2.

(c)

We implement a Metropolis algorithm to approximate $P(\alpha, \beta | x, y)$. After adjusting different proposal distribution, we finally to achieve a reasonable acceptance rate of approximately 72%.

```r
library(MASS)
inv = solve
nest = read.table("msparrownest.dat")
S = 10000
burnin=5000
y = nest[, 1]
n = length(y)
x = cbind(rep(1, n), nest[, 2])
# a fair proposal distribution
prop =  inv(t(x) %*% x)
# Prior
mean_theta = c(0, 0)
sd_theta = sqrt(c(25, 0.25))
a = numeric(S + burnin)
b = numeric(S + burnin)
accpt = 0
theta = c(0, 0)
ratio = function(x, y, theta) {
  p = exp(x %*% theta) / (1 + exp(x %*% theta))
  sum(dbinom(y, 1, p, log = TRUE))+sum(dnorm(theta, mean_theta, sd_theta, log = TRUE))
}
for (i in 1:(S + burnin)) {
  theta_star = mvrnorm(1, theta, prop)
  log_mh = ratio(x, y, theta_star) -ratio(x, y, theta)

  if (log(runif(1)) < log_mh) {
    theta = theta_star
    if (i > burnin) {
      accpt = accpt + 1
    }
  }
  a[i] = theta[1]
  b[i] = theta[2]
}
a = a[burnin:length(a)]
b = b[burnin:length(b)]
accpt / S
```

```
## [1] 0.7222
```

(d)

```r
library(ggplot2)
a_prior = data.frame(
  val = seq(-10, 10, length.out = 1000),
  density = dnorm(seq(-10, 10, length.out = 1000), 0, 5),
  var = 'alpha', dist = 'prior'
)
```

1

```
b_prior = data.frame(
  val = seq(-1, 1, length.out = 1000),
  density = dnorm(seq(-1, 1, length.out = 1000), 0, 0.5),
  var = 'beta', dist = 'prior'
)

a_post = data.frame(
  val = a, var = 'alpha', dist = 'posterior')
b_post = data.frame(
  val = b, var = 'beta', dist = 'posterior')
priors = rbind(a_prior, b_prior)
posts = rbind(a_post, b_post)

ggplot(priors, aes(x = val, y = density, color = dist)) +
  geom_line(size=1.5) +
  geom_density(data=posts, mapping = aes(x = val, y = ..density..),size=1.5) +
  facet_wrap(~ var,scales = 'free')+
  scale_color_manual(values=c("navy", "gold"))+
  ggtitle("Prior and Posterior Densities of Alpha and Beta")
```
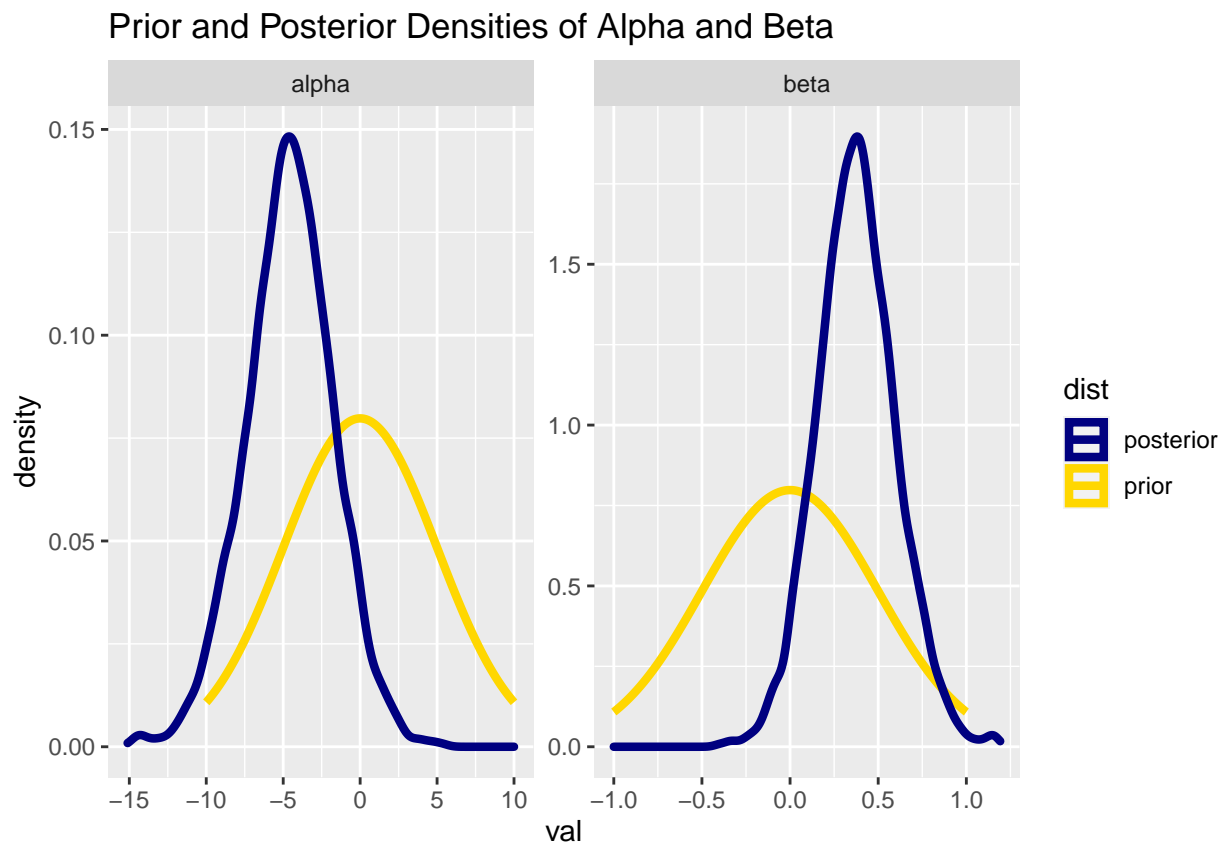

The plot above shows the prior densities of α and β and posterior densities of α and β. We can see that the posterior densities for both α and β are less variate than their prior. The posteriors' center(mean) also changed.