Finite Element Method

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1 Exercise 3: Approximate a three-dimensional vector in a plane

We extend the unit vectors $\boldsymbol{\varphi}_0 = (1,0)$ and $\boldsymbol{\varphi}_1 = (0,1)$ to (1, 0, 0) and (0, 1, 0) respectively. The vector $\mathbf{u} = \sum_{i=0}^{1} c_i \boldsymbol{\varphi}_i$, the error $\mathbf{e} = \mathbf{f} - \mathbf{u}$. We should choose coefficients c_0, c_1 to get the best approximation. According to the Galerkin method, we obtain

$$(\mathbf{e}, \boldsymbol{\varphi}_i) = 0 \tag{1}$$

$$\begin{cases} (\mathbf{f} - c_0\boldsymbol{\varphi}_0 - c_1\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_0) = 0 \\ (\mathbf{f} - c_0\boldsymbol{\varphi}_0 - c_1\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_1) = 0 \end{cases}$$

$$\begin{cases} (1 - c_0, 1 - c_1, 1) \cdot (1,0,0) = 0 \\ (1 - c_0, 1 - c_1, 1) \cdot (0,1,0) = 0 \end{cases}$$

$c_0 = 1$, $c_1 = 1$. $\mathbf{u} = (1,1)$.

The same principle applies to vectors (2,1) and (1,2). We get

$$\begin{cases} (1 - 2c_0 - c_1, 1 - c_0 - 2c_1, 1) \cdot (2,1,0) = 0 \\ (1 - 2c_0 - c_1, 1 - c_0 - 2c_1, 1) \cdot (1,2,0) = 0 \end{cases}$$

$c_0 = \frac{1}{3}$, $c_1 = \frac{1}{3}$. $\mathbf{u} = (1,1)$.