CS100 #05

Boolean Expression Simplification

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Recap: Boolean Expression Simplification

- Digital computers contain circuits that implement Boolean logic.
- The simpler that we can make a Boolean expression, the smaller the circuit that will result.
- With this in mind, we always want to reduce our Boolean expressions to their simplest form.
- There are a number of Boolean identities that help us to do this.

Boolean Identities: Trivial

Logical Inverse	$0' = 1$	$1' = 0$
Involution	$A'' = A$	
Dominance	$A+1=1$	$A \cdot 0=0$
Identity	$A+0=A$	$A \cdot 1=A$
Idempotence	$A+A=A$	$A \cdot A=A$
Complementarity	$A+A'=1$	$A \cdot A'=0$
Commutativity	$A+B=B+A$	$A \cdot B=B \cdot A$
Associativity	$(A+B)+C=A+(B+C)$	$(A \cdot B) \cdot C=A \cdot (B \cdot C)$

Boolean Identities: Non-Trivial

Distributivity	$A \cdot (B+C)=A \cdot B+A \cdot C$	$A+B \cdot C=(A+B) \cdot (A+C)$
Absorption	$A \cdot (A+B)=A$	$A+A \cdot B=A$
DeMorgan's	$A+B=(A' \cdot B')'$	$A \cdot B=(A'+B')'$
Unnamed	$A+A' \cdot B=A+B$	
This one is usefull in assignment	$XY+X'Z+YZ=$ $XY+X'Z$	

Absorption 1

$$\begin{aligned} A + (A \cdot B) &= (A \cdot 1) + (A \cdot B) \\ &= A \cdot (1 + B) \\ &= A \cdot 1 \\ \therefore A + (A \cdot B) &= A \end{aligned}$$

Absorption 2

$$
\begin{aligned}
A \cdot (A + B) &= (A \cdot A) + (A \cdot B) \\
&= A + (A \cdot B) \\
&= (A \cdot 1) + (A \cdot B) \\
&= A \cdot (1 + B) \\
&= A \cdot 1 \\
\therefore A \cdot (A + B) &= A
\end{aligned}
$$

Chain Of Absorptions

- $A+AB+AC+AD+AE+ \ldots = A$

Let's prove last one

$XY+X'Z+YZ = XY+X'Z$

$XY+X'Z+1YZ \Rightarrow XY+X'Z+(X+X')YZ \Rightarrow$

$XY+X'Z+XYZ+X'YZ \Rightarrow (XY+XYZ)+(X'Z+X'YZ) \Rightarrow$

$XY+X'Z$

Example

$$AB + BC(B + C)$$

Example

$AB + BC(B + C)$

$\downarrow$ Distributing terms

$AB + BBC + BCC$

$\downarrow$ Applying identity **AA = A** to 2nd and 3rd terms

$AB + BC + BC$

$\downarrow$ Applying identity **A + A = A** to 2nd and 3rd terms

$AB + BC$

$\downarrow$ Factoring **B** out of terms

$B(A + C)$

Example

$$A + B(A + C) + AC$$

Example

`A + B(A + C) + AC`

Distributing terms

`A + AB + BC + AC`

Applying rule **`A + AB = A`** to 1st and 2nd terms

`A + BC + AC`

Applying rule **`A + AB = A`** to 1st and 3rd terms

`A + BC`

Example

$$\overline{A + \overline{BC}}$$

Example

$$\overline{A + \overline{BC}}$$

$$\downarrow$$

Breaking longest bar
(addition changes to multiplication)

$$\overline{A}\ \overline{\overline{BC}}$$

$$\downarrow$$

Applying identity $\overline{\overline{A}} = A$
to $\overline{\overline{BC}}$

$$\overline{A}BC$$

Example

$$\overline{A + \overline{BC}}$$

Incorrect step!

Breaking long bar between A and B;
Breaking both bars between B and C

$$\downarrow$$

$$\overline{A}\;\overline{\overline{B}} + \overline{\overline{C}}$$

Applying identity $\overline{\overline{A}} = A$
to $\overline{\overline{B}}$ and $\overline{\overline{C}}$

$$\downarrow$$

Incorrect answer: $\overline{A}B + C$

Example

$$\overline{A + \overline{BC}}$$

$\downarrow$ Breaking shortest bar (multiplication changes to addition)

$$\overline{A + (\overline{B} + \overline{C})}$$

$\downarrow$ Applying associative property to remove parentheses

$$\overline{A + \overline{B} + \overline{C}}$$

$\downarrow$ Breaking long bar in two places, between 1st and 2nd terms; between 2nd and 3rd terms

$$\overline{A}\ \overline{\overline{B}}\ \overline{\overline{C}}$$

$\downarrow$ Applying identity $\overline{\overline{A}} = A$ to $\overline{\overline{B}}$ and $\overline{\overline{C}}$

$$\overline{A}BC$$

Example

$$\overline{\overline{A + BC} + \overline{A\overline{B}}}$$

Example

$$\overline{\overline{A + BC} + \overline{A\overline{B}}}$$

↓ Breaking longest bar

$$(\overline{\overline{A + BC}})\ (\overline{\overline{A\overline{B}}})$$

↓ Applying identity $\overline{\overline{A}} = A$ wherever double bars of equal length are found

$$(A + BC)(A\overline{B})$$

↓ Distributive property

$$AA\overline{B} + BCA\overline{B}$$

↓ Applying identity $AA = A$ to left term; applying identity $A\overline{A} = 0$ to B and $\overline{B}$ in right term

$$A\overline{B} + 0$$

↓ Applying identity $A + 0 = A$

$$A\overline{B}$$

Example

$$\overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

Example

$$\overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

Factoring **BC** out of 1st and 4th terms

$$BC(\overline{A} + A) + A\overline{B}C + AB\overline{C}$$

Applying identity $A + \overline{A} = 1$

$$BC(1) + A\overline{B}C + AB\overline{C}$$

Applying identity $1A = A$

$$BC + A\overline{B}C + AB\overline{C}$$

Factoring **B** out of 1st and 3rd terms

$$B(C + A\overline{C}) + A\overline{B}C$$

Applying rule $A + \overline{A}B = A + B$ to the $C + A\overline{C}$ term

Example

$$B(C + A) + A\overline{B}C$$

Distributing terms

$$BC + AB + A\overline{B}C$$

Factoring **A** out of 2^{nd} and 3^{rd} terms

$$BC + A(B + \overline{B}C)$$

Applying rule $\mathbf{A + \overline{A}B = A + B}$ to the $B + \overline{B}C$ term

$$BC + A(B + C)$$

Distributing terms

$$BC + AB + AC$$

or

$$AB + BC + AC$$

Simplified result

References

- https://www.allaboutcircuits.com/textbook/digital/chpt-7/boolean-algebraic-identities/