具体数学阅读笔记-chap1 exercise

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1 Exercises

1.1 Warmups

练习 1 1 All horses are the same color; we can prove this by induction on the number of horses in a given set. Here's how: "If there's just one horse then it's the same color as itself, so the basis is trivial. For the induction step, assume that there are n horses numbered 1 to n. By the induc- tion hypothesis, horses 1 through n - 1 are the same color, and similarly horses 2 through n are the same color. But the middle horses, 2 through n - 1, can't change color when they're in different groups; these are horses, not chameleons. So horses 1 and n must be the same color as well, by transitivity. Thus all n horses are the same color; QED." What, if anything, is wrong with this reasoning?

题目解答 1 n=1 情况下马有相同颜色

但 n=2 时该假设不一定成立

练习 2 2

题目解答 2 不允许在 A B 之间直接移动, 求最短的移动序列

$$
\begin{array}{lllll}
k=1 & 1 & A \Rightarrow C, C \Rightarrow B & 2 & sum=2 \\
k=2 & 1 & A \Rightarrow C, C \Rightarrow B, & 2 & \\
 & 2 & A \Rightarrow C & 1 & \\
 & 1 & B \Rightarrow C, C \Rightarrow A, & 2 & \\
 & 2 & C \Rightarrow B & 1 & \\
 & 1 & A \Rightarrow C, C \Rightarrow B & 2 & sum=8 \\
k=3 & 1 & A \Rightarrow C, C \Rightarrow B, & 2 & \\
 & 2 & A \Rightarrow C & 1 & \\
 & 1 & B \Rightarrow C, C \Rightarrow A, & 2 & \\
 & 2 & C \Rightarrow B & 1 & \\
 & 1 & A \Rightarrow C, C \Rightarrow B & 2 & sum=8 \\
 & 3 & A \Rightarrow C & 1 & \\
 & 1 & B \Rightarrow C, C \Rightarrow A, & 2 & \\
 & 2 & B \Rightarrow C & 1 & \\
 & 1 & A \Rightarrow C, C \Rightarrow B, & 2 & \\
 & 2 & C \Rightarrow A & 1 & \\
 & 1 & B \Rightarrow C, C \Rightarrow A, & 2 & \\
 & 3 & C \Rightarrow B & 1 & sum=18 \\
 & 1 & A \Rightarrow C, C \Rightarrow B, & 2 & \\
 & 2 & A \Rightarrow C & 1 & \\
 & 1 & B \Rightarrow C, C \Rightarrow A, & 2 & \\
 & 2 & C \Rightarrow B & 1 & \\
 & 1 & A \Rightarrow C, C \Rightarrow B & 2 & sum=26 \\
\vdots & & & & \\
k=n & 1 & A \Rightarrow C, C \Rightarrow B & &
\end{array}
$$

从前面的移动可以看出 $f(n) = 3*f(n-1)+2$, 设 $g(n) = f(n)+1, g(1) = f(1)+1 = 3, g(n) = 3g(n-1)$. $g(n) = 3^n$, $f(n) = 3^n - 1$.

练习 3 3

题目解答 3 是的, 以 n 个圆盘为例正确的叠放方法有 3^n 种将 ABC 视为 3 个序列, 将所有圆盘从大到小依次放置在 3 个序列中, 每个圆盘放置时有 3 种选择, 所共有 3^n 种正确的叠放方法。第二题移动 3^n-1 次, 再加上移动前所有圆盘都在 A 柱上的情况, 共有 3^n 种情况, 所以所有正确的叠放方法均会出现。

我的思考, n 个圆盘在 3 根柱子上任意放的方法有多少种？

练习 4 4

题目解答 4 Are there any starting and ending configurations of n disks on three pegs that are more than 2^n-1 moves apart, under Lucas's original rules?

是否存在 $m > 2^n - 1$

不存在。根据卢卡斯的规则, 将可能出现的移动情况分为两种:
1. 最大的圆盘不需要移动, 根据归纳法, 最多需要移动 $2^{n-1}-1$ 次。
2. 最大的圆盘需要移动, 根据归纳法, 最多需要移动 $2^{n-1}-1+1+2^{n-1}-1$ 即 2^n-1 次

练习 5 5

题目解答 5 3 个给定集合, 共有 8 个可能子集。使用 Venn 图表示

[1] A, B, C, 三个集合的所有子集为 $\{\varnothing, A, B, C, A\cap B, A\cap C, B\cap C, A\cap B\cap C,\}$, $\{A\setminus B, A\setminus C, B\setminus A, B\setminus C, C\setminus A, C\setminus B\}$, $\{A\setminus(B\cup C), B\setminus(C\cup A), C\setminus(A\cup B)\}$, $\{A\cup B, A\cup C, B\cup C, A\cup B\cup C\}. \dots$

我认为这里所将的八个子集应当是 $\{\varnothing\}, \{A\setminus(B\cup C), B\setminus(C\cup A), C\setminus(A\cup B)\}$, $\{(A\cap B)\setminus C, (C\cap A)\setminus B, (B\cap C)\setminus A\}, \{A\cap B\cap C\}$. 空集和 7 个互不相交的真子集。

[1]Venn 图之后会补上

对于 4 个集合, Venn 图不能给出可能的 16 个子集, 因为不同的圆至多交于两点。参考答案中说的卵形 (ovals) 是什么意思？

练习 6 6

题目解答 6 无界区域个数 $2n$
所有区域个数 $\frac{n(n+1)}{2}+1$
二者相减得到有界区域个数 $\frac{(n-1)(n-2)}{2}$

练习 7 7

题目解答 7 设 $H(n)=J(n+1)-J(n)$.
$H(2n)=2$, 对 $n\geqslant 1$ 有

$$\begin{aligned} H(2n+1) &= J(2n+2)-J(2n+1) \\ &= (2J(n+1)-1)-(2J(n)+1) \\ &= 2H(n)-2 \end{aligned}$$

但在 $n=0$ 时, 由此推出

$$H(1)=J(2)-J(1)=1-1=0\neq 2$$

1.2 作业题

练习 8

$$\begin{aligned} Q_0 &= \alpha \\ Q_1 &= \beta \\ Q_n &= \frac{1+Q_{n-1}}{Q_{n-2}}, \quad n>1 \end{aligned}$$

(hint: $Q_4=\frac{1+\alpha}{\beta}$)

题目解答 8

$$
\begin{aligned}
Q_0 &= \alpha && && && = \alpha \\
Q_1 &= \beta && && && = \beta \\
Q_2 &= \frac{1+Q_1}{Q_0} && && && = \frac{1+\beta}{\alpha} \\
Q_3 &= \frac{1+Q_2}{Q_1} &&= \frac{1+\frac{1+\beta}{\alpha}}{\beta} && && = \frac{1+\alpha+\beta}{\alpha\beta} \\
Q_4 &= \frac{1+Q_3}{Q_2} &&= \frac{1+\frac{1+\alpha+\beta}{\alpha\beta}}{\frac{1+\beta}{\alpha}} &&= \frac{\alpha\beta+1+\alpha+\beta}{\beta(1+\beta)} && = \frac{1+\alpha}{\beta} \\
Q_5 &= \frac{1+Q_4}{Q_3} &&= \frac{1+\frac{1+\alpha}{\beta}}{\frac{1+\alpha+\beta}{\alpha\beta}} &&= \frac{\alpha\beta+\alpha(1+\alpha)}{1+\alpha+beta} && = \alpha \\
Q_6 &= \frac{1+Q_5}{Q_4} &&= \frac{1+\alpha}{\frac{1+\alpha}{\beta}} && && = \beta
\end{aligned}
$$

因此解得

$$
\begin{array}{llllllll}
Q_i & =\{ & \alpha, & \beta, & \frac{1+\beta}{\alpha}, & \frac{1+\alpha+\beta}{\alpha\beta}, & \frac{1+\alpha}{\beta} & \} \\
(i\%n) & =\{ & 0, & 1, & 2, & 3, & 4, & \}
\end{array}
$$

练习 9 反向归纳法，从 n 到 $n-1$ 证明命题

$$P(n):\ x_1\ldots x_n \leqslant \left(\frac{x_1+\cdots+x_n}{n}\right)^n, \quad x_i \geqslant 0, i=1,\ldots,n$$

$n=2$ 时为真

$$(x_1+x_2)^2-4x_1x_2=(x_1-x_2)^2\geqslant 0$$

a) $x_n = \frac{x_1+\cdots+x_{n-1}}{n-1}$, 证明只要 $n > 1$ 时 $P(n)$ 蕴含 $P(n-1)$.

b) 证明 $P(n)$ 和 $P(2)$ 蕴含 $P(2n)$

c) 由 a), b) 说明这就蕴含了 $P(n)$ 对所有 n 为真

题目解答 9 a) $P(n)$ 成立, $\forall n > 1$

给定 $x_n = \frac{x_1+\cdots+x_{n-1}}{n-1}$, 则有

$$
\begin{aligned}
x_1 \dots x_{n-1} \cdot \frac{x_1+\cdots+x_{n-1}}{n-1} &\leqslant \left(\frac{x_1+x_{n-1}+\dfrac{x_1+\cdots+x_{n-1}}{n-1}}{n}\right)^n \\
x_1 \dots x_{n-1} \cdot \frac{x_1+\cdots+x_{n-1}}{n-1} &\leqslant \left(\frac{x_1+\cdots+x_{n-1}}{n-1}\right)^n \\
x_1 \dots x_{n-1} &\leqslant \left(\frac{x_1+\cdots+x_{n-1}}{n-1}\right)^{n-1}
\end{aligned}
$$

$P(n)$ 成立

b) 由 $P(n)$ 可得

$$
x_1 \dots x_n \cdot x_{n+1} \dots x_{2n} \leqslant \left(\frac{x_1+\cdots+x_n}{n}\right)^n \cdots \left(\frac{x_{n+1}+\cdots+x_{2n}}{n}\right)^n
$$

记 $A = \left(\dfrac{x_1+\cdots+x_n}{n}\right)$, $B = \left(\dfrac{x_{n+1}+\cdots+x_{2n}}{n}\right)$

由 $P(2)$ 可得

$$\begin{aligned} AB &\leqslant \left(\frac{A+B}{2}\right)^2 \\ A^nB^n = (AB)^n &\leqslant \left(\frac{A+B}{2}\right)^{2n} \\ x_1 \dots x_{2n} &\leqslant \left(\frac{x_1+\cdots+x_{2n}}{2n}\right)^{2n} \end{aligned}$$

由此推知 $P(2n)$ 成立。

c) Cauchy 向前-向后方法。

1. $P(2) \Rightarrow P(4) \Rightarrow \cdots P(2^n)$.
2. $P(n) \Rightarrow P(n-1)$.

$\therefore \forall n \geqslant 1, P(n)$ 成立

练习 10 圆盘只能在 ABC 三根柱子上按照顺时针方向移动。记:

Q_n 为 n 个盘从 A 到 B 最少移动的次数。

R_n 为 n 个盘从 B 到 A 最少移动的次数。

题目解答 10 先列出两种移动方式各自的迭代式:

$$Q_n = \begin{cases} 0, & n=0 \\ 2R_{n-1}+1, & n>0 \end{cases} \qquad R_n = \begin{cases} 0, & n=0 \\ Q_n+Q_{n-1}+1, & n>0 \end{cases}$$

这两个公式是如何得到的？

$$\begin{array}{lll}
k=0 & Q_0=0 & R_0=0 \\
k=1 & A\Rightarrow B & B\Rightarrow C\Rightarrow A \\
 & Q_1=1 & R_1=2 \\
k=2 & 1:A\Rightarrow B\Rightarrow C & 1:B\Rightarrow C\Rightarrow A \\
 & 2:A\Rightarrow B & 2:B\Rightarrow C \\
 & 1:C\Rightarrow A\Rightarrow B & 1:A\Rightarrow B \\
 & & 2C\Rightarrow A \\
 & & 1:B\Rightarrow C\Rightarrow A \\
 & Q_2=5 & R_2=7 \\
k=n & A\Rightarrow B\ Q_n & B\Rightarrow A\ R_n \\
 & (n-1)A\Rightarrow C\ R_{n-1} & (n-1)B\Rightarrow A\ R_{n-1} \\
 & n:A\Rightarrow B\ 1 & n:B\Rightarrow C\ 1 \\
 & (n-1)C\Rightarrow B\ R_{n-1} & (n-1)A\Rightarrow B\ Q_{n-1} \\
 & & n:C\Rightarrow A\ 1 \\
 & & (n-1)B\Rightarrow A\ R_{n-1} \\
 & Q_n=2R_{n-1}+1 & R_n=Q_{n-1}+2R_{n-1}+2=Q_n+Q_{n-1}+1
\end{array}$$

练习 11 双重河内塔 2n 个圆盘, 第 $2k-1$ 个与第 $2k$ 个大小相同。

题目解答 11 a) 不区分相同尺寸

$$\begin{array}{ll}
n=0S_0=0 & \\
n=1S_1=2 & A\Rightarrow B, A\Rightarrow B \\
n=2S_2=6 & A\Rightarrow C, A\Rightarrow C \\
\quad A\Rightarrow B, A\Rightarrow B & \\
\quad C\Rightarrow B, C\Rightarrow B &
\end{array}$$

解得 $S_n=2T_n=2(2^n-1)=2^{n+1}-2$

b) 在最后排列中将圆盘恢复次序需要移动几次？

$$
\begin{array}{lll}
k=0 & R_0=0 & \\
k=1 & R_1=3 & 1.1:\ A\Rightarrow C \\
 & & 1.2:\ A\Rightarrow B \\
 & & 1.1:\ C\Rightarrow B \\
k=2 & R_2=11 & 1.1:\ A\Rightarrow B \\
 & & 1.2:\ A\Rightarrow B \\
 & & 2.1:\ A\Rightarrow C \\
 & & 1.2:\ B\Rightarrow C \\
 & & 1.1:\ B\Rightarrow C \\
 & & 2.2:\ A\Rightarrow B \\
 & & 1.1:\ C\Rightarrow A \\
 & & 1.2:\ C\Rightarrow A \\
 & & 2.1:\ C\Rightarrow B \\
 & & 1.2:\ A\Rightarrow B \\
 & & 1.1:\ A\Rightarrow B \\
k=n & R_n & n-1\ A\Rightarrow B\quad S_{n-1} \\
 & & n.1:\ A\Rightarrow C\quad 1 \\
 & & n-1\ B\Rightarrow C\quad S_{n-1} \\
 & & n.2:\ A\Rightarrow B\quad 1 \\
 & & n-1\ C\Rightarrow A\quad S_{n-1} \\
 & & n.1:\ C\Rightarrow B\quad 1 \\
 & & n-1\ A\Rightarrow B\quad S_{n-1}
\end{array}
$$

$$R_n=4S_{n-1}+3=2^{n+2}-5\quad (n\geqslant 1)$$

练习 12 12 11 推广，m_k 个尺寸为 k 的圆盘，不区分相同尺寸的圆盘移动一个塔最少次数 $A(m_1,\ldots,m_n)$

题目解答 12

$$\begin{aligned}
F(0) &= 0 \\
F(1) &= m_1 \\
F(2) &= 2F(1) + m_2 = 2m_1 + m_2 \\
&\vdots \\
F(n) &= 2F(n-1) + m_n
\end{aligned}$$

$$\begin{aligned}
A(m_1, \ldots, m_n) &= F(n) = 2F(n-1) + m_n \\
&= 2^{n-1}m_1 + 2^{n-2}m_2 + \cdots + m_n \\
&= \sum_{k=1}^{n} 2^{n-k} m_k
\end{aligned}$$

练习 13 13

题目解答 13

$$\begin{aligned}
k &= 1 \quad ZZ_1 = 2 + 0 = 2 \\
k &= 2 \quad ZZ_2 = 4 + 8 = 12 \\
k &= 3 \quad ZZ_3 = 6 + 25 = 31
\end{aligned}$$

对于定义了 L_n 个区域的 n 条直线, 可以用极狭窄的 Z 形线来代替。

例如, 每一对 Z 形线间有 9 个交点

$$\begin{aligned}
ZZ_n &= ZZ_{n-1} + 9n - 8, \quad (n > 0) \\
ZZ_n &= 9S_n - 8n + 1 \\
&= 9\frac{n(n+1)}{2} - 8n + 1 \\
&= \tfrac{9}{2}n^2 - \tfrac{7}{2}n + 1
\end{aligned} \tag{1}$$

练习 14 14

题目解答 14

$$\begin{aligned} n=0 &\quad P_0=1\\ n=1 &\quad P_1=2\\ n=2 &\quad P_2=4\\ n=3 &\quad P_3=8\\ n=4 &\quad P_4=8+6=14 \end{aligned}$$

$$P_n=P_{n-1}+L_{n-1}$$

其中

$$L_n=1+S_n,\quad S_n=\frac{n(n+1)}{2}$$

$\therefore P_n=P_{n-1}+1+\frac{n(n+1)}{2}$

$$\begin{aligned} P_0 &=1 \\ P_1 &=P_0+L_0 &&=1+1+\tfrac{0\cdot1}{2} &&=2\\ P_2 &=P_1+L_1 &&=2+1+\tfrac{1\cdot2}{2} &&=4\\ P_3 &=P_2+L_2 &&=4+1+\tfrac{2\cdot3}{2} &&=8\\ P_4 &=P_3+L_3 &&=8+1+\tfrac{3\cdot4}{2} &&=15\\ P_5 &=P_4+L_4 &&=15+1+\tfrac{4\cdot5}{2} &&=26 \end{aligned}$$

$$\begin{aligned} P_n &= P_{n-1}+L_{n-1}\\ &=0+\sum_{k=0}^{n-1}\left(1+\frac{k(k+1)}{2}\right)\\ &=n+\frac{(n-1)n(n+1)}{6}\\ &=\frac{n(n^2+5)}{6} \end{aligned}$$

练习 15 15 约瑟夫问题, 倒数第二个 $I(n)$

表 1: 约瑟夫问题 J(n) 与 I(n)

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
J(n)	1	1	3	1	3	5	7	1	3	5	7	9	11	13	15	1	3	5
I(n)	∼	2	1	3	5	1	3	5	7	9	11	1	3	5	7	9	11	13

题目解答 15 $n > 1$ 时, $J(n), I(n)$ 有相同递归式

$$\begin{aligned} &I(2) = 2, I(1) = 1 \\ &n = 2^m + 2^{m-1} + k, \quad 0 \leqslant k \leqslant 2^m + 2^{m-1} \\ &I(n) = 2k + 1 \end{aligned}$$

$$n = 2^m + l, \quad I(n) = \begin{cases} J(n) + 2^{m-1}, & 0 \leqslant l < 2^{m-1} \\ J(n) - 2^m, & 2^{m-1} \leqslant l < 2^m \end{cases}$$

练习 16

$$\begin{cases} g(1) &= \alpha \\ g(2n+j) &= 3g(n) + \gamma n + \beta_j, \quad j = 0, 1, n \leqslant 1 \end{cases}$$

(提示, 用 $g(n) = n$)

题目解答 16 Suppose $g(n) = n$

$$g(1) = 1 = \alpha,$$
$$g(2n+j) = 2n + j = 3n + \gamma n + \beta_j.$$

解得 $\alpha = 1, \gamma = -1, \beta_j = \begin{cases} 0, & j = 0 \\ 1, & j = 1 \end{cases}$

(题解)

$$g(n) = a(n)\alpha + b(n)\beta_0 + c(n)\beta_1 + d(n)\gamma$$

$n = (1b_{m-1} \dots b_1 b_0)_2$ 将 n 以基数 2 展开 (写成二进制)。

$$a(n)\alpha + b(n)\beta_0 + c(n)\beta_1 = (\alpha\beta_{m-1}\beta_{m-2} \dots \beta_{b_1}\beta_{b_0})_3$$

$g(n) = n. \quad (\alpha = 1, \beta_0 = 0, \beta_1 = 1, \gamma = -1)$

$$a(n) + c(n) - d(n) = n$$

$g(n) = 1. \quad (\alpha = 1, \beta_0 = -2, \beta_1 = 2, \gamma = 0)$

$$a(n) - 2b(n) - c(n) = 1$$

$$\begin{aligned} d(n) &= a(n) + c(n) - n \\ b(n) &= \frac{1}{2}a(n) - \frac{1}{2}c(n) - \frac{1}{2} \end{aligned}$$

$$\begin{aligned} g(n) = a(n)\alpha + (\frac{1}{2}a(n) - \frac{1}{2}c(n) - \frac{1}{2})\beta_0 \\ + c(n)\beta_1 + (a(n) + c(n) - n)\gamma \end{aligned}$$

若 $\beta_i = 0\ (i = 0, 1), \gamma = 0$

$$\begin{cases} g(1) = \alpha \\ g(2n + j) = 3g(n) \end{cases}$$

$g(1) = \alpha, g(2) = g(3) = 3\alpha, g(4) = g(5) = g(6) = g(7) = 9\alpha, g(8) = 3g(4) = 27\alpha.$
由此推知

$$g(n) = g(2^m + k) = 3^m, \quad a(n) = 3^m$$

继续计算 $d(n)$ 遇到困难

若 $\alpha = 0, \beta_0 = 0, \gamma = 0, g(n) = \beta_1 c(n)$

$$\begin{cases} g(1) = 0 \\ g(2n) = 3g(n) \\ g(2n + 1) = 3g(n) + 1 \end{cases}$$

表 2: m,k 变化规律

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
	0	0	1	0	1	3	4	0	1	3	4	9	10	12	13	0	1	3	4
m	0	1		2				3								4			
k			1		1	2	3		1	2	3	4	5	6	7		1	2	3

$g(2)=0$, $g(3)=1$, $g(4)=0$, $g(5)=1$, $g(6)=3g(3)=3$, $g(7)=3g(3)+1=4$, $g(8)=0$, $g(9)=1\ldots$

$$g(2^m+k)=g((1b_{m-1}b_{m-2}\cdots_1 b_0)_2)$$

复习和重做 16 题的部分暂不录入

练习 17

$$W_{n(n+1)/2}\leqslant 2W_{n(n-1)/2}+T_n,\quad n>0$$

题目解答 17 In general we have $W_m\leqslant 2W_{m-k}+T_k,\ 0\leqslant k\leqslant M$ (This relation corresponds to transferring the top $m-k$. then using only three pegs to move the bottom $k\Rightarrow T_k$, then finishing with the top $m-k\Rightarrow 2\cdot W_{m-k}$)

The stated relation turns out to be based on the unique value of k that minimizes the right-hand side of this general inequality， when $m\frac{n(n+1)}{2}$.

(However, we cannot conclude that equality holds. Many other strategies for transferring the tower are conceivale.)

If we set $Y_n=(W_{n(n+1)/2}-1)/2^n$

we find that $Y_n\leqslant Y_{n-1}+1$. hence $W_{n(n+1)/2}\leqslant 2^n(n-1)+1$

练习 18 证明如下的一组 n 条折线定义 Z_n 个区域

$$
\begin{aligned}
Z_n = L_{2n} - 2n &= \frac{2n(2n+1)}{2} + 1 - 2n \\
&= 2n^2 - n + 1. \quad n \geqslant 0.
\end{aligned} \tag{2}
$$

第 j 条折线 $(1 \leqslant j \leqslant n)$ 的锯齿点在 $(n^{kj}, 0)$. 并向上经过点 $(n^{2j} - n^j, 1)$ 与 $(n^{2j} - n^j - n^{-n}, 1)$

题目解答 18

练习 19 当每一个锯齿的角度为 30° 时,有可能由 n 条折线得到 Z_n 个区域吗?

题目解答 19

练习 20 利用成套方法解递归式:

$$
\begin{cases}
h(1) & = \alpha \\
h(2n) & = 4h(n) \quad +\gamma_0 n \quad +\beta_0 \qquad n \geqslant 1 \\
h(2n) & = 4h(n) \quad +\gamma_1 n \quad +\beta_1
\end{cases}
$$

题目解答 20

$$
h(n) = \alpha A(n) + \beta_0 B(n) + \gamma_0 C(n) + \beta_1 D(n) + \gamma_1 E(n)
$$

1. $h(1) = 1$

$$
\begin{cases}
1 & = \alpha \\
1 & = 4 \times 1 \quad +\gamma_0 n \quad +\beta_0 \\
1 & = 4 \times 1 \quad +\gamma_1 n \quad +\beta_1
\end{cases}
$$

解得 $(\alpha, \beta_0, \gamma_0, \beta_1, \gamma_1) = (1, -3, 0, -3, 0)$

$$
1 = A(n) - 3B(n) - 3D(n)
$$

2. $h(n) = n$

$$
\begin{cases}
1 & = \alpha \\
2n & = 4 \times n \quad +\gamma_0 n \quad +\beta_0 \\
2n+1 & = 4 \times n \quad +\gamma_1 n \quad +\beta_1
\end{cases}
$$

解得 $(\alpha, \beta_0, \gamma_0, \beta_1, \gamma_1) = (1, 0, -2, 1, -2)$

$$n = A(n) - 2C(n) + D(n) - 2E(n)$$

3. $h(n) = n^2$

$$\begin{cases} 1 & = \alpha \\ (2n)^2 & = 4 \times n^2 & +\gamma_0 n & +\beta_0 \\ (2n+1)^2 & = 4 \times n^2 & +\gamma_1 n & +\beta_1 \end{cases}$$

解得 $(\alpha, \beta_0, \gamma_0, \beta_1, \gamma_1) = (1, 0, 0, 1, 4)$

$$n^2 = A(n) + D(n) + 4E(n)$$

最终结果???

练习 21 $2n$ 个人围成圈, 前 n 个好伙计后 n 个坏家伙. 证明: 总存在一个整数 q (与 n 有关), 使得若在绕圆圈走时每隔 $q-1$ 个人处死一个, 那么所有坏家伙首先出局. (例如, $n=3$ 时取 $q=5$, $n=4$ 时取 $q=30$)

题目解答 21 We can let m be the least (or any) common multiple of $2n, 2n-1, \ldots, n+1$. a non-rigorous argument suggests that "random" value of m will succeed with probability

$$\frac{n}{2n}\frac{n-1}{2n-1}\cdots\frac{1}{n+1} = \frac{1}{\binom{2n}{n}} \sim \frac{\sqrt{\pi n}}{4^n}$$

1.3 附加题

练习 22 22

题目解答 22 22

练习 23 假如约瑟夫发现自己处在 j, 但能指定 q, 隔 $q-1$ 人处死一人. 他是否能保全自己?

题目解答 23