CS330 **Homework 1**

Topics covered:	Stable Matching Problem
Deliverables:	Write down your answers neatly on a piece of paper. Your name must be indicated clearly on the piece of paper. Then scan it into a `.pdf` file called cs330_yourid_1`.pdf`. If you have used more than one piece of paper, please scan them in order.
Objectives:	To demonstrate the ability to do simple proofs for different properties of the algorithm and apply the G-S algorithm in a similar but slightly different context.

Homework Questions

1. Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample. (10 points)

   True or false? Consider an instance of the Stable Matching Problem in which there exists a man m and a woman w such that m is ranked first on the preference list of w and w is ranked first on the preference list of m. Then in every stable matching S for this instance, the pair(m,w) belongs to S.
   Hint: Use contradiction

2. Let $M = m_1, m_2, m_3$ and $W = w_1, w_2, w_3$. Suppose that you are given the following preference lists:

   $m_1 < w_3\ w_2\ w_1 >$; $m_2 < w_2\ w_3\ w_1 >$; $m_3 < w_2\ w_3\ w_1 >$
   $w_1 < m_3\ m_1\ m_2 >$; $w_2 < m_1\ m_3\ m_2 >$; $w_3 < m_3\ m_1\ m_2 >$

   1) Give a stable perfect matching, and an instable perfect matching. [10 points]
   2) Find the best valid partner for each member of sets M and W. (Hint: Run G-S algorithm) [10 points]

3. Gale and Shapley published their paper on the Stable Matching Problem in 1962; but a version of their algorithm had already been in use for ten years by the National Resident Matching Program, for the problem of assigning medical residents to hospitals.

   Basically, the situation was the following. There were *m* hospitals, each with a certain number of available positions for hiring residents. There were *n* medical students graduating in a given year, each interested in joining one of the hospitals. Each hospital had a ranking of the students in order of preference, and each student had a ranking of the hospitals in order of preference. We will assume that there were more students graduating than there were slots available in the *m* hospitals.

   The interest, naturally, was in finding a way of assigning each student to at most one hospital, in such a way that all available positions in all hospitals were filled. (Since we

are assuming a surplus of students, there would be some students who do not get assigned to any hospital.)

We say that an assignment of students to hospitals is ***stable*** if neither of the following situations arises.

First type of instability: There are students *s* and *s*', and a hospital *h*, so that

- s is assigned to *h*, and
- *s*' is assigned to no hospital, and
- *h* prefers *s*' to *s*.

Second type of instability: There are students *s* and *s*', and hospitals *h* and *h'*, so that

- *s* is assigned to *h*, and
- *s*' is assigned to *h'*, and
- *h* prefers *s*' to s, and
- *s*' prefers *h* to *h'*.

There is an instance: 5 students S={1, 2, 3, 4, 5} and two hospitals H= {C, M}, each with 2 positions available. The preference is given below. Give a stable matching and an instable matching. (10 points)

C <5 1 2 4 3 >; M <5 3 1 2 4 >;
1 <C M >; 2 <C M >; 3 <C M >; 4 <M C>; 5 <M C>

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1) The statement is true. This is because W is ranked first on the preferred list of M and M is also ranked first on the preferred list of W. According to G-S Algorithm, regardless of which gender propose first, they will propose to the opposite gender in descending order of preference. Therefore, they will always be matched together first due to highest preference in each other's list and will reject the subsequence proposal due to lower preference. As a result, they will always be returned as pair (w, m) for every stable matching S.

2.1a) Stable perfect matching = {(M1, W3), (M2, W1), (M3, W2)}

Working:

- M1 propose to W3, W3 single. S = {(M1, W3)}
- M2 propose to W2, W2 single. S = {(M1, W3), (M2, W2)}
- M3 propose to W2, W2 engaged but W2 prefer M3 > M2. W2 break with M2. S = {(M1, W3), (M3, W2)}

- M2 single, propose to W3. W3 engage but prefer M1 > M2, nothing happens.
- M2 continue propose to W1, W1 single. S = {(M1, W3), (M2, W1), (M3, W2)}

2.1b) Instable perfect matching = {(M1, W1), (M2, W2), (M3, W3)}

Explanation:

- (M1, W1) is an element of the Set S
- (M2, W2) is an element of the Set S
- However, M1 prefers W2> W1 while W2 prefers M1 > M2
- Therefore, the above set is instable perfect matching

2.2) M = {(M1, W3), (M2, W1), (M3, W2)}

Working:

- M1 propose to W3, W3 single. S = {(M1, W3)}
- M2 propose to W2, W2 single. S = {(M1, W3), (M2, W2)}
- M3 propose to W2, W2 engaged but W2 prefer M3 > M2. W2 break with M2. S = {(M1, W3), (M3, W2)}

- M2 single, propose to W3. W3 engage but prefer M1 > M2, nothing happens.
- M2 continue propose to W1, W1 single. S = {(M1, W3), (M2, W1), (M3, W2)}

W = {(W1, M2), (W2, M1), (W3, M3)}

Working:

- W1 propose to M3, M3 single. S = {(W1, M3)}
- W2 propose to M1, M1 single. S = {(W1, M3), (W2, M1)}

- W3 propose to M3, M3 engaged but prefer W3 > W1. M3 break with W3.
  S = {(W2, M1), (W3, M3)}

- W1 single, propose to M1. M1 engaged but prefer W2 > W1, nothing happens.
- W1 continue propose to M2, M2 single. S = {(W1, M2), (W2, M1), (W3, M3)}

3) Stable Matching = {(1, C), (2, C), (3, M), (5, M)}

Working:

- 1 propose to C. C accept 1, Cleft 1 slot. S = {(1, C)}
- 2 propose to C. C accept 2, Cleft 0 slot. S = {(1, C), (2, C)}
- 3 propose to C. C left 0 slot but prefer 1 & 2 > 3. C rejected 3.
- 3 propose to M. M accept 3, M left 1 slot. S = {(1, C), (2, C), (3, M)}
- 4 propose to M. M accept 4, M left 0 slot. S = {(1, C), (2, C), (3, M), (4, M)}
- 5 propose to M. M left 0 slot but prefer 5 & 3 > 4. M accept 5 and kicks 4.
  S = {(1, C), (2, C), (3, M), (5, M)}

Instable Matching = {(1, C), (2, C), (3, M), (4, M)}

Explanation:

- Student 4 is assigned to hospital M but student 5 is not assigned to any hospital.
- However, hospital M prefers student 5 to 4.
- Therefore, the above set is instable.

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Assignment 1 - Stable Marriage Problem

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Feedback

Grade	40.00 / 40.00
Graded on	Friday, 4 June 2021, 5:31 PM
Graded by	Hong Wei CHUA
Feedback comments	Good Job!
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