# TOKENFORMER: RETHINKING TRANSFORMER SCALING WITH TOKENIZED MODEL PARAMETERS

——Wang et al.,2024

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2024.12.31

# Contents

# Introduction

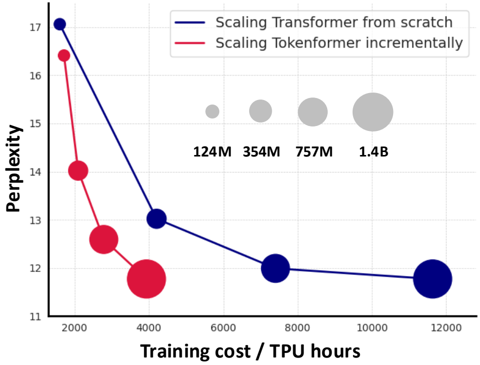
Transformers have become the predominant architecture in foundation models due to their excellent performance across various domains. However,

- scaling these models requires substantial costs;
- architectural modifications typically require the entire model to be retrained from scratch.

The **Tokenformer**, a natively scalable architecture that totally leverages the attention mechanism, was proposed.

- Treat model parameters as tokens and replace all linear projections in Transformers with token-parameter attention layers;
- Input tokens act as queries and model parameters as keys and values;
- Allow for progressive and efficient scaling without retraining from scratch;
- Achieve performance comparable to Transformers while greatly reducing the training costs.

# Transformer vs Tokenformer



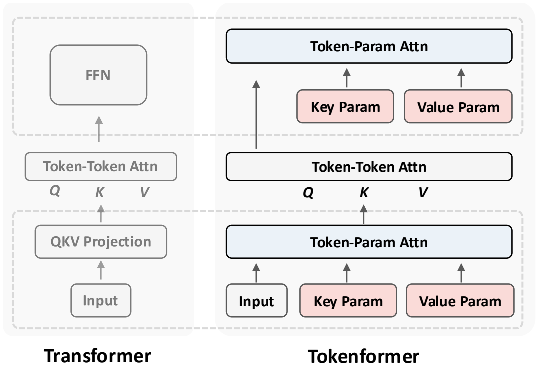
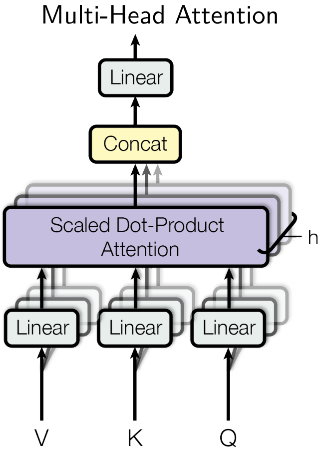




图: A simple comparasion between Transformers and Tokenformers.

# Transformer




Input tokens: $X \in \mathbb{R}^{T \times d}$

$$Q = X \cdot W^Q, \quad K = X \cdot W^K, \quad V = X \cdot W^V; \quad (1)$$

$W^Q, W^K \in \mathbb{R}^{d \times d_k}, W^V \in \mathbb{R}^{d \times d_v}.$

$$\text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right)V. \quad (2)$$

Output:

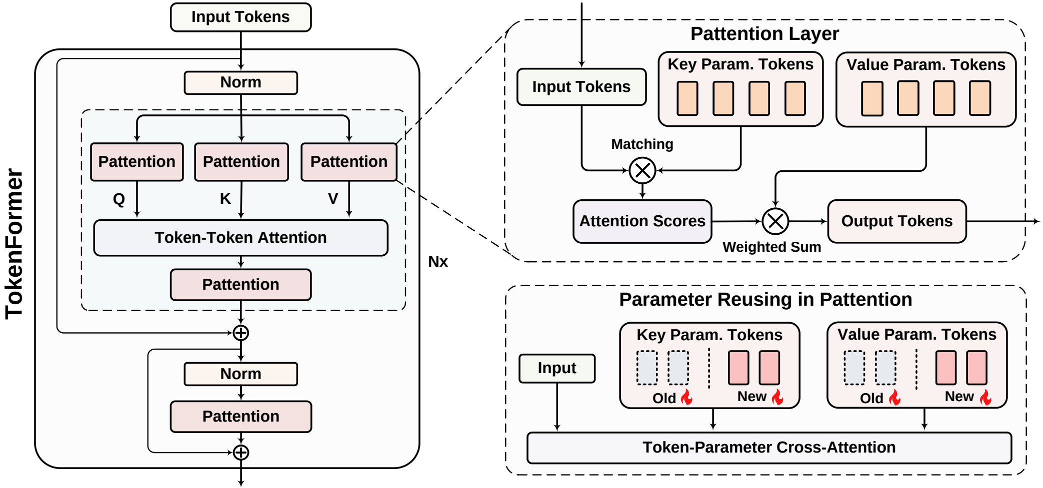
$$O = X_{\text{att}} \cdot W^O, \quad W^O \in \mathbb{R}^{d_v \times d}.$$

FFN:

$$X_{\text{ffn}} = \text{Activation}(O \cdot W_1 + b_1)W_2 + b_2.$$

$W_1 \in \mathbb{R}^{d \times d_{\text{ffn}}}, W_2 \in \mathbb{R}^{d_{\text{ffn}} \times d}.$

# TokenFormer



# Pattention Layer

Let $K_P \in \mathbb{R}^{n\times d_1}$ and $V_P \in \mathbb{R}^{n\times d_2}$ represent the learnable parameter tokens.($n$ is the number of key-value pairs)

$$\text{Pattention}(X, K_P, V_P) = \theta(X \cdot K_P^T) \cdot V_P, \tag{3}$$

where $\theta$ is the modified softmax function. The output Pattention scores, $S \in \mathbb{R}^{n\times n}$, are computed as:

$$S_{ij} = \text{GeLU}\left(\frac{A_{ij} \times \tau}{\sqrt{\sum_{k=1}^{n} |A_{ik}|^2}}\right), \quad \forall\ i, j \in 1 \ldots n, \tag{4}$$

where $A = X \cdot K_P^T$ and $\tau$ is a scale factor.

In transformer, we have $Q = X \cdot W_Q$.
In tokenformer, we have $Q = \text{Pattention}(X, K_P^Q, V_P^Q)$.

# Overall Architecture

- QKV Pattention:

$$Q = \text{Pattention}(X, K_P^Q, V_P^Q), \quad K = \text{Pattention}(X, K_P^K, V_P^K), \quad V = \text{Pattention}(X, K_P^V, V_P^V).$$

- Token-Token Attention:

$$X_{\text{att}} = \text{softmax}\left(\frac{Q \cdot K^T}{\sqrt{d}}\right) \cdot V,$$

$$O_{\text{att}} = \text{Pattention}(X_{\text{att}}, K_P^O, V_P^O).$$

- FFN:

$$O_{\text{ffn}} = \text{Pattention}(O_{\text{att}}, K_P^{\text{ffn}}, V_P^{\text{ffn}}).$$

# Progressive Model Scaling

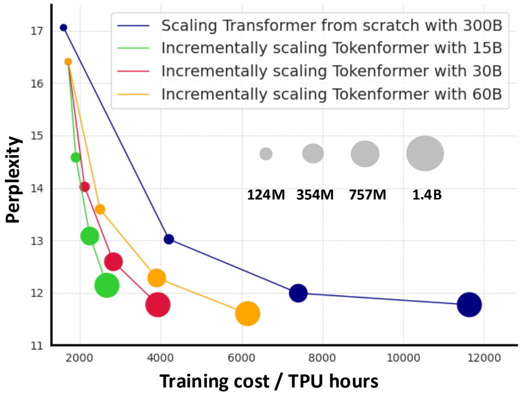
Consider an existing Tokenformer model equipped with a set of pre-trained key-value parameter tokens, denoted as $K_P^{\text{old}}, V_P^{\text{old}} \in \mathbb{R}^{n \times d}$. To scale the model, just augment this set by appending new key-value parameter tokens $K_P^{\text{new}}, V_P^{\text{new}} \in \mathbb{R}^{m \times d}$ as

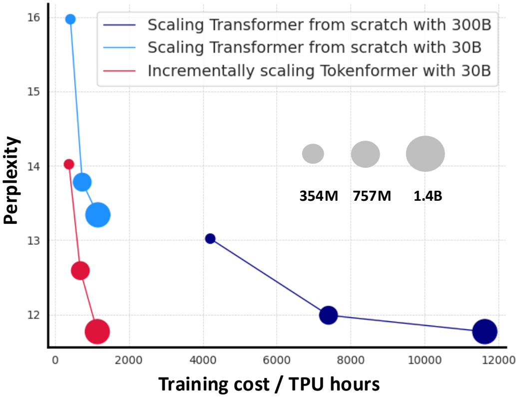
$$K_P^{\text{scale}} = \left[K_P^{\text{old}}, K_P^{\text{new}}\right], \qquad V_P^{\text{scale}} = \left[V_P^{\text{old}}, V_P^{\text{new}}\right]. \tag{5}$$

$$O = \text{Pattention}\left(X, K_P^{\text{scale}}, V_P^{\text{scale}}\right). \tag{6}$$

Importantly, by initializing $K_P^{\text{new}}$ with zero, the model can perfectly resume the model state from the pre-training phase without losing the well-learned knowledge, facilitating faster convergence and accelerating the overall scaling process.

# Experiments—Progressive Modeling Scaling



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# Experiments—Benchmarking

| Model | #Param | Pile ppl ↓ | LAMBADA ppl ↓ | LAMBADA acc ↑ | HellaSwag acc ↑ | PIQA acc ↑ | Arc-E acc ↑ | Arc-C acc ↑ | WinoGrande acc ↑ | Average acc ↑ |
|---|---|---|---|---|---|---|---|---|---|---|
| Pythia-160M | 160M | 29.64 | 37.25 | 35.4 | 30.3 | 62.3 | 43.6 | 23.6 | **51.3** | 40.1 |
| **Ours (TokenFormer-150M)** | 150M | **10.45** | **16.38** | **45.0** | **35.5** | **64.9** | **47.3** | **24.9** | 50.4 | **44.7** |
| Pythia-410M | 410M | 9.95 | 10.84 | 51.4 | 40.6 | 66.9 | 52.1 | 24.6 | 53.8 | 48.2 |
| **Ours (TokenFormer-450M)** | 450M | **8.28** | **7.69** | **57.3** | **47.5** | **69.5** | **56.2** | **26.7** | **54.6** | **52.0** |
| Pythia-1B | 1B | 7.82 | 7.92 | 56.1 | 47.2 | 70.7 | 57.0 | 27.1 | 53.5 | 51.9 |
| **Ours (TokenFormer-900M)** | 900M | **7.38** | **5.46** | **64.0** | **55.3** | **72.4** | **59.9** | **30.6** | **56.4** | **56.4** |
| GPT-Neo 1.3B | 1.3B | - | 7.50 | 57.2 | 48.9 | 71.1 | 56.2 | 25.9 | 54.9 | 52.4 |
| OPT-1.3B | 1.3B | - | 6.64 | 58.0 | 53.7 | 72.4 | 56.7 | 29.6 | 59.5 | 55.0 |
| Pythia-1.3B | 1.3B | 7.51 | 6.08 | 61.7 | 52.1 | 71.0 | 60.5 | 28.5 | 57.2 | 55.2 |
| GPT-Neo 2.7B | 2.7B | - | 5.63 | 62.2 | 55.8 | 71.1 | 61.1 | 30.2 | 57.6 | 56.5 |
| OPT-2.7B | 2.7B | - | 5.12 | 63.6 | **60.6** | 74.8 | 60.8 | 31.3 | **61.0** | 58.7 |
| Pythia-2.8B | 2.8B | - | **5.04** | 64.7 | 59.3 | 74.0 | 64.1 | **32.9** | 59.7 | 59.1 |
| **Ours (TokenFormer-1.5B)** | 1.5B | **6.91** | 5.24 | **64.7** | 60.0 | **74.8** | **64.8** | 32.0 | 59.7 | **59.3** |

表: **Zero-shot Evaluations.**

# Experiments—Benchmarking

| Method | Image Size | #Param | Top-1 acc |
|---|---|---|---|
| ViT-B/16 | $384^2$ | 86M | 77.9 |
| DeiT-B/16 | $224^2$ | 86M | 81.8 |
| ViT-B/16 (MAE) | $224^2$ | 86M | 82.3 |
| Ours (TokenFormer-B/16$^\dagger$ ) | $224^2$ | 86M | 82.1 |
| **Ours (TokenFormer-B/16)** | $224^2$ | 109M | **82.5** |
| ViT-L/16 | $384^2$ | 307M | 76.5 |
| ViT-L/16 (MAE) | $224^2$ | 307M | 82.6 |
| Ours (TokenFormer-L/16$^\dagger$) | $224^2$ | 307M | 83.0 |
| **Ours (TokenFormer-L/16)** | $224^2$ | 407M | **83.1** |

表: **Image Classification.**