# Exercise sheet: Review of Probability

1. Suppose that the two RVs $X$ and $Z$ are statistically independent. Show that the mean and variance of their sum satisfies

$$E\{X + Z\} = E\{X\} + E\{Z\}$$
$$\text{var}\{X + Z\} = \text{var}\{X\} + \text{var}\{Z\}.$$

2. Consider a discrete RV $X$ whose pmf is given as

$$P(X) = \begin{cases} \frac{1}{3}, & \text{if } x = -1, 0, 1, \\ 0 & \text{otherwise} \end{cases}$$

   Find the mean and variance of $X$.

3. The RV $X$ can take values $x_1 = 1$ and $x_2 = 2$. Likewise, the RV $Y$ can take values $y_1 = 1$ and $y_2 = 2$. The joint pmf of the RVs $X$ and $Y$ is given as

$$P(X, Y) = \begin{cases} k(2x_i + y_j), & \text{for } i = 1, 2 \, ; j = 1, 2 \\ 0 & \text{otherwise,} \end{cases}$$

   where $k$ is a constant.

   (a) Find the value of $k$.
   (b) Find the marginal pmf for $X$ and $Y$.
   (c) Are $X$ and $Y$ independent?

4. The joint pdf of the RVs $X$ and $Y$ is given by

$$p(x, y) = \begin{cases} k(x + y), & \text{for } 0 < x < 2, 0 < y < 2 \\ 0 & \text{otherwise,} \end{cases}$$

   where $k$ is a constant.

   (a) Find the value of $k$.
   (b) Find the marginal pdf for $X$ and $Y$.
   (c) Are $X$ and $Y$ independent?

5. Suppose that we have three coloured boxes $r$ (red), $b$ (blue), and $g$ (green). Box $r$ contains 3 apples, 4 oranges, and 3 limes, box $b$ contains 1 apple, 1 orange, and 0 limes, and box $g$ contains 3 apples, 3 oranges, and 4 limes. If a box is chosen at random with probabilities $P(r) = 0.2, P(b) = 0.2, P(g) = 0.6$, and a piece of fruit is removed from the box (with equal probability of selecting any of the items in the box), then what is the probability of selecting an apple? If we observe that the selected fruit is in fact an orange, what is the probability that it came from the green box?