Assignment 2 (5.1)&(5.2)

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5.1

```
#load the data and preparation
school1 <- read.table("school1.dat")
school2 <- read.table("school2.dat")
school3 <- read.table("school3.dat")
time1 <- school1$V1
time2 <- school2$V1
time3 <- school3$V1
mu0 <- 5
sigmasq <- 4
k0 <- 1
v0 <- 2
```

(a)

```
#school1
y_bar1 <- mean(time1)
var1 <- var(time1)
sd1 <- sd(time1)
n1 <- length(time1)
kn1 <- k0 + n1
mu_n1 <- (k0*mu0+n1*y_bar1)/kn1
vn1 <- v0 + n1
sigmasq_n1 <- (1/vn1)*(v0*sigmasq+(n1-1)*var1+((k0*n1)/kn1)*(y_bar1-mu0)^2)
gamma1 <- rgamma(10000, vn1/2,(vn1*sigmasq_n1)/2)
invgamma1 <- 1/gamma1
theta1 <- rnorm(10000,mu_n1,sqrt(invgamma1/kn1))
mean_theta1 <- mean(theta1)
CI_theta1 <- quantile(theta1,c(0.025,0.975))
mean_sigma1 <- mean(sqrt(invgamma1))
CI_sigma1 <- quantile(sqrt(invgamma1),c(0.025,0.975))

#school2
y_bar2 <- mean(time2)
var2 <- var(time2)
sd2 <- sd(time2)
n2 <- length(time2)
kn2 <- k0 + n2
mu_n2 <- (k0*mu0+n2*y_bar2)/kn2
vn2 <- v0 + n2
sigmasq_n2 <- (1/vn2)*(v0*sigmasq+(n2-1)*var2+((k0*n2)/kn2)*(y_bar2-mu0)^2)
```

```r
gamma2 <- rgamma(10000, vn2/2,(vn2*sigmasq_n2)/2)
invgamma2 <- 1/gamma2
theta2 <- rnorm(10000,mu_n2,sqrt(invgamma2/kn2))
mean_theta2 <- mean(theta2)
CI_theta2 <- quantile(theta2,c(0.025,0.975))
mean_sigma2 <- mean(sqrt(invgamma2))
CI_sigma2 <- quantile(sqrt(invgamma2),c(0.025,0.975))

#school3
y_bar3 <- mean(time3)
var3 <- var(time3)
sd3 <- sd(time3)
n3 <- length(time3)
kn3 <- k0 + n3
mu_n3 <- (k0*mu0+n3*y_bar3)/kn3
vn3 <- v0 + n3
sigmasq_n3 <- (1/vn3)*(v0*sigmasq+(n3-1)*var3+((k0*n3)/kn3)*(y_bar3-mu0)^2)
gamma3 <- rgamma(10000, vn3/2,(vn3*sigmasq_n3)/2)
invgamma3 <- 1/gamma3
theta3 <- rnorm(10000,mu_n3,sqrt(invgamma3/kn3))
mean_theta3 <- mean(theta3)
CI_theta3 <- quantile(theta3,c(0.025,0.975))
mean_sigma3 <- mean(sqrt(invgamma3))
CI_sigma3 <- quantile(sqrt(invgamma3),c(0.025,0.975))

#put the results together
result_a1 <- data.frame(
  "School" <- c("School 1","School 2","School 3"),
  "Post_Mean_theta" <- c(mean_theta1,mean_theta2,mean_theta3),
  "CI_theta_lower" <- c(CI_theta1[[1]],CI_theta2[[1]],CI_theta3[[1]]),
  "CI_theta_upper" <- c(CI_theta1[[2]],CI_theta2[[2]],CI_theta3[[2]])
)
colnames(result_a1) <- c("School","Post Mean theta","CI theta lower","CI theta upper")

result_a2 <- data.frame(
  "School" <- c("School 1","School 2","School 3"),
  "Post_Mean_sigma" <- c(mean_sigma1,mean_sigma2,mean_sigma3),
  "CI_sigma_lower" <- c(CI_sigma1[[1]],CI_sigma2[[1]],CI_sigma3[[1]]),
  "CI_sigma_upper" <- c(CI_sigma1[[2]],CI_sigma2[[2]],CI_sigma3[[2]])
)
colnames(result_a2) <- c("School","Post Mean sigma","CI sigma lower","CI sigma upper")
result_a1
```

```
##     School Post Mean theta CI theta lower CI theta upper
## 1 School 1        9.300497       7.782337      10.806364
## 2 School 2        6.949001       5.147284       8.757193
## 3 School 3        7.815673       6.172716       9.439072
```

```r
result_a2
```

```
##     School Post Mean sigma CI sigma lower CI sigma upper
## 1 School 1        3.909486       3.005835       5.180484
## 2 School 2        4.399866       3.337183       5.902947
## 3 School 3        3.732385       2.792235       5.071948
```

The posterior means and 95% confidence intervals for the mean theta and standard deviation sigma from each school are shown above.

(b)

```
result_b <- data.frame(
  "i,j,k" <- "P(thetai < thetaj < thetak)",
  "1,2,3" <- mean(theta1<theta2 & theta2<theta3),
  "1,3,2" <- mean(theta1<theta3 & theta3<theta2),
  "2,1,3" <- mean(theta2<theta1 & theta1<theta3),
  "2,3,1" <- mean(theta2<theta3 & theta3<theta1),
  "3,1,2" <- mean(theta3<theta1 & theta1<theta2),
  "3,2,1" <- mean(theta3<theta2 & theta2<theta1)
)
colnames(result_b) <- c("(i,j,k)","(1,2,3)","(1,3,2)",
                        "(2,1,3)","(2,3,1)","(3,1,2)","(3,2,1)")
result_b
```

```
##                       (i,j,k) (1,2,3) (1,3,2) (2,1,3) (2,3,1) (3,1,2) (3,2,1)
## 1 P(thetai < thetaj < thetak)  0.0044  0.0029  0.0846  0.6749  0.0177  0.2155
```

The posterior probability that theta i < theta j < theta k for all six permutations are shown above.

(c)

```
ypred1 <- rnorm(10000, theta1, sqrt(invgamma1))
ypred2 <- rnorm(10000, theta2, sqrt(invgamma2))
ypred3 <- rnorm(10000, theta3, sqrt(invgamma3))

result_c <- data.frame(
  "i,j,k" <- "P(Y_i < Y_j < Y_k)",
  "1,2,3" <- mean(ypred1<ypred2 & ypred2<ypred3),
  "1,3,2" <- mean(ypred1<ypred3 & ypred3<ypred2),
  "2,1,3" <- mean(ypred2<ypred1 & ypred1<ypred3),
  "2,3,1" <- mean(ypred2<ypred3 & ypred3<ypred1),
  "3,1,2" <- mean(ypred3<ypred1 & ypred1<ypred2),
  "3,2,1" <- mean(ypred3<ypred2 & ypred2<ypred1)
)
colnames(result_c) <- c("(i,j,k)","(1,2,3)","(1,3,2)",
                        "(2,1,3)","(2,3,1)","(3,1,2)","(3,2,1)")
result_c
```

```
##              (i,j,k) (1,2,3) (1,3,2) (2,1,3) (2,3,1) (3,1,2) (3,2,1)
## 1 P(Y_i < Y_j < Y_k)  0.1012  0.1052  0.1912   0.265  0.1403  0.1971
```

The posterior probability of sample from the posterior predictive distribution of schools that Y_pred_i < Y_pred_j < Y_pred_k for all six permutations are shown above.

(d)

```
result_d <- data.frame(
   "P(theta1 > theta2 & theta1 > theta3)" <- mean(theta1>theta2 & theta1>theta3),
   "P(Y_pred1 > Y_pred2 & Y_pred1 > Y_pred3)" <- mean(ypred1>ypred2 & ypred1>ypred3)
)
colnames(result_d) <- c("P(theta1 > theta2 & theta3)",
```

```
                         "P(Y_pred1 > Y_pred2 & Y_pred3)")
result_d
```

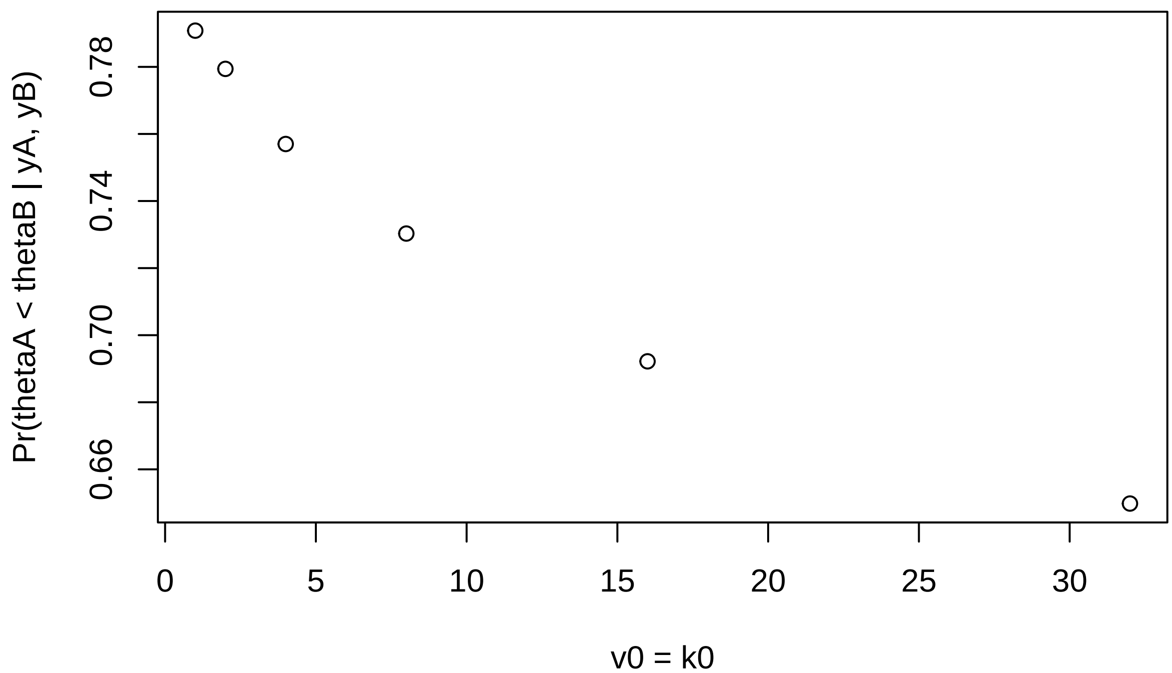
```
##   P(theta1 > theta2 & theta3) P(Y_pred1 > Y_pred2 & Y_pred3)
## 1                      0.8904                         0.4621
```

The posterior probability that theta 1 is bigger than both theta 2 and theta 3, and the posterior probability that Y_pred_1 is bigger than both Y_pred_2 and y_pred_3 are shown above.

5.2

```
mu_zero <- 75
sigmasq0 <- 100
ya <- 75.2
sda <- 7.3
n_a <- 16
yb <- 77.5
sdb <- 8.1
n_b <- 16
v_zero=c(1,2,4,8,16,32)
k_zero=c(1,2,4,8,16,32)
#a
lst_a=list(list(),list(),list(),list(),list(),list())
vn_a <- v_zero+n_a
kn_a <- k_zero+n_a

for (i in 1:length(k_zero)){
  mu_na <- (k_zero[i]*mu_zero+n_a*ya)/kn_a[i]
  sigmasq_na <- (1/vn_a[i])*
    (v_zero[i]*sigmasq0+(n_a-1)*(sda^2)+((k_zero[i]*n_a)/kn_a[i])*(ya-mu_zero)^2)
  invgamma_a <- 1/rgamma(10000, vn_a[i]/2,(vn_a[i]*sigmasq_na)/2)
  theta_a <- rnorm(10000,mu_na,sqrt(invgamma_a/kn_a[i]))
  lst_a[[i]] <- theta_a
}

#b
vn_b <- v_zero+n_b
kn_b <- k_zero+n_b

lst_b=list(list(),list(),list(),list(),list(),list())
for (i in 1:length(k_zero)){
  mu_nb <- (k_zero[i]*mu_zero+n_b*yb)/kn_b[i]
  sigmasq_nb <- (1/vn_b[i])*
    (v_zero[i]*sigmasq0+(n_b-1)*(sdb^2)+((k_zero[i]*n_b)/kn_b[i])*(yb-mu_zero)^2)
  invgamma_b <- 1/rgamma(10000, vn_b[i]/2,(vn_b[i]*sigmasq_nb)/2)
  theta_b <- rnorm(10000,mu_nb,sqrt(invgamma_b/kn_b[i]))
  lst_b[[i]] <- theta_b
}

#Pr(thetaA < thetaB)
result <- c()
for (i in 1:length(v_zero)){
  prob <- mean(lst_a[[i]]<lst_b[[i]])
```

```
  result[i]<- prob
}
#plot
plot(v_zero,result,xlab="v0 = k0",ylab = "Pr(thetaA < thetaB | yA, yB)")
```



From the plot, we can see that as prior sample size increases, the probability that theta A < theta B decreases. The data is showing that there is a difference between theta A and theta B, while the prior sample is showing that mean of two groups are the same. Therefore, as we increase the size of the prior sample, the belief in the data will be weakened by the prior sample, and the prior sample will play a bigger role in this process.