Finite Element Method

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1 Exercise 20: Compute the deflection of a cable with 1 P2 elements

The shape functions are

$$\varphi_A = 2(x-0.5)(x-1), \quad \varphi_B = -4x(x-1), \quad \varphi_C = 2x(x-0.5)$$

Based on the scheme in Exercise 17, the linear system is

$$\begin{pmatrix} 7/3 & -8/3 & 1/3 \\ -8/3 & 16/3 & -8/3 \\ 1/3 & -8/3 & 7/3 \end{pmatrix} \begin{pmatrix} u_A \\ u_B \\ u_C \end{pmatrix} = \begin{pmatrix} -1/6 \\ -2/3 \\ -1/6 \end{pmatrix}$$

Since $u_A = 0$, the linear system become

$$\begin{pmatrix} 16/3 & -8/3 \\ -8/3 & 7/3 \end{pmatrix} \begin{pmatrix} u_B \\ u_C \end{pmatrix} = \begin{pmatrix} -2/3 \\ -1/6 \end{pmatrix}$$

$$u_B = -0.375, \qquad u_C = -0.5$$

$$\hat{u} = -0.375\varphi_B - 0.5\varphi_C = 0.5x^2 - x.$$

We find that $\hat{u}$ equals to the exact solution.