

表 1: 有限微积分和无限微积分中的运算对比

无限微积分	有限微积分
D 逆运算 $\int$ (积分算子, 逆微分算子)	$\Delta$ 逆运算 $\sum$ (求和算子, 逆差分算子)
微积分基本定理	
$g(x) = Df(x) \iff \int g(x)dx = f(x) + C$	$g(x) = \Delta f(x) \iff \sum g(x)\delta(x) = f(x) + C$
定积分	和式
若 $g(x) = Df(x)$ 那么	若 $g(x) = \Delta f(x)$ 那么
$\int_a^b g(x)dx = f(x) _a^b = f(b) - f(a)$	$\sum_a^b g(x)\delta x = f(x) _a^b = f(b) - f(a)$
$\int_b^a g(x)dx = -\int_a^b g(x)dx$	$\sum_b^a g(x)\delta x = -\sum_a^b g(x)\delta x$
$\int_a^b + \int_b^c = \int_a^c$	$\sum_a^b + \sum_b^c = \sum_a^c$
$\int_0^n x^m = \frac{x^{m+1}}{m+1}\Big _0^n = \frac{n^{m+1}}{m+1}, m \neq -1$	$\sum_0^n k^{\overline{m}} = \frac{k^{\overline{m+1}}}{m+1}\Big _0^n = \frac{n^{\overline{m+1}}}{m+1}, m \neq -1$
	$\sum_0^n k^{\overline{m}} = \frac{k^{\overline{m+1}}}{m+1}\Big _0^n = \frac{n^{\overline{m+1}}}{m+1}, m \neq -1$
$(x+y)^2 = x^2 + 2xy + y^2$	$(x+y)^{\underline{2}} = x^{\underline{2}} + 2x^{\underline{1}}y^{\underline{1}} + y^{\underline{2}}$
	$(x+y)^{\overline{2}} = x^{\overline{2}} + 2x^{\overline{1}}y^{\overline{1}} + y^{\overline{2}}$
$m = -1, \int_a^b x^{-1} = \ln x \Big _a^b$	$m = -1, \sum_a^b k^{\overline{-1}} = H_k \Big _a^b$
$\int_a^b x^m = \frac{x^{m+1}}{m+1}\Big _a^b, m \neq -1$	$\sum_a^b k^{\overline{m}} = \frac{k^{\overline{m+1}}}{m+1}\Big _a^b, m \neq -1$
$\ln n \Big _a, m = -1$	$H_k \Big _a^b, m = -1$
	$\sum_a^b k^{\overline{m}} = \frac{k^{\overline{m+1}}}{m+1}\Big _a^b, m \neq -1$
	$H_{(k+1)} \Big _a^b, m \neq -1$
连续性问题的解中会出现自然对数	快速排序这样的问题中会出现调和数的原因
$e^x$ , 性质 $De^x = e^x$	$\Delta f(x) = f(x), f(x) = 2^x$ 离散指数函数