数学分析习题课讲义上册习题

我的解答

A.G 不等式 $a_1, a_2, \dots, a_n$, n 个非负实数

$$\frac{a_1+a_2+\cdots+a_n}{n} \ge \sqrt[n]{a_1 \dots a_n} \tag{1}$$

$\ge$ in inequation became $= \iff a_1 = a_2 = \cdots = a_n$

Proof

Proof 0.1 *1. induction method*

$$\begin{aligned}
k=1 &\qquad a_1 = a_1 \\
k=2 &\qquad \frac{a_1+a_2}{2} \ge \sqrt{a_1 a_2} \\
k=n &\qquad \text{suppose} \quad \frac{a_1+a_2+\cdots+a_n}{n} \ge \sqrt[n]{a_1 \dots a_n} \\
k=n+1 &\qquad \frac{a_1+a_2+\cdots+a_{n+1}}{n+1} - \frac{a_1+a_2+\cdots+a_n}{n} \\
&= \frac{n(a_1+a_2+\cdots+a_{n+1}) - (n+1)(a_1+a_2+\cdots+a_n)}{n(n+1)} \\
&= \frac{na_{n+1} - (a_1+a_2+\cdots+a_n)}{n(n+1)}
\end{aligned}$$

Set $A = \frac{a_1+a_2+\cdots+a_n}{n}, B = \frac{na_{n+1}-(a_1+a_2\cdots+a_n)}{n(n+1)}$

$$(\frac{a_1+a_2+\cdots+a_{n+1}}{n+1})^{n+1} = (A+B)^{n+1}$$

$A > 0, B \ge 0$

$$(A+B)^{n+1} \ge A^{n+1} + (n+1)A^n B$$

$$A^{n+1} + (n+1)A^n B = A^n(A + (n+1)B)$$