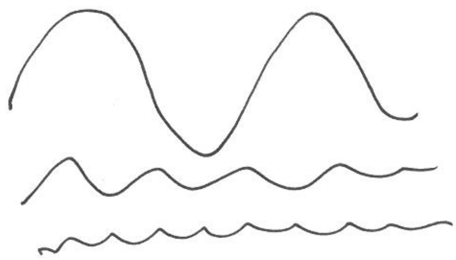
Fourier Transform

Image → Freq Domain

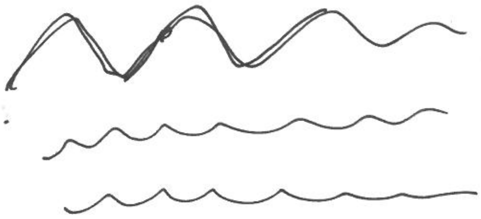
F. Series

periodic.



F. Transf.

non-periodic

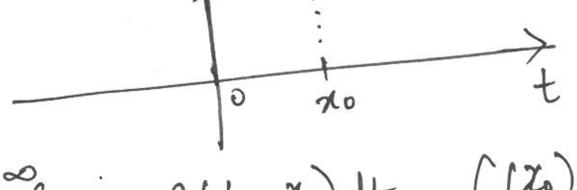


periodic.

Unit Impulse

$$\delta(t) = \begin{cases} \infty & t=0; \\ 0 & \text{otherwise.} \end{cases}$$

$$\int \delta(t)\, dt = 1$$



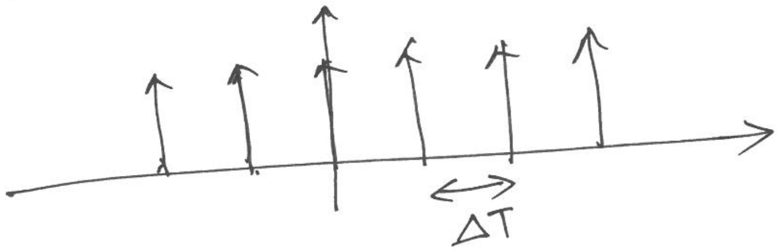
$$\int_{-\infty}^{\infty} f(t)\cdot\delta(t-x_0)\, dt = f(x_0).$$

$$\delta(t) = \begin{cases} 1 & t=0; \\ 0 & \text{otherwise.} \end{cases}$$

$$\sum \delta(t) = 1$$

$$\sum_{t=-\infty}^{\infty} f(t)\, \delta(t-x_0) = f(x_0).$$

Impulse Train:



$$s_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\Delta T).$$

F.S of I.T:

$$s_{\Delta T} = \sum_{n=-\infty}^{\infty} \frac{1}{\Delta T} e^{\frac{j2\pi n}{\Delta T} t}.$$

F.T of I.T:

$$S(u) = \frac{1}{\Delta T} \sum_{n=-\infty}^{\infty} \delta\left(u - \frac{n}{\Delta T}\right)$$

Convolution.

$f(t)$ $\quad$ $h(t)$

$\downarrow$ Image. $\quad$ $\downarrow$ Kernel.

$$\boxed{f(t) * h(t) = \int_{-\infty}^{\infty} f(\tau)\, h(t-\tau)\, d\tau.}$$

$$\mathbf{F}\{f(t) * h(t)\} = H(u)\cdot F(u) \quad \text{—— (1)}$$

$$\mathbf{F}\{f(t)\} = F(u) = \int_{-\infty}^{\infty} f(t)\cdot e^{-j2\pi u t}\cdot dt.$$

$$\mathbf{F}(h(t)) = H(u)$$

$$\mathbf{F}\{f(t)\cdot h(t)\} = F(u) * H(u) \quad \text{—— (2)}$$

Expand ?

$$\mathbf{F}\{f(t) * h(t)\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\tau)\cdot h(t-\tau)\cdot d\tau \cdot e^{-j2\pi u t} \cdot dt$$

Assume: $\mathbf{F}\{h(t-\tau)\} = H(u)\cdot e^{-j2\pi u \tau}$.

$$= \int_{-\infty}^{\infty} f(\tau) \left[\int_{-\infty}^{\infty} h(t-\tau)\cdot e^{-j2\pi u t}\cdot dt\right]\cdot d\tau$$

$$\mathbf{F}\{h(t-\tau)\} = H(u)\cdot e^{-j2\pi u \tau}$$

$$= \int_{-\infty}^{\infty} f(\tau)\, H(u)\cdot e^{-j2\pi u \tau}\cdot d\tau$$

$$= H(u)\cdot F(u).$$

$$\tilde{F}(u) = F\left\{\underbrace{\overbrace{f(t) \cdot s_{\Delta T}(t)}^{\tilde{f}(t)}}_{\text{Sample fn. of } f(t)}\right\} = \underline{F(f(t))} * \underline{F(s_{\Delta T}(t))}$$

F.T. of Sampled function of f(t)

↓

periodic → infinite.

↓

many F(U) copies