Neural Network Basics

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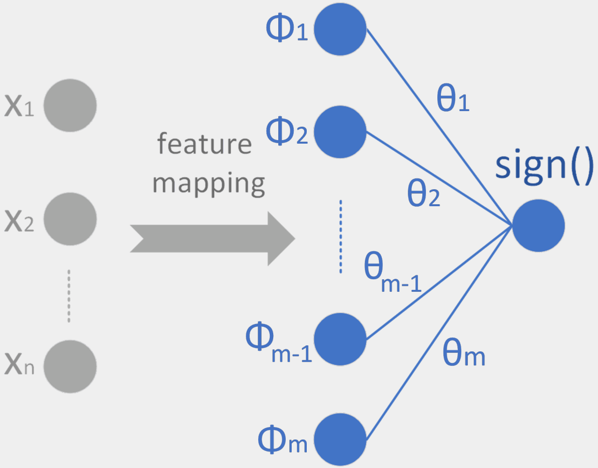
- Recall non-linear classifier via feature mapping:

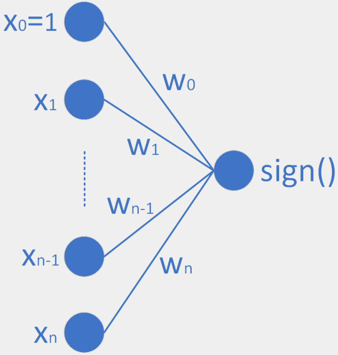
$$h(x;\theta,\theta_0) = sign(\theta\phi(x))$$

- $x = [x_1, x_2, \ldots, x_n]$
- $\phi(x) = [\phi_1(x), \phi_2(x), \ldots, \phi_m(x)]$
- $\theta = [\theta_1, \theta_2, \ldots, \theta_m]$

Non-Linear Classifier Graph View



- x can be raw data but more commonly, features extracted from raw data



- If $\sum_{i=0}^{n} x_i w_i > 0$, x is $+$, otherwise x is $-$

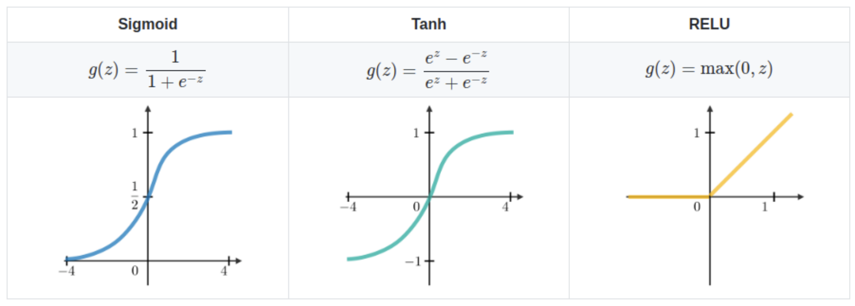
- A simple neural network that delivers a linear classifier for binary classification
- Guarantee to solve any binary classification task if data are linearly separable

Algorithm 1 Perceptron Learning Algorithm

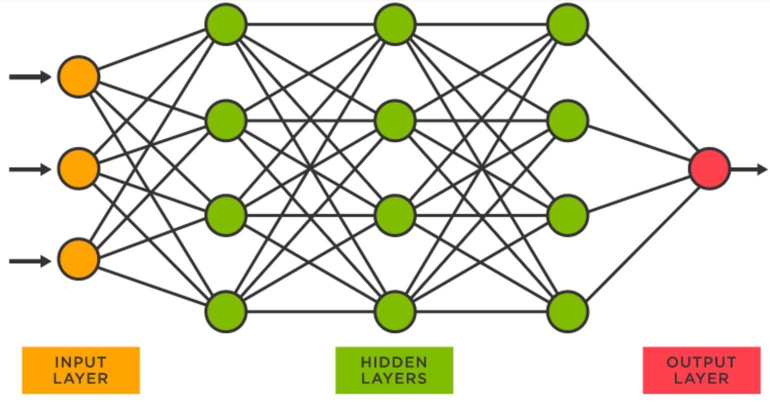
Require: $\{(X^{(i)}, y^{(i)})\}_{i=1,\ldots,n}$; T

for $t = 1, \ldots, T$ **do**
  for $i = 1, \ldots, n$ **do**
    if $y^{(i)}(wX^{(i)} + w_0) \leq 0$ **then**
      $w_0 = w_0 + y^{(i)}$
      $w = w + y^{(i)} X^{(i)}$
    end if
  end for
end for
return w_0, w

- Use a non-linear activation function instead of simple *sign*()
- $g(z = \sum_{i=0}^{n} x_i w_i)$: Sigmoid, Tanh, ReLU, etc.



- Neural networks with multiple hidden layers; backbone for "deep learning"



- Number of hidden layers
- Number of units of each layer
- Activation function in the units (usually the same function is used for the entire network)
- Ideal architecture is guided by monitoring validation error

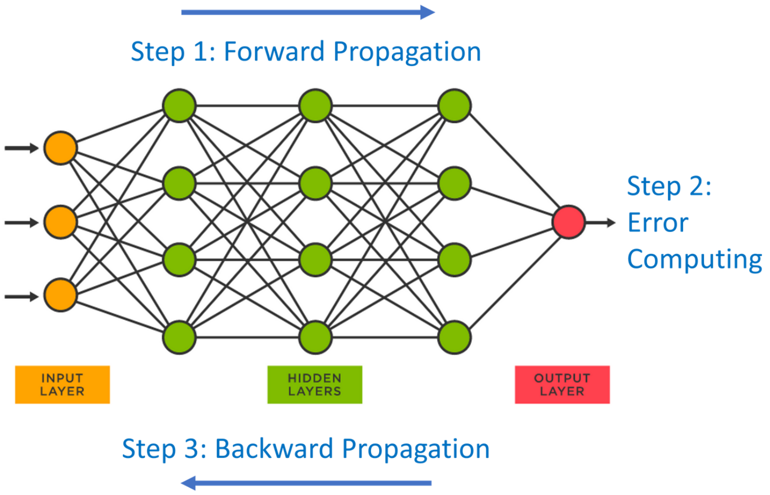
- Playground (link)

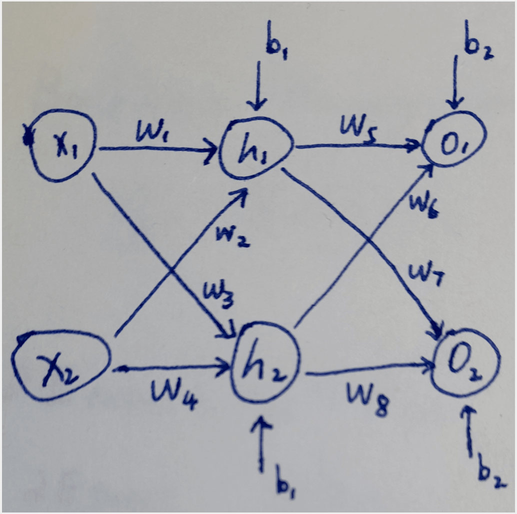
- Fully-connected: all units of a layer connect to all units of the next layer
- Convolutional neural networks are not fully connected

- Feedforward: connections among units do not form a cycle
  - Fully-connected, feedforward neural networks are also called multilayer perceptron (MLP)
- Recurrent: connections among units form a cycle

- The concepts of "multiple layers", "connected", and "activation function" are loosely from neuroscience
- Modern neural networks are guided by mathematical and engineering principles
- Better to think modern neural networks as function approximators to achieve statistical generalizability

Neural Network Training




b1
b2
x1
w1
h1
w5
O1
w2
w6
w3
w7
x2
w4
h2
w8
O2
b1
b2

- Activation function (h_1, h_2, o_1, o_2): Sigmoid
- Input: $x_1 = 0.05, x_2 = 0.1$
- Weights: $w_1 = 0.15, w_2 = 0.2, w_3 = 0.25, w_4 = 0.3, w_5 = 0.4, w_6 = 0.45, w_7 = 0.5, w_8 = 0.55$
- Bias: $b_1 = 0.35, b_2 = 0.6$
- Label: $y_1 = 0.01, y_2 = 0.99$
- Learning rate: $\alpha = 0.5$
- Loss function: $\frac{1}{2}(y - \hat{y})^2, y = [y_1, y_2], \hat{y} = [\hat{y}_1, \hat{y}_2]$

- Input and output of unit h_1: $IN_{h_1} = 0.3775, OUT_{h_1} = 0.5933$
- Input and output of unit h_2: $IN_{h_2} = 0.3925, OUT_{h_2} = 0.5969$
- Input and output of unit o_1: $IN_{o_1} = 1.1059, OUT_{o_1} = 0.7514$
- Input and output of unit o_2: $IN_{o_2} = 1.2249, OUT_{o_2} = 0.7729$

- Error at unit o_1: $E_{o_1} = \frac{1}{2}(y_1 - OUT_{o_1}) = 0.2748$
- Error at unit o_2: $E_{o_2} = \frac{1}{2}(y_2 - OUT_{o_2}) = 0.0236$
- Total error: $E_{total} = E_{o_1} + E_{o_2} = 0.2984$

- An example: $w_5 = w_5 - \alpha \frac{\partial E_{total}}{\partial w_5} = 0.3589$
- Use one training data point to update all parameters (i.e., from w_1 to w_8); then move on to the next training data point
- Epoch: going through the entire training set