MODULE-I: LINEAR ALGEBRA

1. Define singular and non-singular matrix.
2. Define Echelon form of a matrix.
3. Define Eigen Value and Eigen vector.
4. What is diagonal matrix? Give an example.
5. Define the Rank of a matrix.
6. Define Equivalent matrices.
7. Write the necessary and sufficient conditions for Echelon form of a matrix.
8. Find the Rank of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$.
9. Find the rank of a matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.
10. Find the rank of the matrix $A = \begin{bmatrix} 101 & 2 \\ 102 & 3 \end{bmatrix}$
11. Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$.
12. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.
13. Find the rank of the matrix A $= \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 1 \\ 4 & 1 & 3 \end{bmatrix}$.
14. Write the working procedure to find the rank of a matrix.
15. Write are the conditions for the system of equations to have infinite solutions.
16. What are the conditions for the matrix to be consistent.
17. What are the conditions for the equation $AX = B$ have unique solution and infinite solution.
18. Given a matrix $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$, find the characteristic equation.
19. Find the characteristic equation for A $= \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.
20. Find the characteristic equation of A $= \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.
21. Calculate the Eigenvalue of the matrix A $= \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$
22. Find the Eigen value of the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$
23. Find the Eigen Value of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.
24. If 2 and 8 are two of the eigen values of $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & 1 \\ 2 & -1 & 3 \end{bmatrix}$. Find the third Eigen value.

25. If -2 and 6 are two Eigen values of A = $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. Find the third Eigen value of A.

26. If $\lambda = 2$ is one of the eigen value of the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ then find the corresponding eigen vector.

27. Find the Eigen values and Eigen vectors of the matrix A = $\begin{bmatrix} 5 & -1 \\ 4 & 9 \end{bmatrix}$.

28. Find the sum and product of eigen values and of eigen vectors of $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 0 & 3 \\ -2 & -1 & -3 \end{bmatrix}$.

29. Using Gauss-Seidel iteration method find 1st iteration 5x-y=9; x-5y+2=-4; y-5z=6

30. Solve using Gauss elimination method: $\begin{matrix} 2x + 3y = 7 \\ x - 2y = -3 \end{matrix}$.

31. Solve the following system of linear equations using Gauss elimination method

$$x - y = 2$$
$$2x + y = 5.$$

32. Use the Rayleigh power method to find the eigenvalue & eigen vector of the matrix: $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

33. Use Rayleigh power method to find the new vector $X^{(1)}$ for $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$.

MODULE-2: DIFFERENTIAL CALCULUS

1. Write the n^{th} derivative of the following functions
   i. e^{ax+b}
   ii. a^{mx}
   iii. $(ax+b)^m$
   iv. $\log(ax+b)$
   v. $\sin(ax+b)$
   vi. $\cos(ax+b)$
2. Find the nth order derivative for the following functions
   i. $y = e^{3x}$
   ii. $y = a^{3x}$
   iii. $y = 3^{5x}$
   iv. $y = \frac{1}{3x+2}$
   v. $y = e^{2x+3}$
   vi. $y = x^2 e^x$.
   vii. $y = \log(1-x)$
   viii. $y = \frac{1}{(ax+b)^2}$
   ix. $y = x^n \log x$
   x. $y = e^{ax}\cos(bx)$
   xi. $y = \sqrt{\frac{(1-\cos 2x)}{1+\cos 2x}}$
   xii. $y = \log(ax+b)$
   xiii. $y = e^{ax}\sin(bx+c)$
   xiv. $f(x) = \cos(cx)$
3. Find $\frac{dy}{dx}$, if $y^2 + x^3 - xy + \cos y = 0$.
4. If $y = \tan^{-1}x$, find the second order derivative.
5. If $f(x) = (x^2+3x)e^x$, find the 2rd order derivative.
6. Write the formula for the angle between radius vector and tangent vector.
7. Write the condition for the polar curves cut orthogonally.
8. What is the condition for orthogonal intersection of two curves.
9. Determine the angle between the polar curves $r = \sin\theta$ and $r = \cos\theta$ at the origin.
10. Find the angle between the radius vector and tangent to the curve $r = a\theta$ at any point.
11. Determine the angle between the radius vector and the tangent to the curve $r = e^{\theta}$ at any point.
12. Find the angle between radius vector and the tangent for $r = a(1-\cos\theta)$.
13. If $r = ae^{b\theta}$, find the angle between radius vector and tangent.
14. Find the angle between the radius vector and tangent to the curve $r = 2a\cos\theta$ at $\theta = \pi/4$.

15. Find the angle of intersection of the following pair of curves $i)\ r = sin\theta + cos\theta$
    $ii)\ r = 2\ sin\theta$.
16. Write the Pedal equation in polar form.
17. Find the pedal equation of the curve $r = \alpha e^{\cot\,\alpha}$.
18. Find the pedal equation of the curve r= $\frac{a}{\theta}$.

MODULE 3: PARTIAL DIFFERENTIATION

1. Define Partial derivative of a function $u = f(x, y)$.
2. State Euler's theorem for homogeneous function of two variables.
3. If $u = f(x, y)$ where $x = x(t)$ and $y = y(t)$ then write the total derivative of u with respect to t.
4. If $u = 3x^2y + 6xy^2 + 7$, find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.
5. If $u = \sin(xy)$, find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.
6. If $u = e^{4x+3y}$, find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.
7. Find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ for the function $u = x^3 - 3xy^2 + x + e^x \cos y + 1$.
8. For the given function $u = 3x^2y + 6xy^2 + 7$, find the partial derivative 'u' with respect to x and y.
9. If $f(x, y) = x^2y - 3y^2$, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.
10. If $u = x^y$ then find $\frac{\partial^2 y}{\partial x\, \partial y}$.
11. If $u = f(x + ay) + g(x - ay)$ then find $\frac{\partial^2 u}{\partial y^2}$.
12. If $f(x, y) = x^2y + 3xy^2$, find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$.
13. If $f(x, y) = \cos\,(xy)$, find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$.
14. If u = e^{4x+3y} then find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.
15. Calculate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for $f(x, y) = \frac{xy}{x^2+y^2}$.
16. Determine the partial derivatives of $f(x, y, z) = xe^{yz}$ with respect to x and z.
17. Compute $\frac{\partial^2 t}{\partial x\, \partial y}$ for $f(x, y) = \log\,(x^2 + y^2)$.
18. Calculate $\frac{\partial^2 f}{\partial y^2}$ for $f(x, y) = x\cos\,(y)$.
19. If $u = e^x cosy + 1$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$
20. If $u = x^2 + y^2$, find $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$
21. Define symmetric functions with examples.
22. Find the total derivative of the function, $z = xy^2 + x^2y; x = at; y = 2at$.
23. Write the total derivative of a function $u = f(x, y)$.
24. If $(i) x^3 + xy^2 + y^3 \quad (ii)\ x = r\, sin\theta\, sin\Phi$, then find the value of total derivative.

25. If $z = u^2 + v^2$ and $v = at^2$. Find $\frac{dz}{dt}$.

26. If $z = x^2y + y^2x, x = at^2, y = 2at$, find $\frac{dz}{dt}$.

27. Find the total derivative of $z = x^2y + y^2x$ with respect to x.

28. If $u = f(x, y)$, where $x = x(t)$ and $y = y(t)$, then $\frac{du}{dt}$ =?

29. If $z = xy^2 + x^2y$, where $x = at, y = 2at$, find the total derivative of $\frac{dz}{dt}$.

30. If $z = xe^{y/x}$, show that $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y} = z$.

31. If $g(u)$ is continuously differentiable, show that $w = g(x^2 - y^2)$ is a solution of $y\frac{\partial w}{\partial x}+x\frac{\partial w}{\partial y}$=0.

32. If $u = \frac{x}{y}$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$.

33. If $u = x^2 + 2xy + y^2 + x + y$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u$.

34. State Euler's theorem on homogeneous function.

35. Verify Euler's Theorem for $f(x, y) = x^3 - 3x^2y + y^3$

36. If $z = e^{ax+by}f(ax + by)$ then show that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$.

37. If $y = \alpha e^{ax} + \beta e^{-ax}$, prove that $\frac{\partial^2 u}{\partial x^2} - a^2y = 0$.

38. If $u = \frac{y}{z} + \frac{z}{x}$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$.

39. Write the formula for
   i. Jacobian of functions u *and* v with two independent variables x *and* y.
   ii. Jacobian of functions u, v *and* w with three independent variables x, y *and* z.

40. Find the Jacobian of u, v, w with respect to x, y, z given $u = x + y + z$, $v = y + z, w = z$.

41. Write the formula for Jacobian $J\left[\frac{x,y,z}{u,v,w}\right]$.

42. Find $\frac{\partial(u,v)}{\partial(x,y)}$. Find $\frac{\partial u}{\partial x}$, if $x^2 + xy + y^2 = 1$.

43. If $u = x + 3y^2 - z^3, v = 4x^2yz, w = 2z^2 - xy$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

44. If $u = x + y, v = x - y$, find $J\left[\frac{u,v}{x,y}\right]$.

45. If $u = x + y, v = x^2 + y^2$, find $\frac{\partial(u,v)}{\partial(x,y)}$

46. If $x = u + v$ and $y = u - v$, find $J\left(\frac{x,y}{u,v}\right)$.

47. If $x = u^2 + v^2$ & $y = uv$, find $J\left(\frac{x,y}{u,v}\right)$.

48. If $x = r\cos\,\theta$ & $y = r\sin\,\theta$ find $J\left(\frac{x,y}{r,\theta}\right)$.

49. If $u = xy,\ v = x^3$, find $\frac{\partial(u,v)}{\partial(x,y)}$.

50. If $x = uv$ and $y = \frac{u}{v}$ then find $J\left(\frac{x,y}{u,v}\right)$.

51. Given $u = x^2 + y^2$ and $v = 2xy$, find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$

MODULE 4: INTEGRAL CALCULUS

1. Write the Reduction formula for $\int \sin^n x \, dx$.
2. Write the Reduction formula for $\int \cos^n x \, dx$.
3. Write the Reduction formula for $\int \sin^m x \cos^n x \; dx$, where m & n are positive integers.
4. Write the Reduction formula for $\int_0^{\frac{\pi}{2}} \sin^{\mathrm{m}} x \cos^{\mathrm{n}} x \, dx$.
5. Find
   i. $\int_0^{\frac{\pi}{2}} \sin^4 x \, dx$.
   ii. $\int_0^{\pi/2} \; sin^5 \, x \, dx$.
   iii. $\int_0^{\pi/2} \sin^7 x \, dx$.
   iv. Evaluate $\int_0^{\frac{\pi}{2}} \sin^8 x \, dx$.
   v. $\int_0^{\pi/2} \cos^4 x \, dx$.
   vi. $\int_0^{\frac{\pi}{2}} \cos^5 x \, dx$.
   vii. $\int_0^{\pi/2} \cos^6 x \, dx$.
   viii. $\int_0^{\frac{\pi}{2}} \sin^7 x \cos^5 x \, dx$.
   ix. $\int_0^{\frac{\pi}{2}} \sin^5 x \cos^4 x \;\; dx$.
   x. $\int_0^{\frac{\pi}{2}} \sin^6 x \cos^5 x \;\; dx$.
   xi. $\int_0^{\pi/2} sin^3 \, x \; cos^7 \, x \, dx$.
   xii. $\int_0^{\pi/2} \cos x \sin^{99} x \, dx$.
   xiii. $\int_0^{\pi} \sin^4 \left(\frac{x}{2}\right) dx$.
   xiv. $\int_0^{\pi/8} \cos^3(4x) \, dx$.
6. Evaluate $\int \sin^3 x \, dx$ using the reduction formula.
7. Evaluate $\int \frac{x^2-x+2}{x^2+1}$dx.
8. Find $\iint (x+y) \, dxdy$.
9. Evaluate $\iint (x-y) dxdy$.
10. Find $\int_1^2 \int_1^3 xy^2 \, dxdy$.
11. Evaluate $\int_0^{\pi} x \; sin^8 \, x \, dx$.
12. Evaluate $\int_{y=0}^{2} \int_{x=0}^{1} xy \, dx \, dy$.
13. Evaluate $\int_0^1 \int_0^x (x^2+y^2) dy \, dx$.
14. Evaluate $\int_1^3 \int_2^4 9x^3y^2 \, dydx$.
15. Evaluate $\int_0^1 \int_1^2 (x^2+y^2) \, dx \, dy$.
16. Evaluate $\int_0^2 \int_1^2 (x^2+y^2) \, dx \, dy$.

17. Evaluate $\int_0^1 \int_0^2 (x + y)\, dx\, dy$.

18. Compute the double integral $\int_0^1 \int_x^{x^2} (x + y)\, dy\, dx$.

19. Evaluate $\int_0^1 \int_1^2 (x + 3)\, dx\, dy$.

20. $\int_2^1 \int_0^1 (y + 4)\, dy\, dx$.

21. Evaluate $\int_0^1 \int_0^x (x^2 + y^2)\, dy\, dx$.

22. Verify that $\int_0^2 \int_0^1 (x^2 + y^2)\, dx\, dy = \int_0^1 \int_1^2 (x^2 + y^2)\, dy\, dx$.

23. Find the value of $\int_0^1 \int_0^x (2x + y)\, dy\, dx$.

24. Evaluate $\iiint xy\, dxdydz$

25. Evaluate $\int_{z=0}^3 \int_{y=0}^2 \int_{x=0}^1 dx\, dy\, dz$.

26. Evaluate $\int_0^1 \int_0^1 \int_0^1 xyzdxdydz$.

27. Evaluate $\int_{-1}^1 \int_0^1 \int_0^x y\, dy\, dx\, dz$.

MODULE 5: DIFFERENTIAL EQUATIONS

1. What is homogeneous and non-homogeneous differential equation.
2. Find the order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^2 = c\frac{d^2y}{dx^2}$.
3. Find the order and degree of the differential equation $\sqrt{\frac{dy}{dx}} = (4x+y+1)$.
4. Write the order and degree of the differential equation $\left(\frac{dy}{dx}\right)^2 + 3\frac{dy}{dx} + 2 = 0$.
5. Determine order and degree of differential equation $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$.
6. Find the order and degree for the following differential equation $y''' + y'' + e^{y'} = y^2$.
7. Find the general solution of the differential equation $\frac{dy}{dx} = 1 + \frac{y^2}{1+x^2}$.
8. For each of the given differential equation find a particular solution satisfying the given condition $\frac{dy}{dx} = y\ tanx; y = 1$ when $x = 0$.
9. Solve $(D^2 + 2D + 1)y = 0$.
10. Solve the homogeneous differential equation $y'' + y = 0$.
11. Write the necessary and sufficient condition for differential equation to be exact.
12. Write the formula for Linear differential equation and Bernoulli's equation.
13. Solve the differential equation $\frac{dy}{dx} + y = e^x$.
14. Reduce the given equation to linear differential equation $\frac{dy}{dx} + \frac{y}{x} = y^2x$.
15. What is the general solution of linear differential equation.
16. Find the integrating factor for $ydx - xdy = 0$.
17. Find the integrating factor for $\frac{dy}{dx} + y\cot\ x = \cos\ x$.
18. Find the integrating factor for the differential equation $(2x+3y)\ dx - 2xy\ dy = 0$.
19. Find the integrating factor for the differential equation $(2x^2+3y^2)\ dx - 4xy\ dy = 0$.
20. Find the Integral factor of (i) $\frac{dy}{dx} + 2y = 6x$ (ii) $\frac{dy}{dx} - \frac{2y}{x} = \sin\ (x)$.
21. Find the integrating factor for $x\frac{dy}{dx} - y = 2x^2$.
22. Mention the necessary and sufficient condition for the differential equation and write the solution for the $M(x,y) + N(x,y)dy = 0$.
23. Check if the differential equation $(2x+3y)dx + (3x+2y)dy = 0$ is exact.
24. Write the complimentary function for $(D^3 - 2D^2 + 4D - 8)y = 0$.
25. Solve $(D^2 - 2D + 1)y = 0$.
26. Mention the nature of the roots of the auxiliary equation.
27. If $1, 1, 1 \pm i$ are the roots of the auxiliary equation, write the complimentary function.
28. Find the complimentary function for the following differential equation

    $$y''' - 5y'' + 7y' - 3y = 2e^x.$$
29. Give the complementary function for $(D^2 + 6D + 9)y = 0$.

30. Check the differential equation $(3x^2 - 2xy)dx + (4xy - y^2)dy = 0$ is exact or not.

31. Find the auxiliary equation for the differential equation $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 4y = 0$.

32. Find the complementary function for the differential equation $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + 4y - 8 = 0$.

33. $\frac{dy}{dx} + \frac{y}{x} = y^2x$. Convert this equation into a linear differential equation.

34. What is the condition for the Differential equation to be exact.

35. Find the Integratory factor of the Differential equation $\frac{dy}{dx} + y\cot\ x = \cos\ x$.

36. Solve $(D^3 - 3D + 2)\,y\ =\ 0$.

37. Solve $\frac{d^{2y}}{dx^2} - 6\frac{dy}{dx} + 9y = 3e^{-4x}$.