具体数学阅读笔记-chap1 exercise

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1 Exercises

1.1 Warmups

练习 1 All horses are the same color; we can prove this by induction on the number of horses in a given set. Here's how: "If there's just one horse then it's the same color as itself, so the basis is trivial. For the induction step, assume that there are n horses numbered 1 to n. By the induc- tion hypothesis, horses 1 through n 1 are the same color, and similarly horses 2 through n are the same color. But the middle horses, 2 through n 1, can't change color when they're in different groups; these are horses, not chameleons. So horses 1 and n must be the same color as well, by transitivity. Thus all n horses are the same color; QED." What, if anything, is wrong with this reasoning?

解 1 n=1 情况下马有相同颜色

但 n=2 时该假设不一定成立

解 2 不允许在 A B 之间直接移动, 求最短的移动序列

$$
\begin{array}{lllll}
k=11 & A \rightarrowtail C, C \rightarrowtail B & 2 & sum=2 \\
k=21 & A \rightarrowtail C, C \rightarrowtail B, & & 2 \\
\quad 2 & A \rightarrowtail C & & 1 \\
\quad 1 & B \rightarrowtail C, C \rightarrowtail A, & & 2 \\
\quad 2 & C \rightarrowtail B & & 1 \\
\quad 1 & A \rightarrowtail C, C \rightarrowtail B & 2 & sum=8 \\
k=31 & A \rightarrowtail C, C \rightarrowtail B, & & 2 \\
\quad 2 & A \rightarrowtail C & & 1 \\
\quad 1 & B \rightarrowtail C, C \rightarrowtail A, & & 2 \\
\quad 2 & C \rightarrowtail B & & 1 \\
\quad 1 & A \rightarrowtail C, C \rightarrowtail B & 2 & sum=8 \\
\quad 3 & A \rightarrowtail C & & 1 \\
\quad 1 & B \rightarrowtail C, C \rightarrowtail A, & & 2 \\
\quad 2 & B \rightarrowtail C & & 1 \\
\quad 1 & A \rightarrowtail C, C \rightarrowtail B, & & 2 \\
\quad 2 & C \rightarrowtail A & & 1 \\
\quad 1 & B \rightarrowtail C, C \rightarrowtail A, & & 2 \\
\quad 3 & C \rightarrowtail B & 1 & sum=18 \\
\quad 1 & A \rightarrowtail C, C \rightarrowtail B, & & 2 \\
\quad 2 & A \rightarrowtail C & & 1 \\
\quad 1 & B \rightarrowtail C, C \rightarrowtail A, & & 2 \\
\quad 2 & C \rightarrowtail B & & 1 \\
\quad 1 & A \rightarrowtail C, C \rightarrowtail B & 2 & sum=26 \\
\quad \vdots & & & k=n1 \quad A \rightarrowtail C, C \rightarrowtail B \quad ?
\end{array}
$$

从前面的移动可以看出 $f(n) = 3*f(n-1)+2$, 设 $g(n) = f(n)+1, g(1) = f(1)+1 = 3, g(n) = 3g(n-1)$. $g(n) = 3^n$, $f(n) = 3^n - 1$.