Reinforcement Learning for Business, Economics, and Social Sciences

Unit 4-2: Deep Neural Networks

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How to improve flexibility of approximation?

Deep Neural Networks

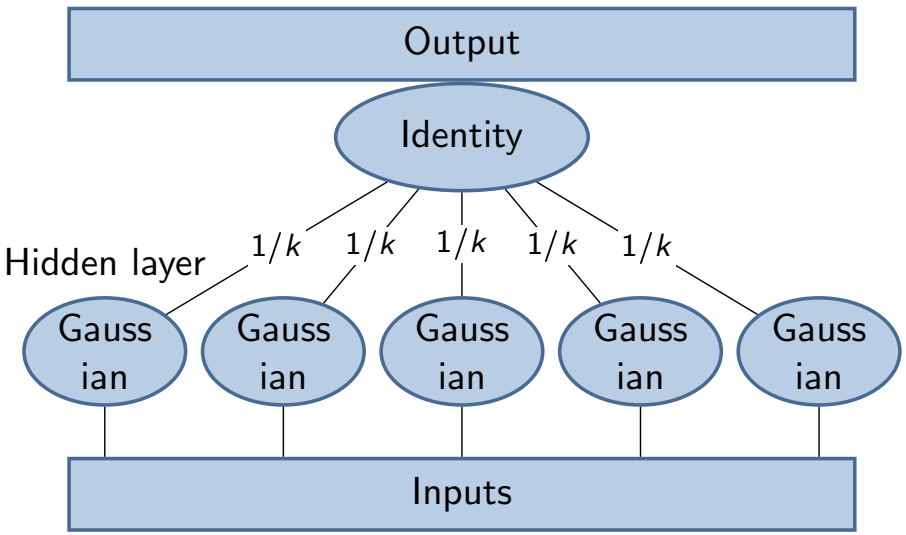
Deep Neural Networks

- Definition: neural network with many hidden layers
- Advantage: high expressivity
- Challenges:
  - How should we train a deep neural network?
  - How can we avoid overfitting?

(Goodfellow, 2016)

Mixture of Gaussians

- Shallow neural network (flat mixture)



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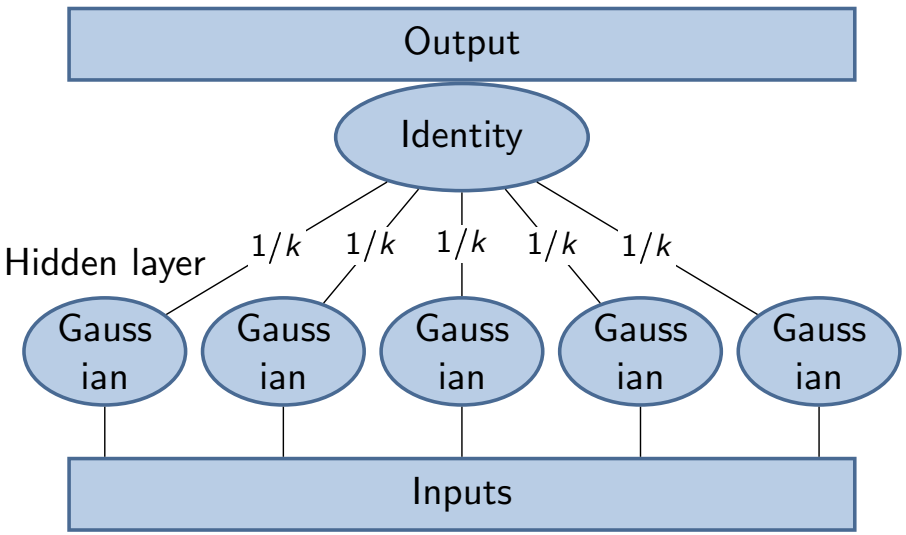
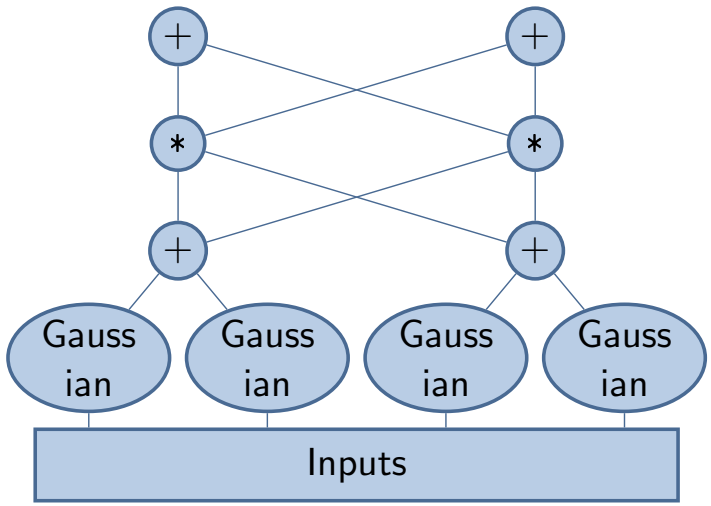


- Deep neural network (hierarchical mixture)



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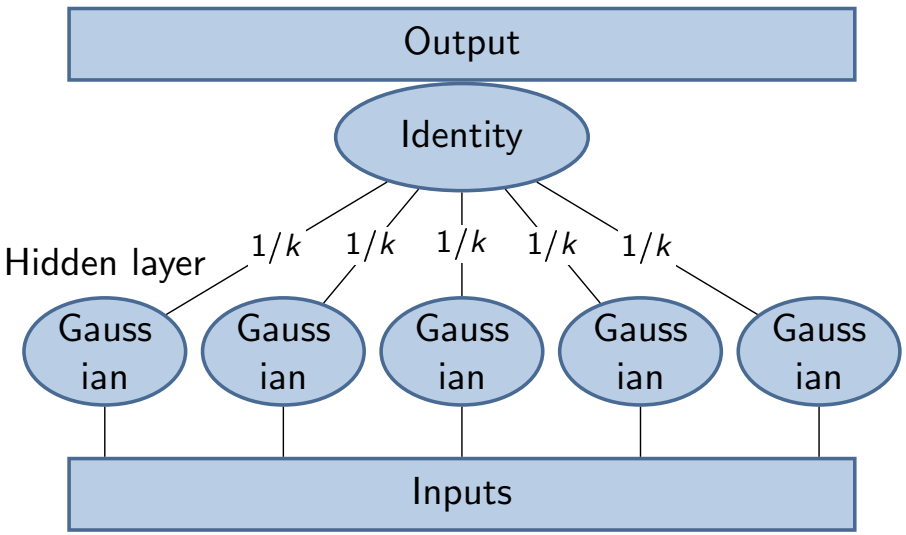

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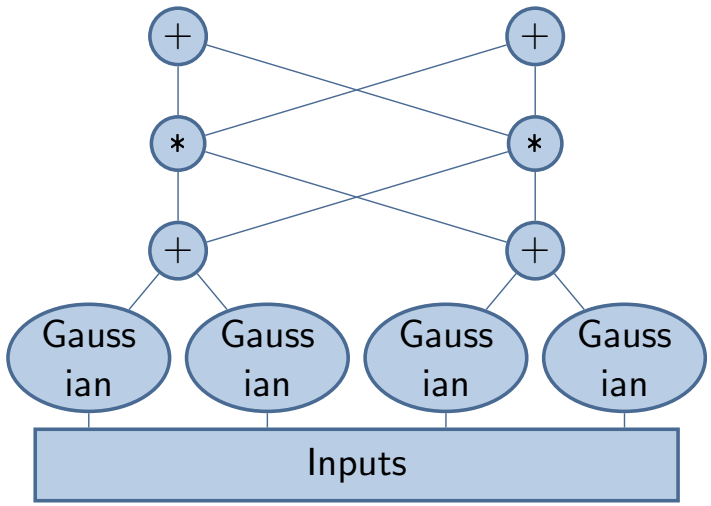
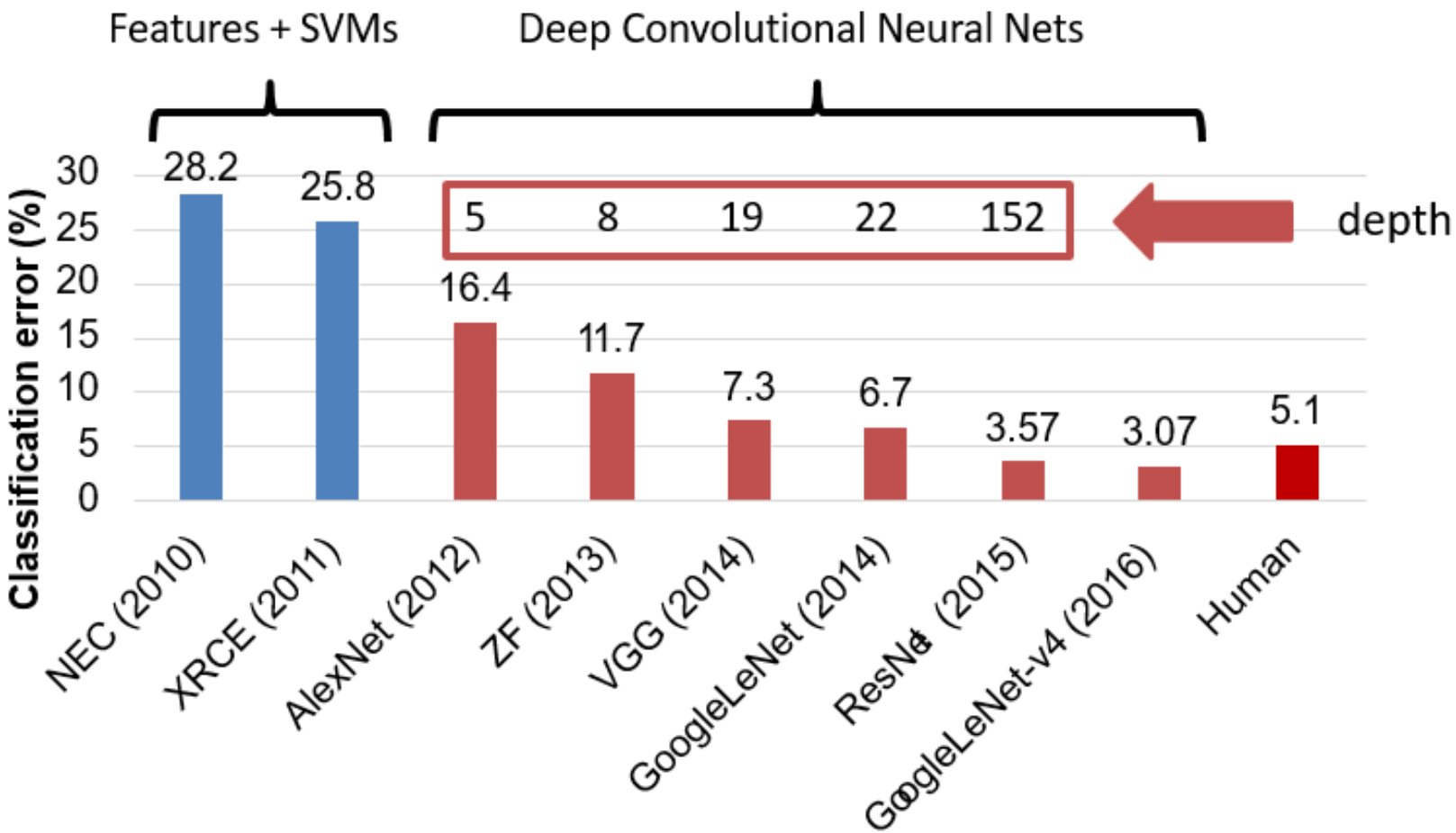


Sum-Product Network (Exponentially large mixture of Gaussians but linear hierachy)

Image Classification

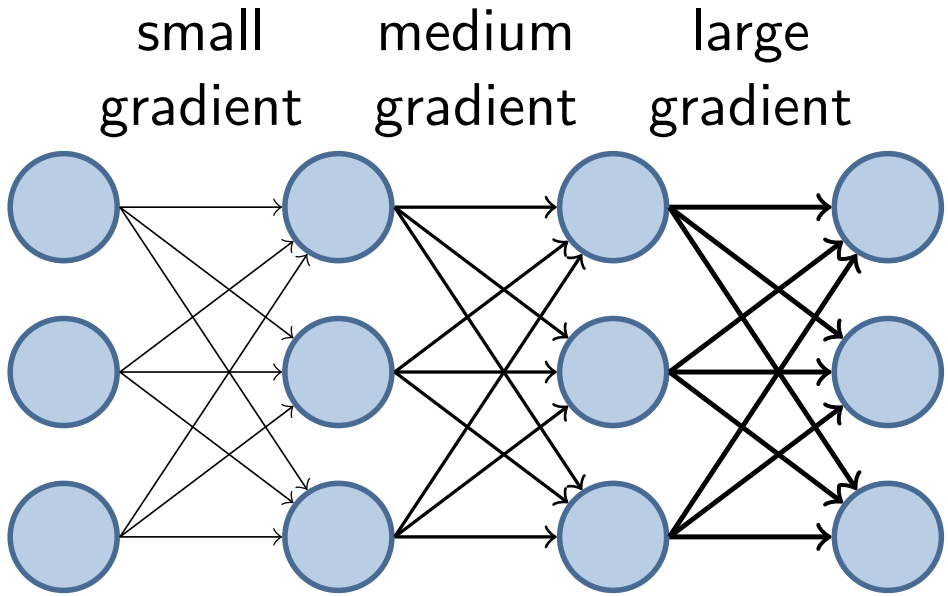
- ImageNet Large Scale Visual Recognition Challenge



Vanishing Gradients

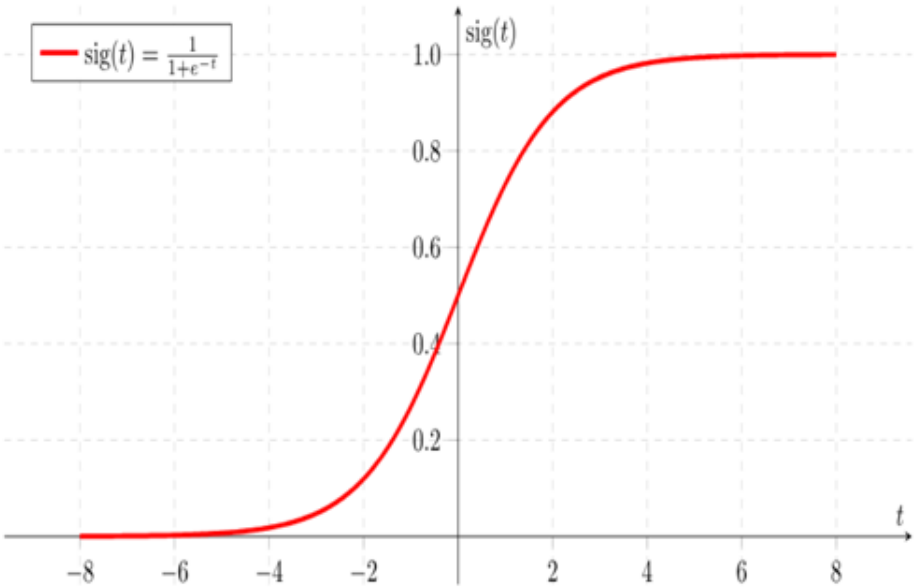
Vanishing Gradients

- Deep neural networks of sigmoid and hyperbolic units often suffer from vanishing gradients

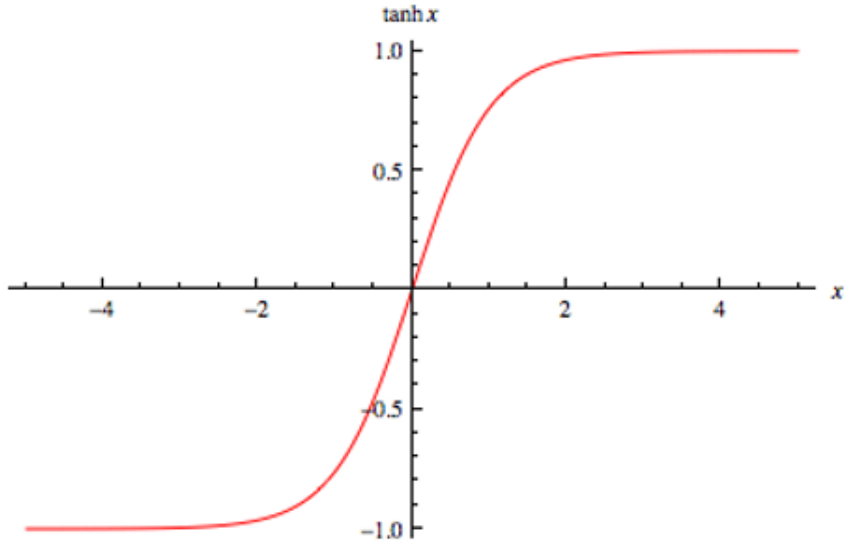


Sigmoid and hyperbolic units

- Derivative is always less than 1



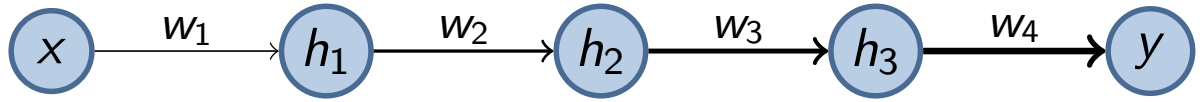
sigmoid



hyperbolic

Simple Example

- $y = \sigma\left(w_4\sigma\left(w_3\sigma\left(w_2\sigma\left(w_1 x\right)\right)\right)\right)$



- Common weight initialization in (-1,1)
- Sigmoid function and its derivative always less than 1
- This leads to vanishing gradients:

$$\frac{\partial y}{\partial w_4} = \sigma'\left(a_4\right)\sigma\left(a_3\right)$$
$$\frac{\partial y}{\partial w_3} = \sigma'\left(a_4\right)w_4\sigma'\left(a_3\right)\sigma\left(a_2\right) \leq \frac{\partial y}{\partial w_4}$$
$$\frac{\partial y}{\partial w_2} = \sigma'\left(a_4\right)w_4\sigma'\left(a_3\right)w_3\sigma'\left(a_2\right)\sigma\left(a_1\right) \leq \frac{\partial y}{\partial w_3}$$
$$\frac{\partial y}{\partial w_1} = \sigma'\left(a_4\right)w_4\sigma'\left(a_3\right)w_3\sigma'\left(a_2\right)w_2\sigma'\left(a_1\right)x \leq \frac{\partial y}{\partial w_2}$$

Mitigating Vanishing Gradients

- Some popular solutions:
  - Pre-training
  - **Rectified linear units**
  - Batch normalization
  - Skip connections

Rectified Linear Units

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  - Gradient is 0 or 1
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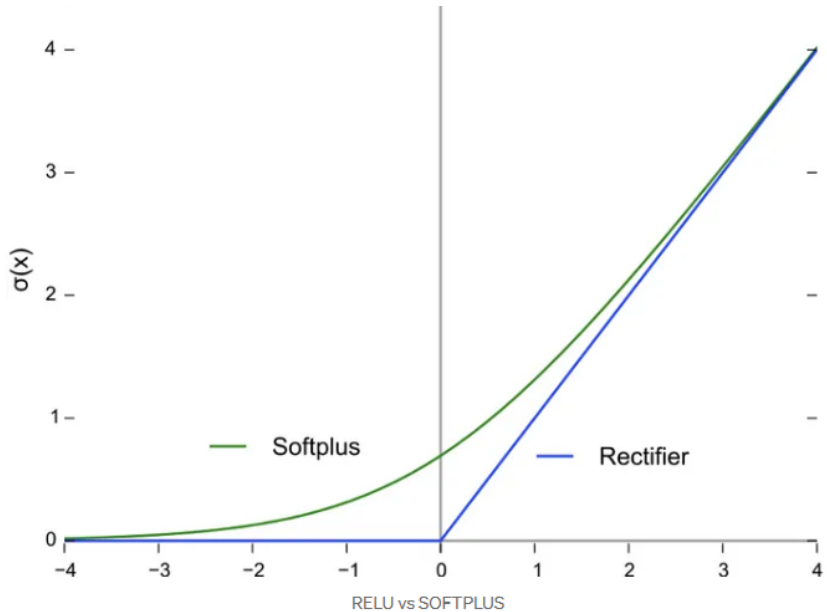
  ("Softplus"):
  $h(a) = \log(1 + e^a)$



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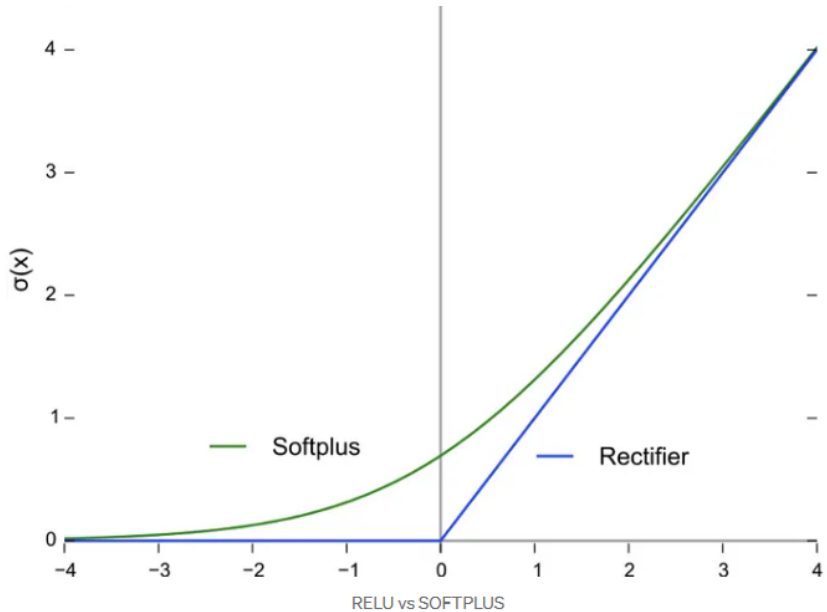

- Warning: softplus does not prevent gradient vanishing
  (gradient < 1)

References I

Goodfellow, I. (2016): *Deep learning*, vol. 196. MIT press, Available at http://deeplearningbook.org/.

Takeaways

How do Deep Neural Networks Help Modeling Complex Data?

- Use multiple hidden layers
- They enable complex function approximation
- A key challenge is the vanishing gradient problem
- Solutions include ReLU activation functions, batch normalization, skip connections, and pre-training
- Rectified Linear Units (ReLU) help mitigate vanishing gradients