

Task - $\text{Search}(x)$ = Comparing " x " with other elements

Q. Can we do $\text{Search}(x)$ without any comparison?

Locate/Search - $O(k)$ k is constant

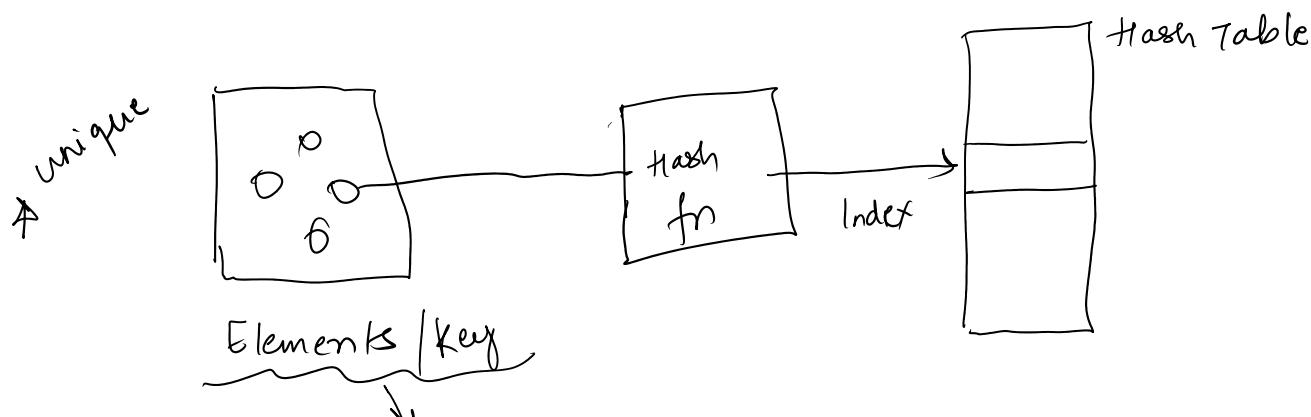
HASHING

#g. Array of 10 integers, 0 - 99; unique integers

[1, 3, 9, 15, 25, 87, 99, 2, 42, 50]

HASH TABLE

0	1	9	15	25	87	99
0	2	3	9	15	25	



Elements / Key

We use value as the YP or index of hash function

KEY BASED ALGORITHMS → DIRECT ADDRESSING

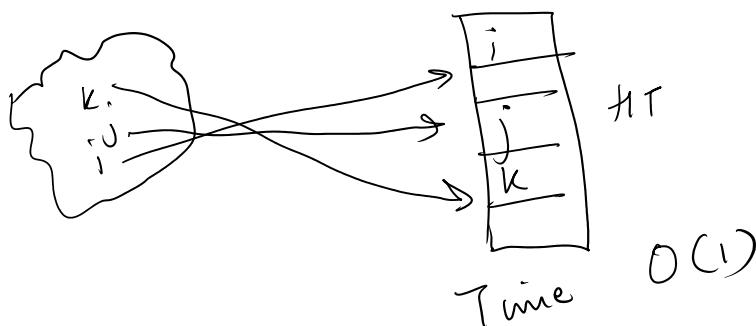
Given a key value we can find the item
in $O(1)$

☐ Hash function

$$y = h(x)$$

↓ ↗ unique
I/P / key operation that maps
↓ one large set of values
O/P index into another set of values
might not be unique

Time vs Space complexity.



#g Student Ids - 10,000 Students

hash function - Last 4 digits → index of the HT
7213

HT - 10,000

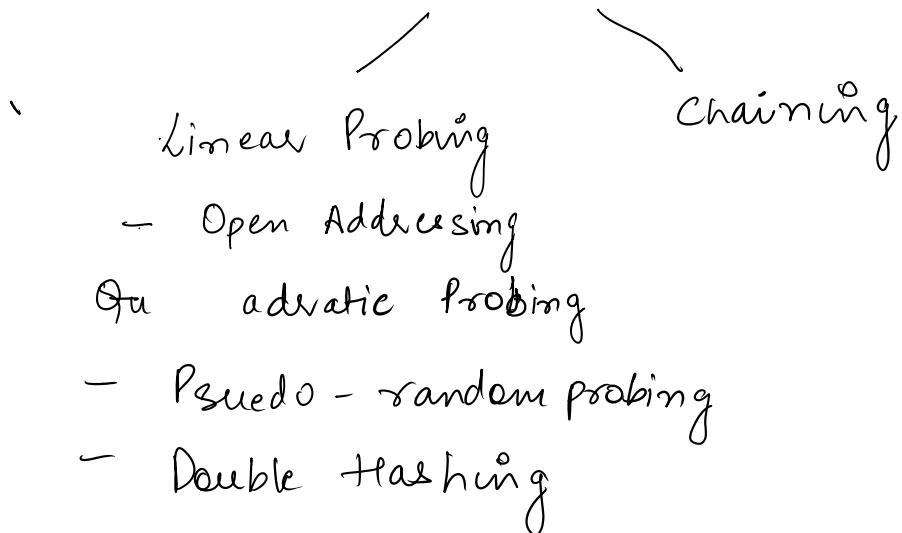
=> multiple student ids pointing to the same slot in HT

↳ COLLISION

If hash function points to the same index in the hash table

- Better hash functions
- Deal with collisions

* COLLISION RESOLUTION



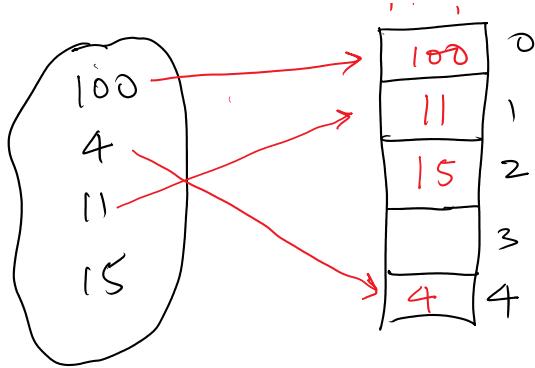
□ Linear Probing

#g 100, 4, 11, 15

$h(x) = x \% 5$; size of

HT = 5





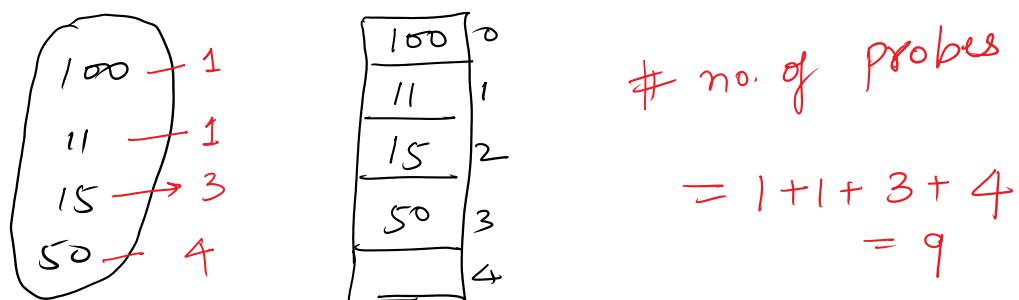
Many to one function - Collisions are inevitable

Find the index of the HIT using hash function.

If that index is not empty, traverse the hash table linearly (go to the next index).

Keep on doing the above recursively, till an empty slot is found.

$$\#g \quad 100, 11, 15, 50 \quad h(x) = x \% 5$$



#g

19	1
16	1
9	2
14	1
1	1
12	2
20	7

14	0
1	1
16	2
9	3
20	4
19	5
12	6

$$h(x) = x \% 7$$

$$LF = \frac{7}{7} = 1$$

$$\Rightarrow \boxed{\text{LOAD FACTOR}} = \frac{\# \text{ of items stored in HT}}{\text{Total size of HT}}$$

HT

#g	
5	
10	
15	

$LF = \frac{3}{5}$

If HT is $\frac{2}{3}$ full - spend most of the time doing collision resolution. For an average cluster size of 3, avg 2 probes collisions tend to form clusters
 \downarrow
group of items.

clusters grow quickly - SNOWBALL EFFECT

\Rightarrow HT is sparse - Linear probing is faster

HIT

$$\frac{1 + \frac{1}{1-x}}{2}$$

Find an item

Knuth's

formula

Avg. no. of
probes

MISS

$$\frac{1 + \frac{1}{(1-x)^2}}{2}$$

Discover that the item
does not exist

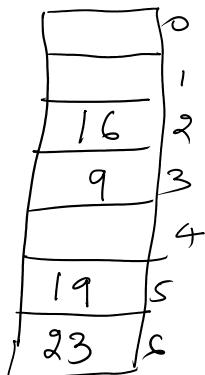
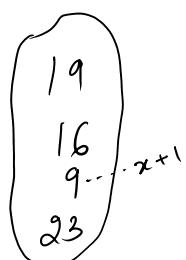
* Quadratic Probing

$$\text{Probes} = \{1, 4, 9, 16, \dots, y\}$$

$$\rightarrow \downarrow y+1, y+2^2, y+3^2, \dots$$

index from
the hash fn

#g.



$$h(x) = x \% 7$$

$$\text{for } 9 - h(x) = 2 = y$$

collision
 $y+1 = 3$

$$\text{for } 23 - y = 2$$

$$\begin{aligned} \text{Collision } y+1 &= 3 \\ y+2^2 &= 6 \end{aligned}$$

* Pseudo-Random probing

$$\{1, 5, 7, 6, \cancel{8}, 3, 4, 0, 2\}$$

* Double Hashing — $P(k)$
— $S(k)$

1st Probe $P(k) + S(k)$

2nd Probe $P(k) + 2S(k)$

3rd Probe $P(k) + 3S(k)$

#g 6, 12, 18 $h(x) = x \% 6$; $g(x) = (x \% 5) + 1$



	$h(x)$	$g(x)$	
for 12	0	$0 + 3 = 3$	
for 18	0	$0 + 4 = 4$	

#g 11, 22, 33, 12, 13, 25, 18

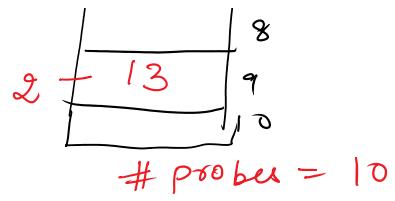
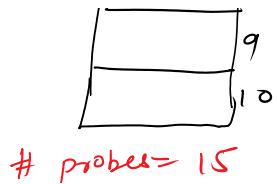
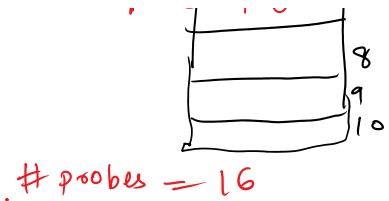
$$h(x) = x \% 11; g(x) = (x \% 7) + 1$$

	L.P
1	11
2	22
3	33
3	12
3	13
3	25
1	18

	Q.P
1	11
2	22
2	12
2	13
3	33
3	25
2	18

	DH
1	11
1	12
2	22
1	25
2	33
1	18
2	13

$$\begin{aligned} 22 &= 0 + 2 \\ 33 &= 0 + 6 \\ 13 &= 2 + 7 \end{aligned}$$



④ Deletion from Hash Table

Mark
Pack

Every slot in HT O/U/D

occupied	un-	Deleted
	occupied	

→ Mark - O/U/D , If inserting U/D

#g 19, 16, 9, 14, 1, 12 $h(k) = k \% 7$

O	14	0
O	1	1
O	16	2
O	9	3
U		4
D	X	5
O	12	6

Delete 19

↓
remove it from HT

change/ Mark - D

Find 12 - 12%7 = 5 O → U

↳ slot 5 - D → look for the next slot

So if slot 5 - U → 12 does not exist in HT

U → If nothing is stored in that slot

- O → If there is a key value in that slot
- D → If there was a key value previously stored in that slot, & now that value has been removed from HT

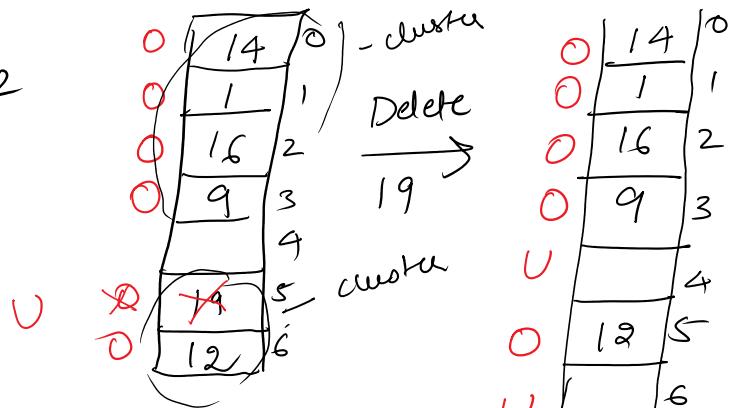
Insert '26' - index 5 - mark D

insert 26 in Slot 5 & change mark O

⇒ Pack - Reordering

- for each item in HT after the deleted item that is in the cluster, mark its slot as unoccupied and reinsert it in the HT.

#g. 19, 16, 9, 14, 1, 12



Delete 19 from HT

change slot 5 mark O → U

Reinsert 12 - find the index for 12 in HT again

Insert '26' - in slot 6 because slot 5 is O

* CHAINING

Linked List

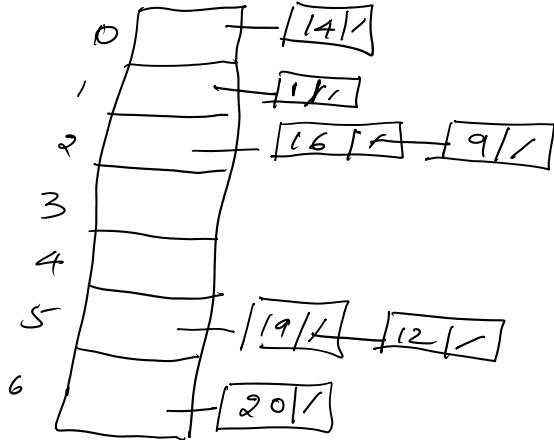
Data is stored outside of the table

#g 19, 16, 9, 14, 1, 12, 20
1 1 2 1 1 2 1

$$h(k) = k \% 7$$

probes = 9

$$LF = \frac{7}{7} = 1$$



Load factor \rightarrow Avg length of LL

No concept of 2/3 FULL.