Histogram Processing -2

Recap

- What is a Histogram?
- Histogram Normalization
- What is Random variable
- Histogram Equalization

Lecture Objectives

- Histogram matching / Histogram specification
- Local histogram processing

Key Stages in DIP

Image Acquisition → Image Enhancement → Image Restoration → Image Compression

Image Processing

Morphological Processing

Image Analysis

Image pattern classification ← Object Recognition ← Image Segmentation

Computer Vision (making sense)

Histogram Matching
(Histogram Specification)

Histogram Matching

- Recall that **histogram equalization** yields an image whose pixels are (in theory) uniformly distributed among all gray levels.

- When **automatic enhancement** is desired, **histogram equalization** is a good approach to consider because the results from this technique are predictable and the method is simple to implement.

- Sometimes, this may not be desirable. Instead, we may want a transformation that yields an output image with a **pre-specified histogram**. This technique is called *histogram matching* **OR** *histogram specification*.

Histogram Matching

- Given Information :
  - *Input image* from which we can compute its histogram.
  - *User specified histogram* of the output image.

- Goal:
  - Derive a point operation, $\mathbf{h(r)}$, that *maps* the input image into an output image that has the user-specified histogram.

- We will assume again, for the moment, continuous gray values.

Histogram Matching

- Consider for a moment *continuous intensities* ***r*** and ***z*** which, we treat as random variables with PDFs $p_r(r)$ and $p_z(z)$, respectively.
  - Here, ***r*** and ***z*** denote the intensity levels of the input and output (processed) images, respectively.

- We can *estimate* $p_r(r)$ from the given ***input image***, and $p_z(z)$ is the ***user specified PDF*** that we wish the *output image to have*.

Histogram Matching

- Let ***s*** be a random variable with the property:

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw$$

— where ***w*** is dummy variable of integration.

- Define a function ***G*** on variable ***z*** with the property:

$$G(z) = (L-1)\int_0^z p_z(v)\,dv = s$$

— where ***v*** is a dummy variable of integration.

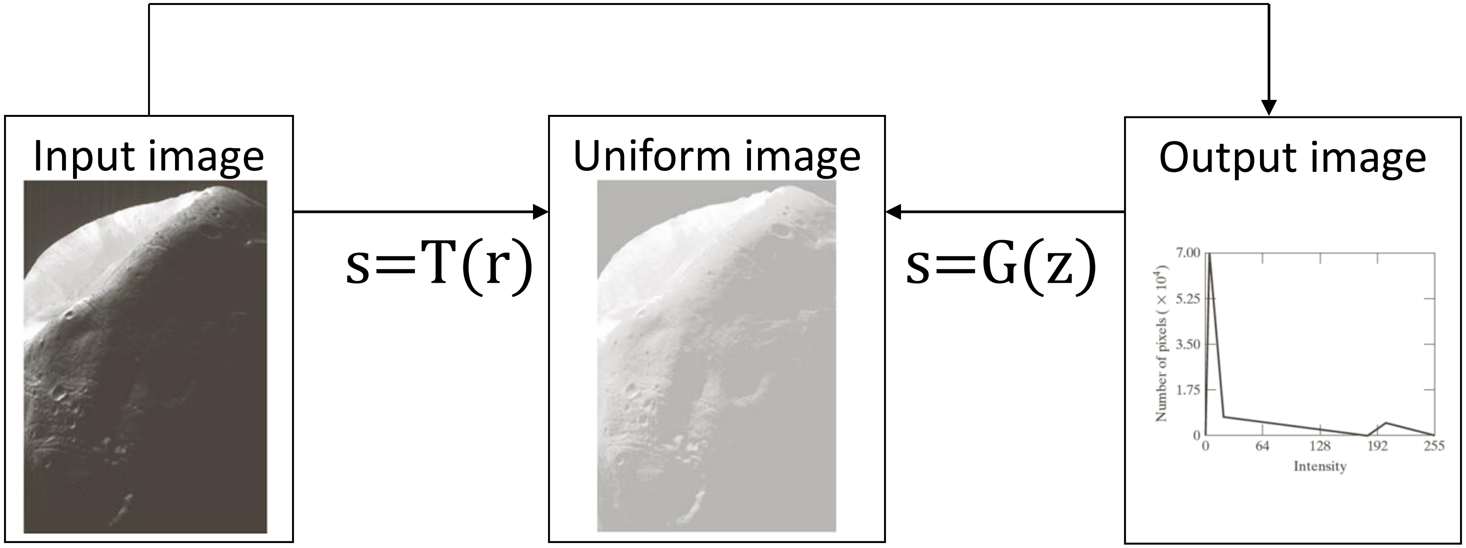
- It follows from the preceding two equations that $G(z) = s = T(r)$ and, therefore, that ***z*** must satisfy the condition:

$$z = G^{-1}(s) = G^{-1}[T(r)]$$

Histogram Matching

- **Approach of derivation**

$$Z = G^{-1}(s=T(r))$$



$p_R(r)$ $p_S(s)$ $p_Z(z)$

$p_R(r)$(computed), $p_Z(z)$(Given), $s=T(r)=G(z)$(Given)

Histogram Matching

- Input: $p_R(r)$(computed), $p_Z(z)$(Given), $s=T(r)=G(z)$(Given)
- Objective: compute **z**
- First apply the transformation: $s = T(r) = (L-1)\int_0^r p_r(w)\,dw$
  - This gives an *uniform image* with a *uniform probability density*.
- Apply the transformation: $G(z) = (L-1)\int_0^z p_z(v)\,dv = s$
  - This would generate an image with the *desired uniform density*.

Histogram Matching

- From the grayscale values ***s***, we can obtain the grayscale values ***z*** by using the inverse transformation:

$$z = G^{-1}(s) = G^{-1}(T(r))$$

- It will generate an image with the specified PDF $p_Z(z)$ from an input image with PDF of $p_R(r)$.

Example

Given:

$$P_R(r) = \begin{cases} \frac{2r}{(L-1)^2} & \text{if } 0 \le r \le L-1 \\ 0 & \text{otherwise} \end{cases}$$

$$s = T(r) = (L-1)\int_0^r P_R(w)dw$$

$$P_Z(z) = \begin{cases} \frac{3z^2}{(L-1)^3} & \text{if } 0 \le z \le L-1 \\ 0 & \text{otherwise} \end{cases}$$

$$s = G(z) = (L-1)\int_0^z P_Z(t)dt$$

Derive $z = G^{-1}(s) = G^{-1}(T(r))$

Example

$$P_R(r) = \begin{cases} \frac{2r}{(L-1)^2} & \text{if } 0 \le r \le L-1 \\ 0 & \text{otherwise} \end{cases}$$

$$s = T(r) = (L-1)\int_0^r p_r(w)dw = \frac{2}{(L-1)}\int_0^r w\,dw = \frac{r^2}{(L-1)}$$

Example

$$P_Z(z) = \begin{cases} \frac{3z^2}{(L-1)^3} & \text{if } 0 \leq z \leq L-1 \\ 0 & \text{otherwise} \end{cases}$$

$$G(z) = (L-1)\int_0^z p_z(w)dw = \frac{3}{(L-1)^2}\int_0^z w^2\,dw = \frac{z^3}{(L-1)^2}$$

Example

$$s = \frac{r^2}{(L-1)} \qquad G(z) = \frac{z^3}{(L-1)^2}$$

Finally, we require that *G(z)=s*, but $G(z) = z^3 / (L-1)^2$

So $z^3 / (L-1)^2 = s$ and we have:

$$z = G^{-1}(s) = \left[(L-1)^2 s\right]^{1/3}$$

Back to Discrete Case

- We have to *convert the continuous result just derived into a discrete form*. This means that we work with **histograms** instead of **PDFs**.

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw \quad \Rightarrow \quad s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) \qquad k = 0,1,2,\ldots,L-1$$

Similarly, given a specific value of ***sk***,

$$G(z) = (L-1)\int_0^z p_z(v)\,dv = s \quad \Rightarrow \quad G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i)$$

for a value of ***q*** so that $G(z_q) = s_k$,where $p_z(z_i)$ is the i^{th} value of the specified histogram.

- Finally, we obtain the desired value $\boldsymbol{z_q}$ from the *inverse transformation*:

$$z_q = G^{-1}(s_k)$$

Algorithm for Histogram Matching

Step-1: Compute the histogram, $p_R(r)$ of the input image, and use it in following equation to map the intensities in the input image to the intensities in the *histogram-equalized image*. Round the resulting values s_k , to the integers in the range **[0, *L* – 1]**.

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) \quad k = 0,1,2,\ldots,L-1$$

Step-2: Compute all values of function $G(z_q)$ using the following equation for ***q* = 0,1, 2,…,*L* – 1**, where $p_z(z_i)$ are the values of the *specified histogram*. Round the values of ***G*** to integers in the range **[0, *L* – 1]**. Store the rounded values of ***G*** in a **lookup table**.

$$G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i)$$

Algorithm for Histogram Matching

Step-3: For every value of s_k ,for **k = 0,1, 2,…, L– 1**, use the stored values of ***G*** from Step-2 to find the corresponding value of z_q so that $G(z_q)$ is closest to s_k .

- Store these mappings from ***s*** to ***z***.
- When more than one value of z_q gives the same match in $G(z_q)$ (i.e., the mapping is not unique), choose the smallest value by convention.

Step-4: Form the histogram-specified image by *mapping every equalized pixel* with value s_k to the corresponding pixel with value z_q in the histogram-specified image, using the mappings found in Step-3.

Histogram Matching Illustration - Example

Given:

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

z_q	Specified $p_z(z_q)$
$z_0 = 0$	0.00
$z_1 = 1$	0.00
$z_2 = 2$	0.00
$z_3 = 3$	0.15
$z_4 = 4$	0.20
$z_5 = 5$	0.30
$z_6 = 6$	0.20
$z_7 = 7$	0.15

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} P_R(r_j)$$

$$G(z_q) = (L-1)\sum_{i=0}^{q} P_Z(z_i)$$

Histogram Matching Illustration - Example

Step-1: Compute the histogram, $p_R(r)$ of the input image, and use it in following equation to map the intensities in the input image to the intensities in the histogram-equalized image. Round the resulting values, s_k , to the integer range **[0, *L*– 1]**.

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} P_R(r_j)$$

$$\begin{aligned} s_0 &= 1.33 \rightarrow 1 & s_4 &= 6.23 \rightarrow 6 \\ s_1 &= 3.08 \rightarrow 3 & s_5 &= 6.65 \rightarrow 7 \\ s_2 &= 4.55 \rightarrow 5 & s_6 &= 6.86 \rightarrow 7 \\ s_3 &= 5.67 \rightarrow 6 & s_7 &= 7.00 \rightarrow 7 \end{aligned}$$

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

Histogram Matching Illustration - Example

Step-2: Compute all values of function $G(z_q)$ using the following equation for $\boldsymbol{q = 0, 1, 2, \ldots, L-1}$, where $p_z(z_i)$ are the values of the specified histogram. Round the values of $\boldsymbol{G}$ to integers in the range $\mathbf{[0, L-1]}$. Store the rounded values of $\boldsymbol{G}$ in a lookup table.

$$G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i)$$

$$G(z_0) = 0.00 \rightarrow 0 \qquad G(z_4) = 2.45 \rightarrow 2$$

$$G(z_1) = 0.00 \rightarrow 0 \qquad G(z_5) = 4.55 \rightarrow 5$$

$$G(z_2) = 0.00 \rightarrow 0 \qquad G(z_6) = 5.95 \rightarrow 6$$

$$G(z_3) = 1.05 \rightarrow 1 \qquad G(z_7) = 7.00 \rightarrow 7$$

z_q	Specified $p_z(z_q)$
$z_0 = 0$	0.00
$z_1 = 1$	0.00
$z_2 = 2$	0.00
$z_3 = 3$	0.15
$z_4 = 4$	0.20
$z_5 = 5$	0.30
$z_6 = 6$	0.20
$z_7 = 7$	0.15

z_q	$G(z_q)$
$z_0 = 0$	0
$z_1 = 1$	0
$z_2 = 2$	0
$z_3 = 3$	1
$z_4 = 4$	2
$z_5 = 5$	5
$z_6 = 6$	6
$z_7 = 7$	7

Histogram Matching Illustration - Example

Step-3: For every value of s_k ,for $k = 0,1, 2,\ldots, L-1$, use the stored values of G from Step-2 to find the corresponding value of z_q so that $G(z_q)$ is closest to s_k .

- Store these mappings from s to z.
- When more than one value of z_q gives the same match in $G(z_q)$ (i.e., the mapping is not unique), choose the smallest value by convention.

r_k	->	s_k	->	z_q
0	->	1	->	3
1	->	3	->	4
2	->	5	->	5
3	->	6	->	6
4	->	6	->	6
5	->	7	->	7
6	->	7	->	7
7	->	7	->	7

z_q	$G(z_q)$
$z_0 = 0$	0
$z_1 = 1$	0
$z_2 = 2$	0
$z_3 = 3$	1
$z_4 = 4$	2
$z_5 = 5$	5
$z_6 = 6$	6
$z_7 = 7$	7

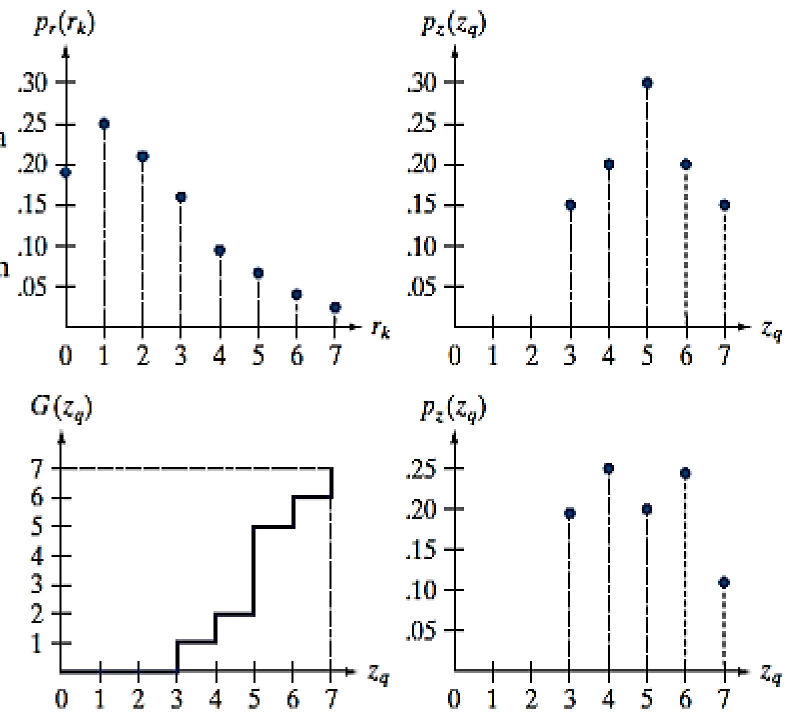
s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7

Histogram Matching Illustration - Example

Step-4: Form the histogram-specified image by *mapping every equalized pixel* with value s_k to the corresponding pixel with value z_q in the histogram-specified image, using the mappings found in Step-3.

r_k	->	s_k	->	z_q
0	->	1	->	3
1	->	3	->	4
2	->	5	->	5
3	->	6	->	6
4	->	6	->	6
5	->	7	->	7
6	->	7	->	7
7	->	7	->	7

z_q	Specified $p_z(z_q)$	Actual $p_z(z_k)$
$z_0 = 0$	0.00	0.00
$z_1 = 1$	0.00	0.00
$z_2 = 2$	0.00	0.00
$z_3 = 3$	0.15	0.19
$z_4 = 4$	0.20	0.25
$z_5 = 5$	0.30	0.21
$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11



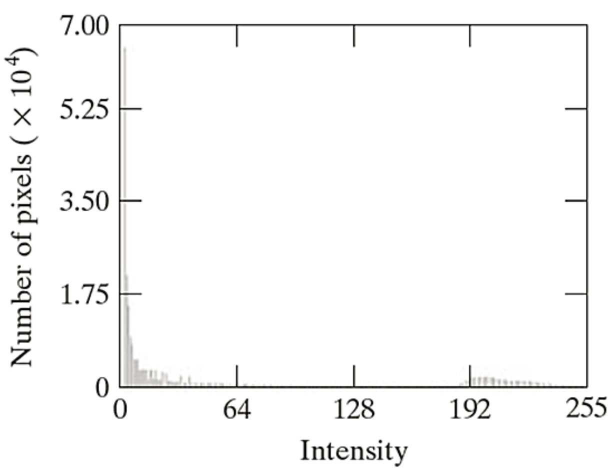
a b
c d

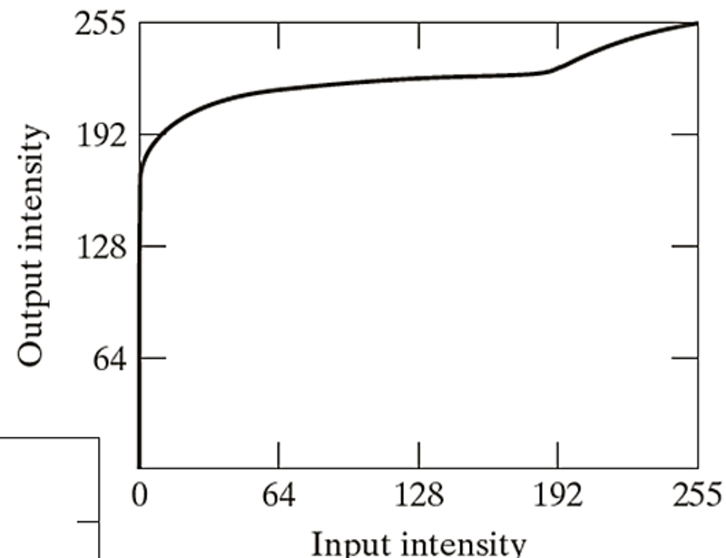
(a) Histogram of a 3-bit image. (b) Specified histogram. (c) Transformation function obtained from the specified histogram. (d) Result of histogram specification. Compare the histograms in (b) and (d).

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

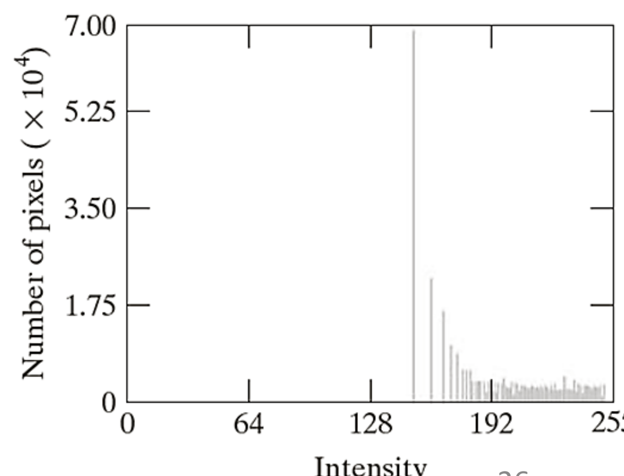
Equalization or Specification?

Mars moon





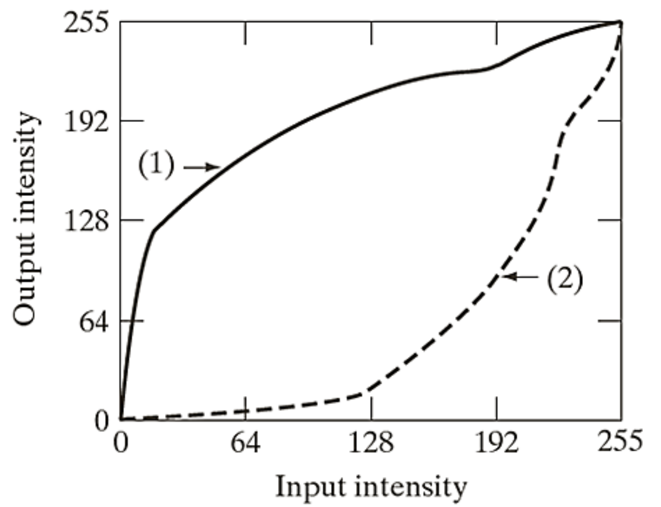
Histogram equalization transform obtained

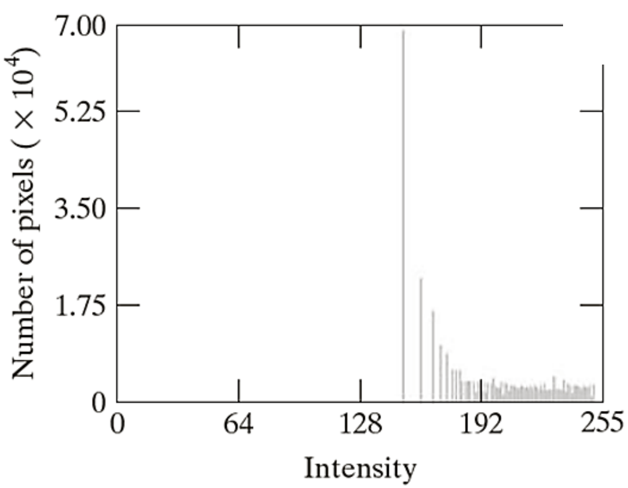


Histogram Equalization

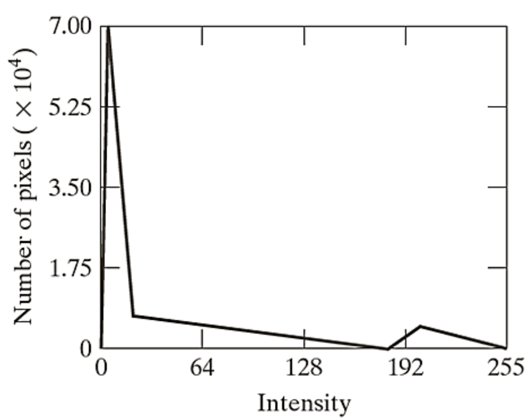
Equalization or Specification?

Transformation $G(z_q)$, labeled (1), and $G^{-1}(s_k)$, labeled (2)

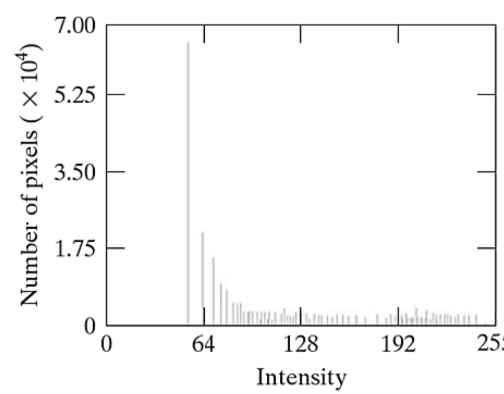




Histogram equalization



Specified histogram



Histogram specification

Local Histogram Processing

- The histogram processing methods discussed thus far are *global*, in the sense that pixels are modified by a transformation function based on the intensity distribution of an entire image.

- The *global approach is suitable for overall enhancement*, but generally **fails** when the objective is to enhance details over small areas in an image.

- This is because the number of pixels in small areas have *negligible* influence on the computation of global transformations.

- The **solution** is to devise transformation functions based on the *intensity distribution of pixel neighborhoods*.

Local Histogram Processing

- In ***local histogram processing methods ,*** we follow these steps:

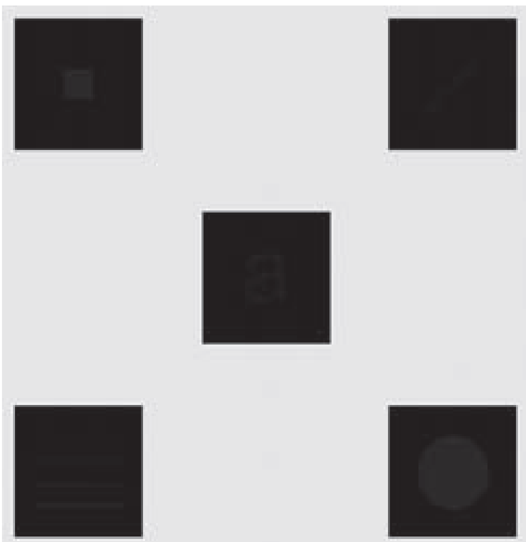
1. Define a neighborhood and move its **center** from pixel to pixel in a horizontal or vertical direction.

2. At each location, the histogram of the points in the neighborhood is computed, and either a **histogram equalization** **or** **histogram specification** transformation function is obtained.

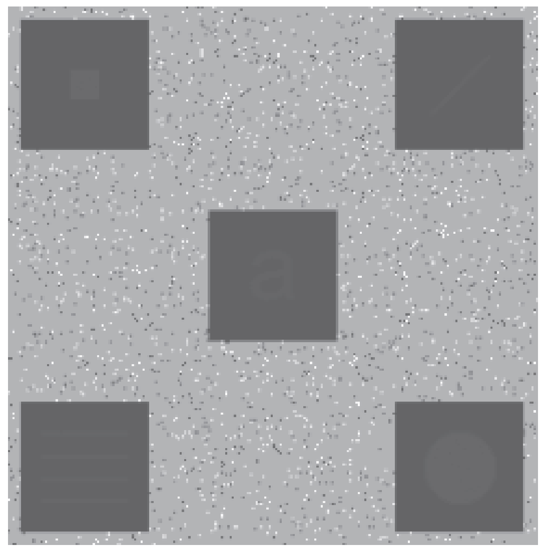
3. This function is used to map the intensity of the pixel centered in the neighborhood.

4. The center of the neighborhood is then moved to an adjacent pixel location and the procedure is repeated.

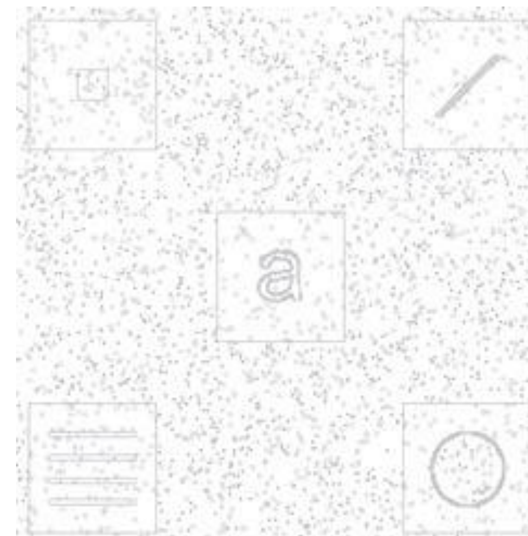
Local Histogram Processing



Original
image



Result of global
Histogram equalization



Result of local
Histogram equalization

Assignment 2

- Implement operations including:
  - Addition, subtraction, product, negative, log transform, power (Gamma) transform
  - Histogram equalization
  - Smoothing filters and sharpening filters
  - Connected component labeling

Next Lecture

- Fundamentals of Spatial Filtering
- Correlation and Convolution
- How to construct Spatial filter masks?
- Smoothing (Lowpass) spatial filters
  - Box filter kernels
  - Gaussian filter kernels
  - Smoothing Non-linear Filters