Finite Element Method

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1 Exercise 19: Compute the deflection of a cable with 2 P1 elements

We use 2 P1 elements. See figure1.
For element1, the shape functions are

$$\begin{cases} \varphi_A^{(1)} = 1 - 2x & 0 \le x \le 0.5 \\ \varphi_B^{(1)} = 2x & 0 \le x \le 0.5 \end{cases} \tag{1}$$

For element2, the shape functions are

$$\begin{cases} \varphi_A^{(2)} = -2x + 2 & 0.5 < x \le 1 \\ \varphi_B^{(2)} = 2x - 1 & 0.5 < x \le 1 \end{cases} \tag{2}$$

For element1, the stiffness matrix

$$A^{(1)} = \begin{pmatrix} \int_0^{0.5} \varphi_A'^{(1)} \varphi_A'^{(1)} dx & \int_0^{0.5} \varphi_B'^{(1)} \varphi_A'^{(1)} dx \\ \int_0^{0.5} \varphi_A'^{(1)} \varphi_B'^{(1)} dx & \int_0^{0.5} \varphi_B'^{(1)} \varphi_B'^{(1)} dx \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$$

$$b^{(1)} = \begin{pmatrix} -0.25 \\ -0.25 \end{pmatrix}$$

For element2, the stiffness matrix

$$A^{(2)} = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}, \qquad b^{(2)} = \begin{pmatrix} -0.25 \\ -0.25 \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & -2 & 0 \\ -2 & 4 & -2 \\ 0 & -2 & 2 \end{pmatrix}, \qquad b = \begin{pmatrix} -0.25 \\ -0.5 \\ -0.25 \end{pmatrix}$$

From the boundary condition, we know that $u_A = 0$, the linear system change into

$$\begin{pmatrix} 4 & -2 \\ -2 & 2 \end{pmatrix} \begin{pmatrix} u_B \\ u_C \end{pmatrix} = \begin{pmatrix} -0.5 \\ -0.25 \end{pmatrix}$$

$$u_B = -0.375, \qquad u_C = -0.5.$$

$$\hat{u} = \begin{cases} -0.75x & 0 \le x \le 0.5 \\ -0.25x - 0.25 & 0.5 < x \le 1 \end{cases} \tag{3}$$

The results are shown in Figure 2.

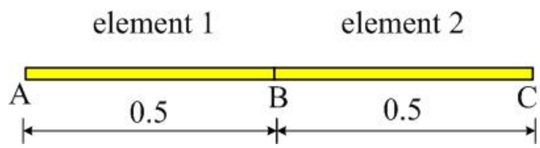

Figure 1: 2 P1 elements

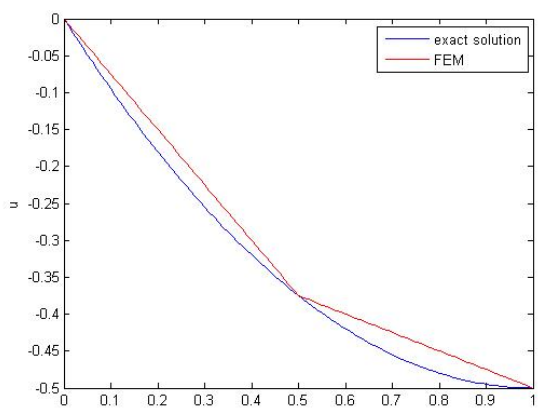

Figure 2: Finite element approximation for $u'' = 1$ using 2 P1 elements