Midterm Assessment – Question Paper

1. There are 3 men {Alan, Bob, John} and 3 women {Andrea, Cindy, May}. Each woman lists her preference list of men in order from the most to the least preferred. Each man lists his preference list of women in the same order. Their preference lists are given below.

   Woman Andrea: <Alan, Bob, John>.
   Woman Cindy: <Bob, John, Alan>.
   Woman May: <Bob, Alan, John>.

   Man Alan: <Cindy, May, Andrea>.
   Man Bob: <May, Andrea, Cindy>.
   Man John: <Andrea, Cindy, May>.

   a) Please state the returned output (matching) after applying the G-S algorithm with **women proposing**. Process should be given in the following way. (6 marks)

iteration	**Woman**	**Man**	**Engagement**	**Free women**
0				Andrea, Cindy, May
1 (e.g.)	Andrea	Alan	(Andrea, Alan)	Cindy, May
2				
…				

   b) Please state the returned output (matching) after applying the G-S algorithm with **men proposing**. Process is required by filling the following form. Process should be given also. (4 marks)

   c) Give an **instable** perfect matching and state which pair of couples cause the instable. (5 marks)

2. Please list the following functions according to their order of growth from the lowest to the highest. (15 marks)

   f1(n) = 1
   f2(n) = $\sqrt[3]{n}$
   f3(n) = $0.01n^4 + n^2 + 3$
   f4(n) = $log_2(n)$
   f5(n) = $(n-1)!$
   f6(n) = 2^n
   f7(n) = 2^{lgn}

3. Use **characteristic equation** to solve the following recurrence and give its order of growth. Process is required. (10 marks)

$$T(n) = 4T(n-1) - 4T(n-2), T(0) = 1, T(1) = 3$$

4. Solve the following recurrence and give its order of growth. Use **Recursion tree** and **Master's theorem** to solve it respectively. Process is required. (20 marks)

$$T(n) = 2T(n/3) + 1, T(1) = 1$$

5. A group of students have been asked to develop algorithms for finding the k-th smallest element in an input array containing n positive integer values. For example, the 5-th smallest in the array A = [10, 18, 16, 28, 25, 32, 11, 18] is 18, since there are greater than or equal to 5 values less than or equal to it. One student proposes the following search-based counting approach

$$
\begin{array}{l}
\textbf{for } i \leftarrow 0 \textbf{ to } n-1 \\
\quad lt \leftarrow 0 \\
\quad le \leftarrow 0 \\
\quad \textbf{for } j \leftarrow 0 \textbf{ to } n-1 \\
\quad\quad \textbf{if } A[j] < A[i] \\
\quad\quad\quad lt \leftarrow lt + 1 \\
\quad\quad \textbf{if } A[j] \leq A[i] \\
\quad\quad\quad le \leftarrow le + 1 \\
\quad \textbf{if } lt < k \textbf{ and } le \geq k \\
\quad\quad \textbf{return } A[i]
\end{array}
$$

a) Running through the given algorithm on the given array A = [10, 18, 16, 28, 25, 32, 11, 18], k = 5, step by step. For each step, show the values of ***i***, ***j***, ***le*** and ***lt***. (10 marks)

b) Using the O notation, what is the running time of this algorithm in the **best case**? (5 marks)

c) Using the O notation, what is the running time of this algorithm in the **worst case**? (5 marks)

1a)

Iteration	Women	Men	Engagement	Free Women
0				Andrea, Cindy, May
1	Andrea	Alan	(Andrea, Alan)	Cindy, May
2	Cindy	Bob	(Andrea, Alan) (Cindy, Bob)	May
3	May	Bob	(Andrea, Alan) (May, Bob)	Cindy
4	Cindy	John	(Andrea, Alan) (Cindy, John) (May, Bob)	

Bob prefer May > Cindy

1b)

Iteration	Men	Women	Engagement	Free Men
0				Alan, Bob, John
1	Alan	Cindy	(Alan, Cindy)	Bob, John
2	Bob	May	(Alan, Cindy) (Bob, May)	John
3	John	Andrea	(Alan, Cindy) (Bob, May) (John, Andrea)	

c) Women Propose : $S = \{$ (Andrea, Alan), (Cindy, Bob), (May, John) $\}$

(Cindy, Bob) Cindy prefer Bob > John

(May, John) May prefer Bob > John

Bob prefer May > Cindy, Bob wants to break marriage with Cindy and May and pair with May. Therefore, (Cindy, Bob) and (May, John) is instable.

2.

$f1(10) = 1$

$f2(10) = \sqrt[3]{10} = 2.1544$

$f3(10) = 0.01(10)^4 + 10^2 + 3 = 203$

$f4(10) = \log_2 10 = 3.32192$

$f5(10) = (10-1)! = 9! = 362880$

$f6(10) = 2^{10} = 1024$

$f7(10) = 2^{\lg n} = 2^{\lg 10} = 2$

Order of Growth (lowest to highest): $f1(n)$, $f7(n)$, $f2(n)$, $f4(n)$, $f3(n)$, $f6(n)$, $f5(n)$

3. $T(n) = 4T(n-1) - 4T(n-2)$ $\quad T(0) = 1$, $T(1) = 3$

$r^2 - 4r + 4 = 0$

$r_1 = r_2 = 2$

$T(n) = \alpha_1 r_1^n + \alpha_2 n r_2^n$

$= \alpha_1 (2)^n + \alpha_2 n (2)^n$

$= 2^n \alpha_1 + 2^n \alpha_2 n$

$T(0) = 2^0 \alpha_1 + 2^0 \alpha_2 (0)$

$1 = \alpha_1$ —— (1)

$T(1) = 2^1 \alpha_1 + 2^1 \alpha_2 (1)$

$3 = 2\alpha_1 + 2\alpha_2$

$3 = 2(1) + 2\alpha_2$

$2\alpha_2 = 1$

$\alpha_2 = 0.5$

$T(n) = 2^n \alpha_1 + 2^n \alpha_2 n$

$= 2^n (1) + 2^n (0.5) n$

$= 2^n + 0.5n(2^n)$

$= 2^n (1 + 0.5n)$

$\in \Theta(2^n)$

4. Master Theorem

$T(n) = 2T\left(\frac{n}{3}\right) + 1$ $\qquad a = 2,\ b = 3$

$f(n) = 1 \qquad n^{\log_3 2} = n^{0.6309}$

$f(n) \in \Omega\left(n^{0.6309 + \varepsilon}\right)$, let $\varepsilon = 0.1$

$= f(n) \in \Omega\left(n^{0.73}\right)$

$af\left(\frac{n}{b}\right) \leq (1 - \varepsilon')f(n)$

$2f\left(\frac{n}{3}\right) \leq (1 - \varepsilon')f(n)$

$2(1) \leq (1 - \varepsilon')(1)$

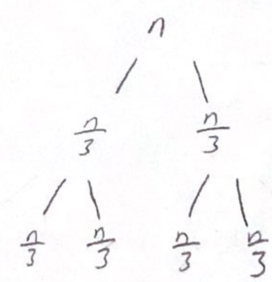
$2 \leq 1 - \varepsilon'$

$\varepsilon' \leq -1$

$\therefore T(n) \in \Theta(1)$ //

Recursion Tree

$T(n) = 2T\left(\frac{n}{3}\right) + 1 \quad , \quad T(1) = 1$



level	#	size
1	2^0	$\frac{n}{3^0}$
2	2^1	$\frac{n}{3^1}$
3	2^2	$\frac{n}{3^2}$
⋮		
n	2^{n-1}	$\frac{n}{3^{n-1}}$

$T(n) = 2^0\left(\frac{n}{3^0}\right) + 2^1\left(\frac{n}{3^1}\right) + 2^2\left(\frac{n}{3^2}\right) + \ldots\ 2^{n-1}\left(\frac{n}{3^{n-1}}\right)$

$= n\left[\left(\frac{2}{3}\right)^0 + \left(\frac{2}{3}\right)^1 + \left(\frac{2}{3}\right)^2 + \ldots \left(\frac{2}{3}\right)^{n-1}\right]$

$= n \cdot \left[\frac{1 - \left(\frac{2}{3}\right)^{n-1+1}}{1 - \frac{2}{3}}\right]$

$= n \cdot \left[\frac{1 - \left(\frac{2}{3}\right)^n}{\frac{1}{3}}\right]$

$= n \cdot \left[3\left(1 - \left(\frac{2}{3}\right)^n\right)\right]$

$= n \cdot \left[3 - 3\left(\frac{2}{3}\right)^n\right]$

$= 3n - 3n\left(\frac{2}{3}\right)^n$

$= 3n\left[1 - \left(\frac{2}{3}\right)^n\right]$

$=$

5a. $A = [10, 18, 16, 28, 25, 32, 11, 18]$, $n = 8$

```
for (int i = 0 ; i < n-1 ; ++i)      7
{
  int lt = 0 ;
  int le = 0 ;
  for (int j = 0 ; i < n-1 ; ++j)    7
  {
    if A[j] < A[i]
        lt = lt + 1
    if A[j] ≤ A[i]
        le = le + 1
  }
  if (lt < k) and (le ≥ k)
    return A[i]
```

$i = 0$, $lt = 0$, $le = 0$, $j = 0$, $A[0] = 10 \le A[0] = 10$, $le = 0 + 1 = 1$, $lt = 0$

$k = 5$

$= 1$, $A[1] = 18 < 10$ ✗

$= 2$ $A[2] = 16 < 10$ ✗

$= 3$, $A[3] = 28 < 10$ ✗

$= 4$, $A[4] = 25 < 10$ ✗

$= 5$, $A[5] = 32 < 10$ ✗

$= 6$, $A[6] = 11 < 10$ ✗

$= 7$, $A[7] = 18 < 10$ ✗ $le = 1$, $lt = 0$

$lt < 5$ ✓ $le \ge 5$ ✗

$i = 1$, $lt = 0$, $le = 0$, $j = 0$, $A[0] = 10 < A[1] = 18$, $lt = 1$, $le = 1$

$= 1$, $A[1] = 18 \le A[1] = 18$, $le = 2$

$= 2$, $A[2] = 16 < 18$, $lt = 2$, 3

$= 3$, $A[3] = 28 < 18$ ✗

$= 4$, $A[4] = 25 < 18$ ✗

$= 5$, $A[5] = 32 < 18$ ✗

$= 6$, $A[6] = 11 < 18$ ✓ , $lt = 3$, $le = 4$

$= 7$, $A[7] = 18 \le 18$ ✓ , $lt = 3$, $le = 5$

$lt < 5$ ✓ $le \ge 5$ ✓

$A[1] = 18$

5b) $O(n)$

5c) $O(n^2)$

cs330su21-a.sg

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Mid-term Assessment

cs330_midterm_questions-su2021.pdf 17 June 2021, 11:57 AM

Submission status

Submission status	Submitted for grading
Grading status	Graded
Due date	Thursday, 17 June 2021, 12:40 PM
Time remaining	Assignment was submitted 1 min 54 secs early
Last modified	Thursday, 17 June 2021, 12:38 PM
File submissions	CS330_GOHWEIZHE_MIDTERMS.pdf 17 June 2021, 12:38 PM
Submission comments	Comments (0)

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Feedback

Grade	63.00 / 80.00
Graded on	Monday, 21 June 2021, 12:51 PM
Graded by	Yidi WANG

Feedback comments

+

2) (-6 marks)

f1(n)<**f4(n)<f7(n)<f2(n)**<f3(n)<f6(n)<f5(n)

3)partical correct answer (-1 mark)

Time function: T(n)= 2^n + 0.5n2^n∈**O(n2^n)**

4)

For...

◄ Attendance cs330su21-a.sg Thursday 17/06/2021 11:00am-12:40pm

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Woman Andrea: <Alan, Bob, John>.
Woman Cindy: <Bob, John, Alan>.
Woman May: <Bob, Alan, John>.

Man Alan: <Cindy, May, Andrea>.
Man Bob: <May, Andrea, Cindy>.
Man John: <Andrea, Cindy, May>.

iteration	Woman	Man	Engagement	Free women
0				Andrea, Cindy, May
1	Andrea	Alan	(Andrea, Alan)	Cindy, May
2	Cindy	Bob	(Andrea, Alan), (Cindy, Bob)	May
3	May	Bob	(Andrea, Alan), (May, Bob)	Cindy
4	Cindy	John	(Andrea, Alan), (May, Bob) (Cindy John)	Null

⇐ Let women propose.

matching.

Let men propose.

iteration	Man	Woman	Engagement	Free men
0				Alan, Bob, John
1	Alan	Cindy	(Alan, Cindy)	Bob, John
2	Bob	May	(Alan, Cindy) (Bob, May)	John
3	John	Andrea	(Alan, Cindy) (Bob, May) (John, Andrea)	Null

stable Matching.

Woman Andrea: <Alan, Bob, John>.
Woman Cindy: <Bob, John, Alan>.
Woman May: <Bob, Alan, John>.

Man Alan: <Cindy, May, Andrea>.
Man Bob: <May, Andrea, Cindy>.
Man John: <Andrea, Cindy, May>.

In {(Cindy Bob), (May Alan), (Andrea John)},
(Cindy Bob) and (May Alan) cause the instable, also
(Cindy Bob) and (Andrea John) could cause the instable.

In {(Andrea Alan), (Cindy Bob), (May John)},
(Andrea Alan) and (May John) cause the instable.

In {(Cindy Alan), (Andrea Bob), (May John)},
(Cindy Alan) and (May John) cause the instable, also
(Andrea Bob) and (May John) could cause the instable.

① Instable Matching.

② causes.

$\langle m_1 \quad w_1 \rangle$

$\langle m_2 \quad w_2 \rangle$

Cindy Bob

May Alan

$a^{\log_b n} \Leftrightarrow n^{\log_b a}$

$1, \log_2 n, n^{\lg 2}, n^{\frac{1}{3}}, n^4, 2^n, (n-1)!$

f1(n) = 1
f2(n) = $\sqrt[3]{n}$
f3(n) = $0.01n^4 + n^2 + 3$
f4(n) = $log_2(n)$
f5(n) = $(n-1)!$
f6(n) = 2^n
f7(n) = 2^{lgn}

1
$\boxed{n^{\frac{1}{3}}}$
$\boxed{n^4}$
$\log_2 n$
$(n-1)! \Rightarrow n!$
2^n
$\boxed{n^{\lg 2}} \Rightarrow$

$\boxed{1 \quad 4 \quad 7 \quad 2 \; 3 \quad 6 \quad 5}$

$\lg n = \log_{10} n$

$0.3337 \approx \frac{1}{3} \quad \lg 2 \approx 0.3$

f1(n) = 1 < f4(n) = $log_2(n)$ < f7(n) = 2^{lgn}(= n^{lg2} = $n^{0.3}$)<f2(n) = $\sqrt[3]{n}$ < f3(n) = $0.01n^4 + n^2 + 3$ < f6(n) = 2^n < f5(n) = $(n-1)!$

$T(n) = 4T(n-1) - 4T(n-2), T(0) = 1, T(1) = 3$

$k = 2.$

$r^2 - 4r + 4 = 0$

$(r-2)^2 = 0$

$r_1 = r_2 = 2.$

$T(n) = \alpha_1 \cdot 2^n + \alpha_2 \cdot n \cdot 2^n$

$T(0) = \alpha_1 \cdot 2^0 + \alpha_2 \cdot 0 \cdot 2^0 = 1$

$\alpha_1 = 1$ ①

$T(1) = \alpha_1 \cdot 2^1 + \alpha_2 \cdot 1 \cdot 2^1 = 3$

$2\alpha_1 + 2\alpha_2 = 3$ ②

$2\alpha_2 = 1$

solve ① and ②

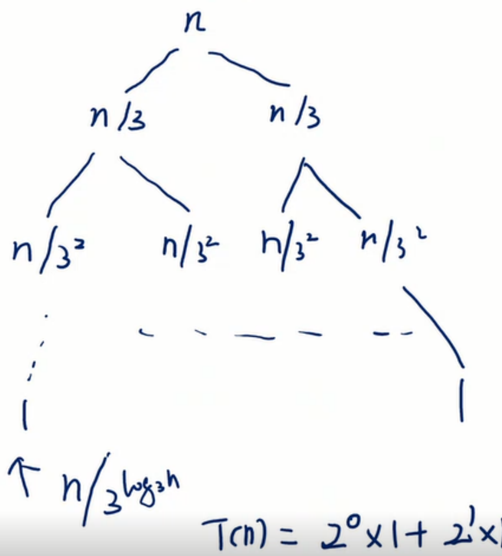
for α_1, α_2

$\alpha_1 = 1 \quad \alpha_2 = 0.5$

$T(n) = 2^n + \cancel{0.5}\, n \cdot 2^n$

$\in O(n \cdot 2^n)$

$T(n) = 2T(n/3) + 1, T(1) = 1$



Recursion Tree. $a^{\log_b n} == n^{\log_b a}$

Level #	# subproblems	cost on f(n)
0	2^0	1
1	2^1	1
2	2^2	1
$\vdots$	$\vdots$	$\vdots$
$\log_3 n$	$2^{\log_3 n}$	1

$$T(n) = 2^0 \times 1 + 2^1 \times 1 + \cdots 2^{\log_3 n} \times 1 = \boxed{2^{\log_3 n}}^{+1} - 1 \in O(n^{\log_3 2})$$

$T(n) = 2T(n/3) + 1, T(1) = 1$

Master's method

$a = 2 \quad b = 3$

$n^{\log_b a} = n^{\log_3 2} > 0.$ $\qquad f(n) = 1 = n^0$

case 1 matched. $\quad f(n) \in O(n^{\log_b a})$ is true.

$1 \in O(n^{\log_3 2})$

$T(n) \in O(n^{\log_3 2})$

A = [10, 18, 16, 28, 25, 32, 11, 18], k=5

for $i \leftarrow 0$ **to** $n-1$
  $lt \leftarrow 0$
  $le \leftarrow 0$
  for $j \leftarrow 0$ **to** $n-1$
    if $A[j] < A[i]$
      $lt \leftarrow lt + 1$
    if $A[j] \leq A[i]$
      $le \leftarrow le + 1$
  if $lt < k$ **and** $le \geq k$
    return $A[i]$

I	J	Lt	Le
0	0..7	0	1
1	0	1	1
1	1	1	2
1	2	2	3
1	3	2	3
1	4	2	3
1	5	2	3
1	6	3	4
1	7	3<K	5>=5

k-th largest element

0 1 2 3 4 5 6 7
A = [10, 18, 16, 28, 25, 32, 11, 18], k=5
i j

for $i \leftarrow 0$ **to** $n-1$ n
  $lt \leftarrow 0$
  $le \leftarrow 0$
  for $j \leftarrow 0$ **to** $n-1$ n
    if $A[j] < A[i]$ 18<18
      $lt \leftarrow lt + 1$ 18≤18.
    if $A[j] \leq A[i]$
      $le \leftarrow le + 1$
  if $lt < k$ **and** $le \geq k$
    return $A[i]$

I	J	Lt	Le
0	0..7	0	1
1	0	1	1
1	1	1	2
1	2	2	3
1	3	2	3
1	4	2	3
1	5	2	3
1	6	3	4
1	7	3<K	5>=5

← lt = le = 0

best case A[0] is the k-th element. ∈ $O(n)$

worst case. A[n-1] is the k-th element. ∈ $O(n^2)$