Module - 4
Superconductivity

Syllabus: Introduction to Superconductors- temperature dependence of resistivity of metals and superconductors-critical temperature- Meissner's effect- critical field- temperature dependence of critical field-types of superconductors- BCS theory (Qualitatively) - flux quantization, DC Josephson Junction (Qualitatively). Application of superconductivity in DC SQUIDs. Numerical problems.

Introduction:

The phenomenon of superconductivity was discovered by Kemerling Onnes in 1911, when he was measuring the resistivity of mercury at low temperatures. He observed that the electrical resistivity of pure mercury drops to zero at 4.2K. The material has passed into a new state called the superconducting state.

Superconductors are those materials whose electrical resistance is zero below critical temperature. Superconductivity is the phenomenon of obtaining superconductors below critical temperature.

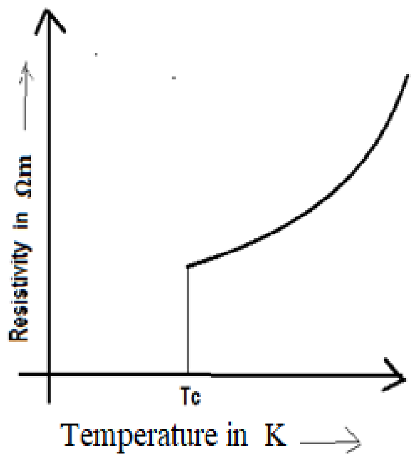
Critical temperature (T_c): It is a particular temperature at which the materials loses all its resistance and acts as superconductors.

Examples for critical temperature:

Sl No	Material	Critical Temperature (T_c in K)
1	Mercury (Hg)	4.2 (-268.8 ^{0}C)
2	Lead (Pb)	7.2 (-265 ^{0}C)
3	Niobium (Nb)	9.3 (-263.7^{0}C)
4	Y-Ba-Cu-Oxide	92 (-181^{0}C)

Note: Highest critical temperature ever recorded is 203 K (-70^{0}C) using Hydrogen compound for research purpose.

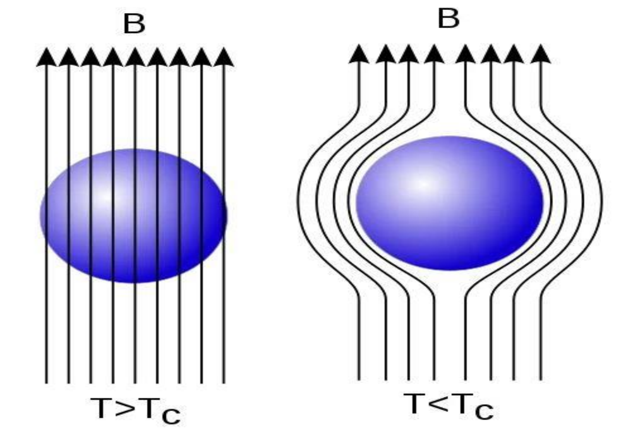
Temperature Dependence of Resistivity of Superconductors:



The variation of resistivity versus temperature is as shown in the graph. As temperature of a material is decreased, its resistivity also decreased. At a particular temperature known as critical temperature ($T = T_c$), the resistivity of the material suddenly drops to zero and possess zero resistance to the electric current. Hence the material acts a superconductor below critical temperature. As the temperature of the material is increased above the critical temperature, it acts as normal conductor ($R \neq 0$). Critical temperature is also called as transition temperature

MeissnerEffect:

When a normal conductor is cooled in the presence of external magnetic at a temperature above the critical temperature ($T > T_c$), the magnetic field lines will pass through the specimen or material as shown in first figure.



Magnetic field lines through a conductor vs. field lines through a superconductor

But when the temperature of the material is decreased below critical temperature ($T<T_c$) then the normal conductor loses all its resistance and becomes superconductor and hence the magnetic field lines (flux) will expel (moving away) near the vicinity of superconductor and exhibits perfect diamagnetism as shown second figure. This effect is known as Meissner's effect. The magnetic flux density inside the normal material is given by,

$$B = \mu_0 (M + H) \quad \ldots\ldots\ldots(1)$$

Where, M→ Magnetization

H→ External applied magnetic field (field strength)

μ_0 → Absolute permeability of free space

When $T<T_c$ then magnetic flux density inside the material is zero. Hence $B = 0$

Eqn (1) ⇒ $\mu_0 (M + H) = 0$

⇒ $M = -H$

i.e., Susceptibility $\chi = \frac{M}{H} = -1$ indicates perfect diamagnetism.

Critical Field (H_c):

The minimum value of the applied magnetic field required to destroy superconductivity is called the critical field (H_c). or

The minimum magnetic field required to switch a material from its superconducting state to its normal state is called **critical field (Hc).**

Temperature dependence, of Critical field

Critical field is a function of temperature and it is given by

$$\mathbf{H_c = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]} \quad \text{------------ (1)}$$

Where, T→ Temperature in K

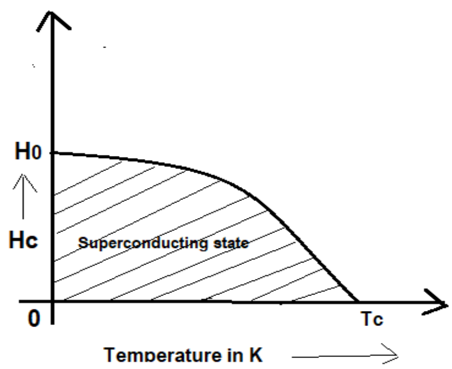
T_c→ Critical Temperature in

H_0→ Critical Filed at 0 K.

Case (i) at $T = T_c$, $H_c = 0$

Case (ii) at T= 0 K, $H_c = H_0$

The graphical variation of H_c versus Temperature T is as shown in figure.



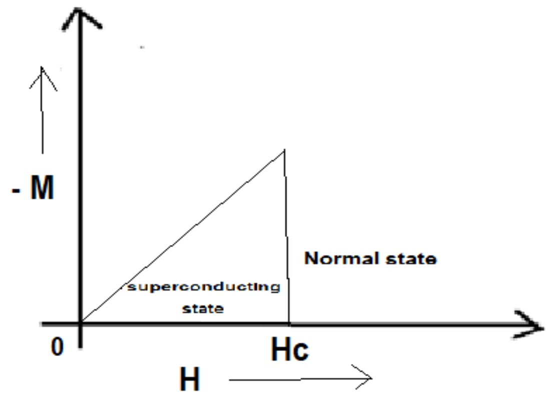
Types of Superconductors:

Based on the critical field, superconductors are mainly classified in to two types. They are

1. Type–I superconductors
2. Type–II superconductors

Type–I superconductors:

The graph of Magnetization (M) v/s applied magnetic field (H) for Type-1 superconductor is as shown in the figure.



As the value applied filed H, increases from zero, magnetization also increases and becomes maximum at critical field. That is from 0 to H_c the magnetization varies directly with the applied magnetic field and the specimen acts like perfect diamagnetic and obeys Meissner's effect.

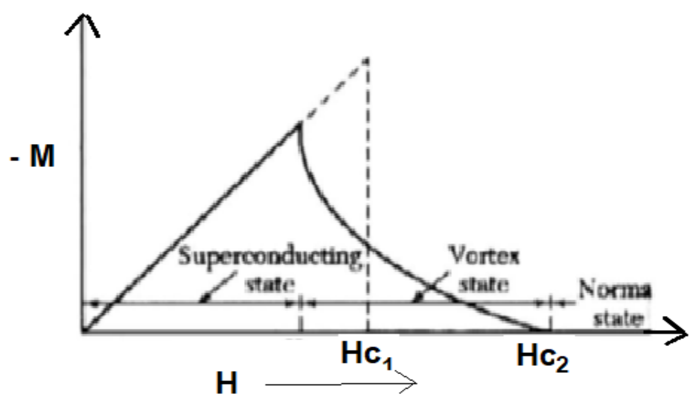
As the magnetic is increased beyond H_c , then the field lines passes through the specimen and the specimen acts as normal conductor.

Type -1 superconductor are also called as soft superconductors because they cannot withstand for high magnetic fields and they cannot be used for making superconducting magnets.

Example: Mercury (Hg), Lead (Pb) and Zinc (Zn), other pure elements.

Type–II Superconductors:

The graph of magnetization (M) v/s applied magnetic field (H) for Type -I1 superconductor is as shown in the figure.



Type- II superconductor possesses two critical fields H_{C1} and H_{C2}.

Between 0 to H_{C1}, the specimen acts as perfect diamagnetic and obeys complete Meissner's effect. Thus the magnetization is proportional to the applied magnetic field.

Between H_{C1} to H_{C2} the magnetization decreases with the increase in applied magnetic field. Between H_{C1} to H_{C2} the specimen is electrically superconductor but not magnetically. This state is called as mixed state or Vortex state because in this state the Meissner's effect is incomplete.

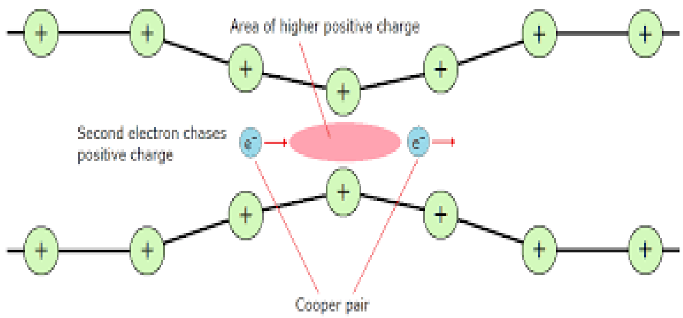
Above the value of critical field $H_{C2,}$ the specimen loses its magnetization and acts as normal conductor.

Type- II superconductors are usually alloys or transition metals with high value of critical field. Hence these superconductors are also known as Hard superconductors.

Example: Nb_3Sn, Nb_3Ge etc

BCS Theory:

Three scientists Bardeen, Cooper and Schrieffer (BCS) explained the superconductivity on the basis of quantum Theory.



Consider an electron approaching a positive ion core and suffers attractive coulomb interaction. Due to this attraction ion core is set in motion and thus distorts the lattice. Let a second electron come in the way of distorted lattice and interaction between the two occurs which lowers the energy of the second electron.

The two electrons there- fore interact indirectly through the lattice distortion or the phonon field which lowers the energy of the electrons. The above interaction is interpreted as electron - Lattice - electron interaction through phonon field. It was shown by Cooper that, this attractive force becomes maximum if two electrons have opposite spins and momentum. The attractive force may exceed coulombs repulsive force between the two electrons below the critical temperature. The first electron emits a phonon and second electron absorbs it. Hence these two electron coupled together and forms **Cooper pairs** (bound pair of electrons).

Attraction force between two electrons in a Cooper pair is very weak and can be separated by increase in temperature due to thermal or Lattice vibration.

Below the critical temperature the density of Cooper pair is large and move collectively through the lattice, which minimize the collisions and resistance becomes zero. The movement of Cooper pairs shows less velocity even for large current.

Flux quantization:

The flux quantization in superconductors is the phenomena where magnetic flux passing through a super conducting loop is quantized in discrete unit of quantization of magnetic flux, which is important for the development of flux quantization.

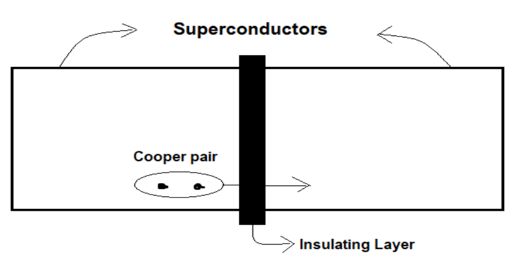
This quantization of magnetic flux is observed in superconductors. The quantization flux is considered a key identification of electron pairing in super conductors. Flux quantization is crucial in the operation of SQUIDs

Josephson junction:

Two superconductors separated by a thin layer of insulator is called as a weak link or Josephson Junction.

The Cooper pairs tunnel through the insulating from one superconductor to another. The following two are observed due to tunnelling of electrons pairs

1. DC Josephson Effect
2. AC Josephson Effect



DC Josephson Effect:

It consists of two superconducting metals separated by a thin layer of insulator (oxides) of thickness 10 Å to 20 Å. The movement of Cooper pairs in a superconductor is represented by a wave function. This insulating layer introduces a phase difference between wave functions of Cooper pairs on both sides of insulating layer. Because of this phase difference super current flows across the junction, even when the applied voltage is zero. This is known as DC Josephson effect.

The super current flowing through the junction is given by

$$\mathbf{I_S = I_C \sin\phi_0}$$

Where, $I_C \rightarrow$ critical current at zero voltage condition. It depends on the thickness of insulating layer and temperature.

$\phi_0 \rightarrow$ the Phase difference between the wave function on either side of insulator.

Applications:

SQUID:

SQUIDs are the acronym for "**S**uperconducting **QU**antum **I**nterference **D**evice".

It is used to measure tiny or weak magnetic fields of the order of 10^{-14} T and also used as diagnostic tool in the detection of brain signals.

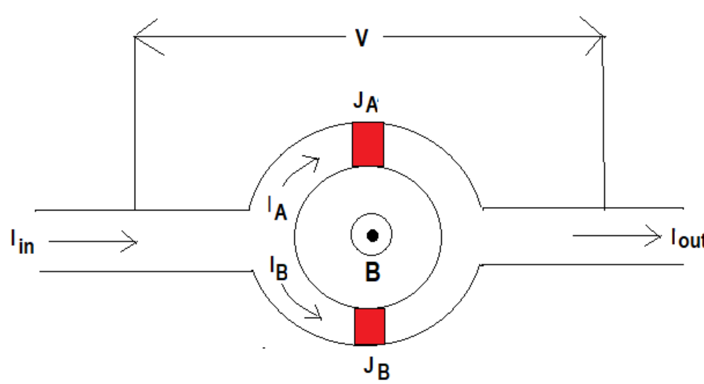
It mainly consists of superconducting ring with one or two Josephson junctions. The magnetic flus interacting with superconducting loop is quantized in steps of $\phi_0 = \frac{h}{2e}$.

There are two types of SQUIDs

1. DC SQUIDs
2. AC SQUIDs (RF SQUIDs)

DC SQUIDS:

It works on the principle of DC Josephson effect. It consists of two josepson junctions J_A and J_B. current entering the superconducting ring (I_{in}) is divided in to two parts I_A and I_B as shown in the figure.



If an external magnetic field is applied it modifies the phase of two currents I_A and I_B which in turn modifies the output current (I_{out}) due to interference. The output current varies periodically with applied magnetic field or flux. Thus if the magnetic field (B) is applied to a SQUID, its output current changes. This change in the current is used to measure the magnetic flux with help of feedback current (voltage V).

Numerical on Superconductivity

Solved Numerical:

1. The critical temperature of Nb is 9.15K. At 0 Kelvin, the critical field is 0.196T. Calculate the critical field at 8K.

Solution:

Given Data: Critical magnetic field at zero Temperature H_o = 0.0196 T,

Critical temperature T_c = 9.15 K, and

Critical field at T = 8K ,H_c = ?

$$H_c = H_0\left[1 - \left(\frac{T}{T_C}\right)^2\right]$$

$$H_c = 0.0196\left[1 - \left(\frac{8}{9.15}\right)^2\right]$$

$H_c = 0.0046$ T

$$H_c = 4.6 \times 10^{-3}\ \text{T}$$

2. A superconducting tin has a critical temperature of 3.7 K at zero magnetic field and a critical field of 0.0306 Tesla at 0 K. Find the critical field at 2 K.

Solution:

Given Data: Critical field at zero Temperature Ho = 0.0306 T,

Critical temperature T c = 3.7k,

Critical field at T = 2K, H_C=?

$$H_c = H_0\left[1 - \left(\frac{T}{T_C}\right)^2\right]$$

$$H_c = 0.0306\left[1 - \left(\frac{2}{3.7}\right)^2\right]$$

H_c= 0.0216 T or 2.16×10^{-3}T

3. Lead has a superconducting transition temperature of 7.26 K. If the initial critical field at 0K is 50×10^3 Am^{-1}, Calculate the critical field at 6K.

Solution:

Given Data: Initial critical field at zero Temperature, H_o=50×10^3 Am^{-1} ,

Transition temperature T_c = 7.26K,

Critical field at T = 6K, H_C= ?

$$H_c = H_0\left[1 - \left(\frac{T}{T_C}\right)^2\right]$$

$$H_c = 50 \times 10^3\left[1 - \left(\frac{6}{7.26}\right)^2\right]$$

$$H_c = 15.84 \times 10^3 Am^{-1}$$

4. At the temperature of 6 K critical magnetic field is $5\text{x}10^3$ Am^{-1}, Calculate the transition temperature when a critical magnetic field is $2\text{x}10^4$ A/m.

Solution:

Given Data: Initial critical field at T = 6 K is ,H_c =5 × 10^3 Am^{-1} ,

Field at 0 K, H_o = 2 × 10^4 Am^{-1} ,

Transition temperature T_c= ?,

$$H_c = H_0\left[1-\left(\frac{T}{T_C}\right)^2\right]$$

$$5 \times 10^3 = 2 \times 10^4\left[1-\left(\frac{6}{Tc}\right)^2\right]$$

$$\left[1-\left(\frac{6}{T_C}\right)^2\right] = \frac{5 \times 10^3}{(2 \times 10^4)}$$

T_c = 6.93K

5. The superconducting transition temperature of Lead is 7.26K. The initial critical field at 0K is 64×10^3 Am^{-1}. Calculate the critical magnetic field at 5K.

Solution:

Given Data: Critical magnetic field at 0 K, H_o =64 × 10^3 Am^{-1}

Transition temperature T_c = 7.26 K,

Critical magnetic field at T = 5K, H_C=?

$$H_c = H_0\left[1-\left(\frac{T}{T_C}\right)^2\right]$$

$$H_c = 64 \times 10^3\left[1-\left(\frac{5}{7.26}\right)^2\right]$$

$$H_c = 33.64 \times 10^3 Am^{-1}$$

6. A superconducting material has a critical temperature of 3.7 K in zero magnetic field and a critical magnetic field of 0.02 Am^{-1} at 0 K. Find the critical magnetic field at 3 K.

Solution:

Given Data: critical Field at 0 K, H_o = 0.02 T

Transition temperature T_c = 3.7 K,

Critical field at T = 3K, H_C= ?

: $$H_c = H_0\left[1-\left(\frac{T}{T_C}\right)^2\right]$$

$$H_c = 0.02\left[1-\left(\frac{3}{3.7}\right)^2\right]$$

$$H_c = 0.0068 \text{ T}$$

7. A superconducting sample has a critical temperature of 3.722 K in zero magnetic field and a critical field of 0.0305 T at 0 K. Evaluate the critical field at 2 K.

Solution:

Given Data: Field at zero Temperature, H_o =0.0305T

Transition temperature T_c = 3.722 K,

Critical field at T = 2K, H_C=?

$$H_c = H_0\left[1-\left(\frac{T}{T_C}\right)^2\right]$$

$$H_c = 0.0305\left[1 - \left(\frac{2}{3.722}\right)^2\right]$$

$$H_c = 0.0216\ \mathrm{T}$$

8. The material lead (Pb) behaves as a superconductor at a temperature of T_c = 7.26 K. Ifthe value of critical magnetic field of lead at 0 K is $H_c = 8 \times 10^5$ A/m, find the critical magnetic field of lead at 4 K.

Solution:

Given Data: Field at zero Temperature, $H_o = 8 \times 10^5$ A/m
Transition temperature T_c = 7.26 K,
Critical field at T = 4K, H_C=?

$$H_c = H_0\left[1 - \left(\frac{T}{T_C}\right)^2\right]$$

$$H_c = 8 \times 10^5\left[1 - \left(\frac{4}{7.26}\right)^2\right]$$

$$H_c = 5.571 \times 10^5 Am^{-1}$$

9. Determine the transition temperature and critical field at 4.2 K for a given specimen of a superconductor if the critical fields are 1.41×10^5 and 4.205×10^5 A/m at 14.1 K and 12.9 K, respectively.

Solution:

Given Data: H_C^1 and T_C=? At T^1 =4.2K
If $H_{c1} = 1.41 \times 10^5$ A/m at $T_1 = 14.1K$
$H_{c2} = 4.205 \times 10^5$ A/m at $T_1 = 12.9K$

$$H_{C1} = H_0\left[1 - \left(\frac{T_1}{T_c}\right)^2\right]$$

$$1.41 \times 10^5 = Ho\left[1 - \left(\frac{14.1}{T_c}\right)^2\right] \quad \text{------------- (1)}$$

$$H_{C2} = Ho\left[1 - \left(\frac{T_2}{T_c}\right)^2\right]$$

$$4.205 \times 10^5 = Ho\left[1 - \left(\frac{12.9}{T_c}\right)^2\right] \quad \text{------------- (2)}$$

To find T_C
Equation (1) / (2) => T_C=14.66K
By using Eqn (1) or (2)

$$1.41 \times 10^5 = Ho\left[1 - \left(\frac{14.1}{14.66}\right)^2\right]$$

$H_0 = 18.81 \times 10^5$ A/m

Caluclation of H_C^1 using $H_O = 18.81 \times 10^5$ A/m and T^1 =4.2K

$$H_{c1} = H_0\left[1 - \left(\frac{T^1}{T_c}\right)^2\right]$$

$$H_{c1} = 18.81 \text{ x } 10^5 \left[1 - \left(\frac{4.2}{14.66}\right)^2 \right]$$

\ $$H_{c1} = 17.266 \text{ x } 10^5 \text{A/m}$$

10. The critical field for lead is 1.2 × 10 ^{5} A/m at 8 K and 2.4 × 10 ^{5} A/m at 0 K. Find the critical temperature of the material.

Given Data: Field at 0 K Temperature, H_o =2.4 × 10 ^{5} A/m
Critical temperature T_c= ?
Critical field at T = 8K, H_C=1.2 × 10 ^{5} A/m

Solution: $$H_c = H_0 \left[1 - \left(\frac{T}{T_C}\right)^2 \right]$$

$$1.2 \times 10^5 = 2.4 \times 10^5 \left[1 - \left(\frac{8}{T_C}\right)^2 \right]$$

$$\Rightarrow \quad \frac{1}{2} = 1 - \left(\frac{8}{T_C}\right)^2$$

$$\Rightarrow \quad \frac{1}{2} = 1 - \frac{64}{T_C^2}$$

$\Rightarrow$ $T_C^2 = 128$ => $\boldsymbol{T_C}$**=11.31K**

Exercise Problems:

1. Lead has superconducting transition temperature of 7.26K. If the initial critical field at 0 K is 75 × 10 ^{3} Am $^{-1}$ Calculate the critical field at 7K.

2. A superconducting tin has a critical temperature of 3.4K at zero critical magnetic field and a critical magnetic field of 0.0206 tesla at 0K. Find the critical magnetic field at 5K.

3. The superconducting transition temperature of Lead is 7.26K. Calculate the initial critical field at 0K.Given the critical magnetic field at 8K as 33.644 x 10 ^{3} Am $^{-1}$

4. Calculate the ratio of critical fields for a superconductor at 7K and 5K given thecritical temperature 8K.

5. The critical field for niobium is 1 × 10^5 Am^{-1} at 8K and 2 x 10^5 Am^{-1} at 0 K.Calculate the transition temperature of the element.