

## Master Theorem Examples

example 1:  $T(n) = 8T(n/2) + n$

$$f(n) = n, \quad a=8, \quad b=2, \quad n^{\log_2 8} = n^3$$

$$f(n) = n \in O(n^{3-\epsilon}) \quad \text{when } \epsilon = 1$$

case 1 method:  $T(n) = \Theta(n^3)$

$$T(n) \in \Theta(n^{\log_2 8})$$

example 2:  $T(n) = 8T(n/2) + n^3$

$$f(n) = n^3, \quad a=8, \quad b=2, \quad n^{\log_2 8} = n^3$$

case 1:  $f(n) = n^3 \in O(n^{3-\epsilon})$  for some  $\epsilon > 0$  X

case 2:  $f(n) = n^3 \in \Theta(n^3 \cdot (\lg n)^k)$  Let  $k=0$

$$n^3 \in \Theta(n^3) \quad \text{true.} \quad \checkmark$$

$$T(n) = \Theta(n^{\log_2 8} \cdot (\lg n)^{k+1}) = \Theta(n^3 \lg n)$$

example 3:  $T(n) = T(n/4) + 1$

$$f(n) = 1, \quad a=1, \quad b=4, \quad n^{\log_4 1} = n^0 = 1$$

case 2:  $f(n) = 1 \in \Theta(n^0 \cdot (\lg n)^k)$   $k=0$

$$f(n) = 1 \in \Theta(1) \quad T(n) = \Theta(\lg n)$$

example 4:  $T(n) = 2T(n/4) + 1$

$$f(n) = 1, \quad a=2, \quad b=4, \quad n^{\log_4 2} = n^{0.5}$$

case 1:  $f(n) = 1 \in O(n^{0.5-\epsilon})$   $\epsilon = 0.1$

$$T(n) = \Theta(n^{0.5})$$