Partial Differential Equations (PDE)

Formation of PDE by eliminating arbitrary constants and arbitrary functions (5 marks)

1. Form the PDE by eliminating the arbitrary constants
   $z = xy + y\sqrt{x^2 - a^2} + b$.

2. Form the PDE by eliminating the arbitrary constants $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

3. Form the PDE by eliminating the arbitrary functions $z = f(x^2 + y^2)$.

4. Form the PDE by eliminating the arbitrary constants $z = x + y + f(xy)$

5. Find the PDE of the family of all spheres whose centres lie on the plane $z = 0$ and have a constant value 'r'.

6. Form the PDE by eliminating the arbitrary functions $z = e^y f(x + y)$

7. Form the PDE by eliminating the arbitrary functions
   $lx + my + nz = \emptyset(x^2 + y^2 + z^2)$.

Formation of PDE by eliminating arbitrary functions (7 marks)

8. Form the PDE by eliminating the arbitrary functions in the following:
   $$\emptyset(x + y + z, x^2 + y^2 - z^2) = 0.$$

9. Form the PDE by eliminating the arbitrary functions in the following:
   $$\emptyset(xy + z^2, x + y + z) = 0.$$

10. Form the PDE by eliminating the arbitrary functions in the following:
    $$\emptyset(x^2 + y^2 + z^2, z^2 - 2xy) = 0.$$

11. Form the PDE by eliminating the arbitrary functions in the following: $z = f(y + x) + g(y + 2x)$.

Solution of the Lagrange's Linear PDE (5 or 7 marks)

12. Solve : $p\cot x + q\cot y = \cot z$
13. Solve: $x^2 p + y^2 q = z$
14. Solve: $y^2 p - xyq = x(z - 2y)$
15. Solve: $y^2 zp = x^2(zq + y)$
16. Solve: $(y^2 + z^2)p + x(yq - z) = 0$.
17. Solve: $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$.
18. Solve: $(mz - ny)\frac{\partial z}{\partial x} + (nx - lz)\frac{\partial z}{\partial y} + (mx - ly) = 0$.
19. Solve: $(x^2 - y^2 - z^2)p + 2xyq = 2xz$.

Solution of homogeneous PDE involving derivatives with respect to one independent variable only (7 marks)

20. Solve $\dfrac{\partial^2 z}{\partial x^2} + z = 0$ given that when $x = 0, z = e^y$ and $\dfrac{\partial z}{\partial x} = 1$.

21. Solve $\dfrac{\partial^2 z}{\partial x^2} = a^2 z$ given that $\dfrac{\partial z}{\partial x} = a\sin y$ and $z = 0$ when $x = 0$.

22. Solve $\dfrac{\partial^2 z}{\partial x^2} + 3\dfrac{\partial z}{\partial x} - 4z = 0$ given that $z = 1$ and $\dfrac{\partial z}{\partial x} = y$ when $x = 0$.

23. Solve $\dfrac{\partial^3 z}{\partial x^3} + 4\dfrac{\partial z}{\partial x} = 0$ given that $z = 0, \dfrac{\partial z}{\partial x} = 0, \dfrac{\partial^2 z}{\partial x^2} = 4$ when $x = 0$.

24. Solve $\dfrac{\partial^2 z}{\partial y^2} = z$, given that $z = 0$ and $\dfrac{\partial z}{\partial y} = \sin x$ when $y = 0$.