

Quiz-3Answer **ALL** the questions

Total marks - 20

* Always round off your results to two places after the decimal point whenever necessary. For example, 12.34556 becomes 12.35, and 12.34456 becomes 12.34.

1. Compute DFT of the following function: $f(x) = 5 \delta(x)$ for $x = 0, 1, 2$ and 3 where $\delta(x)$ is unit impulse function. Fill the following table. **(0.5 x 8 = 4 marks)**

DFT of $f(x)$

x	$f(x)$	$F(u)$
0	5	5
1	0	5
2	0	5
3	0	5

Hint: Unit impulse is 1 when $x=0$ and it is 0 elsewhere.

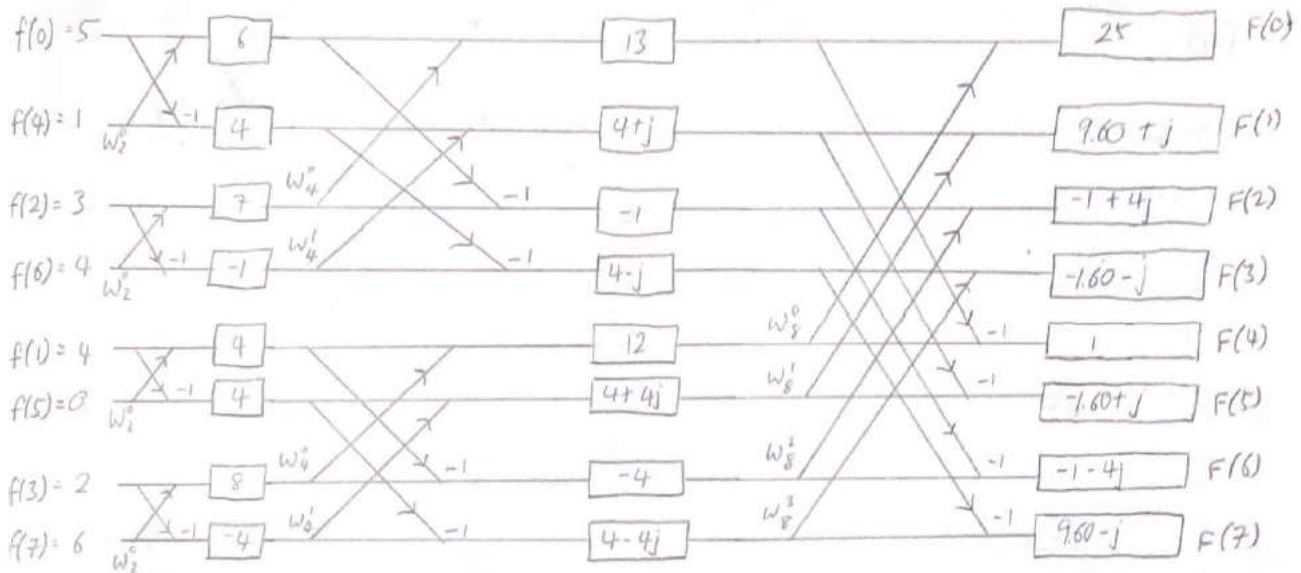
2. Compute DFT for the following $f(x)$ and fill in the table below: **(0.5 x 16 = 8 marks)**

$$f(x) = [j, 0, j, 1].$$

$F(u)$	Magnitude	Phase	Power spectrum
$F(0) = 1 + 2j$	$\text{Sqrt}(2^2 + 1^2) = \text{Sqrt}(5) = 2.24$ (2d.p)	$\text{Tan}^{-1}(2/1) = 1.11$ (2d.p)	$2^2 + 1^2 = 5$
$F(1) = j$	$\text{Sqrt}(1^2 + 0^2) = \text{Sqrt}(1) = 1$	$\text{Tan}^{-1}(1/0) = 0$	$1^2 + 0^2 = 1$
$F(2) = -1 + 2j$	$\text{Sqrt}(2^2 + (-1)^2) = \text{Sqrt}(5) = 2.24$ (2d.p)	$\text{Tan}^{-1}(2/(-1)) = -1.11$ (2d.p)	$2^2 + (-1)^2 = 5$
$F(3) = -j$	$\text{Sqrt}((-1)^2 + 0^2) = \text{Sqrt}(1) = 1$	$\text{Tan}^{-1}((-1)/0) = 0$	$(-1)^2 + 0^2 = 1$

3. Compute Fourier Transform for the following $f(x)$ using FFT method. Show all the stages, intermediate results, and the final results in the Butterfly approach clearly. Write your $F(u)$ in two decimal places. (1 x 8 = 8 marks)

x	0	1	2	3	4	5	6	7
$f(x)$	5	4	3	2	1	0	4	6
$F(u)$	25	$9.60 + j$	$-1 + 4j$	$-1.60 - j$	1	$-1.60 + j$	$-1 - 4j$	$9.60 - j$



$$W_2^0 = W_4^0 = W_8^0 = 1 \quad W_4^1 = -j \quad W_8^1 = 0.707 - 0.707j \quad W_8^2 = j \quad W_8^3 = -0.707 - 0.707j$$

Write your Butterfly approach diagram here

-----Rough Work Sheet-----

(Do not detach it)

Q1.

$$\begin{aligned}
 F(0) &= \sum_{x=0}^3 f(x) = f(0) + f(1) + f(2) + f(3) \\
 &= 5(1) + 5(0) + 5(0) + 5(0) \\
 &= 5 \\
 F(1) &= \sum_{x=0}^3 f(x) e^{-j2\pi(1)x/4} = 5(1)(1) + 5(0)e^{-j\pi/2} + 5(0)e^{-j\pi} + 5(0)e^{-j3\pi/4} \\
 &= 5 + 0 + 0 + 0 = 5 \\
 F(2) &= \sum_{x=0}^3 f(x) e^{-j2\pi(2)x/4} \\
 &= \sum_{x=0}^3 f(x) e^{-j\pi x} = 5(1)(1) + 5(0)e^{-j\pi} + 5(0)e^{-j2\pi} + 5(0)e^{-j3\pi} \\
 &= 5 + 0 + 0 + 0 = 5 \\
 F(3) &= \sum_{x=0}^3 f(x) e^{-j2\pi(3)x/4} \\
 &= \sum_{x=0}^3 f(x) e^{-j3\pi x/2} = 5(1)(1) + 5(0)e^{-j3\pi/2} + 5(0)e^{-j3\pi} + 5(0)e^{-j9\pi/2} \\
 &= 5 + 0 + 0 + 0 \\
 &= 5
 \end{aligned}$$

Q2.

$$\begin{aligned}
 f(x) &= [j, 0, j, 1] \\
 F(0) &= \sum_{x=0}^3 f(x) = f(0) + f(1) + f(2) + f(3) \\
 &= j + 0 + j + 1 \\
 &= 2j + 1 \\
 F(1) &= \sum_{x=0}^3 f(x) e^{-j2\pi(1)x/4} \\
 &= f(0)e^{-j2\pi(0)/4} + f(1)e^{-j2\pi(1)/4} + f(2)e^{-j2\pi(2)/4} + f(3)e^{-j2\pi(3)/4} \\
 &= j + 0 + je^{j\pi} + e^{-j3\pi/2} \\
 &= j + j(\cos \pi - j \sin \pi) + (\cos \frac{3\pi}{2} - j \sin \frac{3\pi}{2}) \\
 &= j + j(-1 - j(0)) + (0 - j(-1)) \\
 &= j - j + j \\
 &= j \\
 F(2) &= \sum_{x=0}^3 f(x) e^{-j2\pi(2)x/4} \\
 &= \sum_{x=0}^3 f(x) e^{-j\pi x} \\
 &= f(0)e^{-j\pi(0)} + f(1)e^{-j\pi(1)} + f(2)e^{-j\pi(2)} + f(3)e^{-j\pi(3)} \\
 &= j + 0 + je^{-j2\pi} + e^{-j3\pi} \\
 &= j + j[\cos 2\pi - j \sin 2\pi] + [\cos 3\pi - j \sin 3\pi] \\
 &= j + j[1 - j(0)] + [-1 - j(0)] \\
 &= j + j - 1 \\
 &= 2j - 1 \\
 F(3) &= \sum_{x=0}^3 f(x) e^{-j2\pi(3)x/4} \\
 &= \sum_{x=0}^3 f(x) e^{-j3\pi x/2} \\
 &= f(0)e^{-j3\pi(0)/2} + f(1)e^{-j3\pi(1)/2} + f(2)e^{-j3\pi(2)/2} + f(3)e^{-j3\pi(3)/2} \\
 &= j + 0 + je^{-j3\pi} + e^{-j\frac{9\pi}{2}} \\
 &= j + j(\cos 3\pi - j \sin 3\pi) + (\cos \frac{9\pi}{2} - j \sin \frac{9\pi}{2}) \\
 &= j + j[(-1) - j(0)] + [0 - j(1)] \\
 &= j - j - j \\
 &= -j
 \end{aligned}$$

Q3.

$F(0) = x[0] + x[4] \omega_2^0$ $= 5 + 1(1)$ $= 6$	$F(4) = x[0] - x[4] \omega_2^1$ $= 5 - 1(1)$ $= 4$
$F(2) = x[2] + x[6] \omega_2^0$ $= 3 + 4(1)$ $= 7$	$F(6) = x[2] - x[6] \omega_2^1$ $= 3 - 4(1)$ $= -1$
$F(1) = x[1] + x[5] \omega_2^0$ $= 4 + 0(1)$ $= 4$	$F(5) = x[1] - x[5] \omega_2^1$ $= 4 - 0(1)$ $= 4$
$F(3) = x[3] + x[7] \omega_2^0$ $= 2 + 6(1)$ $= 8$	$F(7) = x[3] - x[7] \omega_2^1$ $= 2 - 6(1)$ $= -4$

$F(0) = x[0] + x[2] \omega_4^0$ $= 6 + 7(1)$ $= 13$	$F(2) = x[0] - x[2] \omega_4^2$ $= 6 - 7(1)$ $= -1$
$F(4) = x[4] + x[6] \omega_4^1$ $= 4 + (-1)(-j)$ $= 4 + j$	$F(6) = x[4] - x[6] \omega_4^3$ $= 4 - (-1)(-j)$ $= 4 - j$
$F(1) = x[1] + x[3] \omega_4^0$ $= 4 + 8(1)$ $= 12$	$F(3) = x[1] - x[3] \omega_4^2$ $= 4 - 8(1)$ $= -4$

$F(5) = x[5] + x[7] \omega_6^1$ $= 4 + (-4)(-j)$ $= 4 + 4j$	$F(7) = x[5] - x[7] \omega_6^5$ $= 4 - (-4)(j)$ $= 4 - 4j$
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$F(0) = x[0] + x[1] \omega_8^0$ $= 13 + 12(1)$ $= 25$	$F(1) = x[0] - x[1] \omega_8^8$ $= 13 - 12(1)$ $= 1$
$F(4) = x[4] + x[5] \omega_8^1$ $= (4+j) + (4+4j)(0.707 - 0.707j)$ $= 4+j + 2.8 - 2.8j + 2.8j - 2.8j^2$ $= 4+j + 2.8 - 2.8j^2$ $= -2.8j^2 + j + 6.8$ $= -2.8(-1) + j + 6.8 = 9.6 + j$	$F(5) = x[4] - x[5] \omega_8^7$ $= (4+j) - (4+4j)(0.707 - 0.707j)$ $= 4+j - (2.8 - 2.8j + 2.8j - 2.8j^2)$ $= 4+j - (2.8 - 2.8j^2)$ $= 4+j - 2.8 + 2.8j^2$ $= 2.8j^2 + j + 1.2$ $= 2.8(-1) + j + 1.2 = -1.6 + j$
$F(2) = x[2] + x[3] \omega_8^2$ $= (-1) + (-4)(-j)$ $= -1 + 4j$	$F(3) = x[2] - x[3] \omega_8^6$ $= (-1) - (-4)(-j)$ $= -1 - 4j$
$F(6) = x[6] + x[7] \omega_8^3$ $= (4-j) + (4-4j)(-0.707 - 0.707j)$ $= (4-j) + (-2.8 - 2.8j + 2.8j + 2.8j^2)$ $= 4-j - 2.8 + 2.8j^2$ $= 2.8j^2 - j + 1.2$ $= 2.8(-1) - j + 1.2 = -1.6 - j$	$F(7) = x[6] - x[7] \omega_8^5$ $= (4-j) - (4-4j)(-0.707 - 0.707j)$ $= (4-j) - (-2.8 - 2.8j + 2.8j + 2.8j^2)$ $= 4-j - (-2.8 + 2.8j^2)$ $= 4-j + 2.8 - 2.8j^2$ $= -2.8j^2 - j + 6.8$ $= -2.8(-1) - j + 6.8 = 2.8 - j + 6.8 = 9.6 - j$

-----**Rough Work Sheet**-----

(Do not detach it)