Algorithm Analysis

LECTURE 1 Introduction to Algorithm Design and Analysis

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Scene A

	Students		Status
1	John	Applied: EB, BubiSoft (sorted)	Waiting …
2	John	Gets offer from BubiSoft	Accept (John BubiSoft)
3	John	Gets offer from EB	**Withdraw (Null, BubiSoft)** **Accept (John, EB)**

Unstable

Scene B

	Students		Status
1	Alan	Applies Geeks	waiting
2	Alan	Gets offer from Geeks	Accept (Alan Geeks)
3	Cindy	Applies Geeks Geeks: Cindy Alan (sorted)	**Withdraw (Alan, Null)** **Accept (Cindy, Geeks)**

Unstable

Is the *self-enforcing* possible?

- whenever a student is assigned to a company the self-interest of both the student and the company prevents them from breaking the match.

stable

	Students		Status
1	Alan	Applies Geeks	waiting
2	Alan	Gets offer from Geeks	Accept (Alan Geeks)
3	Cindy	Applies Geeks Geeks: Alan Cindy (sorted)	(Alan Geeks)
4	Cindy	Applied: BubiSoft EB (sorted)	Waiting …
5	Cindy	Gets offer from BubiSoft	Accept (Cindy BubiSoft)
6	Cindy	Gets offer from EB	(Cindy BubiSoft)

Problem Formulating

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Stable Marriage Problem

- John’s preference list
  - Cindy
  - Mary
- Alan‘s
  - Mary
  - Cindy
- Cindy’s
  - John
  - Alan
- Mary’s
  - Alan
  - John

Terms

- M={John Alan}
- F = {Mary Cindy}
- M × F =
  - { (John Mary)
  - (John Cindy)
  - (Alan Mary)
  - (Alan Cindy) }

- Matching
  - S = {(John Mary) (Alan Cindy)}
  - S = {(John Cindy)}
- Perfect Matching
  - S = {(John Mary) (Alan Cindy)}
  - S = {(John Cindy) (Alan Mary)}
- Stable/Instable Matching
  - Stable: {(John Cindy) (Alan Mary)}
  - Instable: {(John Mary) (Alan Cindy)}

Terms - Definition

- Matching
  - A matching S is a set of ordered pairs, each from $M \times F$, with the property that each member of M and each member of F appears in at most one pair in S.
- Perfect Matching
  - A perfect matching S' is a matching with the property that each member of M and each member of F appears in exactly one pair in S'.
- Stable/Instable Matching
  - A matching S is considered to be instable if there are pairs $(m_1, f_1) \in S$ and $(m_2, f_2) \in S$ while m_1 prefers f_2 to f_1 and f_2 prefers m_1 to m_2.
  - A matching is stable if no such pairs exists.

Goal

- Given
  - a set of males M,
  - a set of women F and
  - their individual preference lists,

  - To find a perfect and stable matching.

Designing the algorithm

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	M Proposes to	F	Engagement
1	A	1	(A 1)
2	B	1	(A 1)
3	B	2	(A 1) (B 2)
4	C	1	(A 1) (B 2)
5	C	2	(A 1) (B 2)
6	C	3	(A 1) (B 2) (C 3)

Note: there may be other sequences .
It depends on the input(free man choosing) order.

M	Preference List	F	Preference List
A	1 2 3	1	A B C
B	1 2 3	2	A B C
C	1 2 3	3	A B C

	M Proposes to	F	Engagement	Free Males
0				{A B C}
1	A	1	(A 1)	{B C}
2	B	1	(B 1)	{C A}
3	C	1	(C 1)	{A B}
4	A	2	(C 1) (A 2)	{B}
5	B	2	(C 1) (B 2)	{A}
6	A	3	(C 1) (B 2) (A 3)	{}

Note: there may be other sequences .
It depends on the input(free man choosing) order.

M	Preference List	F	Preference List
A	1 2 3	1	C B A
B	1 2 3	2	C B A
C	1 2 3	3	C B A

Algorithm Analysis

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Questions?

- Does it work with any test case/instance?
  - Is the output S a perfect and stable matching if we change the instance?
- Is it fair, for men and woman?
- Dose the free man choosing influence the pairing result?
- How long will it take?
  - How many proposes should be made?

- Theorem 1. The while loop of the presented G-S algorithm terminates after at most n^2 iterations where n is the number of men.

Iteration	M Proposes to	F	Status of Engagement
1	A	1	(A 1)
2	B	1	(A 1)
	B	2	(A 1) (B 2)
3	C	1	(A 1) (B 2)
	C	2	(A 1) (B 2)
	C	3	(A 1) (B 2) (C 3)

$1 + 2 + 3 = 6$ $\longrightarrow$ Consider n men $1 + 2 + 3 + \cdots + n$

Worst case: each man proposes to all women, $n \times n$

- Observation 1. Consider any woman w. Once engaged, w remains engaged and progressively gets engaged to a better and better man (in terms of her preference list) if possible.
    - The woman labeled with 1

- Observation 2. The women a man proposes to becomes worse and worse (in terms of his preference list).
    - The man labeled with A

- Let’s turn to discussion with abstract notation!