one numbers and games reading notes

weiyuan

2021.11.14

Chapter 1

All numbers gerat and small

Construction

Two sets of numbers L, R

no member of L is $\geq$ any members of $\mathbb{R}$

then there is a number$\{L|R\}$

All numbers are constructed in this way.

Convension

If $x \in \{L|R\}$ we write x^L for the typical member of L, and x^R for the typical member of $\mathbb{R}$. For x itself we then write $\{x^L|x^R\}$

$$x = \{a, b, c, \dots | d, e, f, \dots\} \tag{1.1}$$

means that $x \in \{L|R\}$, where $a, b, c, \dots$ are the typical members of L, and $d, e, f, \dots$ the typical members of R

Definition 1. $x \geq y, x \leq y$

$$x \geq y \quad \Longleftrightarrow \quad no\, x^R \leq y\, and\, x \leq no\, y^L$$

and $x \leq y \iff y \geq x$ We write $x \ngeq y$ to mean that $x \leq y$ does not hold

Definition 2. $x = y, x > y, x < y$

$$x = y \iff x \geq y\, and\, y \geq x$$
$$x > y \iff x \geq y\, and\, y \ngeq x$$
$$x < y \iff y > x$$

Definition 3. $x + y$

$$x + y = \{x^L + y, x + y^L | x^R + y, x + y^R\}$$

Definition 4. $-x$

$$-x = \{-x^R | -x^L\}$$

Definition 5. $x \cdot y$

$$x \cdot y = \{x^L + y + xy^L - x^L y^L, x^R y + xy^R - x^R y^R |$$
$$x^L y + xy^R - x^L y^R, x^R y + xy^L - x^R y^L\}$$

证明.

$$xy > x^L y + xy^L - x^L y^L \tag{1.2}$$
$$\because (x - x^L)(y - y^L) > 0 \tag{1.3}$$

$\square$