observation 1: women: sequence of man she's engaged to ↗

observation 2: men: sequence of women he proposes to ↙

Correctnes Analysis

if <u>m</u> is free, there exists a <u>w</u> to whom he hasn't proposed yet.	→	perfect matching	→	Stable Perfect matching
Lemma 1		Lemma 2		Lemma 3

Lemma1: if <u>m</u> is free, there exists a <u>w</u> to whom <u>m</u> hasn't proposed yet.

contradiction: m is free and he's already proposed to every women. → (implies)

every women are engaged →

n pairs in the matching, n ~~m~~ women, n−1 men → (m', w) (m', w')

contradict with "no two women marry to one man"

□

Lemma2: G-S results in a perfect matching S.

Contradiction: S is not a perfect matching →

m is free and/or w is free

Look at condition of loop termination. Either

① no free man → no free woman → contradicts with a w is free □ or

② there is free man, but he's propsosed to every woman → contradicts with Lemma 1

□

Lemma 3: G-S results in a stable perfect matching S.

contradiction: S is instable $\rightarrow$

(m, w) s.t. (subject to) m prefers w' to w

(m', w') w' prefers m to m'

in terms of m's preference list, m proposes to w' first, and ~~at~~ later, m is rejected by w'.

rejection1: (m, w'), and then m' proposes to w', w' reject m, accept m' (m', w')

$\rightarrow$ ~~con~~ w' prefers m' to m

$\rightarrow$ contradicts with assumption. w' prefers m to m' $\square$

rejection2: (m'', w'), w' engaged with m'' and reject m $\rightarrow$ w': $m'' > m$

w': $m > m'$ $\rightarrow$ $m'' > m > m'$ $\rightarrow$

sequence of man w' engaged to $(m'', \ldots m')$?.

$\rightarrow$ contradict with observation 1 $\rightarrow$

(woman should get better and better man) $\square$

Extention. Analysis

Theorem 3 G-S results in ~~a~~ matching S^*.

In S^*

$(m, w = best(m))$

~~$\Rightarrow m: w$~~

~~Exe~~cution $\mathcal{E}$ returns S

* m is rejected by w because of a better man m'

$\rightarrow$ $w: (m' > m)$

* it is also the first time a man is rejected by a woman

$\rightarrow$ $m': (w > w')$

any other woman

S' by G-S, also

$(m \; w)$

$(m' \; w')$

$w: (m' > m)$ and $m': (w > w')$

$\rightarrow$ Instability.

$\rightarrow$ contradicts with Lemma 3