

#### CS100 #05

# Boolean Expression Simplification

**Vadim Surov** 



## Recap: Boolean Expression Simplification

- Digital computers contain circuits that implement Boolean logic.
- The simpler that we can make a Boolean expression, the smaller the circuit that will result.
- With this in mind, we always want to reduce our Boolean expressions to their simplest form.
- There are a number of Boolean identities that help us to do this.



#### **Boolean Identities: Trivial**

| Logical Inverse | 0' = 1          | 1' = 0                  |
|-----------------|-----------------|-------------------------|
| Involution      | A'' = A         |                         |
| Dominance       | A+1=1           | A · 0=0                 |
| Identity        | A+0=A           | A · 1=A                 |
| Idempotence     | A+A=A           | A·A=A                   |
| Complementarity | A+A'=1          | A · A'=0                |
| Commutativity   | A+B=B+A         | A·B=B·A                 |
| Associativity   | (A+B)+C=A+(B+C) | (A · B) · C=A · (B · C) |



#### **Boolean Identities: Non-Trivial**

| Distributivity                    | A · (B+C)=A · B+A · C | $A+B\cdot C=(A+B)\cdot (A+C)$ |
|-----------------------------------|-----------------------|-------------------------------|
| Absorption                        | A · (A+B)=A           | A+A·B=A                       |
| DeMorgan's                        | A+B=(A'·B')'          | A · B=(A'+B')'                |
| Unnamed                           | A+A'·B=A+B            |                               |
| This one is usefull in assignment | XY+X'Z+YZ=<br>XY+X'Z  |                               |



# Absorption 1

$$A + (A \cdot B) = (A \cdot 1) + (A \cdot B)$$
  
 $= A \cdot (1 + B)$   
 $= A \cdot 1$   
 $\therefore A + (A \cdot B) = A$ 



## Absorption 2

$$A \cdot (A + B) = (A \cdot A) + (A \cdot B)$$

$$= A + (A \cdot B)$$

$$= (A \cdot 1) + (A \cdot B)$$

$$= A \cdot (1 + B)$$

$$= A \cdot 1$$

$$\therefore A \cdot (A + B) = A$$



#### Chain Of Absorptions

• A+AB+AC+AD+AE+ ... = A



#### Let's prove last one

$$XY+X'Z+YZ = XY+X'Z$$

$$XY+X'Z+1YZ => XY+X'Z+(X+X')YZ =>$$

$$XY+X'Z+XYZ+X'YZ => (XY+XYZ)+(X'Z+X'YZ) =>$$

$$XY+X'Z$$



$$AB + BC(B + C)$$





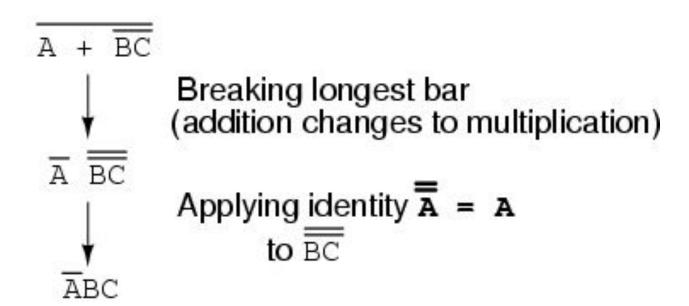
$$A + B(A + C) + AC$$



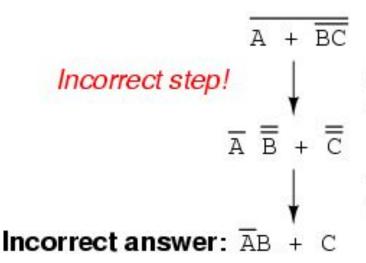


$$\overline{A + BC}$$





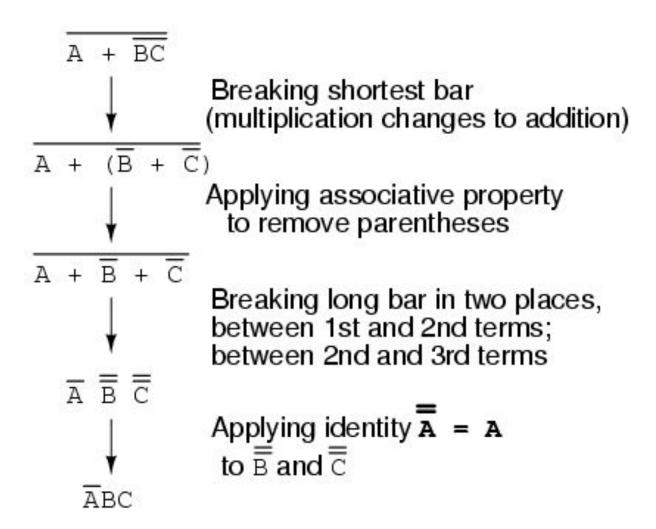




Breaking long bar between A and B; Breaking both bars between B and C

Applying identity 
$$\overline{\overline{A}} = A$$
 to  $\overline{\overline{B}}$  and  $\overline{\overline{C}}$ 







$$\overline{\overline{A} + BC} + \overline{AB}$$



Breaking longest bar

Applying identity 
$$\overline{A} = A$$
wherever double bars of equal length are found

(A + BC) (AB)

Distributive property

AAB + BCAB

Applying identity  $AA = A$ 
to left term; applying identity  $AA = A$ 
to left term; applying identity  $AA = A$ 
to left term; applying identity  $AA = A$ 
to left term

AB + 0

Applying identity  $AA = A$ 
to left term; applying identity  $AA = A$ 
to left term; applying identity  $AA = A$ 
to left term

AB + 0

Applying identity  $AA = A$ 



$$\overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$



$$\overline{A}BC + \overline{A}\overline{B}C + \overline{A}B\overline{C} + \overline{A}B\overline{C}$$

Factoring BC out of 1<sup>st</sup> and 4<sup>th</sup> terms

 $BC(\overline{A} + \overline{A}) + \overline{A}\overline{B}C + \overline{A}B\overline{C}$ 

Applying identity  $\overline{A} + \overline{A} = 1$ 
 $BC(1) + \overline{A}B\overline{C} + \overline{A}B\overline{C}$ 

Applying identity  $1A = A$ 
 $BC + \overline{A}B\overline{C} + \overline{A}B\overline{C}$ 

Factoring B out of 1<sup>st</sup> and 3<sup>rd</sup> terms

 $B(C + \overline{A}C) + \overline{A}B\overline{C}$ 

Applying rule  $A + \overline{A}B = A + B$  to the  $C + \overline{A}C$  term





#### References

 https://www.allaboutcircuits.com/textbook/digital/chpt-7/bo olean-algebraic-identities/