Pedal Equation:

Length of the perpendicular from the pole to the tangent. $\rightarrow$ pedal Eq^n. / (p-r Eq^{n}s)

$$\boxed{p = r\sin\phi}$$

(1). $\frac{2a}{r} = (1+\cos\theta) \rightarrow (1)$.

$\Rightarrow r(1+\cos\theta) = 2a$.

$\Rightarrow r.(-\sin\theta) + (1+\cos\theta)\frac{dr}{d\theta} = 0$

$(1+\cos\theta)\frac{dr}{d\theta} = r\sin\theta$

$\frac{dr}{d\theta} = \frac{r\sin\theta}{1+\cos\theta}$

$\frac{d\theta}{dr} = \frac{1+\cos\theta}{r\sin\theta} \Rightarrow \cancel{r\frac{d\theta}{dr}} = \cancel{\frac{r\sin\theta}{1+\cos\theta}}$

from (1), we have

$r(1+\cos\theta) = 2a$.

$\frac{2a}{r} = (1+\cos\theta)$

$\frac{\cancel{2}a}{r} = \cancel{2}\cos^2\theta/2$

$\frac{a}{r} = \cos^2\theta/2$

$\Rightarrow r\frac{d\theta}{dr} = \frac{(1+\cos\theta)}{\sin\theta} = \frac{\cancel{2}\cos^{\cancel{2}}\theta/2}{\cancel{2}\sin\theta/2\,\cancel{\cos\theta/2}} = \cot\theta/2$

$\tan\phi = \cot\theta/2 \Rightarrow \tan\phi = \tan(\frac{\pi}{2} - \theta/2)$

$\phi = \frac{\pi}{2} - \frac{\theta}{2}$

$p = r\sin\phi$

$= r\sin(\frac{\pi}{2} - \frac{\theta}{2}) = r\cos\theta/2$

$\frac{p}{r} = \cos\theta/2 \Rightarrow \frac{p^2}{r^2} = \frac{a}{r}$ $\boxed{\Rightarrow p^2 = ar}$

(2). $r^n = a^n \cos n\theta$. $\qquad \cos n\theta = \frac{r^n}{a^n}$

$\therefore n \cdot r^{n-1} \cdot \frac{dr}{d\theta} = a^n(-n \sin n\theta)$

$\Rightarrow r^{n-1} \cdot \frac{dr}{d\theta} = -a^n \sin n\theta$

$$\frac{dr}{d\theta} = -\frac{a^n \sin n\theta}{r^{n-1}}$$

$$\frac{d\theta}{dr} = -\frac{r^{n-1}}{a^n \sin n\theta}$$

$$r\frac{d\theta}{dr} = -\frac{r^{n-1} \cdot r}{a^n \sin n\theta} = -\frac{r^n}{a^n \sin n\theta} = -\frac{\cancel{a^n} \cos n\theta}{\cancel{a^n} \sin n\theta}$$

$$= -\cot n\theta$$

$\tan\phi = -\cot n\theta$

$\Rightarrow \tan\phi = \tan\left(\frac{\pi}{2} + n\theta\right)$

$\phi = \frac{\pi}{2} + n\theta$

$p = r\sin\phi$

$= r\sin\left(\frac{\pi}{2} + n\theta\right) \Rightarrow r\cos n\theta$

$\therefore p = r \cdot \frac{r^n}{a^n} \Rightarrow \boxed{a^n p = r^{n+1}}$

(3). $r^m = a^m(\cos m\theta + \sin m\theta)$.

Sol: $m \cdot r^{m-1} \cdot \frac{dr}{d\theta} = a^m(-\sin m\theta \cdot m + \cos m\theta \cdot m)$

$r^{m-1}\frac{dr}{d\theta} = a^m(\cos m\theta - \sin m\theta)$.

$$\frac{1}{r^{m-1}}\frac{d\theta}{dr} = \frac{1}{a^m(\cos m\theta - \sin m\theta)}$$

$$\frac{d\theta}{dr} = \frac{r^{m-1}}{a^m(\cos m\theta - \sin m\theta)} \Rightarrow r\frac{d\theta}{dr} = \frac{r^m}{a^m(\cos m\theta - \sin m\theta)}$$

$$\Rightarrow r\frac{d\theta}{dr} = \frac{\cancel{a^m}(\cos m\theta + \sin m\theta)}{\cancel{a^m}(\cos m\theta - \sin m\theta)}$$

$$\Rightarrow \tan\phi = \frac{1 + \tan m\theta}{1 - \tan m\theta} \Rightarrow$$

$$\boxed{\phi = \frac{\pi}{4} + m\theta}$$

$$p = r\sin\phi$$

$$p = r\sin\left(\frac{\pi}{4} + m\theta\right)$$

$$= r\left[\sin\left(\frac{\pi}{4}\right) \cdot \cos m\theta + \cos\left(\frac{\pi}{4}\right) \cdot \sin(m\theta)\right)$$

$$= r\left[\frac{\cos m\theta}{\sqrt{2}} + \frac{\sin m\theta}{\sqrt{2}}\right)$$

$$= \frac{r}{\sqrt{2}}\left[\cos m\theta + \sin m\theta\right)$$

$$= \frac{r^{m+1}}{\sqrt{2}} \quad \text{cor)} \quad \boxed{a^m \sqrt{2}\, p = r^{m+1}}$$