# 具体数学阅读笔记-chap1 exercise

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## 1 Exercises

### 1.1 Warmups

**练习 1** All horses are the same color; we can prove this by induction on the number of horses in a given set. Here's how: "If there's just one horse then it's the same color as itself, so the basis is trivial. For the induction step, assume that there are n horses numbered 1 to n. By the induc- tion hypothesis, horses 1 through n 1 are the same color, and similarly horses 2 through n are the same color. But the middle horses, 2 through n 1, can't change color when they're in different groups; these are horses, not chameleons. So horses 1 and n must be the same color as well, by transitivity. Thus all n horses are the same color; QED." What, if anything, is wrong with this reasoning?

**解 1** n=1 情况下马有相同颜色

但 n=2 时该假设不一定成立

**解 2** 不允许在 A B 之间直接移动, 求最短的移动序列

$$
\begin{array}{rlrr}
k=1\ 1 & A \rightarrowtail C, C \rightarrowtail B & 2 & sum=2 \\
k=2\ 1 & A \rightarrowtail C, C \rightarrowtail B, & & 2 \\
2 & A \rightarrowtail C & & 1 \\
1 & B \rightarrowtail C, C \rightarrowtail A, & & 2 \\
2 & C \rightarrowtail B & & 1 \\
1 & A \rightarrowtail C, C \rightarrowtail B & 2 & sum=8 \\
k=3\ 1 & A \rightarrowtail C, C \rightarrowtail B, & & 2 \\
2 & A \rightarrowtail C & & 1 \\
1 & B \rightarrowtail C, C \rightarrowtail A, & & 2 \\
2 & C \rightarrowtail B & & 1 \\
1 & A \rightarrowtail C, C \rightarrowtail B & 2 & sum=8 \\
3 & A \rightarrowtail C & & 1 \\
1 & B \rightarrowtail C, C \rightarrowtail A, & & 2 \\
2 & B \rightarrowtail C & & 1 \\
1 & A \rightarrowtail C, C \rightarrowtail B, & & 2 \\
2 & C \rightarrowtail A & & 1 \\
1 & B \rightarrowtail C, C \rightarrowtail A, & & 2 \\
3 & C \rightarrowtail B & 1 & sum=18 \\
1 & A \rightarrowtail C, C \rightarrowtail B, & & 2 \\
2 & A \rightarrowtail C & & 1 \\
1 & B \rightarrowtail C, C \rightarrowtail A, & & 2 \\
2 & C \rightarrowtail B & & 1 \\
1 & A \rightarrowtail C, C \rightarrowtail B & 2 & sum=26 \\
\vdots & & &
\end{array}
$$

$$k=n\ 1 \quad A \rightarrowtail C, C \rightarrowtail B \qquad ?$$

从前面的移动可以看出 $f(n) = 3 * f(n-1) + 2$, 设 $g(n) = f(n) + 1, g(1) = f(1) + 1 = 3, g(n) = 3g(n-1)$. $g(n) = 3^n, f(n) = 3^n - 1$.

**解 3** 是的，以 n 个圆盘为例正确的叠放方法有 $3^n$ 种将 ABC 视为 3 个序列，将所有圆盘从大到小依次放置在 3 个序列中，每个圆盘放置时有 3 种选择，所共有 $3^n$ 种正确的叠放方法。第二题移动 $3^n - 1$ 次，再加上移动前所有圆盘都在 A 柱上的情况，共有 $3^n$ 种情况，所以所有正确的叠放方法均会出现。

我的思考，n 个圆盘在 3 根柱子上任意放的方法有多少种？

**解 4** Are there any starting and ending configurations of n disks on three pegs that are more than $2^n 1$ moves apart, under Lucas's original rules?

是否存在 $m > 2^n - 1$

不存在。根据卢卡斯的规则，将可能出现的移动情况分为两种：
1. 最大的圆盘不需要移动，根据归纳法，最多需要移动 $2^{n-1} - 1$ 次。
2. 最大的圆盘需要移动，根据归纳法，最多需要移动 $2^{n-1} - 1 + 1 + 2^{n-1} - 1$ 即 $2^n - 1$ 次

**解 5** 3 个给定集合，共有 8 个可能子集。使用 Venn 图表示

[1] $A, B, C$, 三个集合的所有子集为 $\{\varnothing, A, B, C, A \cap B, A \cap C, B \cap C, A \cap B \cap C, \}$ , $\{A \setminus B, A \setminus C, B \setminus A, B \setminus C, C \setminus A, C \setminus B\}, \{A \setminus (B \cup C), B \setminus (C \cup A), C \setminus (A \cup B)\}, \{A \cup B, A \cup C, B \cup C, A \cup B \cup C\}. \ldots$

我认为这里所将的八个子集应当是 $\{\varnothing\}, \{A \setminus (B \cup C), B \setminus (C \cup A), C \setminus (A \cup B)\}, \{(A \cap B) \setminus C, (C \cap A) \setminus B, (B \cap C) \setminus A\}, \{A \cap B \cap C\}$

[1]Venn 图之后会补上