2 + 1:size//2 + 1, -size//2 + 1:size//2 + 1]
    g = numpy.exp(-((x**2 + y**2)/(2.0*sigma**2)))
    return g/g.sum()

```

I.3 Feature Extraction I

I.3.1 Regions

```
clear;
Icol    = imread('tissue.png');
I       = single(rgb2gray(Icol));

%% ===== Diff-of-Gaussians (DOG)
Fs1    = fspecial('Gaussian',[11 11], 3); % fine low-pass filter
Fs2    = fspecial('Gaussian',[21 21], 6); % coarse low-pass filter

Is1    = conv2(I, Fs1, 'same');           % convolution with image
Is2    = conv2(I, Fs2, 'same');

Idog   = Is2 - Is1;                     % diff-of-Gaussians
BWblobs = Idog > 15;                  % thresholding for blobs

%% ===== Laplacian-of-Gaussian (LoG)
Flog   = fspecial('log',[15 15],5);
Ilog   = conv2(I, Flog, 'same');
BWlog  = Ilog > .6;

%% ----- Plotting
figure(1);clf;
[nr nc] = deal(3,2);
subplot(nr,nc,1), imagesc(Icol);
subplot(nr,nc,2), imagesc(I);
subplot(nr,nc,3), imagesc(Idog);colorbar;
subplot(nr,nc,4), imagesc(BWblobs);
subplot(nr,nc,5), imagesc(Ilog);colorbar;
subplot(nr,nc,6), imagesc(BWlog);
```

```
from skimage import data
from skimage.color import rgb2gray
from fspecialGauss import *
from scipy.signal import convolve2d
from matplotlib.pyplot import *

#%%% ----- The image -----
Icol    = data.immunohistochemistry()
I       = rgb2gray(Icol);

#%%% ===== Diff-of-Gaussians (DOG) =====
Fs1    = fspecialGauss(11, 3)          # fine low-pass filter
Fs2    = fspecialGauss(21, 6)          # coarse low-pass filter

Is1    = convolve2d(I, Fs1, 'same')     # convolution with image
Is2    = convolve2d(I, Fs2, 'same')

Idog   = Is2 - Is1;                   # diff-of-Gaussians
BWblobs = Idog > 0.02               # thresholding for blobs

#%%% ----- Plotting -----
figure(figsize=(15,15))
nr, nc = 3,2
subplot(nr,nc,1), imshow(Icol)
subplot(nr,nc,2), imshow(I)
subplot(nr,nc,3), imshow(Idog);colorbar
subplot(nr,nc,4), imshow(BWblobs)
```

I.3.2 Edge Detection

```

clear; format compact;
sgm      = 1; % scale, typically 1-5

%% ----- Load an Image
I        = double(imread('cameraman.tif'));

%% ===== Blurring
Nb      = [2 2]+sgm;
Fsc     = fspecial('gaussian', Nb, sgm); % 2D gaussian
Iblr    = conv2(I, Fsc, 'same');

%% ===== Edge Detection
BWrob   = edge(Iblr, 'roberts');
BWsob   = edge(Iblr, 'sobel');
BWpwt   = edge(Iblr, 'prewitt'); % similar to Sobel
BWlog   = edge(Iblr, 'log', [], sgm); % laplacian of Gaussian
BWzex   = edge(Iblr, 'zerocross');
% For Canny we apply to original image I (because it will perform the
% blurring)
BWCny   = edge(I, 'canny', [], sgm);

%% ----- Plotting
figure(1); clf; colormap(gray); [nr nc] = deal(3,2);
subplot(nr,nc,1); imagesc(BWrob); title('Roberts');
subplot(nr,nc,2); imagesc(BWsob); title('Sobel');
subplot(nr,nc,3); imagesc(BWpwt); title('Prewitt');
subplot(nr,nc,4); imagesc(BWlog); title('Log');
subplot(nr,nc,5); imagesc(BWCny); title('Canny');
subplot(nr,nc,6); imagesc(BWzex); title('Zero-Cross');

```

```

from skimage import data
from skimage.feature import canny
from skimage.filters import roberts, sobel, prewitt

%% ----- Load -----
I      = data.camera() # uint8

%% ----- One-scale edge detection -----
Mrob   = roberts(I)
Msob   = sobel(I)
Mpwt   = prewitt(I)

%% ----- Canny for two scales -----
ME1    = canny(I, sigma=1)
ME2    = canny(I, sigma=3)

%% ----- Plotting -----
from matplotlib.pyplot import *
figure(figsize=(12,10)); nr,nc = 2,3;
subplot(nr,nc,1); imshow(I); title('Input Image')
subplot(nr,nc,2); imshow(Mrob); title('Roberts')
subplot(nr,nc,3); imshow(Msob); title('Sobel')
subplot(nr,nc,4); imshow(Mpwt); title('Prewitt')
subplot(nr,nc,5); imshow(ME1); title('Canny $\sigma=1$')
subplot(nr,nc,6); imshow(ME2); title('Canny $\sigma=3$')

```

I.3.3 Edge Following

```

clear;
% ----- Stimulus -----
M          = zeros(15,15); % an empty map
M([5 10],5:10) = 1;      % upper & lower sides of a rectangle
M(5:10,[5 10]) = 1;      % left & right sides of a rectangle
M(3:8, 3)       = 1;      % straight line

%% ===== Labeling,Tracing =====
aBon     = bwboundaries(M,'noholes');
BonR    = bwtraceboundary(M, [5 5], 'N');

%% ----- Plotting -----
figure(1);clf;
imagesc(M); hold on;
nB = length(aBon);
for i = 1:nB
    Bon = aBon{i};
    plot(Bon(:,2), Bon(:,1), 'r');
end
plot(BonR(:,2)+.2, BonR(:,1)+.2,'g');

```

```

from numpy import zeros, arange
from skimage.measure import find_contours
from matplotlib.pyplot import *

#% ----- Stimulus -----
M          = zeros((15,15)) # an empty map
M[[5,10],5:10] = 1         # upper & lower sides of a rectangle
M[5:10,[5,10]] = 1        # left & right sides of a rectangle
M[3:8, 3]       = 1        # straight line

#%%% ===== Labeling,Tracing =====
aBon     = find_contours(M, 0.8) # closer to pixels
aBonE   = find_contours(M, 0.2) # closer to exterior pixels

#%%% ----- Plotting -----
figure
imshow(M, interpolation='nearest')
nB = len(aBon)
for i in arange(0,nB):
    Bon     = aBon[i]
    BonE   = aBonE[i]
    plot(Bon[:,1], Bon[:,0], 'r')
    plot(BonE[:,1], BonE[:,0], 'y')

```

I.3.4 Texture Filters

```
% A filter bank for texture. (The Leung-Malik filter bank)
% see also: http://www.robots.ox.ac.uk/~vgg/research/texclass/filters.html
%
function F = f_GenTxtFilt(figNo)
sz          = 49;           % filter size
Sc1Blb     = sqrt(2).^(1:4); % sigma for blob filters
Sc1Ori     = sqrt(2).^(1:3); % sigma for oriented filters
nSc1Blb    = length(Sc1Blb);
nSc1Ori    = length(Sc1Ori);
nBlb       = 12;            % # of blob filters (first 12 filters)
nOri       = 6;             % # of orientations
nFlt        = nSc1Ori*nOri*2 + nBlb; % # of total filters

%% ----- Init Memory & Points
F          = zeros(sz,sz,nFlt);
rd         = (sz-1)/2;
[X Y]      = meshgrid(-rd:rd, rd:-1:-rd);
PTS        = [X(:) Y(:)]';

%% ----- Blob Filters (first 12 filters)
for i = 1:nSc1Blb
    F(:,:,i) = ff_Norm(fspecial('gaussian', sz, Sc1Blb(i)));
    F(:,:,4*i) = ff_Norm(fspecial('log', sz, Sc1Blb(i)));
    F(:,:,8*i) = ff_Norm(fspecial('log', sz, Sc1Blb(i)*4));
end

%% ----- Edge & Bar Filters (filters 13-48)
cc        = 1;
for s = 1:nSc1Ori,
    for o = 0:nOri-1,
        ang          = pi*a/nOri; % Not 2pi as filters have symmetry
        ca           = cos(ang);
        sa           = sin(ang);
        PTSrot      = [ca -sa; sa ca] * PTS;
        F(:,:,12+cc) = ff_Edg(PTSrot, Sc1Ori(s), sz); % edge
        F(:,:,30+cc) = ff_Bar(PTSrot, Sc1Ori(s), sz); % bar
        cc          = cc+1;
    end
end

%% ----- Plotting
figure(figNo); clf; colormap(gray);
for i = 1:nFlt
    subplot(8,6,i)
    I = F(:,:,i);
    imagesc(I);
    set(gca,'fontsize',4);
    title(num2str(i),'fontsize',6,'fontweight','bold');
end

end % MAIN

%% ===== SUB FUNCTIONS =====
function F = ff_Bar(PTS, sgm, sz)
Gx          = normpdf(PTS(1,:), 0, sgm*3);
Gy          = normpdf(PTS(2,:), 0, sgm);
vnc         = sgm^2;
Gy          = Gy.*((PTS(2,:).^2-vnc)/(vnc^2)); % 2nd derivative
F          = ff_Norm(reshape(Gx.*Gy, sz, sz));
end % SUB

function F = ff_Edg(PTS, sgm, sz)
Gx          = normpdf(PTS(1,:), 0, sgm);
Gy          = normpdf(PTS(2,:), 0, sgm);
Gy          = -Gy.*((PTS(2,:)/(sgm^2))); % 1st derivative
F          = ff_Norm(reshape(Gx.*Gy, sz, sz));
end % SUB

function D = ff_Norm(D)
D          = D - mean(D(:));
D          = D / sum(abs(D(:)));
end % SUB
```

I.4 Loading the MNIST dataset

Note that this function contains two subfunctions, `ff_LoadImg` and `ff_ReadLab`.

```
% Loads the MNIST data - the 4 files on the following website -
% and converts them from ubyte to single.
%      http://yann.lecun.com/exdb/mnist/
% IN   - (no input arguments)
% OUT  TREN   training images column wise [60000 28*28]
%       TEST   testing (sample) images [10000 28*28]
%       Lbl    struct with training and testing class labels as matrices
%              .Tren [60000 10] binary matrix with training labels
%              .Test [60000 10] binary matrix with testing labels
function [TREN TEST Lbl] = LoadMNIST()
fprintf('Loading MNIST...');

filePath = 'c:\kzimg_down\MNIST\';
FnTrainImg = [filePath 'train-images.idx3-ubyte'];
FnTrainLab = [filePath 'train-labels.idx1-ubyte'];
FnTestImg = [filePath 't10k-images.idx3-ubyte'];
FnTestLab = [filePath 't10k-labels.idx1-ubyte'];

TREN = ff_LoadImg(FnTrainImg); % [60000 28*28]
TEST = ff_LoadImg(FnTestImg); % [10000 28*28]
Lbl.Tren = ff_ReadLab(FnTrainLab); % [60000 10]
Lbl.Test = ff_ReadLab(FnTestLab); % [10000 10]

TREN = single(TREN)/255.0;
TEST = single(TEST)/255.0;
fprintf('done. Normalized\n');
end % MAIN FUNCTION

%% ====== Load Digits
function IMGS = ff_LoadImg(imgFile)
fid = fopen(imgFile, 'rb');
idf = fread(fid, 1, '*int32', 0, 'b'); % identifier
nImg = fread(fid, 1, '*int32', 0, 'b');
nRow = fread(fid, 1, '*int32', 0, 'b');
nCol = fread(fid, 1, '*int32', 0, 'b');
IMGS = fread(fid, inf, '*uint8', 0, 'b');
fclose( fid );
assert(idf==2051, '%s is not MNIST image file.', imgFile);
IMGS = reshape(IMGS, [nRow*nCol, nImg]);
for i=1:nImg
    Img = reshape(IMGS(i,:), [nRow nCol]);
    IMGS(i,:) = reshape(Img, [1 nRow*nCol]);
end
end % SUB FUNCTION

%% ====== Load Labels
function Lab = ff_ReadLab(labFile)
fid = fopen(labFile, 'rb');
idf = fread(fid, 1, '*int32', 0, 'b');
nLabs = fread(fid, 1, '*int32', 0, 'b');
ind = fread(fid, inf, '*uint8', 0, 'b');
fclose(fid);
assert(idf==2049, '%s is not MNIST label file.', labFile);
Lab = false(nLabs,10);
ind = ind + 1;
for i=1:nLabs
    Lab(i,ind(i)) = true;
end
end % SUB FUNCTION
```

```

# Loads the MNIST data - the 4 files on the following website -
# and converts them from ubyte to float.
#      http://yann.lecun.com/exdb/mnist/
# IN   -      (no input arguments)
# OUT  TREN   training images as 3-dimensional array [60000 28 28]
#       TEST   testing (sample) images as 3-dim array [10000 28 28]
#       Lbl    struct with training and testing class labels as vectors and
#               matrices:
#               .Tren   [60000 1] vector with training labels
#               .Test   [10000 1] vector with testing labels
#               .TrenMx [60000 10] binary matrix with training labels
#               .TestMx [10000 10] binary matrix with testing labels
def LoadMNIST():
    from numpy import fromfile, int8, uint8
    from collections import namedtuple
    import struct
    from keras.utils import to_categorical

    print('Loading MNIST...')
    filePath    = 'c:/kzimg_down/MNIST/'
    Lbl         = namedtuple('Lbl', ['Tren', 'Test', 'TrenMx', 'TestMx'])
    #%% ----- TRAINING DATA -----
    fipaImg     = filePath + 'train-images.idx3-ubyte'
    fipaLab     = filePath + 'train-labels.idx1-ubyte'

    with open(fipaLab, 'rb') as Flab:
        idf, nLab = struct.unpack(">II", Flab.read(8))
        Lbl.Tren = fromfile(Flab, dtype=int8)

    with open(fipaImg, 'rb') as Fimg:
        idf, nImg, nRow, nCol = struct.unpack(">IIII", Fimg.read(16))
        TREN = fromfile(Fimg, dtype=uint8).reshape(nLab, nRow, nCol)

    #%% ----- TESTING DATA -----
    fipaImg     = filePath + 't10k-images.idx3-ubyte'
    fipaLab     = filePath + 't10k-labels.idx1-ubyte';

    with open(fipaLab, 'rb') as Flab:
        idf, nLab = struct.unpack(">II", Flab.read(8))
        Lbl.Test = fromfile(Flab, dtype=int8)

    with open(fipaImg, 'rb') as Fimg:
        idf, nImg, nRow, nCol = struct.unpack(">IIII", Fimg.read(16))
        TEST = fromfile(Fimg, dtype=uint8).reshape(nLab, nRow, nCol)

    #%% ----- NORMALIZATION -----
    TREN      = TREN.astype('float32')/255.0
    TEST      = TEST.astype('float32')/255.0

    #%% ----- Convert class vectors to binary class matrices -----
    Lbl.TrenMx = to_categorical(Lbl.Tren, 10)
    Lbl.TestMx = to_categorical(Lbl.Test, 10)

    print('done. Normalized\n')
    return TREN, TEST, Lbl

```

I.5 CNN Example for CIFAR10

I.5.1 Loading the CIFAR-10 files

```
% Loads images & category labels for CIF10 database.  
% IN -  
% OUT IMGS      images [60000 32*32*3]  
%       Lbl      category labels [60000 1] E [1..10]  
%       CatNames strings {10 x 1}  
function [IMGS Lbl CatNames] = LoadImgCIF10()  
global PATH  
  
%% ---- Train  
bt1 = [PATH.IMGS 'data_batch_1']; BT1 = load(bt1);  
bt2 = [PATH.IMGS 'data_batch_2']; BT2 = load(bt2);  
bt3 = [PATH.IMGS 'data_batch_3']; BT3 = load(bt3);  
bt4 = [PATH.IMGS 'data_batch_4']; BT4 = load(bt4);  
bt5 = [PATH.IMGS 'data_batch_5']; BT5 = load(bt5);  
  
TREN = [BT1.data; BT2.data; BT3.data; BT4.data; BT5.data];  
LblTren = [BT1.labels; BT2.labels; BT3.labels; BT4.labels; BT5.labels];  
  
%% ---- Test  
btt = [PATH.IMGS 'test_batch']; BTT = load(btt);  
TEST = BTT.data;  
LblTest = BTT.labels;  
  
%% ---- Concat Train & Test  
IMGS = [TREN; TEST];  
Lbl = single([LblTren; LblTest])+1; % add one because labeling E [0..9]  
  
%% ----  
bm = [PATH.IMGS 'batches.meta.mat']; BM = load(bm);  
CatNames = BM.label_names;  
end
```

```
# Loads the CIFAR10 data set. Here we load the Matlab files (!)  
# For the python files see 'http://www.cs.toronto.edu/~kriz/cifar-10-python.tar.gz'  
# IN - (no input arguments)  
# OUT TREN training images as 3-dimensional array [50000 32 32]  
#       TEST testing (sample) images as 3-dim array [10000 32 32]  
#       Lbl struct with training and testing class labels as vectors and  
#             matrices:  
#             .Tren [50000 1] vector with training labels  
#             .Test [10000 1] vector with testing labels  
#             .TrenMx [50000 10] binary matrix with training labels  
#             .TestMx [10000 10] binary matrix with testing labels  
#from __future__ import absolute_import  
from numpy import shape, zeros, reshape, transpose  
import os  
import scipy.io as sio  
#import pickle  
from collections import namedtuple  
from keras.utils import to_categorical  
  
def LoadImgCIF10():  
  
    path = 'c:/kzimg_down/CIFAR10/'  
    Lbl = namedtuple('Lbl', ['Tren', 'Test'])  
  
    #%% ----- TRAINING DATA -----  
    TREN = zeros((50000, 32*32*3), dtype='uint8')  
    Lbl.Tren = zeros((50000, 1), dtype='uint8')  
  
    for i in range(1, 6):  
        fpath = os.path.join(path, 'data_batch_' + str(i))
```

```

DAT      = sio.loadmat(fpath)
TREN[   (i-1)*10000 : i*10000, :]  = DAT['data']
Lbl.Tren[(i-1)*10000 : i*10000]     = DAT['labels']

#%% ----- TESTING DATA -----
fpath    = os.path.join(path, 'test_batch')
DAT      = sio.loadmat(fpath)

TEST     = DAT['data']
Lbl.Test = DAT['labels']

#%% ----- Labels -----
fpath    = os.path.join(path, 'batches.meta')
DAT      = sio.loadmat(fpath)
LbStr   = DAT['label_names']

#%% ----- NORMALIZATION -----
TREN     = TREN.astype('float32')/255.0
TEST     = TEST.astype('float32')/255.0

#%% ----- Reshape -----
TREN     = reshape(TREN,(50000,3,32,32))
TREN     = TREN.transpose(0, 2, 3, 1)
TEST     = reshape(TEST,(10000,3,32,32))
TEST     = TEST.transpose(0, 2, 3, 1)

#%% ----- Convert class vectors to binary class matrices -----
Lbl.TrenMx = to_categorical(Lbl.Tren, 10)
Lbl.TestMx = to_categorical(Lbl.Test, 10)

return TREN, TEST, Lbl, LbStr

```

I.5.2 Example of a Network for the CIFAR-10 set

```

# Train a simple CNN on the CIFAR10 (32x32 pixels, 10 classes)
from __future__ import print_function
import keras
from keras.models import Sequential
from keras.layers import Dense, Dropout, Activation, Flatten
from keras.layers import Conv2D, MaxPooling2D
from LoadImgCIF10 import *
batchSz   = 32
nEpoch   = 20

#%% ----- Load Database & Labels -----
Isz       = (32, 32, 3)
(TREN, TEST, Lbl, LbCat) = LoadImgCIF10()

#%% ----- Build Network      -----
N = Sequential()

N.add(Conv2D(32, (3,3), padding='same', input_shape=Isz))
N.add(Activation('relu'))
N.add(Conv2D(32, (3,3)))
N.add(Activation('relu'))
N.add(MaxPooling2D(pool_size=(2,2)))
N.add(Dropout(0.25))

N.add(Conv2D(64, (3,3), padding='same'))
N.add(Activation('relu'))
N.add(Conv2D(64, (3,3)))
N.add(Activation('relu'))
N.add(MaxPooling2D(pool_size=(2,2)))
N.add(Dropout(0.25))

```

```

N.add(Flatten())
N.add(Dense(512))
N.add(Activation('relu'))
N.add(Dropout(0.5))
N.add(Dense(10))
N.add(Activation('softmax'))

fOptim = keras.optimizers.rmsprop(lr=0.0001, decay=1e-6)
N.compile(loss      = 'categorical_crossentropy',
           optimizer = fOptim,
           metrics   = ['accuracy'])

#%% ===== Learning =====
N.fit(TREN, Lbl.TrenMx,
       batch_size = batchSz,
       epochs     = nEpoch,
       validation_data = (TEST, Lbl.TestMx),
       shuffle    = True)

#%% ===== Evaluation =====
score = N.evaluate(TEST, Lbl.TestMx, verbose=0)
print('Model Accuracy = %.2f' % (score[1]))

```

I.5.3 Example of Transfer Learning

As introduced in Section 5.2. The string variable `dirImg`s is the directory to your database, with two folders: 'train' and 'valid' for training and validation. In each one of those two folders, there is the exact same list of folder names representing the category labels. For those in turn, we provide different images. For small data sets place two thirds of the images into the corresponding training folder (ie. `train/dog/`) and one third into the corresponding validation folder (`valid/dog/`). To obtain optimal results, one would calculate the (normalized) mean and standard deviation of the entire dataset - here we take arbitrary values.

```

from numpy import zeros, array

import torch
from torchvision import models, transforms
from torchvision.datasets import ImageFolder
from torch.utils.data import DataLoader
from torch.optim import SGD, lr_scheduler
import torch.nn as nn

import time
from os.path import join
import copy

dirImg      = 'C:/ImgCollection/'
# computed mean values for all images of entire database
ImgMean     = [0.50, 0.50, 0.50]      # we assume them to be 0.5
ImgStdv     = [0.22, 0.22, 0.22]      # we assume them to be 0.22
szImgTarg   = 224

#%% ===== Parameters =====
nEpo        = 6                      # number of epochs (learning steps)
lernRate    = 0.001                  # learning rate
momFact     = 0.9                    # momentum factor
szStep      = 7                      # step size
gam         = 0.1                    # gamma
szBtch      = 4                      # number of images per batch
# --- the model
MOD         = models.resnet18(pretrained=True)

#% ##### DATA PREPARATION #####
#% ----- Augmentation & Normalization -----
AUGNtrain   = transforms.Compose([

```

```

        transforms.RandomResizedCrop(szImgTarg),
        transforms.RandomHorizontalFlip(),
        transforms.ToTensor(),
        transforms.Normalize(ImgMean, ImgStdv) ])
AUGNvalid = transforms.Compose([
    transforms.Resize(256),
    transforms.CenterCrop(szImgTarg),
    transforms.ToTensor(),
    transforms.Normalize(ImgMean, ImgStdv) ])
# %% ----- Folder and Loader -----
FOLDStrain = ImageFolder(join(dirImgs,'train'), AUGNtrain)
FOLDSvalid = ImageFolder(join(dirImgs,'valid'), AUGNvalid)
LOADERtrain = DataLoader(FOLDStrain, batch_size=szBtch, shuffle=True)
LOADERvalid = DataLoader(FOLDSvalid, batch_size=szBtch, shuffle=True)

nImgTrain = len(FOLDStrain)      # total number of images for training set
nImgValid = len(FOLDSvalid)      # total number of images for validation set

aLbClass = FOLDStrain.classes    # class labels
nClss = len(aLbClass)

# %% ##### Select Device (GPU | CPU) #####
device = torch.device("cuda:0" if torch.cuda.is_available() else "cpu")

# %% ##### PREPARE MODEL #####
# freezing layers until last one (to avoid backward computations)
for param in MOD.parameters():
    param.requires_grad = False
# replaces last fully-connected layer with new random weights:
MOD.fc = nn.Linear(MOD.fc.in_features, nClss)

MOD = MOD.to(device)

# only parameters of final layer are being optimized
optim = SGD(MOD.fc.parameters(), lr=lernRate, momentum=momFact)

# Decrease learning rate every szStep epochs by a factor of gamma
sched = lr_scheduler.StepLR(optim, step_size=szStep, gamma=gam)

crit = nn.CrossEntropyLoss()

# %% ##### TRAIN AND EVALUATE #####
t0 = time.time()
WgtsBest = copy.deepcopy(MOD.state_dict())
accuBest = 0.0
PrfVal, PrfTrn = zeros((nEpo,2)), zeros((nEpo,2))
# SSSSSSSSSSSSSSSS      LOOP EPOCHS      SSSSSSSSSSSSSSSSSSSSS
for i in range(nEpo):
    print(f'---- epoch {i}/{nEpo} ----')
    # ===== TRAIN =====
    sched.step()
    MOD.train()           # set model to training mode
    # ===== LOOP Batches =====
    lossRun = 0.0
    cCrrRun = 0
    for ImBtc, LbBtc in LOADERtrain:
        ImBtc = ImBtc.to(device)          # [nImgBtch 3RGB height width]
        LbBtc = LbBtc.to(device)          # [nImgBtch]
        with torch.set_grad_enabled(True):
            # forward
            Post = MOD(ImBtc)           # posteriors [nImgBtch nClasses]
            _,LbPred = Post.max(1)       # predicted labels
            loss = crit(Post, LbBtc)
            # backward + optimize
            optim.zero_grad()           # zero the parameter gradients
            loss.backward()
            optim.step()

```

```

# statistics
lossRun += loss.item() * szBtch
cCrrRun += torch.sum(LbPred==LbBtc.data)

# --- performance per epoch
lossEpo = lossRun / nImgTrain
accuEpo = cCrrRun.double() / nImgTrain
print(f'train loss: {lossEpo:.4f} acc: {accuEpo:.4f}')
PrfTrn[i,:] = [accuEpo, lossEpo] # record performance

# ===== VALIDATE =====
MOD.eval() # set model to evaluate mode
# ===== LOOP Batches =====
lossRun = 0.0
cCrrRun = 0
for ImBtc, LbBtc in LOADERvalid:
    ImBtc = ImBtc.to(device) # [nImgBtch 3RGB height width]
    LbBtc = LbBtc.to(device) # [nImgBtch]
    with torch.set_grad_enabled(False):
        # forward
        Post = MOD(ImBtc) # posteriors [nImgBtch nClasses]
        _,LbPred = Post.max(1) # predicted labels
        loss = crit(Post, LbBtc)

    # statistics
    lossRun += loss.item() * szBtch
    cCrrRun += torch.sum(LbPred==LbBtc.data)

# --- performance per epoch
lossEpo = lossRun / nImgValid
accuEpo = cCrrRun.double() / nImgValid
print(f'valid loss: {lossEpo:.4f} acc: {accuEpo:.4f}')
PrfVal[i,:] = [accuEpo, lossEpo] # record performance
# keep weights if better than previous weights
if accuEpo>accuBest:
    accuBest = accuEpo
    WgtsBest = copy.deepcopy(MOD.state_dict())

# ---- Concluding
tElaps = time.time() - t0
print('Training duration {:.0f}min {:.0f}sec'.format(tElaps // 60, tElaps % 60))
print(f'Max accuracy {accuBest:.4f}')

MOD.load_state_dict(WgtsBest) # loads best performing weights for application

# %% ---- Plot Learning Curves -----
import matplotlib.pyplot as plt
from matplotlib.pyplot import *
figure
plot(PrfVal[:,0], 'b')
plot(PrfTrn[:,0], 'b')
plot(PrfVal[:,1], 'r')
plot(PrfTrn[:,1], 'r')
gca().legend(('valid acc', 'train acc', 'valid err', 'train err'))

```

I.6 Image Processing II: Segmentation

```

clear;
I = imread('coins.png');

%% ----- Thresholding with Otsu and Median
tOts = graythresh(I);
BWots = im2bw(I,tOts);
I = single(I); % the following computation is simpler in single
tMed = median(I(:))/max(I(:));
BWmed = im2bw(uint8(I),tMed);

%% ----- K-means
Ix = kmeans(I(:,2));
BWkmen = false(size(I));
BWkmen(Ix==2) = true;

%% ----- Watershed
Ilpf = conv2(I,fspecial('Gaussian',[5 5],2));
W = watershed(Ilpf);

%% ----- Plotting
figure(1);clf;[nr nc] = deal(3,2);
subplot(nr,nc,1);
    imagesc(I); colormap(gray);
subplot(nr,nc,2);
    bar(histc(I(:,[0:255])); hold on;
    tOts = tOts*256;
    plot([tOts tOts],[0 1000],'b:');
    title('Histogram');
subplot(nr,nc,3);
    imagesc(BWots); title('Otsu');
subplot(nr,nc,4);
    imagesc(BWmed); title('Median Value');
subplot(nr,nc,5);
    imagesc(BWkmen); title('K-means');
subplot(nr,nc,6);
    imagesc(W); title('Watershed');

```

```

from numpy import array, arange, zeros, ones, kron, shape, median
from scipy.signal import convolve2d
from scipy.cluster.vq import kmeans
from scipy.ndimage import label
from skimage import data
from skimage.feature import peak_local_max
from skimage.filters import threshold_otsu
from skimage.segmentation import watershed
from fspecialGauss import fspecialGauss
# %% ----- Load -----
I = data.coins() # uint8
(m,n) = shape(I)
nPix = m*n
# %% ----- Thresholding with Otsu and Median
tOts = threshold_otsu(I) # E [0 255] uint8
BWots = I > tOts
tMed = median(I) # E [0 255] uint8
BWmed = I > tMed

# %% ----- k-Means
Ivec = array(I.flatten(order='C'), dtype=float) # as a vector
Kls = kmeans(Ivec,2) # group centers
Cen = Kls[0] # first entry contains centers
# Distances & nearest neibors
DIS = zeros((nPix,2),float)
for i in range(0,2):
    DIS[:,i] = (kron(ones(nPix), Cen[i]) - Ivec) ** 2 # squared distance

```

```

IxNN      = DIS.argmin(axis=1)          # nearest neighbor
BWkmen   = IxNN.reshape((m,n))         # reshape to image for plotting

# %% ----- Watershed
Ilpf     = convolve2d(I, fspecialGauss(5,2), mode='same')
Mmx     = peak_local_max(Ilpf, labels=I, footprint=ones((3,3)), indices=False)
Mmrk    = label(Mmx)[0]
W       = watershed(Ilpf, Mmrk, mask=I);

# %% ----- Plotting
from matplotlib.pyplot import *
figure(figsize=(12,9));(nr,nc)=(3,2)
# image in upper left
subplot(nr,nc,1)
imshow(I, cmap=cm.gray);
# histogram in upper right
subplot(nr,nc,2)
hist(I.flatten(),arange(0,256))
plot([t0ts,t0ts],[0,1000], 'b:')
plot([tMed,tMed],[0,1000], 'r:')
title('Histogram')
# output of otsu
subplot(nr,nc,3)
imshow(BWots); title('Otsu')
# output of median
subplot(nr,nc,4)
imshow(BWmed); title('Median Value')
# output of kmeans
subplot(nr,nc,5)
imshow(BWkmen); title('K-means')
# output of watershed
subplot(nr,nc,6)
imshow(W); title('Watershed')

```

I.7 Shape

I.7.1 Generating Shapes

```
clear;
I      = ones(150,760)*255;          % empty image
sz     = 28;
Ix     = 65:90;                  % indices for A to Z
nShp   = length(Ix);
figure(1);clf;
imagesc(I); hold on; axis off;
for i = 1:nShp
    ix = Ix(i);
    text(i*sz,20,char(ix),'fontweight','bold');
    text(i*sz,50,char(ix),'fontweight','bold','fontsize',12);
    text(i*sz,110,char(ix),'fontweight','bold','fontsize',12,'rotation',45);
    text(i*sz,80,char(ix),'fontweight','bold','fontsize',14);
    text(i*sz,140,char(ix),'fontweight','bold','fontsize',16);
end
print('ShapeLetters',' -djpeg ',' -r300');
```

```
from numpy import ones, arange
from matplotlib.pyplot import *

I      = ones((150,760))          # empty image
I[0,0] = 0                      # to ensure that the colormap uses its range
sz     = 28;
Ix     = arange(65,91)           # indices for A to Z
nShp   = len(Ix)
figure(figsize=(12,5))
imshow(I,cmap=cm.gray); axis('off')
xoff   = 8
for i in arange(0,nShp):
    ix = Ix[i]
    text(i*sz+xoff,20, chr(ix),fontweight='bold')
    text(i*sz+xoff,50, chr(ix),fontweight='bold',fontsize=12)
    text(i*sz+xoff,110,chr(ix),fontweight='bold',fontsize=12,rotation=45)
    text(i*sz+xoff,80, chr(ix),fontweight='bold',fontsize=14)
    text(i*sz+xoff,140,chr(ix),fontweight='bold',fontsize=16)

savefig('ShapeLettersPy.jpeg', facecolor='w', bbox_inches='tight')
```

I.7.2 Simple Measures

```
clear;
BW     = rgb2gray(imread('ShapeLetters.jpg')) < 80;
%% ===== Shape Properties
RG     = regionprops(BW, 'all');
Ara   = cat(1,RG.Area);
Ecc   = cat(1,RG.Eccentricity);
EqD   = cat(1,RG.EquivDiameter);
BBx   = cat(1,RG.BoundingBox);
Vec   = [Ara Ecc EqD];           % [nShp 3] three dimensions
nShp  = length(Ara);
fprintf('# Shapes %d\n', nShp);

%% ===== Pairwise Distance Measurements
DM     = squareform(pdist(Vec)); % distance matrix [nShp nShp]
DM(diag(true(nShp,1))) = nan; % inactivate own shape
[D0 O] = sort(DM,2,'ascend'); % sort along rows

%% ----- Plotting First nSim Similar Shapes for Each Found Shape
```

```

% Bounding box
UL      = floor(BBx(:,1:2));    % upper left corner
Wth     = BBx(:,3);           % width
Hgt     = BBx(:,4);           % height
mxWth   = max(Wth)+2;
mxHgt   = max(Hgt)+2;
nSim    = 20;                 % # similar ones we plot
nShp2   = ceil(nShp/2);
[ID1 ID2] = deal(zeros(nShp2*mxWth,nSim*mxHgt));
for i = 1:nShp

    % --- given/selected shape in 1st row
    Row     = (1:Hgt(i))+UL(i,2);    % rows
    Col     = (1:Wth(i))+UL(i,1);    % columns
    Sbw     = BW(Row,Col);
    Szs     = size(Sbw);
    if i<=nShp2, ID1((1:Szs(1))+(i-1)*mxHgt, 1:Szs(2)) = Sbw*2;
    else          ID2((1:Szs(1))+(i-nShp2)*mxHgt,1:Szs(2)) = Sbw*2;
    end

    % --- similar shapes in rows 2nd to nShp+1
    for k = 1:nSim
        ix     = 0(i,k);
        Row     = (1:Hgt(ix))+UL(ix,2);    % rows
        Col     = (1:Wth(ix))+UL(ix,1);    % columns
        Sbw     = BW(Row,Col);
        Szs     = size(Sbw);
        if i<=nShp2, ID1((1:Szs(1))+(i-1)*mxHgt, (1:Szs(2))+k*mxWth) = Sbw;
        else          ID2((1:Szs(1))+(i-nShp2)*mxHgt,(1:Szs(2))+k*mxWth) = Sbw;
        end
    end
end

figure(1);clf;
subplot(1,2,1); imagesc(ID1); title('First Half of Letters');
subplot(1,2,2); imagesc(ID2); title('Second Half of Letters');

```

```

from numpy import asarray, asmatrix, concatenate, nan, diagflat, floor, ceil
from numpy import arange, zeros, ones, shape, ix_, sort, argsort
from skimage.io          import imread
from skimage.color         import rgb2gray
from skimage.measure       import label, regionprops
from scipy.spatial.distance import pdist, squareform

BW      = rgb2gray(imread('ShapeLettersPy.jpeg')) < 0.3
BW      = BW.astype(int)

#%%% ===== Shape Properties =====
LB      = label(BW);
RG      = regionprops(LB)
nShp   = len(RG)
print('# Shapes', nShp)

#%%% ===== Create Attribute Vectors =====
Ara    = asmatrix([r.area for r in RG])
Ecc    = asmatrix([r.eccentricity for r in RG])
EqD    = asmatrix([r.equivalent_diameter for r in RG])
Vec    = concatenate((Ara,Ecc,EqD)).conj().T  # [nShp 3] three dimensions

#%%% ===== Pairwise Distance Measurements
DM     = squareform(pdist(Vec))                # distance matrix [nShp nShp]
DM[diagflat(ones((1,nShp),dtype=bool))] = nan  # inactivate own shape
DO     = sort(DM,axis=1)                         # sort along rows
O      = argsort(DM,axis=1)                      # obtain indices separately

#%%% ----- Plotting First nSim Similar Shapes for Each Found Shape
BBx    = asarray([r.bbox for r in RG])          # Bounding box

```

```

UL      = floor(BBx[:,0:2])           # upper left corner
#LR     = floor(BBx[:,2:4])           # lower rite corner
Hgt    = BBx[:,2]-BBx[:,0]           # width
Wth    = BBx[:,3]-BBx[:,1]           # height
mxHgt  = max(Hgt)+2
mxWth  = max(Wth)+2
nSim   = 20                         # similar ones we plot
nShp2  = int(ceil(nShp/2))
ID1    = zeros((nShp2*mxHgt,nSim*mxWth))
ID2    = ID1.copy()
for i in arange(0,nShp):
    # --- given/selected shape in 1st row
    Row   = arange(0,Hgt[i])+UL[i,0]-1    # rows
    Col   = arange(0,Wth[i])+UL[i,1]-1    # columns
    Sbw   = BW[ix_(Row.astype(int), Col.astype(int))]
    Szs   = shape(Sbw)
    if i < nShp2:
        ID1[ix_(arange(0,Szs[0])+i*mxHgt,       arange(0,Szs[1]))] = Sbw*2
    else:
        ID2[ix_(arange(0,Szs[0])+(i-nShp2)*mxHgt,arange(0,Szs[1]))] = Sbw*2

    # --- similar shapes in rows 2nd to nShp+1
    for k in range(0,nSim-1):
        ix   = 0[i,k]
        Row  = arange(0,Hgt[ix])+UL[ix,0]-1    # rows
        Col  = arange(0,Wth[ix])+UL[ix,1]-1    # columns
        Sbw  = BW[ix_(Row.astype(int), Col.astype(int))]
        Szs  = shape(Sbw)
        if i < nShp2:
            ID1[ix_(arange(0,Szs[0])+i*mxHgt,       arange(0,Szs[1])+(k+1)*mxWth)] = Sbw
        else:
            ID2[ix_(arange(0,Szs[0])+(i-nShp2)*mxHgt,arange(0,Szs[1])+(k+1)*mxWth)] = Sbw

from matplotlib.pyplot import *
figure(figsize=(20,20))
subplot(1,2,1); imshow(ID1); title('First Half of Letters')
subplot(1,2,2); imshow(ID2); title('Second Half of Letters')

```

I.7.3 Shape: Radial Signature

```

clear;
BW      = imread('text.png');          % the stimulus
aBonImg = bwboundaries(BW);
nShp   = length(aBonImg);
fprintf('# Shapes %d\n', nShp);

%% ===== Shape Properties
Vec     = zeros(nShp,4);
aBon   = cell(nShp,1);
for i = 1:nShp
    Bon   = aBonImg{i};
    nPix  = length(Bon);
    cenPt = mean(Bon,1);
    Rsig  = sqrt(sum(bsxfun(@minus,Bon,cenPt).^2,2));
    Fdsc  = abs(fft(Rsig));           % fast Fourier
    FDn   = Fdsc(2:end)/Fdsc(1);     % normalization by 1st FD
    fprintf('%3d #pix %2d FFD %2d\n', i, nPix, length(FDn));
    Vec(i,:) = FDn(1:4);
    aBon{i} = bsxfun(@minus,Bon,cenPt-[10 10]); % move into [1..20 1..20]
    if 0
        figure(1); imagesc(zeros(20,20)); hold on;
        plot(aBon{i}(:,2), aBon{i}(:,1));
        pause();
    end
end

```

```

clear aBonImg

%% ===== Pairwise Distance Measurements
DM      = squareform(pdist(Vec)); % distance matrix [nShp nShp]
DM(diag(true(nShp,1))) = nan; % inactivate own shape
[DO O] = sort(DM,2,'ascend'); % sort along rows

%% ----- Plotting First nSim Similar Shapes for Each Found Shape
sz      = 19;
nSim    = 20; % # similar ones we plot
nShp2   = ceil(nShp/2);
[ID1 ID2] = deal(zeros(nShp2*sz,nSim*sz));
figure(1);clf;
subplot(1,2,1); imagesc(ID1); title('First Half of Letters'); hold on;
subplot(1,2,2); imagesc(ID2); title('Second Half of Letters'); hold on;
for i = 1:nShp

    % --- given/selected shape in 1st row
    Bon     = aBon{i};
    if i<=60,
        subplot(1,2,1);
        plot(Bon(:,2), Bon(:,1)+(i-1)*sz,'b');
    else
        subplot(1,2,2);
        plot(Bon(:,2), Bon(:,1)+(i-61)*sz,'b');
    end

    % --- similar shapes in rows 2nd to nShp+1
    for k = 1:nSim
        ix      = O(i,k);
        Bon     = aBon{ix};
        if i<=60
            subplot(1,2,1);
            plot(Bon(:,2)+k*sz,Bon(:,1)+(i-1)*sz,'k');

            % ID1((1:Szs(1))+(i-1)*mxHgt, (1:Szs(2))+k*mxWth) = Sbw;
        else
            subplot(1,2,2);
            plot(Bon(:,2)+k*sz,Bon(:,1)+(i-61)*sz,'k');
            %ID2((1:Szs(1))+(i-45)*mxHgt,(1:Szs(2))+k*mxWth) = Sbw;
        end
    end
end

```

I.8 Tracking

To play the movie for illustration, uncomment the line starting with '`movie(Moc,...)`' (line 14 approximately).

```

clear;
sR = 7; % radius of search region (block matching)
wR = sR*2+1; % diameter of search region
%% Load Movie
ObjVid = VideoReader('xylophone.mpg'); % movie 'handler'
FRAMES = read(ObjVid); % actual movie data
[nRow nCol dmy nFrm] = size(FRAMES); % rows/columns/colors/frames
MOC(1:nFrm) = struct('cdata', zeros(nRow, nCol, 3, 'uint8'), 'colormap', []);
MOG = zeros(nRow,nCol,nFrm,'uint8'); % movie in grayscale
for k = 1:nFrm
    MOC(k).cdata = FRAMES(:,:,:,:k); % movie in color
    MOG(:,:,:k) = rgb2gray(MOC(k).cdata); % movie in grayscale
end
% movie(Moc, 1, ObjVid.FrameRate); % plays movie
%% ===== LOOPING FRAMES
b_pause = 1; figure(1); clf;
for i = 2:nFrm

    %% ----- Difference Frames
    Fprv = MOG(:,:,i-1); % previous frame
    Fnow = MOG(:,:,i); % current frame
    FDf = abs(Fprv-Fnow); % difference image [nrow ncol]
    FDorig = FDf; % keep original for plotting
    FDf(FDf<20) = 0; % threshold to eliminate 'noise'
    %% ----- Localize patches that change
    [LF nMot] = bwlabel(FDf); % find connected components
    [IXPCH Sz] = deal({}, []); % pixel indices and sizes
    for k = 1:nMot % loop thru detected motion patches
        bR = LF==k;
        IXPCH{k} = find(bR); % pixel indices
        Sz(k) = nnz(bR); % patch size
    end
    fprintf('Frm %d # of motions %3d,\t patch sizes %2d-%4d,\t ', i, nMot, min(Sz), max(Sz));
    %% ----- Select Large Patches/Changes
    [Sz Ixs] = sort(Sz, 'descend'); % sort patch sizes
    bLrg = Sz>10; % detect large ones
    nLrg = nnz(bLrg); % # of large ones
    IXPCH = IXPCH(Ixs(bLrg)); % reduce to detected large changes
    fprintf('reduced to %d large patches\n', nLrg);
    %% ----- Bounding Box of patches
    PchBB = []; cP = 0;
    for k = 1:nLrg
        IxPix = IXPCH{k};
        [Row Col] = ind2sub([nRow nCol], IxPix);
        [leb rib] = deal(min(Col), max(Col)); % left and right boundary
        [upb lob] = deal(min(Row), max(Row)); % upper and lower boundary
        [leb rib] = deal(max(leb,sR+1), min(rib,nCol-sR-1));
        [upb lob] = deal(max(upb,sR+1), min(lob,nRow-sR-1));
        % --- check width and height (maybe 0 or even negative)
        width = rib-leb;
        height = lob-upb;
        if width<1 || height<1, continue; end % if any 0, then move on
        % --- patch bounding box (coordinates)
        cP = cP+1;
        PchBB(cP,:) = [upb lob leb rib]; % upper/lower/left/right
    end
    %% ----- Match each motion patch with its neighborhood
    MchBB = [];
    for k = 1:cP
        Co = PchBB(k,:); % coordinates [1 4]
        rRow = Co(1):Co(2); % range rows
        rCol = Co(3):Co(4); % range cols
        Pprv = Fprv(rRow,rCol); % patch in previous frame
        ps = size(Pprv);

```

```

% --- correlation with neiboring patches
CorrPtch = zeros(wR,wR);
for m = -sR:1:sR
    for n = -sR:1:sR
        Pnow = Fnow(rRow+m, rCol+n);
        CorrPtch(m+sR+1,n+sR+1) = corr2(Pprv,Pnow);
    end
end
CorrPtch(sR+1,sR+1) = 0; % set own match to 0
% --- selection
[vl ix] = max(CorrPtch(:)); % select highest correlation
[rr cc] = ind2sub([wR wR], ix); % linear index to subindices
MchBB(k,1:2) = Co(1:2)+rr-sR-1; % store bounding box of best match
MchBB(k,3:4) = Co(3:4)+cc-sR-1;
if 1
    figure(10); clf
    imagesc(CorrPtch); colormap(gray); hold on;
    plot(cc,rr,'*')
    pause();
end
%% ----- Plotting
MotPresFrm2(i) = sum(FDf(:));
if b_pause,
    figure(1); [rr cc] = deal(2,2);
    subplot(rr,cc,1), imagesc(Fprv); colormap(gray); title(['Frame ' num2str(i)]);
    subplot(rr,cc,2), imagesc(255-FDforig); title('Difference Image');
    for k = 1:cP
        Ix = PchBB(k,:); Lo1 = [Ix(3) Ix(1) Ix(4)-Ix(3) Ix(2)-Ix(1)];
        Ix = MchBB(k,:); Lo2 = [Ix(3) Ix(1) Ix(4)-Ix(3) Ix(2)-Ix(1)];
        rectangle('position', Lo1, 'edgecolor', 'b');
        rectangle('position', Lo2, 'edgecolor', 'r');
    end
    subplot(rr,cc,3), imagesc(255-FDf); title('Thresholded Diff Img');
    subplot(4,2,6), hist(FDforig(:),1:255); title('Histogram of Differences');
    subplot(4,2,8), hist(Sz,1:10:1000); title('Histogram of Patch Sizes');
    pause();
end
end

```

I.9 2D Transformations

```
% Examples of 2D transforms and their motion estimation. See pdf 36, 312.
clear;
%% Simple Pattern
Co = [0 0; 0 .8; 0.6 0.75; 1.2 .8; 1.0 0.9; 1.2 0; 0 0]*4+2; % distorted rectangle
np = size(Co,1); % # of points
cpt = mean(Co,1); % center point
Cpt = repmat(cpt,np,1); % replicated center point
%% ===== Transformations =====
r = 0.25; % rotation/scale
tx = 0.5; % x-translation
ty = 1.5; % y-translation
a = 0.25; % scale (for similarity)
b = 0.25; % rotation (for similarity)
[a00 a01 a10 a11] = deal(-0.5, 0.25, 0.125, -0.75); % for affinity
% ----- Transformation Matrices
TRot = [cos(r) -sin(r); sin(r) cos(r)]; % clockwise rotation
TEuc = [TRot [tx ty]'; % Euclidean (rot&trans) aka rigid body motion
TEuc0 = [TEuc; [0 0 1]]; % with row extended
TShear = [1 0; 0.5 1]; % shear
TScal = [r 0; 0 r]; % scaling
TSim = [1+a -b tx; b 1+a ty]; % similarity
TSim0 = [TSim; 0 0 1]; % with row extended
TAff = [1+a00 a01 tx; a10 1+a11 ty; 0 0 1]; % affinity (with row extended)
% ----- perform transformations
% transf. mx are transposed to match column-wise coordinates:
Coc = Co-Cpt; % subtract centpt: coordinates 0,0 centered
% --- rot, shear, scale:
Crot = Coc * TRot' + Cpt; % rot transformation and add centpt
Csh = Coc * TShear' + Cpt; % shear
Csc = Coc * TScal' + Cpt; % scale
% For aff/euc/sim: add row of 1s, then transform
Caff = [Coc ones(np,1)] * TAff'; % affinity " here we add a col of 1s
Ceuc = [Coc ones(np,1)] * TEuc0'; % Euclidean " here we add a col of 1s
Csim = [Coc ones(np,1)] * TSim0'; % similarity " here we add a col of 1s
Caff(:,[1 2]) = Caff(:,[1 2])+Cpt; % THEN add centpt
Ceuc(:,[1 2]) = Ceuc(:,[1 2])+Cpt;
Csim(:,[1 2]) = Csimg(:,[1 2])+Cpt;
%% ===== Plotting =====
figure(1); clf; [rr cc] = deal(1,1); hold on;
plot(Co(:,1), Co(:,2), 'color', 'k', 'linewidth', 2);
plot(Crot(:,1), Crot(:,2), 'color', 'g');
plot(Csh(:,1), Csh(:,2), 'color', 'r');
plot(Csc(:,1), Csc(:,2), 'color', 'b');
plot(Caff(:,1), Caff(:,2), 'color', 'm');
plot(Ceuc(:,1), Ceuc(:,2), 'color', 'c');
plot(Csim(:,1), Csim(:,2), 'color', ones(3,1)*.5);
axis equal
legend('original', 'rotation', 'shear', 'scaling', 'affine', 'rigid', 'similar',...
'location', 'northwest');
set(gca, 'xlim', [0 8], 'ylim', [0 8]);
%% ===== Estimating Motion =====
%% Building A and b
JSim = inline('[1 0 x -y; 0 1 y x]');
JAff = inline('[1 0 x y 0 0; 0 1 0 0 x y]');
% JEuc = inline('[1 0 -sin(r)*x -cos(r)*y; 0 1 -cos(r)*x -sin(r)*y]');
Asim = zeros(4,4); bsim = 0; % init A and b for similarity case
Aaff = zeros(6,6); baaff = 0; % init A and b for affinity case
DtSim = Csimg(:,[1 2])-Coc; % delta (transformed-original 0,0 centered)
DtAff = Caff(:,[1 2])-Coc; % delta (transformed-original 0,0 centered)
for i = 1 : np
    pt = Coc(i,:); % original 0,0 centered
    Jp = JSim(pt(1),pt(2));
    Asim = Asim + Jp'*Jp;
    bsim = bsim + DtSim(i,:)*Jp;
    Jp = JAff(pt(1),pt(2));
    Aaff = Aaff + Jp'*Jp;
    baaff = baaff + DtAff(i,:)*Jp;
end
%% Least-Square for A and b for Similarity
disp('Similarity');
[Prm1 resm1] = lsqnonneg(Asim, bsim); % Prm1(3:4) contain estimates for a and b
[Prm2 resm2] = lsqlin(Asim, bsim);
Prm1;
Prm2;
(Prm1(1:2)-cpt'); % translation parameters (tx, ty)
%% Least-Square for A and b for Affinity
disp('Affinity');
[Prm resm] = lsqlin(Aaff, baaff); % Prm(3:6) contain estimates for a00, a01, a10, a11
Prm';
(Prm(1:2)-cpt') % tx and ty
```

I.10 RanSAC

```
% Random Sampling Consensus for affine transformation.
% No correspondence determined - assumes list entries correspond already.
% IN: - Pt1 list of original points [np1,2]
%      - Pt2 list of transformed points [np2,2]
%      - Opt options
% OUT: - TP struct with estimates
%        - PrmEst [nGoodFits x 6] parameters from lsqlin
%        - ErrEst [nGoodFits x 1] error
function TP = f_RanSac(Pt1, Pt2, Opt, b_plot)
TP.nGoodFits = 0;
if isempty(Pt1) || isempty(Pt2), return; end
JAff = inline('1 0 x y 0 0; 0 1 0 0 x y');
np1 = size(Pt1,1);
np2 = size(Pt2,1);
fprintf('Original has %d points, transformed has %d points\n', np1, np2);
nMinPt = Opt.nMinPts; % # of minimum pts required for transformation
nCom = np1-nMinPt; % # of complement pts
if ~nCom, warning('no complementing points'); end
cpt = mean(Pt1); % center point of original
Pt1Cen = Pt1 - repmat(cpt,np1,1); % original 0,0 centered

%% ===== Iterating =====
cIter = 0;
nGoodFits = 0;
[Prm Err] = deal([]);
while cIter < Opt.nMaxIter

    %% ----- Random Subset (Samples)
    Ixrs = randsample(np1, nMinPt); % random sample indices
    Ixcm = setdiff(1:np1, Ixrs); % complement

    %% ----- Estimation with Random Samples
    Dlt = Pt2(Ixrs,:)-Pt1Cen(Ixrs,:); % delta (transformed-original 0,0 centered)
    Atot = zeros(6,6); b = 0; % init Atot and b
    for i = 1:nMinPt
        pt = Pt1Cen(Ixrs(i),:);
        Jp = JAff(pt(1),pt(2));
        Atot = Atot + Jp'*Jp; % building A
        b = b + Dlt(i,:)*Jp; % building b
    end
    [prm err] = lsqlin(double(Atot), double(b'));% prm(3:6) contain a00...
    [tx ty] = deal(prm(1)-cpt(1), prm(2)-cpt(2)); % tx and ty
    fprintf('tx %.4f ty %.4f ', tx, ty);

    %% ----- Transform Complement
    [a00 a10 a11] = deal(prm(3), prm(4), prm(5), prm(6));
    TAff = [1:a00 a01 tx; a10 1*a11 ty; 0 0 1]; % affinity (with row extended)
    Pt1Com = Pt1Cen(Ixcm,:);
    Gaff = [Pt1Com ones(nCom,1)] * TAff'; % we add a col of 1
    Caff(:,1:2) = Caff(:,1:2)+repmat(cpt,nCom,1); % THEN add centpt

    %% ----- Close enough?
    DfCo = Caff(:,1:2)-Pt2(Ixcm,:); % [nCom,2]
    Di = sqrt(sum(DfCo.^2,2));
    bNear = Di < Opt.thrNear;
    nNear = nnz(bNear);
    fprintf('%d near of %d err %.12.10f', nNear, nCom, err);
    if nNear >= Opt.nNear
        nGoodFits = nGoodFits + 1;
        % Refit =====
        Pt2n = Pt2(Ixcm(bNear,:));
        Pt1Cenn = Pt1Cen(Ixcm(bNear),:);
        Dltn = Pt2n - Pt1Cenn;
        % --- building A and b
        A2 = zeros(6,6); b = 0; % init Atot and b
        for i = 1:nNear
            pt = Pt1Cenn(i,:);
            Jp = JAff(pt(1),pt(2));
            A2 = A2 + Jp'*Jp; % building A
            b = b + Dltn(i,:)*Jp; % building b
        end
        % --- least squares:
        [prm2 err2] = lsqlin(double(A2), double(b'));
        [tx ty] = deal(prm(1)-cpt(1), prm(2)-cpt(2)); % tx and ty
        Prm = [Prm; [tx ty prm2(3:end)']];
        Err = [Err; err2];
    end

    %% ----- Plotting
    if b_plot.dist
        Dis = sort(Di, 'ascend');
        figure(10); clf; [rr cc] = deal(1,2);
        subplot(rr,cc,1);
        plot(Dis);
        set(gca, 'ylim', [0 1500]);
        title([Opt.xlsb ' num2str(cIter)]);
        xlabel(Opt.Match, 'fontweight', 'bold', 'fontsize', 12);
        subplot(rr,cc,2);
        plot(Dis); hold on;
        set(gca, 'xlim', [0 nCom*0.25]);
        set(gca, 'ylim', [0 Opt.thrNear*2]);
        plot([0 nCom], ones(1,2)*Opt.thrNear, 'k:');
        pause();
    end

    cIter = cIter + 1;
    fprintf('\n');
end
TP.PrmEst = Prm;
TP.ErrEst = Err;
TP.nGoodFits = nGoodFits;
fprintf('#GoodFits %d out of %d iterations\n', nGoodFits, Opt.nMaxIter);
end % function
```

This is the testing script calling the above function:

```
%> ===== TESTING SCRIPT t_Ransac
% Testing Ransac.
clear;
nP      = 40;
%% Simple Pattern
CoOrig = rand(nP,2)*3+5;
np      = size(CoOrig,1);          % # of points
cpt     = mean(CoOrig,1);         % center point
Cpt     = repmat(cpt,np,1);       % replicated center point

%% ===== Transform Original =====
tx = 4;           % x-translation
ty = 1.5;          % y-translation
a  = 0.25;         % scale (for similarity)
b  = 0.35;         % rotation (for similarity)
% ----- Transformation Matrices
TSim   = [1*a -b tx; b 1*a ty];    % similarity
TSim0  = [TSim; 0 0 1];             % with row extended
% ----- Perform Transformations
Coc    = CoOrig-Cpt;               % subtract centpt: coordinates 0,0 centered
Csim   = [Coc ones(np,1)] * TSim0'; % similarity " here we add a col of 1s
Csim(:,[1 2]) = Csim(:,[1 2])+Cpt;
CoTrns = Csim(:,[1 2]) + randn(nP,2)*1.2; % adding some noise

%% ===== Plotting =====
figure(1); clf; hold on;
plot(CoOrig(:,1), CoOrig(:,2), 'g'); % original set of points
plot(CoTrns(:,1), CoTrns(:,2), 'r'); % transformed set of points
axis equal
legend('original', 'transformed', 'location', 'northwest');

%% ===== RanSaC =====
disp('Verifying f_RanSaC');
Opt.nMinPts = 6;
Opt.nMaxIter = 10;
Opt.thrNear = 0.2;
Opt.nNear = 3;
Opt.xlb = ':';
Opt.Match = ':';
b.plot.dist = 1;
RSpnm = f_RanSaC(CoOrig, CoTrns, Opt, b.plot);
RSpnm.PrmEst
```

Python offers a function in `skimage.measure.ransac`