Finite Element Method

Wei Zhang

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1 Exercise 18: Check integration by parts

The problem is

$$u'' = 1, x \in (0,1), \qquad u(0) = 0, \quad u'(1) = 0 \tag{1}$$

$$u \approx \hat{u} = 0 + \sum_{j=1}^{N} c_j N_j(x)$$

with $N_j(0) = 0$.

We choose weighting function $W_i = N_i$. This leads to:

$$\sum_{j=1}^{N} \left(\int_0^1 N_i N_j''(x) dx \right) c_j = \int_0^1 N_i dx, \qquad i = 1, \ldots, N. \tag{2}$$

Integrating by parts on the left side

$$-\sum_{j=1}^{N} \left(\int_0^1 N_i' N_j'(x) dx \right) c_j + N_i(1)\hat{u}'(1) - N_i(0)\hat{u}'(0) = \int_0^1 N_i dx, \quad i = 1, \ldots, N. \tag{3}$$

Since the integral only with respect to x, c_j are independent of whether we perform integration by part or not.