

Assignment 4

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Since $y_1, y_2, \dots, y_n \sim \text{Poisson}(\theta)$, we know the Jeffrey's prior is then $\theta \sim \text{Gamma}(1/2, b \rightarrow 0)$. And we can derive the posterior distribution as $\text{Gamma}(1/2 + \sum y_i, n)$ for which $n = 10$ and $\sum y_i = 200$. We then pick three different importance density functions to generate 1000 samples and use them to generate θ . In the end, we calculate the probability for $\theta > 20$. The results shows that the posterior probability $\theta > 20$ is approximately 50%.

```
n=10
sample=1000
sum_yi=200
# Case 1: abs(Normal(0,1))
x1=abs(rnorm(sample))
theta1=rgamma(x1,sum_yi+1/2,n)
sum(theta1>20)/sample
```

```
## [1] 0.49
```

```
#Case 2: uniform(0,1000)
x2=runif(sample,0,1000)
theta2=rgamma(x2,sum_yi+1/2,n)
sum(theta2>20)/sample
```

```
## [1] 0.493
```

```
# Case 3: abs(Cauchy(0,1))
x3=abs(rcauchy(sample))
theta3=rgamma(x3,sum_yi+1/2,n)
sum(theta3>20)/sample
```

```
## [1] 0.499
```