Image Restoration-2

Recap

- The image degradation/restoration model
- Noise models
  - Important noise probability density functions
  - Periodic noise
  - Estimating noise parameters
- Restoration using spatial filters
  - Mean filters
  - Order-static filters
  - Adaptive filters

Lecture Objectives

- Periodic noise reduction using frequency domain filtering
  - Notch filtering
  - Optimum notch filtering (self study)
- Linear, position-invariant degradations
- Estimating degradation function (H)

Key Stages in DIP

Image Acquisition → Image Enhancement → Image Restoration → Image Compression → Morphological Processing → Image Segmentation → Object Recognition → Representation & Description

Periodic Noise Reduction Using Frequency Domain Filtering

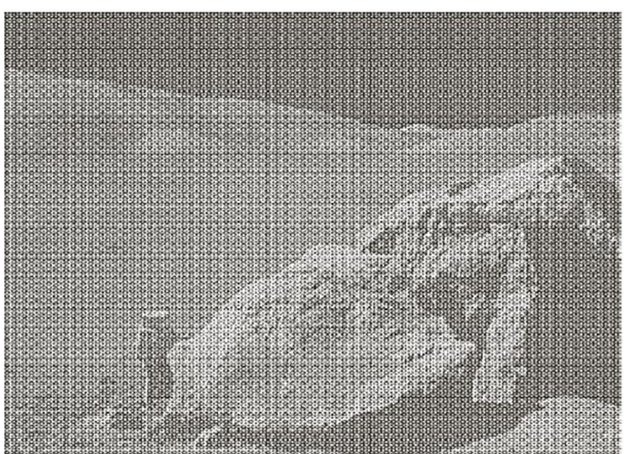
Periodic Noise

- Arises from *electrical* or *electromechanical interference* during image acquisition.

Spatially dependent

- Appears as *concentrated bursts of energy* in the *Fourier transform*, at locations corresponding to the frequencies of the *periodic interference*.

- Periodic noise in *spatial domain* ≈ points of intensity in the *frequency domain.*



Periodic Noise Removal – Selective Filtering

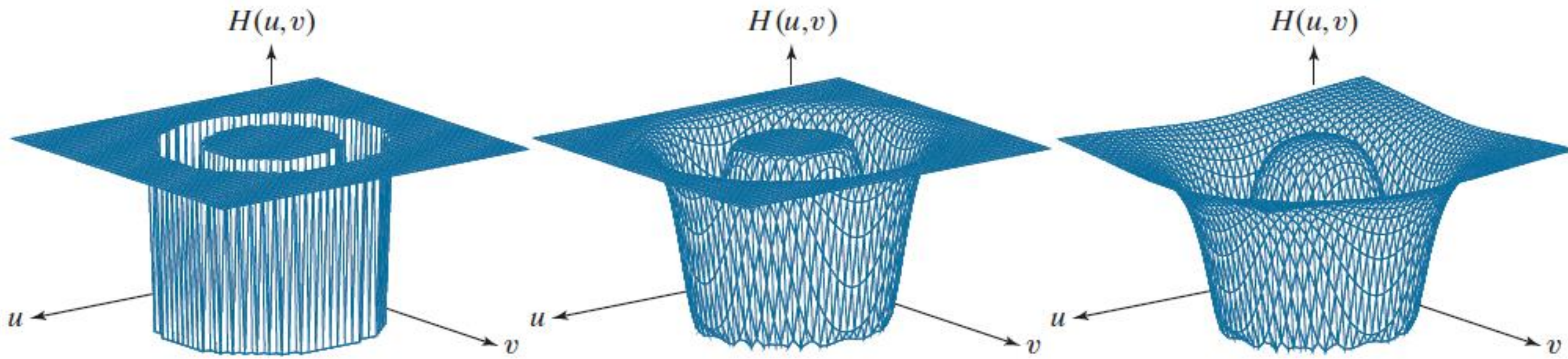
- **Basic approach:** Use selective filters to isolate the noise:
  - Bandreject
  - Bandpass
  - Notch Filters

- Selective filters process *specific bands of frequencies* (bandreject/bandpass filters) or *small regions of the frequency rectangles* (notch filters).

- In restoration of images corrupted by *periodic interference*, the tool of choice is a *notch filter*.

Bandreject Filters

Ideal (IBRF)	Gaussian (GBRF)	Butterworth (BBRF)
$H(u,v) = \begin{cases} 0 & \text{if } C_0 - \frac{W}{2} \le D(u,v) \le C_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = 1 - e^{-\left[\frac{D^2(u,v) - C_0^2}{D(u,v)W}\right]^2}$	$H(u,v) = \dfrac{1}{1 + \left[\dfrac{D(u,v)W}{D^2(u,v) - C_0^2}\right]^{2n}}$

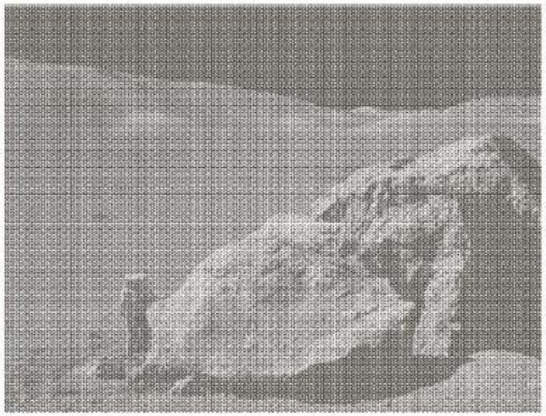


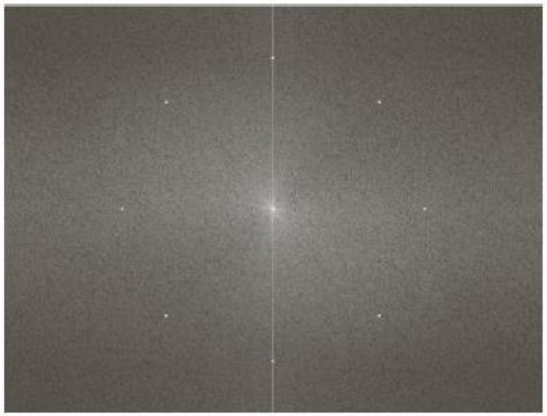
Ideal | **Modified Gaussian** | **Butterworth (Order 1)**

W – Width of the band
D – Distance D(u, v) from the center of the filter
C_0 – Center of the band
n – Order of the Butterworth filter

Bandreject Filter - example

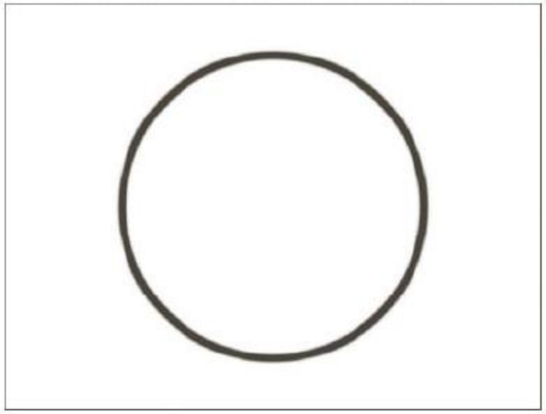
Image
+
Periodic Noise





Fourier Transform of Input Image

Butterworth Bandreject Filter (Order 4)

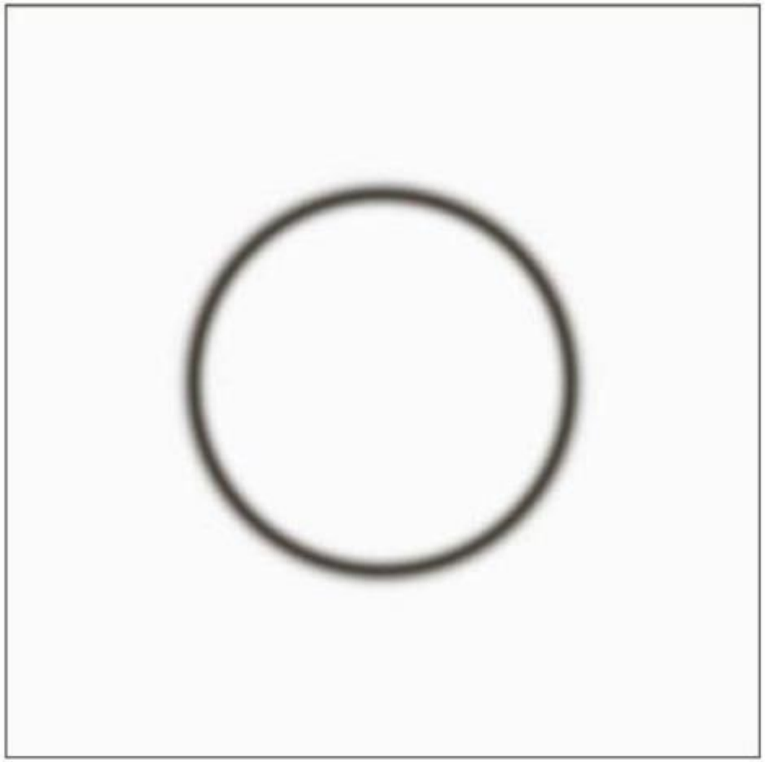


Restored Image

Not possible to achieve this result by spatial filtering

Bandpass Filters

$$H_{BP}(u,v)=1-H_{BR}(u,v)$$



Bandreject Gaussian Filter

Bandpass Gaussian Filter

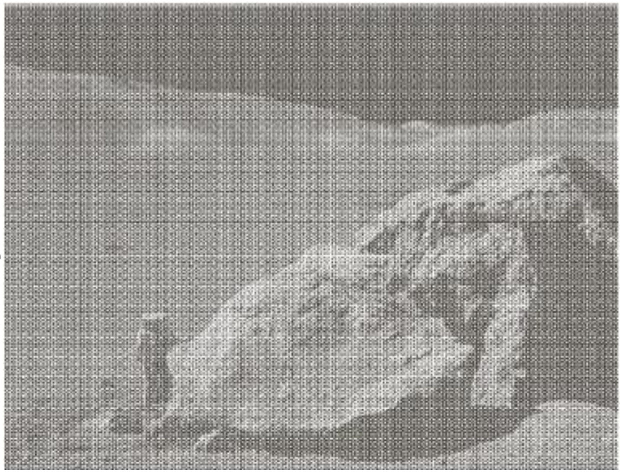
Bandpass Filters

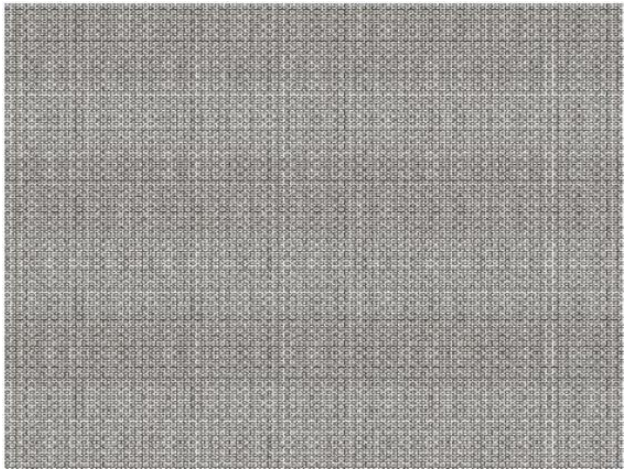
- *Opposite* function of *Bandreject* filter.

- Not usually used for filtering out noise.

- Useful to isolate the noise component from the image details for the further *analysis of the noise*.
  - Only allows the noise to pass through

Image
+
Periodic Noise





Noise Component in input Image

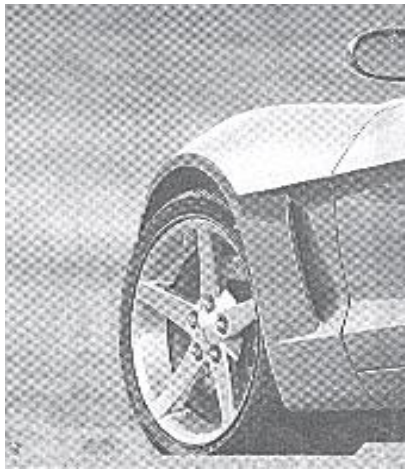
Notch Filters

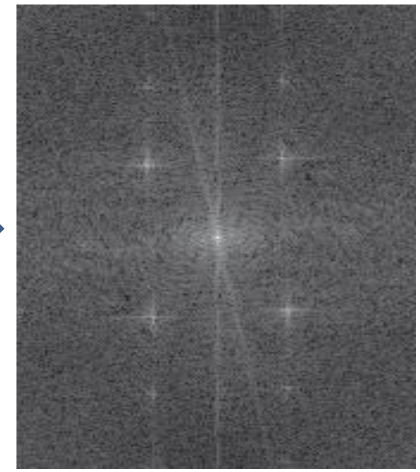
- Most useful selective filters.
- *Rejects/Passes frequencies* in a predefined neighborhood about the center of the frequency rectangle.
- These are symmetric filters about the origin
  - Can be of any shape
  - Must occur in pairs

Notch Filtering Example

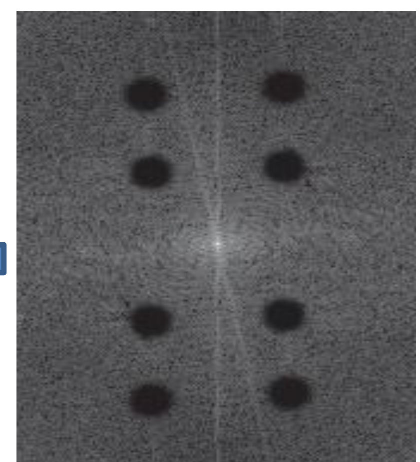
removing moiré patterns from digitized printed media images

Newspaper image showing a moiré pattern
f(x,y)

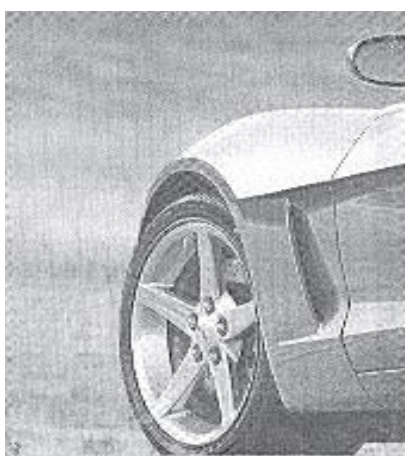




F(u,v)



F(u,v) multiplied by a pair of Butterworth notch reject filter transfer function



Filtered image

Notch Filters

- *Notch reject filter* transfer functions are constructed as *products of highpass filter transfer functions* whose centers have been translated to the centers of the notches:

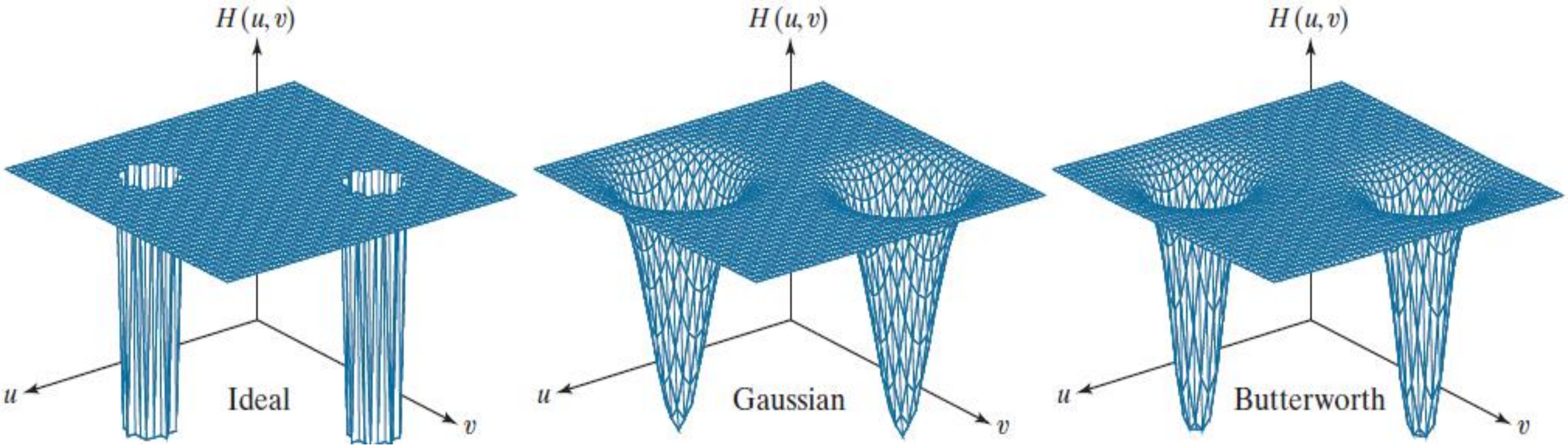
$$H_{\mathrm{NR}}(u,v) = \prod_{k=1}^{Q} H_k(u,v)\, H_{-k}(u,v)$$

where $H_k(u,v)$ and $H_{-k}(u,v)$ are highpass filter transfer functions whose centers are at (u_k, v_k) and $(-u_k, -v_k)$ respectively.

- A *notch pass filter* transfer function is obtained from a notch reject function:

$$H_{\mathrm{NP}}(u,v) = 1 - H_{\mathrm{NR}}(u,v)$$

Notch Reject Filters



Notch Filter: Example-1

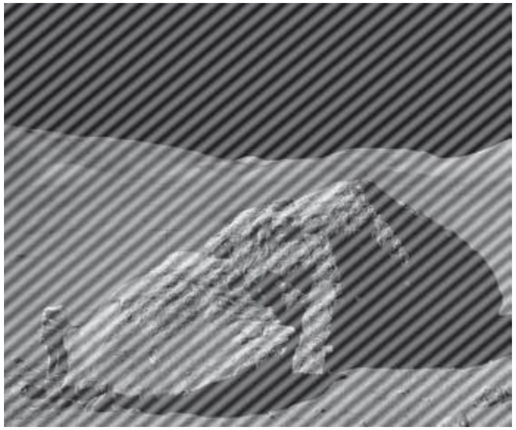
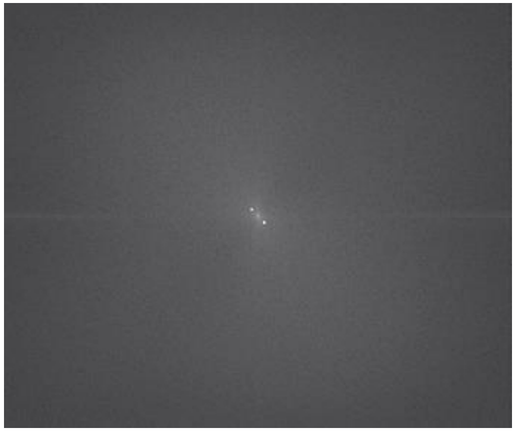
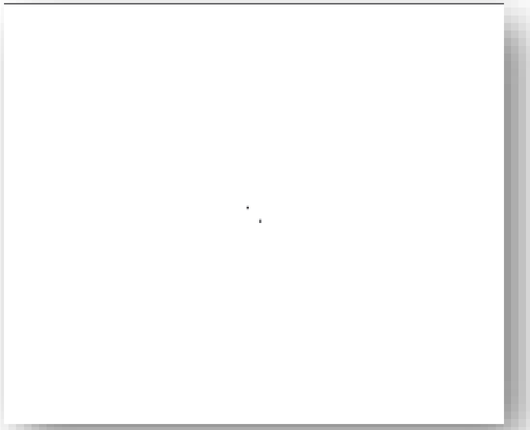


Image corrupted by sinusoidal interference (a)



Spectrum showing the bursts of energy caused by the interference



Notch reject filter transfer function $H_{NR}(u,v)$

Result of notch reject filtering

Notch Filter: Example-1

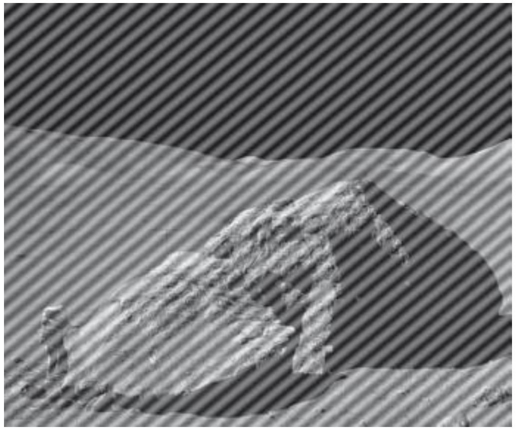
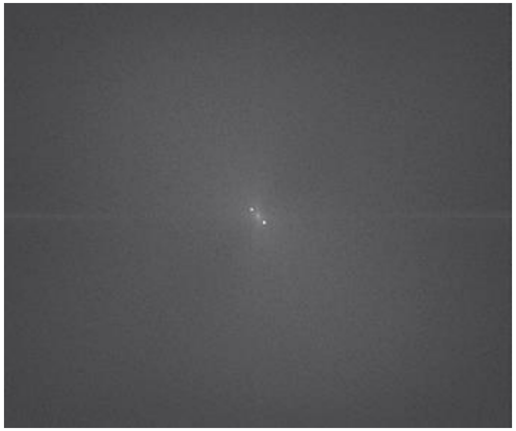
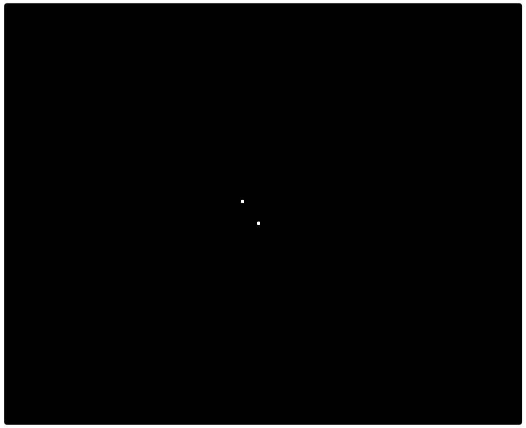


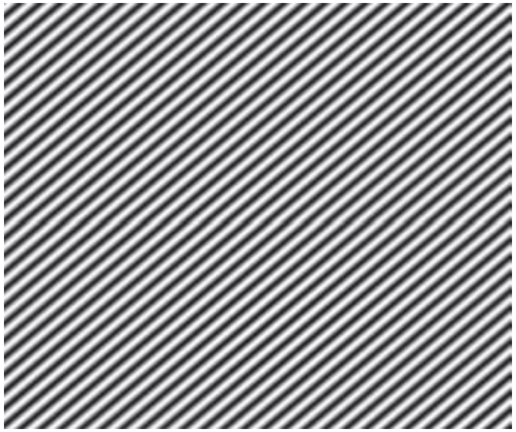
Image corrupted by sinusoidal interference (a)



Spectrum showing the bursts of energy caused by the interference



Notch reject filter transfer function $H_{NR}(u,v)$



Sinusoidal pattern extracted from the DFT of Fig. (a) using a notch pass filter $H_{NP}(u,v) = 1 - H_{NR}(u,v)$

Notch Filter: Example-2

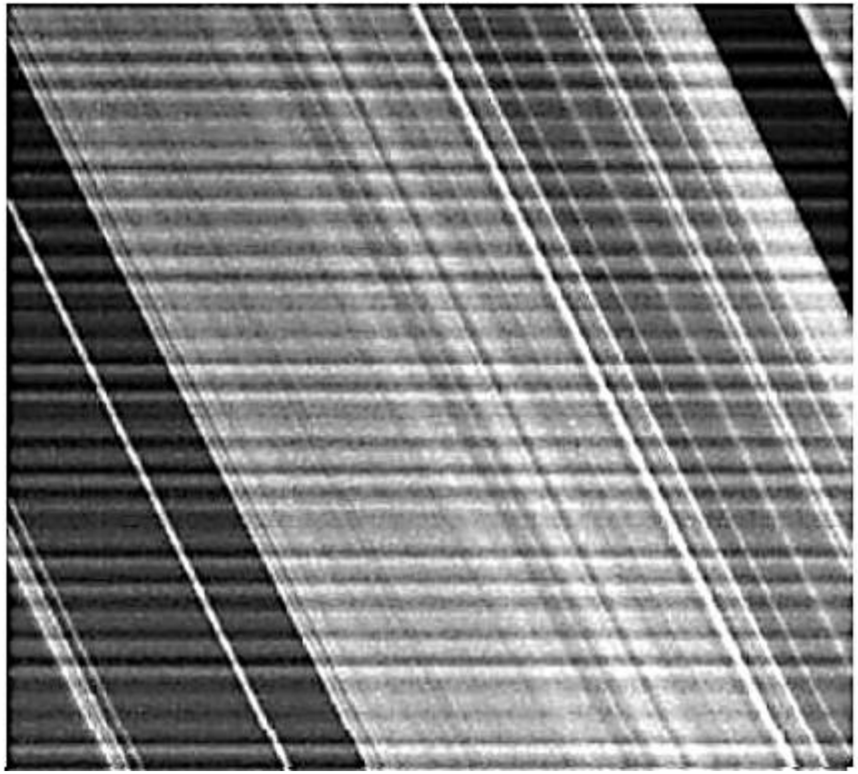
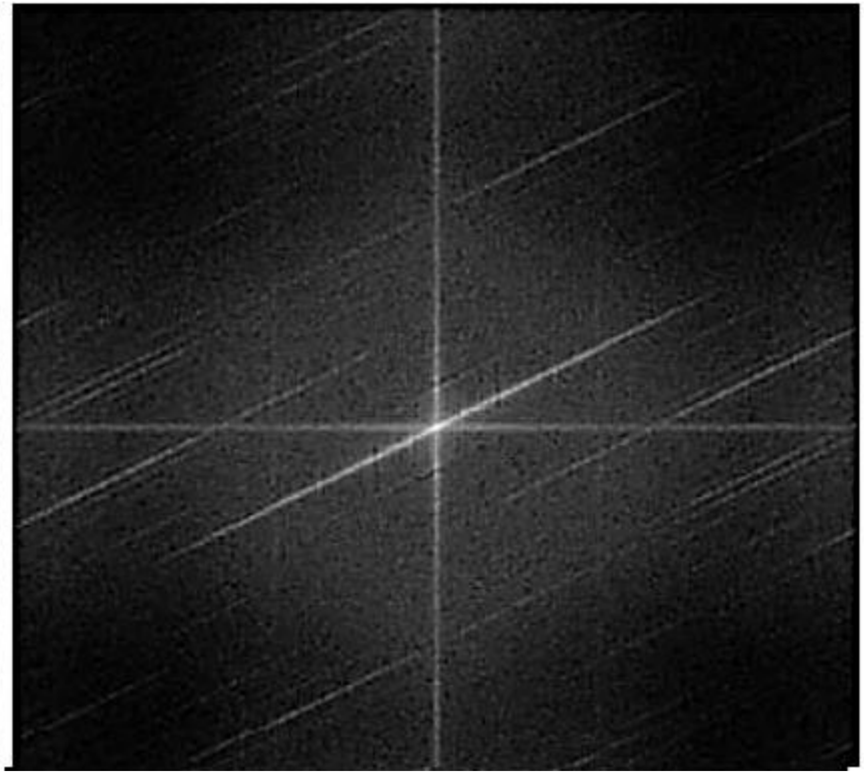
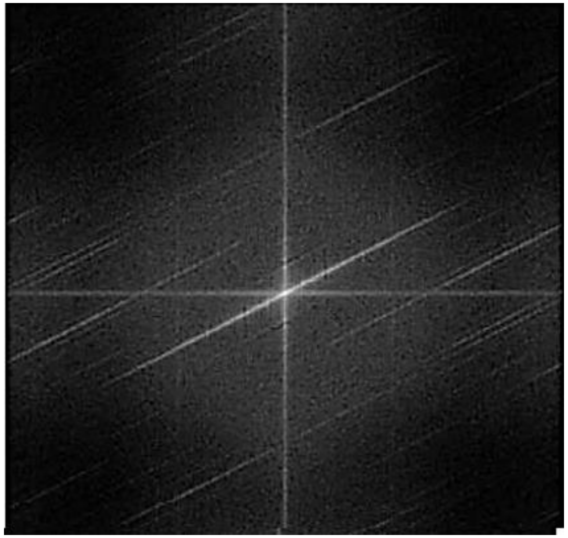


Image corrupted by periodic interference **(a)**

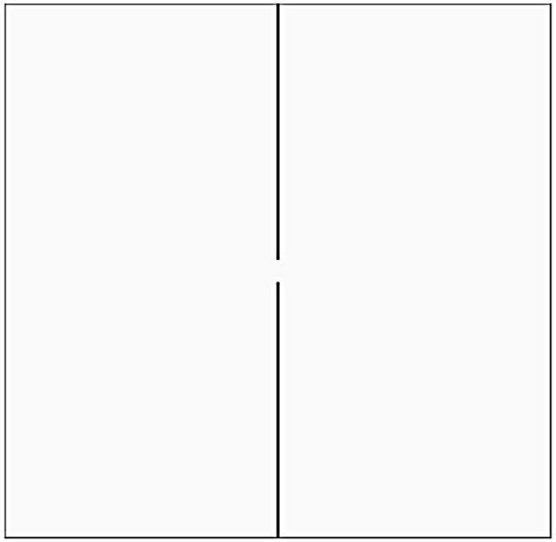


Spectrum showing the bursts of energy caused by the interference

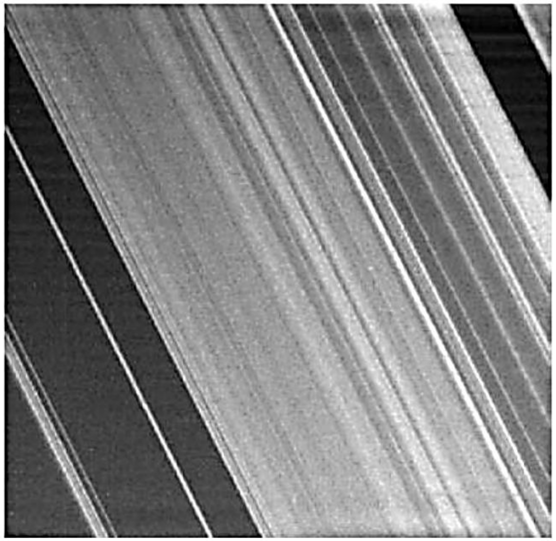
Notch Filter: Example-2



Spectrum showing the bursts of energy caused by the interference



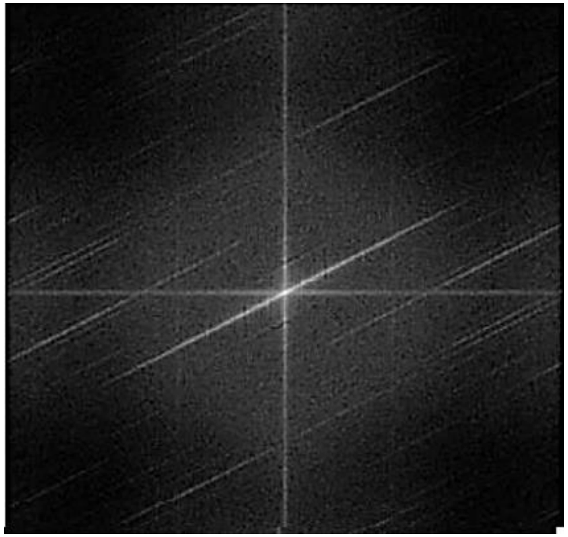
Notch reject filter transfer function $H_{NR}(u,v)$



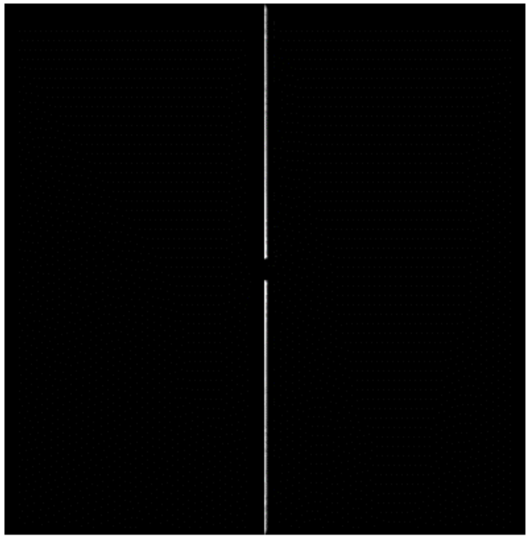
Result of notch reject filtering

We do not filter near the origin to avoid eliminating the dc term and low frequencies, which are responsible for the intensity differences between smooth areas.

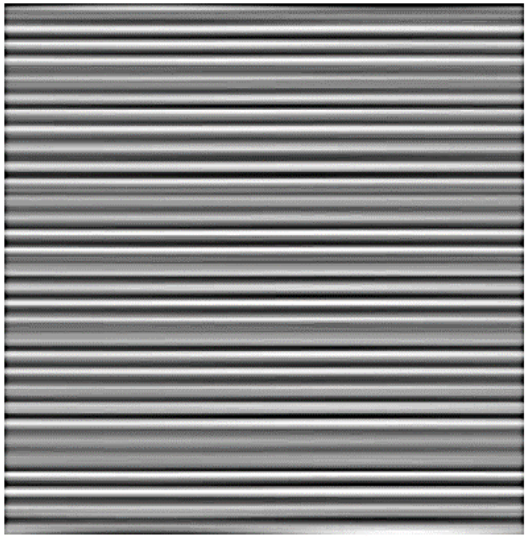
Notch Filter: Example-2



Spectrum showing the bursts of energy caused by the interference

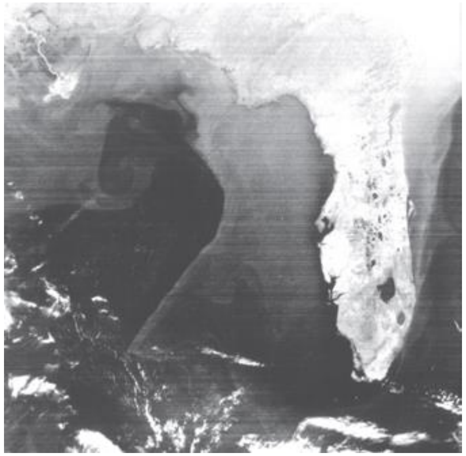


Notch pass filter transfer function $H_{NP}(u,v) = 1 - H_{NR}(u,v)$

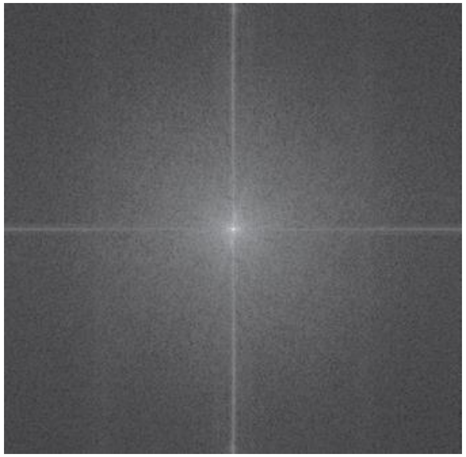


Sinusoidal pattern extracted from the DFT of Fig. **(a)** using a notch pass filter

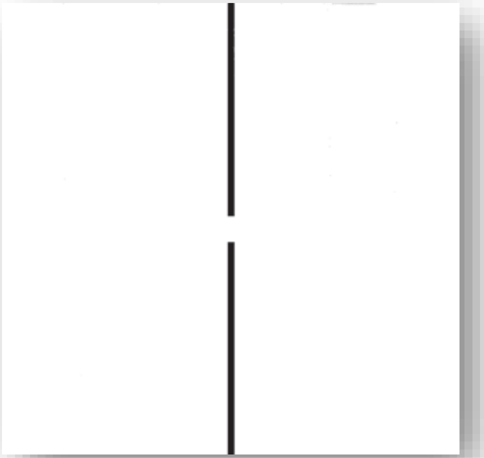
Notch Filter: Example-3



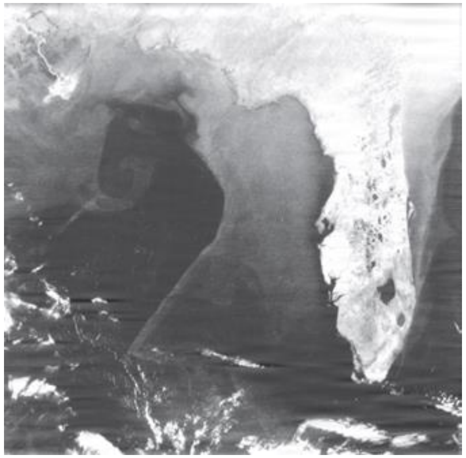
Distorted satellite image **(a)**



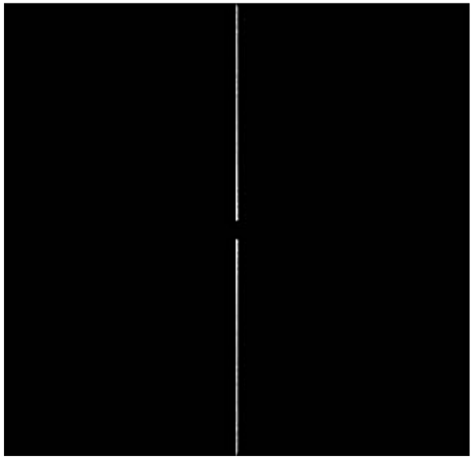
Spectrum of (a)



Notch reject filter transfer function



Filtered image



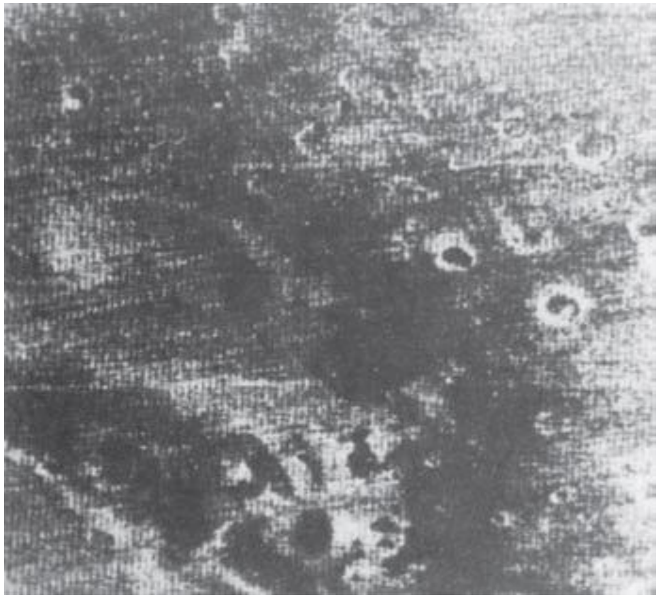
Notch pass filter transfer function

Noise pattern extracted from Fig. **(a)** by notch pass filter

Optimal Notch Filtering

- Preceding examples assume that the noise is caused by one (or two) sinusoidal components.

- In practice, the noise may have *several interference components*.



Martian Terrain (corrupted by semi-periodic interference pattern)

Fourier spectrum

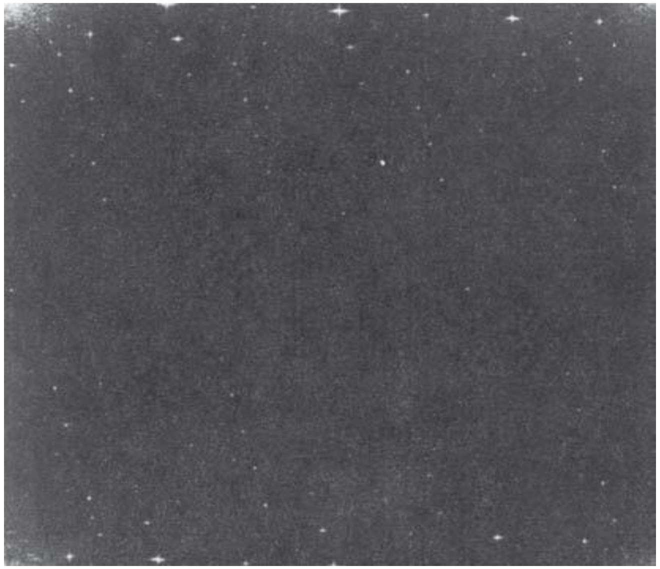
Heuristic specifications of filter transfer functions are not always acceptable because they may remove too much image information in the filtering process.

Semi-periodic Noise Patterns

- Noise patterns are not readily identifiable.

- Many *star-like* structures are present
  - More than one sinusoidal components present
- What is actual noise and what is image content?
- Interactively manipulate the mask?
  - **Tedious activity**

Spectrum without centering. The more prominent dc term and low frequencies are "out of way," providing a clearer view of interference components.



Optimal Notch Filtering - Overall Approach

1. Isolate the principal noise component.

2. Subtract a variable, weighted portion of the principal noise pattern from the corrupted image.

- Approach is *very generic* and can be adapted to other restoration tasks with multiple periodic interference problem.

Optimal Notch Filtering

1. Isolate the principal noise component

Given:
$g(x,y)$ - the corrupted image
$G(u,v)$ - Fourier transform of $g(x,y)$
$H_{NP}(u,v)$ - Notch pass filter

Fourier transform of the noise pattern is given by :

$$N(u,v) = H_{NP}(u,v)G(u,v)$$

Corresponding pattern in spatial domain is given by :

$$\eta(x,y) = \mathbb{F}^{-1}\left\{H_{NP}(u,v)G(u,v)\right\}$$

Optimal Notch Filtering

- Corrupted image ***g(x, y)*** is assumed to be formed by the addition of the uncorrupted image ***f*** **(*x, y*)** and the interference, ***η(x, y)*** :

$$g(x,y)=f(x,y)+\eta(x,y)$$

- If ***η(x, y)*** is **known completely**, then we can restore the corrupted image by:

$$f(x,y)=g(x,y)-\eta(x,y)$$

- ***Problem !!!***
  - We do not know what caused the degradation precisely (unknown component)
  - Filtering procedure usually yields only an approximation of the true noise pattern
  - The effect of this unknown component is not present in the estimate of ***η(x, y)***

Optimal Notch Filtering

2. Subtract a variable, weighted portion of the principal noise pattern from the corrupted image

- Minimize the effect of unknown components by:
  - subtracting a *weighted portion* of ***η(x, y)***

$$\hat{f}(x,y) = g(x,y) - w(x,y)\eta(x,y) \qquad \text{--------} \ \mathbf{(1)}$$

Where,

$\hat{f}$**(x, y)** – is the estimate of **f(x, y)**

w – weighting/modulation function (*to be determined*)

- Choose $w(x, y)$ so that the **local variance** of $\hat{f}$**(x, y)** is minimized over specified neighborhood S_{xy} of every point (x, y).

Optimal Notch Filtering – Estimating weighting function $\boldsymbol{w}(\boldsymbol{x}, \boldsymbol{y})$

- Consider a neighborhood $\boldsymbol{S_{xy}}$ of (odd) size $m \times n$, centered on (x, y).

- The "local" variance of $\hat{\boldsymbol{f}}$**(x, y)** at point (x, y) can be estimated using the samples in $\boldsymbol{S_{xy}}$, as follows:

$$\sigma^2(x,y) = \frac{1}{mn} \sum_{(r,c)\in S_{xy}} [\hat{f}(r,c) - \bar{\hat{f}}]^2 \quad \text{-------- } \mathbf{(2)}$$

Where $\bar{\hat{f}}$ is the average value of $\hat{\boldsymbol{f}}$ in neighborhood $\boldsymbol{S_{xy}}$ **:**

$$\bar{\hat{f}} = \frac{1}{mn} \sum_{(r,c)\in S_{xy}} \hat{f}(r,c) \quad \text{-------- } \mathbf{(3)}$$

Optimal Notch Filtering – Estimating weighting function $w(x, y)$

- Substituting Eq. **(1)** into Eq. **(2)** we get:

$$\sigma^2(x,y) = \frac{1}{mn} \sum_{(r,c)\in S_{xy}} \left\{ [g(r,c) - w(r,c)\eta(r,c)] - [\bar{g} - \overline{w\eta}] \right\}^2 \quad \text{---- (4)}$$

where $\bar{g}$ and $\overline{w\eta}$ denote the average values of g and of the product $w\eta$ in neighborhood S_{xy}, respectively.

- Assume $w(x, y)$ is constant over a neighborhood, then:

$w(r,c) = w(x,y)$ **and** $\bar{w} = w(x,y)$ **so** $\overline{w\eta} = w(x,y)\,\bar{\eta}$ ---- **(5)**

- Substituting approximations **(5)** into Eq. **(4)** we get:

$$\sigma^2(x,y) = \frac{1}{mn} \sum_{(r,c)\in S_{xy}} \left\{ [g(r,c) - w(x,y)\eta(r,c)] - [\bar{g} - w(x,y)\,\bar{\eta}] \right\}^2$$

Optimal Notch Filtering – Estimating weighting function $\boldsymbol{w(x, y)}$

- To minimize $\boldsymbol{\sigma^2(x, y)}$ with respect to $\boldsymbol{w(x, y)}$ we solve:

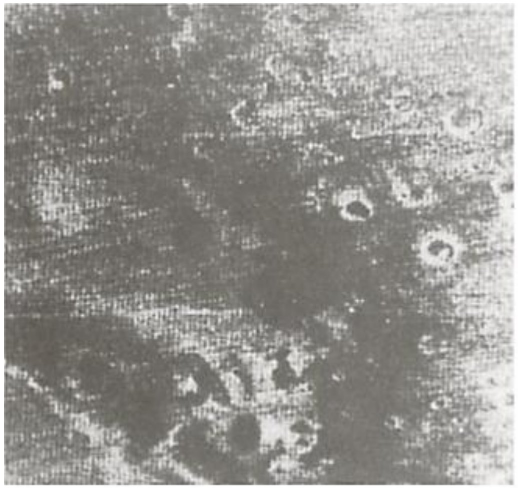
$$\frac{\partial \sigma^2(x, y)}{\partial w(x, y)} = 0$$

and we get

$$w(x, y) = \frac{\overline{g\eta} - \bar{g}\bar{\eta}}{\overline{\eta^2} - \bar{\eta}^2}$$

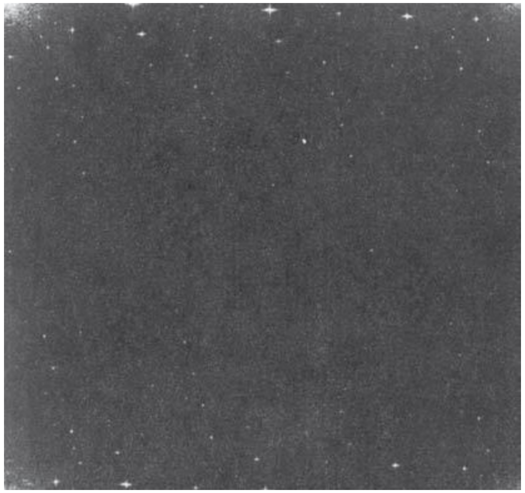
- To obtain the restored image we substitute $\boldsymbol{w(x, y)}$ at every point in the noisy image $\boldsymbol{g(x, y)}$.

Optimal Notch Filtering - Result

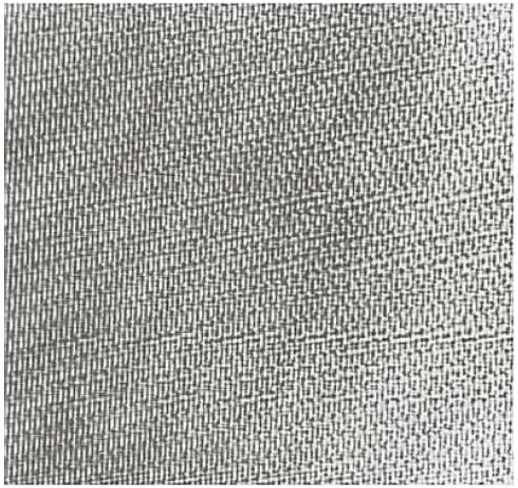


Original Image

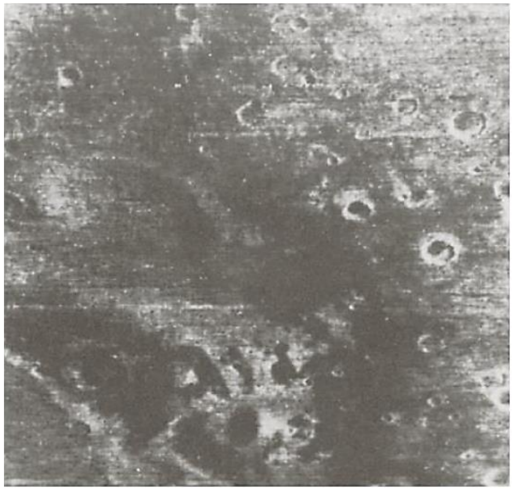
Fourier Spectrum



Fourier Spectrum
(without centering)



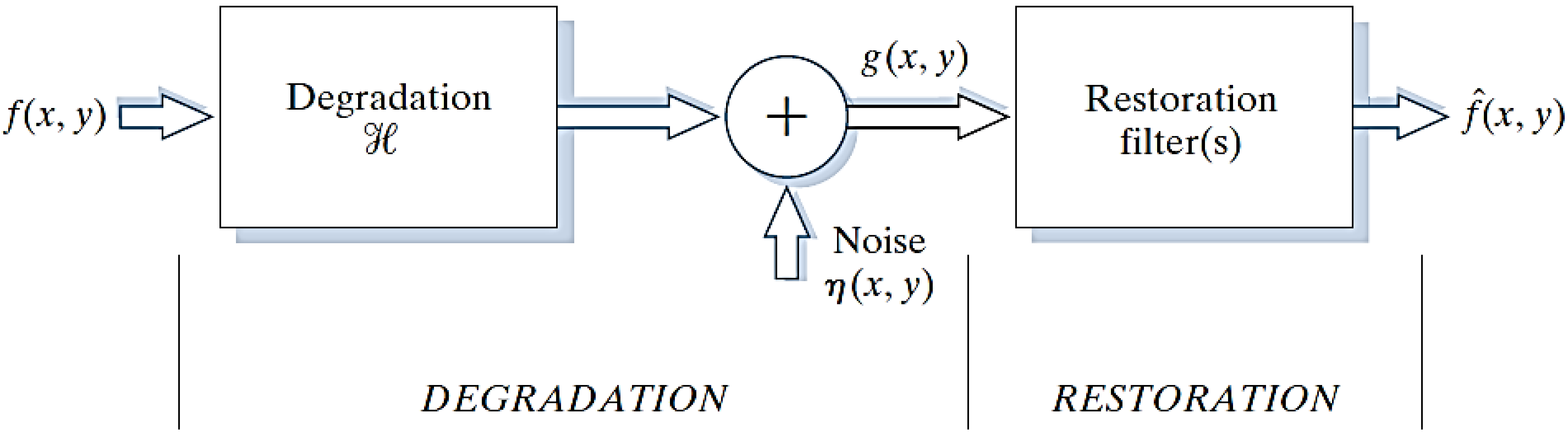
η(x,y) after applying
notch pass filter



Restored Image $\hat{f}(x, y) = \mathbf{g(x, y) - w(x, y)} * \boldsymbol{\eta(x, y)}$

Linear, Position-invariant Degradations

Linear Additive Noise and Degradation



$f(x, y)$: Input image
$H(x, y)$: Degradation filter
$g(x, y)$: Degraded image
$\eta(x, y)$: Noise
$\hat{f}(x, y)$: Restored Image

The input-output relationship in this Figure **before the restoration** stage is:

$$g(x, y) = \mathcal{H}\left[f(x, y)\right] + \eta(x, y)$$

Linear, Invariant Degradation

let us assume that $\eta(x, y) = 0$ so that $g(x, y) = \mathcal{H}[f(x, y)]$

If $\mathcal{H}$ is *linear* then $\mathcal{H}$ satisfies the following property:

$$\mathcal{H}[af_1(x,y) + bf_2(x,y)] = a\mathcal{H}[f_1(x,y)] + b\mathcal{H}[f_2(x,y)] \quad \text{..... } \mathbf{(1)}$$

If $a = b = 1$, Eq. **(1)** becomes:

$$\mathcal{H}[f_1(x,y) + f_2(x,y)] = \mathcal{H}[f_1(x,y)] + \mathcal{H}[f_2(x,y)]$$

- Additivity property

If $f_2(x, y) = 0$, Eq. **(1)** becomes:

$$\mathcal{H}[af_1(x,y)] = a\mathcal{H}[f_1(x,y)]$$

- Homogeneity property

A linear operator possesses both the property of additivity and the property of homogeneity.

Linear, Invariant Degradation

If $g(x, y) = H[f(x, y)]$ and

$$\boxed{H[f(x-\alpha, y-\beta)] = g(x-\alpha, y-\beta)}$$

The operator $\mathcal{H}$ is said to be Position(or Space) Invariant for any ***f* (*x*, *y*)** and any two scalars $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$.

Impulse Response

Using the *sifting property* of the 2-D *continuous impulse* we can write value of ***f* (*x*, *y*)** ***at some points*** $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ as:

$$f(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\;\;\delta(x-\alpha,y-\beta)\quad d\alpha\;\;d\beta$$

$$g(x,y)=H[f(x,y)]=$$

$$H[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\;\;\delta(x-\alpha,y-\beta)\quad d\alpha\;\;d\beta]$$

If we extend the **additivity** property to integrals,

$$g(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H[f(\alpha,\beta)\;\;\delta(x-\alpha,y-\beta)]\quad d\alpha\;\;d\beta$$

Impulse Response

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H[f(\alpha,\beta)\ \ \delta(x-\alpha, y-\beta)]\quad d\alpha\ \ d\beta$$

f(α, β) is independent wrt. **x** and **y**

Using **homogeneity** property,

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\ \boxed{H[\delta(x-\alpha, y-\beta)]}\quad d\alpha\ \ d\beta$$

Impulse Response of H

Impulse Response

$$h(x, \alpha, y, \beta) = H[\delta(x - \alpha, y - \beta)]$$

Impulse Response of H

$h(x, \alpha, y, \beta)$ is the response of H to an impulse at location (x,y)

Substituting in equation for g(x,y), we get:

$$\boxed{g(x, y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha, \beta) \;\; h(x, \alpha, y, \beta) \quad d\alpha \;\; d\beta}$$

Superposition Integral of the First Kind

Which states that:

If we know the response of H to an impulse, then we can calculate the response of H to ANY function $f(\alpha, \beta)$ using the above equation

Including the Noise Term

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta) \;\; h(x,\alpha,y,\beta) \quad d\alpha \;\; d\beta + \eta(x,y)$$

Adding noise term

If **H** is position invariant then,

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta) \;\; h(x-\alpha,y-\beta) \quad d\alpha \;\; d\beta + \eta(x,y)$$

Using the notation of ***convolution***, we can write ***g(x, y)*** in **spatial domain** as,

$$g(x,y) = (h \star f)(x,y) \; + \; \eta(x,y)$$

OR

$$G(u,v) = H(u,v)F(u,v) \; + \; N(u,v)$$ in **frequency domain**

Estimation of the Degradation Function ($\mathcal{H}$)

Estimation of Image Degradation

- Three principal approaches:
  - Observation
  - Experimentation
  - Mathematical Modeling

Estimation by Observation

- We are given a degraded image *without any knowledge* about the degradation function $\mathcal{H}$.

- To estimate $\mathcal{H}$, work on a **sub-image** (rectangular section of image) with some *assumption* of *noise* and *signal.*

- The **sub-image** contains sample structures like, *known objects* and *background*.
  - In order to *reduce the effect of noise*, we would look for a sub-image area in which the *signal content is strong* (e.g., an area of high contrast), so that the **noise is negligible.**

- The next step would be to process the **sub-image** to arrive at a result that is **as unblurred as possible**.

Estimation by Observation

Actual Procedure

Let the observed subimage be denoted by $g_s(x, y)$, and let the processed subimage (which in reality is our estimate of the original image in that area) be denoted by $\hat{f}_s(x, y)$.

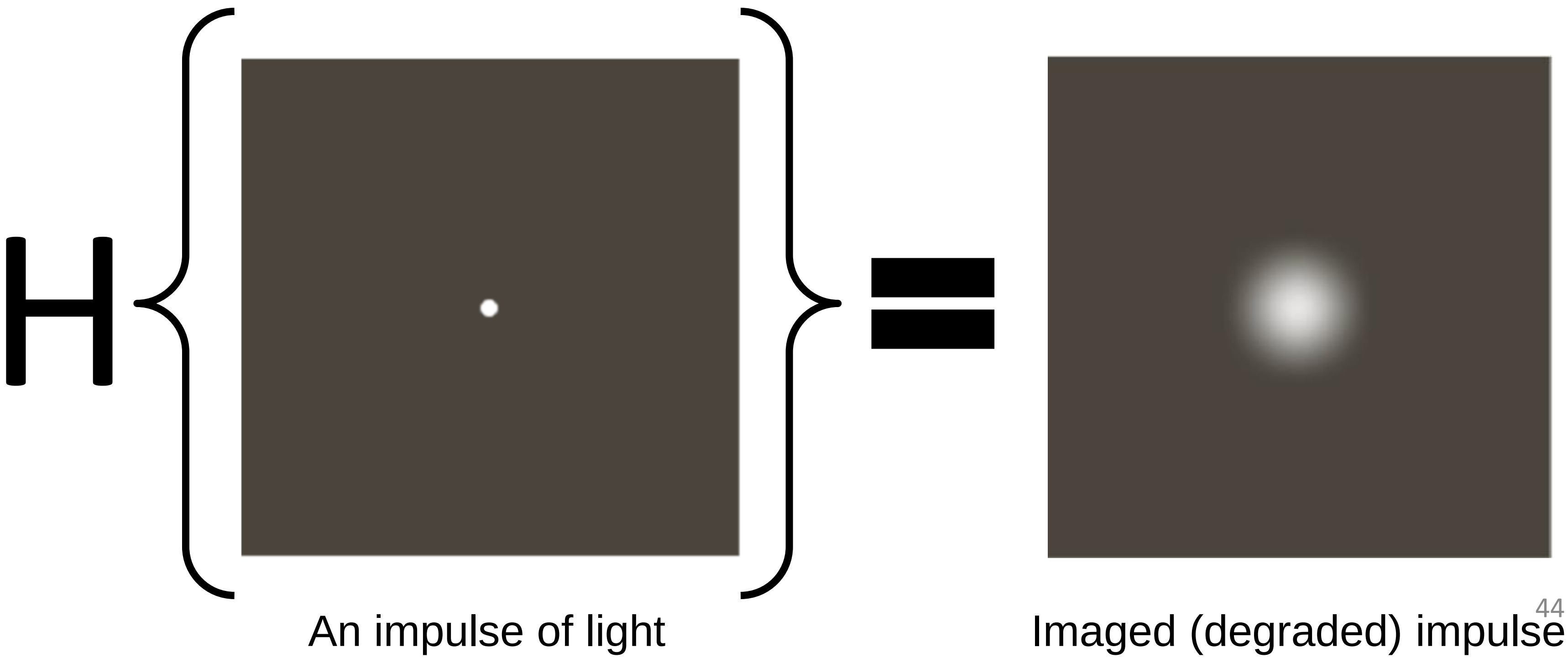
Then, assuming that the effect of noise is negligible because of our choice of a strong-signal area, it follows from $G(u,v) = H(u,v)F(u,v) + N(u,v)$ that

$$H_s(u, v) = \frac{G_s(u, v)}{\hat{F}_s(u, v)}$$

$H_s(u, v)$ denotes the degradation in the image. We can extrapolate the behavior of H from the sub-image to the entire image.

Estimation by Experimentation

- Use a similar equipment that was used to acquire the degraded image.
- Using various settings of this equipment, try to acquire images as closely as possible to the degraded image we wish to restore.
- Then obtain the *impulse response of the degradation* by imaging an impulse (*small dot of bright light*) using the same system settings.



Estimation by Experimentation

- Since Fourier Transform of an impulse is a **constant function** , it follows that:

$$H(u,v) = \frac{G(u,v)}{A}$$

We select the dot of light, as bright as possible to reduce the effect of noise to negligible values

where,

$G(u,v)$ = Fourier Transform of the observed image,

A = constant describing the strength of the impulse

Estimation by Modeling
Approach-1

- Degradation model based on atmospheric turbulence [Hufnagel and Stanley, 1964].

$$H(u,v) = e^{-k(u^2 + v^2)^{5/6}} \quad \text{where} \quad k = \text{constant of turbulence}$$

No visible turbulence

Severe turbulence, **k=0.0025**

Mild turbulence, **k=0.001**

Low turbulence, **k=0.00025**

Estimation by Modeling

Approach-2

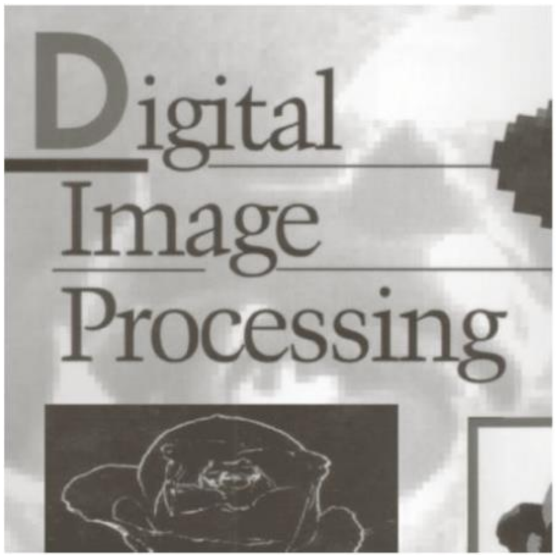
- Image has been blurred by *uniform linear motion* between the image and the sensor during image acquisition.

Image $\boldsymbol{f(x, y)}$ undergoes planar motion and that $\boldsymbol{x_0(t)}$ and $\boldsymbol{y_0(t)}$ are the time-varying components of motion in the x- and y-directions.

Estimation by Modeling

Approach-2

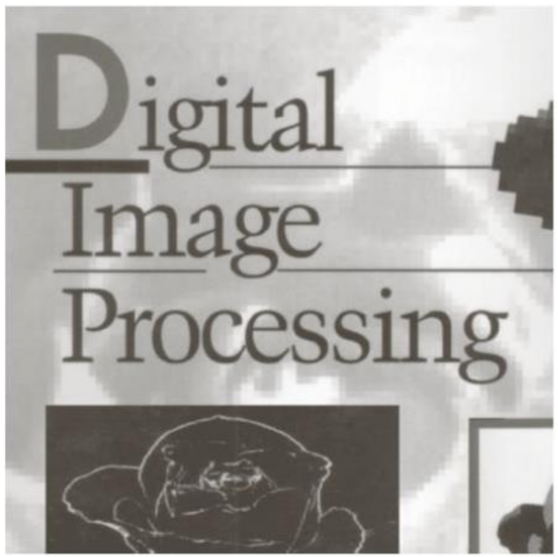
$H\{$



$\} =$

- Resulting blurred image for $x_0(t)=at/T$, $y_0(t)=bt/T$, where **a=b=0.1**, **T=1**
- We assume that the shutter opening/closing time **(t)** is instantaneous (**t=1**)

Estimation by Modeling
Approach-2

H


=


By defining

$$H(u,v) = \int_0^T e^{-j2\pi[ux_0(t) + vy_0(t)]}\, dt$$

$$\boxed{H(u,v) = \frac{T}{\pi(ua + vb)} \sin\left[\pi(ua + vb)\right] e^{-j\pi(ua + vb)}}$$

FINAL EXAM

Complete Syllabus – Important topics name are given in Moodle