习题：

1. $n = 2^m + l$ $\quad 0 \le l < 2^m$

$\underline{m = \lfloor \log_2 n \rfloor}$ $\quad l = n - 2^m$

$\downarrow$

答案是 $\lfloor \lg n \rfloor$? $\rightarrow$ 本书 $\lg n$ 代表 $\log_2 n$.

$\log n$ 代表 $\log_{10} n$. 行吧.

2.

$\lfloor x + 0.5 \rfloor$ $\quad \lfloor 2.7 + 0.5 \rfloor = 3$

$\lceil x - 0.5 \rceil$ $\quad \lfloor 2.5 + 0.5 \rfloor = 3$

$\lceil 2.5 - 0.5 \rceil = 2$

3. $\left\lfloor \frac{\lfloor m\alpha \rfloor n}{\alpha} \right\rfloor$ $\quad m, n \in N^+$. $\alpha > n$ 无理数

$\left\lfloor \frac{\lfloor m\alpha \rfloor n}{\alpha} \right\rfloor$ $\quad \lfloor m\alpha \rfloor = m\alpha - \{m\alpha\}$

$\frac{\lfloor m\alpha \rfloor n}{\alpha} = \frac{m\alpha n}{\alpha} - \frac{\{m\alpha\} n}{\alpha}$

$$\left\lfloor \frac{\lfloor m\alpha \rfloor n}{\alpha} \right\rfloor = \left\lfloor mn - \frac{\{m\alpha\} n}{\alpha} \right\rfloor = mn$$

5 $\quad \lfloor nx \rfloor = n\lfloor x \rfloor$ 充要条件

$$\lfloor nx \rfloor = \lfloor n\lfloor x \rfloor + n\{x\} \rfloor = n\lfloor x \rfloor$$

$$0 \leqslant n\{x\} < 1 \qquad 0 \leqslant \{x\} < \frac{1}{n}$$

(Notice that $\lfloor nx \rfloor \geqslant n\lfloor x \rfloor$ for all x in this case)

6. 例 $f(x) = -x$

$\lfloor f(x) \rfloor = \lfloor -x \rfloor$

$$\lfloor f(x) \rfloor = \lfloor f(\lceil x \rceil) \rfloor$$

7. $\begin{cases} X_n = n & 0 \leqslant n < m \\ X_n = X_{n-m} + 1 & n \geqslant m \end{cases}$

$X_0=0,\quad X_1=1 \cdots \quad X_{m-1}=m-1$

$X_m= X_0+1 = 0+1 = 1 \qquad X_{2m}=X_m+1=2$

$X_{m+1}= X_1+1 = 1+1 = 2$

设 $n \bmod m = d \qquad n = km+d$

$X_n = X_{km+d} = X_d + k = k+d$

$= \lfloor \frac{n}{m} \rfloor + n \bmod m$

8.

若所有盒中物品数均 $< \lceil \frac{n}{m} \rceil$

$n \le m(\lceil \frac{n}{m} \rceil - 1)$

$\frac{n}{m}+1 \le \lceil \frac{n}{m} \rceil. \qquad$ 矛盾

若所有盒中物品数均 $> \lfloor \frac{n}{m} \rfloor$

$n \geqslant m(\lfloor \frac{n}{m} \rfloor + 1)$

$\frac{n}{m} - 1 \geqslant \lfloor \frac{n}{m} \rfloor \qquad$ 矛盾.

$\Longrightarrow$ Dirichlet 抽屉原理

n个物体放进m个盒子中. 那么某个盒子中必定有 $\geq \lceil \frac{n}{m} \rceil$ 个物体, 且某个盒子中必定含有 $\leq \lfloor \frac{n}{m} \rfloor$ 个物体

9. $0 < \frac{m}{n} < 1$.

$$\frac{m}{n} = \frac{1}{q} + \left\{ \frac{m}{n} - \frac{1}{q} \text{的表示} \right\}. \quad q = \left\lceil \frac{n}{m} \right\rceil$$

$$q = \left\lceil \frac{n}{m} \right\rceil \qquad \frac{1}{q} = \frac{1}{\lceil n/m \rceil}$$

$$\frac{m}{n} - \frac{1}{q} = \frac{m}{n} - \frac{1}{\lceil n/m \rceil}$$

$$= \frac{m\left\lceil \frac{n}{m} \right\rceil - n}{n\left\lceil \frac{n}{m} \right\rceil}$$

$$\frac{m}{n} - \frac{1}{q} = \frac{\{n \text{ mumble } m\}}{qn}$$

The process must terminate, because

$$0 \leq n \text{ mumble } m < m$$

The denominators of the representation are strictly increasing, hence distinct, because

$$\frac{qn}{n \text{ mumble } m} > q.$$