SAPTHAGIRI NPS UNIVERSITY

Department of Mathematics

MODEL QUESTION PAPER -I

Semester	**:**	**1st sem B.Tech**	**Maximum marks**	**:**	**100**
Course Title	**:**	**Linear Algebra and Calculus**	**Duration**	**:**	**3 hours**
Course Code	**:**	**24BEELY102**	(P Cycle)		

Part-A

Answer any Ten questions **10X02=20**

1	What are the conditions for the equation $AX = B$ have unique solution and infinite solution.	**02**
2	Find the rank of the matrix $\begin{bmatrix} 1 & 5 \\ 3 & 9 \end{bmatrix}$.	**02**
3	Find the eigen vector of $X^{(1)}$ for $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ by using Rayleigh power method.	**02**
4	Write the n^{th} derivative of the function e^{ax+b} .	**02**
5	State Leibnitz theorem for the n^{th} derivative of a product of two functions.	**02**
6	Find the n^{th} derivative of the function $y = e^{3x}$.	**02**
7	Define Partial derivative of a function $u = f(x, y)$.	**02**
8	If $x = uv$ and $y = \frac{u}{v}$ then find $J\left(\frac{x,y}{u,v}\right)$.	**02**
9	If $u = \sin(xy)$ find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.	**02**
10	Evaluate $\int_1^2 \int_1^3 xy^2\, dxdy$.	**02**
11	Find the complimentary function for $(D^2 - 6D + 9)y = 0$.	**02**
12	Find the Integrating factor for $\frac{dt}{dx} - \frac{t}{x} = -x$.	**02**

Part-B

Answer any seven questions **07X05=35**

13	Solve the following system of equations by using Gauss Elimination Method $3x + 4y + 5z = 18; 2x - y + 8z = 13; 5x - 2y + 7z = 20$.	**05**
14	Solve the following system of equations by using Gauss-Seidel Method $28x + 4y - z = 32;\ 2x + 17y + 4z = 35;\ x + 3y + 10z = 24$ carryout 3 iterations correct to 3 decimal places.	**05**
15	Determine the angle between the radius vector and tangent to the curve $r = a(1 - \cos\theta)$.	**05**
16	Determine the pedal equation of the curve $r^m = a^m(\cos m\theta + \sin m\theta)$.	**05**
17	If $u = log\left(\frac{x^2+y^2}{x+y}\right)$, show that $xu_x + yu_y = 1$.	**05**

18		If $u = x^2 + y^2 + z^2, v = xy + yz + zx, w = x + y + z$, find the value of $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.	05
19		Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} x^3\, y\, dx\, dy$.	05
20		Solve $xy(1 + xy^2)\frac{dy}{dx} = 1$ by using Bernoulli's differential equation.	05
21		If $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + y} = 0$, verify the differential equation is exact or not.	05
Part-C **Answer any Three full questions**			**03X15=45**
22	a)	Find the Rank of the matrix $\begin{bmatrix} 91 & 92 & 93 & 94 & 95 \\ 92 & 93 & 94 & 95 & 96 \\ 93 & 94 & 95 & 96 & 97 \\ 94 & 95 & 96 & 97 & 98 \\ 95 & 96 & 97 & 98 & 99 \end{bmatrix}$.	07
	b)	Find the largest eigenvalue and the corresponding eigenvector of the matrix $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by Rayleigh's Power Method by taking the initial approximation to the eigen vector as $[1, 0.8, -0.8]^T$. Perform 5 iterations.	08
23	a)	If $y = \log\left(x + \sqrt{1 + x^2}\right)$, prove that $(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + n^2 y_n = 0$.	07
	b)	Show that the following pair of curves intersect each other orthogonally: $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$.	08
24	a)	If $u = \log(\tan x + \tan y + \tan z)$, show that $\sin 2x\, u_x + \sin 2y\, u_y + \sin 2z\, u_z = 2$.	07
	b)	If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$, show that $6\frac{\partial u}{\partial x} + 4\frac{\partial u}{\partial y} + 3\frac{\partial u}{\partial z} = 0$.	08
25	a)	Evaluate $\int_0^{\frac{\pi}{6}} cos^4\, 3x\, sin^2\, 6x\, dx$.	07
	b)	Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2 + y^2 + z^2)\, dz\, dy\, dx$.	08
26	a)	Solve $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = x^2 - 4x - 6$.	07
	b)	Solve $\frac{dy}{dx} - y\tan x = \frac{\sin x\, cos^2\, x}{y^2}$.	08