Module - 3

Semiconductor Physics

Syllabus: Fermi level in intrinsic and extrinsic semiconductors, expression for concentration electrons in conduction band and holes concentration in valance band (mention), Law of mass action, relation between Fermi energy and energy gap in intrinsic semiconductors, Expression for Electrical conductivity of a semiconductor (derivation), Hall effect - expression for Hall coefficient and its applications, Numerical Problems

Introduction:

Materials are classified into three types based on the energy band gap (E_g) between valence band and conduction band.

i.e., conductors ($E_g = 0$), insulators ($E_g > 3eV$) and semiconductors ($E_g < 3eV$).

Semiconductors: "Semiconductors are those materials whose conductivity lies between a Conductor and insulator". Narrow energy band gap is the characteristic feature of semiconductors, because of which a significant number of thermally excited electrons are available at room temperature.

Ex: Silicon, Germanium, Gallium arsenide and Gallium phosphide etc.

Semiconductors are classified into two types

a) Intrinsic semiconductor: Intrinsic semiconductors are the pure semiconductor without doping. The conductivity of these semiconductors depends only on the thermal excitation. At Zero Kelvin temperature intrinsic semiconductor acts as an insulator because, no free electrons are available for conductivity.

b) Extrinsic semiconductor: Extrinsic semiconductors are the impure semiconductors. Some impurity is added to these semiconductors to enhance its electrical conductivity.

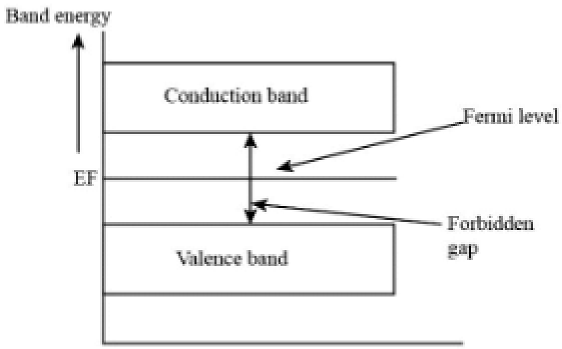
Doping – It is a process of addition of small amount of impurities (atoms) to a pure semiconductor in the controlled manner. The atom or impurity which is added to pure semiconductor called dopant.

Extrinsic semiconductor further classified into

i) P-type semiconductors: P-type semiconductors can be obtained when Ge or Si (Tetravalent atoms) are doped with Trivalent impurities. In these semiconductors holes are the majority charge carriers and electrons are the minority charge carriers.

ii) N-type semiconductor: N-type semiconductors can be obtained when Ge or Si (Tetravalent atoms) are doped with pentavalent impurities. In these semiconductors electrons are the majority charge carriers and holes are the minority charge carriers.

Fermi level in intrinsic semiconductors:



In intrinsic semiconductors, the Fermi level is lies in the mid-point of Forbidden energy gap and is given by $E_F = \frac{E_g}{2}$

In intrinsic semiconductor, the number of holes in the valence band is equal to the number of electrons in the conduction band. Hence, the probability of occupation of energy levels in conduction band and valence band are equal. As temperature increases, the electrons present at the top of valance band (VB) jump in to the bottom of conduction bands (CB) due to thermal excitation.

Expression for electron concentration (N_e):

The number of electrons in the conduction band per unit volume of the material is called the electron concentration. The expression for electron concentration is given by

$$N_e = \frac{4\sqrt{2}}{h^3}(\pi m_e^* kT)^{3/2} e^{-\left(\frac{E_F - E_g}{kT}\right)}$$

Where, h → Planck's constant,
m_e^* → Effective mass of Electrons in the conduction band,
k → Boltzmann constant,
T → Temperature in kelvin,
E_F → Fermi Energy,
E_g → Energy gap.

,

Expression for hole concentration (N_h):

The number of holes in the valence band per unit volume of the material is called the hole concentration. The expression for Holes concentration is given by

$$N_h = \frac{4\sqrt{2}}{h^3}(\pi m_h^* kT)^{3/2} e^{-\left(\frac{E_F}{kT}\right)}$$

Where, h → Planck's constant,
m_h^* → Effective mass of Holes in the valance band,
k → Boltzmann constant,
T → Temperature in kelvin,
E_F → Fermi Energy,

.

Law of mass action

"Law of mass action states that the product the number electrons in the conduction band and the number of holes in the valence band is constant at constant temperature and it is independent of number of donor and acceptor atoms added".

According to law of mass action, $\boxed{n_e . n_h = n_i^2}$

Where, n_e → Number of electrons in the conduction band
n_h → Number of holes in valence band
n_i → intrinsic Charge concentration

Relation between Fermi energy and energy gap in intrinsic semiconductors

In case of intrinsic semiconductors, Number of electrons per unit volume in the conduction band (N_e)is equal to Number of holes per unit volume in the valance band (N_h).

$$\therefore N_e = N_h \qquad \text{----------------- (1)}$$

The expression for electron concentration is given by

$$N_e = \frac{4\sqrt{2}}{h^3}(\pi m_e^* kT)^{3/2}\, e^{-\left(\frac{E_F - E_g}{kT}\right)} \qquad \text{----------------- (2)}$$

And, the expression for holes concentration is given by

$$N_h = \frac{4\sqrt{2}}{h^3}(\pi m_h^* kT)^{3/2}\, e^{-\left(\frac{E_F}{kT}\right)} \qquad \text{----------------- (3)}$$

Where, h → Planck's constant,
m_h^* → Effective mass of Holes in the valance band,
m_e^* → Effective mass of Electrons in the conduction band,
k → Boltzmann constant,
T → Temperature in kelvin,
E_F → Fermi Energy,
E_g → Energy gap

Put Eqns (2) & (3) in Eqn (1)

$$\text{Eqn (1)} \Longrightarrow \quad \frac{4\sqrt{2}}{h^3}(\pi m_e^* kT)^{3/2} e^{-\left(\frac{E_F - E_g}{kT}\right)} = \frac{4\sqrt{2}}{h^3}(\pi m_h^* kT)^{3/2} e^{-\left(\frac{E_F}{kT}\right)}$$

$$\Longrightarrow (m_e^*)^{3/2} e^{-\left(\frac{E_F - E_g}{kT}\right)} = (m_h^*)^{3/2} e^{-\left(\frac{E_F}{kT}\right)}$$

$$\Longrightarrow \quad e^{\left(\frac{E_F - E_g}{kT} + \frac{E_F}{kT}\right)} (m_e^*)^{3/2} = (m_h^*)^{3/2}$$

$$\Longrightarrow \quad e^{\left(\frac{2E_F - E_g}{kT}\right)} = \left(\frac{m_h^*}{m_e^*}\right)^{\frac{3}{2}}$$

Taking Natural log on both sides

We get
$$\left(\frac{2E_F - E_g}{KT}\right) = \ln\left[\left(\frac{m_h^*}{m_e^*}\right)^{\frac{3}{2}}\right] = \frac{3}{2}\ln\left(\frac{m_h^*}{m_e^*}\right)$$

$$\Longrightarrow E_F = \frac{3KT}{4}\ln\left(\frac{m_h^*}{m_e^*}\right) + \frac{E_g}{2} \qquad \text{----------------- (4)}$$

As $m_h^* = m_e^*$ for practical consideration, the first term in the RHS of equation (4) becomes zero.

$$\therefore \boxed{E_F = \frac{E_g}{2}} \qquad \text{----------------- (5)}$$

Eqn (5) is the Relation between Fermi energy and energy gap in intrinsic semiconductors.

Expression for electrical conductivity (σ) of a semiconductor:

The expression for electric current (I) in terms of drift velocity (v_d) is given by

$$I = N_e\, e\, A v_d \qquad \text{-------------- (1)}$$

Where, A is the Area of semiconductor.

In Semiconductors, the conductivity is due to motion of electrons and holes

∴ Current due to motion of electrons is

$$\text{Eqn (1)} \Longrightarrow \qquad I_e = N_e\, e\, A v_e \qquad \text{-------------- (2)}$$

Where N_e→ density of electrons
e →charge of an electron and
v_e→drift velocity of electrons

∴ Current due to motion of holes is

Eqn (1) ⟹ $I_h = N_h\, e\, A v_h$ -------------- (3)

Where $N_h \rightarrow$ density of holes

$e \rightarrow$ charge of an hole and

$v_h \rightarrow$ drift velocity of holes

∴ The total current due to both electrons and holes is $I = I_e + I_h$ -------------- (4)

Put Eqns (2) & (3) in Eqn (4),

Eqn (4) ⟹ $I = N_e\, e\, A v_e + N_h e\, A v_h$

$I = eA\,(N_e v_e + N_h v_h)$

$\frac{I}{A} = e\,(N_e v_e + N_h v_h)$ -------------- (5)

We know $J = \frac{I}{A}$

Eqn (5) ⟹ $J = e\,(N_e v_e + N_h v_h)$ -------------- (6)

But $J = \sigma E$ -------------- (7)

From equation (6) and (7)

$\sigma E = e\,(N_e v_e + N_h v_h)$

⟹ $\sigma = e\left[N_e\left(\frac{v_e}{E}\right) + N_h\left(\frac{v_h}{E}\right)\right]$ -------------- (8)

Let mobility of electrons, $\mu_e = \frac{v_e}{E}$ and mobility of holes, $\mu_h = \frac{v_h}{E}$

Eqn (8) ⟹ $\boldsymbol{\sigma = e\,[N_e \mu_e + N_h \mu_h]}$ -------------- (9)

For intrinsic semiconductor, $N_e = N_h = N_i$

$\boxed{\boldsymbol{\sigma_i = e\, N_i\,(\mu_e + \mu_h)}}$ -------------- (10)

This is the expression for electrical conductivity of semiconductor in terms of mobility of electrons and holes.

In Extrinsic Semiconductors

For P-type superconductors, $N_h \gg N_e$

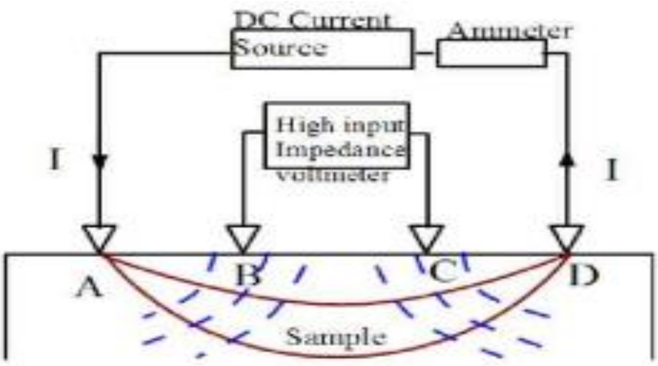
∴ Eqn (10) ⟹ $\boldsymbol{\sigma_p = e\, N_h \mu_h}$

For N-type superconductors, $N_e \gg N_h$

∴ Eqn (10) ⟹ $\boldsymbol{\sigma_e = e\, N_e \mu_e}$

Four Probe Method:

A four point probe is typically used to measure the sheet resistivity of a thin layer or substrate by passing current through two outer probes and reading the voltage across the two inner probes. The 4-point probe set up (Fig) consists of four equally spaced tungsten metal tips with finite radius. Each tip is supported by springs on the end to minimize sample damage during probing. The four metal tips are part of an auto-mechanical stage which travels up and down during measurements.



A high impedance current source is used to supply current through the outer two probes. A voltmeter measures the voltage across the inner two probes to determine the sample resistivity. Typical probe spacing is nearly 2 mm. These inner probes draw no current because of the high input impedance voltmeter in the circuit. Thus unwanted voltage drop (*IR* drop) at point B and point C caused by contact resistance between probes and the sample is eliminated from the potential measurements. Since these contact resistances are very sensitive to pressure and to surface condition (such as oxidation of either surface).

The resistivity of the bulk material of semi conductor can be caluclated by using the formula

$$\boxed{\rho = 2\pi s \left(\frac{V}{I}\right)}$$

Where,

s →connstant distance between two consecutive probes

V→voltage developed between probes B and C

I → Current applied between probes A and D

The resistivity of the thin sheet of thickness **'t'** of semiconductor can be caluclated by using the formula

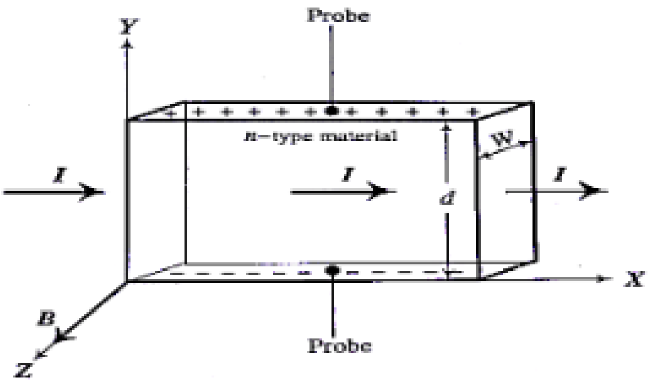
$$\boxed{\rho = \frac{\pi t}{\ln 2}\left(\frac{V}{I}\right)}$$

Hall Effect:

"When a current carrying semiconductor is placed in a transverse magnetic field, an electric field is produced inside the conductor in a direction perpendicular to both the current and the magnetic field. This phenomenon is known as Hall Effect". The generated voltage is called Hall voltage.

Hall effect- expression for Hall coefficient and its applications

Consider an n-type rectangular semiconductor having width **w** and thickness **d** with cross section area **wxd**, placed in a transverse magnetic field **B** along Z- axis. Let current **I** be passing through the semiconductor along X-axis .the Hall voltage developed along Y-axis which can be measured at terminals of the probes as shown in figure.



GENERATION OF ELECTRIC FIELD DUE TO HALL EFFECT

Under equilibrium condition,

The force on the charge q in magnetic field 'B' is balanced by force on the charge due to electric field 'E' is

$$qE_H = qvB$$

$$\Rightarrow \quad E_H = vB \qquad \text{----------------- (1)}$$

But

$$E_H = \frac{V_H}{d} \qquad \text{----------------- (2)}$$

where V_H is Hall voltage and d is the thickness of specimen

From (1) and (2)

$$\frac{V_H}{d} = vB$$

$$\Rightarrow \quad V_H = vBd \qquad \text{---------------- (3)}$$

But $I = nqAv$

$\Rightarrow \quad v = \frac{I}{nqA}$ ---------------- (4)

But, area of cross section, A= w.d

$\Rightarrow \quad \frac{1}{w} = \frac{d}{A}$ --------------- (5)

Put Equ (4) in (3)

Eqn (4) $\Rightarrow \quad V_H = \frac{IBd}{nqA} = \frac{IB}{nq}\left(\frac{d}{A}\right)$ --------------- (6)

Put Eqn (5) in (6)

Eqn (6) $\Rightarrow \quad V_H = \frac{IB}{nq}\left(\frac{1}{w}\right)$

$\Rightarrow \quad V_H = \left(\frac{1}{nq}\right)\left(\frac{IB}{w}\right)$

$\Rightarrow \quad \boxed{V_H = R_H\left(\frac{IB}{w}\right)}$ --------------- (7)

Eqn (7) is the expression for Hall Voltage (V_H)

Where $\boxed{R_H = \frac{1}{nq}}$ --------------- (8)

Eqn (8) is the expression for Hall Coefficient.

Note(1):*Eqn (7) can also be written in terms of current density (J),*

$$V_H = R_H\left(\frac{IB}{w}\right) = R_H\left(\frac{IB}{A/d}\right) = R_H Bd\left(\frac{I}{A}\right) = \frac{BdJ}{ne}$$

Note(2): *Hall Coefficient* $R_H = \frac{1}{nq} = \frac{1}{\rho}$

Where $\rho \rightarrow$ *Charge carrier density*

Applications of Hall effect:

Hall Effect is used

1. To determine the charge carrier density
2. To determine mobility of charge carriers
3. As magnetic field sensing device.
4. In phase angle measurement
5. In Hall effect sensors and probes.
6. Whether the charge carriers are positive or negative which in turn Used to determine p type or n type material.

Numerical on Semiconductors Physics

Solved Numerical:

1. **For intrinsic GaAs, the room temperature electrical conductivity is $10^{-6}/\Omega$m; the electron and hole mobilities are respectively 0.85m^2/Vs and 0.04 m^2/Vs. Compute the intrinsic carrier concentration at room temperature.**

Sol: Given that, $\sigma = 10^{-6}\ (\Omega\text{-m})^{-1}$

$\mu_e = 0.85\ \text{m}^2/\text{V-s}$

$\mu_p = 0.04\ \text{m}^2/\text{V-s}$

$e = 1.6 \times 10^{-19}$ C

Formula: Conductivity, $\sigma = n_i\, e\, (\mu_e + \mu_p)$

$$10^{-6} = n_i \times 1.6 \times 10^{-19} \times (0.85 + 0.04)$$

⇨ $n_i = \dfrac{10^{-6}}{1.6\times10^{-19}\times0.89}$

⇨ $n_i = 7\text{x}10^{12}/\text{m}^3$

2. **The following data are given for intrinsic Ge at 300K, n_i = 2.4 X $10^{19}/\text{m}^3$, μ_e=0.39m^2/Vs, μ_h = 0.19 m^2/Vs. Calculate the resistivity of the sample.**

Sol: Given that, $n_i = 2.4\ \text{X}\ 10^{19}/\text{m}^3$

$\mu_e = 0.39\text{m}^2/\text{Vs}$

$\mu_h = 0.19\ \text{m}^2/\text{Vs}$

$e = 1.6 \times 10^{-19}$ C

Resistivity, $\rho = \dfrac{1}{\sigma} = \dfrac{1}{n_i e(\mu_e + \mu_h)}$

$$\rho = \frac{1}{2.4\ \text{X}\ 10^{19}\times1.6\times10^{-19}(0.39+0.19)}$$

$\rho = 0.449\ \Omega$m

3. **The resistivity of the intrinsic Ge at 27^{0}C is equal to 0.47 Ωm. Assuming electron and hole mobilities as 0.38 and 0.18m^2/Vs respectively, calculate the intrinsic carrier density.**

Sol: Given that

$\rho = 0.47\ \Omega$m
$\mu_e = 0.38\ \text{m}^2/\text{Vs}$
$\mu_h = 0.18\text{m}^2/\text{Vs}$
$e = 1.6 \times 10^{-19}$ C
Resistivity, $\rho = \dfrac{1}{\sigma} = \dfrac{1}{n_i e\,(\mu_e + \mu_h)}$

Re arranging, $n_i = \frac{1}{\rho e\,(\mu_e + \mu_h)}$

$$n_i = \frac{1}{0.47 \times 1.6 \times 10^{-19}\,(0.38 + 0.18)}$$

n_i= 2.4 X $10^{19}/m^3$

4. **The Hall co-efficient of a material is -3.68X10^{-5} m^3/C. What is the type of charge carriers? Also calculate the carrier concentration.**

Sol: Given: Hall Co-efficient of the material, R_H = -3.68X10^{-5} m^3/C

Carrier Concentration, n=?

Since the sign of the R_H is negative as per data, the charge carriers are electrons.

We have, $R_H = \frac{1}{\rho}$

$R_H = \frac{1}{ne}$

The carrier concentration, $n = \frac{1}{R_H\, e}$

$$n = \frac{1}{3.68X10^{-5}\ X\ 1.602X10^{-19}}$$

n =1.7x$10^{23}/m^3$

5. **The hall co-efficient of a specimen of a doped Si is found to be 3.66X$10^{-4}$$m^3/C$. The resistivity of the specimen is 8.93X10^{-3}Ωm. Find the mobility and density of the charge carrier, assuming single carrier conduction.**

Sol: Given: Hall Co-efficient of the doped silicon, R_H = 3.66X$10^{4}$$m^3/C$.

Resistivity of the specimen, ρ_i = 8.93X10^{-3} Ωm

i. Mobility of the charge carrier μ =?
Since the sign of the R_H is positive as per the data, the charge carriers must be holes. Hence we have to find mobility of holes, i.e., μ_h= ?

ii. Charge carrier density, N_h= ?

We Know, $R_H = \frac{1}{ne}$

$= \frac{1}{N_h\, e}$

$N_h = \frac{1}{R_H\, e}$

$$= \frac{1}{3.66X10^{-4}X1.602X10^{-19}}$$

N_h = 1.7055X $10^{22}/m^3$.

We know that the mobility of the charge carrier μ_h is given by,

$\mu_h = \sigma R_H$

Where, σ is the conductivity.

$\sigma = \frac{1}{\rho}$ and $R_H = \frac{1}{N_h\, e}$

Combining, $\mu_h = \frac{1}{\rho\, N_h\, e}$

$$\mu_h = \frac{1}{(8.93X10^{-3})(1.7055X10^{22})(1.602X10^{-19})}$$

$\mu_h = 0.041 m^2V^{-1}s^{-1}$.

6. **The conductivity and the Hall coefficient of an n-type silicon specimen are 112/Ωm and 1.25 x 10^{-3} m^3/C, respectively. Calculate the charge carrier concentration and electron mobility.**

Sol: Given that, σ= 112/Ωm

R_H= 1.25 X $10^{-3}$$m^3$/C

e= 1.6 × 10^{-19} C

$R_H = \frac{1}{n_e\, e}$ and $\sigma = n_e\ e\ \mu_e$

On rearranging the equations

$$n_e = \frac{1}{R_H\, e} \text{ and } \mu_e = \frac{\sigma}{n_e\, e} = \sigma R_H$$

On Substitution,

Carrier concentration, $n_e = \frac{1}{1.25\ \mathrm{X}\ 10^{-3} \times 1.6 \times 10^{-19}}$

n_e= 5 x 10^{21} m^{-3}

Mobility, $\mu_e = 112\ X\ 1.25\ \mathrm{X}\ 10^{-3}$

μ_e = 0.14 m^2/VS.

Exercise problems:

1. Calculate the conductivity of pure silicon if the intrinsic carrier concentration is 1.6×10^{10} cm^{-3}. Given electron and hole mobilities are 0.15 $m^2V^{-1}s^{-1}$ and 0.5 $m^2V^{-1}s^{-1}$ respectively.
2. Calculate the conductivity of extrinsic silicon doped with boron atoms at a concentration of 10×10^{21} m^{-3}. Given μ_h= 0.05 $m^2V^{-1}s^{-1}$
3. Mobilities of electrons and holes in an intrinsic germanium at 300K are 0.36 $m^2V^{-1}s^{-1}$ and 0.14 $m^2V^{-1}s^{-1}$ respectively. If the resistivity of Ge is 2.5Ωm, compute the intrinsic carrier density.
4. The electron concentration in an N-type semiconductor is 5x$10^{17}$$m^{-3}$. Neglecting the hole current, calculate the conductivity of the material if the drift velocity of electrons is 350m/s in an electric field of 1000V/m.
5. The Hall coefficient of a material is −3.68x$10^{-5}$$m^3$/C. What is the type of charge carriers? Also calculate the carrier concentration.
6. The Hall coefficient of a specimen of doped silicon is found to be 3.66x$10^{-4}$$m^3$/C. The resistivity of the specimen is 9.93x10^{-3} Ωm. Find the mobility and the charge carrier density assuming single carrier conduction.