具体数学阅读笔记-chap2

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$$A = \begin{bmatrix} a_1a_1 & a_1a_2 & \cdots & a_1a_n \\ a_2a_1 & a_2a_2 & \cdots & a_2a_n \\ \vdots & \vdots & & \vdots \\ a_na_1 & a_na_2 & \cdots & a_na_n \end{bmatrix} \tag{1}$$

求 $S_{\triangleleft} = \sum_{1 \leqslant j \leqslant k \leqslant n} a_j a_k$ [1]

解 1 $\because a_j a_k = a_k a_j$, $\therefore$ 矩阵 A 沿主对角线对称, $S_{\triangleleft} = S_{\triangleright}$.

$$[1 \leqslant j \leqslant k \leqslant n] + [1 \leqslant k \leqslant j \leqslant n] = [1 \leqslant j, k \leqslant n] + [1 \leqslant j = k \leqslant n]$$

$$\begin{aligned} 2S_{\triangleleft} = S_{\triangleleft} + S_{\triangleright} &= S_A + S_{diag(A)} \\ &= \sum_{1 \leqslant j,k \leqslant n} a_j a_k + \sum_{1 \leqslant j = k \leqslant n} a_j a_k \\ &= \left(\sum_{j=1}^{n} a_j\right)\left(\sum_{k=1}^{n} a_k\right) + \sum_{k=1}^{n} a_k^2 \\ &= \left(\sum_{k=1}^{n} a_k\right)^2 + \sum_{k=1}^{n} a_k^2 \end{aligned}$$

$\therefore S_{\triangleleft} = \frac{1}{2}[(\sum_{k=1}^{n} a_k)^2 + \sum_{k=1}^{n} a_k^2]$

[1]下三角形矩阵的符号是一个右上部分的直角三角形, 目前我还不会输入

$$S = \sum_{1 \leqslant j < k \leqslant n} (a_k - a_j)(b_k - b_j) \tag{2}$$

解 2 交换 j, k 仍有对称性.

$$S = \sum_{1 \leqslant j < k \leqslant n} (a_k - a_j)(b_k - b_j) = \sum_{1 \leqslant j < k \leqslant n} (a_j - a_k)(b_j - b_k)$$

$$[1 \leqslant j < k \leqslant n] + [1 \leqslant k < j \leqslant n] = [1 \leqslant j, k \leqslant n] - [1 \leqslant j = k \leqslant n]$$

$$\begin{aligned}
2S &= 2 \sum_{1 \leqslant j < k \leqslant n} (a_k - a_j)(b_k - b_j) \\
&= \sum_{1 \leqslant j < k \leqslant n} (a_k - a_j)(b_k - b_j) + \sum_{1 \leqslant k < j \leqslant n} (a_k - a_j)(b_k - b_j) \\
&= \sum_{1 \leqslant j, k \leqslant n} (a_k - a_j)(b_k - b_j) - \sum_{1 \leqslant j = k \leqslant n} (a_k - a_j)(b_k - b_j) \\
&(a_j - a_k = 0, b_j - b_k = 0, [j = k]) \qquad = \sum_{1 \leqslant j, k \leqslant n} (a_k b_k - a_j b_k - \\
&= \sum_{j=1}^{n} \sum_{k=1}^{n} a_k b_k - \sum_{j=1}^{n} \sum_{k=1}^{n} a_j b_k - \sum_{j=1}^{n} \sum_{k=1}^{n} a_k b_j + \sum_{j=1}^{n} \sum_{k=1}^{n} a_j b_j \\
&= n \sum_{k=1}^{n} a_k b_k - \sum_{j=1}^{n} \sum_{k=1}^{n} a_j b_k - \sum_{j=1}^{n} \sum_{k=1}^{n} a_k b_j + n \sum_{j=1}^{n} a_j b_j \\
&= 2n \sum_{k=1}^{n} a_k b_k - 2 \sum_{j=1}^{n} a_j \sum_{k=1}^{n} b_k
\end{aligned}$$

$$S = n \sum_{k=1}^{n} a_k b_k - \left(\sum_{k=1}^{n} a_k \right) \left(\sum_{k=1}^{n} b_k \right)$$