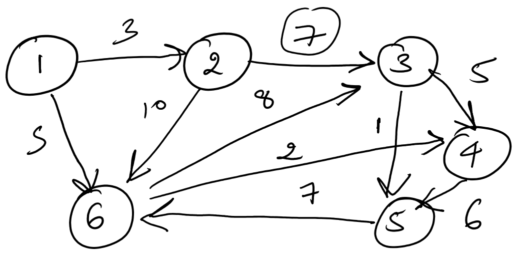
Shortest Path

[redacted]

#9



- Paths from (1) to (5)

Path	Cost
1 2 3 4 5	21
1 2 3 5	11
1 2 6 3 5	22
1 2 6 3 4 5	32
1 2 6 4 5	21
1 6 3 5	14
1 6 3 4 5	24
1 6 4 5	13

How to find shortest path from (1) to (5) ?

DIJKSTRA'S ALGO

Find the shortest path from source node to all other nodes in the graph with no negative edge costs

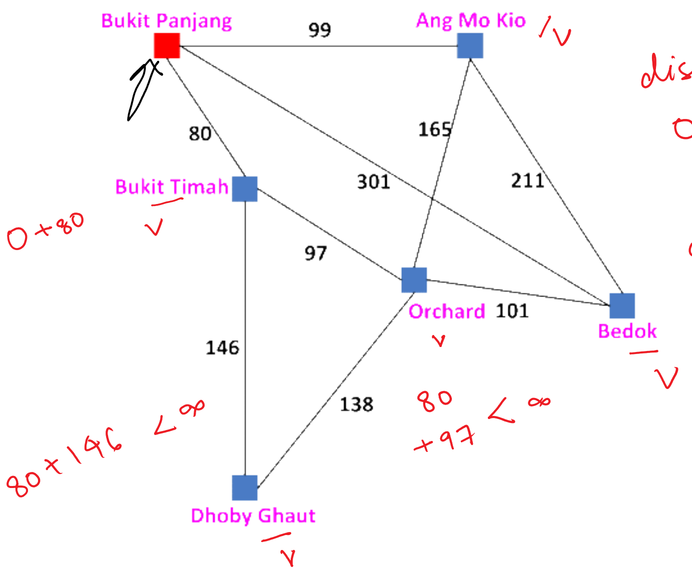
```
Dijkstra (Graph G, Source) {
        dist [source] = 0
        for (each vertex V in graph) {
Initialization
            if (V ≠ source)
            { dist [V] = ∞
                prev [V] = NA
            PQ. add with priority (V, dist[V]) // $T_{push}$
        }
    while ( PQ. is not empty)
    {
```

→ dist [A]
↓
cost of path from source to node 'A'

→ prev [A]
↓
records the prev node on the path source to

the path
Source to
node 'A'

```
while( PQ. is not empty)
{
    U = PQ. extractMin (); //Tpop
    for(each neighbor 'v' of u)
    { if( dist [u] + cost (U,V) < dist[v])
        { dist[v] = dist[u] + cost(u,v);
          prev [v] = u
        }
    PQ. decrease key [v, dist[v]) // Tupdate
}
```



$dist[u] + cost(u,v)$

$0 + 99 < \overset{dist(v)}{\infty}$

$99+165 < 177$

$99+211 < 301$

previous[BP]	=	NA
previous[AMK]	=	~~NA~~ BP
previous[BT]	=	~~NA~~ BP
previous[O]	-	~~NA~~ BT
previous[B]	=	~~NA~~ BP
previous[DG]	-	~~NA~~ BT

dist[BP]	=	0
dist[AMK]	=	~~+∞~~ 99
dist[BT]		~~+∞~~ 80
dist[O]	=	~~+∞~~ 177
dist[B]	-	~~+∞~~ 301
dist[DG]	=	~~+∞~~ 226

$PQ=\{\overset{u}{BP}(0), AMK(+\overset{99}{\cancel{\infty}}), BT(+\overset{80}{\cancel{\infty}}), O(+\infty), B(+\overset{301}{\cancel{\infty}}), DB(+\infty)\}$

$PQ=\{ \underset{\uparrow u}{BT(80)}, AMK(99), B(301), O(\overset{177}{\infty}), DB(\overset{226}{\infty})\}$

$PQ= \{ \underline{AMK(99)}, O(177), DB(226), B(301)\}$

Time complexity.

General = |V| * Tpush + |V| Tpop + |E| Tupdate.

exp

Arrays

$|V| * C + |V| * V$

$+ |E| * C$

$O(V^2)$

Heaps

$|V| * \log(V) + |V| * C$

$+ |E| * \log V$

$O(E \log V)$