SAPTHAGIRI NPS UNIVERSITY

BASIC ELECTRICAL & ELECTRONICS ENGG

1 SEM

MODULE 1

NUMERICALS ON SINGLE-PHASE AC CIRCUIT

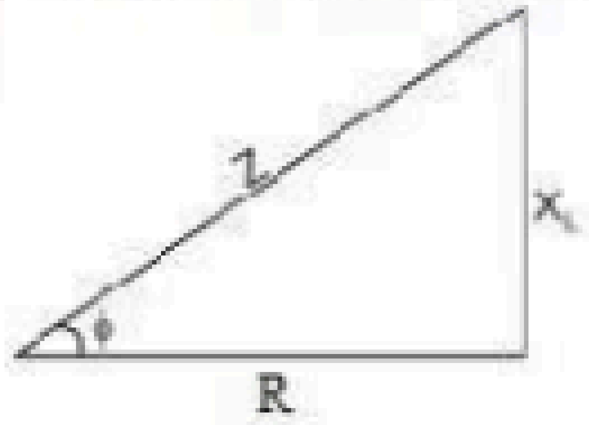
Power Factor

The power factor (*i.e.* $\cos\phi$) of a circuit can be defined in one of the following wavs :

(*i*) Power factor = $\cos\phi$ = cosine of angle between V and I

(*ii*) Power factor $= \frac{R}{Z} = \frac{\text{Resistance}}{\text{Impedance}}$

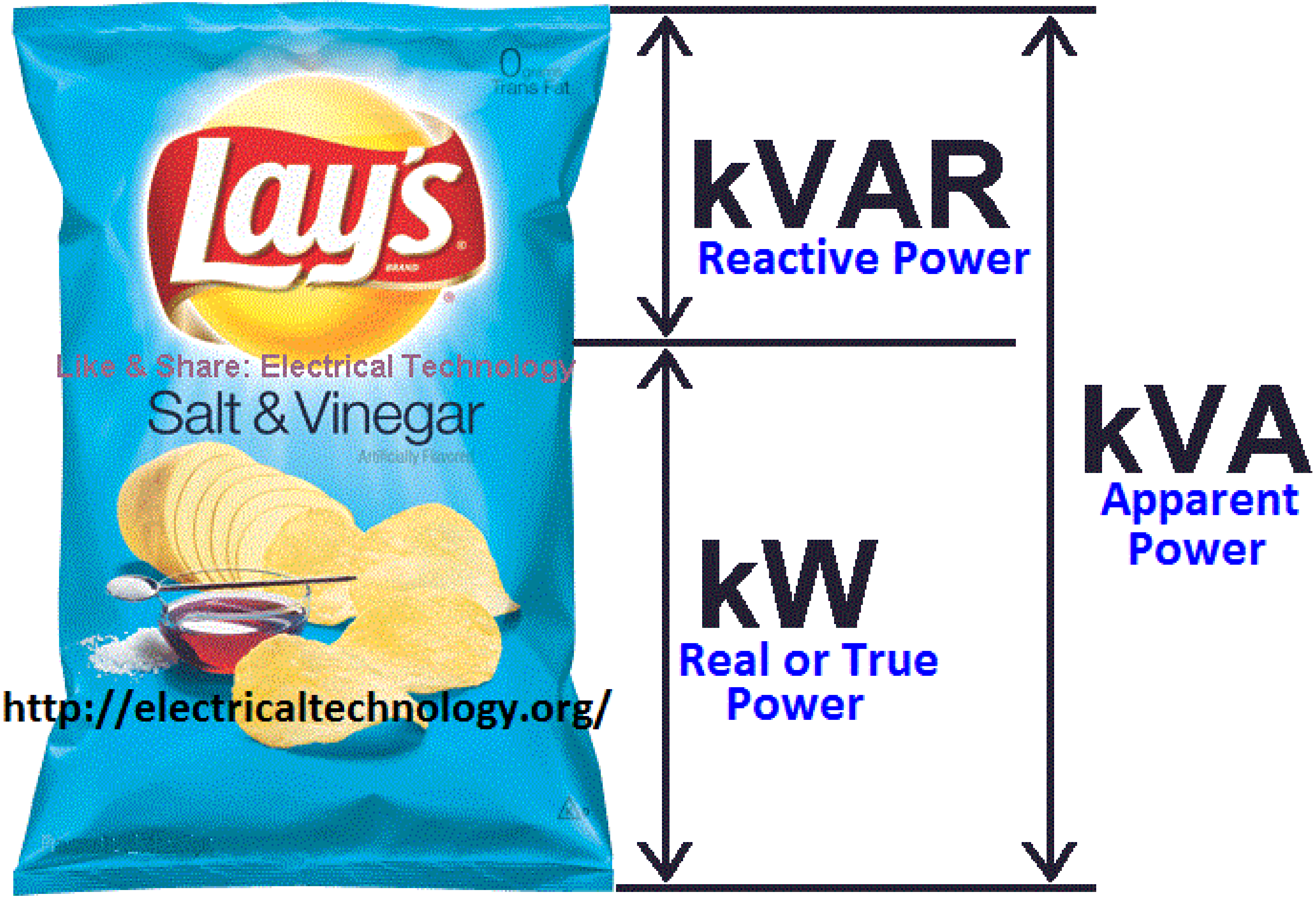
(*iii*) Power factor $= \frac{V\,I\cos\phi}{V\,I} = \frac{\text{True power}}{\text{Apparent power}}$



For example, in a resistor, the current and voltage are in phase *i.e.* $\phi = 0°$. Therefore, power factor of a pure resistive circuit is $\cos 0° = 1$. Similarly, phase difference between voltage and current in a pure inductance or capacitance is 90°. Hence power factor of pure L or C is zero. This is the reason that power consumed by pure L or C is *zero. For a circuit having R, L and C in varying proportions, the value of power factor will lie between 0 and 1. It may be noted that power factor can never have a value greater than 1.

(*a*) It is a usual practice to attach the word 'lagging' or 'leading' with the numerical value of power factor to signify whether the current lags or leads the voltage. Thus if a circuit has a p.f. of 0.5 and the current lags the voltage, we generally write p.f. as 0.5 lagging.

(*b*) Sometimes power factor is expressed as a percentage. Thus 0.8 lagging power factor may be expressed as 80% lagging.


kVAR
Reactive Power
kW
Real or True Power
kVA
Apparent Power
Lay's
Like & Share: Electrical Technology
Salt & Vinegar
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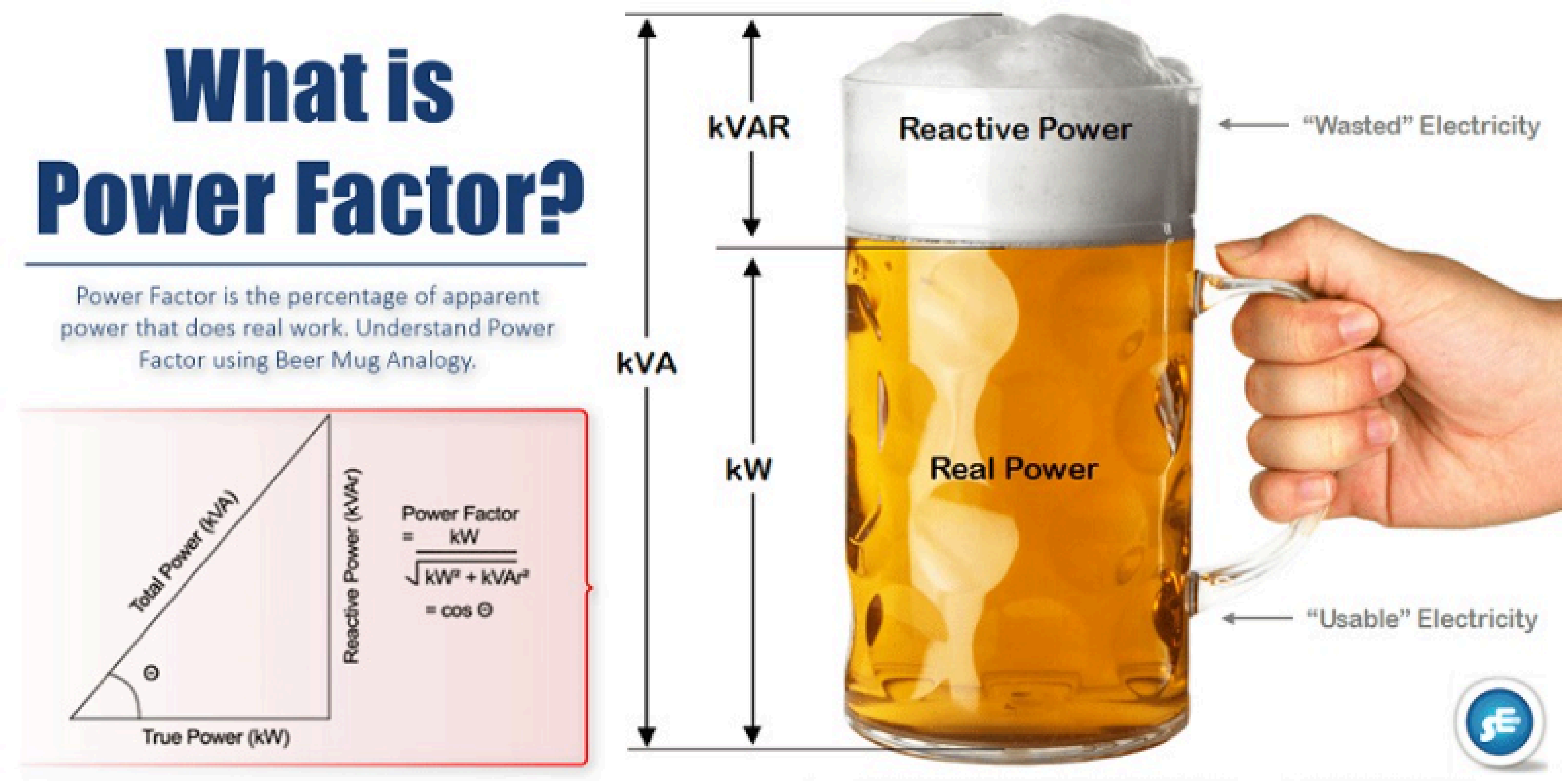
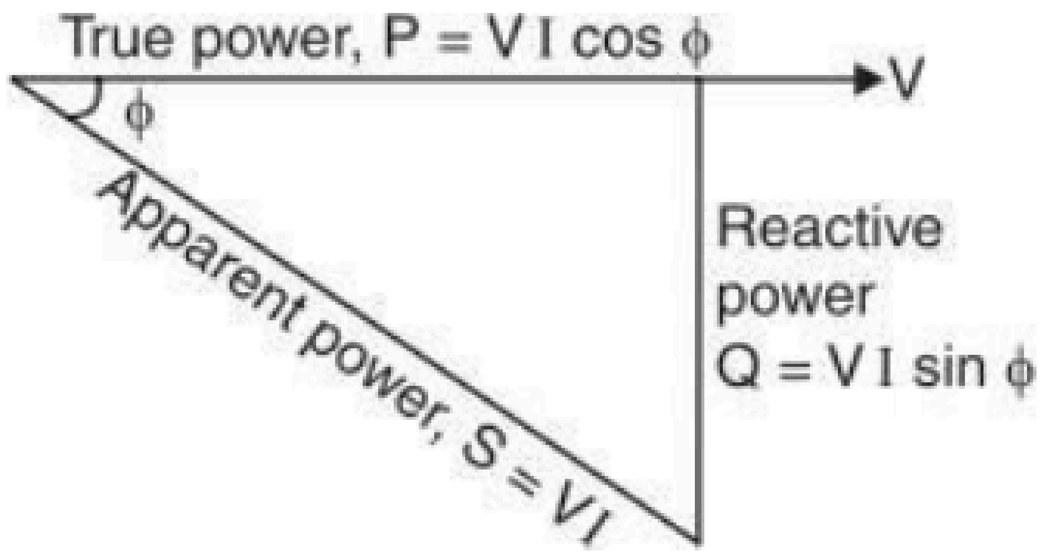


Simply stated, the Power Factor is the percentage of Apparent Power that does real work.

Power triangle.



(*i*) Power factor, $\cos\phi = \dfrac{\text{True power}}{\text{Apparent power}} = \dfrac{VI\cos\phi}{VI}$

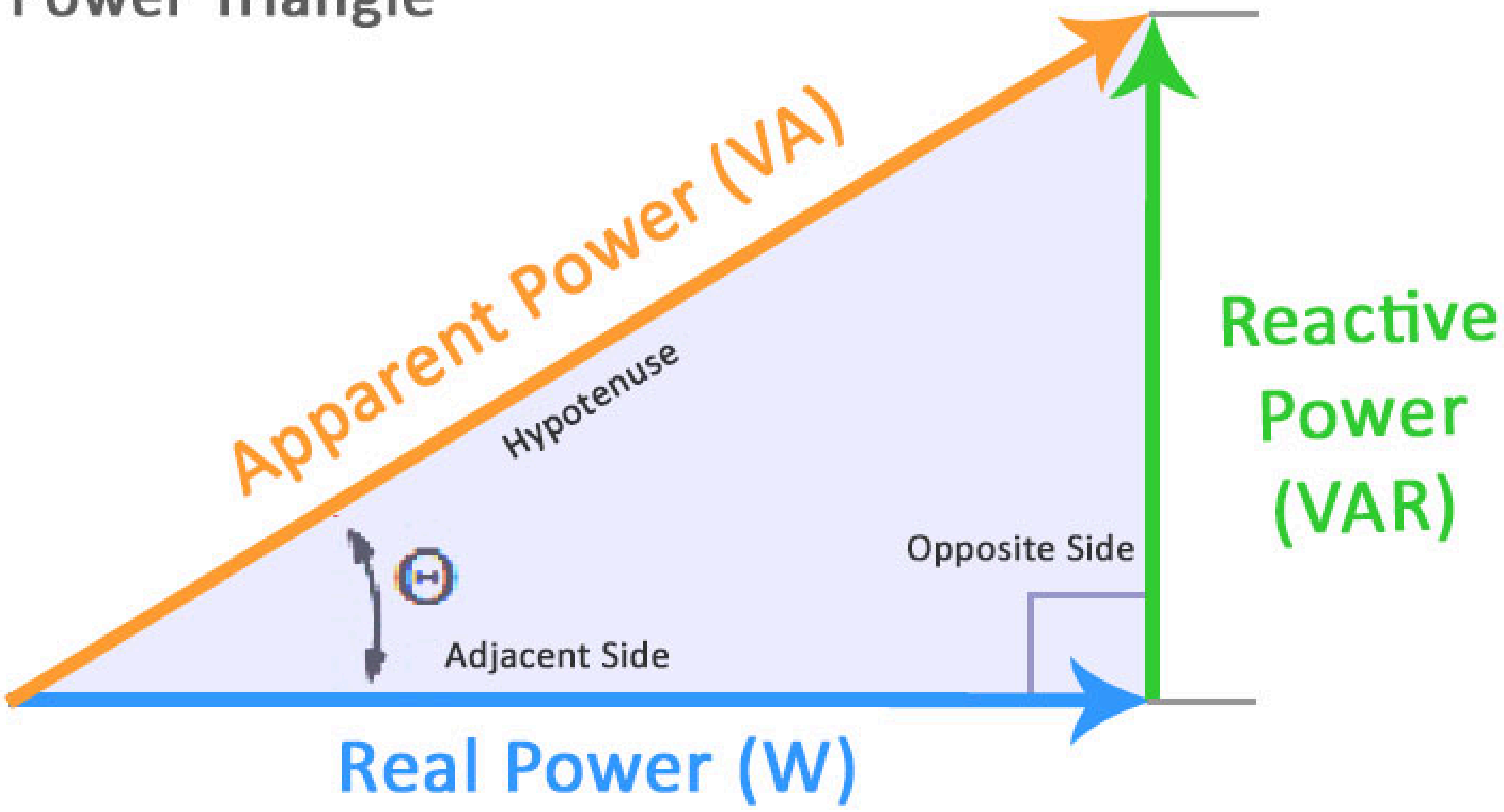
(*ii*) $(\text{Apparent power})^2 = (\text{True power})^2 + (\text{Reactive power})^2$

or $S^2 = P^2 + Q^2$

(*iii*) True power, P = Apparent power $\times \cos\phi = VI\cos\phi$

Reactive power, Q = Apparent power $\times \sin\phi = VI\sin\phi$

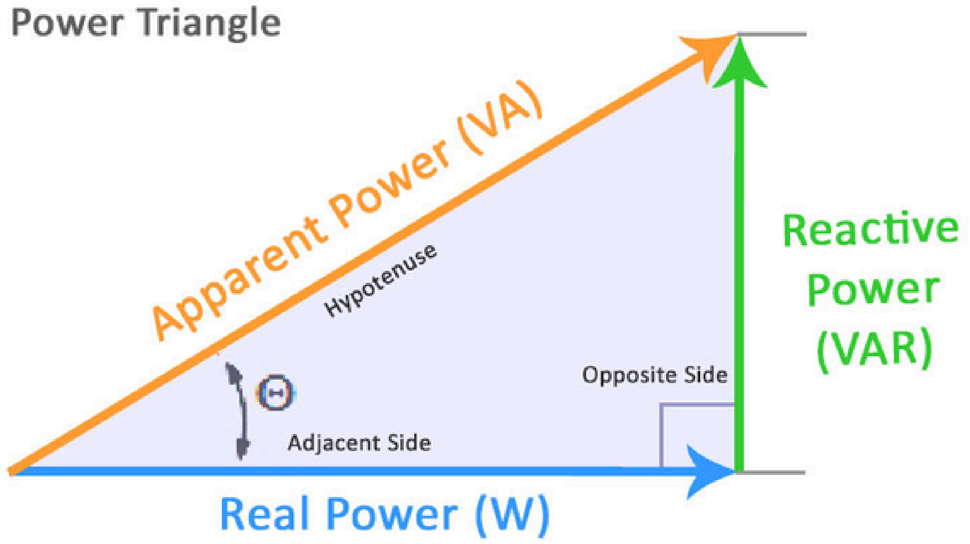
Power Triangle



Formulas
Real Power (P) = VIcos θ, Watts (W)
Reactive Power (Q) = VIsin θ, Volt-amperes Reactive (VAr)
Apparent Power (S) = VI, Volt-amperes (VA)

The power triangle is a useful way to visualize the relationship between Active power (P), reactive power (Q), and apparent power (S) in AC circuits.

Explanation:



- Real or Active Power (P): Measured in watts (W), this is the actual power consumed by the circuit to do useful work. Real Power (P) = VIcos θ, Watts (W)

- Reactive Power (Q): Measured in volt-amperes reactive (VAR), this is the power that oscillates between the source and the reactive components (inductors and capacitors) in the circuit.

  Reactive Power (Q) = VIsin θ, Volt-amperes Reactive (VAr)

- Apparent Power (S): Measured in volt-amperes (VA), this is the combination of active and reactive power.

  Apparent Power (S) = VI, Volt-amperes (VA)

Numerical on pure resistive circuit

1. *An a.c. circuit consists of a pure resistance of 10 Ω and is connected across an a.c. supply of 230 V, 50 Hz. Calculate (i) current (ii) power consumed and (iii) equations for voltage and current.*

Solution. (*i*) Current, $I = V/R = 230/10 = \mathbf{23\ A}$

(*ii*) Power, $P = VI = 230 \times 23 = \mathbf{5290\ W}$

(*iii*) Now,

$$V_m = \sqrt{2}V = \sqrt{2} \times 230 = 325.27 \text{ volts}$$

$$I_m = \sqrt{2}I = \sqrt{2} \times 23 = 32.52 \text{ A}$$

$$\omega = 2\pi f = 2\pi \times 50 = 314 \text{ rad/s}$$

∴ Equations of voltage and current are :

$$v = 325{\cdot}27 \sin 314\, t \quad ; \quad i = 32{\cdot}52 \sin 314\, t$$

2. *The* current in a 2·2 kW resistor is i = 5 sin (2pi ×100t + 45°) mA

(i) Write the mathematical expression for the voltage across the resistor.

(ii) What is the r.m.s. value of the resistor voltage?

(iii) What is the instantaneous value of resistor voltage at t = 0·4 ms ?

Solution. $I_m = 5 \text{ mA} = 5 \times 10^{-3}$ A ; $R = 2.2 \text{ k}\Omega = 2.2 \times 10^3\ \Omega$

(i) $V_m = I_m R = 5 \times 10^{-3} \times 2.2 \times 10^3 = 11$ V

Since voltage across the resistor R is in phase with current,

$\therefore$ $v = V_m \sin(2\pi \times 100\,t + 45°)$

or $v = 11 \sin(2\pi \times 100\,t + 45°)$V **Ans.**

(ii) R.M.S. value of resistor voltage, $V_{r.m.s.} \dfrac{V_m}{\sqrt{2}} = \dfrac{11}{\sqrt{2}} = = \mathbf{7.78\ V}$

(iii) The instantaneous value of resistor voltage at $t = 0{\cdot}4$ ms is

$$v(0{\cdot}4\text{ ms}) = 11\sin[2\pi \times 100 \times 0{\cdot}4 \times 10^{-3} + 45°]\text{V}$$

$$= 11\sin[0{\cdot}2513\text{ rad} + 45°]\text{V}$$

$$= 11\sin[{*}14{\cdot}4° + 45°]\text{ V} = 11\sin 59{\cdot}4° = \mathbf{9{\cdot}47\ V}$$

Numerical on pure Inductive Circuit:

Example 1 *A pure inductive coil allows a current of 10 A to flow from a 230 V, 50 Hz supply. Find (i) inductive reactance (ii) inductance of the coil (iii) power absorbed. Write down the equations for voltage and current.*

Solution. *(i)* Circuit current, $I = V/X_L$ $(V_L = V)$

$\therefore$ Inductive reactance, $X_L = V/I = 230/10 = \mathbf{23\ \Omega}$

(ii) Now, $X_L = 2\pi f L \quad \therefore \quad L = \dfrac{X_L}{2\pi f} = \dfrac{23}{2\pi \times 50} = \mathbf{0.073\ H}$

(iii) Power absorbed $=$ **Zero**

$V_m = 230 \times \sqrt{2} = 325.27\ \text{V}$; $I_m = 10 \times \sqrt{2} = 14.14\ \text{A}$; $\omega = 2\pi \times 50 = 314\ \text{rad/s}$

Since in a pure inductive circuit, current lags behind the voltage by $\pi/2$ radians, the equations are :

$$v = 325.27 \sin 314\,t \quad ; \quad i = 14.14 \sin(314\,t - \pi/2)$$

Numericals on pure Inductive Circuit:

Example 2 *The current through an 80 mH inductor is 0·1 sin (440 t − 25°) A. Write the mathematical expression for the voltage across it.*

Solution. Inductive reactance is

$$X_L = 2\pi f L = 400 \times 80 \times 10^{-3} = 32\ \Omega$$

$$V_m = I_m X_L = 0{\cdot}1 \times 32 = 3{\cdot}2\ \text{V}$$

Since the voltage leads the current by 90°, we must add 90° to the phase angle of voltage.

$\therefore$ $$v = V_m \sin(400\,t - 25^\circ + 90^\circ)$$

or $$\boldsymbol{v = 3{\cdot}2 \sin(400\,t + 65^\circ)\ \text{V}}$$

Numerical on pure Inductive Circuit:

Example 3 *The voltage across and current through a circuit element are :*

$v = 100 \sin(314t + 45°)$ volts ; $i = 10 \sin(314t + 315°)$ amperes

(i) Identify the circuit element. (ii) Find the value. (iii) Obtain expression for power.

Solution. $v = 100 \sin(314t + 45°)$; $i = 10 \sin(314t + 315°)$

The expression for i can be written as : $i = 10 \sin(314t - 45°)$.

(*i*) From the equations of voltage and current, it is clear that i lags behind v by 90°. Therefore, the circuit element is **pure inductor.**

(*ii*) Inductive reactance of the element is

$$X_L = \frac{V_m}{I_m} = \frac{100}{10} = 10\Omega$$

$$\therefore \quad L = \frac{X_L}{\omega} = \frac{10}{314} = 0.0318 \text{ H} = \mathbf{3.18\ mH}$$

(*iii*) Expression for instantaneous power p is

$$p = -\frac{V_m I_m}{2}\sin 2\omega t = -\frac{100\times 10}{2}\sin(2\times 314t)$$

$$\therefore \quad p = \mathbf{-500 \sin 628t}$$

Numerical on pure Capacitive Circuit or Capacitor

Example 1. *A 318 μF capacitor is connected across a 230 V, 50 Hz system. Determine (i) the capacitive reactance (ii) r.m.s. value of current and (iii) equations for voltage and current.*

Solution. (*i*) Capacitive reactance, $X_C = \dfrac{1}{2\pi fC} = \dfrac{10^6}{2\pi \times 50 \times 318} = 10\Omega$

(*ii*) R.M.S. value of current, $I = V/X_C = 230/10 =$ **23 A**

(*iii*) $V_m = 230 \times \sqrt{2} = 325{\cdot}27$ volts ; $I_m = \sqrt{2} \times 23 = 32{\cdot}53$ A ; $\omega = 2\pi \times 50 = 314$ rad/s

∴ Equations for voltage and current are :

$$v = 325{\cdot}27 \sin 314\,t \quad ; \quad i = 32{\cdot}53 \sin(314\,t + \pi/2)$$

Numerical on pure Capacitive Circuit or Capacitor

Example 2. *The voltage across a 0.01 μ F capacitor is 240 sin (1·25 × 10^4 t − 30°) V. Write the mathematical expression for the current through it.*

Solution. Capacitive reactance, $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(1\cdot 25\times 10^4)\times(0\cdot 01\times 10^{-6})} = 8000\ \Omega$

$$\text{Peak current, } I_m = \frac{V_m}{X_C} = \frac{240}{8000} = 0.03\text{ A}$$

∴ Expression for current through the capacitor is

$$i = I_m \sin(1\cdot 25\times 10^4 t - 30° + 90°)\text{ A}$$

or $$i = 0\cdot 03 \sin(1\cdot 25\times 10^4 t + 60°)\text{ A}$$

Numerical on R-L

Example 12.1. *A coil having a resistance of 7 Ω and an inductance of 31·8 mH is connected to 230 V, 50 Hz supply. Calculate (i) the circuit current (ii) phase angle (iii) power factor (iv) power consumed and (v) voltage drop across resistor and inductor.*

Solution. (*i*) Inductive reactance, $X_L = 2\pi f L = 2\pi \times 50 \times 31.8 \times 10^{-3} = 10\ \Omega$

$$\text{Coil impedance, } Z = \sqrt{R^2 + X_L^2} = \sqrt{7^2 + 10^2} = 12.2\ \Omega$$

$\therefore$ Circuit current, $I = V/Z = 230/12.2 = $ **18·85 A**

(*ii*) $\tan\phi = X_L/R = 10/7$

$\therefore$ Phase angle, $\phi = \tan^{-1}(10/7) = $ **55° lag**

(*iii*) Power factor $= \cos\phi = \cos 55° = $ **0.573 lag**

(*iv*) Power consumed, $P = VI\cos\phi = 230 \times 18.85 \times 0.573 = $ **2484·24 W**

(*v*) Voltage drop across $R = IR = 18.85 \times 7 = $ **131.95V**

Voltage drop across $L = IX_L = 18.85 \times 10 = $ **188.5 V**

Example 12.2. *An inductor coil is connected to a supply of 250 V at 50 Hz and takes a current of 5 A. The coil dissipates 750 W. Calculate (i) power factor (ii) resistance of coil and (iii) inductance of coil.*

Solution. (*i*) Power consumed, $P = VI\cos\phi$

$$\therefore \quad \text{Power factor, } \cos\phi = \frac{P}{VI} = \frac{750}{250 \times 5} = \mathbf{0.6\ lag}$$

(*ii*) Impedance of coil, $Z = V/I = 250/5 = 50\ \Omega$

Resistance of coil, $R = Z\cos\phi = 50 \times 0.6 = $ **30** Ω

$$(iii) \quad \text{Reactance of coil, } X_L = \sqrt{Z^2 - R^2} = \sqrt{(50)^2 - (30)^2} = 40\ \Omega$$

$$\therefore \quad \text{Inductance of coil, } L = \frac{X_L}{2\pi f} = \frac{40}{2\pi \times 50} = \mathbf{0.127\ H}$$

Numerical on R-L-C

Example 12.38. *A 230 V, 50 Hz a.c. supply is applied to a coil of 0.06 H inductance and 2.5Ω resistance connected in series with a 6·8 μF capacitor. Calculate (i) impedance (ii) current (iii) phase angle between current and voltage (iv) power factor and (v) power consumed.*

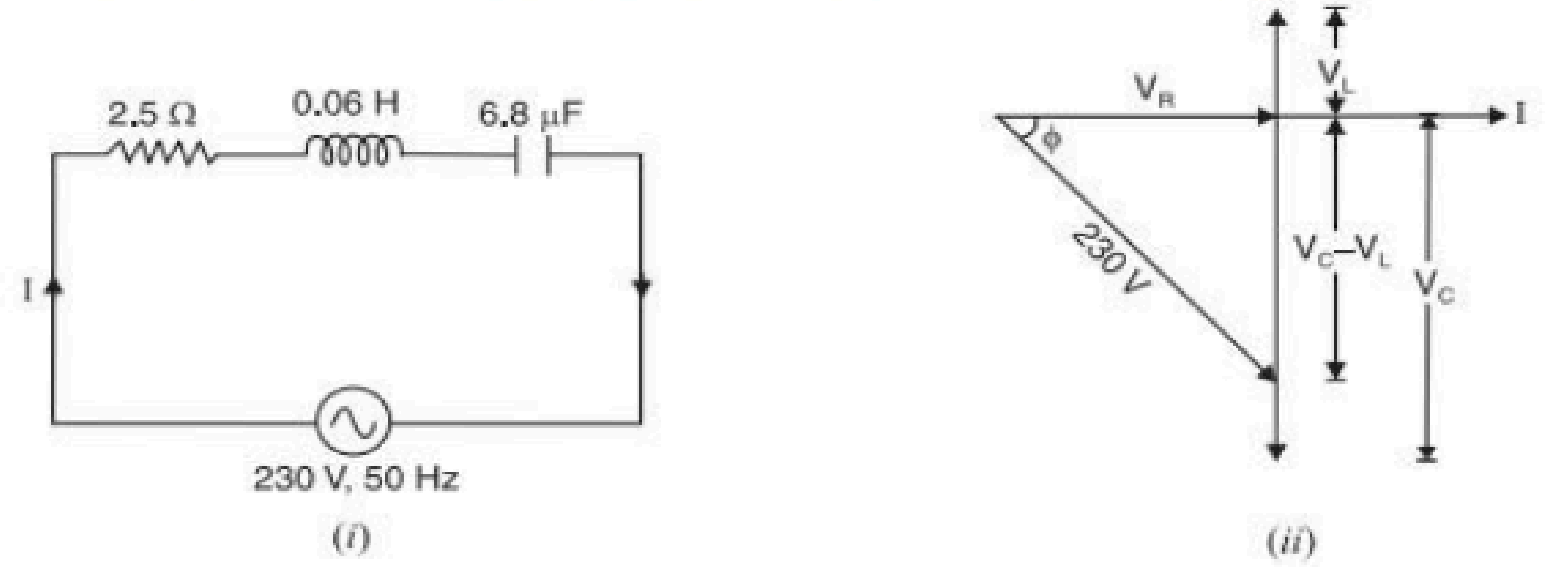


Fig. 12.35

Solution. Fig. 12.35 (*i*) shows the conditions of the problem.

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.06 = 18.85\ \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{10^6}{2\pi \times 50 \times 6.8} = 468\ \Omega$$

(*i*) Circuit impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(2.5)^2 + (18.85 - 468)^2} =$ **449.2 Ω**

(*ii*) Circuit current, $I = V/Z = 230/449.2 =$ **0.512 A**

(*iii*) $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{18.85 - 468}{2.5} = -179.66$

∴ Phase angle, $\phi = \tan^{-1} - 179.66 = -89.7° =$ **89.7° *lead***

The negative sign with φ shows that current is leading the voltage [See the phasor diagram in Fig. 12.35 (*ii*)].

(*iv*) Power factor, $\cos\phi = \dfrac{R}{Z} = \dfrac{2.5}{449.2} =$ **0.00557 *lead***

(*v*) Power consumed, $P = VI\cos\phi = 230 \times 0.512 \times 0.00557 =$ **0.656 W**

Numerical on R-L-C

Example 12.39. *A coil of p.f. 0.8 is connected in series with a 110 μF capacitor. The supply frequency is 50 Hz. The p.d. across the coil is found to be equal to the p.d. across the capacitor. Calculate the resistance and inductance of the coil.*

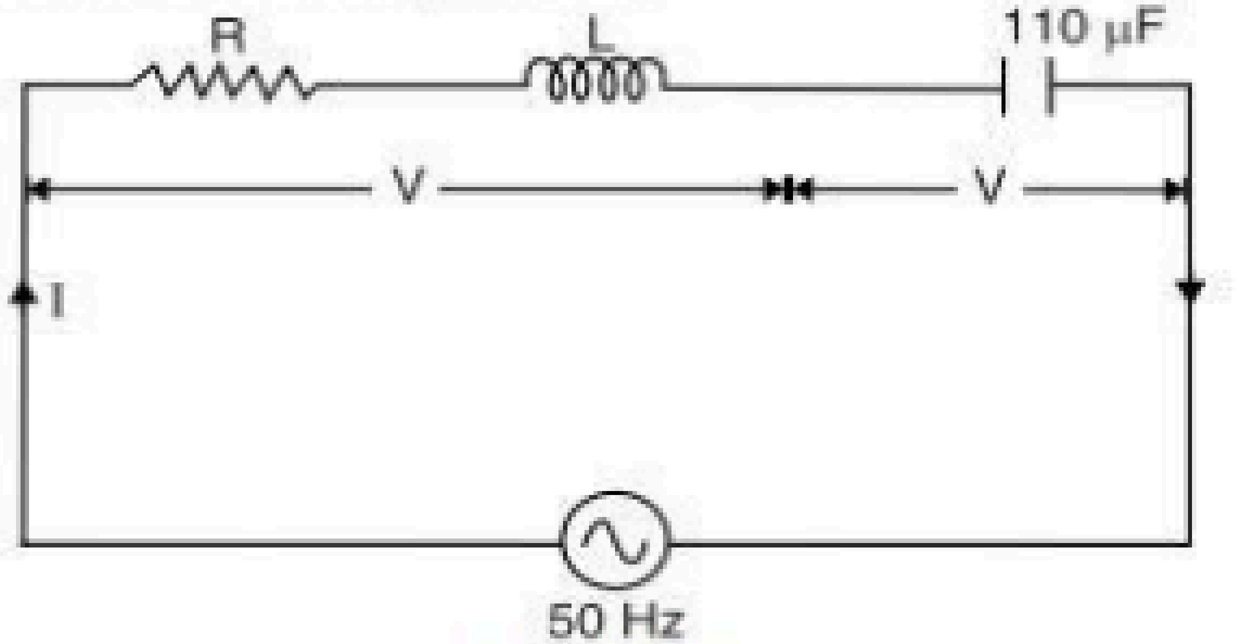


Fig. 12.36

Solution. Fig. 12.36 shows the conditions of the problem.

$$\text{Reactance of capacitor, } X_C = \frac{1}{2\pi f C} = \frac{10^6}{2\pi\times 50\times 110} = 29\ \Omega$$

$$\text{Now,} \quad I Z_{coil} = I X_C \quad \therefore \quad Z_{coil} = X_C = 29\ \Omega$$

$$\text{For the coil, } \cos\phi = R/Z_{coil} \quad \therefore \quad R = Z_{coil}\cos\phi = 29\times 0.8 = \mathbf{23.2\ \Omega}$$

$$\text{Reactance of coil, } {}^*X_L = Z_{coil}\sin\phi = 29\times 0.6 = 17.4\ \Omega$$

$$\therefore \quad \text{Inductance of coil, } L = \frac{X_L}{2\pi f} = \frac{17.4}{2\pi\times 50} = \mathbf{0.055\ H}$$

THANK YOU