CS280-Data Structures

Introductory Sorting Part 1

Recap

- Algorithm complexity analysis
- Big *O*h notation
  - Asymptotic analysis
  - Common growth rates
- Program complexity derivation
  - Sequences
  - Conditionals
  - Loops
  - Function calls

Sorting Algorithm

- Input: A sequence of n numbers $\{a_1,a_2,\ldots,a_n\}$
- Output: A permutation $\{a_1',a_2',\ldots,a_n'\}$ of the input sequence such that $a_1' \le a_2' \le \ldots \le a_n'$ or $a_1' \ge a_2' \ge \ldots \ge a_n'$
  - Increasing order
  - Decreasing order

Basics of Sorting

- $A = \{a_1, a_2,\ldots,a_n\}$, $n \geq 0$ elements from some universal set U
- Sorting algorithms aim to rearrange A to produce A' where each $a_i \in A'$, $i=1,2,\ldots,n$ are in order.

In Order

- What does it mean by in order?
- Numbers:
  - 1, 2, 8, 20,10000
- Strings:
  - Alvin, Edward, Michael, Prabhu, Prasanna, Zhaoyan
- struct Student {

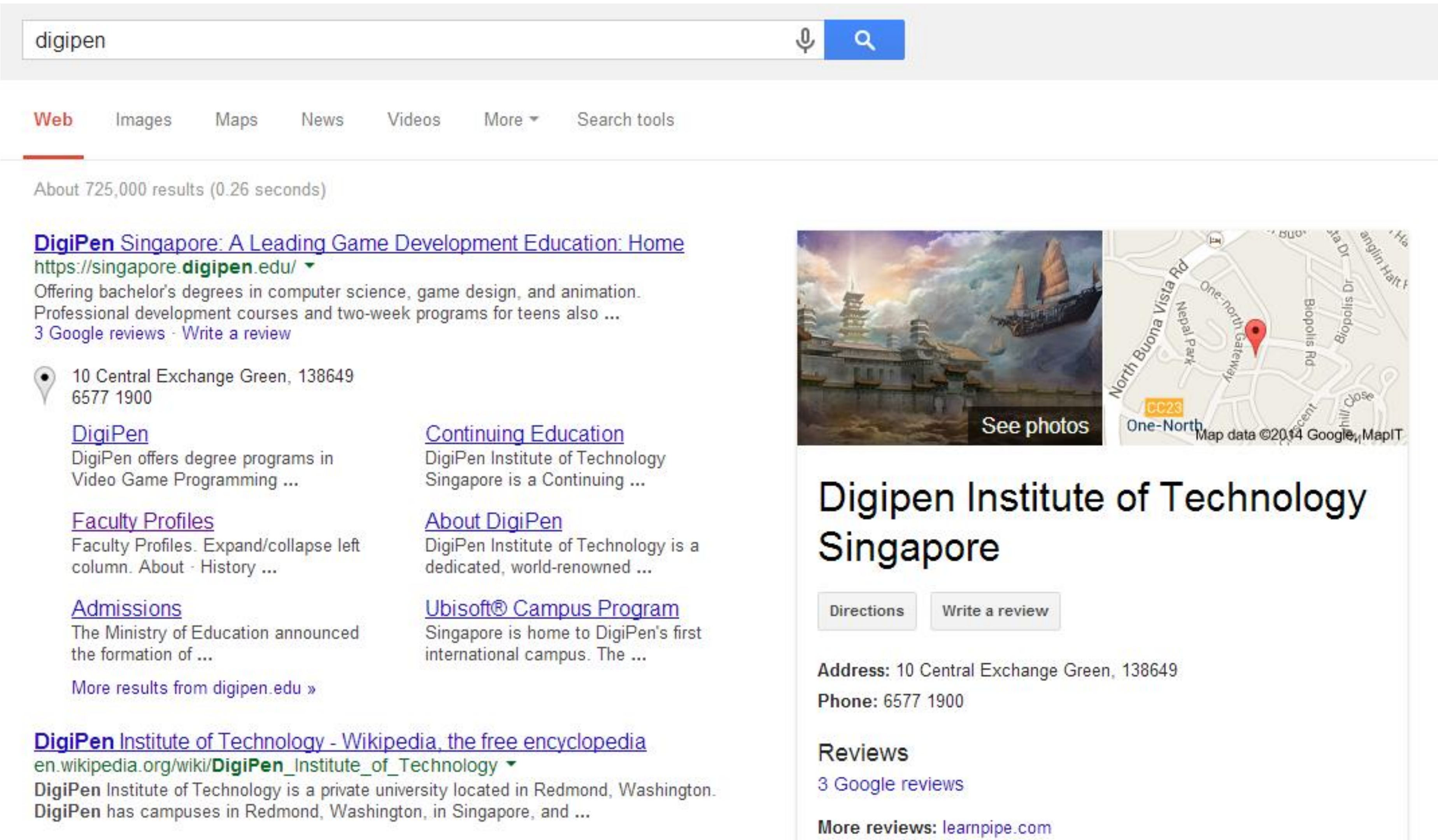
```
struct Student {
    int Age;
    long Year;
    float GPA;
    long ID;
};
```

Usually the **Keys** of Items are in numerical or alphabetical order.

Usefulness in Practice

- Google search
  - Relevance: Similarity between the query and the webpage
  - Importance: PageRank
- Text search
- Image search
  - Content-based image retrieval
  - Semantic-based image retrieval
- Recommendations
- etc.

Usefulness in Practice



Usefulness in Practice

- Google search
  - Relevance: Similarity between the query and the webpage
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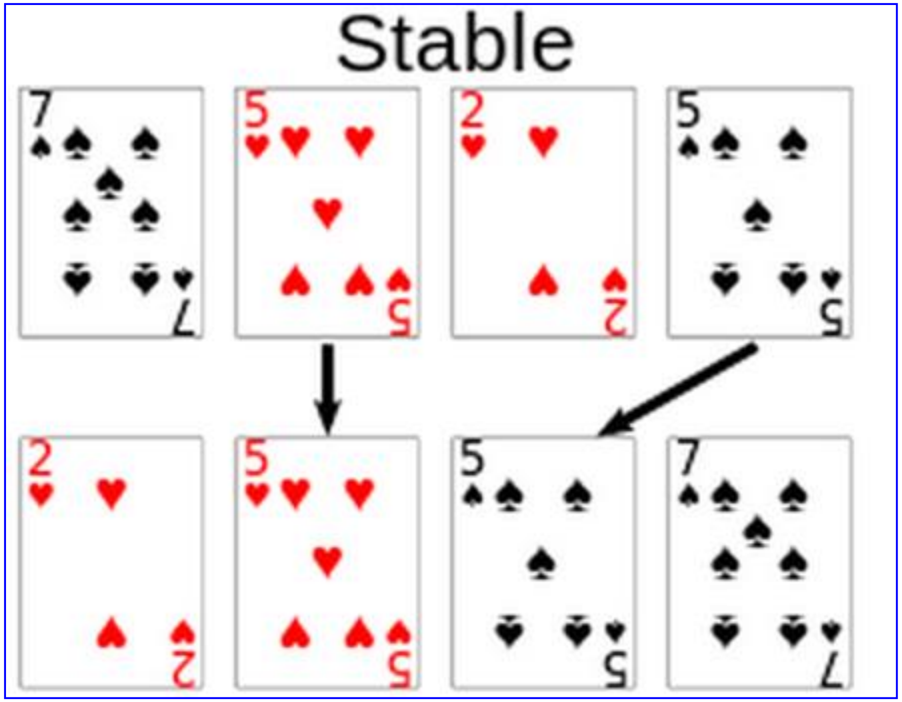
- etc.

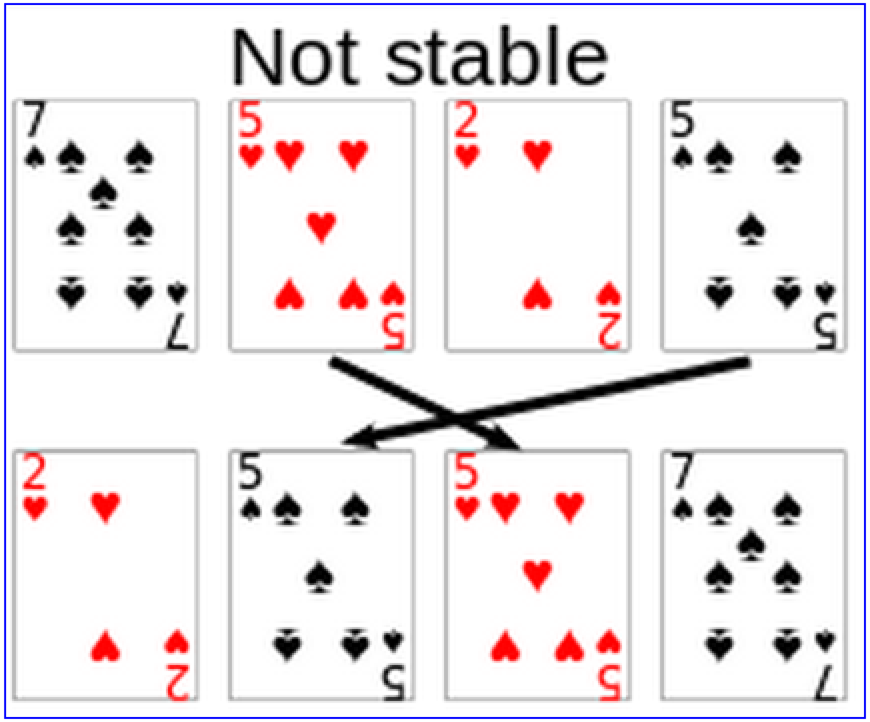
Attributes of Sorting Algorithms

- Stable/unstable sorting algorithms
- Loop invariant
- In-place/Out-of-place algorithms
- Adaptive

Stable Sorting Algorithm

- A sorting method is said to be stable if it preserves the relative order of items with equal values.





Loop Invariant

- Loop invariant is a property that holds before (and after) each repetition

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```
int max(int n, const int a[n]) {
  int m = a[0];
  // m equals the maximum value in a[0...0]
  int i = 1;
  while (i != n) {
       // m equals the maximum value in a[0...i-1]
       if (m < a[i])
           m = a[i];
       // m equals the maximum value in a[0...i]
       ++i;
       // m equals the maximum value in a[0...i-1]
   }
   // m equals the maximum value in a[0...i-1], and i==n
 return m;
}
```

In-place/Out-of-place algorithms

```
void f1(int a[],int n){
   int* b = new int[n];
   int i;
   for(i=0;i<n;++i)
       b[n-i-1] = a[i];
   for(i=0;i<n;++i)
       a[i]=b[i];
   delete [] b;
}
```

```
void f2(int a[], int n){
   int i;
   int m=n/2;
   for(i=0;i<m;++i)
       Swap(a[i],a[n-i-1]);
}
```

Adaptive

- It takes advantage of existing order in its input
- It benefits from the pre-sortedness in the input sequence

- $A_1[9]=\{9,8,7,6,5,4,3,2,1\}$
- $A_2[9]=\{2,1,3,4,5,6,7,8,9\}$

Sorting Algorithms

- Bubble sort
- Selection sort
- Insertion sort
- Merge sort
- Quick sort
- Lower bounds for sorting
- Counting sort

Sorting Algorithms

- Bubble sort
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Sorting Algorithms

- Bubble sort
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Bubble Sort

- Exchange Sort: switch a pair of neighboring elements
- Main idea: Keep bubbling the largest element to the end.

1. Get the current i and i+1 elements.
2. Swap them into the correct order.
3. Start from front again when reach the end.

Example

- A[9]={7, 4, 1, 3, 8, 6, 9, 5, 2}
- objective: A’[9]={1, 2, 3, 4, 5, 6, 7, 8, 9}

Bubble Sort

```
void BubbleSort(int a[], int N){
  for (int i = 0; i < N - 1; ++i)
    for (int j = 0; j < N - i - 1; ++j)
      if (a[j] > a[j + 1])
            Swap(a[j], a[j + 1]);
}

void Swap(int &a, int &b){
  int temp = a;
  a = b;
  b = temp;
}
```

Stable?

Loop invariant?

In-place?

Adaptive?

Bubble Sort

```
void BubbleSort(int a[], int N){
  for (int i = 0; i < N - 1; ++i)
    for (int j = 0; j < N - i - 1; ++j)
      if (a[j] > a[j + 1])
            Swap(a[j], a[j + 1]);
}

void Swap(int &a, int &b){
  int temp = a;
  a = b;
  b = temp;
}
```

Stable: Y

Loop invariant: Y

In-place: Y

Adaptive: N

Bubble Sort

```
void BubbleSort(int a[], int N){
  for (int i = 0; i < N - 1; ++i)
    for (int j = 0; j < N - i - 1; ++j)
      if (a[j] > a[j + 1])
            Swap(a[j], a[j + 1]);
}

void Swap(int &a, int &b){
  int temp = a;
  a = b;
  b = temp;
}
```

Best case complexity: ?

Worst case complexity: ?

Question: How can we improve the algorithm?

(hint: consider the best case)

Bubble Sort

```
void BubbleSort(int a[], int N){
  for (int i = 0; i < N - 1; ++i)
    for (int j = 0; j < N - i - 1; ++j)
      if (a[j] > a[j + 1])
            Swap(a[j], a[j + 1]);
}

void Swap(int &a, int &b){
  int temp = a;
  a = b;
  b = temp;
}
```

Best case complexity: O(N)

Worst case complexity: O(N^2)

Question: How can we improve the algorithm?

(hint: consider the best case)

Adaptive Bubble Sort

- If the algorithm stops when no swaps, then best case is O(N)

```
void BubbleSort(int a[], int N){
  for (int i = 0; i < N - 1; ++i){
    bool swap = false;
    for (int j = 0; j < N - i - 1; ++j)
      if (a[j] > a[j + 1]){
        swap=true;
        Swap(a[j], a[j + 1]);
      }
    }
    if(!swap)
      return;
}
```

Sorting Algorithms

- Bubble sort
- Selection sort
- Insertion sort

Selection Sort

- Main idea: Increase the sorted sequence by selecting the smallest element from the unsorted.

1. Select the smallest element from the unsorted side
2. Append it at the end of the sorted side (or the first position of the unsorted)

Find the Smallest, then the second smallest from the rest…

Selection Sort Example

- Input: A[9]={7, 4, 1, 3, 8, 6, 9, 5, 2}
- Output: A'[9]={1, 2, 3, 4, 5, 6, 7, 8, 9}

Selection Sort

```
void SelectionSort(int a[], int N){
  for (int i = 0; i < N-1; ++i){
    int min = i;
    int j;

    for (j = i + 1; j < N; ++j)
      if (a[j] < a[min])
        min = j;

    Swap(a[min], a[i]);
  }
}
```

Stable?

Loop invariant?

In-place?

Adaptive?

Selection Sort

```
void SelectionSort(int a[], int N){
  for (int i = 0; i < N-1; ++i){
    int min = i;
    int j;

    for (j = i + 1; j < N; ++j)
      if (a[j] < a[min])
        min = j;

    Swap(a[min], a[i]);
  }
}
```

Stable: N

Loop invariant: Y

In-place: Y

Adaptive: N

Selection Sort

```
void SelectionSort(int a[], int N){
  for (int i = 0; i < N-1; ++i){
    int min = i;
    int j;

    for (j = i + 1; j < N; ++j)
      if (a[j] < a[min])
        min = j;

    Swap(a[min], a[i]);
  }
}
```

Best case complexity:?

Worst case complexity: ?

Question: How could you modify this algorithm to sort the array in half the time? (Not really half the time, but half the number of passes.)

Selection Sort

```
void SelectionSort(int a[], int N){
  for (int i = 0; i < N-1; ++i){
    int min = i;
    int j;

    for (j = i + 1; j < N; ++j)
      if (a[j] < a[min])
        min = j;

    Swap(a[min], a[i]);
  }
}
```

Best case complexity: O(N^2)

Worst case complexity: O(N^2)

Question: How could you modify this algorithm to sort the array in half the time? (Not really half the time, but half the number of passes.)

Improved Selection Sort

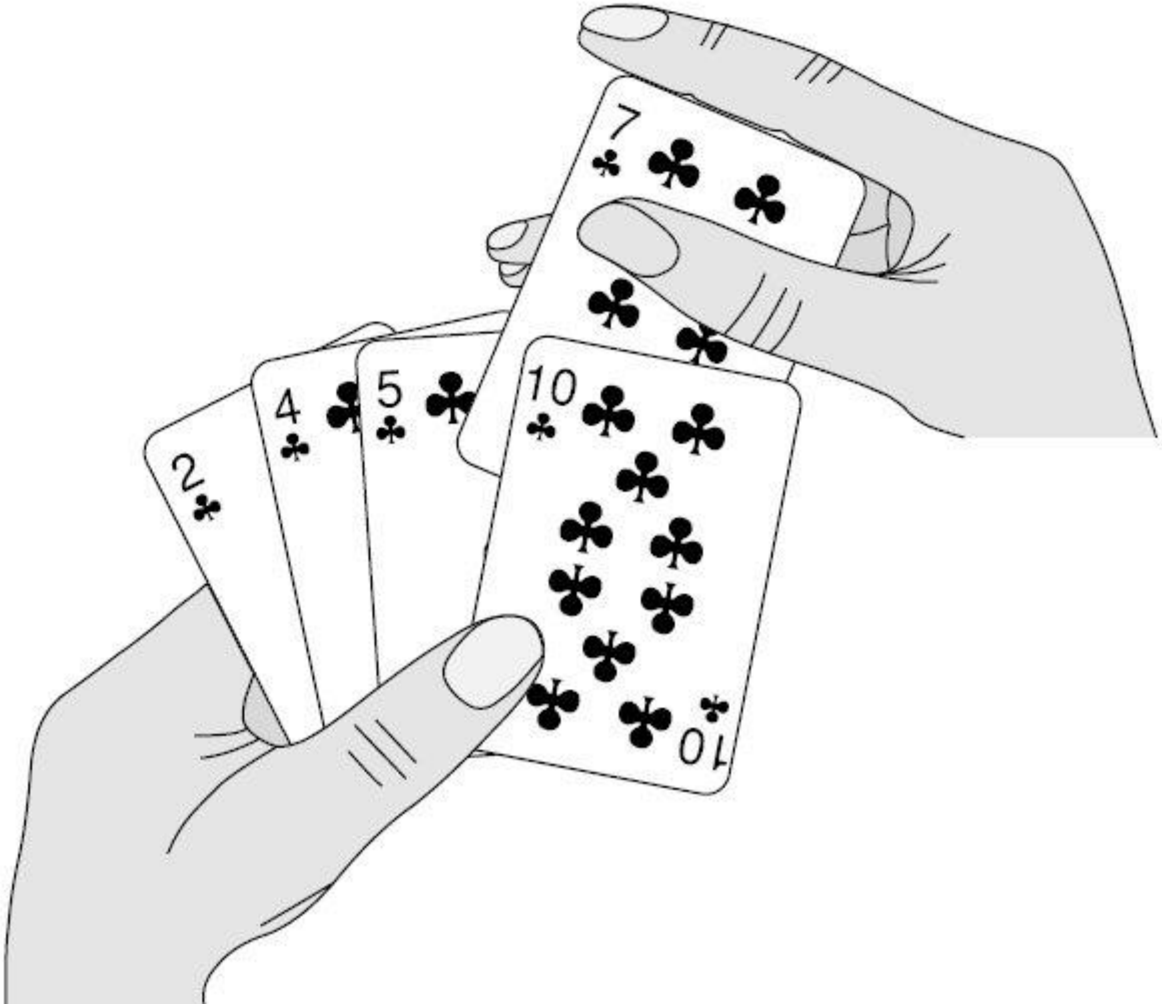
- Find the minimum and maximum at the same time!

```
void BiSelectionSort(int a[], int N){
  for (int i = 0; i < N/2; ++i){
    int min = i;
    int max = N-i-1;
    int j;
    for (j = i; j < N-i; ++j){
      if (a[j] < a[min])
         min = j;
      if (a[j] > a[max])
         max = j;
    }
    Swap(a[min], a[i]);
    if (i==max)
       Swap(a[min], a[N-i-1]);
    else
       Swap(a[max], a[N-i-1]);
 } }
```

Sorting Algorithms

- Bubble Sort
- Selection Sort
- Insertion Sort

Insertion Sort



Insertion Sort

- Main idea: Keep inserting next element into the sorted sequence.

1. Get the 1st element from the unsorted side
2. Find the correct position in the sorted side
3. Shift elements to make room

Insertion Sort Example

- A[9]={7, 4, 1, 3, 8, 6, 9, 5, 2}
- Objective: A'[9]={1, 2, 3, 4, 5, 6, 7, 8, 9}

Insertion Sort

```
void InsertionSort(int a[], int N){
  for (int i = 1; i < N; ++i){
    int j = i;
    int current = a[i];

    while ((j > 0) && (a[j-1] > current)){
      a[j] = a[j-1];
      --j;
    }
    a[j] = current;
  }
}
```

Stable?

Loop invariant?

In-place?

Adaptive?

Insertion Sort

```
void InsertionSort(int a[], int N){
  for (int i = 1; i < N; ++i){
    int j = i;
    int current = a[i]; // the item to be inserted

    while ((j > 0) && (a[j-1] > current)){
      a[j] = a[j-1];
      --j;
    } // find the position for insertion: j
    a[j] = current;
  }
}
```

Stable: Y

Loop invariant: Y

In-place: Y

Adaptive: Y

Insertion Sort

```
void InsertionSort(int a[], int N){
  for (int i = 1; i < N; ++i){
    int j = i;
    int current = a[i];

    while ((j > 0) && (a[j-1] > current)){
      a[j] = a[j-1];
      --j;
    }
    a[j] = current;
  }
}
```

Best case complexity?

Worst case complexity?

Insertion Sort

```
void InsertionSort(int a[], int N){
  for (int i = 1; i < N; ++i){
    int j = i;
    int current = a[i];

    while ((j > 0) && (a[j-1] > current)){
      a[j] = a[j-1];
      --j;
    }
    a[j] = current;
  }
}
```

Best case complexity: $O(N)$

Worst case complexity: $O(N^2)$

Summary

- Elementary Sorting Methods
  - Bubble sort
  - Selection sort
  - Insertion sort

Comparison

- Bubble sort and variants
  - rarely used in practice
  - commonly found in teaching and theoretical discussions.
- Insertion sort
  - generally faster than selection sort in practice, due to fewer comparisons and good performance on almost-sorted data
  - preferred in practice.
  - is relatively efficient for small lists and mostly sorted lists.
- Selection sort
  - Does no more than *n* swaps, and thus is useful where swapping is very expensive.
  - uses fewer writes, and thus is used when write performance is a limiting factor.