



SCHOOL OF ENGINEERING AND TECHNOLOGY

Hesarghatta Main Road, Bengaluru – 560057

BASIC ELECTRICAL AND ELECTRONICS ENGINEERING

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SEMESTER – 1

COURSE TITLE: BASICS OF ELECTRICAL AND ELECTRONICS ENGINEERING

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MODULE-I

DC Circuits: Basic circuit elements and sources—Ohm's Law and its limitations. KCL & KVL series parallel—series-parallel circuits Simple Numerical.

AC Circuits: Alternating voltages and currents—RMS—average—maximum values—Single Phase RLRC—RLC series circuits—Power in AC circuits—Power Factor.

1. DC Circuits:

1.1 INTRODUCTION:

Electric charge is a fundamental concept within physics, serving as a cornerstone for comprehending the physical world. This article will delve into the intricate nature of the electric charge and its accompanying properties. We will unveil the essence of electric charge through meticulous examination, providing a comprehensive definition and exploring its different types and governing principles. Additionally, our article will encompass a comprehensive overview of the various methodologies employed in the charging process, including friction, conduction, and induction.

Electric Charge Definition

Electric charge can be defined as a fundamental property of subatomic particles that gives rise to the phenomenon of experiencing force in the presence of electric and magnetic fields. These fields exert influence on charged particles, resulting in observable effects.

Types of Electric Charge

Electric charge comes in two main types: **positive and negative charges**. Positive charges are associated with protons, which are subatomic particles residing in the nucleus of an atom. They are represented by the symbol "+". On the other hand, negative charges are linked to electrons, which orbit the atomic nucleus and are denoted by the symbol "-". as shown in figure 1.1.

The distinction between positive and negative charges plays a vital role in comprehending the behaviour of electrically charged objects. Opposite charges, such as positive and negative, attract each other, while like charges, such as positive and positive or negative and negative, repel each other. This fundamental principle is the foundation for various concepts in electromagnetism and is pivotal in understanding the interaction of charged particles. An electric circuit is an interconnection of electrical elements such as resistors, capacitors, inductors, voltage source etc. In electrical engineering, transfer of energy takes place from one point to another, which requires interconnection of electrical devices. Such interconnection is known as electric circuit and each component of the circuit is known as an element. When an object carries a negative charge, it possesses an excess of electrons compared to protons. Conversely, a positive charge indicates an excess of protons relative to electrons.

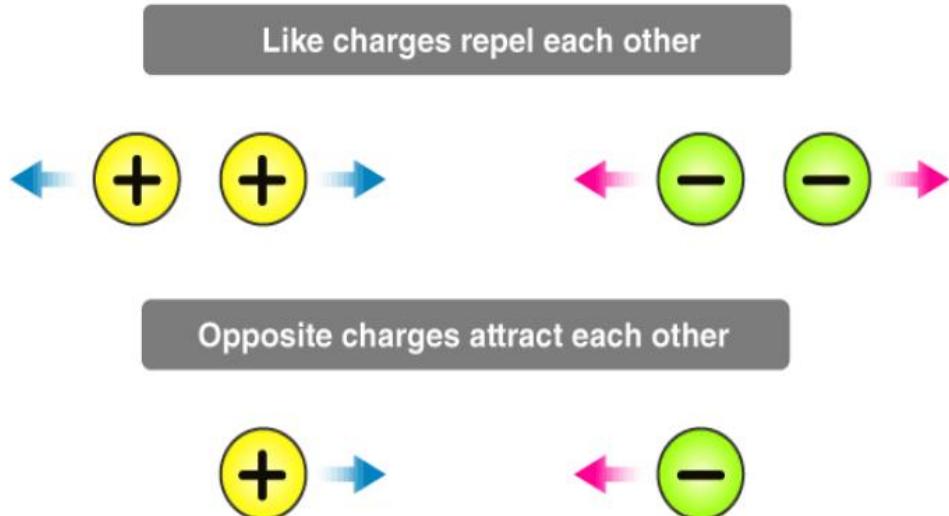


Figure 1.1

Basic circuit elements and sources:

EXAMPLE: Consider an electrical circuit as shown in the figure (1.2). This electric circuit consists of four elements a battery, a lamp, switch & connecting wires. Circuit and network theorem is the study of the behaviour of the circuit: Its behaviour tells us how it responds to a given input how the interconnected elements do and devices in the circuit interact.

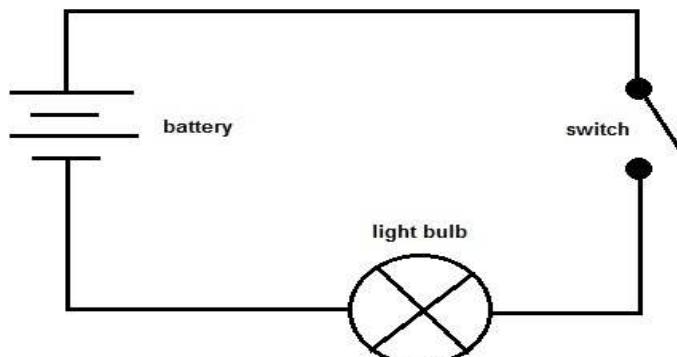


Figure 1.2

CURRENT:

Electric current may be defined as the time rate of net motion of an electric charge across a cross sectional boundary as shown in the figure given below. A random motion of electrons in a metal does not constitute a current unless there is a net transfer of charge with time i.e. electric current.

$$I = \text{Rate of transfer of electric charge}$$

— $dQ/dt = \text{Quantity of electric charge transferred during a given time duration} / \text{Time duration}$

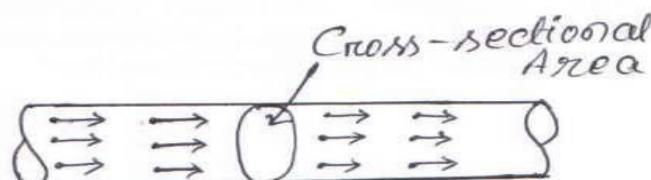


Figure 1.3

Coulomb is the practical as well as SI unit for measurement of electric charge. Since current is the rate of flow of electric charge through conductor and coulomb is the unit of electric charge, the current may be specified in coulombs per second. In practice the ampere is used as the unit of current. Coulomb is the practical as well as SI unit for measurement of electric charge. Since current is the rate of flow of electric charge through conductor and coulomb is the unit of electric charge, the current may be specified in coulombs per second. In practice the ampere is used as the unit of current.

VOLTAGE:

The voltage is the potential difference between two points of a conductor carrying a current of one ampere when the power dissipated between these two points is equal to one watt. The practical unit of voltage is volt.

POWER:

Power is defined as the rate of doing work or rate at which it can perform work. So Power = work done/ Time in seconds.

$$P = \frac{dw}{dt} = \frac{dw}{dq} \frac{dq}{dt} = V I \quad (1)$$

Absolute unit of power is watt. One watt is that power which is required to perform one Joule of work in one second. The practical unit of power is horse power (HP). This value in metric system is 75kg meters per second and in British system is 550 Foot Pounds/second.

Therefore

1 HP (Metric) = 75 Kg meters per second= 735.5 watt

1 HP (British) = 550 Foot Pound/ second = 746 watt

ENERGY:

Energy of a body is its capacity of doing work.

$$E = \int_0^t P dt \quad (2)$$

The unit of energy in MKS system is joule and in SI system is KWH. A system can have this energy in various forms, such as electrical, mechanical, heat, chemical, atomic energy etc. Energy of one form can be transformed to other form, but cannot be created nor be destroyed. If one form of energy disappeared, it reappears in another form. This principle is known as law of conservation of energy.

1.2 CIRCUIT ELEMENTS/PARAMETERS:

a. RESISTANCE:

Resistance restricts the flow $R=V/I$

When an electric current flows through any conductor, heat is generated due to collision of free electrons with atoms. If I amp is the strength of current for potential difference V volts across a conductor, the power observed by resistor is:

$$P=VI=I^2 R \text{ watts}$$

Energy lost in the resistor in form of heat is then

$$E = \int_0^t p \cdot dt = \int_0^t I^2 R \cdot dt = I^2 R t = \frac{V^2}{R} \times t \quad (3)$$

b. INDUCTANCE:

It opposes any change of magnitude or direction of electric current passing through the conductor. Unit is Henry (H). When a current will flow through the coils/Inductor an electromagnetic field is created. However in the event of any change of flow or direction of current, the electromagnetic field also changes. This change of

field induces a voltage (V) across the coil & is given by

$$V = L \cdot \frac{di}{dt} \quad (4)$$

Where 'i' is current through the inductor. Voltage across an inductor is zero when current is constant. Hence an inductor acts like short circuit to dc.

Power absorbed by inductor

$$P=V \times i = L i \frac{di}{dt} \text{ watts.} \quad (5)$$

Energy absorbed.

$$E = \int_0^t p \cdot dt = \frac{1}{2} L i^2 \quad (6)$$

From equations The inductor can store finite amount of energy, even the voltage across it may be nil. A pure inductor does not dissipate energy but can only store it.

c. CAPACITANCE:

It is the property of capacitor, which have the capability to store electric charge in its electric field established by the two polarities of charges on the two electrodes of a capacitor. The amount of charge stored by capacitor is

$$\begin{aligned} q &= CV \\ i &= \frac{dq}{dt} \Rightarrow i = C \frac{dv}{dt} \end{aligned} \quad (7)$$

Therefore if voltage across capacitor is constant, current through it is zero. Hence capacitor acts like an open circuit to dc.

Power absorbed	$P=V \cdot I = VC \frac{dv}{dt}$
Energy stored	$E = \int_0^t p \cdot dt = \frac{1}{2} CV^2$

(8)

A capacitor can store finite amount of energy. Even if the current through it is zero. It never dissipates energy.

1.3 TYPES OF ELEMENTS:

ACTIVE AND PASSIVE ELEMENT:

An active element has capability to generating energy while passive elements have not.

Ex: Active Element: Generators, Batteries, And Amplifiers.

Passive Element: Resistor, Inductor, capacitor.

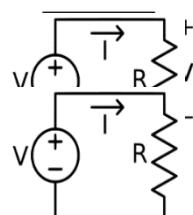
1. Active Elements

The elements that supply energy to the circuit is called active element.

Types of active elements are: **Voltage Source, Current Source.**

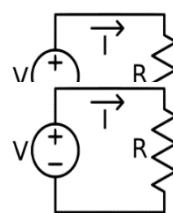
Voltage Source - has a specified voltage across its terminals, independent of current flowing through it.

Ex - Cells, batteries, generators, and other devices that can generate voltage



Current Source - has a specified current through it independent of the voltage appearing across it.

Ex - photoelectric cells, collector currents of transistors.



2. Passive Elements

A passive element is an electrical component that does not generate power, but instead dissipates, stores, and/or releases it.

Ex – Resistor, Capacitor, Inductor

Resistor - If the energy is consumed, then the circuit element is a pure resistor



Capacitor - If the energy is stored in an electric field, the element is a pure capacitor.



BILATERAL AND UNILATERAL ELEMENT:

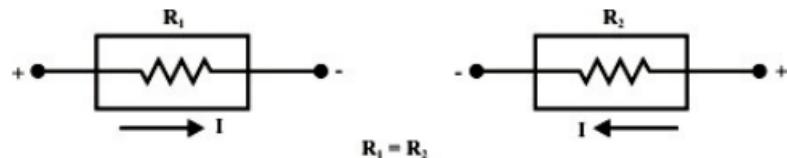


If the magnitude of current passing through the element is affected due to change in the polarity of the applied voltage, the element is called unilateral element. And if the current magnitude remains same, it is called as bilateral element. Ex: Unilateral Element: - Diodes, Transistors.

Bilateral Element: - Resistor, Inductor, Capacitor.

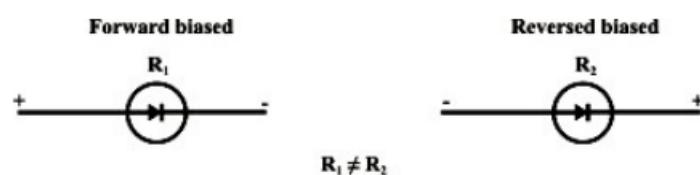
Bilateral - An element is said to be bilateral, when the same relation exists between voltage and current for the current flowing in both directions.

Ex - Voltage source, Current source, resistance, inductance & capacitance.



Unilateral - An element is said to be unilateral, when the same relation does not exist between voltage and current when current flowing in both directions. The circuits containing them are called unilateral circuits.

Ex: Vacuum diodes, Silicon Diodes



LINEAR AND NON-LINEAR ELEMENTS:

A linear element shows linear characteristics of voltage Vs current. Resistors, Inductor, Capacitor are linear elements and their property does not change in applied voltage on circuit current.

For non-linear elements the current passing through it does not change linearly with the time as change in applied voltage at a particular frequency.

Ex: Semiconductor devices.

1.4 ENERGY SOURCES:

Independent Energy sources: The voltage & current sources whose values or strength of voltage and current does not change by any variation in the connected network are called independent sources.

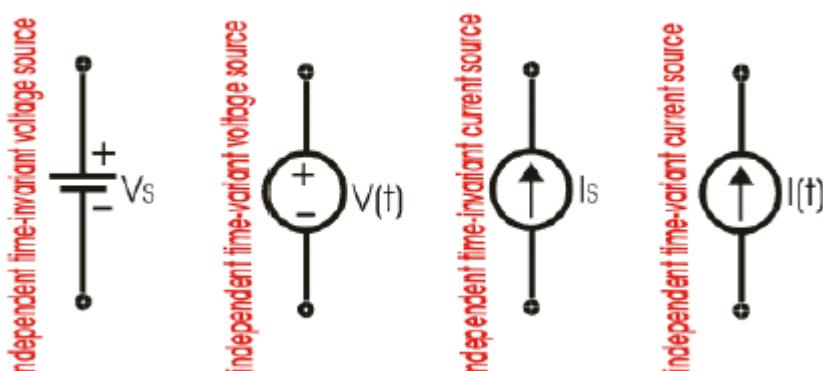


Figure 1.4

Series connected independent sources: Consider the series connection figure (1.4) of two voltage sources as shown in the figure. By KVL the total voltage between the terminals is equal to algebraic sum of individual sources i.e. the voltage sources connected in series may be replaced by a single voltage source whose voltage is equal to the algebraic sum of the individual sources.

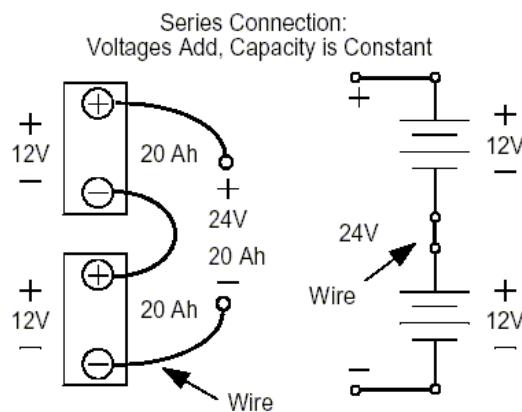


Figure 1.5

Dependent Energy sources: When the strength of voltage and current changes in the sources for any change in the connected network, they are called dependent sources. There four different types of dependent sources refer figure (1.5):

- Voltage controlled voltage source (VCVS)
- Voltage controlled current source (VCCS)
- Current controlled voltage source (CCVS)
- Current controlled current source (CCCS)

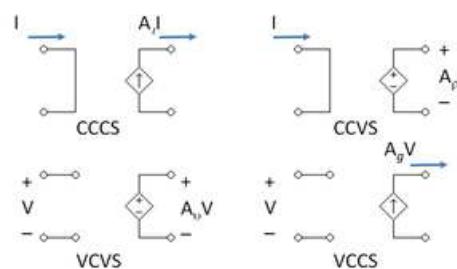


Figure 1.6

SOURCE TRANSFORMATION:

The voltage and current sources are mutually transferable as shown in the figure below.



1.5 OHM'S LAW AND ITS LIMITATIONS

As per Ohm's law, the voltage or potential distinction between the two locations is proportional to the current of electricity moving via the resistance, and the resistance on the circuit is proportional to the current or electricity moving via the resistance. $V=IR$ is the deemed formula used for Ohm's

law Establishing the constant of proportionality, namely the resistance, one enters the general mathematical equation that illustrates this association:

$$I = V/R$$

Over here, I denote the current throughout the conductor. Its units are in terms of amperes.

V denotes the voltage computed across the conductor measured in units of volts, and R denotes the resistance of the conductor measured in units of ohms.

More particularly, Ohm's law determines that the R in this connection is constant, self-sufficient of the current. If there is no constant resistance, the preceding equation will not be called Ohm's law; however, it could still be employed as an explanation of static/DC resistance. Ohm's law is an experiential association that precisely explains the conductivity of the huge bulk of electrically conductive substances over numerous orders of the degree of current. Though a few materials do not comply with Ohm's law, these are known as non-ohmic.

The law was given the name by the German physicist Georg Ohm, who, in a thesis circulated in 1827, explained the dimensions of applied voltage and current via simple electrical circuits comprising different lengths of wire. Ohm clarified his experimental outcomes by a faintly more intricate equation compared to the contemporary form above.

When talked about in physics, the word Ohm's law is as well utilized to designate numerous simplifications of the law; for instance, the vector form of the law employed in material science and electromagnetics:

$$J = \sigma E$$

Over here, J denotes the current density on a particular position in a resistive material, E denotes the electric field on that position, and σ (sigma) denotes a material-dependent factor known as the conductivity. Such reformulation of Ohm's law is because of Gustav Kirchhoff.

Ohm's Law Graphical representation:

Ohm's Law holds good when physical conditions like temperature and others are constant. This is because of the fact that the current flowing through the circuit varies by changing the temperature. Therefore, in such cases when physical factors like temperature come into play, Ohm's law violates. For example, in the case of a Light bulb, where temperature increases when the current flowing through it rises. Here, Ohm's Law doesn't follow. The graph for an ohmic circuit is discussed in the image below,

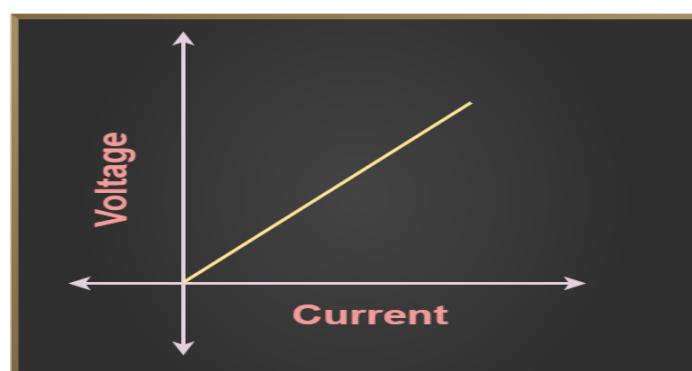


Figure 1.7

Ohm's Law Graph:

1. Plotting a graph taking current I along Y-axis and voltage V along X-axis

By Ohm's law, $V=RI$ (1)

This equation is of the form $y=mx+c$ (2)

which is a straight line equation, where m is slope and c is constant.

2. By comparing equation (1) and equation (2), we get to know

For Y-axis, $y=V$ potential difference

For X-axis, $x=I$ current

The slope, $m=R$ resistance

Hence, Ohm's law graph is a straight line.

Ohm's law limitations:

- The law of Ohm is not applicable to unilateral networks. The current could move in one course in unilateral networks. Transistors, Diodes, and extra electronic components are employed in these kinds of networks.
- Non-linear components are moreover off the hook from Ohm's law. They contain a current which is not proportional to the functional voltage, which connotes that the resistance rate of those components alters relying on the voltage and current. The thyristor is an illustration of this non-linear element.
- Ohm's law is scrutinized on a broad array of length scales. In the initial 20th century, it was considered that Ohm's law would not succeed at the atomic range; however, experiments have not abided out this anticipation. Since 2012, researchers have shown that Ohm's law operates for silicon wires equal to the size of four atoms wide and one atom high.
- Ohm's law is just pertinent in the case of metallic conductors. Consequently, it won't operate in the context of non-metallic conductors.

Ohm's law is not applicable in the following cases:

Case-1

The diode is considered to be a non-ohmic power apparatus. This designates that the current moves by the diode do not boost the linear proportion of the augmented voltage. The voltage throughout the exhaustion layer gets constant on raising the applied voltage. Consequently, no additional boost of the voltage through the PN joint is feasible. Nevertheless, the current throughout the diode boosts with a rise in voltage.

Therefore, it is understandable that the diode current does not augment linearly on a rise of applied voltage. Consequently, in this case, Ohm's law is not valid.

Case-2

When we talk about a luminous lamp, the current throughout the lamp does not augment with a rise in voltage. Why does this occur? The filament resistance augments with the augment in

temperature, and therefore the filament lamp contains non-linear traits. In this, Ohm's law is not appropriate.

1.6 KIRCHHOFF'S LAW:

These laws are more comprehensive than Ohm's law and are used for solving electrical networks which may not be readily solved by latter. Kirchhoff's law is of two types, Kirchhoff's current law and Kirchhoff's voltage law. Kirchhoff's current law is used when voltage is chosen as variable while Kirchhoff's voltage law is used when current is chosen as variable.

KIRCHHOFF'S CURRENT LAW:

According to Kirchhoff's current law the algebraic sum of currents at any node of a circuit is zero. From the figure given below:

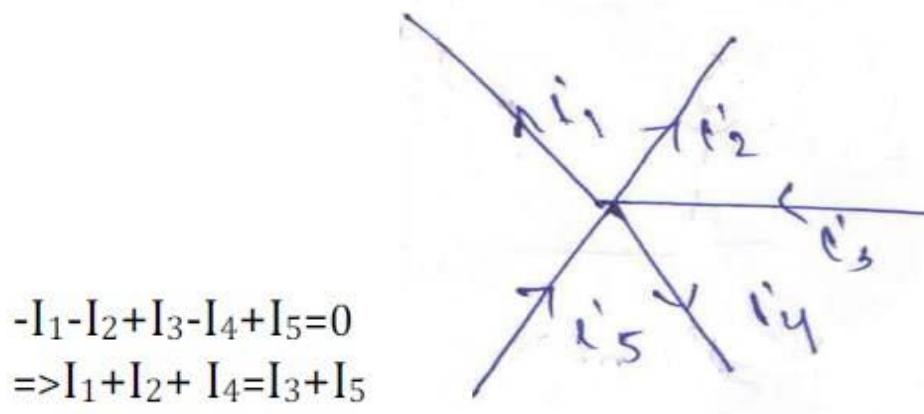


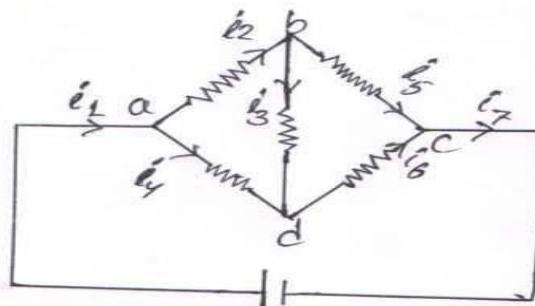
Figure 1.8

Numerical:

Example 1: Find the magnitude and direction of the unknown current as shown in figure given $I_1 = 10\text{ A}$, $I_2 = 6\text{ A}$, $I_5 = 4\text{ A}$

Solution: Assume direction of current in the network

- (i) $I_1 = I_7 = 10\text{ A}$
- (ii) $I_1 = I_2 + I_4 \Rightarrow I_4 = I_1 - I_2 = 10 - 6 = 4\text{ A}$
- (iii) At node b: $I_2 - I_3 - I_5 = 0$
 $\Rightarrow 6 - I_3 - 4 = 0 \Rightarrow I_3 = 2\text{ A}$
- (iv) At node d: $I_4 + I_3 - I_6 = 0$
 $\Rightarrow 4 + 2 - I_6 = 0$
 $\Rightarrow I_6 = 6\text{ A}$



Assume direction of all current are correct because of their positive magnitude. Assume directions of unknown current are arbitrary and any direction can be taken.

Example 2: Find v and the magnitude and direction of the unknown currents in the branch xn, yn and zn as shown in figure.

$$\text{At node } y: 10 + I_x + I_z = I_y + 2$$

$$I_x - I_y + I_z = -8$$

$$\frac{V}{5} + \frac{V}{2} + \frac{V}{4} = -8 \quad [\text{since } I_x = \frac{V}{5}, I_y = -\frac{V}{5}, I_z = \frac{V}{5}]$$

$$V = -8.42 \text{ volt}$$

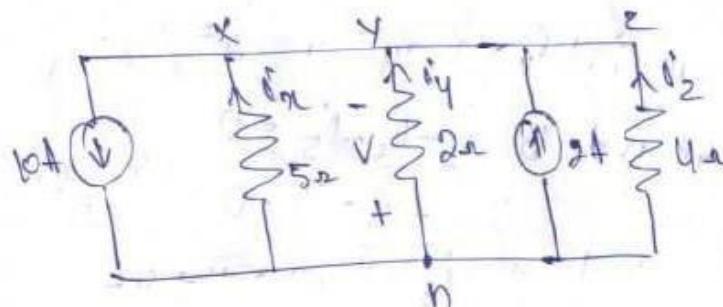
Negative magnitude shows that n to be positive.

$$\text{Therefore } I_x = -\frac{8.42}{5} = -1.684 \text{ A (i.e. from flowing current n to x)}$$

$$I_y = -\frac{(-8.42)}{2} = 4.21 \text{ A (ie Current flowing from n to y)}$$

$$I_z = \frac{-8.42}{4} = -2.1 \text{ A (ie current flowing from n to z)}$$

The circuit can be redrawn as given below

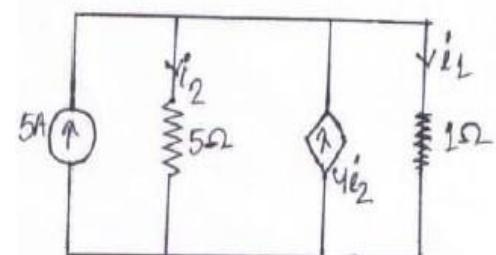


Example 3: Find i_1 and i_2 as shown in figure

Solution: The circuit is redrawn in figure

$$\text{According to KCL: } i_1 + i_2 = 5 + 4i_2 \quad \dots \dots \dots (1)$$

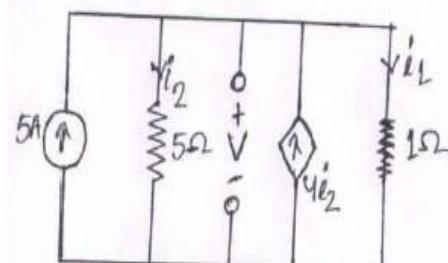
$$i_1 - 3i_2 = 5 \quad \dots \dots \dots (2)$$



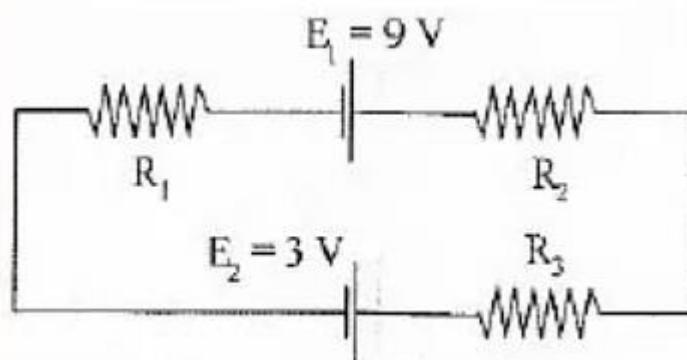
$$\text{Here } i_1 = \frac{V}{1}; i_2 = \frac{V}{5}$$

$$\text{Therefore equation 2: } V - 3\frac{V}{5} = 5 \\ \Rightarrow V = 12.5 \text{ volt}$$

$$\text{Therefore } i_1 = 12.5 \text{ A and } i_2 = 2.5 \text{ A}$$



Example 4: If $R_1 = 2\Omega$, $R_2 = 4\Omega$, $R_3 = 6\Omega$, determine the electric current flows in the circuit below.



Resistor 1 (R_1) = 2Ω ; Resistor 2 (R_2) = 4Ω ; Resistor 3 (R_3) = 6Ω ; Source of emf 1 (E_1) = 9V; Source of emf 2 (E_2) = 3V

From Kirchhoff's law, we arrive at the equation,

$$-I R_1 + E_1 - I R_2 - I R_3 - E_2 = 0$$

$$-2I + 9 - 4I - 6I - 3 = 0$$

$$-12I + 6 = 0$$

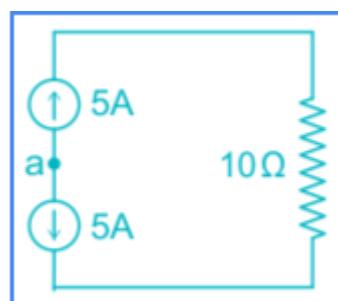
$$-12I = -6$$

$$I = -6 / -12$$

$$I = 0.5$$

Thus current flowing through the circuit is 0.5 A.

Example 5 Find the current in the given circuit.



At a node only one value of current is possible so, here violation of KCL so, no current is possible in the circuit. Consider a circuit with three resistors connected in parallel. The currents flowing through the resistors are I_1 , I_2 , and I_3 . The values of the resistors are $R_1 = 3\Omega$, $R_2 = 5\Omega$, and $R_3 = 2\Omega$. Find the total current flowing into the circuit.

Let's assume the currents as follows:

$$I_1 = 2A, I_2 = 2A$$

$$I_2 = 3A, I_2 = 3A$$

$$I_3 = 1A, I_3 = 1A$$

$$\text{Total current} = I_1 + I_2 + I_3 = 1 + 2 + 3 = 6A$$

Substituting the given values:

Total current = $2A + 3A + 1A$

Total current = $6A$

Example 6: Consider a simple series circuit consisting of a $10V$ battery connected to two resistors R_1 and R_2 . The values of the resistors are $R_1 = 5$ ohms and $R_2 = 3$ ohms. Find the voltage drop across each resistor.

Let's assume the voltage drops across the resistors as follows:

V_1 = Voltage drop across $R_1 R_1$

V_2 = Voltage drop across $R_2 R_2$

Voltage of the battery = Voltage drop across $R_1 R_1$ + Voltage drop across $R_2 R_2$

$$10V = V_1 + V_2 \quad 10V = V_1 + V_2$$

Now, we can apply Ohm's law to find the voltage drops across each resistor:

$$V_1 = I * R_1 \quad V_1 = I * R_1$$

$$V_2 = I * R_2 \quad V_2 = I * R_2$$

Let's assume the current flowing through the circuit is I .

Substituting the values in the equation:

$$10V = (I * R_1) + (I * R_2)$$

$$10V = I * (R_1 + R_2) \quad 10V = I * (R_1 + R_2)$$

$$10V = I * (5 \text{ ohms} + 3 \text{ ohms})$$

$$10V = I * 8 \text{ ohms}$$

Now, we can solve for the current:

$$I = 10V / 8 \text{ ohms}$$

$$I = 1.25A$$

Substituting the value of current back into the voltage drop equations:

$$V_1 = (1.25A) * (5 \text{ ohms}) \quad V_1 = (1.25A) * (5 \text{ ohms})$$

$$V_1 = 6.25V \quad V_1 = 6.25V$$

$$V_2 = (1.25A) * (3 \text{ ohms}) \quad V_2 = (1.25A) * (3 \text{ ohms})$$

$$V_2 = 3.75V$$

NOTE: Steps for KCL

Kirchhoff's Current Law (KCL)

• Kirchhoff's Current Law (KCL)

• The algebraic sum of currents entering any node (junction) is zero

$$\sum_{j=1}^N I_j = 0$$

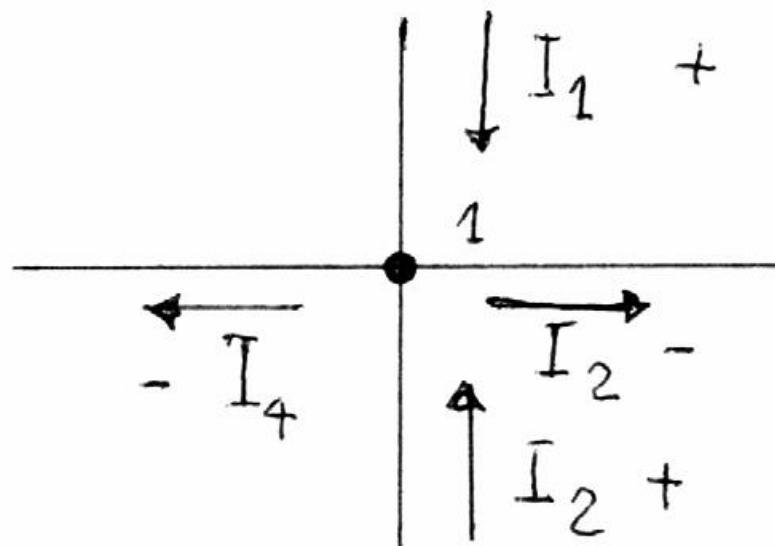
Where N = number of lines entering the node

• NOTE: the sign convention:

• Currents are positive when they enter the node

• Currents negative when leaving

• Or the reverse of this.



KCL is called a Continuity Equation: It says current is not created or destroyed at any node

KIRCHHOFF'S VOLTAGE LAW:

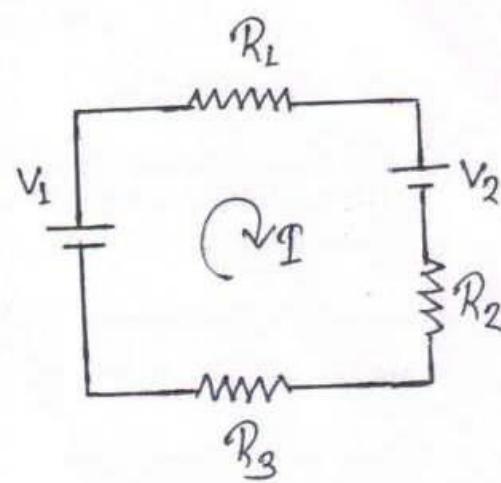
This law can be stated as "The algebraic sum of voltage in any closed path of a network that is traversed in single direction is zero."

Explanation: According to KVL

$$V_1 - IR_1 - V_2 - IR_2 - IR_3 = 0$$

$$IR_1 + IR_2 + IR_3 = V_1 - V_2$$

$$I = V_1 - V_2 / R_1 + R_2 + R_3$$



CURRENT DIVISION RULE:

Two resistors are joined in parallel across a voltage V . The current in each branch, as given in ohm's law is

$$I_1 = V/R_1 \text{ and } I_2 = V/R_2$$

$$\text{Therefore } I_1/I_2 = R_2/R_1 = G_1/G_2$$

Hence the division of current in the branch of parallel circuit is directly proportional to the conductance of the branches or inversely proportional to their resistances. We may also express the branch currents in terms of the total circuit current thus:

Now $I_1 + I_2 = I$

$$\Rightarrow I_2 = I - I_1$$

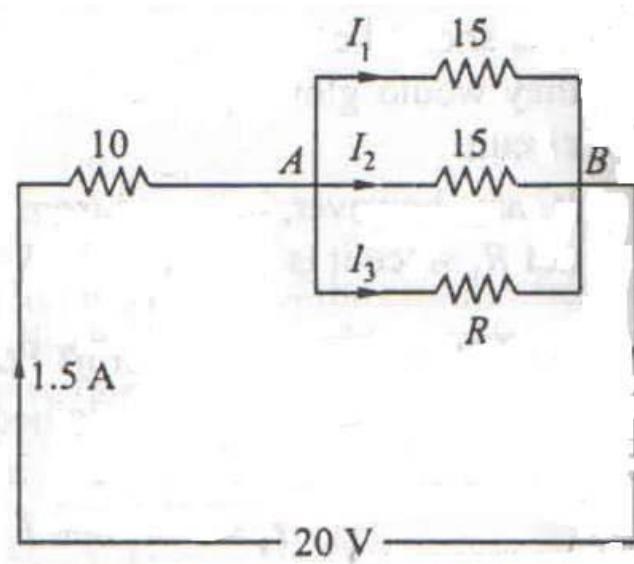
Therefore: $I_1/I - I_1 = R_2/R_1$ or $I_1R_1 = R_2(I - I_1)$

$$I_1 = I R_2 / (R_1 + R_2)$$

$$I_2 = I R_1 / (R_1 + R_2)$$

Thus current division rule is stated as "The current in any of the parallel branches is equal to the ratio of the opposite branch resistance to the total resistance, multiplied by the total current."

Example 1 A resistance of 10 ohm is connected in series with two resistances each of 15 ohm arranged in parallel. What resistance must be shunted across this parallel combination so that the total current taken shall be 1.5 A with 20 volt applied?



The circuit connected in figure

$$\text{Drop across } 10 \text{ ohm resister} = 1.5 * 10 = 15 \text{ V}$$

$$\text{Drop across parallel combination, } V_{AB} = 20 - 15 = 5 \text{ V}$$

Hence voltage across each parallel resistance is 5V.

across each parallel resistance is 5V.

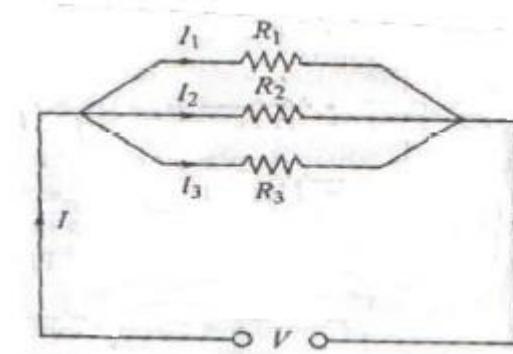
$$I_1 = 5/15 = 1/3 \text{ A}$$

$$I_2 = 5/15 = 1/3 \text{ A}$$

$$I_3 = 1.5 - (1/3 + 1/3) = 5/6 \text{ A}$$

$$\text{Therefore } I_3 R = 5 \text{ or } (5/6) R = 5 \text{ or } R = 6 \text{ ohm}$$

Example 2: Calculate the value of different current for the circuit shown in given figure.



Solution: Total current $I = I_1 + I_2 + I_3$

Let the equivalent resistance be R .

Then $V = I R$

Also $V = I_1 R_1$

Therefore $I R = I_1 R_1$

$$\text{Or } I_1 = I R / R_1 \quad \dots \dots \dots (1)$$

$$\text{Now } (1/R) = (1/R_1) + (1/R_2) + (1/R_3)$$

From equation 1

$$I_1 = R_2 R_3 / (R_1 R_2 + R_2 R_3 + R_3 R_1)$$

$$I_2 = R_1 R_3 / (R_1 R_2 + R_2 R_3 + R_3 R_1)$$

$$I_3 = R_1 R_2 / (R_1 R_2 + R_2 R_3 + R_3 R_1)$$

1.7 VOLTAGE DIVISION RULE:

A voltage divider circuit is a series network which is used to feed other networks with a number of different voltages and is derived from a single input voltage source. Figure shows a simple voltage divider circuit which provide two output voltages V_1 and V_2 . Since no load is connected across the output terminals, it is called an unloaded voltage divider. We may also express the branch voltages in terms of the total circuit voltage thus:

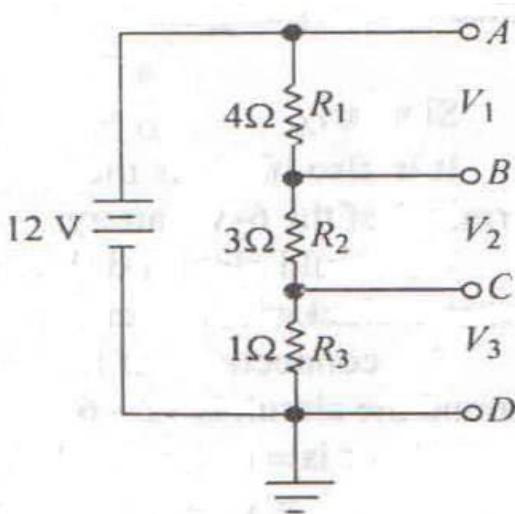
$$\text{Now } V_1 + V_2 = V$$

$$\Rightarrow V_2 = V - V_1$$

$$\text{Therefore } V_1 / V - V_1 = R_1 / R_2 \text{ or } V_1 R_2 = R_1 (V - V_1)$$

$$\text{Therefore } V_1 = V R_1 / (R_1 + R_2) \text{ and } V_2 = V R_2 / (R_1 + R_2)$$

Thus Voltage division rule is stated as “The voltage across a resistor in series circuit is equal to the value of that resistor times the total impressed voltage across the series elements divided by the total resistance of the series elements.”



Example 1: Find the value of different voltages that can be obtained from a 12 V battery with the help of voltage divider circuit of figure.

Solution:

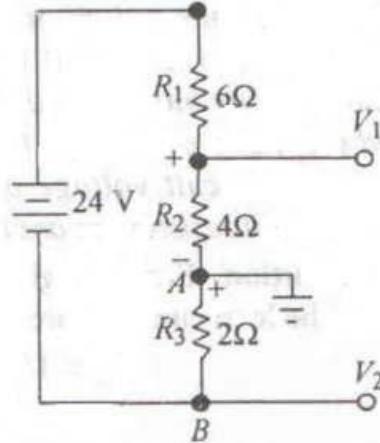
$$R = R_1 + R_2 + R_3 = 4 + 3 + 1 = 8 \text{ ohm}$$

$$\text{Drop across } R_1 = V_{R1} = 12 \text{ V} \times \frac{4}{8} = 6 \text{ volt}$$

$$\text{Drop across } R_2 = V_{R2} = 12 \text{ V} \times \frac{3}{8} = 4.5 \text{ volt}$$

$$\text{Drop across } R_3 = V_{R3} = 12 \text{ V} \times \frac{1}{8} = 1.5 \text{ volt}$$

Example 2: What are the output voltages of the unloaded voltage divider shown in figure what is the direction of current Through AB?



Solution:

It may be remember that both V1 and V2 are with respect to ground.

$$R = 6 + 4 + 2 = 12 \text{ ohm}$$

Therefore

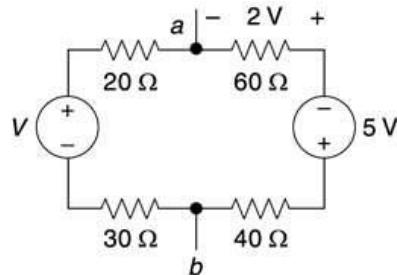
$$V_1 = \text{Drop across } R_2 = 24 \text{ V} \times \frac{4}{12} = 8 \text{ volt}$$

$$V_2 = \text{Drop across } R_3 = -24 \text{ V} \times \frac{2}{12} = -4 \text{ volt}$$

It should be noted that point B is negative potential with respect to the ground. Current flows from A to b i.e. from a point at a higher potential to a point at a lower potential.

Example 3

Find the values of V , V_{ab} and the power delivered by the 5V source. All values of resistances are in ohm.



Solution

$$\text{Current, } i = \frac{2}{60} = \frac{1}{30} \text{ A}$$

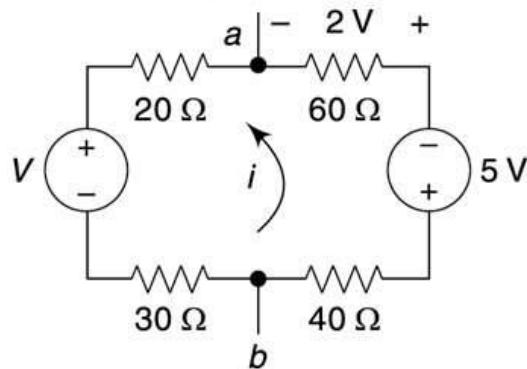
By KVL,

$$20i + 2 + 5 + v + 70i = 0$$

$$v = -7 - 90i = -7 - 90 \times \frac{1}{30} = -10 \text{ V}$$

$$\therefore v_{ab} = 20i + v + 30i = 50i - 10$$

$$= 50 \times \frac{1}{30} - 10 = -8.33 \text{ V}$$



$$\text{Power drawn by the 5V source} = -(\text{Power taken source}) = -5 \times \frac{1}{30} = -0.166 \text{ W}$$

Example 5 for the given circuit find the branch currents and voltages by applying KVL.

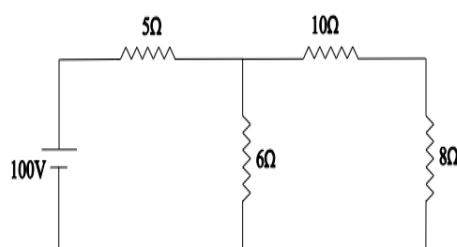


Figure 1.70

Solution:

Consider loop ABEF & Apply KVL in CLK wise direction

$$100 - 5I_1 - 6I_2 = 0$$

$$\text{But } I = I_1 + I_2$$

$$100 - 5(I_1 + I_2) - 6I_1 = 0$$

$$100 - 5I_1 - 5I_2 - 6I_1 = 0$$

$$-11I_1 - 5I_2 + 100 = 0 \quad 11I_1 + 5I_2 = 100 \quad (1)$$

Consider loop BCDEB & Apply KVL in CLK wise direction

$$-10I_2 - 8I_2 + 6I_1 = 0$$

$$-18I_2 + 6I_1 = 0$$

$$6I_1 = 18I_2$$

$$I_1 = 3I_2 \quad (2)$$

Sub I₁ in equ (1)

$$11(3I_2) + 5I_2 = 100$$

$$33I_2 + 5I_2 = 100$$

$$38I_2 = 100$$

$$I = 100/38 = 2.63 \text{ Amps.}$$

$$I_2 = 2.63 \text{ Amps}$$

Sub I₂ in equ (2)

$$I_1 = 3(2.63) = 7.89 \text{ Amps}$$

$$I_1 = 7.89 \text{ Amps}$$

$$I = I_1 + I_2 = 10.52$$

$$I = 10.52 \text{ Amps.}$$

$$\text{Voltage Across } 5\Delta = 5 \times I = 5 \times 10.52$$

$$= 52.6 \text{ volts}$$

$$\text{Voltage Across } 6\Delta = 6 \times I_1 = 6 \times 7.89$$

$$= 47.34 \text{ volts}$$

$$\text{Voltage Across } 10\Delta = 10 \times I_2 = 10 \times 2.63$$

$$= 26.3 \text{ volts}$$

$$\text{Voltage Across } 8\Delta = 8 \times I_2 = 8 \times 2.63$$

$$= 21.04 \text{ volts}$$

(Or)

The above problem can be solved by applying KVL in Anti clock wise directions.

Consider loop ABEF & Apply KVL in anti-clock wise direction

$$6I_1 + 5I - 100 = 0$$

$$\text{But } I = I_1 + I_2$$

$$6I_1 + 5(I_1 + I_2) - 100 = 0$$

$$6I_1 + 5I_1 + 5I_2 = 100$$

$$11I_1 + 5I_2 = 100 \quad (3)$$

Consider loop BCDEB & Apply KVL in anti-clockwise direction

$$8I_2 + 10I_2 - 6I_1 = 0$$

$$18I_2 = 6I_1$$

$$I_1 = 3I_2 \quad (4)$$

equations (3) & (1) are identical

equations (2) & (4) are identical

Hence, we get the same answer irrespective of directions of applying KVL.

Example 6: Calculate the branch current in $15\ \Omega$ resistor by Applying Kirchhoff's

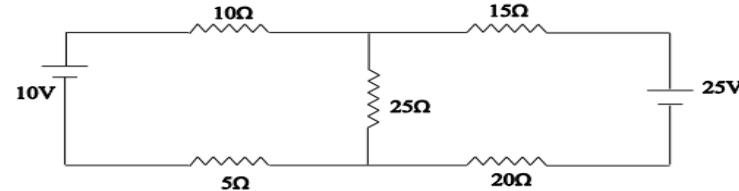


Figure 1.72

Figure 72 battery voltage value 25 volt missing

Solution:

Name the loop and Mark the current directions

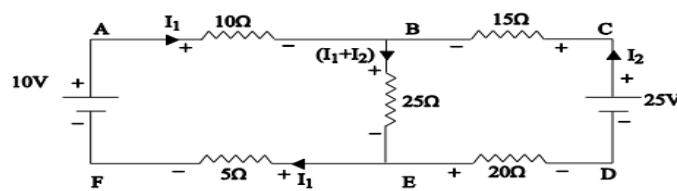


Figure 1.73

Consider the loop ABEFA & apply KVL in CLK wise

$$\begin{aligned} 10 - 10I_1 - 25(I_1 + I_2) &= 5I_1 = 0 \\ 10 - 10I_1 - 25I_1 - 25I_2 - 5I_1 &= 0 \\ -40I_1 - 25I_2 + 10 &= 0 \\ 40I_1 + 25I_2 &= 10 \end{aligned} \quad (1)$$

Cons

Consider the loop BCDEB and Apply KVL in CLK wise direction

$$15I_2 - 25 + 20I_2 + 25(I_1 + I_2) = 0$$

$$15I_2 - 25 + 20I_2 + 25(I_1 + I_2) = 0$$

$$15I_2 - 25 + 20I_2 + 25I_1 + 25I_2 = 0$$

$$25I_1 + 60I_2 - 25 = 0$$

$$25I_1 + 60I_2 = 25 \quad \dots \dots \dots (2)$$

Solve (1) & (2) & find I_2 alone

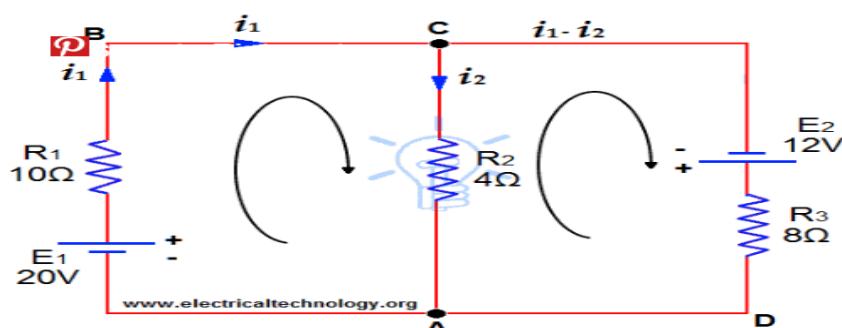
$$(1) \times 25 \Rightarrow 1000L_1 + 625L_2 \leq 25$$

$$(2) \times 40 \Rightarrow 1000I_1 + 2400I_2$$

$$(A) - (B) \Rightarrow -1775 I_1 = -750$$

$I_2 = 0.42$ Amps.

Example 7: Resistors of $R_1 = 10\Omega$, $R_2 = 4\Omega$ and $R_3 = 8\Omega$ are connected up to two batteries (of negligible resistance) as shown. Find the current through each resistor.



Assume currents to flow in directions indicated by arrows.

Apply KCL on Junctions C and A.

Therefore, current in mesh ABC = i_1

Current in Mesh CA = i_2

Then current in Mesh CDA = $i_1 - i_2$

Now, Apply KVL on Mesh ABC, 20V are acting in clockwise direction. Equating the sum of IR products, we get;

$$10i_1 + 4i_2 = 20 \dots (1)$$

In mesh ACD, 12 volts are acting in clockwise direction, then:

$$8(i_1 - i_2) - 4i_2 = 12$$

$$8i_1 - 8i_2 - 4i_2 = 12$$

$$8i_1 - 12i_2 = 12 \dots (2)$$

Multiplying equation (1) by 3;

$$30i_1 + 12i_2 = 60$$

Solving for i_1

$$30i_1 + 12i_2 = 60$$

$$8i_1 - 12i_2 = 12$$

$$\underline{38i_1 = 72}$$

The above equation can be also simplified by Elimination or Cramer's Rule.

$$i_1 = 72 \div 38 = \mathbf{1.895 \text{ Amperes}} = \text{Current in } 10 \text{ Ohms resistor}$$

Substituting this value in (1), we get:

$$10(1.895) + 4i_2 = 20$$

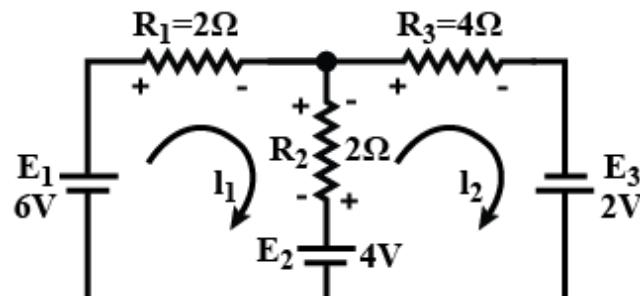
$$4i_2 = 20 - 18.95$$

$$i_2 = \mathbf{0.263 \text{ Amperes}} = \text{Current in } 4 \text{ Ohms Resistors.}$$

Now,

$$i_1 - i_2 = 1.895 - 0.263 = \mathbf{1.632 \text{ Amperes}}$$

Example 8 Using KVL and KCL find the branch currents in the given circuit.



Solution

$$\text{Mesh 1: } -2i_1 + 2(i_1 + i_2) = 6v$$

$$\Rightarrow 4i_1 - 2i_2 = 6v$$

$$\Rightarrow 2i_1 - i_2 = 3 \rightarrow (1)$$

$$\text{Mesh 2: } -4i_2 + 2(i_1 + i_2) = 2v$$

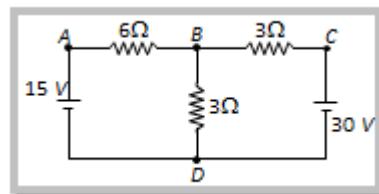
$$\Rightarrow 6i_2 - 2i_1 = 2 \rightarrow (2)$$

Equating (2) and (1)

$$i_2 = 1\text{A}$$

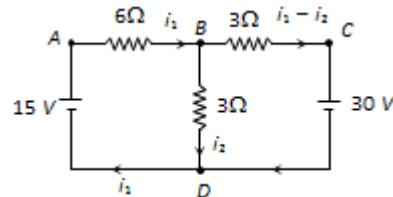
$$i_1 = 2\text{A}$$

Example 9 In the circuit shown in figure, find the current through the branch BD



Solution

The correct option is A 5 A



The current in the circuit are assumed as shown in the fig.

Applying KVL along the loop ABDA, we get

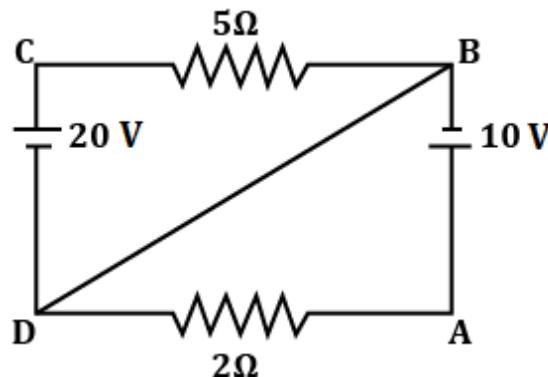
$$-6i_1 - 3i_2 + 15 = 0 \text{ or } 2i_1 + i_2 = 5 \dots\dots\dots (i)$$

Applying KVL along the loop BCDB, we get

$$-3(i_1 - i_2) - 30 + 3i_2 = 0 \text{ or } -i_1 + 2i_2 = 10 \dots\dots\dots (ii)$$

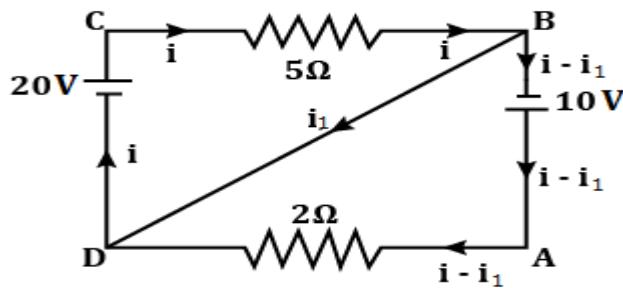
Solving equation (i) and (ii) for i_2 , we get $i_2 = 5\text{A}$

Example 10 For the circuit shown in the figure. Find out the current in the wire BD.



Solution

The correct option is B 1 A from D to B Step 1: Assume some current in branches taking KCL in to account at the junction



Step 2: Apply KVL in closed loops,

In loop DCBD,

$$\Rightarrow 20 - 5i = 0$$

$$i = 4 \text{ A} \dots (1)$$

In loop BADB,

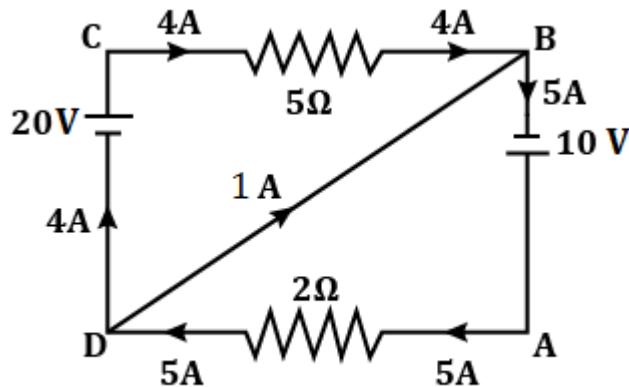
$$+10 - 2(i - i_1) = 0 \dots (2)$$

Using Eq.(1) and (2),

$$10 = 2(4 - i_1)$$

$$i_1 = -1 \text{ A}$$

The diagram showing final distribution of current in branches

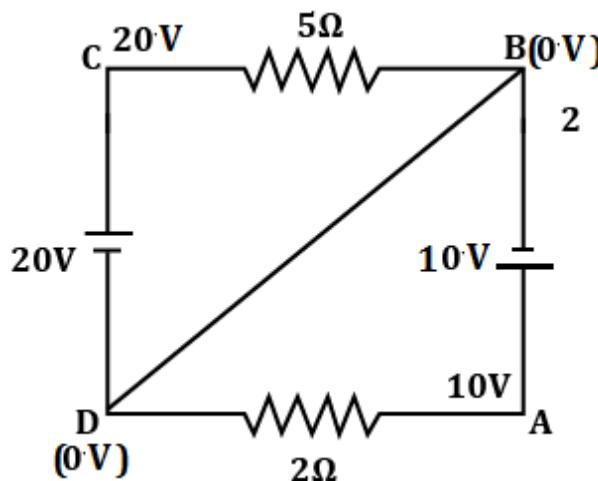


The negative sign of current indicates that its direction will be in the direction opposite to that assumed initially. $\therefore 1 \text{ A}$ current flows from D to B

Method 2

Step 1: Let's assume potential at point B to be zero. Now the points B and D are connected by same conducting wire, hence both will have same potential i.e 0 V.

Step 2: Mark the potential at other points

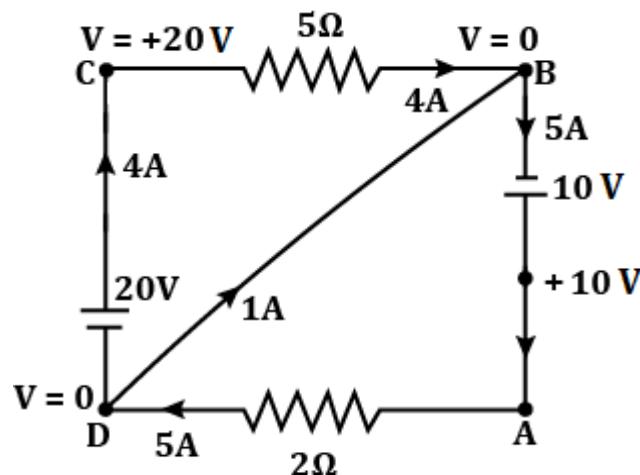


Step 3 : Find the current through each resistance by applying ohm's law

$i = \text{Potential difference} / \text{Resistance}$

$$\Rightarrow i_{CB} = 20 - 0 / 5 = 4 \text{ A} \text{ and } i_{AD} = 10 - 0 / 2 = 5 \text{ A}$$

Step 4 : Apply KCL and obtain required currents in the branches.



Thus in branch BD, 1 A current flows from D to B

Summary of KCL and KVL

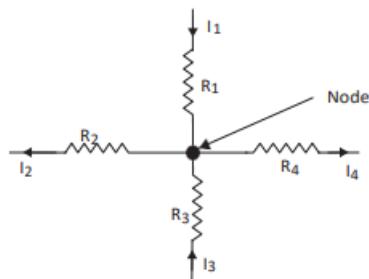
Kirchhoff's Laws

Kirchhoff's laws are more comprehensive than Ohm's law and are used for solving electrical networks which may not be readily solved by the latter.

Kirchhoff's laws, two in number, are particularly useful in determining the equivalent resistance of a complicated network of conductors and for calculating the currents flowing in the various conductors.

1. Kirchhoff's Current Law (KCL)

1. The kirchoff's current law states that the algebraic sum of currents in a node is zero.
 2. It can also be stated that "sum of incoming currents is equal to sum of outgoing currents." Kirchhoff's current law is applied at nodes of the circuit.
 3. A node is defined as two or more electrical elements joined together. The electrical elements may be resistors, inductors capacitors, voltage sources, current sources etc.
 4. Consider an electrical network as shown below.



Four resistors are joined together to form a node. Each resistor carries different currents and they are indicated in the diagram.

$I_1 \rightarrow$ Flows towards the node and it is considered as positive current. (+ I_1)

$I_2 \rightarrow$ Flows away from the node and it is considered as negative current. (- I_2)

$I_3 \rightarrow$ Flows towards the node and it is considered as positive current. ($+I_3$)

$I_4 \rightarrow$ Flows away from the node and hence it is considered as negative current ($-I_4$)

Applying KCL at the node, by definition-1 algebraic sum of currents in a node is zero

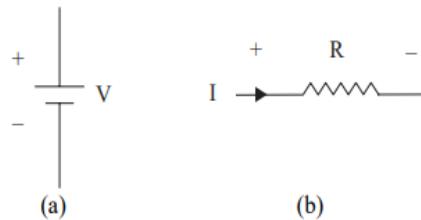
taking the I2 & I4 to other side

From equation (2) we get the definition – 2. Where I_1 & I_3 are positive currents (Flowing towards the node) I_2 & I_4 are negative currents. (Flowing away from the node).

2. Kirchoff's voltage Law: (KVL)

1. Kirchhoff's voltage law states that "sum of the voltages in a closed path (loop) is zero".
 2. In electric circuit there will be closed path called as loops will be present.
 3. The KVL is applied to the closed path only the loop will consists of voltage sources, resistors, inductors etc.
 4. In the loop there will be voltage rise and voltage drop. This voltage rise and voltage drop depends on the direction traced in the loop. So it is important to understand the sign convention and the direction in which KVL is applied (Clock wise Anti clock wise).

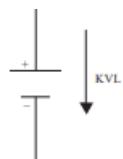
Sign Conventions



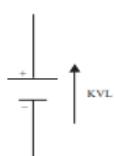
Consider a battery source V as shown in the Figure (a). Here positive of the battery is marked with + sign and negative of the battery is marked with – sign.

When we move from + sign to – sign, it is called voltage drop.

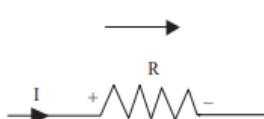
When we move from – sign to + sign, it is called as voltage rise.



When KVL is applied in Anti clockwise direction as shown above it is called as voltage drop. A voltage drop is indicated in a loop with “—” sign ($-V$).

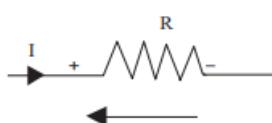


For the same battery source if the KVL is applied in clock wise direction we move from – sign to + sign. Hence it is called as Voltage Rise. A Voltage rise indicated in the loop with + sign. ($+V$).



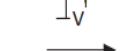
Similarly in the resistor the current entry point is marked as positive (+ sign) and current leaving point is marked as negative sign. (- sign).

For the resistor shown in the diagram above, if KVL is applied in clock wise direction then it is called as voltage drop. Voltage drop in KVL equation must be indicated with negative sign (-). $\therefore -IR$.



For the resistor shown in the diagram above, if KVL is applied in anti-clockwise direction then it is called as voltage rise. A voltage rise is indicated in the KVL equation as positive. i.e. $+ IR$.

In short, the above explanation is summarized below in a Table.

S.No.	Element	KVL in clockwise	KVL in anticlockwise
1.			
2.			

Procedure for KVL:

- Identify the loops and name them.
- Mark the branch currents and name them.
- Apply the sign convention.
- Select a loop & apply KVL either in clockwise or anticlockwise and frame the equation.
- Solve all the equations of the loop.

1.8 SERIES—PARALLEL—SERIES-PARALLEL CIRCUITS

Electrical circuits are circuits made of electric components through which an electric current can pass. The electric circuit includes batteries, wires, resistors, etc. Series and parallel circuits are types of electrical circuits. Series circuits are the circuit in which the circuit components are connected successively. In the series circuit, all the components in the circuit experience the same current. Parallel circuits are in which the components are connected parallel to each other. In the parallel circuit, components are connected with the same point of contact. All components connected in parallel circuit experiences different current for each component.

Series Circuit:

A series circuit refers to a circuit in which all the circuit components are connected in a successive manner. The current in all the components of these circuits is constant. The voltage in all the components of these circuits is variable. In a series circuit, one end of the component is connected to the end of another component. If one component connected with other components in a series does not work the whole circuit fails.



Figure 1.9

- The resultant voltage $V = V_1 + V_2 + V_3$ in the above circuit, where V_1 is the voltage across R_1 , V_2 is the voltage across R_2 , and V_3 is the voltage across R_3 .
- The resultant current $I = I_1 = I_2 = I_3$ in the above circuit, where I_1 is currently flowing through R_1 , I_2 is currently flowing through R_2 and I_3 is currently flowing through R_3 .
- The resultant resistance for the above circuit is given by $R = R_1 + R_2 + R_3$

Parallel Circuit

Parallel circuits refer to the circuit in which all the circuit components are connected parallelly with the same point of contact. The voltage in all the components of these circuits is constant. The current in all the components of these circuits is variable.

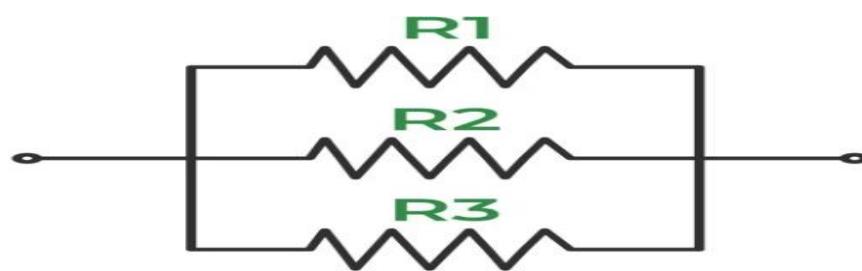


Figure 1.10

1. In a parallel circuit, all the components are connected side by side using the same point.
2. If one component in the circuit does not work, it does not affect the circuit.
3. The resultant voltage $V = V_1 = V_2 = V_3$ in the above circuit, where V_1 is the voltage across R_1 , V_2 is the voltage across R_2 , and V_3 is the voltage across R_3 .
4. The resultant current $I = I_1 + I_2 + I_3$ in the above circuit, where I_1 is currently flowing through R_1 , I_2 is currently flowing through R_2 and I_3 is currently flowing through R_3 .
5. The resultant resistance for the above circuit is given by $R = 1/R_1 + 1/R_2 + 1/R_3$

Similarities between Series and Parallel Circuits:

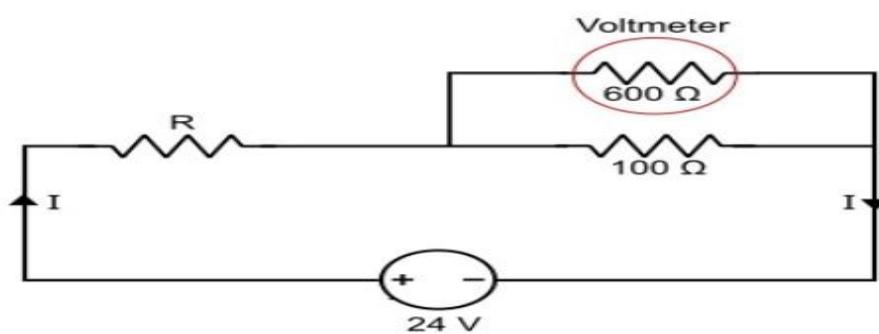
- Both series and parallel circuits are types of electrical circuits.
- Any number of components like resistors etc. can be connected in both series and parallel circuits.
- The purpose of the parallel of both series circuits is to control the flow of current.

Dissimilarities

The variances between series and parallel circuits are demonstrated in the table underneath.

Circuit In Series	Circuit In Parallel
There is a single current pathway	There are multiple current pathways
All components have similar current running through them	All components have similar potential difference across them
The sum of the potential dips across each component is equivalent to the emf of the source.	The sum of the currents flowing into any point in the circuit is equivalent to the sum of the currents flowing out of that point.

Example 1 In the circuit shown below, find the value of unknown resistance R , also determine the total resistance of the circuit and total power dissipated. It is given that voltmeter shows a reading of 10 V.



Solution:

Referring to the circuit shown above,

Total resistance of the parallel combination,

$$R_p = 600 \times 100600 + 100 = 85.71\Omega$$

As, the voltage across the parallel combination is equal to voltmeter reading that is 10 V (given). Therefore,

$$\text{Circuit current } I = 10 / 85.71 = 0.1167 \text{ A}$$

$$\text{Voltage across } R = 24 - 10 = 14 \text{ V}$$

Hence,

$$\text{Value of } R = 14 / 0.1167 = 119.96\Omega$$

Total resistance of the circuit is,

$$R_r = R_p + R = 119.96 + 85.71 = 205.67\Omega$$

Total power dissipated in the circuit is,

$$P_r = I^2 R_r = (0.1167)^2 \times 205.67 = 2.8 \text{ W}$$

Example 2 A battery with a terminal voltage of 9 V is connected to a circuit consisting of four 20Ω resistors and one 10Ω resistor all in series (Figure 6). Assume the battery has negligible internal resistance.

1. Calculate the equivalent resistance of the circuit.
2. Calculate the current through each resistor.
3. Calculate the potential drop across each resistor.
4. Determine the total power dissipated by the resistors and the power supplied by the battery.

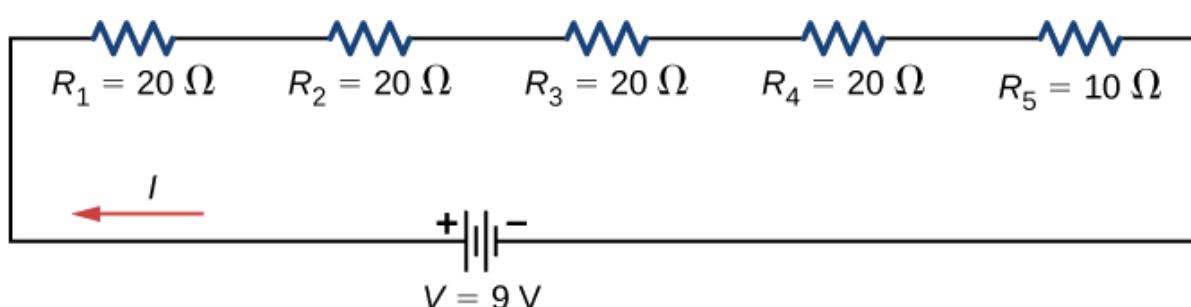


Figure 1.6

Strategy

In a series circuit, the equivalent resistance is the algebraic sum of the resistances. The current through the circuit can be found from Ohm's law and is equal to the voltage divided by the equivalent resistance. The potential drop across each resistor can be found using Ohm's law. The power dissipated by each resistor can be found using $P=I^2RP=I^2R$, and the total power

dissipated by the resistors is equal to the sum of the power dissipated by each resistor. The power supplied by the battery can be found using $P=I\epsilon P=I\epsilon$.

Solution

The equivalent resistance is the algebraic sum of the resistances

$$RS=R_1+R_2+R_3+R_4+R_5=20\Omega+20\Omega+20\Omega+20\Omega+10\Omega=90\Omega. RS=R_1+R_2+R_3+R_4+R_5=20\Omega+20\Omega+20\Omega+20\Omega+10\Omega=90\Omega.$$

The current through the circuit is the same for each resistor in a series circuit and is equal to the applied voltage divided by the equivalent resistance

$$I=VRS=9V90\Omega=0.1A. I=VRS=9V90\Omega=0.1A.$$

Note that the sum of the potential drops across each resistor is equal to the voltage supplied by the battery.

The power dissipated by a resistor is equal to $P=I^2RP=I^2R$, and the power supplied by the battery is equal to $P=I\epsilon P=I\epsilon$.

$$P_1=P_2=P_3=P_4=(0.1A)^2(20\Omega)=0.2W, P_1=P_2=P_3=P_4=(0.1A)^2(20\Omega)=0.2W,$$

$$P_5=(0.1A)^2(10\Omega)=0.1W, P_5=(0.1A)^2(10\Omega)=0.1W,$$

$$P_{dissipated}=0.2W+0.2W+0.2W+0.2W+0.1W=0.9W, d=0.2W+0.2W+0.2W+0.2W+0.1W=0.9W,$$

$$P_{source}=I\epsilon=(0.1A)(9V)=0.9W. P_{source}=I\epsilon=(0.1A)(9V)=0.9W.$$

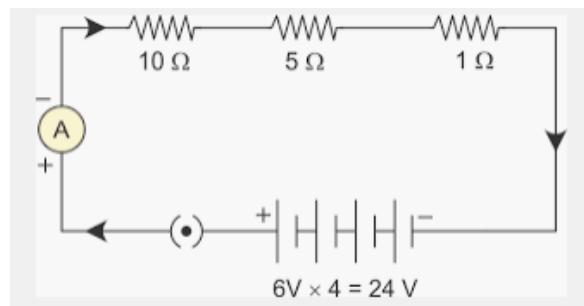
Significance

There are several reasons why we would use multiple resistors instead of just one resistor with a resistance equal to the equivalent resistance of the circuit. Perhaps a resistor of the required size is not available, or we need to dissipate the heat generated, or we want to minimize the cost of resistors. Each resistor may cost a few cents to a few dollars, but when multiplied by thousands of units, the cost saving may be appreciable.

Example 2 Some strings of miniature holiday lights are made to short out when a bulb burns out. The device that causes the short is called a shunt, which allows current to flow around the open circuit. A “short” is like putting a piece of wire across the component. The bulbs are usually grouped in series of nine bulbs. If too many bulbs burn out, the shunts eventually open. What causes this?

The equivalent resistance of nine bulbs connected in series is $9R$. The current is $I=V/9RI=V/9R$. If one bulb burns out, the equivalent resistance is $8R$, and the voltage does not change, but the current increases ($I=V/8R$ ($I=V/8R$). As more bulbs burn out, the current becomes even higher. Eventually, the current becomes too high, burning out the shunt.

Example 3 Draw a schematic diagram of a circuit consisting of 24v battery a 10ohm resistor, a 5 ohm resistor, a 1 ohm resistor an ammeter and a plug key all connected in series.



$$R = 10 + 5 + 1 = 16 \Omega \quad V = 24 \text{ V} \quad I = V/R = 24/16 = 1.5 \text{ A}$$

Example 4 What is the maximum resistance which can be made using five resistors each of $1/5 \Omega$.

We know that,

equivalent resistance is given by

$$R = R_1 + R_2 + R_3 + R_4 + R_5$$

$$R = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$$

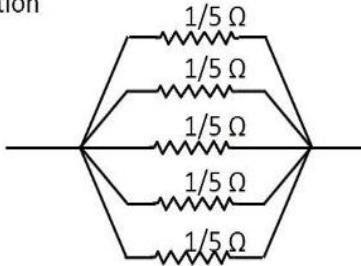
$$R = \frac{5}{5}$$

$$R = 1 \Omega$$

Maximum resistance is 1Ω

Example 5 What is the minimum resistance which can be made using five resistors of $1/5 \Omega$.

equivalent resistance is **minimum** when resistors are in **parallel** combination



We know that,

equivalent resistance is given by

$$R = R_1 + R_2 + R_3 + R_4 + R_5$$

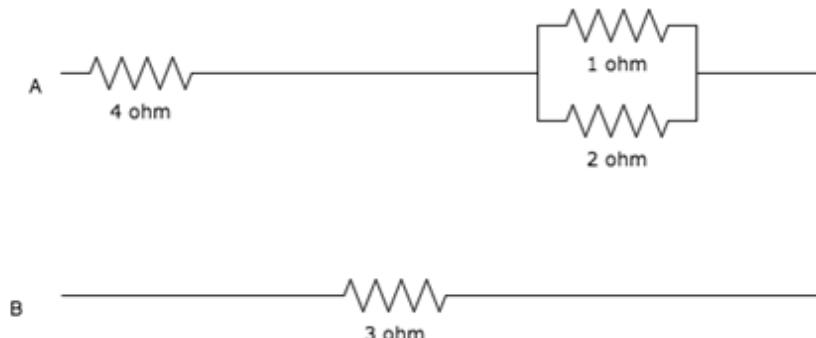
$$\frac{1}{R} = \frac{1}{\frac{1}{5}} + \frac{1}{\frac{1}{5}} + \frac{1}{\frac{1}{5}} + \frac{1}{\frac{1}{5}} + \frac{1}{\frac{1}{5}}$$

$$\frac{1}{R} = \frac{5}{1} + \frac{5}{1} + \frac{5}{1} + \frac{5}{1} + \frac{5}{1}$$

$$\frac{1}{R} = 25$$

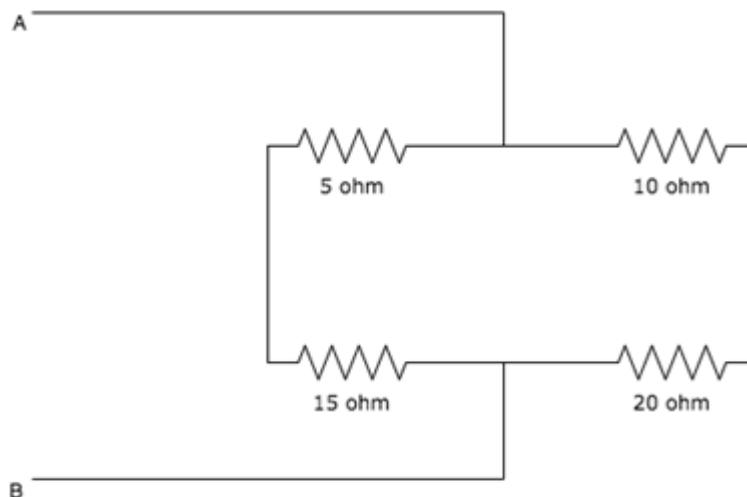
$$R = \frac{1}{25} \Omega$$

Example 6: Calculate the total resistance between the points A and B.



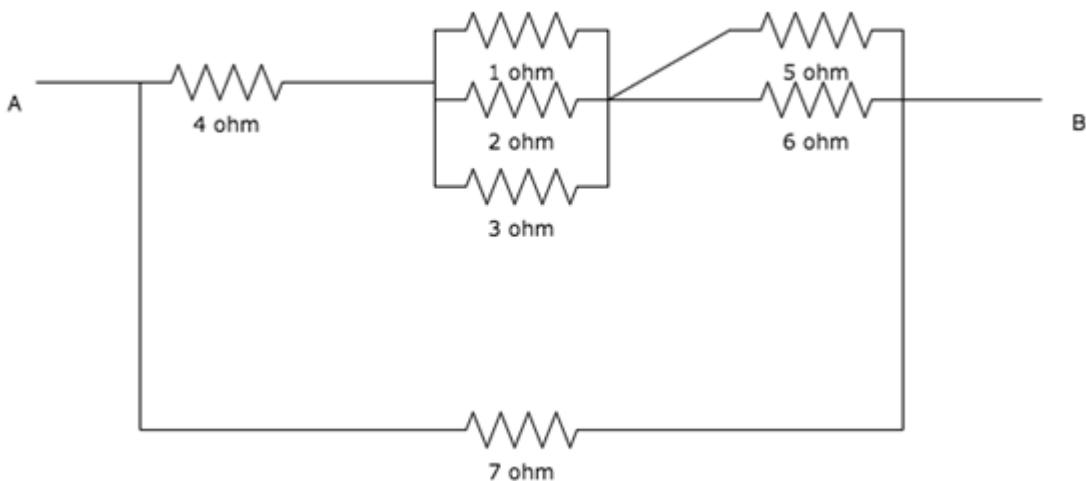
1 ohm in parallel with 2 ohm give $2/3$ ohm equivalent which is in series with 4 ohm and 3 ohm so total resistance between A and B = $4 + 2/3 + 3 = 23/3 = 7.67$ ohm.

Example 7: Calculate the equivalent resistance between A and B.



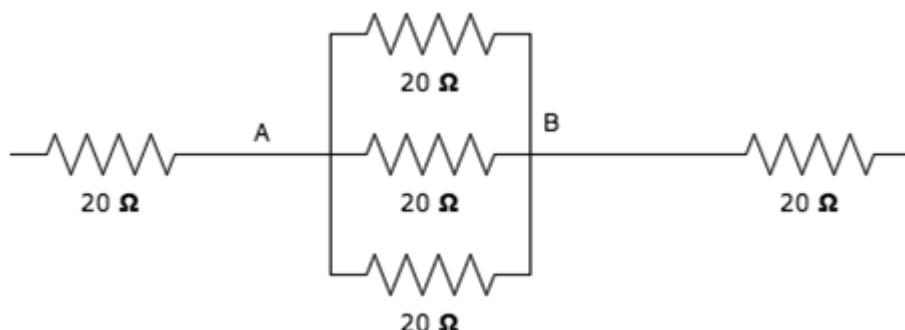
5 ohm and 15 ohm are connected in series to give 20 ohm. 10 ohm and 20 ohm are connected in series to give 30 ohm. Now both equivalent resistances (20 ohm and 30 ohm) are in parallel to give equivalent resistance $20 \times 30 / (20 + 30) = 12$ ohm.

Example 8: Calculate the resistance between A and B.



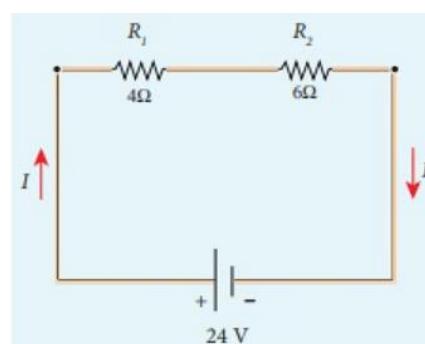
The 1 ohm, 2 ohm and 3 ohm resistors are connected in parallel. Its equivalent resistance is in series with the 4 ohm resistor and the parallel connection of the 5 ohm and 6 ohm resistor. The equivalent resistance of this combination is $80/11$ ohm. This is in parallel with 7 ohm to give equivalent resistance between A and B is 3.56 ohm.

Example 9: Calculate the equivalent resistance between A and B.



$R = 20 \parallel 20 \parallel 20 = 6.67$ ohm. The three 20 ohm resistors are in parallel and re-sistance is measured across this terminal.

Example:10 Calculate the equivalent resistance for the circuit which is connected to 24 V battery and also find the potential difference across 4 Ω and 6 Ω resistors in the circuit.



Ans. Since the resistors are connected in series, the total resistance in the circuit is given by,

$$4 \Omega + 6 \Omega = 10 \Omega$$

$$\text{Current } I \text{ in the circuit} = V / R_{\text{eq}} = 24 / 10 = 2.4 \text{ A}$$

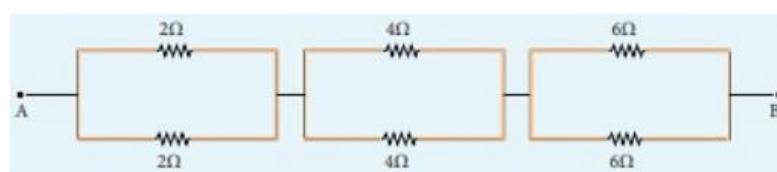
Voltage across 4Ω resistor

$$V_1 = IR_1 = 2.4 \times 4 = 9.6 \text{ V}$$

Voltage across 6 Ω resistor

$$V_2 = IR_1 = 2.4 \times 6 = 14.4 \text{ V}$$

Example 11: Calculate the equivalent resistance between A and B in the given circuit.



Ans. First, we will calculate the parallel resistance of the three units and then calculate the resistances in series.

1st Unit:

$$1/R_1 = 1/2 + 1/2$$

$$1/R_1 = 1 \text{ ohm}$$

$$R_1 = 1 \text{ ohm}$$

2nd Unit:

$$1/R_2 = 1/4 + 1/4$$

$$1/R_2 = 2/4 = \frac{1}{2} \text{ ohm}$$

$$R_2 = 2 \text{ ohm}$$

3rd Unit:

$$1/R_3 = \frac{1}{6} + \frac{1}{3}$$

$$1/R_3 = 2/6 = \frac{1}{3}$$

$$R_3 = 3 \text{ ohm}$$

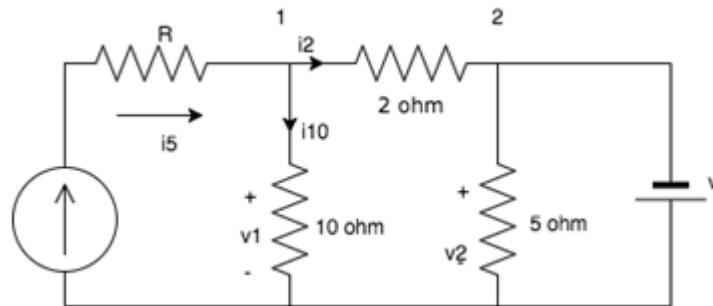
$$\text{Total Resistance in series } R_s = R_1 + R_2 + R_3$$

$$R_s = 1 + 2 + 3$$

$$R_s = 6 \text{ ohm}$$

Additional problems:

- Find the value of v if $v_1=20V$ and value of current source is 6A.



Explanation: The current through the 10 ohm resistor = $v_1/10 = 2A$. Applying KCL at node 1: $i_5 = i_{10} + i_2$. $i_2 = 6 - 2 = 4A$.

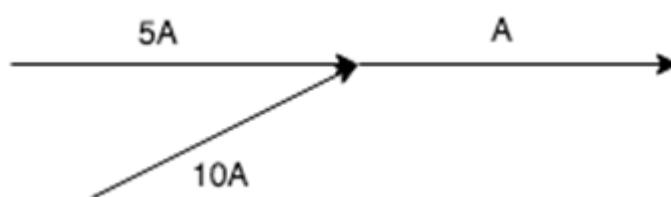
Thus the drop in the 2 ohm resistor = $4 \times 2 = 8V$.

$v_1 = 20V$; hence $v_2 = 20 - v$ across 2 ohm resistor = $20 - 8 = 12V$

$v_2 = v$ since they are connected in parallel.

$v = 12V$.

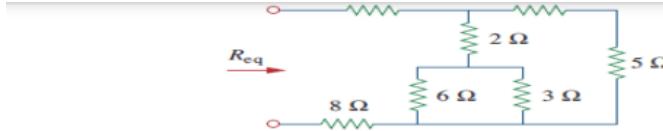
- Calculate the current A.



Explanation: KCL states that the total current leaving the junction is equal to the current entering it.

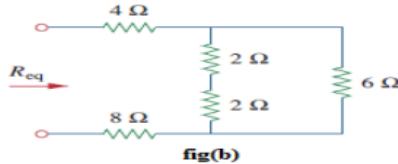
In this case, the current entering the junction is $5A + 10A = 15A$.

3. Find the R_{eq} for the circuit shown in below figure.

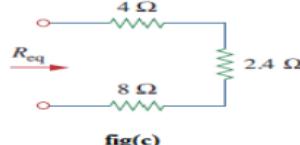


fig(a)

Solution:



fig(b)



fig(c)

To get R_{eq} we combine resistors in series and in parallel. The 6 ohms and 3 ohms resistors are in parallel, so their equivalent resistance is

$$6 \Omega \parallel 3 \Omega = \frac{6 \times 3}{6 + 3} = 2 \Omega$$

Also, the 1 ohm and 5ohms resistors are in series; hence their equivalent resistance is

$$1 \Omega + 5 \Omega = 6 \Omega$$

Thus the circuit in Fig.(b) is reduced to that in Fig. (c). In Fig. (b), we notice that the two 2 ohms resistors are in series, so the equivalent resistance is

$$2 \Omega + 2 \Omega = 4 \Omega$$

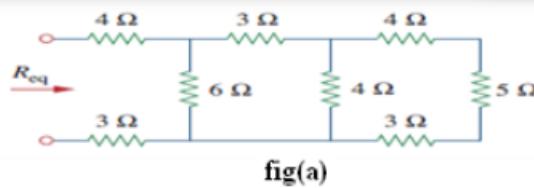
This 4 ohms resistor is now in parallel with the 6 ohms resistor in Fig.(b); their equivalent resistance is

$$4 \Omega \parallel 6 \Omega = \frac{4 \times 6}{4 + 6} = 2.4 \Omega$$

The circuit in Fig.(b) is now replaced with that in Fig.(c). In Fig.(c), the three resistors are in series. Hence, the equivalent resistance for the circuit is

$$R_{eq} = 4 \Omega + 2.4 \Omega + 8 \Omega = 14.4 \Omega$$

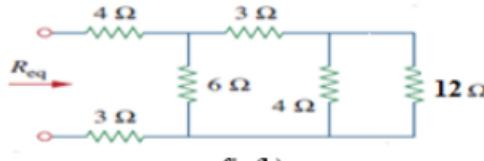
4. Find the R_{eq} for the circuit shown in below figure:



fig(a)

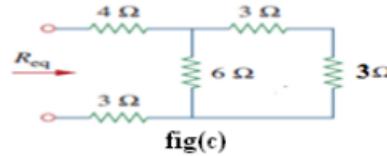
Solution:

In the given network 4 ohms, 5 ohms and 3 ohms comes in series then equivalent resistance is $4+5+3 = 12$ ohms



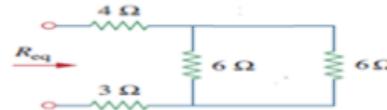
fig(b)

From fig(b), 4 ohms and 12 ohms are in parallel, equivalent is 3 ohms



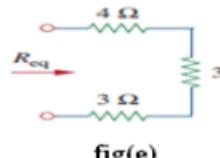
fig(c)

From fig(c), 3 ohms and 3 ohms are in series, equivalent resistance is 6 ohms



fig(d)

From fig(d), 6 ohms and 6 ohms are in parallel, equivalent resistance is 3 ohms



fig(e)

From fig(e), 4 ohms, 3 ohms and 3 ohms are in series .Hence $R_{eq} = 4 + 3 + 3 = 10$ ohms

5. Two lightbulbs, one graded at 40W and one graded at 60W are connected in series to a battery. Which one will be brighter? What if they are connected in parallel?

Ans. First we have to find out how the resistances of light bulb correlate to the power rating. For a resistor, the power dissipated is:

$$P = IV = V^2/R$$

Thus, there is an inverse relationship between the resistance of the lightbulb and the power rating. In a series connection, they share the same current. Whereas in a parallel they share the same voltage. Thus, for the two lightbulbs in series, the one with the higher resistance (lower wattage) will be brighter, and for a parallel configuration the one with the lower resistance (higher wattage) will be brighter.

NOTE: If numerical/problems on any topics are repeated ignore it.

AC Circuits: Alternating voltages and currents—RMS—average—maximum values—Single Phase RL—RC—RLC series circuits—Power in AC circuits—Power Factor.

1.9 Introduction

The flow of electricity can be done in two ways like AC (alternating current) and DC (direct current). Electricity can be defined as the flow of electrons throughout a conductor such as a wire. The main disparity among AC & DC mainly lies within the direction where the electrons supplies. In direct current, the flow of electrons will be in a single direction & in the alternating current; the flow of electrons will change their directions like going forward & then going backward.

In Alternating current, movement of electric charge periodically reverses its direction.

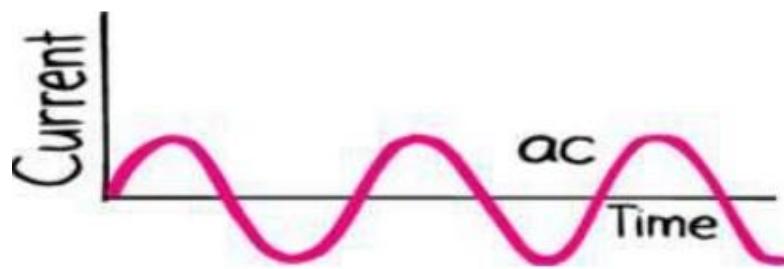


Figure 1.11

Whereas in DC, flow of electric charge is only in one direction.

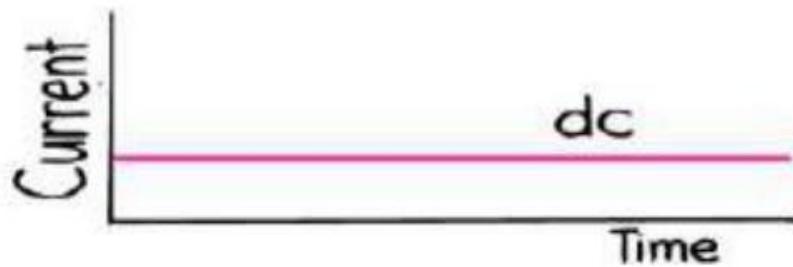


Figure 1.12

Why we use A.C. in homes:

- AC voltage is capable of converting voltage levels just with use of transformers.
- To transmit AC over a long distance, voltage is stepped up to 400 KV at generating stations and stepped down at a low level, 400/230 V for household and commercial utilization.
- AC motors are simple in construction, more efficient and robust as compared to DC motors.

1.10 Alternating voltage and current /The AC generator principle

An electrical generator is a machine which converts mechanical energy (or power) into electrical energy (or power). Induced e.m.f is produced in it according to Faraday's law of electromagnetic induction. This e.m.f cause a current to flow if the conductor circuit is closed. Hence, two basic essential parts of an electrical generator are: a) Magnetic field. b) Conductor or conductors which can move as to cut the flux. Generators are driven by a source of mechanical power, which is usually called the prime mover of the generator (steam turbine, diesel engine, or even an electric motor). Simple loop generator In figure is shown a single turn rectangular copper coil (AA'BB') rotating about its own axis in a magnetic field provided by either permanent magnets or electromagnets. The two end of the coil are joined to two slip-rings which are insulated from each

other and from the central shaft. Two collecting brushes (carbon or copper) press against the slip rings. The rotating coil may be called (armature) and the magnets as (field magnets). One way to generate an AC voltage is to rotate a coil of wire at constant angular velocity in a fixed magnetic field, slip rings and brushes connect the coil to the load. The magnitude of the resulting voltage is proportional to the rate at which flux lines are cut (faraday's law), and its polarity is dependent on the direction the coil sides move through the field.

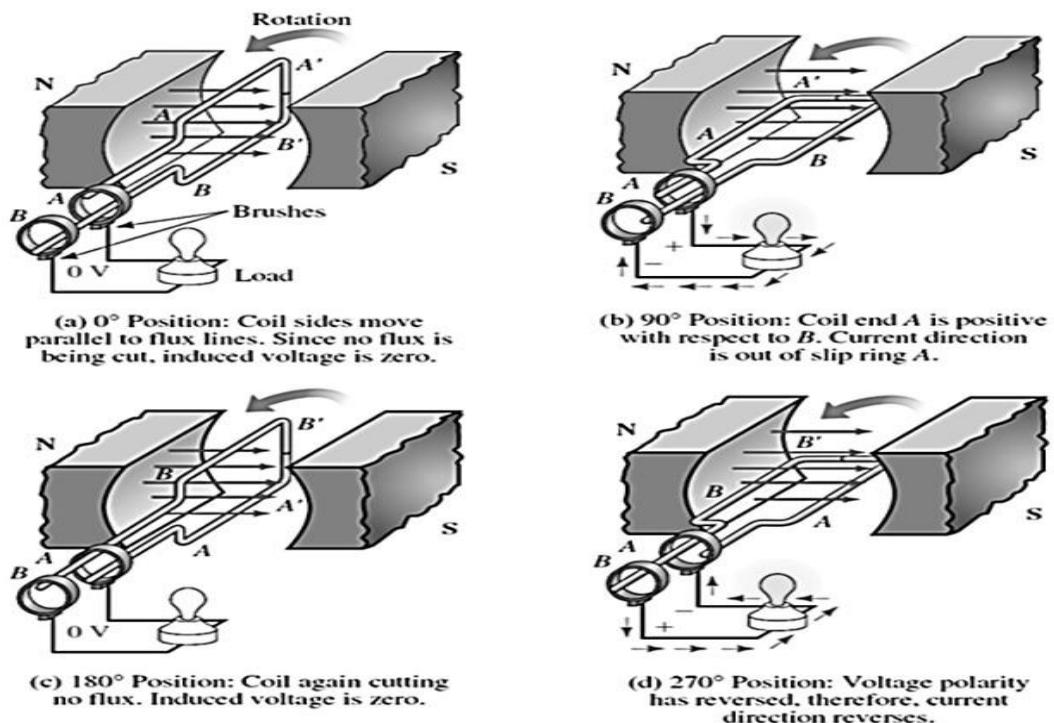
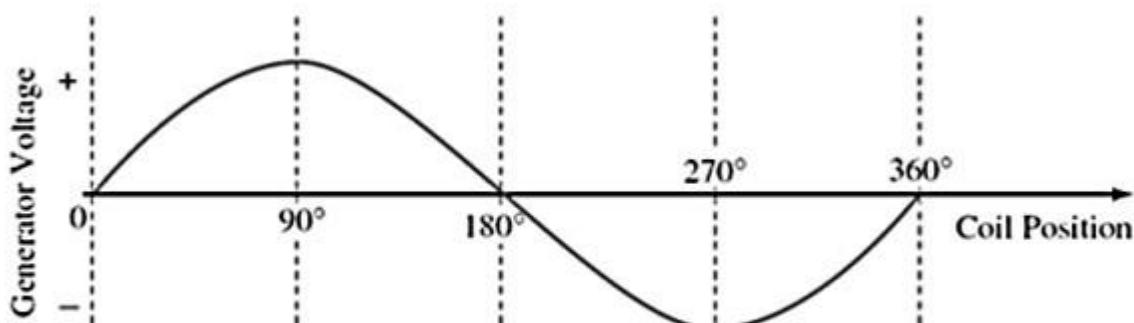


Figure 1.13



Coil voltage versus angular position.

Figure 1.14

A sinusoidal is a signal that has the form of the sine or cosine function. The sinusoidal waveform as shown in figure below.

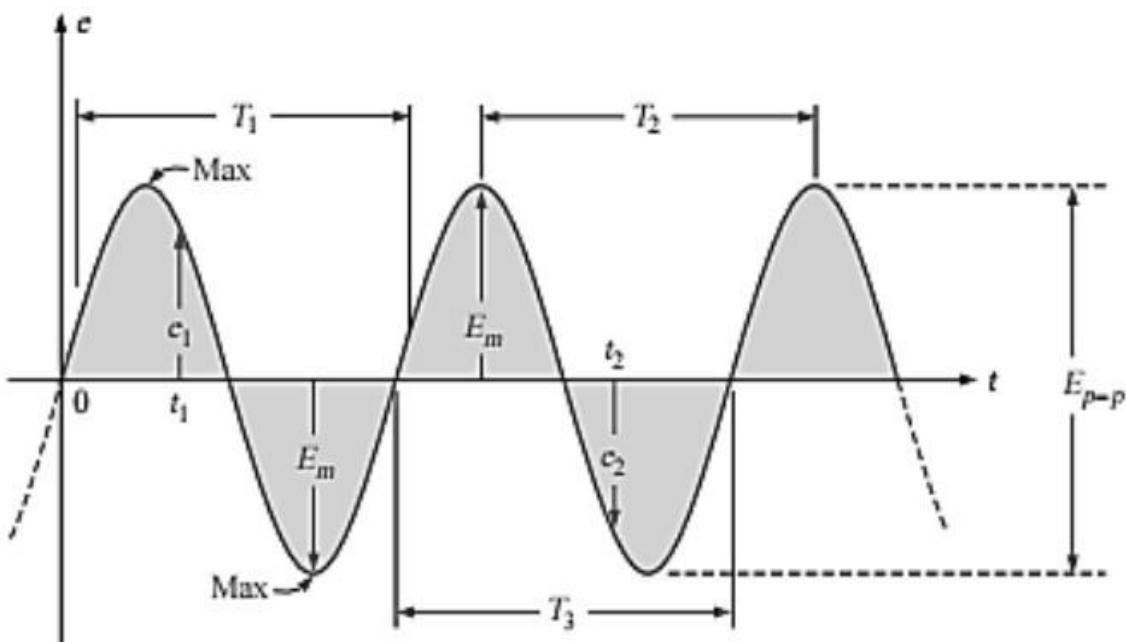


Figure 1. 15

Definitions:

Waveform: The path traced by a quantity, such as the voltage in Figure, plotted as a function of some variable such as time (as above), position, degrees, radians, temperature, and so on.
Instantaneous value: The magnitude of a waveform at any instant of time; denoted by lowercase letters (e_1, e_2).

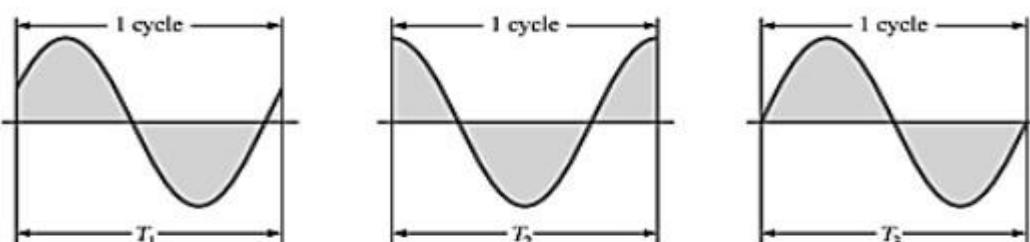
Peak amplitude: The maximum value of a waveform as measured from its average, or mean, value, denoted by uppercase letters (such as E_m for sources of voltage and V_m for the voltage drop across a load). For the waveform as shown in figure, the average value is zero volts.

Peak value: The maximum instantaneous value of a function as measured from the zero-volt level. For the waveform as shown in figure.

Peak-to-peak value: Denoted by E_{p-p} or V_{p-p} , the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks. Similarly, peak-to peak currents are denoted as I_{P-P} .

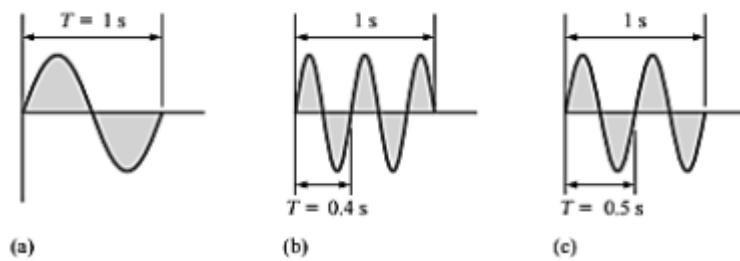
Periodic waveform: A waveform that continually repeats itself after the same time interval. The waveform as shown in figure, is a periodic waveform.

Period (T): The time interval between successive repetitions of a periodic waveform (the period $T_1 = T_2 = T_3$ in Figure), as long as successive similar points of the periodic waveform are used in determining, (T). it is the inverse of frequency. **Cycle:** The portion of a waveform contained in one period of time. The cycles within T_1 , T_2 , and T_3 of figure16



Defining the cycle and period of a sinusoidal waveform.

Frequency (f): The number of cycles that occur in 1 s. The unit of measure for frequency is the hertz (Hz),



Demonstrating the effect of a changing frequency on the period of a sinusoidal waveform.

Figure 1.17

1.11 Average Values:

The magnitude of alternating quantity can be expressed in three ways: 1 Peak Value 2 Average Value 3 Effective or rms value Peak Value: The maximum value attained by an alternating quantity during one cycle is called peak value. This is also called maximum value or amplitude. The peak of an alternating voltage or current is represented by V_m and I_m Average Value: The average value of a periodic waveform whether it is a sine wave, square wave or triangular waveform is defined as: "the quotient of the area under the waveform with respect to time". In other words, the averaging of all the instantaneous values along time axis with time being one full period, (T). For symmetrical waves like sinusoidal current or voltage waveform, the positive half cycle will be exactly equal to negative half cycle. Therefore, the average value over a complete cycle will be zero. The work is done by both, positive and negative cycle and hence the average value is determined without considering the signs. So, the only positive half cycle is considered to determine the average value of alternating quantities of sinusoidal waves.

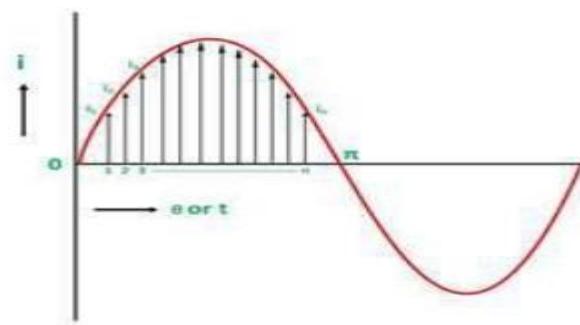


Figure 1.18

Divide the positive half cycle into (n) number of equal parts as shown in the above figure Let $i_1, i_2, i_3, \dots, i_n$ be the mid ordinates The Average value of current $i_{av} = \text{mean of the mid ordinates}$.

$$i_{av} = i_1 + i_2 + i_3 + \dots + i_n / n = \text{area of alternate/base.}$$

$$\begin{aligned}
 \text{Area of alternation} &= \int_0^\pi i d(\omega t) \\
 &= \int_0^\pi I_m \sin \omega t d(\omega t) \\
 &= I_m \int_0^\pi \sin \omega t d(\omega t) \\
 &= I_m [-\cos \omega t] \Big|_0^\pi \\
 &= I_m [-(-1 - 1)] \\
 &= 2I_m
 \end{aligned}$$

Base = π

Therefore, Average value of sinusoidal current = $2I_m/\pi$

Using The Analytical Method

As said previously, the average voltage of a periodic waveform whose two halves are exactly similar, either sinusoidal or non-sinusoidal, will be zero over one complete cycle. Then the average value is obtained by adding the instantaneous values of voltage over one half cycle only. But in the case of an non-symmetrical or complex wave, the average voltage (or current) must be taken over the whole periodic cycle mathematically.

The average value can be taken mathematically by taking the approximation of the area under the curve at various intervals to the distance or length of the base and this can be done using triangles or rectangles as shown.

Approximation of the Area

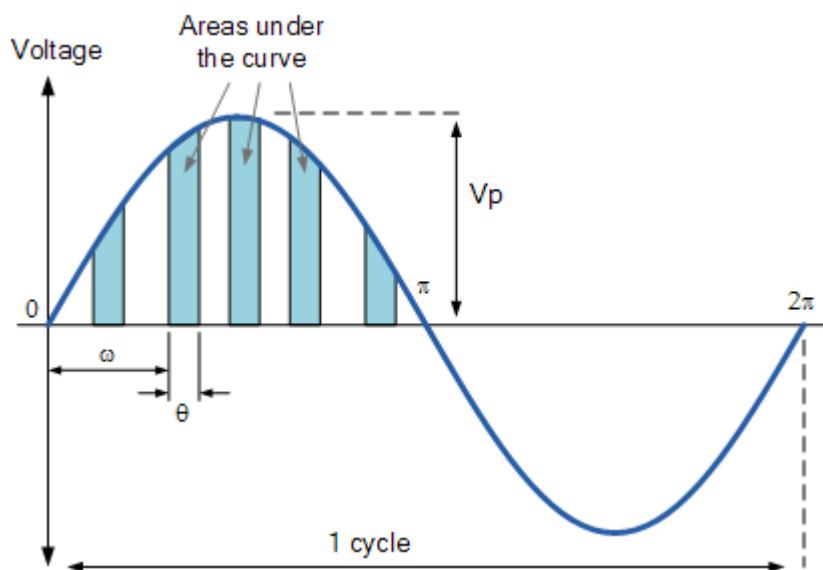


Figure 1.19

Approximating the areas of the rectangles under the curve, we can obtain a rough idea of the actual area of each one. Thus adding together all these areas the average value can be found. If an infinite number of smaller thinner rectangles were used, the more accurate would be the final result as it approaches $2/\pi$.

The area under the curve can be found by various approximation methods such as the *trapezoidal rule*, the *mid-ordinate rule* or *Simpson's rule*. Then the mathematical area under the positive half cycle of the periodic wave which is defined as $V_{(t)} = V_p \cos(\omega t)$ with a period of T using integration is given as:

$$\text{Area} = \int_0^\pi V_p \cos(\omega t) dt$$

Where: 0 and π are the limits of integration since we are determining the average value of voltage over one half a cycle. Then the area below the curve is finally given as Area = $2V_p$. Since we now know the area under the positive (or negative) half cycle, we can easily determine the average value of the positive (or negative) region of a sinusoidal waveform by integrating the sinusoidal quantity over half a cycle and dividing by half the period.

For example, if the instantaneous voltage of a sinusoid is given as: $v = V_p \sin\theta$ and the period of a sinusoid is given as: 2π , then:

$$\begin{aligned} V_{AVE} &= \frac{1}{\pi} \int_0^{\pi} V_p \sin\theta d\theta \\ V_{AVE} &= \frac{V_p}{\pi} (-\cos\theta) \Big|_0^{\pi} \\ &= \frac{2V_p}{\pi} = \frac{2}{\pi} V_p = 0.637 V_p \end{aligned}$$

Which is therefore given as the standard equation for the Average Voltage of a sine wave as:

Average Voltage Equation

$$V_{AVE} = \frac{2V_p}{\pi} = 0.637 V_p$$

The average voltage (V_{AV}) of a sinusoidal waveform is determined by multiplying the peak voltage value by the constant **0.637**, which is two divided by pi (π). The average voltage, which can also be referred to as the mean value, depends on the magnitude of the waveform and is not a function of either the frequency or the phase angle.

Thus this average or mean value (either voltage or current) of a sinusoidal waveform can also be shown as an equivalent DC value of area and time.

1.12 Effective or RMS Value:

"RMS" stands for "Root-Mean-Squared", also called the effective or heating value of alternating current, which would provide the same amount of heat generation in a resistor as the AC voltage would if applied to that same resistor. That steady current which, when flows through a resistor of known resistance for a given period of time than as a result the same quantity of heat is produced by the alternating current when flows through the same resistor for the same period of time is called R.M.S or effective value of the alternating current. In other words, the R.M.S value is defined as the square root of means of squares of instantaneous values. Let I be the alternating current flowing through a resistor R for time t seconds, which produces the same amount of heat as produced by the direct current (I_{eff}). The base of one alteration is divided into n equal parts so that each interval is of t/n seconds as shown in the figure below. The **RMS (Root Mean Square)** value (also known as **effective** or **virtual** value) of an alternating current (AC) is the value of direct current (DC) when flowing through a circuit or resistor for the specific time period and produces same amount of heat which produced by the alternating current (AC) when flowing through the same circuit or resistor for a specific time.

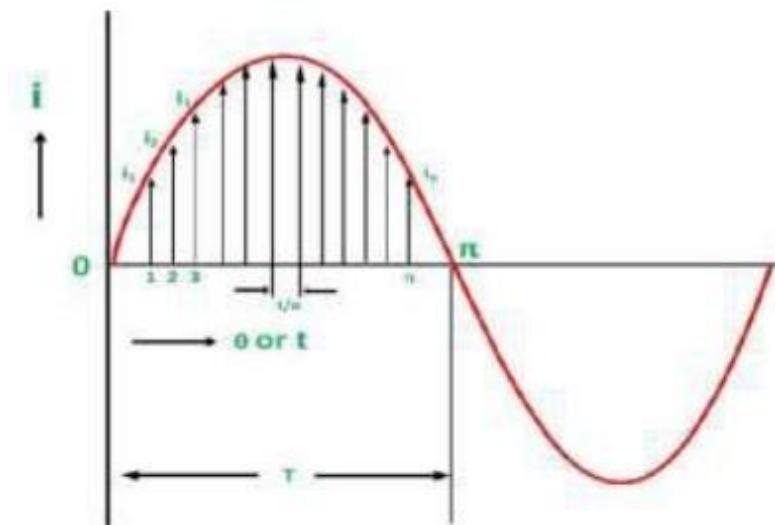


Figure 1.20

The value of an AC which will produce the same amount of heat while passing through in a heating element (such as resistor) as DC produces through the element is called R.M.S Value. In short, The RMS Value of an Alternating Current is that when it compare to the Direct Current. While the mean of square of instantaneous values of current in in half or complete cycle is:

$$= \int_0^{2\pi} \frac{i^2 d\theta}{(2\pi - 0)}$$

The Square root of this value is:

$$= \sqrt{\left(\int_0^{2\pi} \frac{i^2 d\theta}{2\pi} \right)}$$

Hence, the RMS value of the current is (while putting $I = I_m \sin \theta$):

$$I = \sqrt{\left(\int_0^{2\pi} \frac{i^2 d\theta}{2\pi} \right)} = \sqrt{\left(\frac{I_m^2}{2\pi} \int_0^{2\pi} \sin^2 \theta d\theta \right)}$$

Now,

$$\cos 2\theta = 1 - 2 \sin^2 \theta \quad \therefore \sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\begin{aligned} I &= \sqrt{\left(\frac{I_m^2}{4\pi} \int_0^{2\pi} (1 - \cos 2\theta) d\theta \right)} = \sqrt{\left(\frac{I_m^2}{4\pi} \left| \theta - \frac{\sin 2\theta}{2} \right|_0^{2\pi} \right)} \\ &= \sqrt{\frac{I_m^2}{4}} \quad \sqrt{\frac{I_m^2}{2}} \quad \therefore I = \frac{I_m}{\sqrt{2}} = 707 I_m \end{aligned}$$

Therefore, We may find that for a symmetrical sinusoidal current:

$$I_{\text{RMS}} = \text{Max Value of Current} \times 0.707$$

OR

Alternating current (A.C) is the current that changes in magnitude direction continuously with respect to time.

It can be represented as,

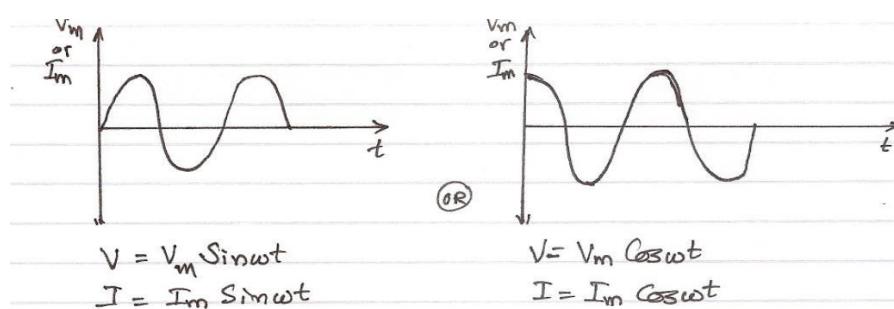


Figure 1.21

The currents and voltages in a.c circuits can be expressed by the following terms:-

- (i) **Instantaneous (I or V)** :- It is the current or voltage that in the circuit at any instant.
- (ii) **Peak (I_m or V_m)**:- It is the maximum available voltage or current in the circuit.
- (iii) **Average (I_{av} or V_{av})**:- It is the arithmetic mean of the instantaneous values of voltages or currents in the circuit.
- (iv) **Room Mean Square (R.M.S) or Effective**:- It is the square root of the average of the squares of the instantaneous values of the voltages or currents in the circuit.

$$V_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \dots}{n}}$$

R.M.S value is the d.c equivalent of a.c

Relation between Peak and R.M.S values

$$\begin{aligned} I_{rms}^2 &= \int_0^T \frac{I^2 dt}{T} = \int_0^T \frac{I_m^2 \sin^2 \omega t dt}{T} \\ &= \int_0^T \frac{I_m^2}{T} \left(\frac{1 - \cos 2\omega t}{2} \right) dt \quad \text{where } \omega = 2\pi/T \\ I_{rms}^2 &= \frac{I_m^2}{2T} \left[\int_0^T dt - \int_0^T \cos 2\omega t dt \right] \end{aligned} \quad \dots \quad (1)$$

$$\text{But } \int_0^T \cos 2\omega t dt = \left[\frac{\sin 2\omega t}{2\omega} \right]_0^T = \frac{1}{2\omega} \left[\sin \frac{4\pi}{T} t \right]_0^T = \frac{1}{2\omega} [\sin 4\pi - \sin 0] = 0$$

Thus equation (1) becomes,

$$I_{rms}^2 = \frac{I_m^2}{2T} [T - 0] \quad \text{So, } I_{rms} = \frac{I_m}{\sqrt{2}}$$

Similarly, it can also be shown that, $V_{rms} = \frac{V_m}{\sqrt{2}}$

Note: the measurement of ac is done by the comparison of the heating effect produced by ac with that by dc. Hence ac measuring instruments are also called ‘hot-wire instruments’.

$$I_{av} = \int_0^T \frac{I dt}{T} = \int_0^T \frac{I_m \sin \omega t dt}{T} = I_m / T \int_0^T \sin \omega t dt = I_m / T \left[\frac{-\cos \omega t}{\omega} \right]_0^T$$

$$= -I_m / \omega T \left[\cos \frac{2\pi}{T} \right]_0^T = -I_m / \omega T [\cos 2\pi - \cos 0] = 0 \text{ (Hence proved)}$$

Note:- But in half a cycle, $I_{av} = \int_0^{\frac{T}{2}} \frac{I dt}{T/2} = \int_0^{\frac{T}{2}} \frac{I_m \sin \omega t dt}{T/2} = 2 I_m / T \int_0^{\frac{T}{2}} \sin \omega t dt = 2 I_m / T \left[\frac{-\cos \omega t}{\omega} \right]_0^{T/2}$

$$= -2I_m / \omega T \left[\cos \frac{2\pi}{T} \right]_0^{T/2} = -I_m / \omega T [\cos \pi - \cos 0] = -I_m / \omega T [-1 - -1]$$

This gives $I_{av} = 2I_m / \pi$ Similary, it can be shown that $V_{av} = 2V_m / \pi$

Show that the average current in the complete cycle of a.c is zero.

1.13 Single Phase RL—RC—RLC series circuits

Single Phase RL:

A circuit that contains a pure resistance R ohms connected in series with a coil having a pure inductance of L (Henry) is known as **RL Series Circuit**. When an AC supply voltage V is applied, the current, I flows in the circuit. So, I_R and I_L will be the current flowing in the resistor and inductor respectively, but the amount of current flowing through both the elements will be same as they are connected in series with each other. The circuit diagram of RL Series Circuit is shown below:

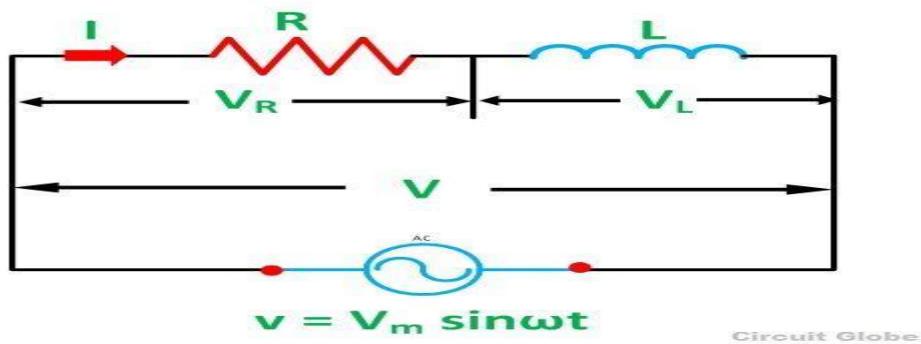


Figure 1.22

Where,

- V_R – voltage across the resistor R
- V_L – voltage across the inductor L
- V – Total voltage of the circuit

Phasor Diagram of the RL Series Circuit

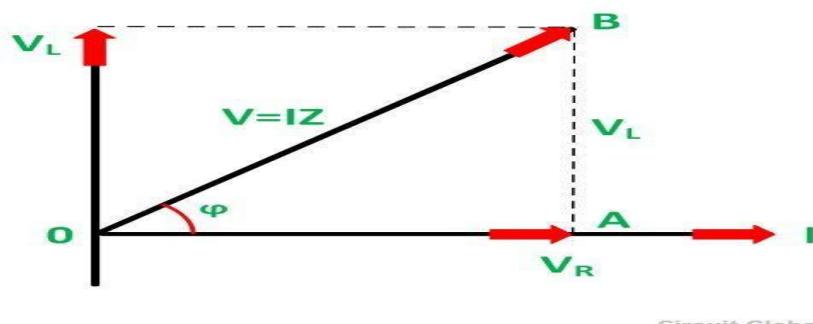


Figure 1. 23

The phasor diagram of the RL Series circuit is shown above.

Steps to draw the Phasor Diagram of RL Series Circuit

The following steps are given below which are followed to draw the phasor diagram step by step:

- Current I is taken as a reference.
- The Voltage drop across the resistance $V_R = IR$ is drawn in phase with the current I.
- The voltage drop across the inductive reactance $V_L = IX_L$ is drawn ahead of the current I. As the current lags voltage by an angle of 90 degrees in the pure Inductive circuit.
- The vector sum of the two voltages drops V_R and V_L is equal to the applied voltage V.

Now,

In right-angle triangle OAB $V_R = IR$ and $V_L = IX_L$ where $X_L = 2\pi fL$

$$V = \sqrt{(V_R)^2 + (V_L)^2} = \sqrt{(IR)^2 + (IX_L)^2}$$

$$V = I\sqrt{R^2 + X_L^2} \quad \text{or}$$

$$I = L = \frac{V}{Z} \quad Z = \sqrt{R^2 + X_L^2}$$

Z is the total opposition offered to the flow of alternating current by an RL Series circuit and is called impedance of the circuit. It is measured in ohms (Ω).

Phase Angle: In RL Series circuit the current lags the voltage by 90 degrees angle known

$$\tan\varphi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} \quad \text{or}$$

$$\varphi = \tan^{-1} \frac{X_L}{R}$$

as phase angle. It is given by

Power in R L Series Circuit

If the alternating voltage applied across the circuit is given by the equation:

$$v = V_m \sin \omega t \dots\dots\dots(1)$$

The equation of current I is given as:

$$i = I_m \sin(\omega t - \varphi) \dots \dots \dots (2)$$

Then the instantaneous power is given by the equation:

Putting the value of v and i from the equation (1) and (2) in the equation (3) we will get

$$P = (V_m \sin \omega t) \times I_m \sin(\omega t - \varphi)$$

$$p = \frac{V_m I_m}{2} 2\sin(\omega t - \varphi) \sin \omega t$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} [\cos \varphi - \cos(2\omega t - \varphi)]$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(2\omega t - \varphi)$$

The average power consumed in the circuit over one complete cycle is given by the equation shown below:

$$P = \text{average of } \frac{V_m}{\sqrt{2}} \frac{V_m}{\sqrt{2}} \cos\varphi - \text{average of } \frac{V_m}{\sqrt{2}} \frac{V_m}{\sqrt{2}} \cos(2\omega t - \varphi) \quad \text{or}$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \text{Zero} \quad \text{or}$$

$$P = V_{r.m.s} I_{r.m.s} \cos\varphi = VI \cos\varphi$$

Where $\cos\phi$ is called the power factor of the circuit.

The power factor is defined as the ratio of resistance to the impedance of an AC Circuit.

Putting the value of V and $\cos\phi$ from the equation (4) the value of power will be:

$$P = (IZ)(I)(R/Z) = I^2 R \dots \dots \dots (5)$$

From equation (5) it can be concluded that the inductor does not consume any power in the circuit.

Single phase RC:

A circuit that contains pure resistance R ohms connected in series with a pure capacitor of capacitance C farads is known as **RC Series Circuit**. A sinusoidal voltage is applied and current I flows through the resistance (R) and the capacitance (C) of the circuit. The RC Series circuit is shown in the figure below:

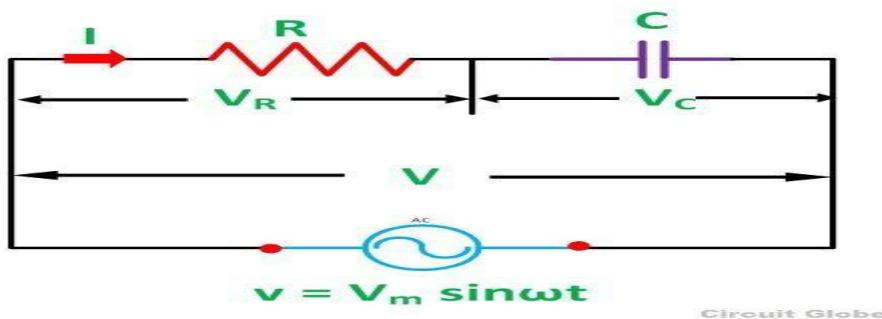


Figure 1.24

Where,

- V_R – voltage across the resistance R
- V_C – voltage across capacitor C
- V – total voltage across the RC Series circuit

Phasor Diagram of RC Series Circuit

The following steps are used to draw the phasor diagram of RC Series circuit

Take the current I (r.m.s value) as a reference vector

Voltage drop in resistance $V_R = IR$ is taken in phase with the current vector

Voltage drop in capacitive reactance $V_C = IXC$ is drawn 90 degrees behind the current vector, as current leads voltage by 90 degrees (in the pure capacitive circuit)

The vector sum of the two voltage drops is equal to the applied voltage V (r.m.s value).

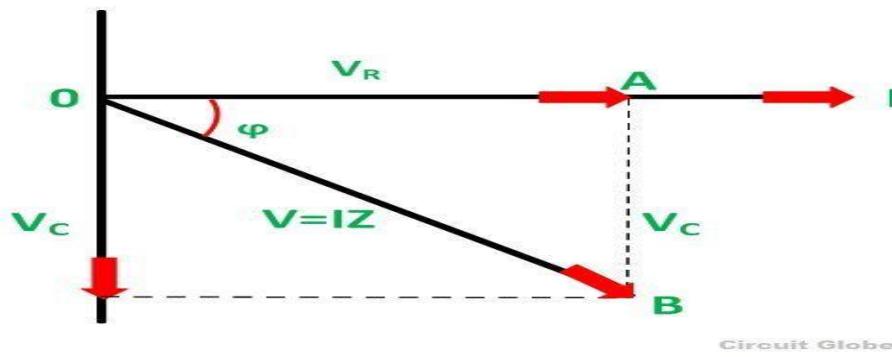


Figure 1.25

$$V_R = I_R \text{ and } V_C = I X_C$$

$$\text{Where } X_C = I/2\pi f C$$

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (IX_C)^2}$$

$$V = I \sqrt{R^2 + X_C^2} \quad \text{or}$$

$$I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}$$

In right triangle OAB,

Where,

$$Z = \sqrt{R^2 + X_C^2}$$

Z is the total opposition offered to the flow of alternating current by an RC series circuit and is called **impedance** of the circuit. It is measured in ohms (Ω).

Phase angle

From the phasor diagram shown above, it is clear that the current in the circuit leads the applied voltage by an angle ϕ and this angle is called the **phase angle**.

$$\tan\varphi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R} \quad \text{or}$$

$$\varphi = \tan^{-1} \frac{X_C}{R}$$

Power in RC Series Circuit

If the alternating voltage applied across the circuit is given by the eq

$$v = V_m \sin\omega t \dots \dots \dots (1)$$

$$i = I_m \sin(\omega t + \varphi) \dots \dots \dots (2)$$

Therefore, the instantaneous power is given by $p = vi$

Putting the value of v and i from the equation (1) and (2) in $p = vi$

$$P = (V_m \sin\omega t) \times I_m \sin(\omega t + \varphi)$$

$$p = \frac{V_m I_m}{2} 2 \sin(\omega t + \varphi) \sin\omega t$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} [\cos\varphi - \cos(2\omega t + \varphi)]$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(2\omega t + \varphi)$$

The average power consumed in the circuit over a complete

$$P = \text{average of } \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \text{average of } \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(2\omega t + \varphi) \quad \text{or}$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \text{Zero} \quad \text{or}$$

$$P = V_{r.m.s} I_{r.m.s} \cos\varphi = VI \cos\varphi$$

cycle is given by

Where $\cos\phi$ is called the power factor of the circuit.

Putting the value of V and $\cos\phi$ from the equation (3) the value of power will be

$$P = (IZ)(I)(R/Z) = I^2 R \dots \dots \dots (4)$$

From the equation (4) it is clear that the power is actually consumed by the resistance only and the capacitor does not consume any power in the circuit.

RLC Series Circuit

When a pure resistance of R ohms, a pure inductance of L Henry and a pure capacitance of C farads are connected together in series combination with each other then **RLC Series Circuit** is formed. As all the three elements are connected in series so, the current flowing through each element of the circuit will be the same as the total current I flowing in the circuit. The **RLC Circuit** is shown below:

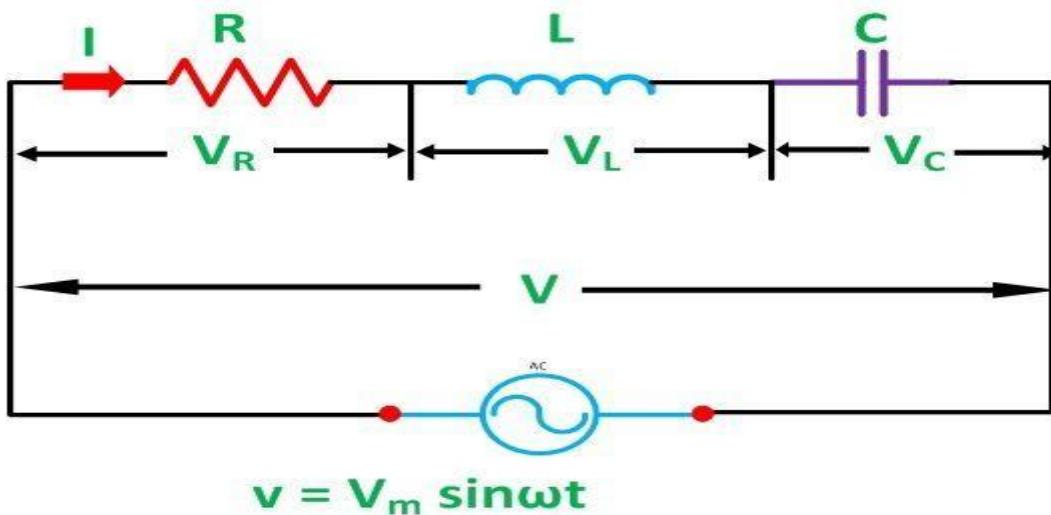


Figure 1.27

In the RLC Series circuit:

$$X_L = 2\pi f L \text{ and } X_C = 1/2\pi f C$$

When the AC voltage is applied through the RLC Series circuit the resulting current I flows through the circuit, and thus the voltage across each element will be:

- $V_R = IR$ that is the voltage across the resistance R and is in phase with the current I.
- $V_L = IX_L$ that is the voltage across the inductance L and it leads the current I by an angle of 90 degrees.
- $V_C = IX_C$ that is the voltage across capacitor C and it lags the current I by an angle of 90 degrees.

Phasor Diagram of RLC Series Circuit

The phasor diagram of the RLC series circuit when the circuit is acting as an inductive circuit that means ($V_L > V_C$) is shown below and if ($V_L < V_C$) the circuit will behave as a capacitive circuit.

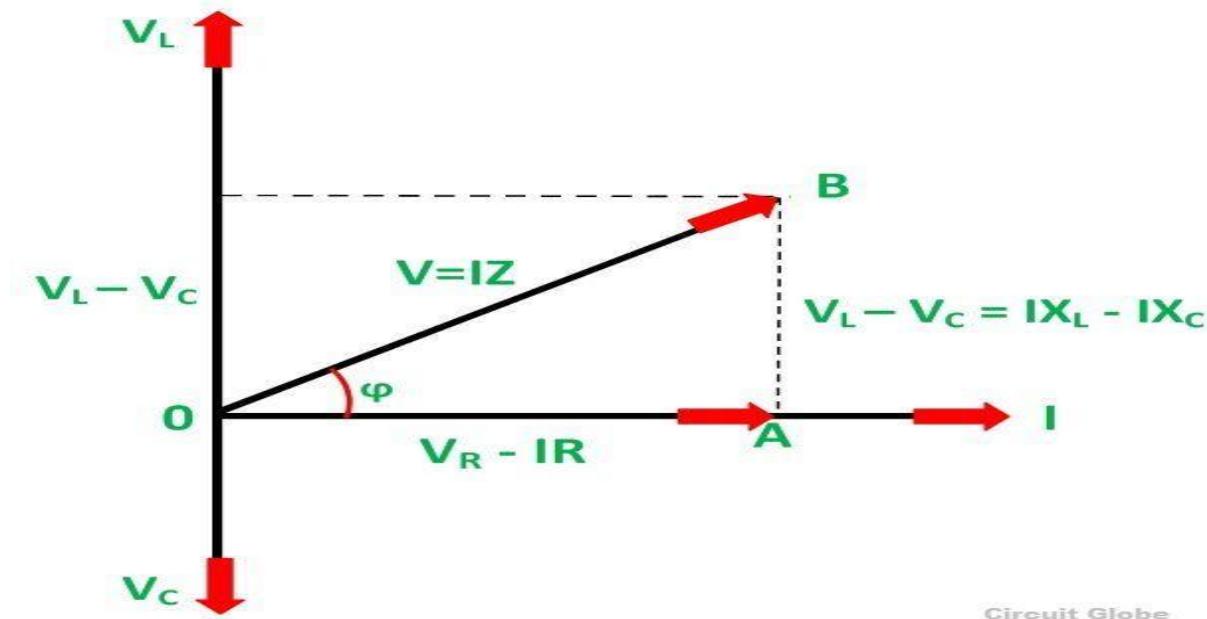


Figure 1.28

Steps to draw the Phasor Diagram of the RLC Series Circuit

- Take current I as the reference as shown in the figure above
- The voltage across the inductor L that is V_L is drawn leads the current I by a 90-degree angle.
- The voltage across the capacitor C that is V_C is drawn lagging the current I by a 90-degree angle because in capacitive load the current leads the voltage by an angle of 90 degrees.
- The two vector V_L and V_C are opposite to each other.

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2} \quad \text{or}$$

$$V = I \sqrt{R^2 + (X_L - X_C)^2} \quad \text{or}$$

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{Z}$$

Where,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

It is the total opposition offered to the flow of current by an RLC Circuit and is known as **Impedance** of the circuit.

Phase Angle

From the phasor diagram, the value of phase angle will be

$$\tan \varphi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} \quad \text{or}$$

$$\varphi = \tan^{-1} \frac{X_L - X_C}{R}$$

Power in RLC Series Circuit

$$P = VI \cos\phi = I^2 R$$

The product of voltage and current is defined as power. Where $\cos\phi$ is the power factor of the circuit and is expressed as:

$$\cos\phi = \frac{V_R}{V} = \frac{R}{Z}$$

The three cases of RLC Series Circuit

- When $X_L > X_C$, the phase angle ϕ is positive. The circuit behaves as RL series circuit in which the current lags behind the applied voltage and the power factor is lagging.
- When $X_L < X_C$, the phase angle ϕ is negative, and the circuit acts as a series RC circuit in which the current leads the voltage by 90 degrees.
- When $X_L = X_C$, the phase angle ϕ is zero, as a result, the circuit behaves like a purely resistive circuit. In this type of circuit, the current and voltage are in phase with each other. The value of the power factor is **unity**.

Impedance Triangle of RLC Series Circuit

When the quantities of the phasor diagram are divided by the common factor I then the right angle triangle is obtained known as impedance triangle. The impedance triangle of the RL series circuit, when ($X_L > X_C$) is shown below:

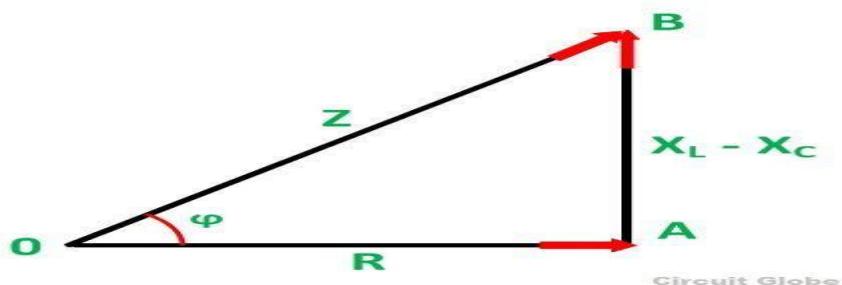


Figure 1.29

If the inductive reactance is greater than the capacitive reactance than the circuit reactance is inductive giving a **lagging phase angle**.

Impedance triangle is shown below when the circuit acts as an RC series circuit ($X_L < X_C$)

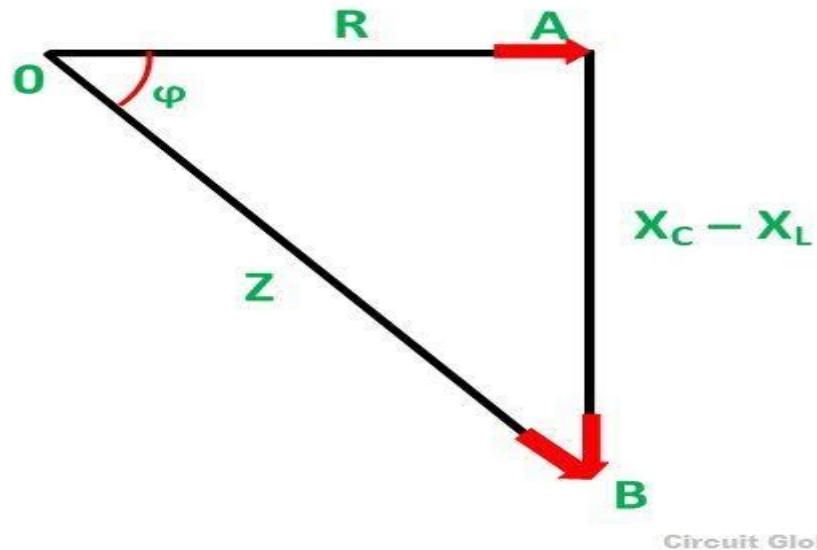


Figure 1.30

When the capacitive reactance is greater than the inductive reactance the overall circuit reactance acts as capacitive and the phase angle will be leading.

Applications of RLC Series Circuit

The following are the application of the RLC circuit:

- It acts as a variable tuned circuit
- It acts as a low pass, high pass, bandpass, bandstop filters depending upon the type of frequency.
- The circuit also works as an oscillator
- Voltage multiplier and pulse discharge circuit

OR

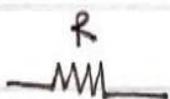
Phasor diagrams

These are the vector diagrams in which the magnitudes represent the peak values and directions represent the phase differences between voltages and currents in the a.c circuits.

A.C Circuits

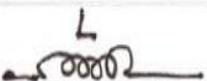
Fundamental components in a.c. circuits.

1. Resistor



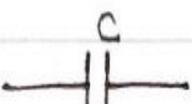
$$V = IR$$

2. Inductor



$$E = -L \frac{dI}{dt}$$

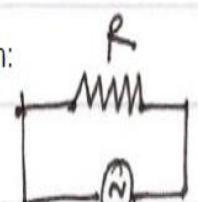
3. Capacitor



$$Q = CV$$

1. Circuit containing only resistor

Circuit diagram:

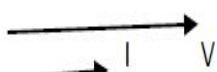


Let $V = V_m \sin \omega t$ Current (I) = $V/R = (V_m \sin \omega t)/R$

i.e., $I = I_m \sin \omega t$. Where $I_m = V_m/R$. $R = V_m/I_m$

Hence the phase difference between voltage and current is, $\phi = 0$.

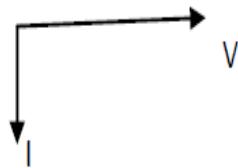
So the phasor diagram can be as shown



The graph showing the variation of current and voltage with time can be represented as:

Here the current lags behind the voltage by $\pi/2$.

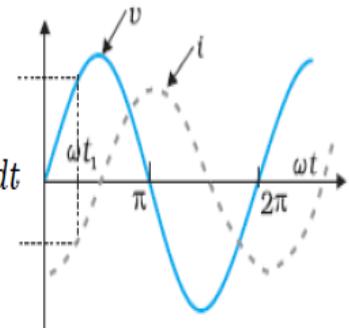
So the phasor diagram can be as shown



The graph showing the variation of current and voltage with time can be represented as:

Power associated with the circuit:

$$\begin{aligned} P_L &= \int_0^T \frac{VIdt}{T} = \frac{1}{T} \int_0^T V_m \sin \omega t \cdot I_m (\sin \omega t - \pi/2) dt = \frac{V_m I_m}{T} \int_0^T \sin \omega t \cdot -\cos \omega t dt \\ &= -\frac{V_m I_m}{T} \int_0^T \left(\frac{\sin 2\omega t}{2} \right) dt = -\frac{V_m I_m}{2T} \cdot 0 \quad \text{as } \int_0^T \frac{\sin 2\omega t dt}{2} = 0 \end{aligned}$$

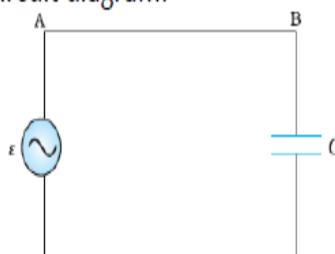


Thus we get, $P_L = 0$ i.e., an ideal inductor does not consume any power.

(Note: this is why they are used as 'choke coils' in ac circuits to reduce current without power loss, in the place of resistors)

3. Circuit containing only capacitor

Circuit diagram:



Let $V = V_m \sin \omega t$

We have $Q = CV$

Differentiating we get, $\frac{dQ}{dt} = C \frac{dV}{dt}$

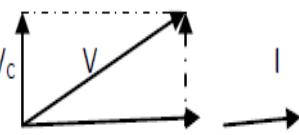
$$\text{i.e. } I = C V_m \cos \omega t \cdot \omega \quad I = \frac{V_m}{1/C\omega} \sin(\omega t + \frac{\pi}{2})$$

$$\text{Hence } I = I_m \sin(\omega t + \pi/2) \quad \text{where } I_m = \frac{V_m}{1/C\omega}$$

Thus $X_C = 1/C\omega$ gives the non-resistive opposition to the flow of current called capacitive reactance.

Hence the phase difference between voltage and current is, $\phi = \pi/2$.

By parallelogram law, $V_L - V_C$



The resultant voltage is given

$$\text{by, } V = \sqrt{V_R^2 + (V_L - V_C)^2} \quad \text{But } V_R = IR, \quad V_L = IX_L \quad \text{and} \quad V_C = IX_C$$

$$\text{Thus } V = I \sqrt{R^2 + (X_L - X_C)^2} \quad \text{Hence } V/I = \sqrt{R^2 + (X_L - X_C)^2}$$

This gives the total opposition to the flow of current in an a.c circuit, including resistance and reactances. It is called the **impedance (Z)** of the circuit.

$$\text{Thus the impedance is given by } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Also the phase difference between the voltage and current is given by

$$\tan\phi = \frac{V_L - V_C}{V_R} \quad \text{i.e. } \tan\phi = \frac{(IX_L - IX_C)}{IR} \quad \text{or} \quad \tan\phi = \frac{(X_L - X_C)}{R}$$

Hence, current in the circuit can be expressed as $I = I_m \sin(\omega t - \phi)$, the negative sign indicates that the inductive reactance dominates capacitive. i.e. current lags behind voltage.

(Refer page 246 of Text book for 'Analytical solution')

Power associated with the circuit:

$$\begin{aligned} P &= \int_0^T \frac{VIdt}{T} = \frac{1}{T} \int_0^T V_m \sin \omega t \cdot I_m (\sin \omega t - \phi) dt = \frac{V_m I_m}{T} \int_0^T \sin \omega t [\sin \omega t \cdot \cos \phi - \cos \omega t \cdot \sin \phi] dt \\ &= \frac{V_m I_m}{T} \left[\cos \phi \cdot \int_0^T \sin^2 \omega t dt - \sin \phi \cdot \int_0^T \sin \omega t \cdot \cos \omega t dt \right] \\ &= \frac{V_m I_m}{T} \left[\cos \phi \cdot \frac{1 - \cos 2\omega t}{2} dt - \sin \phi \cdot \int_0^T \frac{\sin 2\omega t}{2} dt \right] \end{aligned}$$

This gives $P = V_{\text{rms}} \cdot I_{\text{rms}} \cdot \cos\phi$. (Refer above for the integration results)

It shows that power in the circuit depends not only the values of current and voltage, but on the value of 'cosφ' also. Hence it is called the power factor of the circuit.

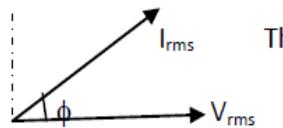
i.e. power factor, $\cos\phi = (V_R/V) = (IR/IZ)$ $\cos\phi = \frac{R}{Z}$

Thus power can also be expressed as $P = V_{rms} \cdot I_{rms} \cdot \frac{R}{Z}$

Wattless current:- It is the current in the a.c circuit in which there is a phase difference of ' $\pi/2$ ' between current and voltage.

We have $P = V_{rms} \cdot I_{rms} \cdot \cos\phi$ Here $\cos\phi = 0$

It can be represented as



Thus by resolving the current into the respective components, we get the

Wattless current = $I_{rms} \cdot \sin\phi$

RESONANCE:- It is the condition at which the inductive and capacitive reactances are equal so that the current in the circuit is maximum.

We know $I_m = V/Z$ Current is maximum, when impedance is minimum.

But $Z = \sqrt{R^2 + (X_L - X_C)^2}$ So 'Z' is minimum when $X_L = X_C$

Hence $Z_{min} = R$

$\omega L = 1/\omega C$ or $\omega^2 = 1/LC$

Advantages of A.C

- (i) Cost of production is low.
- (ii) It can be transmitted to distant places with minimum power loss, with the help of transformers.
- (iii) It can easily be converted into d.c, wherever needed.
- (iv) Current in the circuits can be controlled with the help of choke coils.

Disadvantages of A.C

- (i) It is more dangerous.
- (ii) It cannot be used in the processes like electroplating, making permanent magnets, etc.
- (iii) Energy loss can occur in the case of high frequency a.c due to **skin effect**. [It is the effect in which high frequency a.c confines through the surface of a conductor, when transmitted.]

Numericals:

1. A 255V, $500/\pi$ Hz supply is connected in series with a 100R resistor and a $2\mu\text{F}$ capacitor. Taking the phase of the emf as a reference, find the complex and rms values of (a) the current in the circuit, and (b) the potential difference across each element.

$$\begin{aligned}
 E_m &= 255\sqrt{2} = 361\text{V} \\
 \omega &= 2\pi \times \left(\frac{500}{\pi} \right) = 1000 \text{ rad.s}^{-1} \\
 E &= 361 \exp[j(1000t)] \text{ V} \\
 &= RI - \frac{j}{\omega C} I \\
 &= 100 I - \frac{j}{1000 \times (2 \times 10^{-6})} I \\
 &= (100 - 500j) I \\
 &= \sqrt{100^2 + 500^2} \exp\left[j \tan^{-1} \frac{-500}{100}\right] I \\
 &= 510 \exp[-1.37j] I
 \end{aligned}$$

Example 1: Calculate the frequency and the period of a sinusoidal waveform that complete 80 cycles in 24ms.

Solution:

$$f = \frac{\text{no. of cycle}}{\text{time}} = \frac{80}{24 \times 10^{-3}} = 3333.3333 \text{ (hertz)}$$

$$T \text{ (period)} = \frac{1}{f} = 0.3 \text{ msec.}$$

Example 2: A coil of 100 turns is rotated at 1500 rev/min in a magnetic field having a uniform density of 0.05 T, the axis of rotation being at right angles to the direction of the flux. The mean area per turn is 40 cm^2 . Calculate, (a) the frequency, (b) the period, (c) the maximum value of the generated e.m.f. and (d) the value of the generated e.m.f. when the coil has rotated through 30° from the position of zero e.m.f.

Solution:

$$(a) \text{frequency} = \frac{\text{no.of cycle}}{\text{second}} = \frac{\text{no.of revolution}}{\text{second}} = \frac{1500}{60} = 25 \text{ Hz}$$

(b) period = time of 1 cycle

$$= \frac{1}{\text{frequency}} = \frac{1}{25} = 0.04 \text{ sec}$$

$$(c) E_m = 2\pi ABNn$$

$$= 2\pi \times 0.05 \times 0.004 \times 100 \times 1500/60 = 3.14 \text{ volts}$$

$$(d) e = E_m \sin \theta = 3.14 \times 0.5 = 1.57 \text{ volts}$$

Different form of e.m.f. equation

The standard form of an alternating voltage which would be shown before, is $e = E_m \sin \theta$, which also can be written in the following forms

$$e = E_m \sin \omega t = E_m \sin \frac{2\pi}{T} t = E_m \sin 2\pi f t$$

Example 3: An alternating voltage of frequency 60 Hz has a maximum value of 120 V. Write down the equation for its instantaneous value. Taking time from the instant the current is zero and is becoming positive, find (a) the instantaneous value after 2.8 ms, and (b) the time taken to reach 96 A for the first time.

Solution:

$$v = 120 \sin 2\pi f t = 120 \sin 377t$$

$$(a) v = 120 \sin (377 \times 2.8 \times 10^{-3}) = 104.4 \text{ V.}$$

$$(b) 96 = 120 \sin (377 \times t) \Rightarrow \sin (377 \times t) = 96/120$$

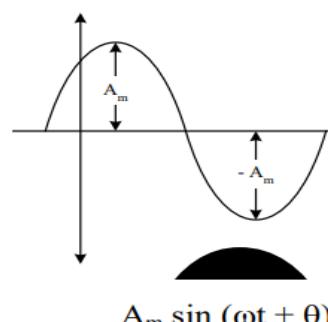
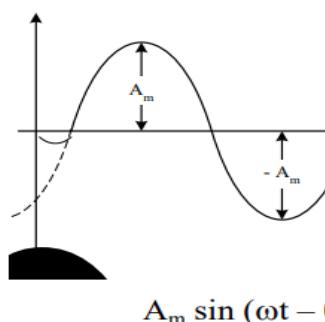
$$\sin (377 \times t) = 0.8 \Rightarrow (377 \times t) = \sin^{-1} 0.8$$

$$(377 \times t) = 0.927$$

$$\therefore t = 0.927/377 = 0.00246 \text{ second.}$$

Phase Relations:

As known from the property of the sine wave that it has a maximum value at $(\pi/2)$ and $3\pi/2$, with a zero value at $(0, \pi, \text{ and } 2\pi)$ as shown in figure 3. If the waveform is shifted



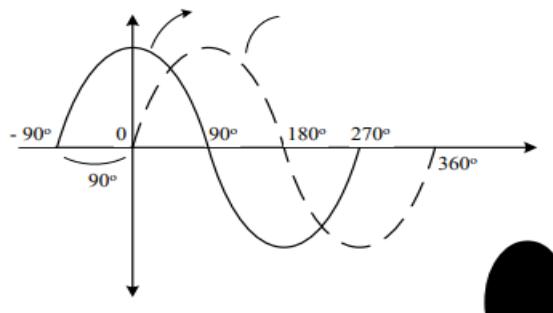
to the right or left of 0 deg, the expression become $A_m \sin(\omega t \pm \theta)$. Where θ is the angle in degree or radians that the waveform has been shifted.

If the waveform shifted with $+ 90^\circ$ ($\pi/2$), the waveform will called cosine wave. That is:

$$\sin(\omega t + 90^\circ) = \cos \omega t$$

while

$$\cos(\omega t - 90^\circ) = \sin \omega t$$



The term lead and lag are used to indicate the relationship between two sinusoidal waveforms of the same frequency. In figure 5, the cosine curve is said to lead the sine curve by 90° , and the sine is said to lag the cosine curve by 90° .

Example 4: what is the phase relationship between the following sinusoidal waveforms?

- a- $v = 10 \sin(\omega t + 30^\circ)$
 $i = 5 \sin(\omega t + 70^\circ)$
- b- $i = 15 \sin(\omega t + 60^\circ)$
 $v = 10 \sin(\omega t - 20^\circ)$
- c- $i = 2 \cos(\omega t + 10^\circ)$
 $v = 3 \sin(\omega t - 10^\circ)$
- d- $i = -\sin(\omega t + 30^\circ)$
 $v = 2 \sin(\omega t + 10^\circ)$
- e- $i = -2 \cos(\omega t - 60^\circ)$
 $v = 3 \sin(\omega t - 150^\circ)$

$$\begin{aligned} \sin(-\alpha) &= -\sin \alpha \\ \cos(-\alpha) &= \cos \alpha \\ -\sin \alpha &= \sin(\alpha \pm 180^\circ) \\ -\cos \alpha &= \cos(\alpha \pm 180^\circ) \end{aligned}$$

Solution:

- a- i leads v by 40° or v lags i by 40° .
- b- i leads v by 80° or v lags i by 80° .
- c- $i = 2 \cos(\omega t + 10^\circ) = 2 \sin(\omega t + 10^\circ + 90^\circ) = 2 \sin(\omega t + 100^\circ)$
 i leads v by 110° or v lags i by 110° .
- d- $-\sin(\omega t + 30^\circ) = \sin(\omega t + 30^\circ - 180^\circ) = \sin(\omega t - 150^\circ)$
 v leads i by 160° or i lags v by 160° .
or
 $-\sin(\omega t + 30^\circ) = \sin(\omega t + 30^\circ + 180^\circ) = \sin(\omega t + 210^\circ)$
 i leads v by 200° or v lags i by 200° .

$$e- -2 \cos(\omega t - 60^\circ) = 2 \cos(\omega t - 60^\circ - 180^\circ) = 2 \cos(\omega t - 240^\circ)$$

however

$$\cos \alpha = \sin(\alpha + 90^\circ)$$

so that

$$2 \cos(\omega t - 240^\circ) = 2 \sin(\omega t - 240^\circ + 90^\circ) = 2 \sin(\omega t - 150^\circ)$$

i and v in phase.

Average Value: the average value over a complete cycle of any current or voltage is equivalent to the D.C. value

$$\text{Average value} = \frac{\text{area under curve of one cycle}}{\text{length of curve}}$$

Example 5: find the average value for the waveforms of 1- figure (a), 2- figure (b), over

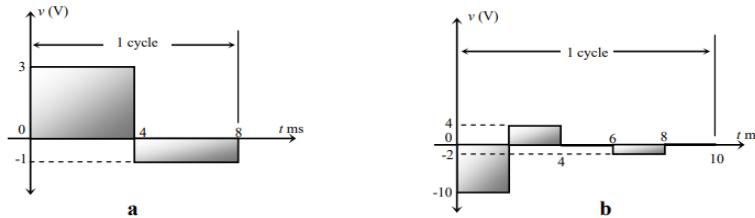


Figure 6

Solution:

$$1-\text{ Average value} = \frac{(3)(4) - (1)(4)}{10} = 1 \text{ volt.}$$

$$2-\text{ Average value} = \frac{-(10)(2) + (2)(4) - (2)(2)}{10} = 1.6 \text{ volt.}$$

one full cycle.

Example 6: Example: Estimate the complex impedance of a RLC circuit ($R=1\text{k}\Omega$, $L=0.1\text{ H}$, $C=10\text{ }\mu\text{F}$) which is powered by a sinusoidal source with frequency $f = 50\text{ Hz}$. The angular frequency is:

$$\omega=2\pi f=314\text{rad/s}$$

Then we can estimate the inductive and capacitive reactance:

$$X_L = 314 \times 0.1 = 31.4 \Omega$$

$$X_C = \frac{1}{314 \times 10 \cdot 10^{-6}} = \frac{10^{-6}}{3140} = 318 \Omega$$

The complex impedance is:

$$Z = 1000 + j(31.4 - 318) = 1000 - j286.6 = 1040e^{-j16^\circ} \Omega$$

From the above equation it is also seen that the impedance is $z=1040\Omega$ and the phase difference is $\varphi=-16\text{deg}$.

Example 7- a coil having a resistance of $6\text{ }\Omega$ and an inductance of 0.03 H is connected across a 50 V , 60 Hz supply. Calculate:

- 1- Current
- 2- The power factor.
- 3- The apparent power.
- 4- The active power.

Solution:

$$1- X_L = 2\pi fL = 2 \times \pi \times 60 \times 0.03 = 11.31\Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{6^2 + 11.31^2} = 12.8\Omega$$

$$I = \frac{50}{12.8} = 3.91 A$$

$$2- \phi = \tan^{-1} \frac{X_L}{R} = \tan^{-1} \frac{11.31}{6} = 62^\circ$$

$$\therefore \text{power factor} = \cos \phi = 0.469$$

$$3- \text{apparent power} = I \times V = 50 \times 3.91 = 195.5 \text{ VA}$$

$$4- \text{active power} = I \times V \cos \phi = 50 \times 3.91 \times 0.469 = 91.7 \text{ W}$$

$$\text{or } I^2 \times R = 3.91^2 \times 6 = 91.7 \text{ W}$$

Example 8: A 0.2 HP induction motor runs at an efficiency of 85 %. If the operating power factor is 0.8 lag, find the reactive power taken by the motor.

Solution:

$$P_{\text{out}} = 0.2 \text{ HP} = \frac{746}{5} = 149.2 \text{ W}$$

$$P_{\text{in}} = \frac{P_{\text{out}}}{\eta} = \frac{149.2}{0.85} = 175.53 \text{ W}$$

$$\text{p.f} = 0.8 = \cos \phi$$

$$\Rightarrow \sin \phi = 0.6$$

$$Q = VI \sin \phi = \frac{P_{\text{in}}}{\cos \phi} \times \sin \phi$$

$$= \frac{149.2}{0.85} \times \frac{1}{0.8} \times 0.6$$

$$= 131.65 \text{ VAR}$$

Example 9 Draw the waveforms & phasor diagrams for the expression given below.

i) $v = 100 \sin(314t + \pi/3)$ volts

ii) $i = 10 \sin(314t - \pi/2)$ amp

Example10 An alternating voltage is given by $v = 141.4 \sin(314t)$.

Find

- (i) Frequency (ii) RMS value (iii) Average value

(v) the time taken for the voltage to reach 100V for the first time after passing through zero value.

Example 11 The instantaneous value of emf is $V = 300 \sin (80 t)$. Determine:-

- (i) Average value
- (ii) RMS value
- (iii) Frequency
- (iv) Period
- (v) Angular Frequency
- (vi) Amplitude (vii) Instantaneous value of emf at $t = 0.1$ sec.

Problems:

1. A pure resistance of 12 is connected across a 240 V, 50 Hz supply. Calculate:-

(i) Current (ii) power consumed and (iii) Write down the equations for voltage and current.

2. A non-resistive inductance of 0.15 H is connected across an ac supply of 250 V, 60 Hz.

Determine: - (i) Inductive reactance (ii) RMS current (iii) Active power and (iv) Voltage and current equations.

3. A pure capacitance of 50 F is connected across a 50 Hz, 400 V supply. Calculate: - (i) capacitive reactance (ii) circuit current (iii) reactive power (iv) obtain equations of voltage and current.

4. A sinusoidal 50 Hz current of maximum value of 100 A flows through a capacitor of 318F capacitance. Calculate

(i) The expression for instantaneous current

(ii) Reactance of the capacitor

(iii) Equation of applied emf

(iv) Rms value of applied emf and current.

5. A voltage $v(t) = 141.4 \sin (314t + 10^\circ)$ is applied to a circuit and a steady current given by $i(t) = 14.14 \sin (314t - 20^\circ)$ is found to flow through it. Determine:-

(i) The p.f. of the circuit

(ii) The power delivered to the circuit

(iii) Draw the phasor diagram.

6. A coil connected to a 250 V, 50 Hz sinusoidal supply takes a current of 10 A at a phase angle of 30° . Calculate the resistance and inductance of coil and also power taken by the coil.

7. A choke coil connected across a 250 V, 50 Hz supply takes a current of 10 A at 0.8 pf.lag. What will be the power taken by the choke when connected across a 220 V, 25 Hz supply?

8. A current of 5A flows through a non-inductive resistance in series with a choking coil when supplied at 250V, 50 Hz. If voltage across the resistance is 125 V and across the coil, 200V, Calculate:-

(i) Impedance, reactance and resistance of the coil

(ii) Power absorbed by the coil

(iii) Total power

Also draw the phasor diagram.

9. Series with a condenser. The current in the circuit is 2.5 A. The power loss in the resistor is 100 watts and that in the condenser is negligible. Calculate the resistance and capacitance.

10. A 240 V, 50 Hz series circuit takes rms current of 20 A. The maximum value of the current occurs 1/900 second before the maximum value of voltage. Calculate: (i) power factor (ii) average power (iii) parameters of circuit.

11. A resistor and capacitor are connected in series across a 150 V, ac supply. When the frequency is 40 Hz, the current is 5A and when the frequency is 50 Hz, the current is 6A Find (i) R (ii) C.

12. A series R-L-C circuit consists of $R = 10$, $L = 0.318$ H, $C = 63.6$ F and emf source $e(t) = 100 \sin 314t$. Calculate:-

i) Expression for $i(t)$

ii) Phase angle between voltage and current

iii) Power factor

iv) Active power consumed

v) Draw the phasor diagram.

13. A coil of power factor 0.6 is in series with a 100 F capacitor. When connected to a 50Hz supply, the voltage across to coil is equal to voltage across capacitor. Find the resistance and inductance of the coil.

14. A 230 V, 50 Hz voltage is applied to a coil of $L = 5 \text{ mH}$ and $R = 2$ in series with a capacitance C. What value must C have so that the p.d. across the coil shall be 250V?

15. A series circuit having a resistance of 10 , an inductance of 0.025 H and a variable capacitance is connected to a 100 V, 25 Hz single phase supply, the current drawn being 8A. Calculate:-

(i) Capacitance (ii) Impedance (iii) power factor (iv) power consumed (v) Sketch the phasor diagram.

16. Draw the waveforms & phasor diagrams for the expression given below.

i) $v = 100 \sin (314 t + /3) \text{ volts}$

ii) $i = 10 \sin (314 t - /2) \text{ amp}$

17. An alternating voltage is given by $v= 141.4 \sin (314t)$.Find

(i) Frequency (ii) RMS value (iii) Average value

(v) The time taken for the voltage to reach 100V for the first time after passing through zero value.

18. The instantaneous value of emf is $V = 300 \sin (80 t)$. Determine:-

(i) Average value

(ii) RMS value

(iii) Frequency

(iv) Period

(v) Angular Frequency

(vi) Amplitude (vii) Instantaneous value of emf at $t = 0.1 \text{ sec.}$

MODULE 2

Three Phase System

2.1 Introduction:

- There are two types of systems in electric circuits: single-phase and three-phase systems. In a single-phase circuit, current flows through one wire with a return path called the neutral line, allowing minimal power transport. Both the generating and load stations are single-phase in this system, which has been used for a long time.
- In 1882, a new polyphone system was invented, allowing more than one phase for generating, transmitting, and loading power. A **three-phase circuit** is a polyphase system where three phases are sent together from the generator to the load.
- Each phase in a three-phase system has a 120-degree electrical phase difference, dividing the total 360 degrees into three equal parts. This ensures continuous power generation as all three phases work together. The sinusoidal waves for a three-phase system illustrate this division.
- The three phases can be used as single phase each. So if the load is single phase, then one phase can be taken from the **three phase circuit** and the neutral can be used as ground to complete the circuit

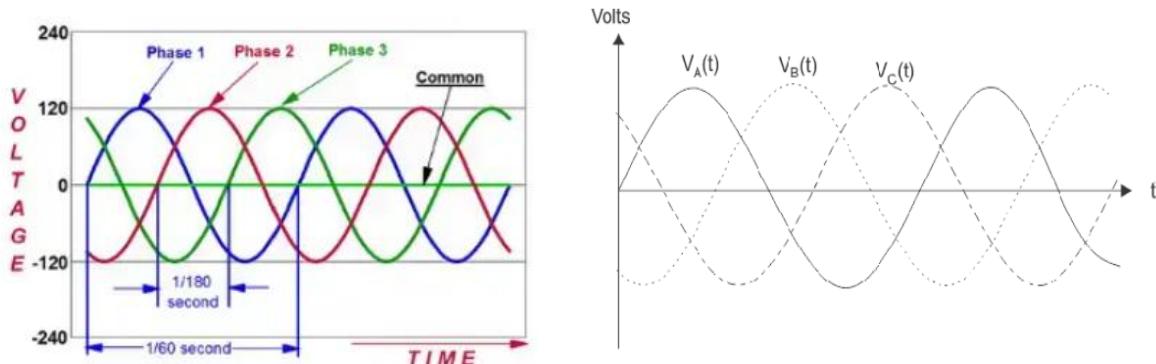


Figure: 2.1 Three phase waveform

2.2 Three Phase Circuit Definition: A three-phase circuit is defined as a system where three electrical phases are used together, each 120 degrees apart, to provide continuous power.

ADVANTAGES OF THREE-PHASE SYSTEM OVER SINGLE PHASE

1. Almost all electric power is generated and distributed in three-phase.
2. When one-phase or two-phase inputs are required, they are taken from the three-phase system rather than generated independently.
3. Even when more than 3 phases are needed, they can be provided by manipulating the available three phases. Example: Aluminum industry, where 48 phases are required for melting purposes.
4. The instantaneous power in a three-phase system can be constant (not pulsating). This results in uniform power transmission and less vibration of three-phase machines.
5. For the same amount of power, the three-phase system is more economical than the single-phase.
6. The amount of wire required for a three-phase system is less than that required for an equivalent single-phase system.

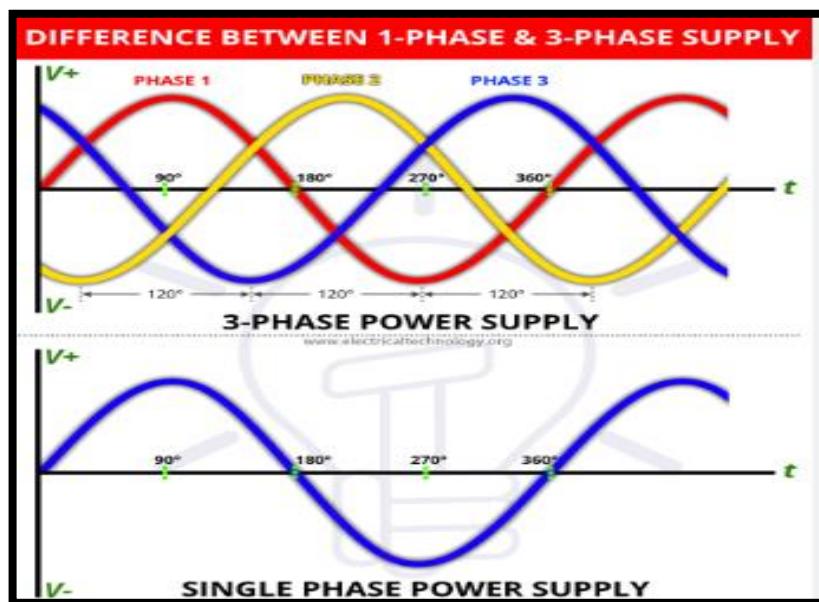


Figure: 2.2 Difference between 1-Phase and 3-Phase supply

2.3 Generation of three phase system: Consider following 3 phase simple loop generator shown in figure 2.3a. Its generated voltage will be as shown in figure 2.3b.

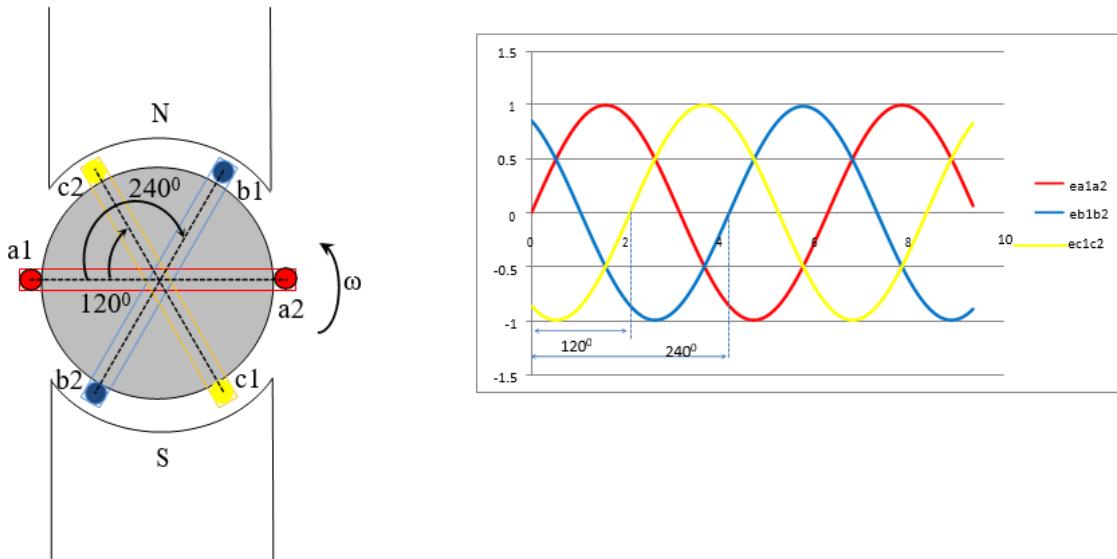


Figure: 2.3 (a)3-phase loop generator

(b)3-phase waveform

$$e_{a1a2} = E_m \sin(\omega t) \quad \text{-----(1)}$$

$$e_{c1c2} = E_m \sin(\omega t - 120^\circ) \quad \text{-----(2)}$$

$$e_{b1b2} = E_m \sin(\omega t - 240^\circ) \quad \text{-----(3)}$$

E_{a1a2} , E_{b1b2} & E_{c1c2} are RMS values.

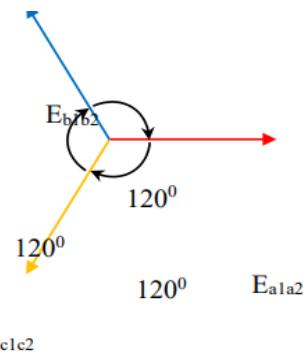


Figure: 2.4 Phasor diagram

Phase Sequence: It is the sequence in which current or voltages in different phases attain their maximum values.

Phase sequence may be R-Y-B or R-B-Y as shown below

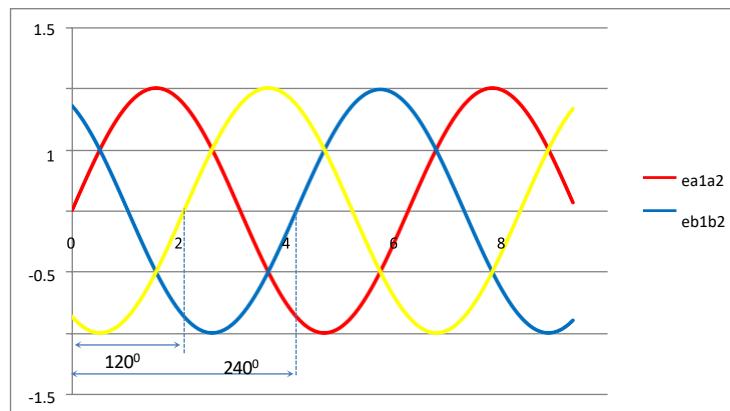


Figure:2.5(a) Phase Sequence R-Y-B

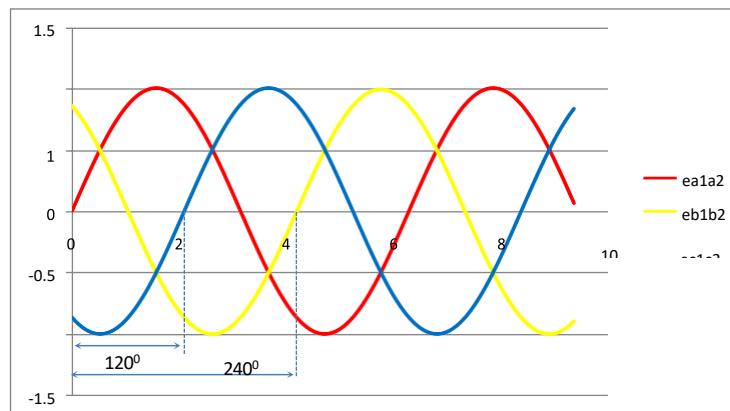


Figure: 2.5(b) Phase Sequence R-B-Y

Interconnection of 3-phase system: The six terminals of three phase winding can be connected to form any of the below.

1. Star or WYE (Y) connected 3- ϕ System
2. Mesh or Delta(Δ) connected 3 - ϕ system

Star or WYE (Y) connected 3-φ System:

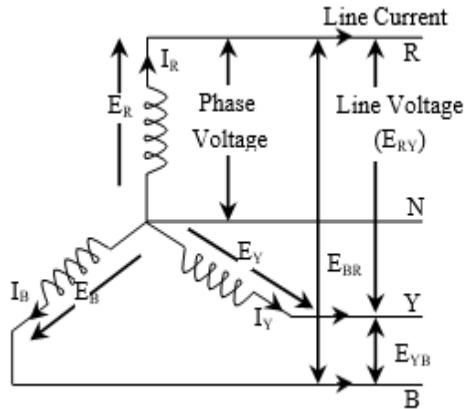


Figure 2.6: Star or WYE (Y) connected 3-φ System

E_R , E_Y & E_B are called phase voltages.

I_R , I_Y & I_B are called phase currents.

E_{RY} , E_{YB} & E_{BR} are called line voltages.

From figure it is clear that

$$I_R + I_Y + I_B = 0$$

Line current (I_L) = Phase Current (I_P)

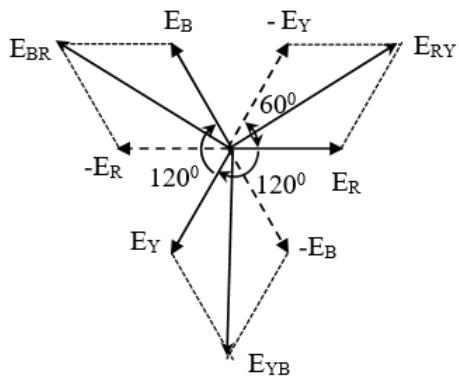


Figure 2.7: phasor diagram

Line voltage from above phasor diagram

$$E_{RY} = E_R - E_Y = E_R + (-E_Y)$$

Its magnitude

$$E_{RY} = \sqrt{E_R^2 + E_Y^2 + 2E_R E_Y \cos 60^\circ}$$

For balanced 3-phase system

$$E_R = E_Y = E_B = E_P \quad (\text{Phase Voltage})$$

$$\text{So } E_{RY} = \sqrt{E_p^2 + E_p^2 + 2E_p E_p \cos 60^\circ} = \sqrt{E_p^2 + E_p^2 + 2E_p^2 \times \frac{1}{2}}$$

$$E_{RY} = \sqrt{3} E_p$$

Similarly

$$E_{YB} = E_Y + (-E_B) = \sqrt{E_Y^2 + E_B^2 + 2E_Y E_B \cos 60^\circ} = \sqrt{3} E_p$$

$$E_{BR} = E_B + (-E_R) = \sqrt{E_B^2 + E_R^2 + 2E_B E_R \cos 60^\circ} = \sqrt{3} E_p$$

Hence for balanced system

$$E_{RY} = E_{YB} = E_{BR} = \sqrt{3} E_p = E_L$$

If ϕ is the angle between phase voltage and phase current then

$$\text{Active power of 3 phase} = 3 E_p I_p \cos \phi = 3 \frac{E_L}{\sqrt{3}} I_L \cos \phi = \sqrt{3} E_L I_L \cos \phi \quad \text{W}$$

$$\text{Reactive power of 3 phase} = 3 E_p I_p \sin \phi = 3 \frac{E_L}{\sqrt{3}} I_L \sin \phi = \sqrt{3} E_L I_L \sin \phi \quad \text{VAR}$$

$$\text{Apparent power of 3 phase} = 3 E_p I_p = 3 \frac{E_L}{\sqrt{3}} I_L = \sqrt{3} E_L I_L \quad \text{VA}$$

Mesh or Delta (Δ) connected 3 - ϕ System

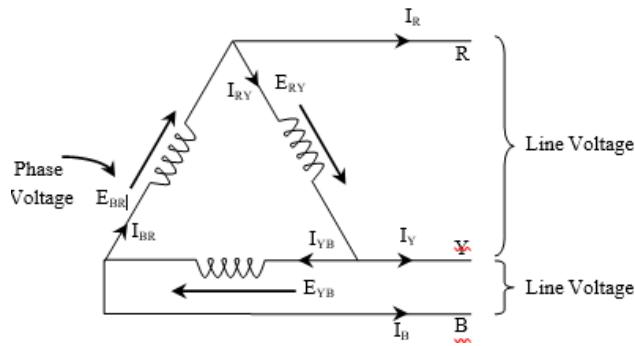


Figure 2.8: Mesh or Delta (Δ) connected 3- ϕ System

E_{RY} , E_{YB} & E_{BR} are called phase voltages.

I_{RY} , I_{YB} & I_{BR} are called phase currents.

From figure it is clear that

$$E_{RY} + E_{BR} + E_{BY} = 0$$

$$\text{Line Voltage } (E_L) = \text{Phase Voltage } (E_P)$$

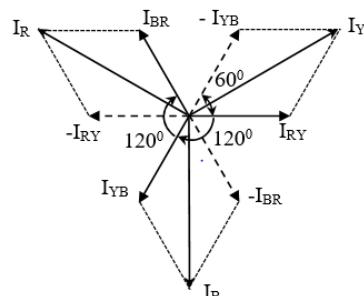


Figure 2.9: phasor diagram

Apply KCL at node R

$$I_R = I_{BR} - I_{RY} = I_{BR} + (-I_{RY})$$

Its magnitude from above phasor diagram

$$I_R = \sqrt{I_{BR}^2 + I_{RY}^2 + 2I_{BR}I_{RY} \cos 60^\circ}$$

For balanced 3-phase system

$$I_{RY} = I_{YB} = I_{BR} = I_P \quad (\text{Phase Current})$$

$$\text{So } I_R = \sqrt{I_p^2 + I_p^2 + 2I_p I_p \cos 60^\circ} = \sqrt{I_p^2 + I_p^2 + 2I_p^2 \times \frac{1}{2}}$$

$$I_R = \sqrt{3}I_p$$

Similarly

$$I_Y = I_{RY} + (-I_{YB}) = \sqrt{I_p^2 + I_p^2 + 2I_p I_p \cos 60^\circ} = \sqrt{3}I_p$$

$$I_Y = I_{YB} + (-I_{BR}) = \sqrt{I_p^2 + I_p^2 + 2I_p I_p \cos 60^\circ} = \sqrt{3}I_p$$

Hence for balanced system

$$I_R = I_Y = I_B = I_L = \sqrt{3}I_p$$

If ϕ is the angle between phase voltage and phase current then

$$\text{Active power of 3 phase} = 3 E_p I_p \cos \phi = 3 E_L \frac{I_L}{\sqrt{3}} \cos \phi = \sqrt{3} E_L I_L \cos \phi \quad \text{W}$$

$$\text{Reactive power of 3 phase} = 3 E_p I_p \sin \phi = 3 E_L \frac{I_L}{\sqrt{3}} \sin \phi = \sqrt{3} E_L I_L \sin \phi \quad \text{VAR}$$

$$\text{Apparent power of 3 phase} = 3 E_p I_p = 3 E_L \frac{I_L}{\sqrt{3}} = \sqrt{3} E_L I_L \quad \text{VA}$$

Numerical on Three phase circuits

Example 1. A balanced star connected load of $(8 + j6)$ phase is connected to a 3 phase, 230V, 50c/s supply. Find the line current, power factor, power, reactive volt amperes and total volt amperes

Solution :

Line voltage = $ E_L = 230$ volts	$\therefore E_\phi = \frac{ E_L }{\sqrt{3}} = \frac{230}{\sqrt{3}} = 133$ volts
Load impedance/phase = $Z_\phi = (8 + j6) \Omega$	$= 10 \angle 36.9^\circ \Omega$
$\therefore Z_\phi = 10 \Omega$	
and phase angle $\phi = 36.9^\circ$ lag	
$I_L = I_\phi = \frac{ E_\phi }{ Z_\phi } = \frac{133}{10} = 13.3$ A	
Power factor = $\cos \phi = \cos 36.9^\circ = 0.8$ lag	$\left[\text{Also } \cos \phi = \frac{ R_\phi }{ Z_\phi } = \frac{8}{10} = 0.8 \right]$
Total volt amperes = $\sqrt{3} E_L I_L = \sqrt{3} E_\phi I_\phi $	$= \sqrt{3} \times 230 \times 13.3 = 5298$ VA
Power = $\sqrt{3} E_L I_L \cos \phi$	$= 5298 \times 0.8$
	$= 4239$ watts
Reactive volt-amperes = $\sqrt{3} E_L I_L \sin \phi$	$= (5298) \times 0.6$
	$= 3179$ VAR

2 A balanced star connected load of $(3 - j4)2$ impedance is connected to 400 V three phase supply. What is the real power consumed?

Solution:

Note: For Star system, unless specified, the given voltage is taken as line voltage.

$$\begin{aligned}
 \text{Here } |E_L| &= 400 \text{ volts} \\
 |Z_\phi| &= (3 - j4) = 5 \angle -53.13^\circ \Omega \\
 \therefore |Z_\phi| &= 5\Omega \text{ and phase angle } \phi = 53.13^\circ \text{ lead} \\
 \text{and } |E_\phi| &= \frac{|E_L|}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231 \text{ volts} \\
 \text{By Ohm's law, } |I_\phi| &= \frac{|E_\phi|}{|Z_\phi|} = \frac{231}{5} = 46.2 \text{ A} \\
 \therefore |I_L| &= |I_\phi| = 46.2 \text{ A} \\
 \text{Real power} &= \sqrt{3} |E_L| |I_L| \cos \phi \\
 &= \sqrt{3} \times 400 \times 46.2 \cos 53.13^\circ \\
 &= 19205 \text{ W} = 19.205 \text{ KW}
 \end{aligned}$$

Case (b): Balanced delta connected load

Example 3. A symmetrical 3 phase, 400 V system supplies a balanced mesh connected load. The current in each branch circuit is 20A and the phase angle is 40° lag. Find (a) the line current (b) the total power.

Solution: Data: Type of connection = delta

$$\begin{aligned}
 |E_L| &= |E_\phi| = 400 \text{ volts} \\
 |I_\phi| &= 20 \text{ A} \\
 \phi &= 40^\circ \text{ lag} \\
 \text{(a)} \quad |I_L| &= \sqrt{3} |I_\phi| \\
 &= \sqrt{3} \times 20 = 34.64 \text{ A} \\
 \text{(b)} \quad \text{Total power} &= \sqrt{3} \times 400 \times 34.64 \times \cos 40^\circ \\
 &= 18386 \text{ W} \\
 &= 18.386 \text{ KW.}
 \end{aligned}$$

Example 4. Three impedances each of 10 resistance and 52 inductive reactance are connected in delta to a 400 V, 36 supply. Determine the current in each phase and in each line. Calculate also the total power drawn from the supply and the p.f of the load.

Solution:

$$\begin{aligned}
 R_\phi &= 10 \Omega, \quad X_\phi = 5 \Omega \\
 |E_L| &= |E_\phi| = 400 \text{ volts, since delta connection} \\
 Z_\phi &= 10 + j5 = 11.18 \angle 26.6^\circ \Omega \\
 \therefore |Z_\phi| &= 11.18 \Omega, \quad \phi = 26.6^\circ \text{ lag} \\
 \therefore |I_\phi| &= \frac{|E_\phi|}{|Z_\phi|} = \frac{400}{11.18} = 35.8 \text{ A} \\
 |I_L| &= \sqrt{3} |I_\phi| = \sqrt{3} \times 35.8 \\
 &= 62.00 \text{ A} \\
 \text{Power factor} &= \cos \phi = \cos 26.6^\circ = 0.89 \text{ lag} \\
 \left. \begin{array}{l} \text{Also } \cos \phi = \frac{R_\phi}{Z_\phi} = \frac{10}{11.18} \end{array} \right\} \\
 \text{Total power} &= \sqrt{3} |E_L| |I_L| \cos \phi \\
 &= 3 \times 400 \times 62 \times 0.89 \\
 &= 38230 \text{ W} \\
 &= 38.23 \text{ KW}
 \end{aligned}$$

DC Generator

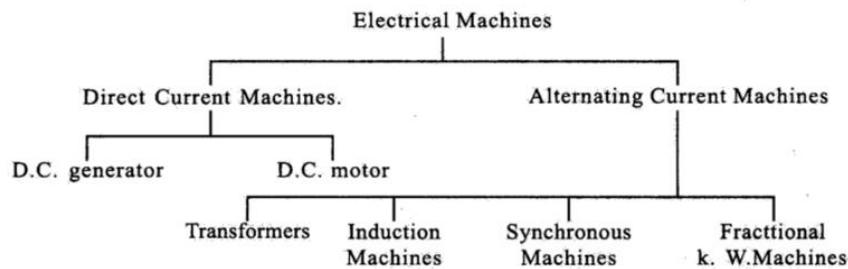


Fig 2.10: Electrical machines types

2.4 INTRODUCTION

- A DC generator is *an electrical machine that converts mechanical energy into electrical energy.*

DC machines Introduction

DC machines are broadly classified into two types DC motor and DC generator. DC motor is a device which convert electrical energy into mechanical energy. When a current carrying conductor placed in a magnetic field, it experienced a force. DC generator is a device which converts mechanical (rotational) energy into electrical energy. It works based on the principle of faradays law of electromagnetic induction. Whenever the conductor cuts by the magnetic field an emf is induced on it

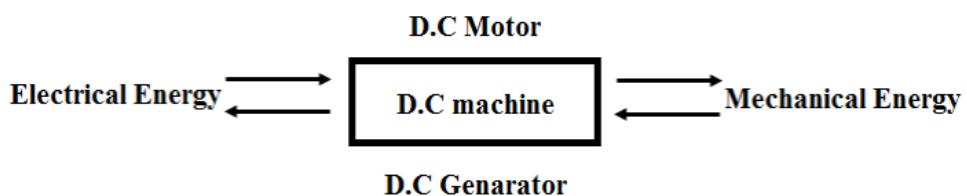


Figure 2.11: DC Machine block diagram

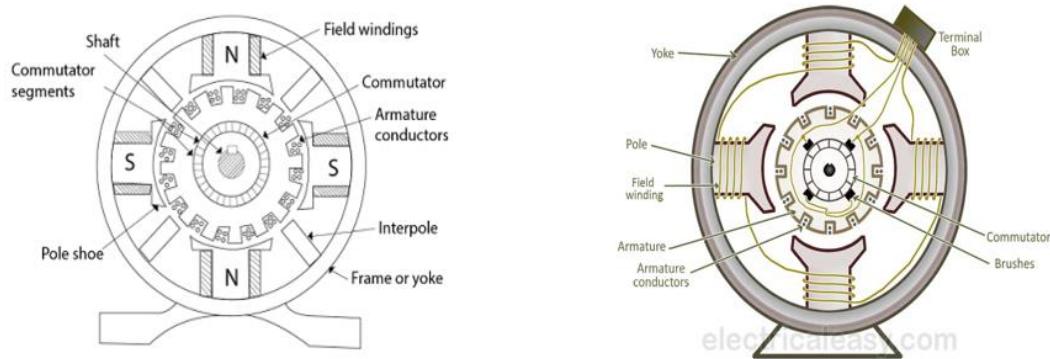


Figure 2.12 DC machine

Construction Of DC Machine

Based on the working principle DC motor & generator, requirement for the both machines are magnetic fields, conductor & mechanical movement. So construction wise both motor & generator are same. Depends upon the types of input (electrical or mechanical) given to the machines it should be differentiated by either motor or generator

Field winding:

Field windings to be used to produce uniform magnetic flux. The winding are placed in a poles which is located in the yoke. The yoke is a circular outer part of DC machines which is made upon cast iron and projecting even number of poles. It provides space for the field windings. Field windings are like aluminium or copper. Poles shoe are extended and widen because of flux part distribution over to the armature.

Armature Systems:

It is the rotating part of the machine. It consists of two major parts. One is armature core and other is armature winding. A shaft carries cylindrical shaped with slot which is used place the rotating (armature) windings. Usually rotating part of the machines are generally called Rotor. It provides house for the armature conductors and also provides low reluctance to magnetic flux created by the field coils

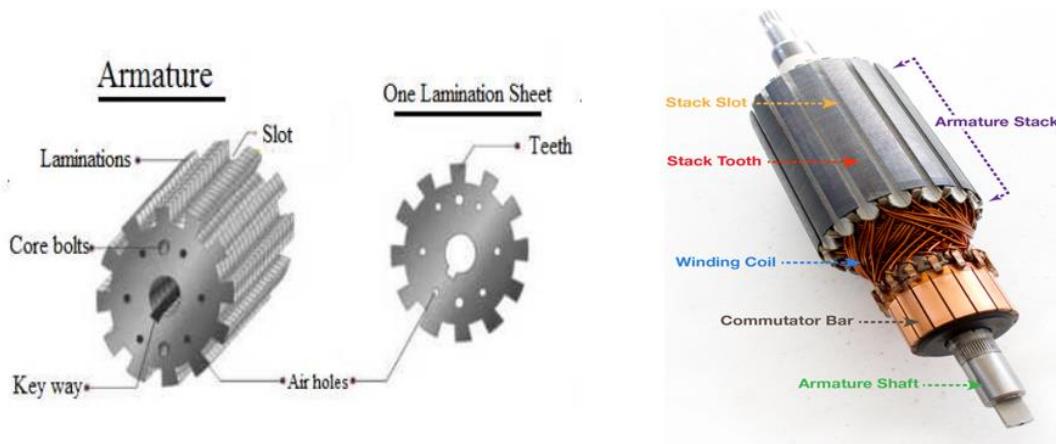


Figure 2.13: Armature

Interconnection of armature conductors/coils is called Armature Windings. It is placed over to the periphery of the armature slots. Main function of the armature coil is to generate e.m.f. also carries current in case of D.C motor. Usually copper are preferred to making materials for this windings.

Commutator:

Induced e.m.f in a D.C generator is alternating nature. Rectification is required so commutator are used for this purpose

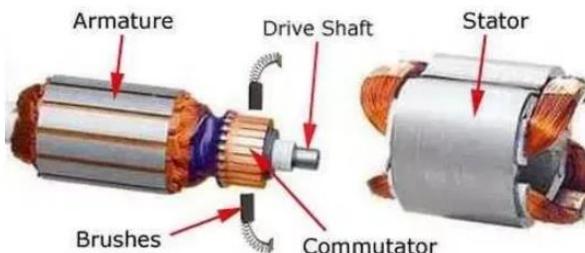


Figure 2.14: commutator

It is cylindrical shaped and each commutator segments are made with copper and connected to armature coil with strips. Connections are shown in figure.

Brushes:

It is stationary part & seated over to the commutator. It collects current from commutator and given to stationary external circuit. Usually, it is made with carbon like soft materials. It is in rectangular shape and made to press on the surface of the commutator with help of a spring. For D.C machines two carbon brushes are required.

2.5 Types of Armature Winding:

More number of conductors arranged in a systematic manner is called windings. Depending upon the conductors connections the armature windings classified as two types,

1.Lap winding

2.Wave winding

Lap winding:

First conductor connected to 3rd conductor for return path and continuation is join or overlap into 2nd conductor and move into so on. In this conductor connection are number of poles is number of parallel paths. i.e., $A=P$ number of parallel paths. Figure shows the Lap winding diagram

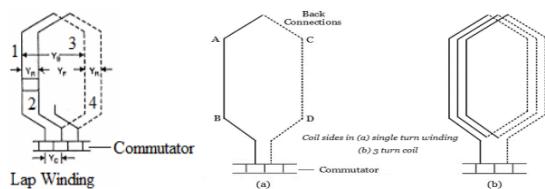


Figure 2.15: Lap winding

Wave winding:

First conductor is move forwarded shown in figure. The conductor moves ahead like wave. For this connection the total conductors are divided into two parallel paths always i.e., $A=2$

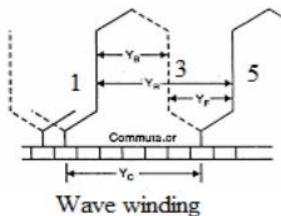


Figure 2.16: wave winding

Working Principle of D.C Generator

D.C generator works based on the principle of Faraday's law of electromagnetic induction." Whenever the conductor cuts by the magnetic field e.m.f (electro motive force) induced on the conductor. Direction of the induced e.m.f can be determined using Fleming's right-hand rule.

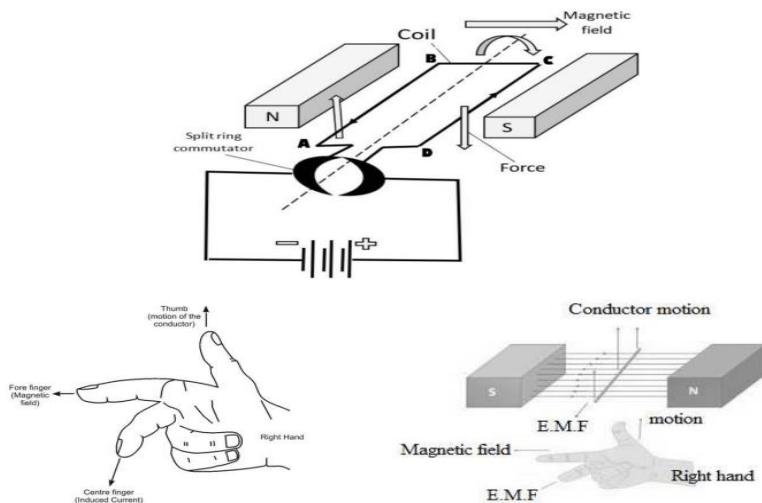


Figure 2.17: Working Principle of D.C Generator

Fleming's Right Hand Rule:

Stretch the thumb, index finger and middle finger of right hand perpendicular to each other. If index finger points out the direction of the magnetic field and thumb represents the direction of the motion conductor means than middle finger gives the direction of the induced e.m.f. In this way we can identify the direction of induced e.m.f using Fleming's right-hand rule.

E.M.F Equation of dc generator

Let

P = Number of poles in the filed systems

Φ = flux per pole (webers)

N = Speed of the armature (revolution per minute –rpm)

Z = Total number of conductors

= Number conductors X slot per conductor

A = parallel paths For Lap winding, A=P; Wave winding A=2

$$induced e.m.f \propto \frac{d\varphi}{dt} \quad \dots \dots \dots 1$$

$$d\varphi = P X \varphi \quad \dots \dots \dots 2$$

$$dt = \frac{60}{N} \quad \dots \dots \dots 3$$

$$induced e.m.f per conductor = 1 \times \frac{d\varphi}{dt} = \frac{P X \varphi}{\frac{60}{N}}$$

According to faraday's law, the rate of change of conductor's cuts by the magnet filed

$$e = \frac{P \varphi N}{60} X \frac{Z}{A} = \frac{\varphi Z N P}{60 A} \text{ volts} \quad \dots \dots \dots 4$$

e.m.f induced on the conductor

For total conductors per parallel path

$$E_{induced} = \frac{\varphi Z N}{60} \text{ volts} \quad \dots \dots \dots 5$$

$$E_{induced} = \frac{\varphi Z N P}{120} \text{ volts} \quad \dots \dots \dots 6$$

Equation-4 general equation of a D.C. generator.

For Lap wound machine, A = P..... **Equation-5**

For wave wound machine, A = 2,..... **Equation-6**

Solved Example Problems of DC Machines (generator)

1. A wave connected armature winding has 19 slots with 54 conductors per slot. If the flux per pole is 0.025 Wb and number of poles is 8, find the speed at which the generator should be run to give 513 V. Also find the speed if the armature is lap connected

Solution:

$$P = 8$$

$$\phi = 0.025 \text{ Wb}$$

$$Z = 19 \times 54 = 1026$$

$$A = 2 \text{ (for wave)}$$

$$E_g = 513 \text{ Volts}$$

$$E_g = \frac{P\phiZN}{60A} \quad (\text{or}) \quad N = \frac{60E_gA}{\phi ZP}$$

$$N = \frac{60 \times 513 \times 2}{0.025 \times 1026 \times 8} \quad (\text{For lap wound } A = P = 8)$$

$N = 1200 \text{ rpm.}$

2. The armature of a 4-pole, 600 rpm, lap wound generator has 100 slots. If each coil has 4 turns, calculate the flux per pole required to generate an emf of 300 V

Solution:

$$\text{No. of poles} = 4; \text{ Speed } 600 \text{ rpm,}$$

$$\text{No. of slots} = 100; E_g = 300 \text{ V}$$

$$\text{No. of conductors } Z = 100 \times 4 \times 2 = 8$$

$$\text{Lap wound generator, } A = P = 4$$

$$\text{Generated emf } E_g = \frac{P\phiZN}{60A}$$

$$\therefore \text{Flux/Pole } \phi = \frac{E_g \times 60A}{PZN}$$

$$= \frac{300 \times 60 \times 4}{4 \times 800 \times 600}$$

$\phi = 37.5 \text{ mWb.}$

2.6 DC Motor:

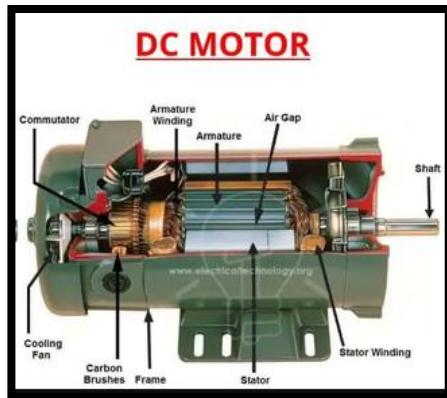


Figure 2.18:DC Motor

A DC motor is an electrical machine that converts electrical energy into mechanical energy

Principle of Operation of a D.C Motor

The principle of operation of d.c motor can be stated in a single statement as "when a current carrying conductor is placed in a magnetic field; it experiences a mechanical force". In de motor, field winding produces a required magnetic field while armature conductors play a role of a current carrying conductors and hence armature conductors experience a force. As conductors are placed in the slots which are on the periphery, the individual force experienced by the conductors acts as a twisting or turning force on the armature which is called a torque.

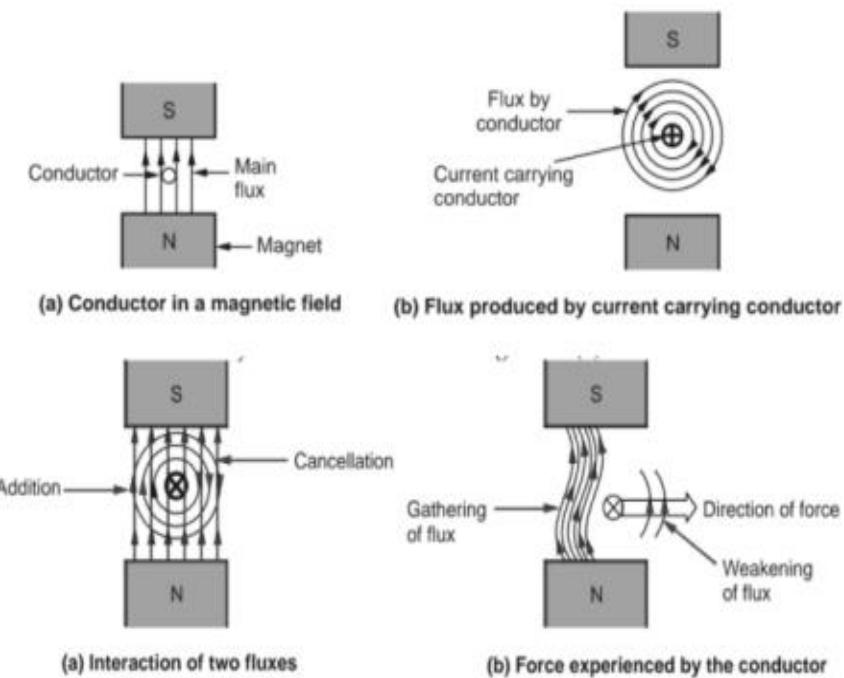


Figure 2.19: Principle of Operation of a D.C Motor

2.7 TORQUE EQUATION: Torque is turning or twisting force about an axis torque is measured by the product of force and the radius at which the force acts. Consider a wheel of radius 'r' metres acted on by a circumferential force 'F' Newton as shown in fig 2.20

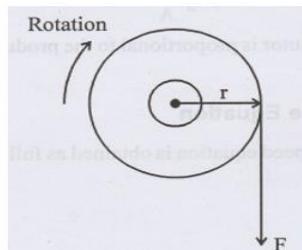


Figure 2.20: Torque

The angular velocity of the wheel is

$$\omega = 2\pi N / 60$$

Torque, $T = F \times r$

Work done per revolution = $F \times$ distance moved

$$= F \times 2\pi r$$

Power developed, $P = \text{Work done} / \text{Time} = F \times 2\pi r / \text{time for 1 rev}$

$$= \frac{F \times 2\pi r}{60/N}$$

$$P = (F \times r) \left(\frac{2\pi N}{60} \right)$$

$$P = T\omega$$

T = Torque in N-m,

ω = Angular speed r/sec

Power in armature = Armature torque $\times \omega$

$$E_b I_a = T_a \times \frac{2\pi N}{60}$$

$$E_b = \frac{\phi ZNP}{60A}$$

$$\frac{\phi ZNP}{60A} I_a = T_a \frac{2\pi N}{60}$$

$$T_a = 0.159 \phi I_a \frac{PZ}{A}$$

2.8 Back Emf and its Significance in DC Motor:

In a DC motor when the armature rotates, the conductors cut the lines of force of magnet field, so that an emf is induced in the armature. This induced emf acts in opposition to the current in the machine and the applied voltage so this emf is called back emf or counter emf. According to lenz's law, the direction of the back emf opposes the supply voltage the back emf is calculated from the equation of induced emf in the generator.

$$E_b = \frac{P\PhiZN}{60A}$$

Where

ϕ - flux/pole in wb

P - Number of poles

Z - Total number of conductors in the armature

N - Speed in rpm

A - No of parallel paths

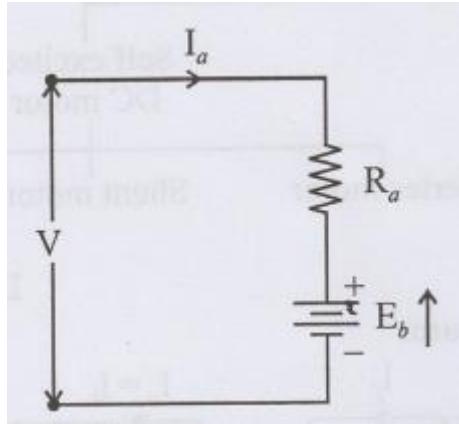


Figure 2.21: equivalent circuit of a motor

The equivalent circuit of a motor is shown in fig . Here, the armature circuit is equivalent to a source of emf E_b , in series with a resistance R_a and a DC supply is applied across, series connection of R_a and E_b . The voltage equation is $V = E_b + I_a R_a$

From the above voltage equation

$$\text{Armature current } I_a = V - E_b / R_a$$

Where V - Applied voltage , E_b - Back emf, I_a - Armature current, R_a - Armature resistance

$V - E_b$ - Net voltage in the armature circuit

- (i) If the motor speed is high, back emf E_b is large and armature current is small.
- (ii) If the motor speed is low, back emf E_b will be less and armature current is more.

2.9 The significance of Back EMF: The presence of back emf makes the d.c. motor a self-regulating machine i.e., it makes the motor to draw as much armature current as is just sufficient to develop the torque required by the load.

Numericals on Torque Equation of DC Motor

1. A 4 pole DC motor has a wave wound armature with 594 conductors. The armature current is 40 A and flux per pole is 7.5 mwb. Calculate the torque developed by the motor

Given : $P = 4$, $Z = 594$, $I_a = 40 \text{ A}$, $\phi = 7.5 \text{ mwb}$

For wave wound $A = 2$

Torque developed

$$T_a = 0.159 \frac{I_a P Z}{A}$$

$$= 0.159 \times 7.5 \times 10^{-3} \times \frac{40 \times 4 \times 594}{2}$$

$$T_a = 56.66 \text{ N-m}$$

2. A 4 pole DC motor takes an armature current of 50 A. The armature has 480 lap connected conductors. The flux per pole is 20 mWb. Calculate the gross torque developed by the motor.

Solution:

No. of poles $P = 4$

Armature current $I_a = 50 \text{ A}$

No. of conductors $Z = 480$

$\phi = 20 \times 10^{-3}$

Lap connection $A = P = 4$.

$$\begin{aligned} T_a &= 0.159 \phi I_a \frac{PZ}{A} \\ &= 0.159 \times 20 \times 10^{-3} \times \frac{50 \times 4 \times 480}{4} \\ T_a &= 76.32 \text{ N-m.} \end{aligned}$$

2.10 TRANSFORMER

- The transformer is a static device that transfers electrical energy from one electrical circuit to another electrical circuit at constant frequency.

Construction & Principle of Working:

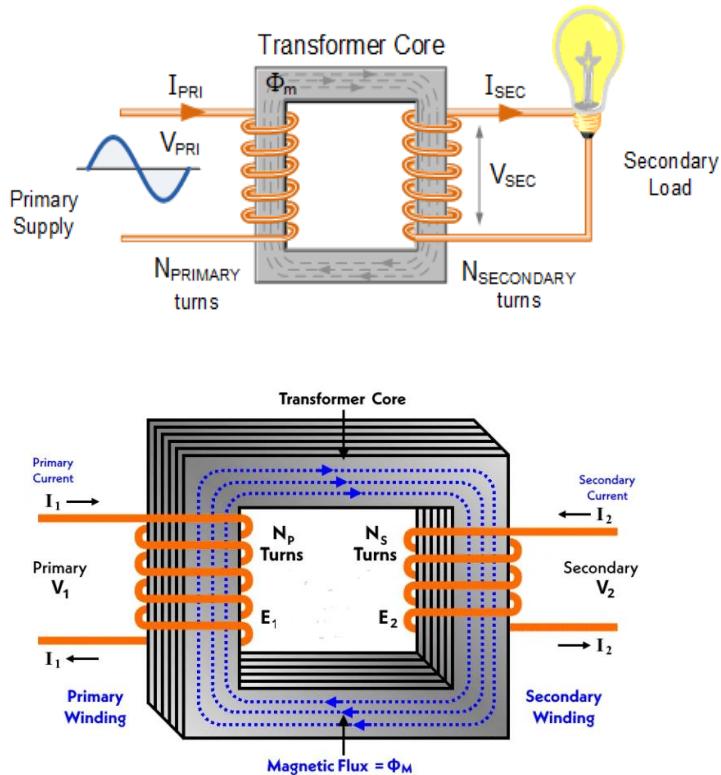


Figure 2.22 Transformer construction

Construction & Principle of Working:

The principle of mutual induction states that when two coils are inductively coupled and if current in one coil is changed uniformly then an e.m.f. gets induced in the other coil. This e.m.f. can drive a current, when a closed path is provided to it. The transformer works on the same principle. In its elementary form, it consists of two inductive coils which are electrically separated but linked through a common magnetic circuit. The two coils have high mutual inductance. The basic transformer is shown in the Fig 2.23

One of the two coils is connected to a source of alternating voltage. This coil in which electrical energy is fed with the help of source is called primary winding (P). The other winding is connected to load. The electrical energy transformed to this winding is drawn out to the load.

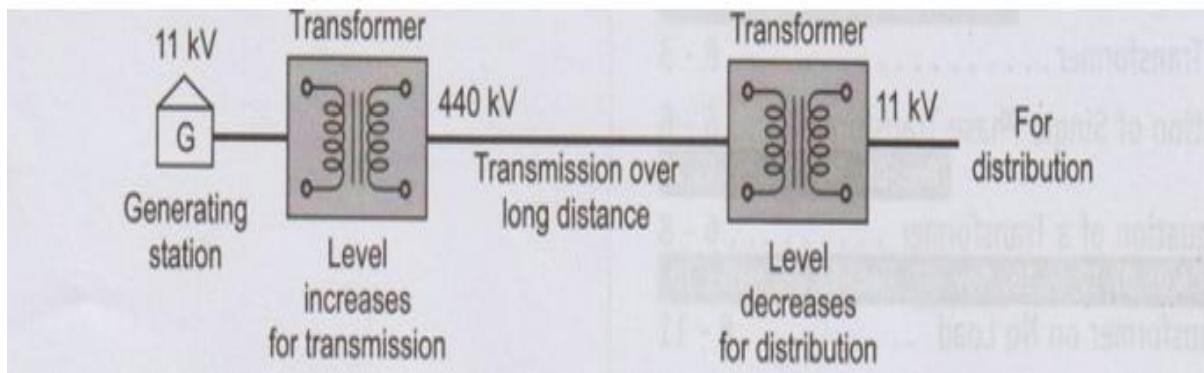


Figure 2.23: use of transformers in transmission systems

This winding is called secondary winding (S). The primary winding has N_1 number of turns while the secondary winding has N_2 number of turns. Symbolically the transformer is indicated as shown in the Fig. 2.24.

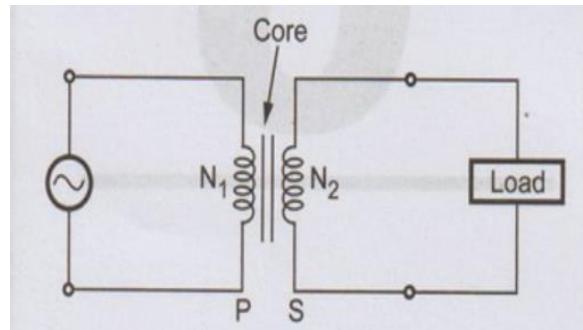


Figure 2.24 symbolic representation

When primary winding is excited by an alternating voltage, it circulates an alternating current. This current produces an alternating flux (ϕ) which completes its path through common magnetic core as shown dotted in the Fig. 2.25. Thus an alternating, flux links with the secondary winding. As the flux is alternating, according to Faraday's law of an electromagnetic induction, mutually induced e.m.f. gets developed in the secondary winding. If now load is connected to the secondary winding, this e.m.f. drives a current through it.

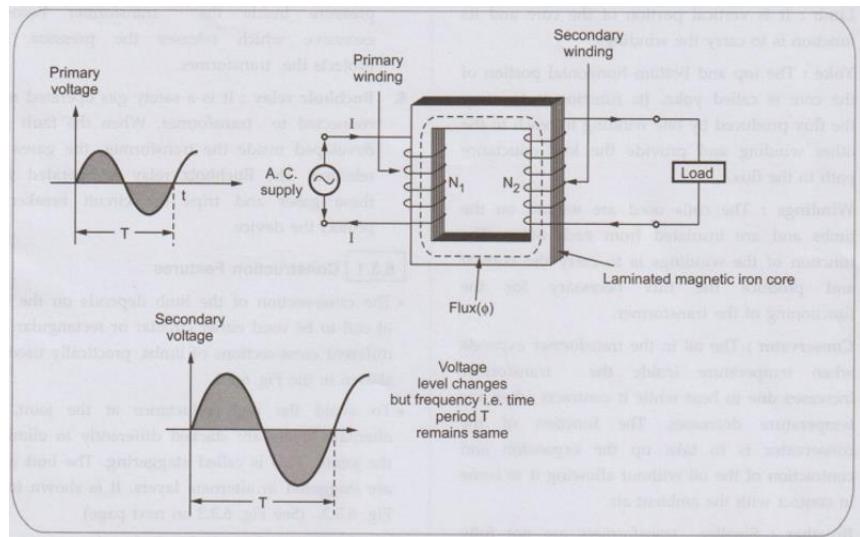


Figure 2.25: Basic transformer

Thus though there is no electrical contact between the two windings, an electrical energy gets transferred from primary to the secondary

Main Parts of the Transformer

The various parts of transformer are,

1. **Core:** It is made up of high grade silicon steel laminations. Its function is to carry the flux, providing low reluctance to it. Generally 'L' es shaped or 'I' shaped laminations are used as shown in the Fig. 2.25
2. **Limb :** It is vertical portion of the core and its function is to carry the windings.
3. **Yoke :** The top and bottom horizontal portion of the core is called yoke. Its function is to carry the flux produced by one winding to reach to the other winding and provide the low reluctance path to the flux.
4. **Windings:** The coils used are wound on the limbs and are insulated from each other. The function of the windings is to carry the current and produce the flux necessary for the functioning of the transformer.

❖ **EMF Equation of Transformer**

N_1 – Number of turns in the primary

N_2 – Number of turns in the secondary

Φ_m – Maximum flux in the weber (Wb)

T – Time period. It is the time taken for 1 cycle.

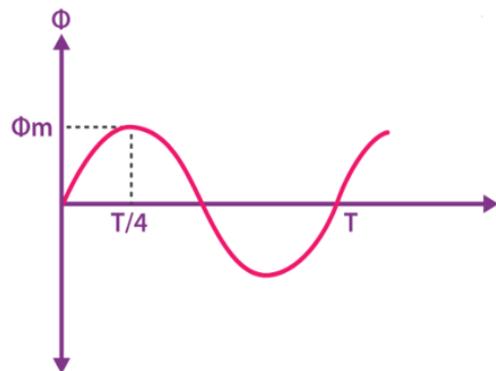
The flux formed is a sinusoidal wave. It rises to a maximum value of Φ_m and decreases to a negative maximum of $-\Phi_m$. So, flux reaches a maximum in one-quarter of a cycle. The time taken is equal to $T/4$.

Average rate of change of flux = $\Phi_m/(T/4) = 4f\Phi_m$

Where, f = frequency

$T = 1/f$

Induced EMF per turn = Rate of change of flux per turn



Form factor = RMS value / average value

RMS value = $1.11 \cdot (4f\Phi_m) = 4.44 f\Phi_m$ [form factor of a sine wave is 1.11]

RMS value of EMF induced in winding = RMS value of EMF per turn x No. of turns

❖ **Primary Winding** RMS value of induced EMF = $E_1 = 4.44 f\Phi_m * N_1$

❖ **Secondary Winding** RMS value of induced EMF = $E_2 = 4.44 f\Phi_m * N_2$

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = 4.44f\Phi_m$$

This is the EMF equation of the transformer.

For an ideal transformer at no load condition,

E_1 = Supply voltage on the primary winding

E_2 = Terminal voltage (theoretical or calculated) on the secondary winding

Voltage Transformation Ratio

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = k$$

K is called the voltage transformation ratio, which is a constant.

Case 1: If $N_2 > N_1$, $K > 1$, it is called a step-up transformer.

Case 2: If $N_2 < N_1$, $K < 1$, it is called a step-down transformer.

Solved Example Problems of Transformer

Example 1. The maximum flux density in the core of a 250/300 V, 50 Hz single phase transformer is 1.2 Wb/m². If the emf per turn is 8 volt, determine (i) Primary and Secondary turns, (ii) Area of the core.

Solution:

$$(i) E_1 = N_1 \times \text{emf induced/turn}$$

$$E_1 = 250/8 = 32; N_2 = |3000/8 = 375.$$

(ii) We may use

$$E_2 = 4.44 f N_2 B_m A$$

$$3000 = 4.44 \times 50 \times 375 \times 1.2 \times A$$

$$A = \frac{3000}{4.44 \times 50 \times 375 \times 1.2}$$

$$A = 0.03 \text{ m}^2.$$

Example 2. A single phase transformer has 400 primary and 1000 secondary turns. The net cross sectional area of the core is 60 cm². If the primary winding be connected to a 50 Hz supply at 520 V. Calculate (i) the peak value of flux density in the core, (ii) the voltage induced in the secondary winding.

Solution:

$$\text{Transformation Ratio: } K = N_2/N_1 = 1000/400 = 2.5:$$

$$(i) \quad E_2/E_1 = K \quad \therefore E_2 = E_1 K = 520 \times 2.5 = 1300 \text{ V}$$

$$E_2 = 1300 \text{ V.}$$

$$(ii) \quad E_1 = 4.44 f N_1 B_m A$$

$$520 = 4.44 \times 50 \times 400 \times B_m \times (60 \times 10^{-4})$$

$$B_m = \frac{520}{4.44 \times 50 \times 400 \times 60 \times 10^{-4}}$$

$$B_m = 0.976 \text{ Wb/m}^2.$$

UNIT 3

PN Diode—Equivalent circuit of diode—Zener diode—Zener diode as voltage regulator—

Rectifiers—Halfwave—Full Wave rectifiers—Bridge rectifier—and capacitor filter circuit.

Photodiode—LED—Photo coupler

3.1 INTRODUCTION

- Based on the electrical conductivity, all the materials in nature are classified as insulators, semiconductors, and conductors

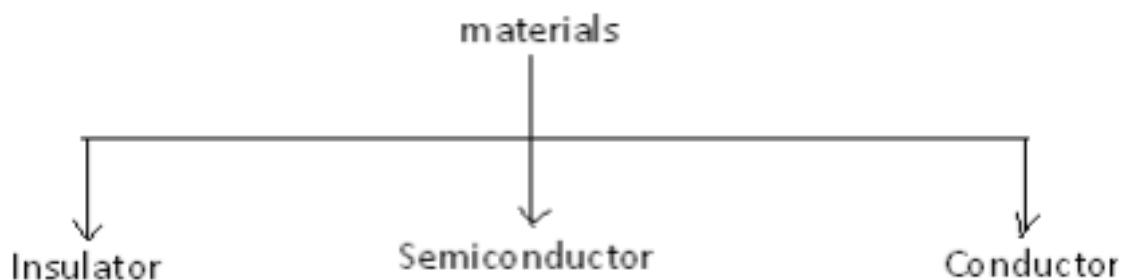


Figure 3.1: materials classification

Insulator: An insulator is a material that offers a very low level (or negligible) of conductivity when voltage is applied.

Eg: Paper, Mica, glass, quartz.

Conductors: A conductor is a material which supports a generous flow of charge when a voltage is applied across its terminals. i.e. it has very high conductivity.

Eg: Copper, Aluminum, Silver, Gold

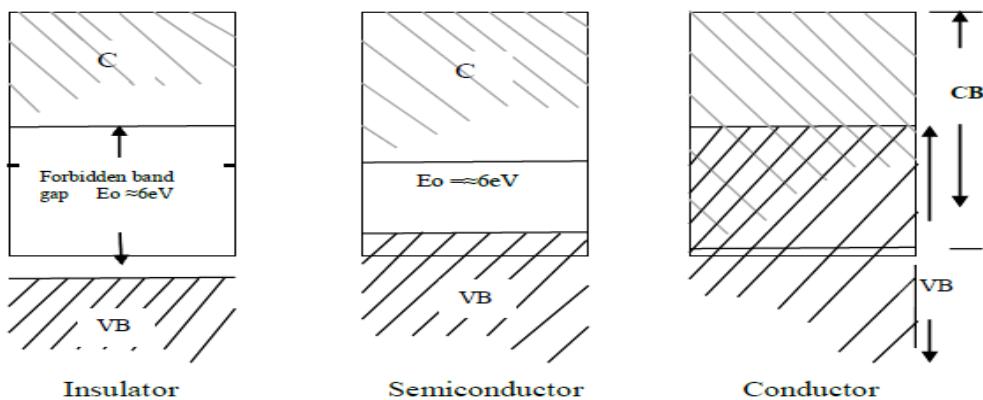


Figure 3.2: Energy band diagram

Semiconductor: A semiconductor is a material that has its conductivity somewhere between the insulator and conductor

Ex: Silicon and germanium.

- Semiconductors are the foundation of modern electronics, including transistors, solar cells, light-emitting diodes (LEDs), quantum dots and digital and analog integrated circuits.

3.2 Semiconductor Types

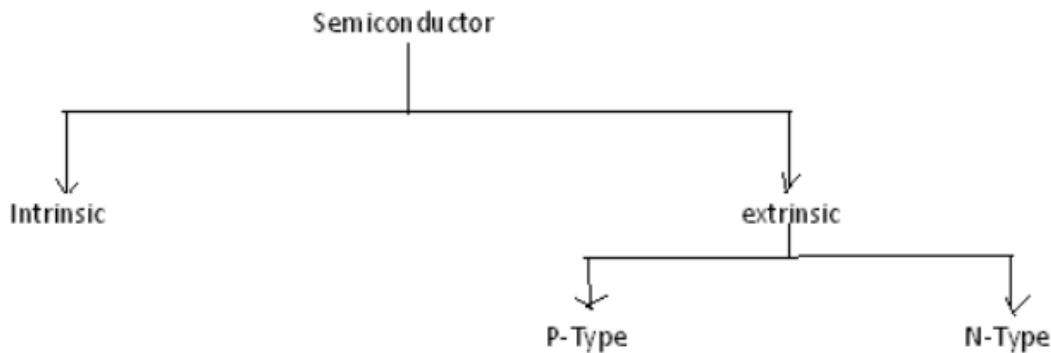


Figure 3.3: Semiconductor classification

- A semiconductor material in its pure form is known as an **intrinsic semiconductor**. Thus, the intrinsic semiconductors are chemically pure, i.e. they are free from impurities
- When a small amount of chemical impurity is added to an intrinsic semiconductor, then the resulting semiconductor material is known as **extrinsic semiconductor**
- Based on the type of doping, the extrinsic semiconductors are classified into two types viz. **N-type semiconductors** and **P-type semiconductors**

N type semiconductor: If the added impurity is a pentavalent atom then the resultant semiconductor is called N-type semiconductor. Examples of pentavalent impurities are Phosphorus, Arsenic, Bismuth, Antimony etc.

P type semiconductor: If the added impurity is a trivalent atom then the resultant semiconductor is called P-type semiconductor. Examples of trivalent impurities are Boron, Gallium , indium etc.

3.3 P-N Junction

- A p-n junction is formed by joining P-type and N-type semiconductors together in very close contact. The term junction refers to the boundary interface where the two regions of the semiconductor meet. Diode is a two-terminal electronic component that conducts electric current in only one direction
- In a n type material has high concentration of free electrons
- p type material has high concentration of free holes
- At junction there is a tendency for electrons to diffuse from n to p, this process is called diffusion.
- As free electrons moves from n to p the donor ions become positively charged, hence positive charge is built on the n side of the junction
- Hence negative charge is built on the p side of the junction
- The net negative charge on the p side prevents the diffusion of electrons from n to p.
- Similarly the positive charge on the n side prevents the holes passing from p to n
- Therefore a barrier is set up near the junction which prevents the movement of charge carriers either electrons or holes
- This is called depletion region, the electro static potential at this junction is called as barrier potential
- the barrier potential for Ge is 0.3V and for Si is 0.6V

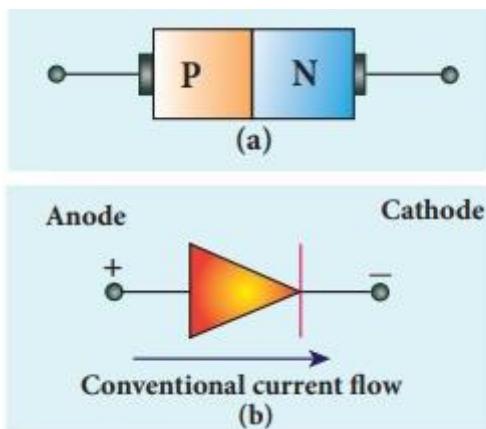


Figure 3.4: pn diode symbol

Figure (a) schematic representation (b) Symbol of PN junction diode

- The term Bias refers to the application of external voltage across the two terminals of the device to extract the response.
- We have two different biasing conditions:
 - Forward Bias
 - Reverse Bias

Zero Bias:

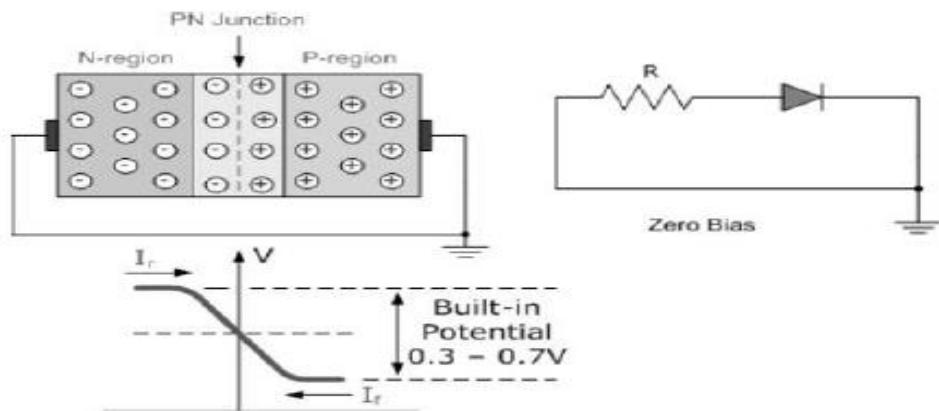


Figure 3.5: pn junction diode under zero bias

When a diode is **Zero Biased** no external energy source is applied and a natural **Potential Barrier** is developed across a depletion layer

Forward Bias condition:

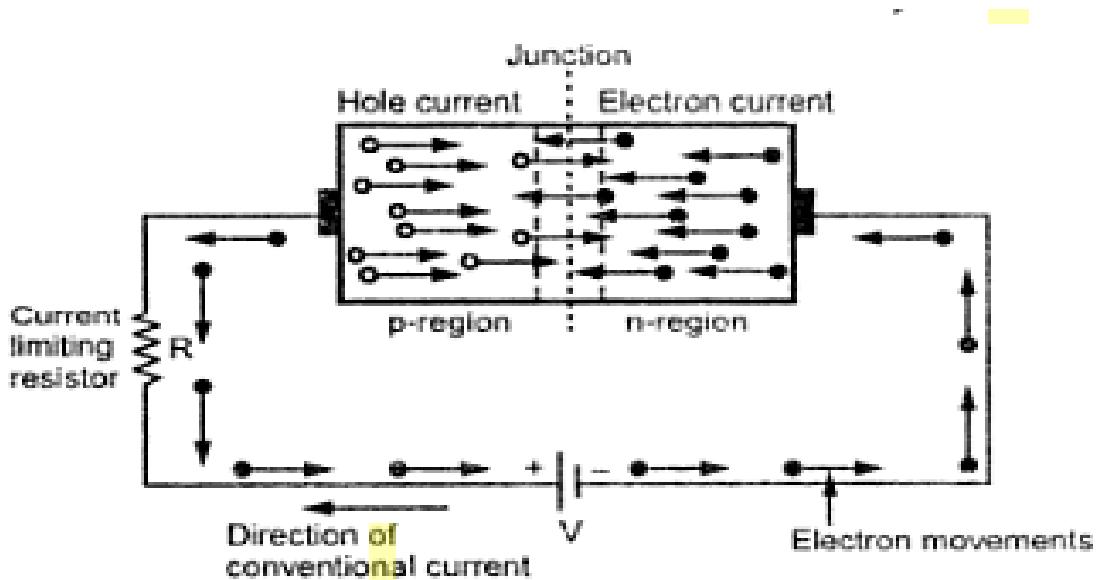


Figure 3.6: Forward biased PN junction: Internal distribution of charge carriers under forward-bias condition

- If the positive terminal of the battery is connected to the p region and negative terminal of the battery is connected to the n region then that biasing is called as forward bias

Operation

- ✓ When p-n junction is in forward bias as long as the applied voltage is less than the barrier potential, then there is no current with in the semiconductor p n Junction diode
- ✓ When applied voltage becomes greater than the barrier potential, the negative terminal of the battery pushes the electrons against the barrier potential from n to p
- ✓ Similarly positive terminal of the battery pushes the holes against the barrier potential from p to n.
- ✓ Thus the applied external voltage overcomes the barrier potential, then it reduces the width of the depletion region
- ✓ Then large number of majority carriers can cross the junction (holes crossing from p to n ,electrons crossing from n to p)
- ✓ These large number of majority carriers constitutes a current known as forward current
- ✓ In p region the current is due to moment of holes which are majority carrier this is called a hole current
- ✓ In n region the current is due to moment of electrons which are majority carrier this is called a electron current
- ✓ The overall forward current is due to both hole current and electron current

Reverse Bias condition:

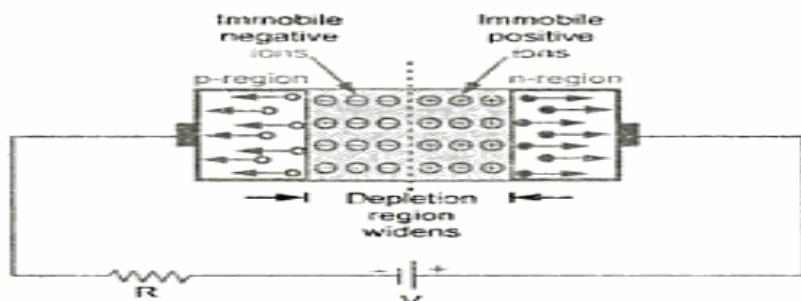


Figure 3.7: Forward biased PN junction: Internal distribution of charge carriers under forward-bias condition

- When the negative terminal of the battery is connected to the p type
- And positive terminal of the battery is connected to the n type the bias is called reverse bias

Operation:

- Under reverse bias the holes which are majority carriers of the p side moves towards the negative terminal of the battery
- Electrons which are majority carriers of n side moves towards the positive terminal of the battery
- Hence width of the depletion region increases and also potential barrier increases which prevents the flow of majority carriers in both directions
- Theoretically no current flows in the circuit
- But practically small current in order of micro ampers or nano ampers Under reverse bias exists

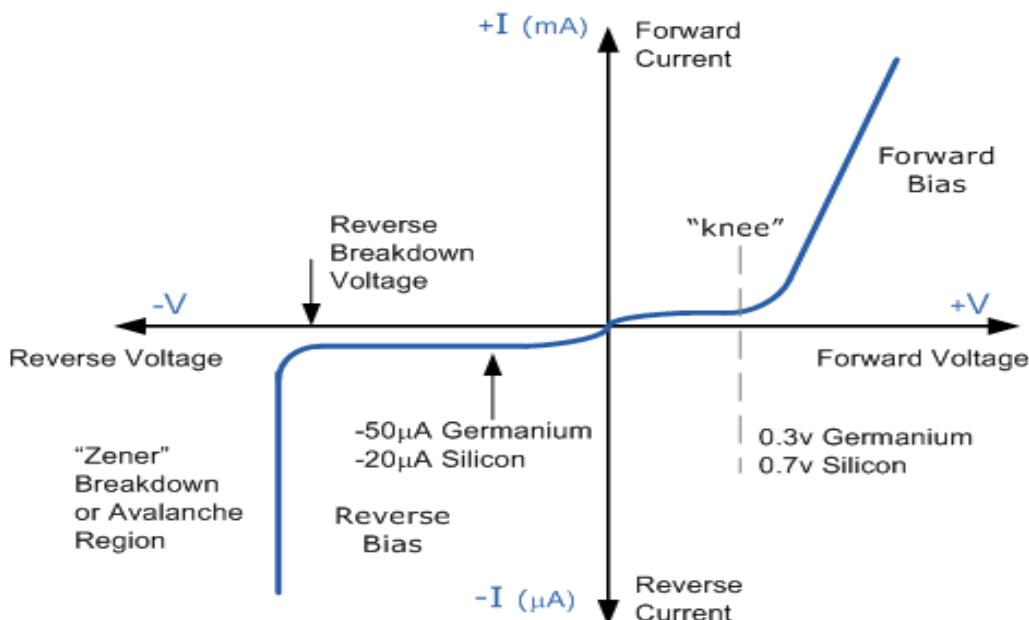


Figure 3.8: V-I characteristics of PN junction Diode

- At certain voltage to the barrier potential the current increases rapidly
- The voltage at which current starts increases is called as cutin voltage
- The cutin voltage for Ge is 0.2 or 0.3V

- For Si is 0.6 or 0.7V

3.4 Diode Current Equation:

- When a diode is subjected to bias there will be a current flow through the diode depending on bias conditions.
- The equation relating pn junction current and voltage levels is called Shockley equation and is represented as

$$I_D = I_o [e^{V_D/nV_T} - 1]$$

Where

I_o - Reverse Saturation Current

V_D - Applied bias voltage across the diode

n - Constant that depends on material

V_T - Thermal voltage called voltage equivalent of temperature

$$V_T = kT/q$$

where

K-Boltzman's constant= 1.38×10^{-23} J/K

T- Absolute Temperature= $(273 + T^\circ C)$ K

q-change of electron= 1.6×10^{-19} C

By substituting the above values V_T can be obtained as

$$V_T = T/11600$$

3.5 Diode Equivalent Circuits:

- For general performance analysis of any device or a system, an equivalent circuit may be used which is an alternate representation of the device or the system.
- An equivalent circuit contains alternate components that best corresponds the actual characteristics of a device or the system under the given operating conditions.

Piecewise Linear Equivalent circuit:

- In order to obtain more precise and an excellent equivalent circuit for a diode approximations for the device characteristics by using straight line segments can be made
- Nonlinear waveforms are often approximated by a number of linear segments in order to simplify the analysis.
- The more the number of linear segments, the more will be the approximations close to the actual waveform such an approximation is called Piecewise Linear approximation.

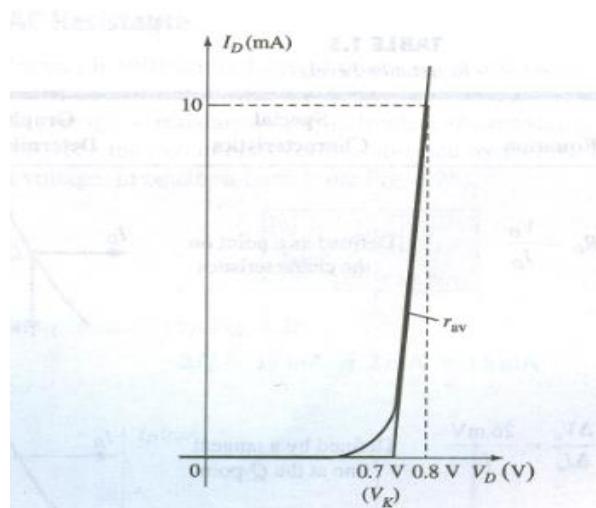


Figure 3.9: Defining the piecewise-linear equivalent circuit using straight-line segments to approximate the characteristic curve.

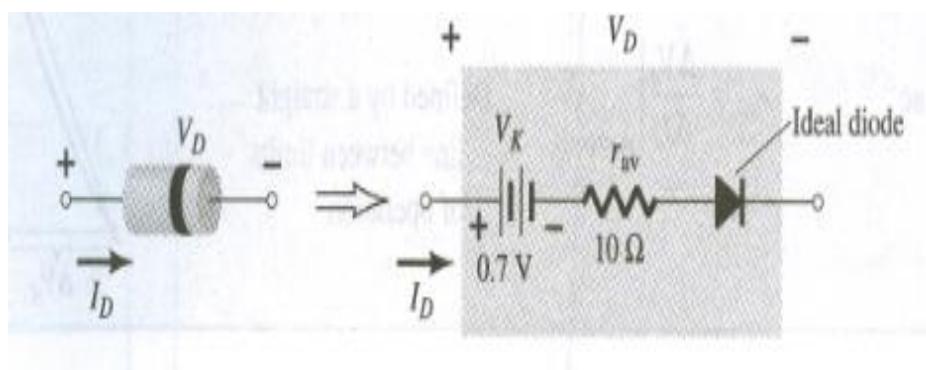


Figure 3.10: Components of piecewise-linear equivalent circuit.

- It should be obvious from the above figure that the straight line segments do not result in exact duplication of the actual characteristics especially in the knee region however, the resulting segments are sufficiently close to the actual curves that establish an equivalent circuit that will provide an excellent first approximation to the actual behaviour of the device.
- The ideal diode is included to establish that there is only one direction of conduction through the device and a reverse bias condition will result in the open circuit state for the device.
- We know that the semiconductor diode does not reach the conduction state until V_D reaches 0.7V with a forward bias, a battery V_k opposing the conduction direction must appear in the equivalent circuit.

3.6 ZENER DIODE

Zener diode is a specially designed ordinary P-N junction diode, which is heavily doped to have a very sharp and almost vertical breakdown.

- They are exclusively operated under reverse bias conditions and designed to operate in breakdown region without damage.
- The device was named after Clarence Zener, who discovered this electrical property.
- The commonly used schematic symbol for Zener diode is shown in Fig

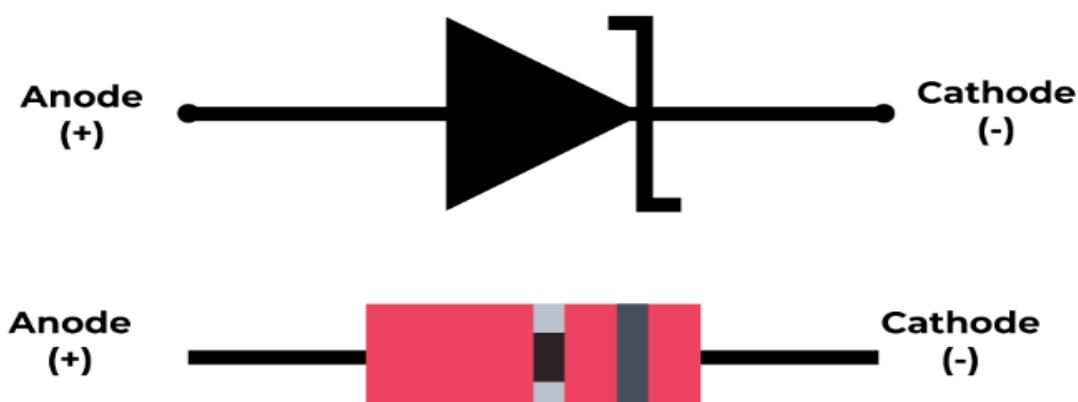


Figure 3.11: symbol of Zener diode

V-I CHARACTERSTICSAND WORKING OF ZENER DIODE

The Forward bias characteristics of Zener diode are same as that of normal PN Junction diode.

- When the applied forward bias voltage ‘ V_F ’ is less than the cut in voltage, the current is negligibly small. When V_F becomes greater than cut in voltage, current starts increasing rapidly.
- In reverse bias mode, current is due to minority charge carriers.
- Since the P and N-regions are heavily doped, the depletion layer at the junction will be very narrow.
- The reverse bias voltage sets up a strong electric field across the narrow depletion layer. This field is strong enough to cause rupture of covalent bonds of atoms. Therefore, there is a generation of a large number of electron-hole pairs, leading to a sharp increase in the reverse current.
- When reverse bias is increased, up to a certain voltage called as breakdown voltage.
- A voltage is reached then the diode starts conducting heavily and the reverse current increases sharply. This voltage is called Zener breakdown voltage(V_z).
- A Zener diode maintains a constant voltage across its terminals when the reverse bias exceeds the breakdown voltage. Therefore, it is used as voltage regulator.

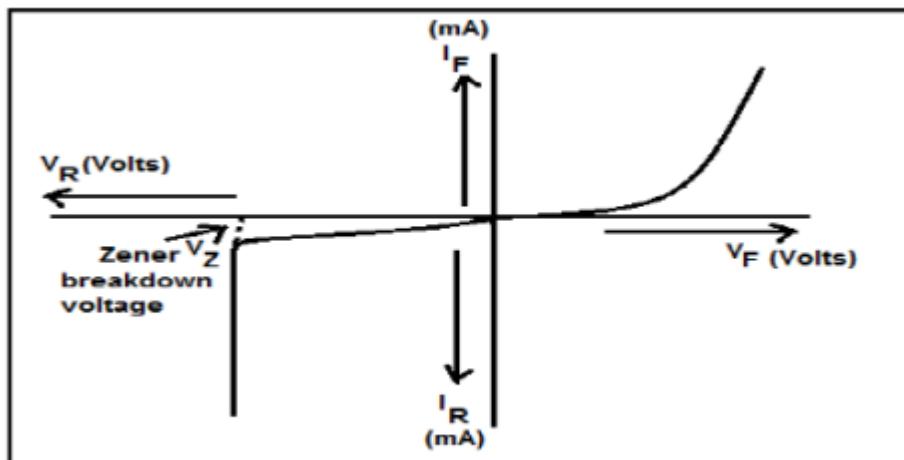


Figure 3.12: V-I CHARACTERSTICS OF ZENER DIODE

In reverse bias condition, two kinds of breakdowns occur for Zener Diode which are

- Avalanche Breakdown
- Zener Breakdown

Avalanche Breakdown

- Avalanche Breakdown occurs due to avalanche multiplication.
- It occurs when the doping concentration is less of order $1 \text{ to } 10^8$.
- Under Reverse bias, the thermally generated carrier crosses the depletion region and acquires Kinetic energy from the applied voltage. This carrier collides with the crystal and disrupts the covalent band. This is known as Impact Ionization.
- The new electron hole pair will be created apart from original carrier. The new carrier in turn collide with another crystal by acquiring enough energy from applied field will create electron hole pair.
- This process continues result in avalanche multiplication. This causes Breakdown known as avalanche Breakdown.

Zener Breakdown

- This breakdown occurs in the heavily doped P and N region.
- When the strong electric field is applied, the direct rupture of covalent bond takes place produce new electron hole pair.
- The new electron hole pair so created will increases the reverse current.
- This reverse current increase at almost 6 volts for heavily doped diode at the field of order $2 \times 10^7 \text{ v/m}$.
- This kind of breakdown occurs in heavily doped PN region is known as zener breakdown.
- Zener Breakdown occur less than 6 V whereas Avalanche Breakdown occur greater than 6V.

Differences between Zener and Avalanche breakdown:

Table 3.1: difference between avalanche and zener breakdown

Avalanche Breakdown	Zener Breakdown
Avalanche breakdown occurs when the high voltage increase the free electron in the semiconductor and a sudden increase in current is seen.	Zener breakdown happens when electrons from the valance band gain energy and reaches the conduction band which then conducts electricity.
Avalanche breakdown is seen in the diodes having breakdown voltage greater than 8 volts.	Zener breakdown is seen in the diodes having breakdown voltage in the range of 5 to 8 volts.
Avalanche breakdown is observed in diodes that are lightly doped.	Zener breakdown is observed in diodes that are highly doped.
In the Avalanche breakdown, the VI characteristics curve is not as sharp as the VI characteristics curve in the Zener breakdown.	Zener Breakdown has a sharp VI characteristics curve.
For Avalanche breakdown increase in temperature increases the breakdown voltage.	For Zener breakdown increase in temperature decreases the breakdown voltage.

3.7 Zener Diode as Voltage Stabiliser

1. Regulation with varying input voltage:

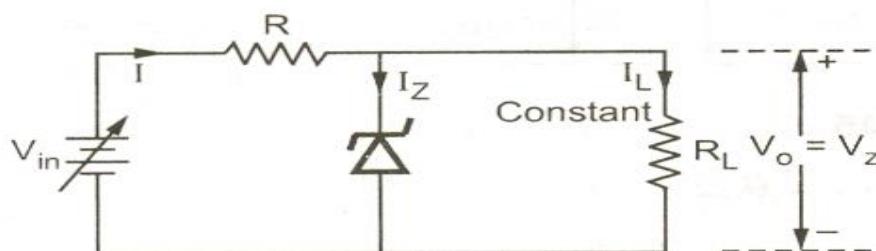


Figure 3.13: Zener Diode as Voltage Stabiliser

- From the above figure $V_o = V_z = \text{constant}$.
- $I_L = V_o/R_L = V_z/R_L = \text{constant}$.
- We can write $I = I_L + I_Z$

- If we increase input voltage V_{in} then the current I increases, but WKT current through the load is constant. Hence current through Zener increases to keep I_L constant.
- As long as I_z is in between $I_{z(min)}$ and $I_{z(max)}$ the V_z i.e. the output voltage V_0 is constant i.e. how the change in input voltage is getting compensated and constant output is maintained.
- When V_{in} decreases the current I decrease. But WKT the current through load is constant the current through zener decreases.
- I_z will be in between $I_z(max)$ to $I_z(min)$ to keep the output voltage constant.

2. Regulation with varying load:

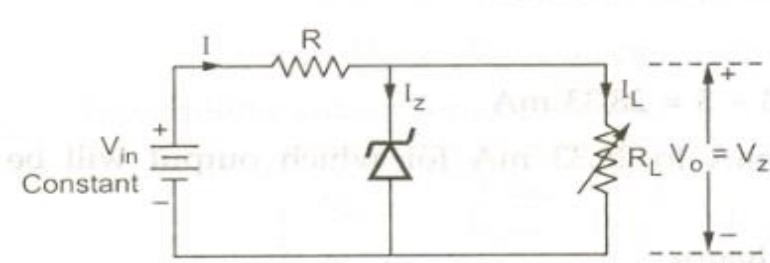
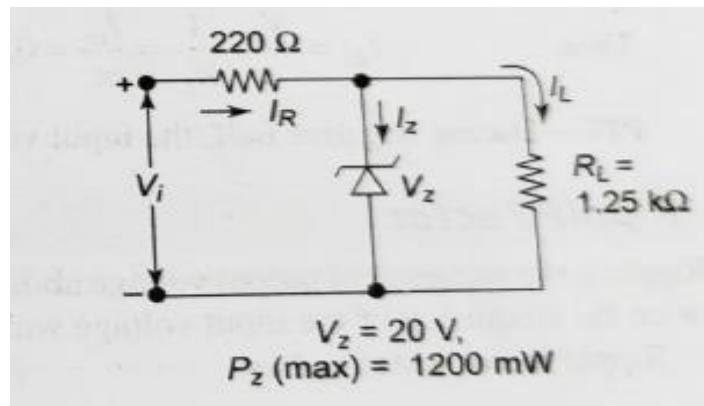


Figure 3.14: Varying load condition

- In the above figure the input voltage V_{in} is kept constant whereas load is varying.
- Here V_{in} is constant and V_0 is also constant
- Current I can be calculated as $I = (V_{in} - V_z)/R$.
- WKT $I_L = V_0/R_L = V_z/R_L = \text{constant}$.
- If R_L increases then current through load I_L decreases, to keep constant I, I_z increases but as long as I_z is in between $I_{z(min)}$ and $I_{z(max)}$ output voltage will be constant.
- If R_L decreases then current through load I_L increases, to keep constant I, I_z decreases but as long as I_z is in between $I_{z(min)}$ and $I_{z(max)}$ output voltage will be constant.

Problem

(1) Determine the range of V_i in which the zener diode of below conducts



Solution

(a) V_z just in conducting state

$$V_z = 20 \text{ V}, I_z = 0$$

$$I_R = I_L = \frac{20}{1.25} = 16 \text{ mA}$$

$$V_i = 20 + 220 \times 16 \times 10^{-3} = 23.52 \text{ V}$$

(b)

$$I_z = I_z(\text{max}) = \frac{1200}{20} = 60 \text{ mA}$$

$$I_L = 16 \text{ mA}$$

$$I_R = 60 + 16 = 76 \text{ mA}$$

$$V_i = 20 + 220 \times 76 \times 10^{-3} = 36.72 \text{ V}$$

Rectifiers

3.8 Introduction:

A **rectifier** is an electrical device that converts alternating current (AC), which periodically reverses direction, to direct current (DC), which flows in only one direction. This process is known as **rectification**.

Classification of Rectifiers:

Using one or more diodes in the circuit, following rectifier circuits can be designed.

- 1) Half - Wave Rectifier
- 2) Full – Wave Rectifier
- 3) Bridge Rectifier

3.9 HALF-WAVE RECTIFIER:

A Half – wave rectifier as shown below , which converts a.c. voltage into a pulsating voltage using only one half cycle of the applied a.c. voltage

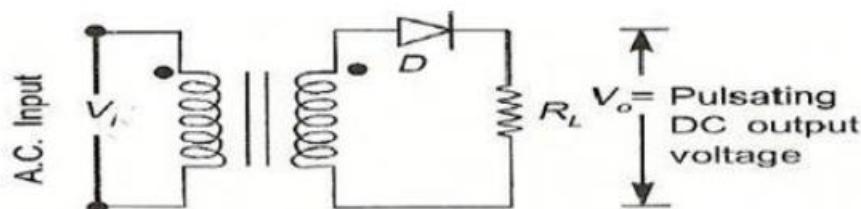


Figure 3.15: Basic structure of half wave rectifier

The a.c. voltage is applied to the rectifier circuit using step-down transformer-rectifying element i.e., p-n junction diode and the source of a.c. voltage, all connected in series. The a.c. voltage is applied to the rectifier circuit using step-down transformer

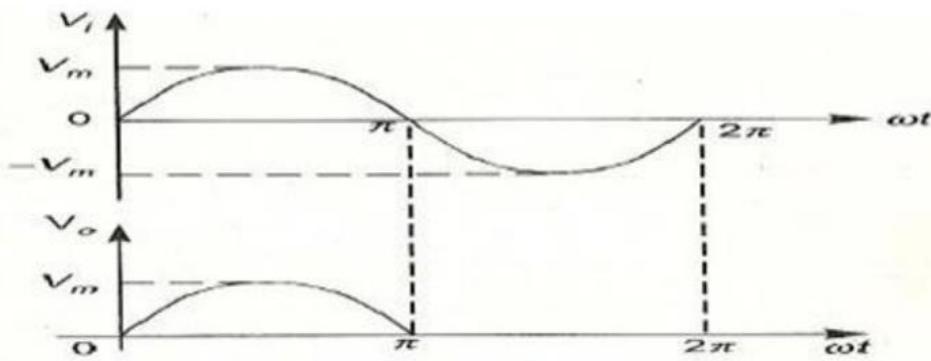


Figure 3.16: input and output waveforms of halfwave rectifier

$$V=V_m \sin(\omega t)$$

The input to the rectifier circuit, Where V_m is the peak value of secondary a.c. voltage.

Operation:

For the positive half-cycle of input a.c. voltage, the diode D is forward biased and hence it conducts. Now a current flows in the circuit and there is a voltage drop across RL . The waveform of the diode current (or) load current is shown in above figure.

For the negative half-cycle of input, the diode D is reverse biased and hence it does not conduct. Now no current flows in the circuit i.e., $i=0$ and $V_o=0$. Thus for the negative half- cycle no power is delivered to the load

Therefore, for an AC voltage given by (1) the output voltage of a half wave rectifier will be (for an ideal diode)

$$V_o(t) = \begin{cases} V_m \sin(\omega t), & 0 \leq t \leq T/2 \\ 0, & T/2 \leq t \leq T \end{cases} \quad \dots \dots \dots (1)$$

Analysis:

Average output voltage of a half wave rectifier

The average value of $V(t)$ over the time period T is defined as

$$\overline{V} = \frac{1}{T} \int_0^T V(t) dt. \quad \dots \dots \dots (2)$$

To calculate the average voltage, V_{dc} , of the pulsating DC output of a half wave rectifier we use the definition (1). Therefore, for the voltage (2) we have

$$\begin{aligned}
 V_{dc} &= \frac{1}{T} \int_0^T V_o(t) dt \\
 &= \frac{1}{T} \int_0^{T/2} V_m \sin(\omega t) dt + \frac{1}{T} \int_{T/2}^T 0 dt \\
 &= \frac{V_m}{T} \int_0^{T/2} \sin(\omega t) dt \\
 &= \frac{V_m}{T} \left[-\frac{\cos(\omega t)}{\omega} \right]_0^{T/2} \\
 &= \frac{V_m}{\omega T} \{-\cos(\omega T/2) + \cos(0)\} \\
 &= \frac{V_m}{\pi}.
 \end{aligned}$$

Here we have used the relation $\omega = 2\pi/T$.

RMS value of the output voltage of a half wave rectifier

To calculate the RMS value of the output voltage, V_{rms} , of the pulsating DC output of a half wave rectifier we use the definition (2). Therefore, for the voltage (1) we have

$$\begin{aligned}
 V_{rms}^2 &= \frac{1}{T} \int_0^T V_o^2(t) dt \\
 &= \frac{V_m^2}{T} \int_0^{T/2} \sin^2(\omega t) dt + \frac{V_m^2}{T} \int_{T/2}^T 0 dt \\
 &= \frac{V_m^2}{2T} \int_0^{T/2} 2 \sin^2(\omega t) dt \\
 &= \frac{V_m^2}{2T} \int_0^{T/2} \{1 - \cos(2\omega t)\} dt \\
 &= \frac{V_m^2}{2T} \int_0^{T/2} dt - \frac{V_m^2}{T} \int_0^{T/2} \cos(2\omega t) dt \\
 &= \frac{V_m^2}{2T} [T]_0^{T/2} - \frac{V_m^2}{2T} \left[\frac{\sin(2\omega t)}{2\omega} \right]_0^{T/2} \\
 &= \frac{V_m^2}{4} - \frac{V_m^2}{\omega T} \{ \sin(2\omega T) - \sin(0) \} \\
 &= \frac{V_m^2}{4}.
 \end{aligned}$$

Hence for the half wave rectifier

$$V_{rms} = \frac{V_m}{2}.$$

Ripple factor of half wave rectifier

Ripple is the unwanted AC component remaining when converting the AC voltage waveform into a DC waveform. Even though we try our best to remove all AC components, there is still some small amount left on the output side which pulsates the DC waveform. This undesirable AC component is called ripple.

To quantify how well the half wave rectifier can convert the AC voltage into DC voltage, we use what is known as the ripple factor (represented by γ). The ripple factor is the ratio between the RMS value of the AC voltage and the DC voltage of the rectifier.

$$\gamma = \frac{\text{RMS value of the AC component}}{\text{value of DC component}} = \frac{V_{r(\text{rms})}}{V_{dc}}.$$

Note that the RMS value of the AC component of the signal is $V_r(\text{rms})$ and V_{rms} is the RMS value of the whole voltage signal.

To calculate $V_r(\text{rms})$, the RMS value of the AC component present in the output of the half wave rectifier we write the output voltage as

$$V_o(t) = V_{ac} + V_{dc};$$

where V_{ac} is the AC component remaining when converting the AC voltage waveform into a DC waveform. The RMS value of the AC component present in the output of the half wave rectifier is given by

$$\overline{V} = \frac{1}{T} \int_0^T V(t) dt.$$

Therefore,

$$\begin{aligned} V_{r(\text{rms})}^2 &= \frac{1}{T} \int_0^T (V_o - V_{dc})^2 dt \\ &= \frac{1}{T} \int_0^T (V_o^2 - 2V_o V_{dc} + V_{dc}^2) dt \\ &= \frac{1}{T} \int_0^T V_o^2 dt - \frac{2V_{dc}}{T} \int_0^T V_o dt + V_{dc}^2 \\ &= V_{\text{rms}}^2 - 2V_{dc}^2 + V_{dc}^2 \\ &= V_{\text{rms}}^2 - V_{dc}^2. \end{aligned}$$

Hence the formula to calculate the ripple factor can be written as

$$\gamma = \frac{V_{r(\text{rms})}}{V_{dc}} = \sqrt{\left(\frac{V_{\text{rms}}}{V_{dc}}\right)^2 - 1}$$

Using the values of $V_{\text{rms}}=V_m/2$ and $V_{dc}=V_m/\pi$ respectively for the half wave rectifier we find the the ripple factor as

$$\gamma = \sqrt{\left(\frac{V_m}{2} \times \frac{\pi}{V_m}\right)^2 - 1} = \sqrt{\left(\frac{\pi}{2}\right)^2 - 1} \approx 1.21.$$

Note that to construct a good rectifier, one should keep the ripple factor as low as possible. This is why capacitors and inductors as filters are used to reduce the ripples in the circuit.

PEAK FACTOR

$$\text{Peak factor} = \frac{\text{peakvalue}}{\text{rmsvalue}}$$

$$\text{Peak Factor} = \frac{V_m}{(V_m/2)}$$

Form Factor

$$\text{Form factor} = \frac{\text{Rmsvalue}}{\text{averagevalue}}$$

$$\text{Form factor} = \frac{(V_m/2)}{V_m/\Pi}$$

$$\text{Form Factor} = 1.57$$

Efficiency:

$$\eta = \frac{o / p\text{power}}{i / p\text{power}} * 100$$

$$\eta = \frac{P_{ac}}{P_{dc}} * 100$$

$$\eta = 40.8$$

Transformer Utilization Factor (TUF):

The d.c. power to be delivered to the load in a rectifier circuit decides the rating of the transformer used in the circuit. Therefore, transformer utilization factor is defined as

$$TUF = \frac{P_{dc}}{P_{ac(rated)}}$$

$$TUF = 0.286.$$

The value of TUF is low which shows that in half-wave circuit, the transformer is not fully utilized. If the transformer rating is 1 KVA (1000VA) then the half-wave rectifier can deliver $1000 \times 0.287 = 287$ watts to resistance load

Peak Inverse Voltage (PIV):

It is defined as the maximum reverse voltage that a diode can withstand without destroying the junction. The peak inverse voltage across a diode is the peak of the negative half-cycle. For half-wave rectifier, PIV is V_m

Advantages of half wave rectifier

1. Simple (lower number of components)
2. Cheaper up front cost

Disadvantages Of Half-Wave Rectifier:

1. The ripple factor is high.
2. The efficiency is low.
3. The Transformer Utilization factor is low.

Numericals:

Problem 1

- (1) The applied input a.c. power to a half-wave rectifier is 100 watts. The d.c. output power obtained is 40 watts. (i) What is the rectification efficiency? (ii) What happens to remaining 60 watts?

Solution

- (i) Rectification efficiency = d.c.output power 40 a.c. input power 100 = = 0.4 = 40%
- (ii) 40% efficiency of rectification does not mean that 60% of power is lost in the rectifier circuit. In fact, a crystal diode consumes little power due to its small internal resistance. The 100 W

3.10 Full Wave Rectifier:

A full-wave rectifier converts an ac voltage into a pulsating dc voltage using both half cycles of the applied ac voltage. In order to rectify both the half cycles of ac input, two diodes are used in this circuit

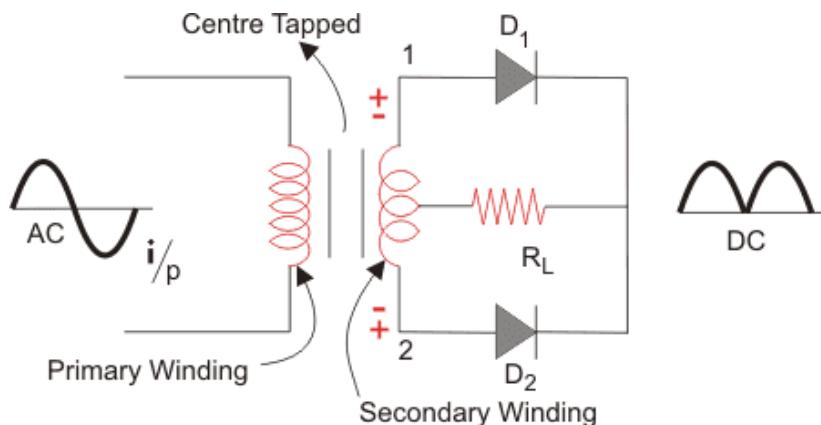


Figure 3.17: Centre tapped full wave rectifier circuit diagram and waveform

During positive half of the input signal, anode of diode D₁ becomes positive and at the same time the anode of diode D₂ becomes negative. Hence D₁ conducts and D₂ does not conduct. The load current flows through D₁ and the voltage drop across R_L will be equal to the input voltage.

During the negative half cycle of the input, the anode of D1 becomes negative and the anode of D2 becomes positive. Hence, D1 does not conduct and D2 conducts. The load current flows through D2 and the voltage drop across RL will be equal to the input voltage. It is noted that the load current flows in the both the half cycles of ac voltage and in the same direction through the load resistance.

For an AC voltage given by (1) the waveform of the output voltage of a full wave rectifier can be written as (for an ideal diode)

$$V_o(t) = \begin{cases} V_m \sin(\omega t), & 0 \leq t \leq T/2 \\ V_m \sin(\omega t - \pi), & T/2 \leq t \leq T \end{cases}$$

Average output voltage of Full wave rectifier:

$$\begin{aligned} V_{dc} &= \frac{1}{T} \int_0^T V_o(t) dt \\ &= \frac{1}{T/2} \int_0^{T/2} V_m \sin(\omega t) dt \\ &= \frac{2V_m}{T} \int_0^{T/2} \sin(\omega t) dt \\ &= \frac{2V_m}{\pi}. \end{aligned}$$

RMS value

RMS value of the output voltage of a full wave rectifier

$$\begin{aligned}
 V_{\text{rms}} &= \left[\frac{1}{T} \int_0^T V_o^2(t) dt \right]^{1/2} \\
 &= \left[\frac{V_m^2}{T/2} \int_0^{T/2} \sin^2(\omega t) dt \right]^{1/2} \\
 &= \left[\frac{V_m^2}{T} \int_0^{T/2} 2 \sin^2(\omega t) dt \right]^{1/2} \\
 &= \frac{V_m}{\sqrt{2}}.
 \end{aligned}$$

Ripple factor

$$\begin{aligned}
 \gamma &= \sqrt{\left(\frac{V_{\text{rms}}}{V_{\text{dc}}} \right)^2 - 1} \\
 &= \sqrt{\left(\frac{\pi}{2\sqrt{2}} \right)^2 - 1} \\
 &\approx 0.48
 \end{aligned}$$

Efficiency of Full wave rectifier:

$$\begin{aligned}
 \eta &= \frac{P_{\text{dc}}}{P_{\text{ac}}} \\
 &= \left(\frac{V_{\text{dc}}}{V_{\text{rms}}} \right)^2 \times \left(1 + \frac{r_f}{R_L} \right) \\
 &\approx 0.8106 \left(1 + \frac{r_f}{R_L} \right)
 \end{aligned}$$

In reality r_f is much smaller than R_L . If we neglect r_f compare to R_L then the efficiency of the rectifier is maximum. Therefore

$$\eta_{\max} \approx 0.8106 = 81.06\%.$$

PEAK FACTOR:

$$\text{Peak factor} = \frac{\text{peak value}}{\text{rms value}}$$

$$\text{Peak Factor} = \frac{V_m}{(V_m / 2)}$$

$$\text{Peak Factor} = 2$$

FORM FACTOR:

$$\text{Form factor} = \frac{\text{Rms value}}{\text{average value}}$$

$$\text{Form factor} = \frac{(V_m / \sqrt{2})}{2V_m / \Pi}$$

$$\text{Form Factor} = 1.11$$

Transformer Utilization Factor (TUF):

The d.c. power to be delivered to the load in a rectifier circuit decides the rating of the transformer used in the circuit. So, transformer utilization factor is defined as

$$TUF = \frac{P_{dc}}{P_{ac(rated)}}$$

Peak Inverse Voltage (PIV):

It is defined as the maximum reverse voltage that a diode can withstand without destroying the junction. The peak inverse voltage across a diode is the peak of the negative half-cycle. For half-wave rectifier, PIV is $2V_m$

Advantages of full wave rectifier

- Full wave rectifiers have higher rectifying efficiency than half-wave rectifiers
- They have low power loss because no voltage signal is wasted in the rectification process.

Disadvantages

- Requires center tapped transformer

3.11 Bridge Rectifier:

- The bridge rectifier circuit is essentially a full wave rectifier circuit, using 4 diodes which are arranged in the form of a bridge and a transformer without centre tapping.
- The figure below shows the bridge rectifier circuit.

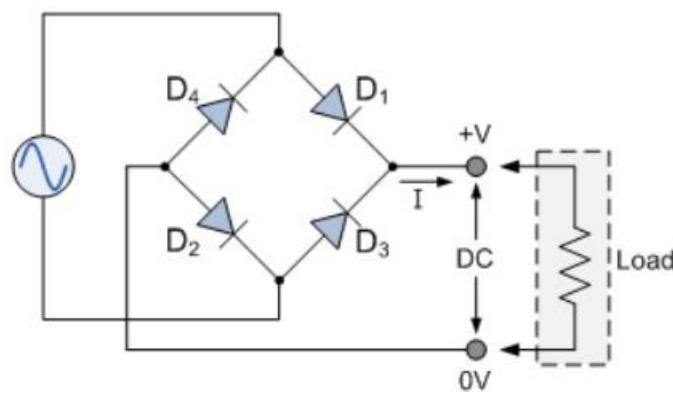


Figure 3.18: Bridge rectifier

The four diodes labeled D1 to D4 are arranged in "series pairs" with only two diodes conducting current during each half cycle. During the positive half cycle of the supply, diodes D1 and D2 conduct in series while diodes D3 and D4 are reverse biased and the current flows through the load

The Positive Half-cycle

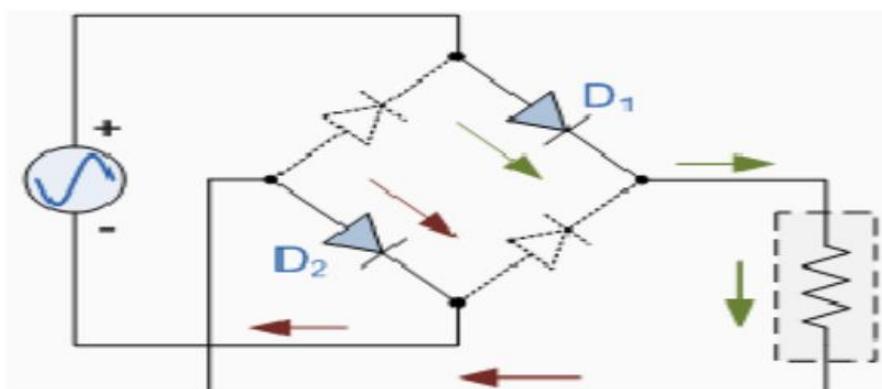


Figure 3.19: Bridge rectifier

The Negative Half-cycle

During the negative half cycle of the supply, diodes D3 and D4 conduct in series (fig 8), but diodes D1 and D2 switch "OFF" as they are now reverse biased. The current flowing through the load is the same direction as before

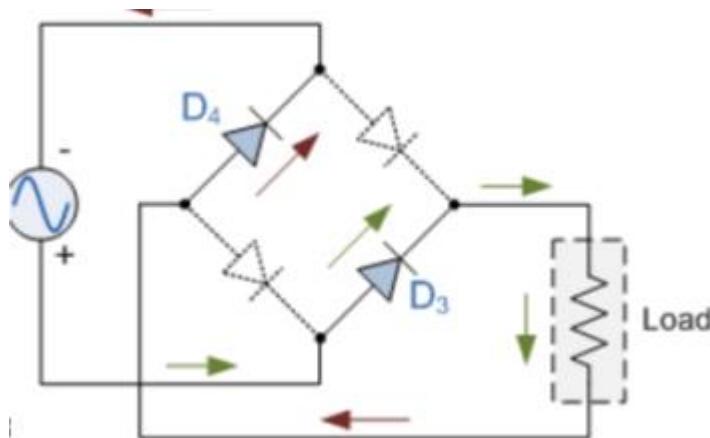


Figure 3.20: Bridge rectifier

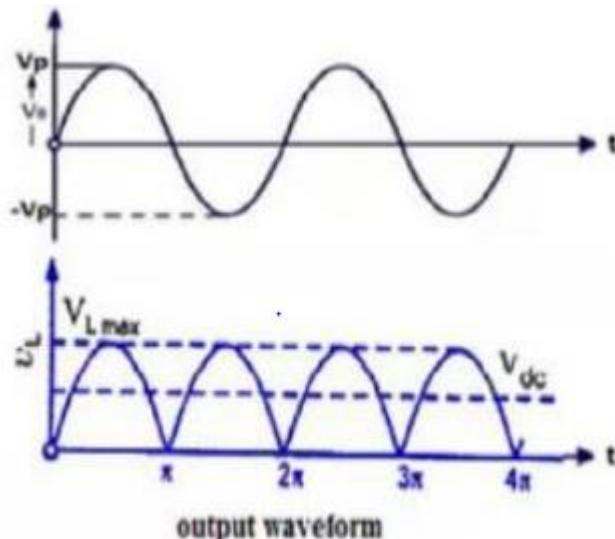


Figure 3.21: Bridge rectifier output waveform

- a) Average current = $\frac{2I_m}{\pi}$
- b) RMS current = $\frac{I_m}{\sqrt{2}}$
- c) DC output voltage $V_{dc} = \frac{2V_m}{\pi}$
- d) Ripple factor = 0.482
- e) Rectification Efficiency = 0.812
- f) DC output full load $V_{DCFL} = \frac{2V_m}{\pi} - I_{dc} (2R_F + R_s)$

Problem 2

(2) A full-wave rectifier uses two diodes, the internal resistance of each diode may be assumed constant at 20Ω . The transformer r.m.s. secondary voltage from centre tap to each end of secondary is 50 V and load resistance is 980Ω . Find : (i) the mean load current (ii) the r.m.s. value of load current

Solution.

$$r_f = 20 \Omega, R_L = 980 \Omega$$

$$\text{Max. a.c. voltage, } V_m = 50 \times \sqrt{2} = 70.7 \text{ V}$$

$$\text{Max. load current, } I_m = \frac{V_m}{r_f + R_L} = \frac{70.7 \text{ V}}{(20 + 980) \Omega} = 70.7 \text{ mA}$$

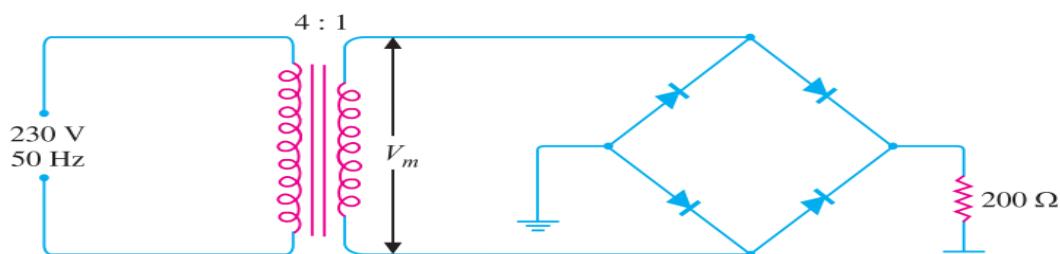
$$(i) \text{ Mean load current, } I_{dc} = \frac{2I_m}{\pi} = \frac{2 \times 70.7}{\pi} = 45 \text{ mA}$$

(ii) R.M.S. value of load current is

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{70.7}{\sqrt{2}} \text{ mA} = 50 \text{ mA}$$

Problem 3

(3) In the bridge type circuit shown in below Fig, the diodes are assumed to be ideal. Find : (i) d.c. output voltage (ii) peak inverse voltage (iii) output frequency. Assume primary to secondary turns to be 4.



Solution.

Primary/secondary turns, $N_1/N_2 = 4$

R.M.S. primary voltage = 230 V

\therefore R.M.S. secondary voltage = $230(N_2/N_1) = 230 \times (1/4) = 57.5$ V

Maximum voltage across secondary is

$$V_m = 57.5 \times \sqrt{2} = 81.3 \text{ V}$$

$$(i) \quad \text{Average current, } I_{dc} = \frac{2V_m}{\pi R_L} = \frac{2 \times 81.3}{\pi \times 200} = 0.26 \text{ A}$$

$$\therefore \text{d.c. output voltage, } V_{dc} = I_{dc} \times R_L = 0.26 \times 200 = 52 \text{ V}$$

(ii) The peak inverse voltage is equal to the maximum secondary voltage i.e.

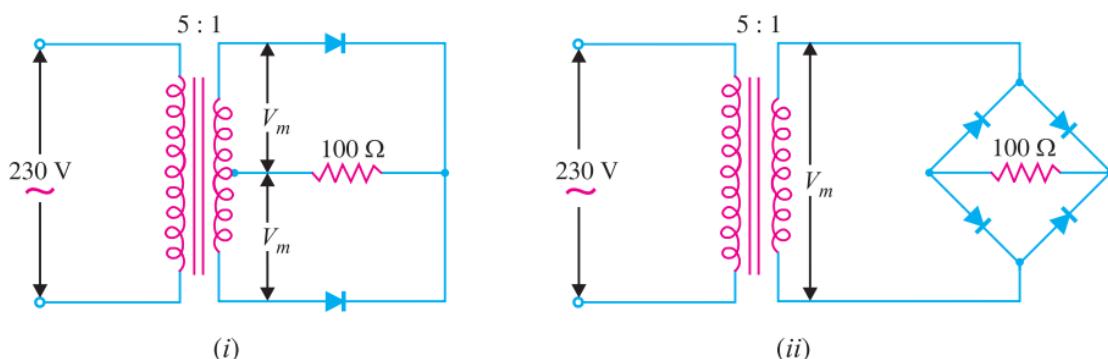
$$PIV = 81.3 \text{ V}$$

(iii) In full-wave rectification, there are two output pulses for each complete cycle of the input a.c. voltage. Therefore, the output frequency is twice that of the a.c. supply frequency i.e.

$$f_{out} = 2 \times f_{in} = 2 \times 50 = 100 \text{ Hz}$$

Problem 4

(4) Fig (i) and Fig(ii) show the centre-tap and bridge type circuits having the same load resistance and transformer turn ratio. The primary of each is connected to 230V, 50 Hz supply. (i) Find the d.c. voltage in each case. (ii) PIV for each case for the same d.c. output. Assume the diodes to be ideal



Solution.
(i) D.C. output voltage
Centre-tap circuit

$$\text{R.M.S. secondary voltage} = 230 \times 1/5 = 46 \text{ V}$$

$$\text{Max. voltage across secondary} = 46 \times \sqrt{2} = 65 \text{ V}$$

Max. voltage appearing across half secondary winding is

$$V_m = 65/2 = 32.5 \text{ V}$$

$$\text{Average current, } I_{dc} = \frac{2V_m}{\pi R_L}$$

$$\begin{aligned} \text{D.C. output voltage, } V_{dc} &= I_{dc} \times R_L = \frac{2V_m}{\pi R_L} \times R_L \\ &= \frac{2V_m}{\pi} = \frac{2 \times 32.5}{\pi} = 20.7 \text{ V} \end{aligned}$$

Bridge Circuit

$$\text{Max. voltage across secondary, } V_m = 65 \text{ V}$$

$$\text{D.C. output voltage, } V_{dc} = I_{dc} R_L = \frac{2V_m}{\pi R_L} \times R_L = \frac{2V_m}{\pi} = \frac{2 \times 65}{\pi} = 41.4 \text{ V}$$

This shows that for the same secondary voltage, the d.c. output voltage of bridge circuit is twice that of the centre-tap circuit.

(iii) PIV for same d.c. output voltage

The d.c. output voltage of the two circuits will be the same if V_m (i.e. max. voltage utilised by each circuit for conversion into d.c.) is the same. For this to happen, the turn ratio of the transformers should be as shown in below Fig

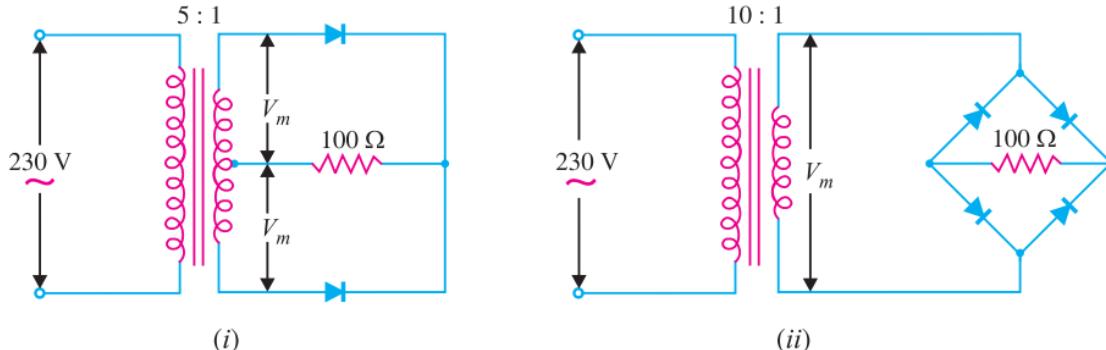
Centre-tap circuit

$$\text{R.M.S. secondary voltage} = 230 \times 1/5 = 46 \text{ V}$$

$$\text{Max. voltage across secondary} = 46 \times \sqrt{2} = 65 \text{ V}$$

Max. voltage across half secondary winding is

$$V_m = 65/2 = 32.5 \text{ V}$$



$$\therefore PIV = 2 V_m = 2 \times 32.5 = 65 \text{ V}$$

Bridge type circuit

R.M.S. secondary voltage = $230 \times 1/10 = 23 \text{ V}$

Max. voltage across secondary, $V_m = 23 \times \sqrt{2} = 32.5 \text{ V}$

$$\therefore PIV = V_m = 32.5 \text{ V}$$

This shows that for the same d.c. output voltage, *PIV* of bridge circuit is half that of centre-tap circuit. This is a distinct advantage of bridge circuit.

COMPARISON:

Table 3.2: comparison of rectifiers

Particulars	Half-wave rectifier	Centre-tapped full-wave rectifier	Bridge rectifier
1. No. of diodes	1	2	4
2. I_{dc}	I_m/Π	$2I_m/\Pi$	$2I_m/\Pi$
3. V_{dc}	V_m/Π	$2V_m/\Pi$	$2V_m/\Pi$
4. I_{rms}	$I_m/2$	$I_m/\sqrt{2}$	$I_m/\sqrt{2}$
5. Efficiency	40.6%	81.2%	81.2%
6. PIV	V_m	$2V_m$	V_m
7. Ripple factor	1.21	0.48	0.48

3.12 Half wave rectifier with capacitor filter

Filters are components used to convert (smoothen) pulsating DC waveforms into constant DC waveforms. They achieve this by suppressing the DC ripples in the waveform. Although half-wave rectifiers without filters are theoretically possible, they cannot be used for any practical applications. As DC equipment requires a constant waveform, we need to smooth out this pulsating waveform for it to be any use in the real world. This is why in reality we use half wave rectifiers with a filter. A capacitor or an inductor can be used as a filter {but half wave rectifier with capacitor filter is most commonly used. The circuit diagram below shows how a capacitive filter is can be used to smoothen out a pulsating DC waveform into a constant DC waveform.

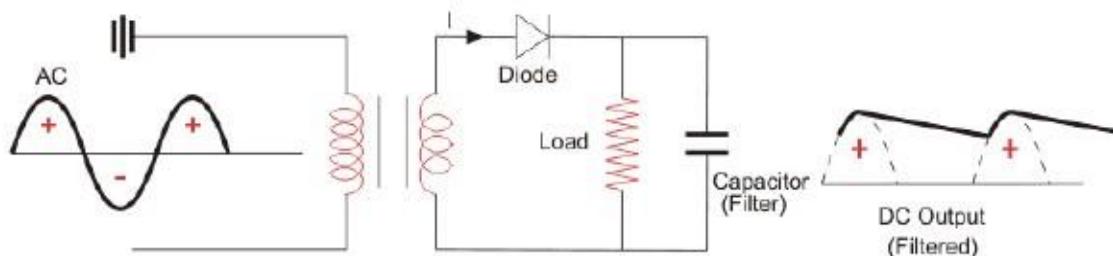


Figure 3.22: Half wave rectifier with capacitor filter and waveform

3.13 PHOTO DIODE

A photodiode is a kind of light detector, which involves the conversion of light into voltage or current, based on the mode of operation of the device.

It is also called as light sensor. The Photodiode works in reverse bias mode.

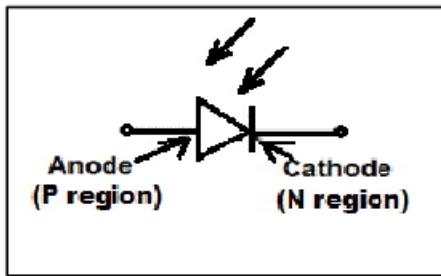


Figure 3.23: symbol of Photodiode

A photodiode is a semi-conductor device, with a p-n junction and an intrinsic layer between p and n layers. It produces photocurrent by generating electron-hole pairs, due to the absorption of light in the intrinsic or depletion region. The photocurrent thus generated is proportional to the absorbed light intensity.

Working Principle of Photodiodes

A photodiode is subjected to photons in the form of light which affects the generation of electron-hole pairs. If the energy of the falling photons ($h\nu$) is greater than the energy gap (E_g) of the semiconductor material, electron-hole pairs are created near the depletion region of the diode. The electron-hole pairs created are separated from each other before recombining due to the electric field of the junction. The direction of the electric field in the diode forces the electrons to move towards the n-side and consequently the holes move towards the p-side. As a result of the increase in the number of electrons on the n-side and holes on the p-side, a rise in the electromotive force is observed. Now when an external load is connected to the system, a current flow is observed through it

The more the electromotive force created, the greater the current flow. The magnitude of the electromotive force created depends directly upon the intensity of the incident light. This effect of the proportional change in photocurrent with the change in light intensity can be easily observed by applying a reverse bias.

Since photodiodes generate current flow directly depending upon the light intensity received, they can be used as photodetectors to detect optical signals. Built-in lenses and optical filters may be used to enhance the power and productivity of a photodiode.

Applications:

Photodiodes find application in the following:

- Cameras
- Medical devices
- Safety equipment
- Optical communication devices
- Position sensors
- Bar code scanners
- Automotive devices
- Surveying instruments

3.14 LED (Light Emitting Diode)

A light-emitting diode (LED) is a semiconductor device that emits light when an electric current flows through it. When current passes through an LED, the electrons recombine with holes emitting light in the process. LEDs allow the current to flow in the forward direction and blocks the current in the reverse direction.

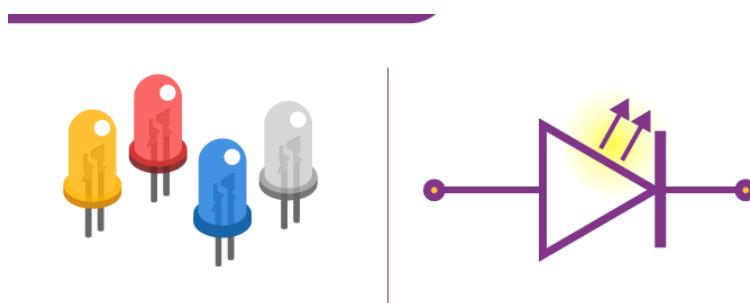


Figure 3.24: LED symbol

Light-emitting diodes are heavily doped p-n junctions. Based on the semiconductor material used and the amount of doping, an LED will emit coloured light at a particular spectral wavelength when forward biased. As shown in the figure, an LED is encapsulated with a transparent cover so that emitted light can come out.

LED Symbol

The LED symbol is the standard symbol for a diode, with the addition of two small arrows denoting the emission of light.

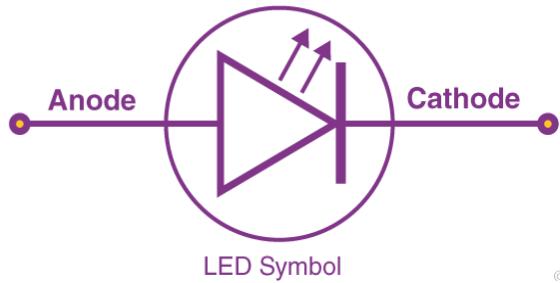


Figure 3.25: LED symbol

Simple LED Circuit

The figure below shows a simple LED circuit

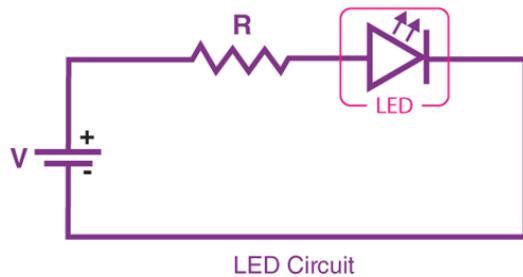


Figure 3.26: LED circuit

The circuit consists of an LED, a voltage supply and a resistor to regulate the current and voltage.

How does an LED work?

When the diode is forward biased, the minority electrons are sent from $p \rightarrow n$ while the minority holes are sent from $n \rightarrow p$. At the junction boundary, the concentration of minority carriers increases. The excess minority carriers at the junction recombine with the majority charge carriers.

WORKING PRINCIPLE OF LED

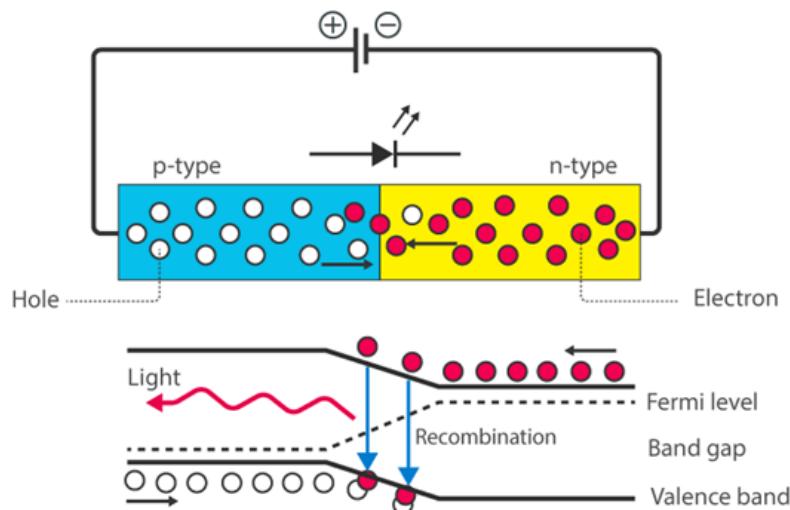


Figure 3.27: LED working principle

The energy is released in the form of photons on recombination. In standard diodes, the energy is released in the form of heat. But in light-emitting diodes, the energy is released in the form of photons. We call this phenomenon electroluminescence. Electroluminescence is an optical phenomenon, and electrical phenomenon where a material emits light in response to an electric current passed through it. As the forward voltage increases, the intensity of the light increases and reaches a maximum.

Applications of LED:

1. Picture phones and digital watches
2. Camera flashes and automotive heat lamps
3. Aviation lighting
4. Digital computers and calculators
5. Traffic signals and Burglar alarms systems
6. Microprocessors and multiplexers
7. Optical Communication
8. Indicator lamps in electric equipment
9. LED television
10. Vehicle head lamps, domestic and decorative illumination, street lighting.

3.15 Photo coupler:

An optoisolator (also called optocoupler) is a device that uses light to couple a signal from its input (a photoemitter e.g., a LED) to its output (a photodetector e.g., a photodiode)

Typically, photo couplers consist of a light emitting device optically coupled with a light detecting device via a transparent galvanic insulator. They are commonly used to transfer an electrical signal between two circuits with different ground potentials by means of light. In the past, electromagnetic relays, isolation transformers, and other devices were used to transfer electrical signals from an integrated circuit or between isolated primary and secondary sides. At present, photo couplers are generally used because they help resolve an impedance mismatch, provide higher isolation between input and output, suppress induced electromotive force, and simplify noise blocking. A photovoltaic-output photo coupler generates electricity on its own in response to light energy from the input light emitting diode (LED). Capable of driving a discrete MOSFET(s) without a power supply, photovoltaic-output photo couplers are expected to replace conventional mechanical relays. This application note provides a description of their electrical characteristics and application circuits for engineers who are unfamiliar with photovoltaic-output photo couplers.

Light emission, light reception, and signal amplification are the three main components of an optical coupler. The input electrical signal causes the light-emitting diode (LED) to emit light of a specific wavelength, which is detected by the photodetector and converted into a photocurrent, which is then output after amplification. This completes the electrical-optical-electrical conversion, acting as input, output, and isolation at the same time. The opto coupler offers good electrical insulation and anti-interference characteristics since its input and output are isolated from each other and the electrical signal transmission is unidirectional.

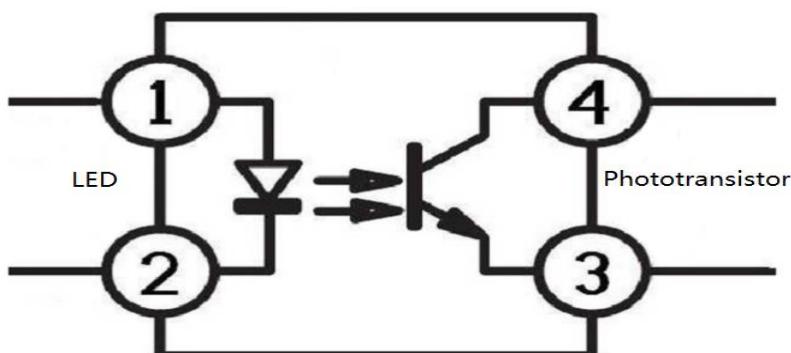


Figure 3.28: Working of photo coupler

The photocoupler's principle is to supply an electrical signal to the photocoupler's input to cause the light-emitting source to emit light. The magnitude of the excitation current determines the light intensity. The photoelectric effect generates a photocurrent once this light is irradiated on the packed light receiver. Is led out of the light receiver's output end, completing the electric-optical-electric conversion. The optocoupler's idea is that an electrical signal drives a light-emitting diode (LED) to emit a specific wavelength of light, which is detected by a photodetector, which generates a photocurrent, which is then amplified and output. This completes the electrical-optical-electrical conversion, acting as input, output, and isolation at the same time.

Where can Photo coupler be used:

(1) Logic circuit application

Optocouplers can be used to create a variety of logic circuits. Because optocouplers have stronger anti-interference and isolation properties than transistors, the logic circuits they generate are more trustworthy.

(2) For use as a solid switch

In the switch circuit, good electrical isolation between the control circuit and the switch is frequently required, which is difficult to achieve with standard electronic switches but relatively simple with a photocoupler.

(3) Use in the trigger circuit

Because the **light-emitting diodes** can be linked in series to the two emitter loops, the photoelectric coupler is employed in the bistable output circuit to effectively handle the problem of output and load isolation.

(4) Pulse amplifier circuit_application

In digital circuits, photocouplers are used to amplify pulse signals.

(5) Linear circuit application

Linear photocouplers are employed in linear circuits because of their strong linearity and electrical isolation.

(6) Use in specific circumstances

Photocouplers can also be utilized for high-voltage control, transformer replacement, contact relay replacement, A/D circuits, and other applications.

UNIT - 4

BJT as an amplifier—BJT as a switch —ON/OFF—LED and LAMP

Feedback amplifiers —types of feedback Voltage series feedback—Gain —stability with feedback.

4.1 Introduction to Transistor

The “transistor” simply referred as ‘transfer of signal from a low resistance to high resistance’. The prefix ‘trans’ means the signal transfer property of the device while ‘istor’ classifies it as a solid element in the same general family with resistors.

There are two basic transistor types: the bipolar junction transistor (**BJT**) and field effect transistor (**FET**).

As we shall see, these two transistor types differ in both their operating characteristics and their internal construction. Note that when we use the term transistor, it means bipolar junction transistor (BJT). The term comes from the fact that in a bipolar transistor, there are two types of charge carriers (viz. electrons and holes) that play part in conduction. Note that bi means two and polar refers to polarities.

A transistor consists of two pn junctions formed by sandwiching either p-type or n-type semiconductor between a pair of opposite types. Accordingly; there are two types of transistors, namely; (i) n-p-n transistor (ii) p-n-p transistor

An n-p-n transistor is composed of two n-type semiconductors separated by a thin section of p-type as shown in Fig. 4.1 (i). However, a p-n-p transistor is formed by two p-sections separated by a thin section of n-type as shown in Fig. 4.1 (ii).



Figure: 4.1 (i) npn transistor, (ii) pnp transistor

In each type of transistor, the following points may be noted:

- (i) These are two PN junctions. Therefore, a transistor may be regarded as a combination of two diodes connected back-to-back.

- (ii) There are three terminals, one taken from each type of semiconductor.
- (iii) The middle section is a very thin layer. This is the most important factor in the function of a transistor.

Naming the Transistor Terminals

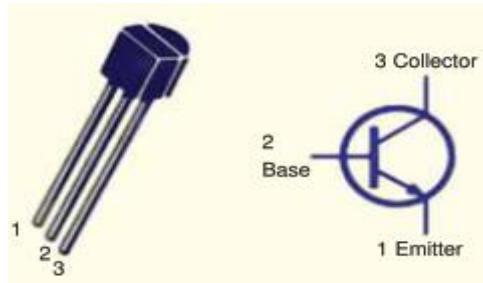


Figure: 4.2 Transistor terminals

A transistor (pnp or npn) has three sections of doped semiconductors. The section on one side is the emitter and the section on the opposite side is the collector. The middle section is called the base and forms two junctions between the emitter and collector.

(i) Emitter: The section on one side that supplies charge carriers (electrons or holes) is called the emitter. The emitter is always forward biased w.r.t. base so that it can supply a large number of majority carriers. In Fig. 4.3 (i), the emitter (p-type) of pnp transistor is forward biased and supplies hole charges to its junction with the base. Similarly, in Fig. 4.3 (ii), the emitter (n-type) of npn transistor has a forward bias and supplies free electrons to its junction with the base.

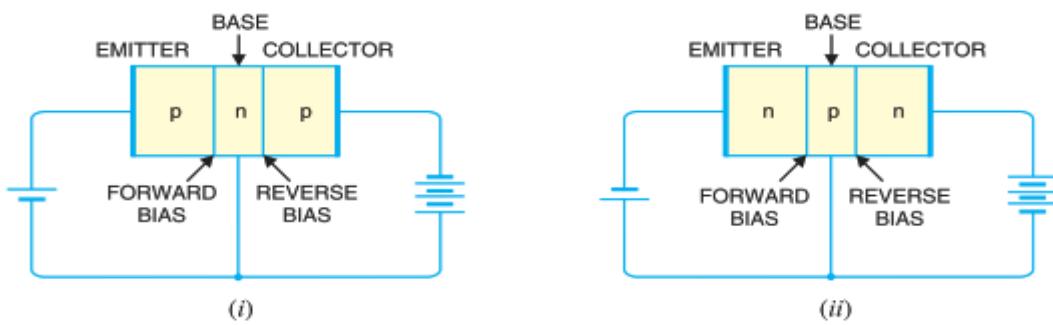


Figure: 4.3 (i) pnp transistor biasing (ii) npn transistor biasing

(ii) Collector: The section on the other side that collects the charges is called the collector. The collector is always reverse biased. Its function is to remove charges from its junction with the base. In Fig. 4.3 (i), the collector (p-type) of pnp transistor has a

reverse bias and receives hole charges that flow in the output circuit. Similarly, in Fig. 4.3 (ii), the collector (n-type) of npn transistor has reverse bias and receives electrons.

(iii) Base: The middle section which forms two pn-junctions between the emitter and collector is called the base. The base-emitter junction is forward biased, allowing low resistance for the emitter circuit. The base-collector junction is reverse biased and provides high resistance in the collector circuit.

Some Facts about the Transistor

(i) The transistor has three regions, namely; emitter, base and collector. The base is much thinner than the emitter while the collector is wider than both as shown in Fig. 4.4. However, for the sake of convenience, it is customary to show emitter and collector to be of equal size.

(ii) The emitter is heavily doped so that it can inject a large number of charge carriers (electrons or holes) into the base. The base is lightly doped and very thin; it passes most of the emitter injected charge carriers to the collector. The collector is moderately doped.



Figure: 4.4 (i) pnp transistor (ii)npn transistor

(iii) The transistor has two pn junctions i.e. it is like two diodes. The junction between emitter and base may be called emitter-base diode or simply the emitter diode. The junction between the base and collector may be called collector-base diode or simply collector diode.

(iv) The emitter diode is always forward biased whereas collector diode is always reverse biased.

(v) The resistance of emitter diode (forward biased) is very small as compared to collector diode (reverse biased). Therefore, forward bias applied to the emitter diode is generally very small whereas reverse bias on the collector diode is much higher.

4.2 Transistor Action

(i) Working of npn transistor, In Fig. 4.5 shows the npn transistor with forward bias to emitter-base junction and reverse bias to collector-base junction. The forward bias causes The electrons in the n-type emitter to flow towards the base. This constitutes the emitter Current I_E . As these electrons flow through the p-type base, they tend to combine with holes. As the base is lightly doped and very thin, therefore, only a few electrons (less than 5%) combine with holes to constitute base current I_B . The remainder (more than 95%) cross over into the collector region to constitute collector current I_C . In this way, almost the entire emitter current flows in the collector circuit. It is clear that emitter current is the sum of collector and base currents i.e. $I_E = I_B + I_C$

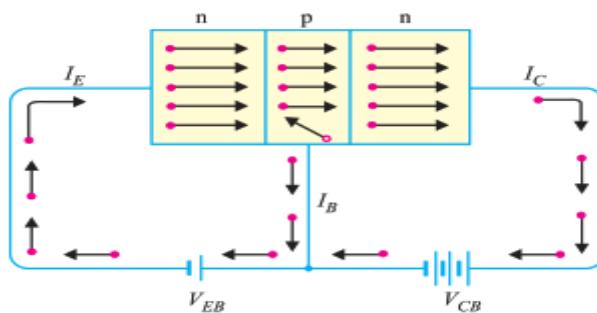


Figure: 4.5 Basic Connection of npn transistor

(ii) Working of pnp transistor. Fig. 4.6 shows the basic connection of a pnp transistor. The forward bias causes the holes in the p-type emitter to flow towards the base. This constitutes the emitter current I_E . As these holes cross into n-type base, they tend to combine with the electrons. As the base is lightly doped and very thin, therefore, only a few holes (less than 5%) combine with the electrons. The remainder (more than 95%) cross into the collector region to constitute collector current I_C . In this way, almost the entire emitter current flows in the collector circuit. It may be noted that current conduction within pnp transistor is by holes. However, in the external connecting wires, the current is still by electrons.

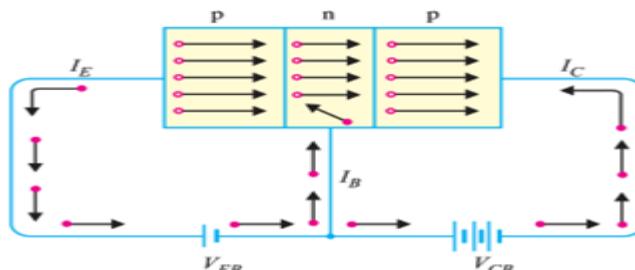


Figure: 4.6 Basic connection of pnp transistor

Importance of transistor action: The input circuit (i.e. emitter-base junction) has low resistance because of forward bias whereas output circuit (i.e. collector-base junction) has high resistance due to reverse bias.

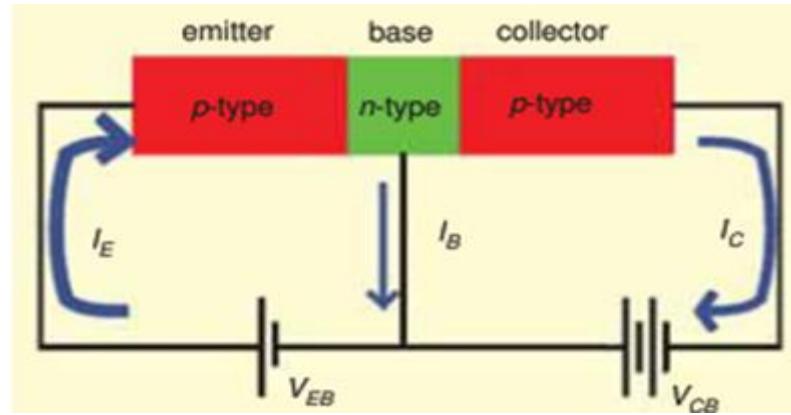


Figure: 4.7 Conventional Currents

As we have seen, the input emitter current almost entirely flows in the collector circuit. Therefore, a transistor transfers the input signal current from a low-resistance circuit to a high-resistance circuit. This is the key factor responsible for the amplifying capability of the transistor.

Transistor Symbols

In the earlier diagrams, the transistors have been shown in diagrammatic form. However, for the sake of convenience, the transistors are represented by schematic diagrams. The symbols used for npn and pnp transistors are shown in Fig. 4.8.

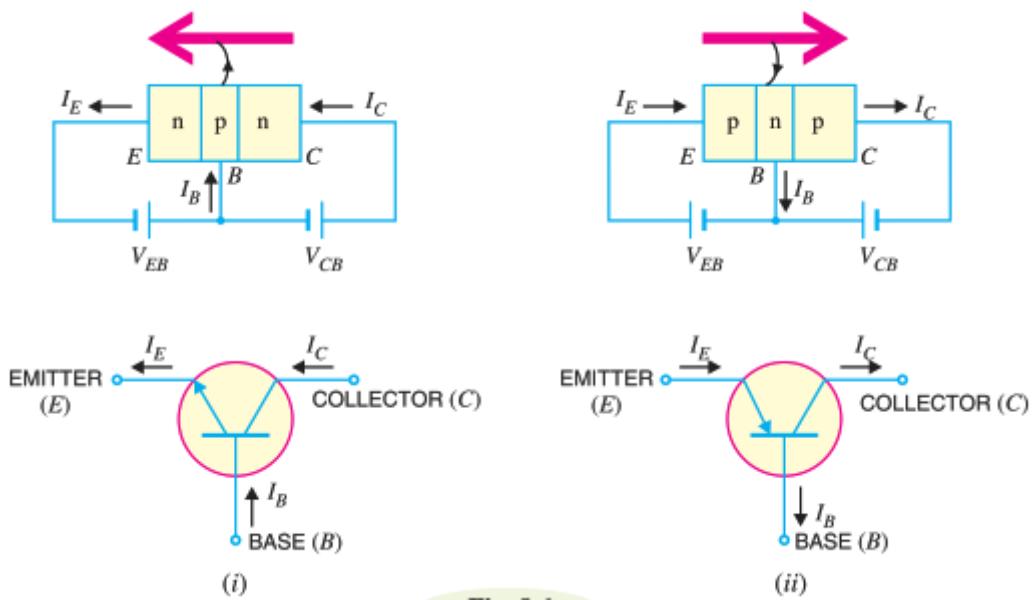


Figure. 4.8 (i) pnp connection (ii) npn connection

Note that emitter is shown by an arrow which indicates the direction of conventional current flow with forward bias. For npn connection, it is clear that conventional current flows out of the emitter as indicated by the outgoing arrow in Fig. 4.8 (i). Similarly, for pnp connection, the conventional current flows into the emitter as indicated by inward arrow in Fig. 4.8(ii).

4.3 Transistor Circuit as an Amplifier

A transistor raises the strength of a weak signal and thus acts as an amplifier. Fig. 4.9 shows the basic circuit of a transistor amplifier. The weak signal is applied between emitter-base junction and output is taken across the load RC connected in the collector circuit. In order to achieve faithful amplification, the input circuit should always remain forward biased. To do so, a d.c. voltage V_{EE} is applied in the input circuit in addition to the signal as shown. This d.c. voltage is known as bias voltage and its magnitude is such that it always keeps the input circuit forward biased regardless of the polarity of the signal.

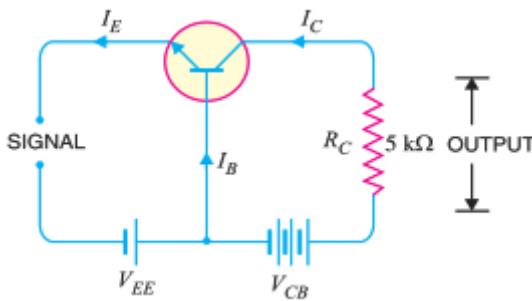


Figure: 4.9 Basic circuit of a transistor amplifier

As the input circuit has low resistance, therefore, a small change in signal voltage causes an appreciable change in emitter current. This causes almost the *same change in collector current due to transistor action. The collector current flowing through a high load resistance R_C produces a large voltage across it. Thus, a weak signal applied in the input circuit appears in the amplified form in the collector circuit. It is in this way that a transistor acts as an amplifier.

Note: The basic amplifying action is produced by transferring a current from a low-resistance to a high-resistance circuit. Consequently, the name transistor is given to the device by combining the two terms given in bolded letters below:

Transfer + Resistor → Transistor

4.4 Transistor Configurations

As the **Bipolar Transistor** is a three-terminal device, there are basically three possible ways to connect it within an electronic circuit with one terminal being common to both the input and output. Each method of connection responding differently to its input signal within a circuit as the static characteristics of the transistor vary with each circuit arrangement.

1. Common Base Configuration - has Voltage Gain but no Current Gain.
2. Common Emitter Configuration - has both Current and Voltage Gain.
3. Common Collector Configuration - has Current Gain but no Voltage Gain.

Common Base Configuration

In this circuit arrangement, input is applied between emitter and base and output is taken from collector and base. Here, base of the transistor is common to both input and output circuits and hence the name common base connection. In Fig. 8.9 (i), a common Base npn transistor circuit is shown whereas Fig. 8.9 (ii) shows the common base pnp transistor circuit.

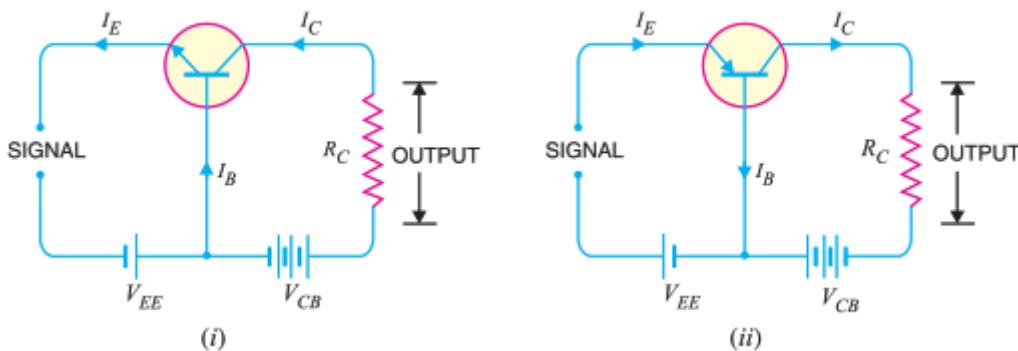


Figure: 4.10 (i) common Base npn transistor circuit (ii) common base pnp transistor circuit

Current amplification factor (α): It is the ratio of output current to input current. In a common base connection, the input current is the emitter current I_E and output current is the collector current I_C .

The ratio of change in collector current to the change in emitter current at constant collector-base voltage V_{CB} is known as current amplification factor i.e.

$$*\alpha = \frac{\Delta I_C}{\Delta I_E} \text{ at constant } V_{CB}$$

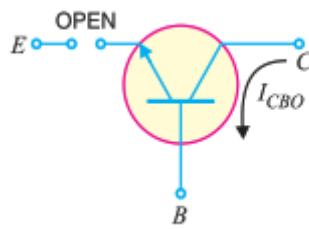


Figure: 4.11 Leakage current in transistor

It is clear that current amplification factor is less than unity. This value can be increased but not more than unity) by decreasing the base current. This is achieved by making the base thin and doping it lightly. Practical values of α in commercial transistors range from 0.9 to 0.99.

Expression for collector current: The whole of emitter current does not reach the collector. It is because a small percentage of it, as a result of electron-hole combinations occurring in base area, gives rise to base current. Moreover, as the collector-base junction is reverse biased, therefore, some leakage current flows due to minority carriers. It follows, therefore, that total collector current consists of:

- (i) That part of emitter current which reaches the collector terminal i.e. αI_E .
- (ii) The leakage current $I_{leakage}$. This current is due to the movement of minority carriers across base-collector junction on account of it being reverse biased. This is generally much smaller than αI_E .

$$\text{Total Collector current, } I_C = I_E + I_{leakage}$$

It is clear that $I_E = 0$ (i.e., emitter circuit is open), a small leakage current still flows in the collector circuit. This $I_{leakage}$ is abbreviated as I_{CBO} , meaning collector-base current with emitter open. The I_{CBO} is indicated in Fig. 8.10.

$$\begin{aligned} \therefore I_C &= \alpha I_E + I_{CBO} && \dots(i) \\ \text{Now } I_E &= I_C + I_B \\ \therefore I_C &= \alpha (I_C + I_B) + I_{CBO} \\ \text{or } I_C (1 - \alpha) &= \alpha I_B + I_{CBO} \\ \text{or } I_C &= \frac{\alpha}{1 - \alpha} I_B + \frac{I_{CBO}}{1 - \alpha} && \dots(ii) \end{aligned}$$

Relation (i) or (ii) can be used to find I_C . It is further clear from these relations that the collector current of a transistor can be controlled by either the emitter or base current.

Fig. 4.12 shows the concept of I_{CBO} . In CB configuration, a small collector current flows even when the emitter current is zero. This is the leakage collector current (i.e. the collector current when emitter is open) and is denoted by I_{CBO} . When the emitter voltage V_{EE} is also applied, the various currents are as shown in Fig. 4.12 (ii).

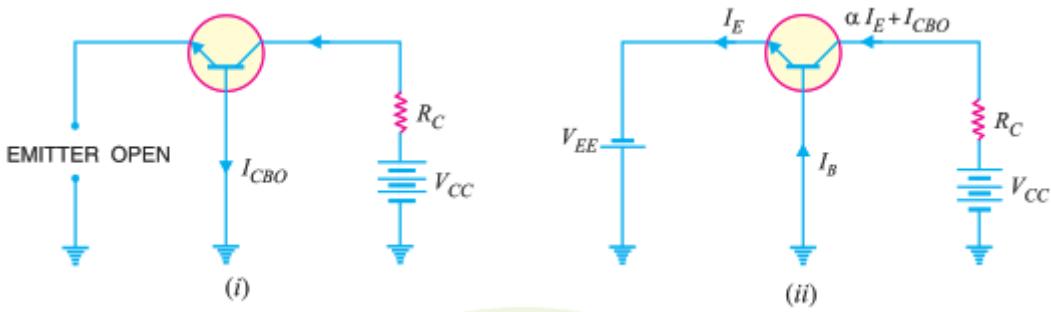


Figure: 4.12 concept of I_{CBO}

Characteristics of Common Base Connection:

The complete electrical behavior of a transistor can be described by stating the interrelation of the various currents and voltages. These relationships can be conveniently displayed graphically and the curves thus obtained are known as the characteristics of transistor. The most important characteristics of common base connection are input characteristics and output characteristics.

Input characteristics: A plot of the variation of i/p emitter current I_E with variation in the EB voltage V_{BE} (i/p) for different values of collector base voltage V_{CB} (o/p) is the i/p characteristics. To obtain the i/p characteristics, V_{CB} is kept constant, V_{BE} is varied and value of I_E is recorded. The active region is the region normally employed for linear amplification. After the cut in voltage $V_{BE} = 0.7$ V, I_E increases with small increase in V_{BE} voltage as seen in Fig: 4.13

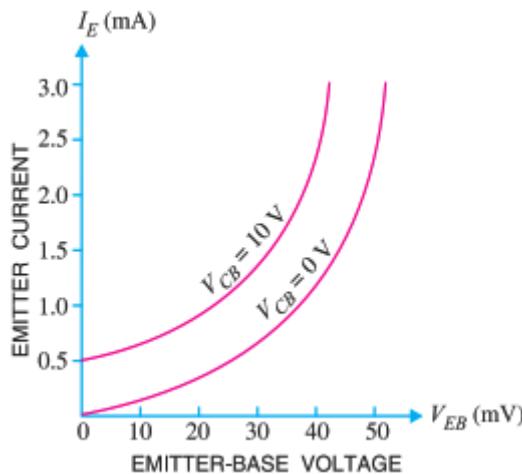


Figure: 4.13 Input characteristics for CB mode

Output characteristics: Relates to o/p current I_C to the o/p v/g V_{CB} for various levels of input current I_E as shown in Fig 4.14. The three region of interest are:-

Active region:- The active region is defined by the biasing arrangements EB—FB and CB—RB. This is the normal operating region of the transistor. At lower end of the active region the emitter current is zero, and the collector current is due to the reverse saturation current which is very small.

Cut off region:- EB—RB,CB—RB. Region where collector current is zero.

Saturation region: EB—FB, CB—FB. The region of the characteristics to the left of V_{CB} is zero.

Early Effect. The effect of increase in reverse voltage reducing the effective base width is called base width modulation or early effect.

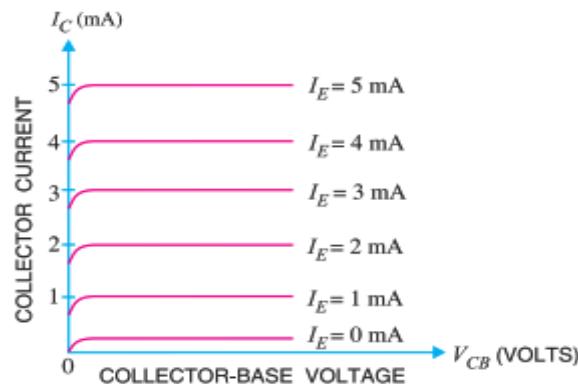


Figure: 4.14 Output characteristics for CB mode

Common Emitter Configuration

In this circuit arrangement, input is applied between base and emitter and output is taken from the collector and emitter. Here, emitter of the transistor is common to both input and output circuits and hence the name common emitter connection. Fig. 4.15 (i) shows common emitter npn transistor circuit whereas Fig. 4.15 (ii) shows common emitter pnp transistor circuit.

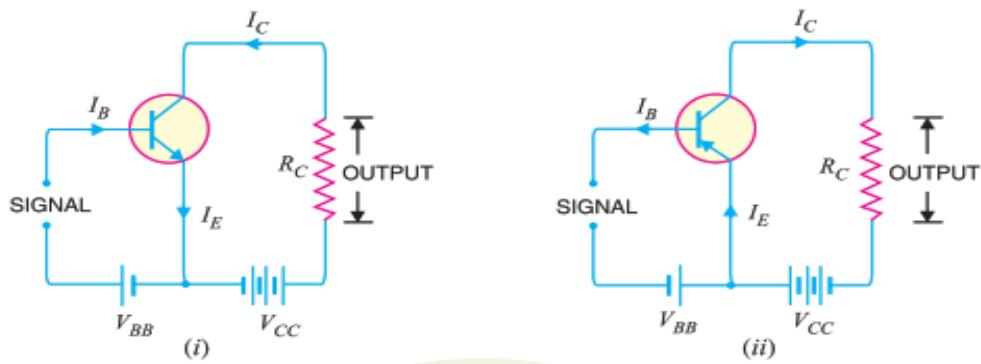


Figure: 4.15 common emitter (i) npn transistor circuit (ii)pnp transistor circuit

Base current amplification factor (β): In common emitter connection, input current is I_B and output current is I_C . The ratio of change in collector current (ΔI_C) to the change in base current (ΔI_B) is known as base current amplification factor (β) i.e.

$$\beta = \frac{\Delta I_C}{\Delta I_B}$$

In almost any transistor, less than 5% of emitter current flows as the base current. Therefore, the value of β is generally greater than 20. Usually, its value ranges from 20 to 500. This type of connection is frequently used as it gives appreciable current gain as well as voltage gain.

Relation between β and α : A simple relation exists between β and α . This can be derived as follows:

$$\beta = \frac{\Delta I_C}{\Delta I_B} \quad \dots(i)$$

$$\alpha = \frac{\Delta I_C}{\Delta I_E} \quad \dots(ii)$$

Now $I_E = I_B + I_C$

or $\Delta I_E = \Delta I_B + \Delta I_C$

or $\Delta I_B = \Delta I_E - \Delta I_C$

Substituting the value of ΔI_B in exp. (i), we get,

$$\beta = \frac{\Delta I_C}{\Delta I_E - \Delta I_C} \quad \dots(iii)$$

Dividing the numerator and denominator of R.H.S. of exp. (iii) by ΔI_E , we get,

$$\begin{aligned} \beta &= \frac{\Delta I_C / \Delta I_E}{\frac{\Delta I_E}{\Delta I_E} - \frac{\Delta I_C}{\Delta I_E}} = \frac{\alpha}{1 - \alpha} & \left[Q \quad \alpha = \frac{\Delta I_C}{\Delta I_E} \right] \\ \therefore \beta &= \frac{\alpha}{1 - \alpha} \end{aligned}$$

It is clear that as α approaches unity, β approaches infinity. In other words, the current gain in common emitter connection is very high. It is due to this reason that this circuit arrangement is used in about 90 to 95 percent of all transistor applications.

Note: If d.c. values are considered, $\beta = I_C/I_B$.

Expression for collector current: In common emitter circuit, I_B is the input current and I_C is the output current.

$$\text{We know } I_E = I_B + I_C \quad \dots(i)$$

$$\text{and } I_C = \alpha I_E + I_{CBO} \quad \dots(ii)$$

From exp. (ii), we get, $I_C = \alpha I_E + I_{CBO} = \alpha(I_B + I_C) + I_{CBO}$

or $I_C(1 - \alpha) = \alpha I_B + I_{CBO}$

or $I_C = \frac{\alpha}{1 - \alpha} I_B + \frac{1}{1 - \alpha} I_{CBO} \quad \dots(iii)$

From exp. (iii), it is apparent that if $I_B = 0$ (i.e. base circuit is open), the collector current will be the current to the emitter. This is abbreviated as I_{CEO} , meaning collector-emitter current with base open.

$$\therefore I_{CEO} = \frac{1}{1 - \alpha} I_{CBO}$$

Substituting the value of $\frac{1}{1 - \alpha} I_{CBO} = I_{CEO}$ in exp. (iii), we get,

$$I_C = \frac{\alpha}{1 - \alpha} I_B + I_{CEO}$$

$$I_C = \beta I_B + I_{CEO} \quad \left(Q \beta = \frac{\alpha}{1 - \alpha} \right)$$

Characteristics of Common Emitter Connection:

Emitter is common to both i/p and o/p terminals.

Input characteristics: A plot of the variation of i/p base current I_B with variation in the EB voltage V_{BE} (i/p) for different values of collector emitter voltage V_{CE} (o/p) is the i/p characteristics as seen in Fig 4.16. To obtain the i/p characteristics, V_{CE} is kept constant, V_{BE} is varied and value of I_B is recorded. For fixed value of V_{BE} , I_B decreases with increase in V_{CE} . This is due to the fact that with increase in V_{CE} , the depletion region of the reverse bias collector base junction widens, reducing base width.

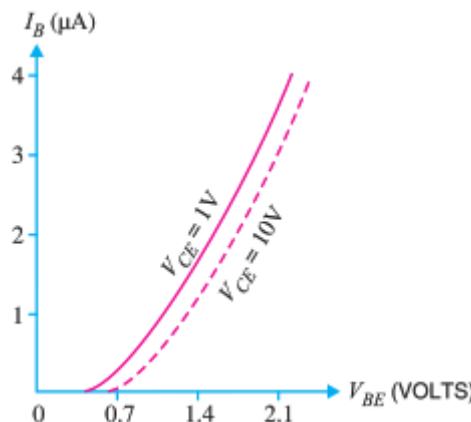


Figure: 4.16 Input characteristics for CE mode

Output characteristics: Relates to o/p current I_C to an o/p v/g V_{CE} for various levels of input current I_B as seen in Fig 4.17. The three region of interest are :-

Active region:-In this region, the collector current responds more readily to any input signal since I_C is more sensitive. So the transistor can be used as an amplifier. The characteristics are not horizontal lines due to fixed I_B , the magnitude of I_C increases with increases in V_{CE} due to early effect.

The collector current I_C can be derived in active region as before.

In common base mode

$$I_C = -\alpha I_E + I_{CO}$$

$$I_C = \alpha(I_B + I_C) + I_{CO}$$

$$I_C = \alpha I_B + \alpha I_C + I_{CO}$$

$$I_C(1-\alpha) = \alpha I_B + I_{CO}$$

$$I_C = \alpha I_B / (1-\alpha) + I_{CO} / (1-\alpha)$$

Substituting $\beta = \alpha / (1-\alpha)$ and $1/(1-\alpha) = 1+\beta$ yields

$$I_C = \beta I_B + I_{CO} / (1+\beta)$$

Cutoff region: the region to the right of the $V_{CE} = 0$ and below $I_B = 0$ is the cutoff region. If $I_B = 0$ then $I_C = I_E = (1+\beta)I_{CO}$.

Saturation region: This region lies extremely close to the zero voltage axis, where all the curves merge and fall rapidly towards the origin. In this region I_C is almost independent of I_B .

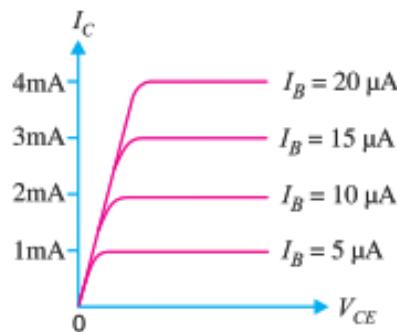


Figure: 4.17 Output characteristics for CE mode

Common Collector Configuration

In this circuit arrangement, input is applied between base and collector while output is taken between the emitter and collector. Here, collector of the transistor is common to both input and output circuits and hence the name common collector connection. Fig. 4.18 (i) shows common collector npn transistor circuit whereas Fig. 4.18 (ii) shows common collector pnp circuit.

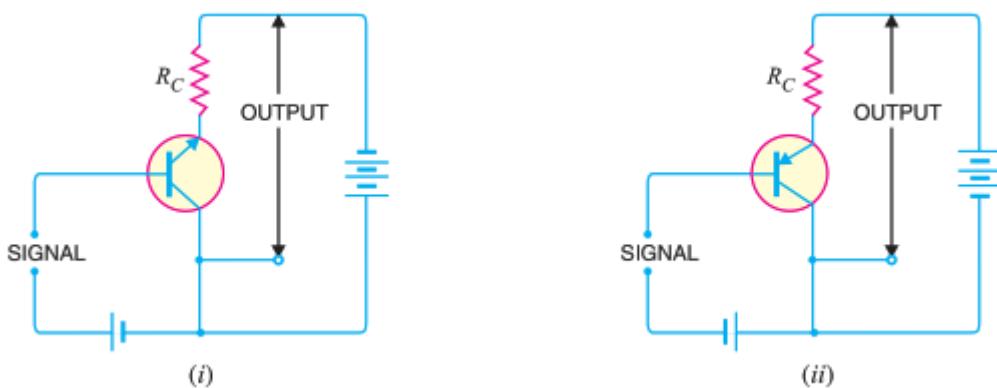


Figure: 4.18 common collector (i) npn transistor circuit (ii)pnp transistor circuit

Current amplification factor (γ): In common collector circuit, input current is the base current I_B and output current is the emitter current I_E . Therefore, current amplification in this circuit arrangement can be defined as under:

The ratio of change in emitter current (ΔI_E) to the change in base current (ΔI_B) is known as current amplification factor in common collector (CC) arrangement i.e.

$$\gamma = \frac{\Delta I_E}{\Delta I_B}$$

This circuit provides about the same current gain as the common emitter circuit as $\Delta I_E = \Delta I_C$. However, its voltage gain is always less than 1.

Relation between γ and α :

$$\gamma = \frac{\Delta I_E}{\Delta I_B} \quad \dots(i)$$

$$\alpha = \frac{\Delta I_C}{\Delta I_E} \quad \dots(ii)$$

$$\begin{aligned} \text{Now} \quad I_E &= I_B + I_C \\ \text{or} \quad \Delta I_E &= \Delta I_B + \Delta I_C \\ \text{or} \quad \Delta I_B &= \Delta I_E - \Delta I_C \end{aligned}$$

Substituting the value of ΔI_B in exp. (i), we get,

$$\gamma = \frac{\Delta I_E}{\Delta I_E - \Delta I_C}$$

Dividing the numerator and denominator of R.H.S. by ΔI_E , we get,

$$\begin{aligned} \gamma &= \frac{\frac{\Delta I_E}{\Delta I_E}}{\frac{\Delta I_E - \Delta I_C}{\Delta I_E}} = \frac{1}{1 - \alpha} \quad \left(\text{Q } \alpha = \frac{\Delta I_C}{\Delta I_E} \right) \\ \therefore \quad \gamma &= \frac{1}{1 - \alpha} \end{aligned}$$

Expression for collector current:

We know

$$I_C = \alpha I_E + I_{CBO}$$

Also

$$I_E = I_B + I_C = I_B + (\alpha I_E + I_{CBO})$$

\therefore

$$I_E(1 - \alpha) = I_B + I_{CBO}$$

or

$$I_E = \frac{I_B}{1 - \alpha} + \frac{I_{CBO}}{1 - \alpha}$$

or

$$I_C + I_E = *(\beta + 1) I_B + (\beta + 1) I_{CBO}$$

Characteristics of Common Collector Connection:

The input characteristic of common-collector configuration is similar with common emitter Configuration. Common-collector circuit configuration is provided with the load resistor connected from emitter to ground. It is used primarily for impedance-matching purpose since it has high input impedance and low output impedance.

For the common-collector configuration, the output characteristics are a plot of I_E vs V_{CE} for a range of values of I_B .

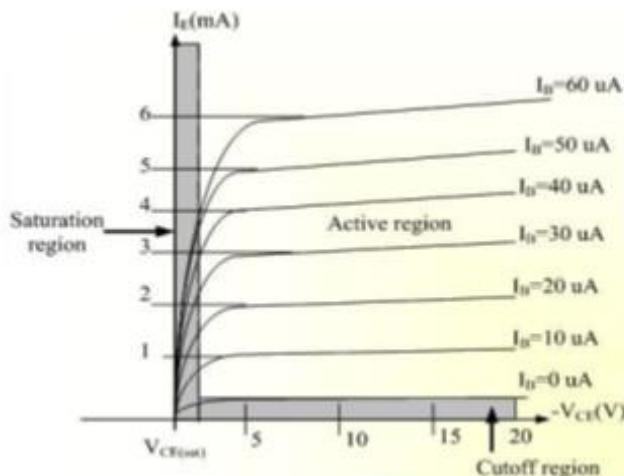


Figure: 4.19 Output characteristics of CC Configuration for npn transistor

The comparison of various characteristics of the three Configurations is given below in the tabular form.

Table 4.1: comparison of the three Configurations

S. No.	Characteristic	Common base	Common emitter	Common collector
1.	Input resistance	Low (about 100Ω)	Low (about 750Ω)	Very high (about $750 k\Omega$)
2.	Output resistance	Very high (about $450 k\Omega$)	High (about $45 k\Omega$)	Low (about 50Ω)
3.	Voltage gain	about 150	about 500	less than 1
4.	Applications	For high frequency applications	For audio frequency applications	For impedance matching
5.	Current gain	No (less than 1)	High (β)	Appreciable

$$\beta = \frac{\alpha}{1-\alpha} \quad \therefore \quad \beta + 1 = \frac{\alpha}{1-\alpha} + 1 = \frac{1}{1-\alpha}$$

4.5 BJT As A Switch

When used as an AC signal amplifier, the transistor's Base biasing voltage is applied in such a way that it always operates within its "active" region, that is the linear part of the output characteristics curves are used. However, both the NPN & PNP type bipolar transistors can be made to operate as "ON/OFF" type solid state switch by biasing the transistors Base terminal differently operating the transistor as a switch.

Solid state switches are one of the main applications for the use of transistor to switch a DC output "ON" or "OFF". Some output devices, such as LED's only require a few milliamps at logic level DC voltages and can therefore be driven directly by the output of a logic gate. However, high power devices such as motors, solenoids or lamps, often require more power than that supplied by an ordinary logic gate so transistor switches are used.

If the circuit uses the **Bipolar Transistor as a Switch**, then the biasing of the transistor, either NPN or PNP is arranged to operate the transistor at both sides of the "I-V" characteristics curves we have seen previously.

The areas of operation for a transistor switch are known as the **Saturation Region** and the **Cut-off Region**. This means then that we can ignore the operating Q-point biasing and voltage divider circuitry required for amplification, and use the transistor as a switch by driving it back and forth between its "fully-OFF" (cut-off) and "fully-ON" (saturation) regions as shown below.

Operating Regions

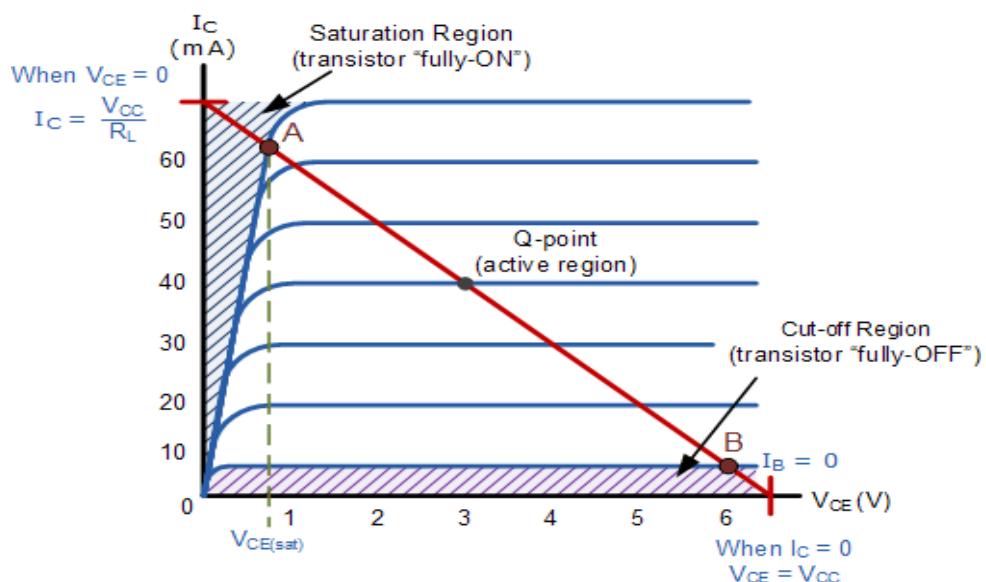


Figure: 4.20 various regions of a transistor

The pink shaded area at the bottom of the curves represents the “Cut-off” region while the blue area to the left represents the “Saturation” region of the transistor. Both these transistor regions are defined as:

1. Cut-off Region

Here the operating conditions of the transistor are zero input base current (I_B), zero output collector current (I_C) and maximum collector voltage (V_{CE}) which results in a large depletion layer and no current flowing through the device. Therefore, the transistor is switched “Fully-OFF”.

Cut-off Characteristics

<p>Fig: 4.21 CE config. For Cut-off region</p>	<ul style="list-style-type: none"> • The input and Base are grounded (0v) • Base-Emitter voltage $V_{BE} < 0.7v$ • Base-Emitter junction is reverse biased • Base-Collector junction is reverse biased • Transistor is “fully-OFF” (Cut-off region) • No Collector current flows ($I_C = 0$) • $V_{OUT} = V_{CE} = V_{CC} = “1”$ • Transistor operates as an “open switch”
--	--

Then we can define the “cut-off region” or “OFF mode” when using a bipolar transistor as a switch as being, both junctions reverse biased, $V_B < 0.7v$ and $I_C = 0$. For a PNP transistor, the Emitter potential must be negative with respect to the Base.

2. Saturation Region

Here the transistor will be biased so that the maximum amount of base current is applied, resulting in maximum collector current resulting in the minimum collector emitter voltage drop which results in the depletion layer being as small as possible and maximum current flowing through the transistor. Therefore, the transistor is switched “Fully-ON”.

Saturation Characteristics

	<ul style="list-style-type: none"> The input and Base are connected to V_{cc} Base-Emitter voltage $V_{BE} > 0.7v$ Base-Emitter junction is forward biased Base-Collector junction is forward biased Transistor is “fully-ON” (saturation region) Max Collector current flows ($I_C = V_{cc}/R_L$) $V_{CE} = 0$ (ideal saturation) $V_{OUT} = V_{CE} = “0”$ Transistor operates as a “closed switch”
<p>Fig: 4.22 CE config. For Saturation region</p>	

Then we can define the “saturation region” or “ON mode” when using a bipolar transistor as a switch as being, both junctions forward biased, $V_B > 0.7v$ and $I_C = \text{Maximum}$. For a PNP transistor, the Emitter potential must be positive with respect to the Base.

Then the transistor operates as a “single-pole single-throw” (SPST) solid state switch. With a zero-signal applied to the Base of the transistor it turns “OFF” acting like an open switch and zero collector current flows. With a positive signal applied to the Base of the transistor it turns “ON” acting like a closed switch and maximum circuit current flows through the device.

The simplest way to switch moderate to high amounts of power is to use the transistor with an open-collector output and the transistors Emitter terminal connected directly to ground. When used in this way, the transistors open collector output can thus “sink” an externally supplied voltage to ground thereby controlling any connected load.

An example of an NPN Transistor as a switch being used to operate a relay is given below. With inductive loads such as relays or solenoids a flywheel diode is placed across the load to dissipate the back EMF generated by the inductive load when the transistor switches “OFF” and so protect the transistor from damage. If the load is of a very high current or voltage nature, such as motors, heaters etc, then the load current can be controlled via a suitable relay as shown.

Basic NPN Transistor Switching Circuit

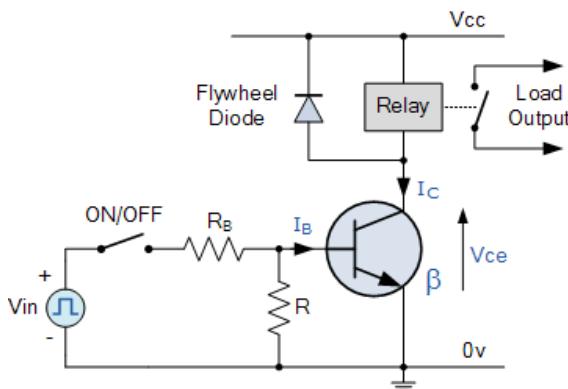


Figure: 4.23 Common Emitter circuit which acts as switch

The circuit resembles that of the *Common Emitter* circuit we looked at in the previous discussions. The difference this time is that to operate the transistor as a switch the transistor needs to be turned either fully “OFF” (cut-off) or fully “ON” (saturated).

An ideal transistor switch would have infinite circuit resistance between the Collector and Emitter when turned “fully-OFF” resulting in zero current flowing through it and zero resistance between the Collector and Emitter when turned “fully-ON”, resulting in maximum current flow.

In practice when the transistor is turned “OFF”, small leakage currents flow through the transistor and when fully “ON” the device has a low resistance value causing a small saturation voltage (V_{CE}) across it. Even though the transistor is not a perfect switch, in both the cut-off and saturation regions the power dissipated by the transistor is at its minimum.

In order for the Base current to flow, the Base input terminal must be made more positive than the Emitter by increasing it above the 0.7 volts needed for a silicon device. By varying this Base-Emitter voltage V_{BE} , the Base current is also altered and which in turn controls the amount of Collector current flowing through the transistor as previously discussed.

When maximum Collector current flows the transistor is said to be **Saturated**. The value of the Base resistor determines how much input voltage is required and corresponding Base current to switch the transistor fully “ON”.

Digital Logic Transistor Switch

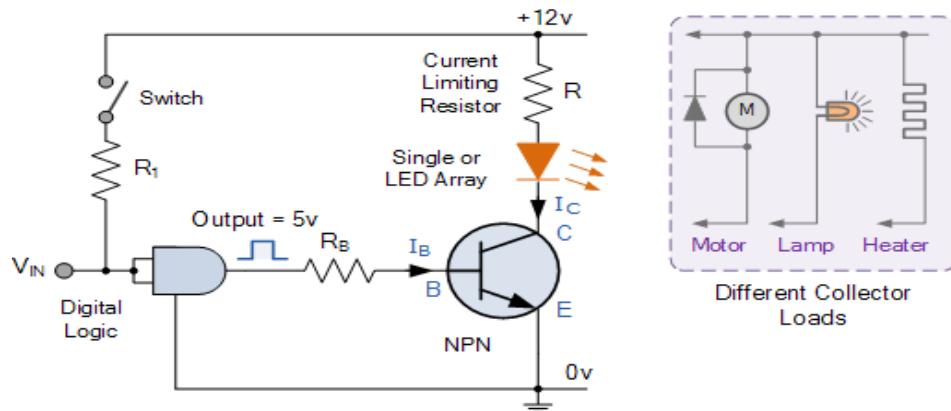


Figure: 4.24 Digital logic transistor (npn) switching circuit

The base resistor, R_B is required to limit the output current from the logic gate.

PNP Transistor Switch

We can also use the PNP Transistors as a switch, the difference this time is that the load is connected to ground (0v) and the PNP transistor switches the power to it. To turn the PNP transistor operating as a switch “ON”, the Base terminal is connected to ground or zero volts (LOW) as shown.

PNP Transistor Switching Circuit

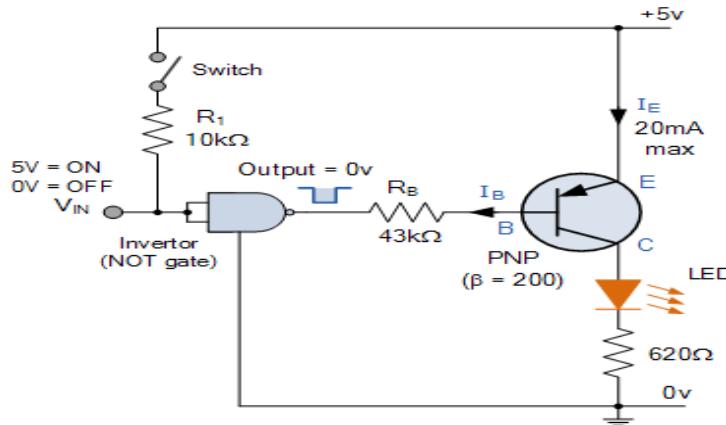


Figure: 4.25 Digital logic transistor (pnp) switching circuit

The equations for calculating the Base resistance, Collector current and voltages are exactly the same as for the previous NPN transistor switch. The difference this time is that we are switching power with a PNP transistor (sourcing current) instead of switching ground with an NPN transistor (sinking current)

4.6 Feedbacks of Amplifier

Concept of Feedback

A feedback amplifier uses feedback from the output of the amplifier back to the input to enhance its performance. This feedback is achieved through a feedback loop and it can be either positive or negative.

Let's discuss each block of feedback amplifier:

- **Signal Source:** The signal source is generally a voltage source, represented by the symbol V_s , with source current I_s . The signal source can be connected either in series or in parallel with the resistor in the electronic circuit.
- **Feedback network:** It is a linear two port network which contains different components of the electronic circuit, like resistors, inductors, capacitors etc.
- **Sampling Network:** A sampling network extracts the output signal from a feedback amplifier and “feeds” it back to the input signal.

Sampling networks are of two types: current sampling network and voltage sampling network.

- **Mixer:** A mixer mixes the source signal and feedback signal to produce either positive feedback or negative feedback. It is also known as a comparator and can be connected in series or parallel.

Gain Stability Relation

The gain of a feedback amplifier is defined as the ratio of output voltage with respect to the input voltage of a circuit.

The gain stability of a feedback amplifier refers to the ability of the amplifier to maintain a constant gain irrespective of the changes in the operating conditions.

For an ideal system of a feedback amplifier, the gain of an amplifier is always infinite.

For a negative feedback amplifier, the closed loop gain is given as

$$A' = \frac{A}{1 + \beta A}$$

Taking logarithms on both the sides of the above equation, gives

$$\log_e A' = \log_e A - \log_e (1 + \beta A)$$

Differentiating the above equation, gives

$$\begin{aligned}\frac{dA'}{A'} &= \frac{dA}{A} - \frac{\beta dA}{(1 + \beta A)} \\ &= \left[\frac{1}{A} - \frac{\beta}{1 + \beta A} \right] dA \\ &= \left[\frac{1 + \beta A - \beta A}{A(1 + \beta A)} \right] dA \quad \text{or} \\ \frac{dA'}{A'} &= \frac{1}{(1 + \beta A)} \left(\frac{dA}{A} \right).\end{aligned}$$

The above Eqn., relates the fractional change in the closed-loop gain of the negative feedback amplifier namely

$$\left(\frac{dA'}{A'} \right)$$

with the fractional change in its open-loop gain, namely

$$\left(\frac{dA}{A} \right)$$

An amplifier circuit simply increases the signal strength. But while amplifying, it just increases the strength of its input signal whether it contains information or some noise along with information. This noise or some disturbance is introduced in the amplifiers because of their strong tendency to introduce **hum** due to sudden temperature changes or stray electric and magnetic fields. Therefore, every high gain amplifier tends to give noise along with signal in its output, which is very undesirable.

The noise level in the amplifier circuits can be considerably reduced by using **negative feedback** done by injecting a fraction of output in phase opposition to the input signal.

Principle of Feedback Amplifier

A feedback amplifier generally consists of two parts. They are the **amplifier** and the **feedback circuit**. The feedback circuit usually consists of resistors. The concept of feedback amplifier can be understood from the following figure.

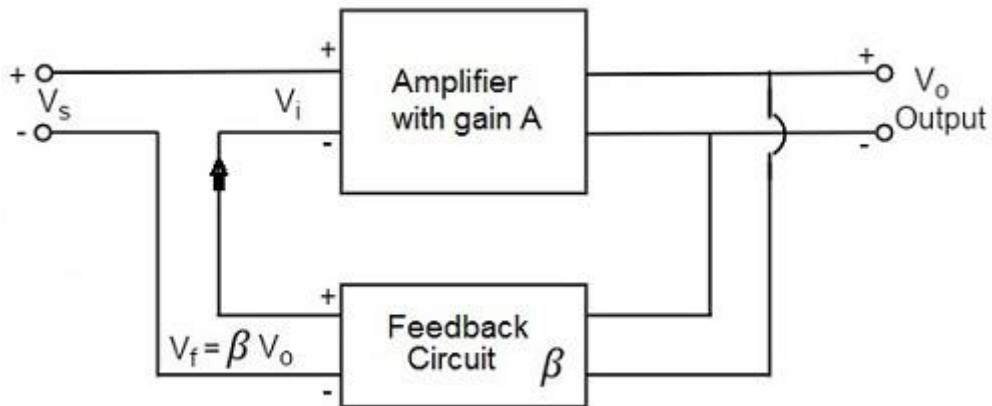


Figure: 4.26 Feedback amplifier system

From the above figure, the gain of the amplifier is represented as A. the gain of the amplifier is the ratio of output voltage V_o to the input voltage V_i . the feedback network extracts a voltage $V_f = \beta V_o$ from the output V_o of the amplifier.

This voltage is added for positive feedback and subtracted for negative feedback, from the signal voltage V_s . Now,

$$V_i = V_s + V_f = V_s + \beta V_o$$

$$V_i = V_s - V_f = V_s - \beta V_o$$

The quantity $\beta = V_f/V_o$ is called as feedback ratio or feedback fraction.

Let us consider the case of negative feedback. The output V_o must be equal to the input voltage $(V_s - \beta V_o)$ multiplied by the gain A of the amplifier.

Hence,

$$(V_s - \beta V_o)A = V_o$$

Or

$$AV_s - A\beta V_o = V_o$$

Or

$$AV_s = V_o(1 + A\beta)$$

Therefore,

$$\frac{V_o}{V_s} = \frac{A}{1 + A\beta}$$

Let A_f be the overall gain (gain with the feedback) of the amplifier. This is defined as the ratio of output voltage V_o to the applied signal voltage V_s , i.e.,

$$A_f = \frac{\text{Output voltage}}{\text{Input signal voltage}} = \frac{V_o}{V_s}$$

So, from the above two equations, we can understand that,

The equation of gain of the feedback amplifier, with negative feedback is given by

$$A_f = \frac{A}{1 + A\beta}$$

The equation of gain of the feedback amplifier, with positive feedback is given by

$$A_f = \frac{A}{1 - A\beta}$$

These are the standard equations to calculate the gain of feedback amplifiers.

Types of Feedbacks

The process of injecting a fraction of output energy of some device back to the input is known as **Feedback**. It has been found that feedback is very useful in reducing noise and making the amplifier operation stable.

Depending upon whether the feedback signal **aids** or **opposes** the input signal, there are two types of feedbacks used.

- **Positive Feedback**

The feedback in which the feedback energy i.e., either voltage or current is in phase with the input signal and thus aids it is called as **Positive feedback**.

Both the input signal and feedback signal introduce a phase shift of 180° thus making a 360° resultant phase shift around the loop, to be finally in phase with the input signal.

Though the positive feedback **increases** the **gain** of the amplifier, it has the disadvantages such as

- Increasing distortion
- Instability

It is because of these disadvantages the positive feedback is not recommended for the amplifiers. If the positive feedback is sufficiently large, it leads to oscillations, by which oscillator circuits are formed. This concept will be discussed in **OSCILLATORS**

- **Negative Feedback**

The feedback in which the feedback energy i.e., either voltage or current is out of phase with the input and thus opposes it, is called as **negative feedback**.

In negative feedback, the amplifier introduces a phase shift of 180° into the circuit while the feedback network is so designed that it produces no phase shift or zero phase shift. Thus, the resultant feedback voltage V_f is 180° out of phase with the input signal V_{in} .

Though the **gain** of negative feedback amplifier is **reduced**, there are many advantages of negative feedback such as:

- Stability of gain is improved
- Reduction in distortion
- Reduction in noise
- Increase in input impedance
- Decrease in output impedance
- Increase in the range of uniform application

It is because of these advantages negative feedback is frequently employed in amplifiers.

4.7 Feedback Amplifier Topologies

There are four types of feedback topology

1. Current series feedback amplifier
2. Voltage series feedback amplifier
3. Current shunt feedback amplifier
4. Voltage shunt feedback amplifier

Let's discuss each of the feedback amplifiers briefly

Voltage series feedback amplifier

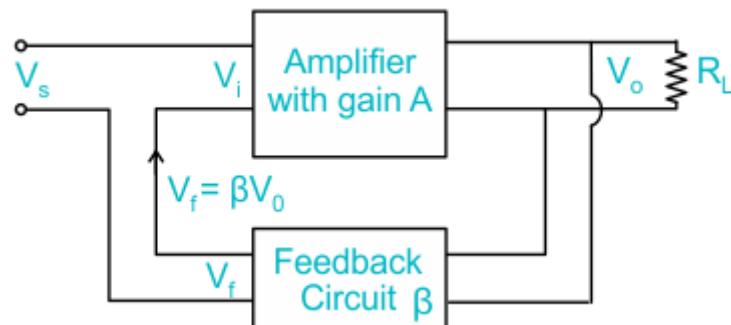


Figure: 4.27 Voltage series feedback amplifier

- The feedback circuit is connected in shunt with the output in such a way that it decreases the output impedance and increases the input impedance.
- In this circuit, it is placed in a shunt with the output but in series with respect to the input signal.

Advantages and Disadvantages of Feedback Amplifier

The followings are the advantages and disadvantages of feedback amplifiers

Advantages

- Improves stability gain of the circuit by negative feedback amplifiers.
- Input resistance can be increased by selective configurations.
- Similarly, output resistance can be decreased.
- Reduces noise and distortion by negative feedback.

Disadvantages

- Increased noise and distortion in positive feedback.
- Can reduce gain.
- Has an increased output gain for current series and current shunt feedback amplifiers.

Application of Feedback Amplifier

The following are the applications of feedback amplifiers

- They can be used in regulated power supplies.
- They are generally used in amplifiers where large bandwidth is required.
- Generally used in electronic amplifiers.

4.8 Oscillators

Definition: Any circuit which is used to generate ac voltage without ac input signal is called an oscillator.

The oscillator circuit is received energy from a DC source to generate AC voltage.

Classification of Oscillators:

What are the various types of Oscillator Circuits?

The oscillator circuits are classified in the following different ways.

[1]Based on the wave forms generated:

- (a) Sinusoidal oscillator
- (b) Relaxation oscillator

4.9 Sinusoidal Oscillator:

If the output voltage of the oscillator circuit is a sine wave function of time, the oscillator is called as a Sinusoidal oscillator or Harmonic oscillator.

Relaxation Oscillator:

This category of oscillator generates voltages or currents which vary abruptly one or more times in a cycle of oscillation. i.e., they generate non-sinusoidal wave forms such as square, rectangular, triangular or saw-tooth waveforms.

[2] Based on the fundamental mechanisms involved:

- (a) Negative resistance oscillators
- (b) Feedback oscillators

Negative Resistance oscillator:

It uses negative resistance of the amplifying device to neutralize the positive resistance of the oscillator.

Feedback Oscillator:

It uses positive feedback in the feedback amplifier to satisfy the Barkhausen Criterion.

[3] Based on the frequency generated:

- (a) Audio Frequency oscillator (AFO) - Upto 20KHz
- (b) Radio Frequency oscillator (RFO) - 20 KHz to 30MHz
- (c) Very High Frequency oscillator (VHF) - 30MHz to 300MHz
- (d) Ultra High Frequency oscillator (UHF) - 300MHz to 3GHz
- (e) Microwave Frequency oscillator - Above 3GHz

[4] Based on the type of circuit used, sine wave oscillators may be classified as

- (a) LC tuned Oscillator
- (b) RC phase shift oscillator

[5] Transistor based oscillators are classified as follows.

- | | |
|--------------------------------|----------------------------|
| (a) Tuned collector oscillator | (b) Colpitt's oscillator |
| (c) Hartley oscillator | (d) Phase shift oscillator |
| (e) Wien Bridge oscillator | (f) Crystal oscillator |

All these transistor-based oscillators are had same function ie, providing undamped waveform as the output. The difference between them is the manner in which they provide the energy to the tank circuit to produce the oscillations.

Conditions for Oscillation: (Barkhausen Criterion)

What are the conditions to be met to generate Oscillator wave forms?

The essential conditions for maintaining oscillations are

1. $|A\beta| = 1$, ie, the magnitude of loop gain must be unity.
2. The total phase shift around the closed loop is zero or 360 degrees.

The condition $|A\beta| = 1$ is the ideal condition.

But in practice, transistor characteristics and other circuit components performance vary with time.

So the value $|A\beta|$ will become greater or less than unity.

To avoid this problem, in all practical oscillator circuits, the value $|A\beta|$ should be set greater than unity so that the amplitude of oscillation will continue to increase.

But such an increase in amplitude is limited by the onset of the non-linearity of operation in the active devices associated with the amplifier as shown in the Block diagram of oscillator.

Applications of Oscillators

Oscillators find widespread applications due to their capacity to produce accurate, repeatable frequencies.

Here are some examples:

1. Timekeeping devices like quartz crystal clocks/watches (crystal oscillators)
2. Audio devices like radios, tuners, sound cards, guitars (sine wave oscillator)
3. RF devices like cell phones, WiFi, radios, radar (voltage-controlled oscillator)
4. Test & measurement equipment like oscilloscopes, function generators

5. Digital electronics like microcontrollers, RAM, CPUs (crystal oscillators)
6. Medical devices like ultrasounds, EKGs, defibrillators
7. Industrial process control systems (crystal, relaxation oscillators)

Unit 5

Digital Electronics fundamentals: Difference between Analog and Digital Circuits—Number system—Binary—Hexadecimal —Conversions —Decimal to Binary—Hexadecimal and vice versa. Sequential and combinational circuits. Logic gates—Universal gates—Full Adder— Half Adder. MUX—DeMUX— Flip Flops (D—T—JK—& SR)

Basic Principle of Communication: Communication system Block diagram & working principles—Evolution of Communication systems

5.1 Digital Electronics Fundamentals

Introduction

Digital electronics is a type of electronics that deals with the digital systems which processes the data/information in the form of binary (0s and 1s) numbers, whereas analog electronics deals with the analog systems which processes the data/information in the form of continuous signals.

Characteristics of Digital systems

- Digital systems manipulate discrete elements of information.
- Discrete elements are nothing but the digits such as 10 decimal digits or 26 letters of alphabets and so on.
- Digital systems use physical quantities called signals to represent discrete elements.
- In digital systems, the signals have two discrete values and are therefore said to be binary.
- A signal in digital system represents one binary digit called a bit. The bit has a value either 0 or 1.

5.2 Difference between Analog Circuit and Digital Circuit

Analog Circuit

An analog circuit is a type electronic circuit that can process any analog signal or data and produce an output in analog form. Analog circuits are composed of **resistors, inductors and capacitors, etc.**

The type of signal which is a continuous function of time is known as **analog signal.**

All the real-world signals are the analog signals; therefore, the analog circuit do not require any conversion of the input signal i.e. the analog input signal can be directly fed to the analog circuit without any loss and it can be directly processed by the given analog circuit. Also, the output signal produced by the analog circuit is an analog signal.

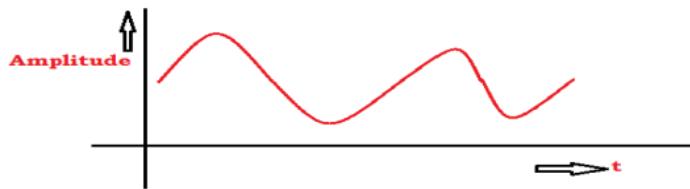


Figure 5.1(a): Analog Signal

Based on the circuit behaviour and the components used, the analog circuit can be of two types:

1. Active Circuit

Ex: Amplifiers

2. Passive Circuit

Ex: Low Pass Filter

The main drawback of the analog circuits is that the analog signals are very susceptible to the noise which may cause distortion of the signal waveform and causing the loss of information.

Digital Circuit

A digital circuit is an electronic circuit that processes digital signals. A signal that is a discrete function of time are known as **digital signals**.

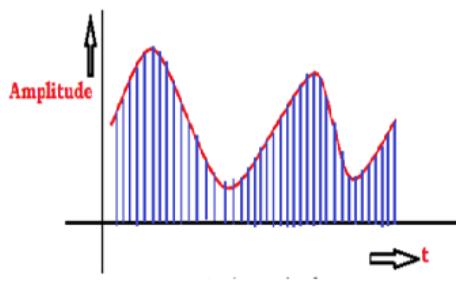


Figure 5.1(b1): Discrete Signal

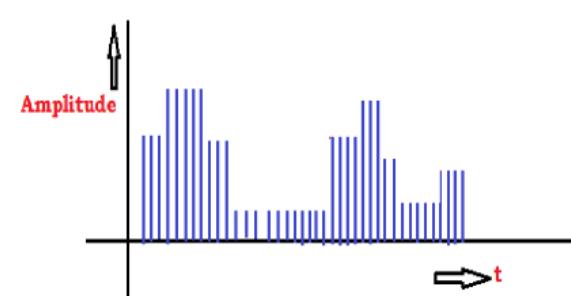


Figure 5.1(b2): Digital Signal

The basic building blocks of digital circuits are digital logic gates. The digital circuit can process only digital signals, but the real-world signals are of analog nature. Therefore, they need to be converted into digital signals using special electronic circuit known as **ADC (Analog to Digital Converter)**. The output of the digital circuits is also digital signals, which is required to be converted back into the analog signal.

There may be loss of information in the digital circuit during sampling process. The digital circuits can only be active circuits which means they require an additional power source to power the circuit.

The following table highlight the major differences between analog circuits and digital circuits

Table 5.1: Difference between Analog Circuit and Digital Circuit

Parameter	Analog Circuit	Digital Circuit
Definition	The electronic circuit which can process only analog signals is known as analog circuit.	The circuit which has ability to process only digital signals is known as digital circuit.
Input signal	The input signal to the analog circuit must be a continuous time signal or analog signal.	The input to the digital circuit is a discrete time signals or digital signal.
Output signal	Analog circuits produce output in the form of analog signals.	The output of the digital circuit is a digital signal.
Circuit components	The circuit components of the analog circuits are resistors, inductors, capacitors, etc.	The main circuit components of the digital circuits are logic gates.
Need of converters	The analog circuits can process the analog signals directly which present in the nature. Therefore, analog circuits do not require signal converters.	As the real world signals are analog, but the digital circuits can process signals only in digital form. Thus, digital circuits require signal converter, i.e. Analog to Digital Converter (ADC) and Digital to Analog Converter (DAC).

Parameter	Analog Circuit	Digital Circuit
Susceptibility to noise	The analog signals are more susceptible to noise.	Digital signals are immune to the noise.
Design	The analog circuits are complex to design because their circuit components need to be placed manually.	The designing of complicated digital circuits is relatively easier by using multiple software.
Flexibility	The implementation of analog circuit is not flexible.	The digital circuits offer more flexible implementation process.
Types	Analog circuits can be of two viz.: active circuit and passive circuit.	The digital circuits are of only one type named active circuit.
Processing speed	The processing speed of analog circuits is relatively low.	The digital circuits have higher processing speed than analog circuits.
Power consumption	The analog circuits consume more power.	The power consumed by the digital circuits is relatively less.
Accuracy & precision	The analog circuits are less accurate and precise.	The digital circuits are comparatively more accurate and precise.
Observational errors	In case of analog circuits, there may be an observational error in the output.	The digital circuits are free from observational errors in the output.
Signal transmission	In case of analog circuits, the signals are transmitted in the form of waves either wirelessly or with wires.	In the digital circuits, the signals can only be transmitted through wires in the digital form.

Parameter	Analog Circuit	Digital Circuit
Form of information storage	The analog circuits store the information in the form of waves.	Digital circuits store the information in binary form.
Logical operations	The analog circuit are not able to perform the logical operations efficiently.	Digital circuit performs logical operations efficiently.

5.3 Number System

Number systems are the technique to represent numbers in the computer system architecture, every value that you are saving or getting into/from computer memory has a defined number system.

Modern computers communicate and operate with binary numbers which use only the digits 0 & 1. Basic number system used by humans is Decimal number system.

For Ex: Let us consider decimal number 18. This number is represented in binary as 10010.

In the digital computer, there are various types of number systems used for representing information.

1. Binary Number System
2. Decimal Number System
3. Hexadecimal Number System
4. Octal Number System

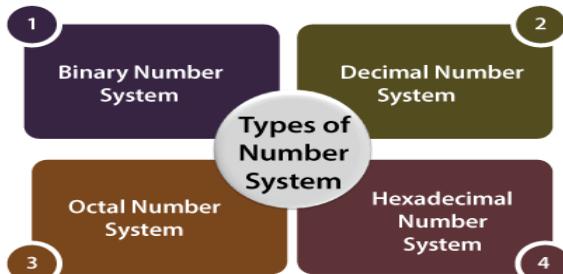


Figure 5.2: Types of Number System

Binary Number System

A Binary number system has only two digits that are **0 and 1**. Every number (value) represents with 0 and 1 in this number system. The base of binary number system is 2, because it has only two digits.

Octal Number System

Octal number system has only eight (8) digits from **0 to 7**. Every number (value) represents with 0,1,2,3,4,5,6 and 7 in this number system. The base of octal number system is 8, because it has only 8 digits.

Decimal Number System

Decimal number system has only ten (10) digits from **0 to 9**. Every number (value) represents with 0,1,2,3,4,5,6, 7,8 and 9 in this number system. The base of decimal number system is 10, because it has only 10 digits.

Hexadecimal Number System

A Hexadecimal number system has sixteen (16) alphanumeric values from **0 to 9** and **A to F**. Every number (value) represents with 0,1,2,3,4,5,6, 7, 8, 9, A, B, C, D, E and F in this number system. The base of hexadecimal number system is 16, because it has 16 alphanumeric values.

Table 5.2: Radix of Number System

Number system	Base(Radix)	Used digits	Example
Binary	2	0,1	$(11110000)_2$
Octal	8	0,1,2,3,4,5,6,7	$(360)_8$
Decimal	10	0,1,2,3,4,5,6,7,8,9	$(240)_{10}$
Hexadecimal	16	0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F	$(F0)_{16}$

Note - Here A is 10, B is 11, C is 12, D is 14, E is 15 and F is 16.

5.4 Number Base Conversion

In our previous section, we learned different types of number systems such as binary, decimal, octal, and hexadecimal. In this part of the tutorial, we will learn how we can change a number from one number system to another number system.

As, we have four types of number systems so each one can be converted into the remaining three systems. There are the following conversions possible in Number System

1. Binary to other Number Systems.
2. Decimal to other Number Systems.
3. Octal to other Number Systems.
4. Hexadecimal to other Number Systems.

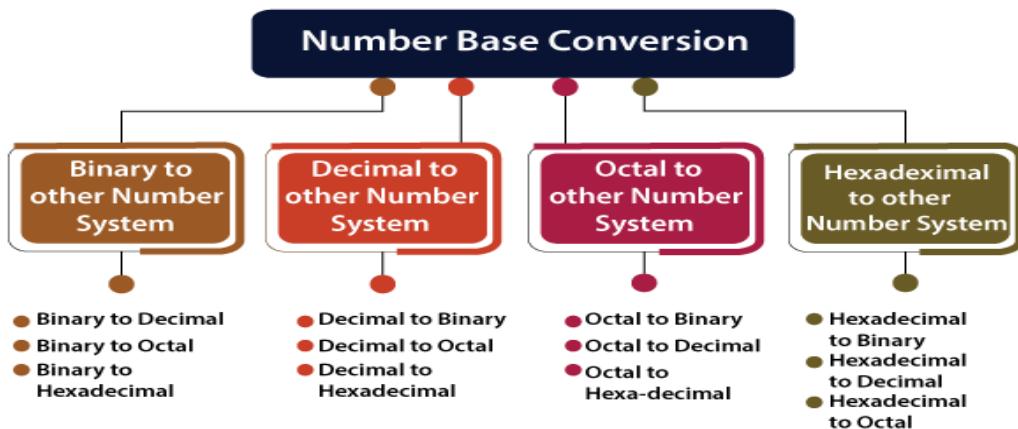


Figure 5.2: Flowchart of Number Base Conversion

5.4.1 Binary to other Number Systems

There are three conversions possible for binary number, i.e., binary to decimal, binary to octal, and binary to hexadecimal. The conversion process of a binary number to decimal differs from the remaining others. Let's take a detailed discussion on Binary Number System conversion.

➤ Binary to Decimal Conversion

The process of converting binary to decimal is quite simple. The process starts from multiplying the bits of binary number with its corresponding positional weights. And lastly, we add all those products.

Let's take an example to understand how the conversion is done from binary to decimal.

Example 1: $(10110.001)_2$

We multiplied each bit of $(10110.001)_2$ with its respective positional weight, and last we add the products of all the bits with its weight.

$$(10110.001)_2 = (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) + \\ (0 \times 2^{-1}) + (0 \times 2^{-2}) + (1 \times 2^{-3})$$

$$(10110.001)_2 = (1 \times 16) + (0 \times 8) + (1 \times 4) + (1 \times 2) + (0 \times 1) + \\ (0 \times \frac{1}{2}) + (0 \times \frac{1}{4}) + (1 \times \frac{1}{8})$$

$$(10110.001)_2 = 16 + 0 + 4 + 2 + 0 + 0 + 0.125$$

$$(10110.001)_2 = (22.125)_{10}$$

➤ Binary to Octal Conversion

The base numbers of binary and octal are 2 and 8, respectively. In a binary number, the pair of three bits is equal to one octal digit. There are only two steps to convert a binary number into an octal number which are as follows:

1. In the first step, we have to make the pairs of three bits on both sides of the binary point. If there will be one or two bits left in a pair of three bits pair, we add the required number of zeros on extreme sides.
2. In the second step, we write the octal digits corresponding to each pair.

Example 1: $(111110101011.0011)_2$

1. Firstly, we make pairs of three bits on both sides of the binary point.

111 110 101 011.001 1

On the right side of the binary point, the last pair has only one bit. To make it a complete pair of three bits, we added two zeros on the extreme side.

111 110 101 011.001 100

2. Then, we wrote the octal digits, which correspond to each pair.

$$(111110101011.0011)_2 = (7653.14)_8$$

➤ Binary to Hexadecimal Conversion

The base numbers of binary and hexadecimal are 2 and 16, respectively. In a binary number, the pair of four bits is equal to one hexadecimal digit. There are also only two steps to convert a binary number into a hexadecimal number which are as follows:

1. In the first step, we have to make the pairs of four bits on both sides of the binary point. If there will be one, two, or three bits left in a pair of four bits pair, we add the required number of zeros on extreme sides.

2. In the second step, we write the hexadecimal digits corresponding to each pair.

Example 1: $(10110101011.0011)_2$

1. Firstly, we make pairs of four bits on both sides of the binary point.

111 1010 1011.0011

On the left side of the binary point, the first pair has three bits. To make it a complete pair of four bits, add one zero on the extreme side.

0111 1010 1011.0011

2. Then, we write the hexadecimal digits, which correspond to each pair.

$(011110101011.0011)_2 = (7AB.3)_{16}$

5.4.2 Decimal to other Number System

The decimal number can be an integer or floating-point integer. When the decimal number is a floating-point integer, then we convert both part (integer and fractional) of the decimal number in the isolated form (individually). There are the following steps that are used to convert the decimal number into a similar number of any base 'r'.

1. In the first step, we perform the division operation on integer and successive part with base 'r'. We will list down all the remainders till the quotient is zero. Then we find out the remainders in reverse order for getting the integer part of the equivalent number of base 'r'. In this, the least and most significant digits are denoted by the first and the last remainders.
2. In the next step, the multiplication operation is done with base 'r' of the fractional and successive fraction. The carries are noted until the result is zero or when the required number of the equivalent digit is obtained. For getting the fractional part of the equivalent number of base 'r', the normal sequence of carrying is considered.

➤ Decimal to Binary Conversion

For converting decimal to binary, there are two steps required to perform, which are as follows:

1. In the first step, we perform the division operation on the integer and the successive quotient with the base of binary (2).
2. Next, we perform the multiplication on the integer and the successive quotient with the base of binary (2).

Example 1: $(152.25)_{10}$

Step 1:

Divide the number 152 and its successive quotients with base 2.

Operation	Quotient	Remainder
$152/2$	76	0 (LSB)
$76/2$	38	0
$38/2$	19	0
$19/2$	9	1
$9/2$	4	1
$4/2$	2	0
$2/2$	1	0
$1/2$	0	1(MSB)

$$(152)_{10} = (10011000)_2$$

Step 2:

Now, perform the multiplication of 0.27 and successive fraction with base 2.

Operation	Result	carry
0.25×2	0.50	0
0.50×2	0	1

OR

Example

$$(10.25)_{10}$$

Integer part :

2	10	0
2	5	1
2	2	0
1		

$$(10)_{10} = (1010)_2$$

Fractional part

$$\begin{array}{l} 0.25 \times 2 = 0.50 \\ 0.50 \times 2 = 1.00 \end{array}$$

$$(0.25)_{10} = (0.01)_2$$

Note: Keep multiplying the fractional part with 2 until decimal part 0.00 is obtained.

$$(0.25)_{10} = (0.01)_2$$

Answer: $(10.25)_{10} = (1010.01)_2$

➤ Decimal to Octal Conversion

For converting decimal to octal, there are two steps required to perform, which are as follows:

1. In the first step, we perform the division operation on the integer and the successive quotient with the base of octal(8).
2. Next, we perform the multiplication on the integer and the successive quotient with the base of octal(8).

Example 1: $(152.25)_{10}$

Step 1:

Divide the number 152 and its successive quotients with base 8.

Operation	Quotient	Remainder
152/8	19	0
19/8	2	3
2/8	0	2

$$(152)_{10} = (230)_8$$

Step 2:

Now perform the multiplication of 0.25 and successive fraction with base 8.

Operation	Result	carry
0.25×8	0	2

$$(0.25)_{10} = (0.2)_8$$

So, the octal number of the decimal number 152.25 is **230.2**

➤ **Decimal to hexadecimal conversion**

For converting decimal to hexadecimal, there are two steps required to perform, which are as follows:

1. In the first step, we perform the division operation on the integer and the successive quotient with the base of hexadecimal (16).
2. Next, we perform the multiplication on the integer and the successive quotient with the base of hexadecimal (16).

Example 1: $(152.25)_{10}$

Step 1:

Divide the number 152 and its successive quotients with base 8.

Operation	Quotient	Remainder
$152/16$	9	8
$9/16$	0	9

$$(152)_{10} = (98)_{16}$$

Step 2:

Now perform the multiplication of 0.25 and successive fraction with base 16.

Operation	Result	carry
0.25×16	0	4

5.4.3 Octal to other Number System

Like binary and decimal, the octal number can also be converted into other number systems. The process of converting octal to decimal differs from the remaining one. Let's start understanding how conversion is done.

➤ **Octal to Decimal Conversion**

The process of converting octal to decimal is the same as binary to decimal. The process starts from multiplying the digits of octal numbers with its corresponding positional weights. And lastly, we add all those products.

Let's take an example to understand how the conversion is done from octal to decimal.

Example 1: $(152.25)_8$

Step 1:

We multiply each digit of **152.25** with its respective positional weight, and last we add the products of all the bits with its weight.

$$(152.25)_8 = (1 \times 8^2) + (5 \times 8^1) + (2 \times 8^0) + (2 \times 8^{-1}) + (5 \times 8^{-2})$$

$$(152.25)_8 = 64 + 40 + 2 + (2 \times 1/8) + (5 \times 1/64)$$

$$(152.25)_8 = 64 + 40 + 2 + 0.25 + 0.078125$$

$$(152.25)_8 = 106.328125$$

So, the decimal number of the octal number 152.25 is **106.328125**

➤ Octal to Binary Conversion

The process of converting octal to binary is the reverse process of binary to octal. We write the three bits binary code of each octal number digit.

Example 1: $(152.25)_8$

We write the three-bit binary digit for 1, 5, 2, and 5.

$$(152.25)_8 = (001101010.010101)_2$$

So, the binary number of the octal number 152.25 is **$(001101010.010101)_2$**

➤ Octal to hexadecimal conversion

For converting octal to hexadecimal, there are two steps required to perform, which are as follows:

1. In the first step, we will find the binary equivalent of number **25**.
2. Next, we have to make the pairs of four bits on both sides of the binary point. If there will be one, two, or three bits left in a pair of four bits pair, we add the required number of zeros on extreme sides and write the hexadecimal digits corresponding to each pair.

Example 1: $(152.25)_8$

Step 1:

We write the three-bit binary digit for 1, 5, 2, and 5.

$$(152.25)_8 = (001101010.010101)_2$$

So, the binary number of the octal number 152.25 is $(001101010.010101)_2$

Step 2:

1. Now, we make pairs of four bits on both sides of the binary point.

0 0110 1010.0101 01

On the left side of the binary point, the first pair has only one digit, and on the right side, the last pair has only two-digit. To make them complete pairs of four bits, add zeros on extreme sides.

0000 0110 1010.0101 0100

2. Now, we write the hexadecimal digits, which correspond to each pair.

$$(0000 \quad 0110 \quad 1010.0101 \quad 0100)_2 = (6A.54)_{16}$$

5.4.4 Hexa-decimal to other Number System

Like binary, decimal, and octal, hexadeciml numbers can also be converted into other number systems. The process of converting hexadeciml to decimal differs from the remaining one. Let's start understanding how conversion is done.

➤ Hexa-decimal to Decimal Conversion

The process of converting hexadeciml to decimal is the same as binary to decimal. The process starts from multiplying the digits of hexadeciml numbers with its corresponding positional weights. And lastly, we add all those products.

Let's take an example to understand how the conversion is done from hexadeciml to decimal.

Example 1: $(152A.25)_{16}$

Step 1:

We multiply each digit of $152A.25$ with its respective positional weight, and last we add the products of all the bits with its weight.

$$(152A.25)_{16} = (1 \times 16^3) + (5 \times 16^2) + (2 \times 16^1) + (A \times 16^0) + (2 \times 16^{-1}) + (5 \times 16^{-2})$$

$$(152A.25)_{16} = (1 \times 4096) + (5 \times 256) + (2 \times 16) + (10 \times 1) + (2 \times 1/16) + (5 \times 1/256)$$

$$(152A.25)_{16} = 4096 + 1280 + 32 + 10 + (2 \times 1/16) + (5 \times 1/256)$$

$$(152A.25)_{16} = 5418 + 0.125 + 0.125$$

$$(152A.25)_{16} = 5418.14453125$$

So, the decimal number of the hexadecimal number 152A.25 is **5418.14453125**

➤ **Hexadecimal to Binary Conversion**

The process of converting hexadecimal to binary is the reverse process of binary to hexadecimal. We write the four bits binary code of each hexadecimal number digit.

Example 1: $(152A.25)_{16}$

We write the four-bit binary digit for 1, 5, A, 2, and 5.

$$(152A.25)_{16} = (0001 \ 0101 \ 0010 \ 1010.0010 \ 0101)_2$$

So, the binary number of the hexadecimal number 152.25 is **$(1010100101010.00100101)_2$**

➤ **Hexadecimal to Octal Conversion**

For converting hexadecimal to octal, there are two steps required to perform, which are as follows:

1. In the first step, we will find the binary equivalent of the hexadecimal number.
2. Next, we have to make the pairs of three bits on both sides of the binary point. If there will be one or two bits left in a pair of three bits pair, we add the required number of zeros on extreme sides and write the octal digits corresponding to each pair.

Example 1: $(152A.25)_{16}$

Step 1:

We write the four-bit binary digit for 1, 5, 2, A, and 5.

$$(152A.25)_{16} = (0001 \ 0101 \ 0010 \ 1010.0010 \ 0101)_2$$

So, the binary number of hexadecimal number 152A.25 is **$(0011010101010.010101)_2$**

Step 2:

3. Then, we make pairs of three bits on both sides of the binary point.

001 010 100 101 010.001 001 010

4. Then, we write the octal digit, which corresponds to each pair.

$$(001010100101010.001001010)_2 = (12452.112)_8$$

So, the octal number of the hexadecimal number 152A.25 is **12452.112**

5.5 Boolean Algebra

Boolean algebra is the category of algebra in which the variable's values are the truth values, true and false, ordinarily denoted 1 and 0 respectively. It is used to analyse and simplify digital circuits or digital gates. It is also called **Binary Algebra** or **Logical Algebra**.

It has been fundamental in the development of digital electronics and is provided for in all modern programming languages.

The basic operations of Boolean algebra are as follows:

1. Conjunction or AND operation
2. Disjunction or OR operation
3. Negation or Not operation

Below is the table defining the symbols for all three basic operations.

Operator	Symbol
NOT	' (or) \neg
AND	. (or) \wedge
OR	+ (or) \vee

Table 5.2: Basic Operations

Suppose A and B are two Boolean variables, then we can define the three operations as;

- A conjunction B or A AND B, satisfies $A \wedge B = \text{True}$, if $A = B = \text{True}$ or else $A \wedge B = \text{False}$.
- A disjunction B or A OR B, satisfies $A \vee B = \text{False}$, if $A = B = \text{False}$, else $A \vee B = \text{True}$.
- Negation A or $\neg A$ satisfies $\neg A = \text{False}$, if $A = \text{True}$ and $\neg A = \text{True}$ if $A = \text{False}$

5.5.1 Boolean Expression

A logical statement that results in a Boolean value, either be True or False, is a Boolean expression. Sometimes, synonyms are used to express the statement such as 'Yes' for 'True' and 'No' for 'False'. Also, 1 and 0 are used for digital circuits for True and False, respectively.

Boolean expressions are the statements that use logical operators, i.e., AND, OR, XOR and NOT. Thus, if we write X AND Y = True, then it is a Boolean expression.

5.5.2 Boolean Algebra Terminologies

Now, let us discuss the important terminologies covered in Boolean algebra.

- ✓ **Boolean algebra:** Boolean algebra is the branch of algebra that deals with logical operations and binary variables.
- ✓ **Boolean Variables:** A Boolean variable is defined as a variable or a symbol defined as a variable or a symbol, generally an alphabet that represents the logical quantities such as 0 or 1.
- ✓ **Boolean Function:** A Boolean function consists of binary variables, logical operators, constants such as 0 and 1, equal to the operator, and the parenthesis symbols.
- ✓ **Literal:** A literal may be a variable or a complement of a variable.
- ✓ **Complement:** The complement is defined as the inverse of a variable, which is represented by a bar over the variable.
- ✓ **Truth Table:** The truth table is a table that gives all the possible values of logical variables and the combination of the variables. It is possible to convert the Boolean equation into a truth table. The number of rows in the truth table should be equal to 2^n , where “n” is the number of variables in the equation. For example, if a Boolean equation consists of 3 variables, then the number of rows in the truth table is 8. (i.e.,) $2^3 = 8$.

5.5.3 Boolean Algebra Truth Table

Now, if we express the above operations in a truth table, we get;

A	B	$A \wedge B$	$A \vee B$
TRUE	TRUE	TRUE	TRUE
TRUE	FALSE	FALSE	TRUE
FALSE	TRUE	FALSE	TRUE
FALSE	FALSE	FALSE	FALSE

A	$\neg A$
TRUE	FALSE
FALSE	TRUE

Table 5.3: Boolean algebra Truth Table

5.5.4 Boolean Algebra Rules

Following are the important rules used in Boolean algebra.

- Variable used can have only two values. Binary 1 for HIGH and Binary 0 for LOW.
- The complement of a variable is represented by an overbar
Thus, complement of variable B is represented as \bar{B} . Thus if $B = 0$ then $\bar{B} = 1$. Thus if $B = 1$ then $\bar{B} = 0$.
- OR-ing of the variables is represented by a plus (+) sign between them. For example, the OR-ing of A, B, and C is represented as $A + B + C$.
- Logical AND-ing of the two or more variables is represented by writing a dot between them, such as $A \cdot B \cdot C$. Sometimes, the dot may be omitted like ABC .

5.5.5 Laws of Boolean algebra

There are six types of Boolean algebra laws. They are:

- Commutative law
- Associative law
- Distributive law
- AND law
- OR law
- Inversion law

➤ Commutative Law

Any binary operation which satisfies the following expression is referred to as a commutative operation. Commutative law states that changing the sequence of the variables does not have any effect on the output of a logic circuit.

- $A \cdot B = B \cdot A$
- $A + B = B + A$

➤ **Associative Law**

It states that the order in which the logic operations are performed is irrelevant as their effect is the same.

- $(A \cdot B) \cdot C = A \cdot (B \cdot C)$
- $(A + B) + C = A + (B + C)$

➤ **Distributive Law**

Distributive law states the following conditions:

- $A \cdot (B + C) = (A \cdot B) + (A \cdot C)$
- $A + (B \cdot C) = (A + B) \cdot (A + C)$

➤ **AND Law**

These laws use the AND operation. Therefore, they are called AND laws.

- $A \cdot 0 = 0$
- $A \cdot 1 = A$
- $A \cdot A = A$
- $A \cdot A^{\prime} = 0$

➤ **OR Law**

These laws use the OR operation. Therefore, they are called OR laws.

- $A + 0 = A$
- $A + 1 = 1$
- $A + A = A$
- $A + A^{\prime} = 1$

➤ **Inversion Law**

In Boolean algebra, the inversion law states that double inversion of variable results in the original variable itself.

- $A^{\prime\prime} = A$

5.5.6 Boolean Algebra Theorems

The two important theorems which are extremely used in Boolean algebra are **De Morgan's First law** and **De Morgan's second law**. These two theorems are used to change the Boolean expression. This theorem basically helps to reduce the given Boolean expression in the simplified form. These two De Morgan's laws are used to change the expression from one form to another form. Now, let us discuss these two theorems in detail.

➤ **De Morgan's First Law:**

- De Morgan's First Law states that $(A \cdot B)' = A' + B'$.
- The first law states that the complement of the product of the variables is equal to the sum of their individual complements of a variable.
- The truth table that shows the verification of De Morgan's First law is given as follows:

Table 5.4: De Morgan's First Law Truth Table

A	B	A'	B'	$(A \cdot B)'$	$A' + B'$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	1	1
1	1	0	0	0	0

The last two columns show that $(A \cdot B)' = A' + B'$.

Hence, De Morgan's First Law is proved.

De Morgan's Second Law:

- De Morgan's Second law states that $(A + B)' = A' \cdot B'$.
- The second law states that the complement of the sum of variables is equal to the product of their individual complements of a variable.

- The following truth table shows the proof for De Morgan's second law.

Table 5.5: De Morgan's Second Law Truth Table

A	B	A'	B'	(A+B)'	A'. B'
0	0	1	1	1	1
0	1	1	0	0	0
1	0	0	1	0	0
1	1	0	0	0	0

The last two columns show that $(A+B)' = A' \cdot B'$.

Hence, De Morgan's second law is proved.

Examples

1. Simplify the following expression:

$$C + \bar{B}C$$

Solution:

Given:

$$C + \bar{B}C$$

According to Demorgan's law, we can write the above expressions as

$$C + (\bar{B} + \bar{C})$$

From Commutative law:

$$(C + \bar{C}) + \bar{B}$$

From Complement law

$$1 + \bar{B} = 1$$

Therefore,

$$C + \bar{B}C = 1$$

2. Draw a truth table for A(B+D).

A	B	D	B+D	A(B+D)
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

5.6 Logic Gates

A **logic gate** is an electronic circuit designed by using electronic components like diodes, transistors, resistors, and more. As the name implies, a logic gate is designed to perform logical operations in digital systems like computers, communication systems, etc.

Therefore, we can say that the building blocks of a digital circuit are logic gates, which execute numerous logical operations that are required by any digital circuit. A logic gate can take two or more inputs but only produce one output. The output of a logic gate depends on the combination of inputs and the logical operation that the logic gate performs.

The logic gates can be classified into the following major types:

5.6.1 Basic Logic Gates

There are three basic logic gates:

1. AND Gate
2. OR Gate
3. NOT Gate

➤ **AND Gate:**

The AND gate performs logical multiplication, commonly known as AND function. The AND gate has two or more inputs and a single output. The output of an AND gate is

HIGH only when all the inputs are HIGH. Even if any one of the inputs is LOW, the output will be LOW. If a & b are input variables of an AND gate and c is its output, then $Y=A \cdot B$.

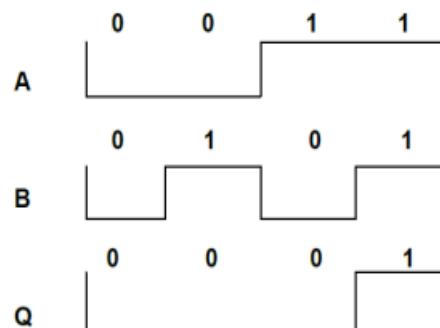
Logic Symbol



Truth Table

Input		Output
A	B	$Y=A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

Timing Diagram

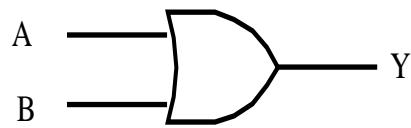


➤ OR Gate

The OR gate performs logical additions commonly known as OR function. The OR gate has two or more inputs and only one output. The operation of OR gate is such that a HIGH (1) on the output is produced when any of the input is HIGH. The output is LOW (0) only when all the inputs are LOW.

- If A & B are the input variables of an OR gate and c is its output, then $A+B$. similarly for more than two variables, the OR function can be expressed as $Y=A+B+C$.

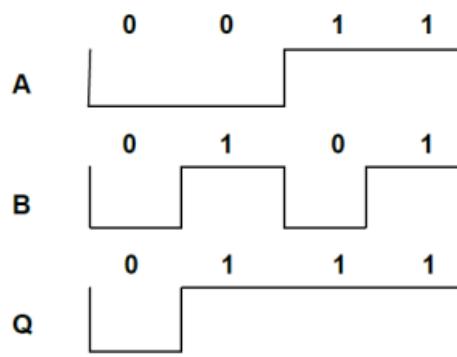
Logic Symbol



Truth Table

Input		Output
A	B	$Y = A+B$
0	0	0
0	1	1
1	0	1
1	1	1

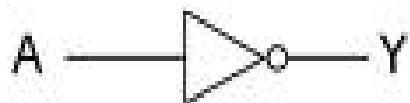
Timing Diagram



➤ **Not Gate (Inverter)**

The NOT gate performs the basic logical function called inversion or complementation. The purpose of this gate is to convert one logic level into the opposite logic level. It has one input and one output. When a HIGH level is applied to an inverter, a LOW level appears at the output and vice-versa.

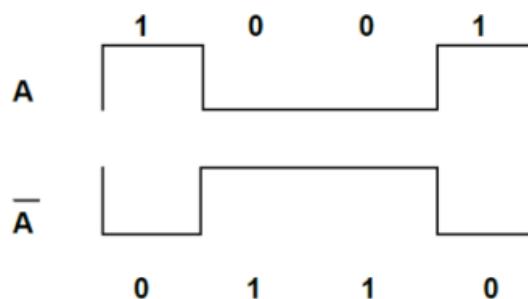
Logic Symbol



Truth Table

Input	Output
A	\bar{A}
0	1
1	0

Timing Diagram



5.6.2 Universal Logic Gates

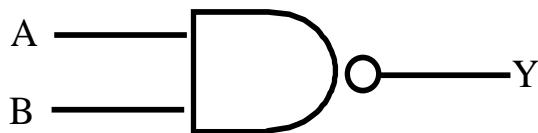
In digital electronics, the following two logic gates are considered as universal logic gates:

1. NAND Gate
2. NOR Gate

➤ **NAND Gate**

The output of a NAND gate is LOW only when all inputs are HIGH and output of the NAND is HIGH if one or more inputs are LOW.

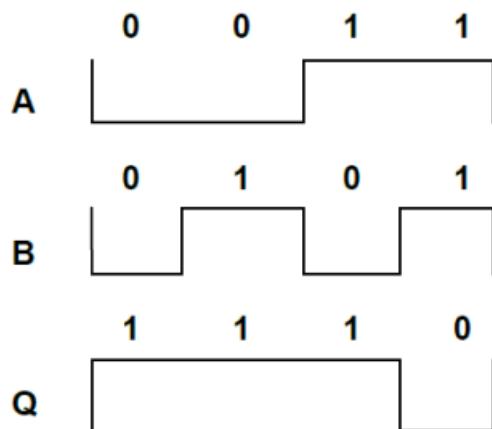
Logic Symbol



Truth Table

INPUT		OUTPUT
A	B	$Q = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

Timing Diagram



➤ **NOR Gate**

The output of the NOR gate is HIGH only when all the inputs are LOW.

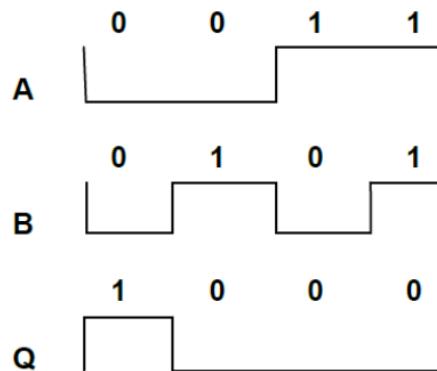
Logic Symbol



Truth Table

INPUT		OUTPUT
A	B	$Q = \overline{A + B}$
0	0	1
0	1	0
1	0	0
1	1	0

Timing Diagram



5.6.3 Derived Logic Gates

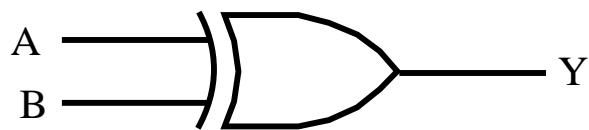
The following two are the derived logic gates used in digital systems:

1. XOR Gate
2. XNOR Gate

➤ XOR Gate or Exclusive OR Gate

In this gate output is HIGH only when any one of the input is HIGH. The circuit is also called as inequality comparator, because it produces output when two inputs are different. When both the inputs are high, then the output is low.

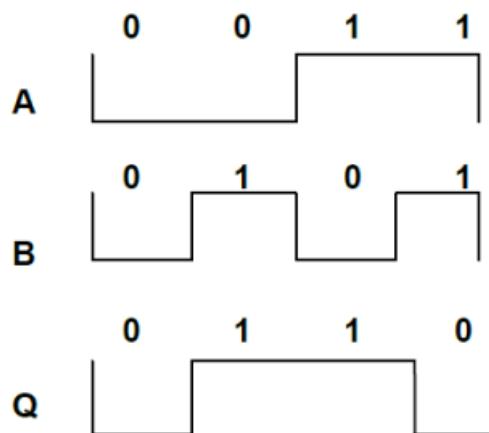
Logic Symbol



Truth Table

INPUT		OUTPUT
A	B	$Q = A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

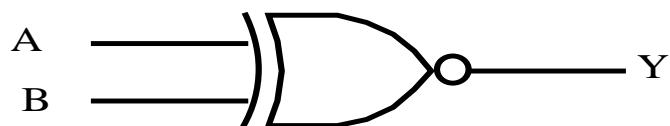
Timing Diagram



XNOR Gate or Exclusive NOR Gate:

An XNOR gate is a gate with two or more inputs and one output. XNOR operation is complimentary of XOR operation. i.e. The output of XNOR gate is High, when all the inputs are identical; otherwise, it is low.

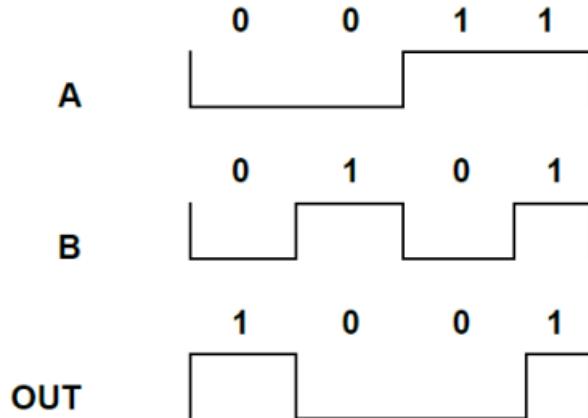
Logic Symbol



Truth Table

INPUT		OUTPUT
A	B	OUT = A XNOR B
0	0	1
0	1	0
1	0	0
1	1	1

Timing Diagram



Logic gates are the fundamental building blocks of all digital circuits and devices like computers. Here are some key digital devices in which logic gates are utilized to design their circuits:

- Computers
- Microprocessors
- Microcontrollers
- Digital and smart watches
- Smartphones, etc.

5.7 Combinational Circuits

Combinational circuit is a circuit in which we combine the different gates in the circuit, for example encoder, decoder, multiplexer and demultiplexer. Some of the characteristics of combinational circuits are following

- The output of combinational circuit at any instant of time, depends only on the levels present at input terminals.
- The combinational circuit do not use any memory. The previous state of input does not have any effect on the present state of the circuit.
- A combinational circuit can have an n number of inputs and m number of outputs.



Figure 5.3: Block Diagram of Combinational Circuits

Adder

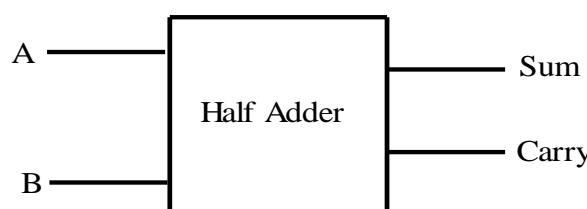
- The Basic operation in digital computer is binary addition. The circuit which perform the addition of binary bits are called as **Adder**.
- The logic circuit which performs the addition of two bit is called **Half adder** and three bit is called **Full adder**.

Rules of 2-Bit Addition

$0 + 0 = 0$
$0 + 1 = 1$
$1 + 0 = 1$
$1 + 1 = 10_2$

5.7.1 Half Adder

A combinational circuit which performs the arithmetic addition of two binary digits is called Half Adder. In the half adder circuit, there are two inputs, one is addend and augends and two outputs are **Sum** and **Carry**.



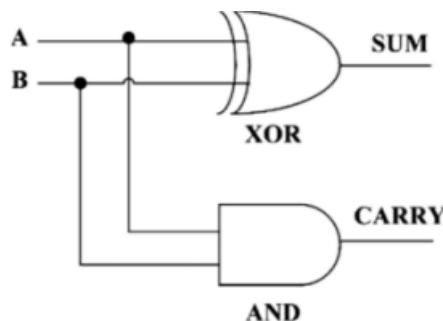
Truth Table

Inputs		Outputs	
A	B	Carry	Sum
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Logic Expression

$$\begin{aligned} \text{Sum, } S &= A'B + AB' = A \oplus B \\ \text{Carry, } C &= A \cdot B \end{aligned}$$

Logic Diagram



Advantages of Half Adder

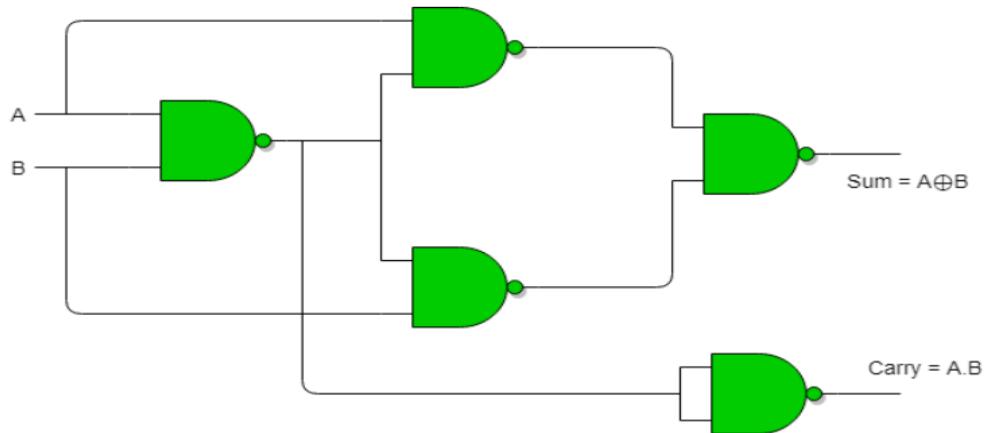
- The half adder and half subtractor circuits use only a few gates, which reduces the cost and power consumption compared to more complex circuits.
- Easy integration: The half adder and half subtractor can be easily integrated with other digital circuits and systems.

Limitations

- The main limitation of a half adder is that it can only add two single bits; it cannot handle a carry bit from the previous step.

- Half adders have no scope of adding the carry bit resulting from the addition of previous bits. This is a major drawback of half adders. This is because real time scenarios involve adding the multiple number of bits which cannot be accomplished using half adders.

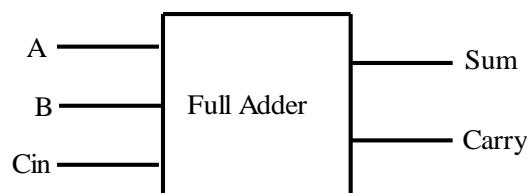
Realization of Half-Adder using NAND Gates



5.7.2 Full Adder

The full adder is a combinational circuit that performs the arithmetic sum of three input bits.

- It consists of three inputs and two outputs. Two of the inputs are variables, denoted by A and B, represent the two significant bits to be added. The third input C_{in} represents carry from the previous lower significant position.



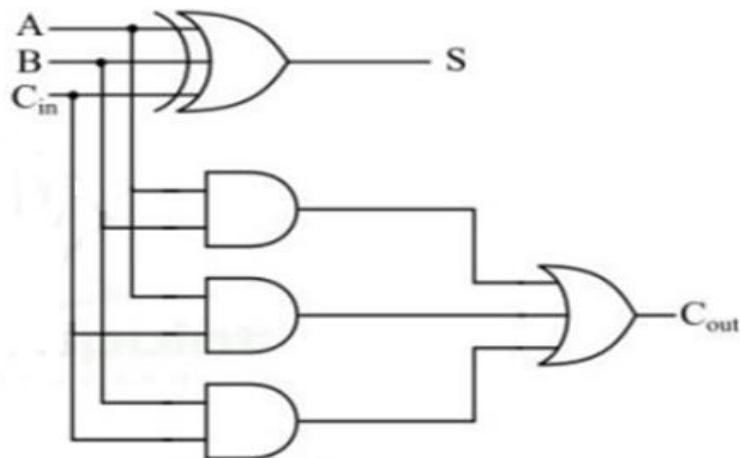
Truth Table

Inputs			Outputs	
A	B	Cin	Cout	Sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

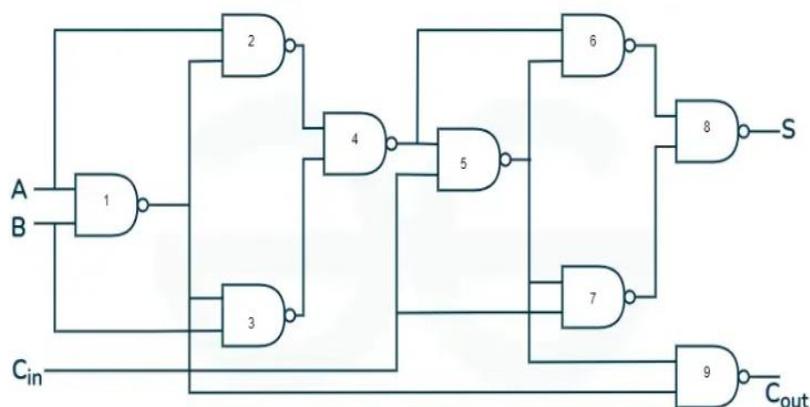
Logic Expression

$$\begin{aligned}\text{Sum, } S &= A'B'C_{in} + A'BC'_{in} + AB'C'_{in} + ABC_{in} \\ \text{Carry, } C_{out} &= AB + AC_{in} + BC_{in}.\end{aligned}$$

Logic Diagram



Realization of Full-Adder using NAND Gates



5.8 Multiplexer

A multiplexer is a combinational circuit that has 2^n input lines and a single output line. Simply, the multiplexer is a multi-input and single-output combinational circuit. The binary information is received from the input lines and directed to the output line. On the basis of the values of the selection lines, one of these data inputs will be connected to the output.

Unlike encoder and decoder, there are n selection lines and 2^n input lines. So, there is a total of 2^N possible combinations of inputs. A multiplexer is also treated as Mux.

There are various types of the multiplexer which are as follows:

- 2-to-1(1selectline)
- 4-to-1(2selectlines)
- 8-to-1(3selectlines)
- 16-to-1(4selectlines)

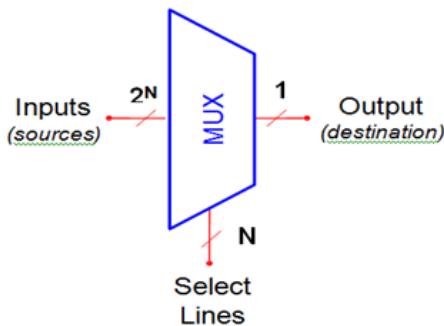
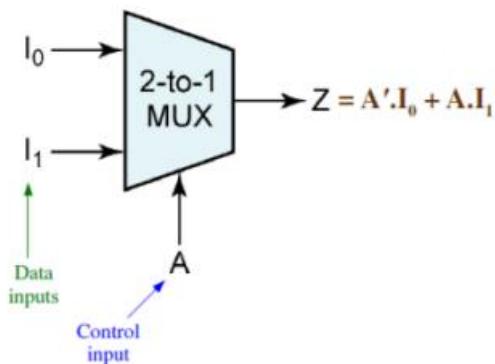


Figure 5.4: Block Diagram of Multiplexer

5.8.1 2x1 Multiplexer

In 2×1 multiplexer, there are only two inputs, i.e., A_0 and A_1 , 1 selection line, i.e., S_0 and single outputs, i.e., Y . On the basis of the combination of inputs which are present at the selection line S_0 , one of these 2 inputs will be connected to the output. The block diagram and the truth table of the 2×1 multiplexer is given below.

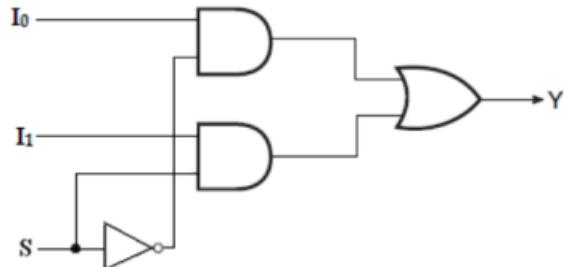
Circuit Diagram



Truth Table

S	Y
0	I_0
1	I_1

Logic Diagram



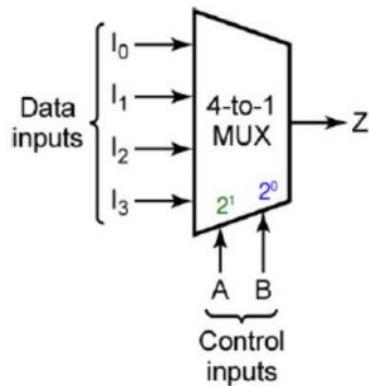
The **Logical Expression** of the term Y is as follows:

$$Y = S_0' \cdot A_0 + S_0 \cdot A_1$$

5.8.2 4x1 Multiplexer:

In the 4×1 multiplexer, there is a total of four inputs, i.e., A_0 , A_1 , A_2 , and A_3 , 2 selection lines, i.e., S_0 and S_1 and single output, i.e., Y . On the basis of the combination of inputs that are present at the selection lines S^0 and S_1 , one of these 4 inputs are connected to the output. The block diagram and the truth table of the 4×1 multiplexer are given below.

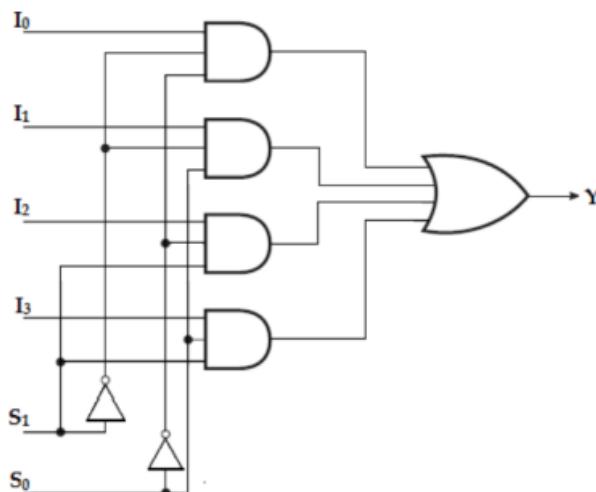
Block Diagram



Truth Table

S_1	S_0	Y
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3

Logic Diagram



The **Logical Expression** of the term Y is as follows:

$$Y = S_1' S_0' A_0 + S_1' S_0' A_1 + S_1 S_0' A_2 + S_1 S_0' A_3$$

5.9 De-Multiplexer

A De-multiplexer is a combinational circuit that has only 1 input line and 2^N output lines. Simply, the multiplexer is a single-input and multi-output combinational circuit. The

information is received from the single input lines and directed to the output line. On the basis of the values of the selection lines, the input will be connected to one of these outputs. De-multiplexer is opposite to the multiplexer.

Unlike encoder and decoder, there are n selection lines and 2^n outputs. So, there is a total of 2^n possible combinations of inputs. De-multiplexer is also treated as **De-mux**.

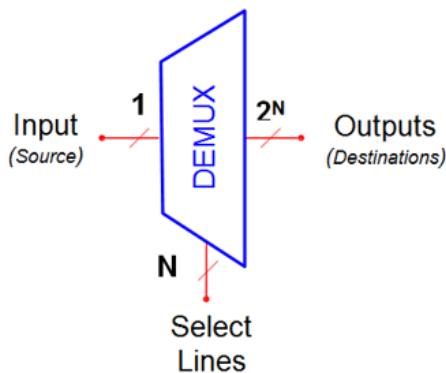
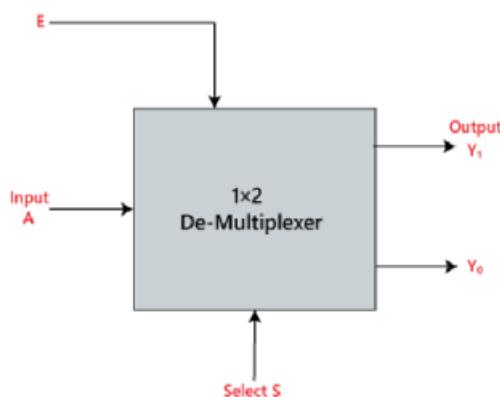


Figure 5.5: Block Diagram of De-Multiplexer

5.9.1 1x2 De-Multiplexer

In the 1 to 2 De-multiplexers, there are only two outputs, i.e., Y_0 , and Y_1 , 1 selection lines, i.e., S_0 , and single input, i.e., A . On the basis of the selection value, the input will be connected to one of the outputs. The block diagram and the truth table of the 1×2 multiplexer is given below.

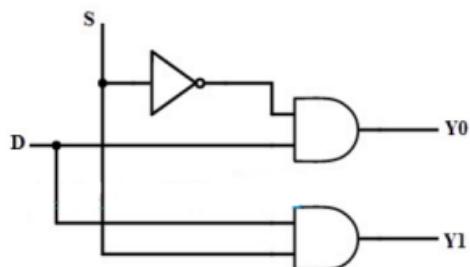
Block Diagram



Truth Table

INPUTS		Output	
S_0		Y_1	Y_0
0		0	A
1		A	0

Logic Diagram



The **Logical Expression** of the term **Y** is as follows:

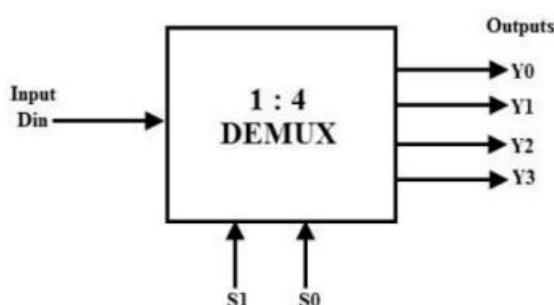
$$Y_0 = S_0' A$$

$$Y_1 = S_0 A$$

5.9.2 1x4 De-multiplexer

In 1 to 4 De-multiplexer, there are total of four outputs, i.e., Y_0 , Y_1 , Y_2 , and Y_3 , 2 selection lines, i.e., S_0 and S_1 and single input, i.e., A. On the basis of the combination of inputs which are present at the selection lines S_0 and S_1 , the input be connected to one of the outputs. The block diagram and the truth table of the 1x4 multiplexer is given below.

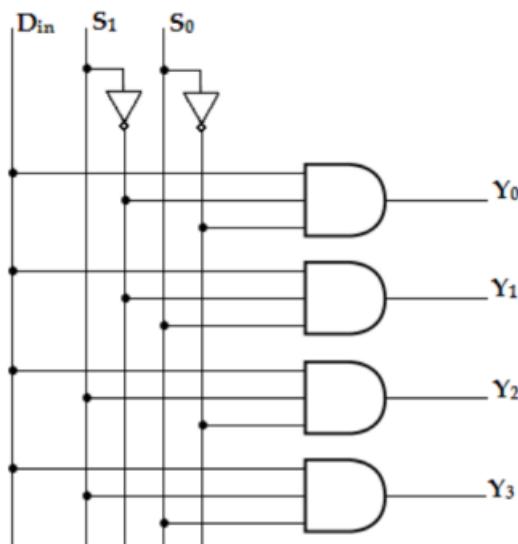
Block Diagram



Truth Table

INPUTS		Output			
S_1	S_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	A
0	1	0	0	A	0
1	0	0	A	0	0
1	1	A	0	0	0

Logic Diagram



The **Logical Expression** of the term Y is as follows:

$$Y_0 = S_1' S_0' A$$

$$Y_1 = S_1' S_0 A$$

$$Y_2 = S_1 S_0' A$$

$$Y_3 = S_1 S_0 A$$

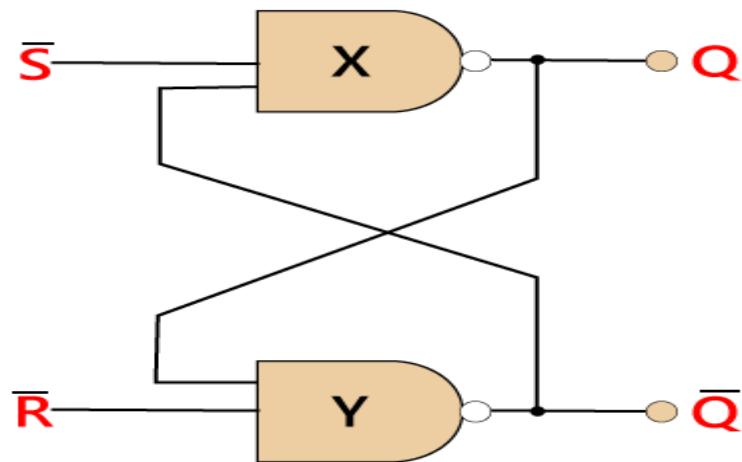
5.10 Flip Flop

A circuit that has two stable states is treated as a **Flip - Flop**. These stable states are used to store binary data that can be changed by applying varying inputs. The flip flops are the fundamental building blocks of the digital system. Flip flops and latches are examples of data storage elements. In the sequential logical circuit, the flip flop is the basic storage element. The latches and flip flops are the basic storage elements but different in working. There are the following types of flip flops:

5.10.1 SR Flip Flop

The **S-R flip flop** is the most common flip flop used in the digital system. In SR flip flop, when the set input "S" is true, the output Y will be high, and Y' will be low. It is required that the wiring of the circuit is maintained when the outputs are established. We maintain the wiring until set or reset input goes high, or power is shutdown.

Logic Diagram



The S-R flip flop is the simplest and easiest circuit to understand.

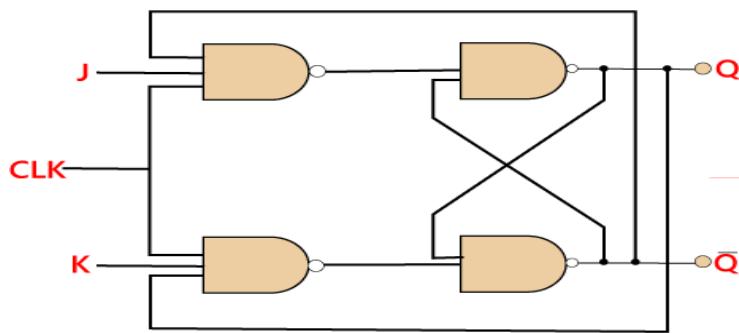
Truth Table

S	R	Y	Y'
0	0	0	1
0	1	0	1
1	0	1	0
1	1	--	--

5.10.2 J-K Flip-Flop

The **JK flip flop** is used to remove the drawback of the S-R flip flop, i.e., undefined states. The JK flip flop is formed by doing modification in the SR flip flop. The S-R flip flop is improved in order to construct the J-K flip flop. When S and R input is set to true, the SR flip flop gives an inaccurate result. But in the case of JK flip flop, it gives the correct output.

Logic Diagram



In J-K flip flop, if both of its inputs are different, the value of J at the next clock edge is taken by the output Y. If both of its input is low, then no change occurs, and if high at the clock edge, then from one state to the other, the output will be toggled. The JK Flip Flop is a Set or Reset Flip flop in the digital system.

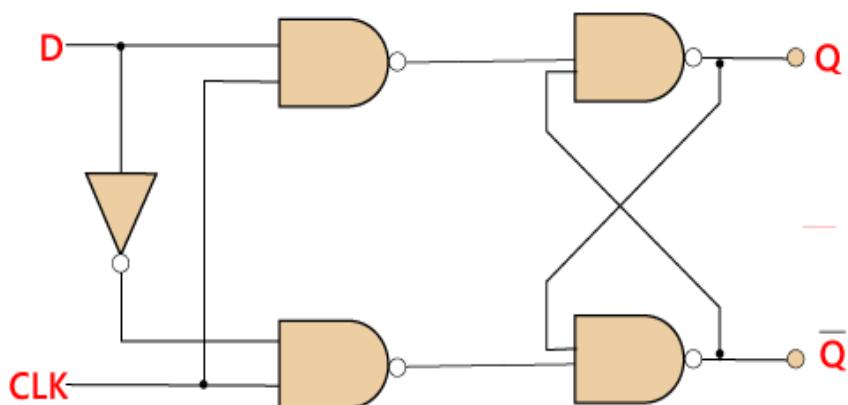
Truth Table

J	K	Y	Y'
0	0	0	0
0	1	0	0
1	0	0	1
1	1	0	1
0	0	1	1
0	1	1	0
1	0	1	1
1	1	1	0

5.10.3 D Flip Flop

D flip flop is a widely used flip flop in digital systems. The D flip flop is mostly used in shift-registers, counters, and input synchronization.

Logic Diagram



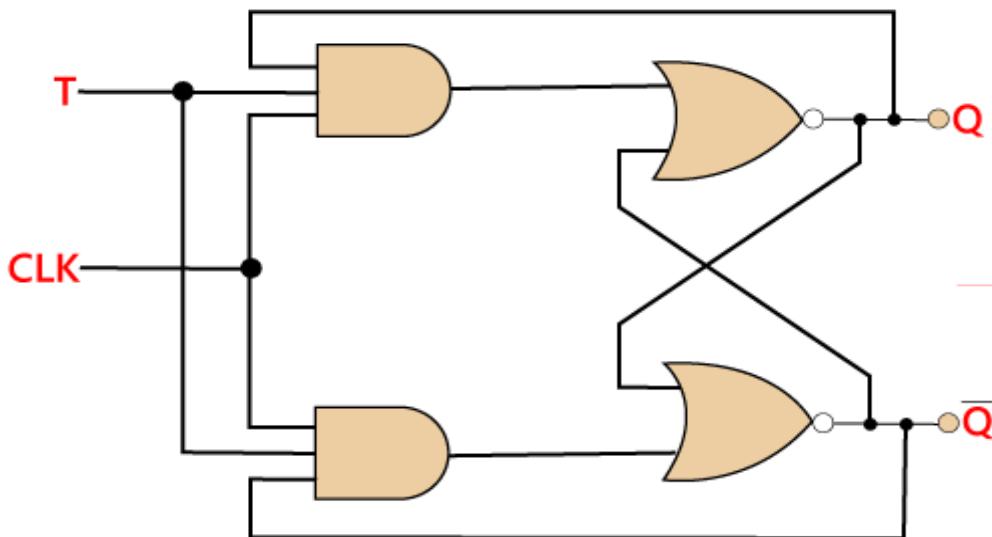
Truth Table

Clock	D	Y	Y'
$\downarrow \gg 0$	0	0	1
$\uparrow \gg 1$	0	0	1
$\downarrow \gg 0$	1	0	1
$\uparrow \gg 1$	1	1	0

5.10.4 T Flip Flop

Just like JK flip-flop, T flip flop is used. Unlike JK flip flop, in T flip flop, there is only single input with the clock input. The T flip flop is constructed by connecting both of the inputs of JK flip flop together as a single input.

Logic Diagram



The **T flip flop is also known as Toggle flip-flop**. These T flip-flops are able to find the complement of its state.

Truth Table

T	Y	Y (t+1)
0	0	0
1	0	1
0	1	1
1	1	0

5.11 Basic Principle of Communication

Sending, receiving, and processing data among two devices are referred to as communication. A communication system is a group of components (devices) that work together to establish a connection between both the sender and recipient. Radio and television, satellite broadcasting, wireless telegraphy, mobile communication, and computer communication are some examples of communication systems.

5.11.1 Principles of Electronic Communication Systems

- Communication is the process of establishing connection or link between two points for information exchange or Communication is simply the basic process of exchanging information.
- The electronics equipment which are used for communication purpose, are called communication equipment. Different communication equipment when assembled together form a communication system.
- Typical example of communication system are line telephony and line telegraphy, radio telephony and radio telegraphy, radio broadcasting, point-to-point communication and mobile communication, computer communication, radar communication, television broadcasting, radio telemetry, radio aids to navigation, radio aids to aircraft landing etc. **Figure 5.6** shows the block diagram of a general communication system, in which the different functional elements are represented by blocks.

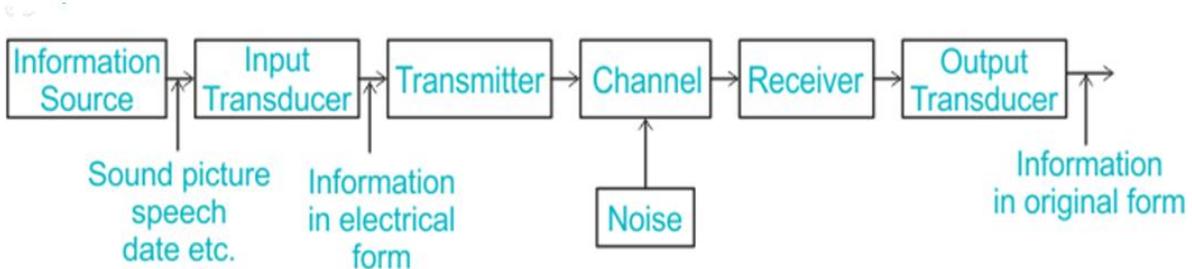


Figure 5.6: Block diagram of Electronic Communication System

The main constituents of basic communication system are:

- Information source and input transducer
- Transmitter
- Channel or medium
- Noise
- Receiver
- Output transducer and final destination

Information Source and Input Transducer

- The physical form of information is represented by a message that is originated by an information source.
- For example, a sentence or paragraph spoken by a person is a message that contains some information. The person, in this case, acts as information source.
- If the information produced by the source is not in an electrical form, it has to be converted into an electrical form using a transducer.

Eg. Microphone.

- The electrical signal produced by transducer is called the baseband signal. It is also called a message signal, an information signal and is usually designated by $s(t)$.
- There are two types of signals:
Analog signal and Digital signal.

Transmitter

- Transmitter processes the base band signal received from transducer prior transmission
- There are two following options for processing signals prior transmission:
 - o the baseband signal, which lies in the low frequency spectrum, is translated to a higher frequency spectrum (Carrier communication system).

o the baseband signal is transmitted without translating it to a higher frequency spectrum (Baseband communication system).

- The carrier communication system is based on the principle of translating a low frequency baseband signal to high frequency spectrum. This process is **Modulation**.

Channel or Medium

- After the required processing, the transmitter section passes the signal to the transmission medium. The signal propagates through the transmission medium and is received at the other side by the receiver section.
- The transmission medium between the transmitter and the receiver is called a channel.
- Most of the noise is added to the signal during its transmission through the channel.
- Depending on physical implementations, channels can be classified into two groups:

Hardware Channels: These channels are manmade structure. The three possible implementations of the hardware channels are: Transmission lines, Waveguides, and Optical Fiber Cables (OFC)

Software Channels: These are certain natural resources. The natural resources that can be used as software channels are: air or open space and sea water.

Noise

- Noise is defined as unwanted electrical energy of random and unpredictable nature.
- Noise is an electrical disturbance, which does not contain any useful information.
- Noise is a highly undesirable part of a communication system, and has to be minimized.
- When noise is mixed with transmitted signal, it rides over it & deteriorates its waveform.
- This results in alteration of original information so that wrong information is received.
- The designer provides adequate signal strength at the time of transmission so that a high SNR (Signal to Noise Ratio) is available at the receiver.

Receiver

- The function of the receiver section is to separate the noise from the received signal, and then recover the original baseband signal by performing demodulation process.
- A voltage amplifier first amplifies the received signal so that it becomes strong enough for further processing, and then recovers the original information.
- The demodulation process removes the high frequency carrier from the received signal and retrieves the original baseband.

Output Transducer & Final Destination

- The recovered baseband signal is handed over to the final destination, which uses a transducer to convert this electrical signal to its original form.
- Prior to handing over the recovered baseband signal to its final destination, the voltage and power are amplified by the amplifier stages.

5.11.2 Evolution of Communication systems

1G – First-Generation System

1G laid the foundation and opened the door to wireless telephone communication before us. It used analog technology, was limited to voice calls, and offered a maximum speed of 2.4 kbps. During 1G, cell phones were big, heavy, and expensive. Also, battery drainage and poor voice quality were other limitations.

2G – Second-generation Communication System (GSM)

The Global System for Mobile Communications (GSM), the second generation (2G) standard developed by the European Telecommunications Standards Institute (ETSI), is based on Time Division Multiple Access (TDMA).

The 2G mobile phones used digital modulation and enabled a maximum speed of 14.4 kbps. Voice calls and SMS were supported, and mobile phones got smaller and more secure.

2.5G and 2.75G Communication System

The transition from 2G to 2.5G marked an advancement in mobile communication technology, where enhancements were introduced to the existing 2G networks. Also, 2.5G represented a notable shift from primarily catering to voice communication in 2G, as it integrated packet-switched data services, enabling basic internet usage and data applications alongside traditional circuit-switched voice services. Further, in 2.5G GPRS, the subscriber data transfer rates got enhanced up to 171 kbps.

EDGE (Enhanced Data Rates for GSM Evolution) or 2.75G is an enhancement of GPRS for data transmission. Also, it works on GSM networks, an extension of GPRS and allows for speeds up to 384 kbps.

3G – Third-generation Communication System

Later, second-generation (2G) cellular technology evolved into third-generation (3G) cellular technology based on the Universal Mobile Telecommunications System (UMTS). Furthermore, this technology changed the primary focus from voice and text to mobile data.

The advent of 3G networks in the first decade of the century paved the foundation for high-speed internet and wireless applications. Further, it resulted in a digitally powered era in communications.

UMTS is a third-generation cellular technology. It allows 2G GSM networks to migrate to 3G. UMTS uses Wideband CDMA (WCDMA) for its radio interface. Further, it enables peak download data rates of up to 2 Mbps and average download speeds of around 384 kbps.

Also, 3G paved the way for video call and streaming services. With the latest enhancements in High-Speed Packet Access (HSPA and HSPA+), UMTS networks can enable peak data rates of up to 71.6 Mbps.

How Does the 3G UMTS Network Function?

The User Equipment (UE) includes two components such as,

1. i) Mobile equipment and
2. ii) Universal Subscriber Identity Module (USIM)

Using the Air interface, the User Equipment connects to the UTRAN (Universal Terrestrial Radio Access Network). UTRAN consists of Node Bs and RNCs, and each RNC (Radio Network Controller) manages multiple Node Bs. In UMTS, there exists a circuit-switched core and a packet-switched core.

In the circuit-switched core, the Mobile Switching Center (MSC) is responsible for voice calls, delivering text messages and tracking down mobile locations.

Gateway MSC (GMSC) offers the connection to other service providers (mobile or fixed). The Home Location Register (HLR) keeps a repository of all the subscribers belonging to a service provider.

The Serving GPRS Support Node (SGSN) in the packet-switched core manages the data connection between the mobile and the Packet Data Network. It also tracks the location of the mobile for data services.

The Gateway GPRS Support Node (GGSN) provides the connection to external data networks. Also, it is an anchor point as the user moves to a different SGSN due to mobility.

How is 4G Different from 3G?

4G was commercially deployed in 2009. The 3G network only uses IP for data, enabling voice with a circuit-switched network. On the other hand, 4G is an all-IP-based standard for both voice and data. For this reason, 4G is more efficient for mobile network providers to operate and optimize instead of managing different network technologies for data and voice.

There are two flavors of 4G – LTE and WiMax.

The Long-Term Evolution (LTE) is fully packet-switched, which uses Orthogonal Frequency Division Multiple Access (OFDMA). LTE is designed to provide connectivity between a user's equipment and a Packet Data network with a data rate of up to 100 Mbps. LTE Advanced (LTEA) is an enhancement that improves the original LTE technology and could deliver up to 1000 Mbps. In addition, LTE supports VoIP, video conferencing, HD Mobile TV, online gaming, mobile broadband and mobile apps.

The key components include the E-UTRAN and the Evolved Packet Core (EPC), collectively forming the LTE network. The evolved NODE B (eNodeB) manages scheduling, handovers and security. The Serving gateway(S-GW) handles mobility between E-UTRAN and EPC.

The PDN Gateway (P-GW) connects the EPC to the Packet Data Network. The controlling entity is the Mobility Management Entity (MME), which tracks the location of UE. Also, it is responsible for session management. The HSS is the central repository of subscriber information.

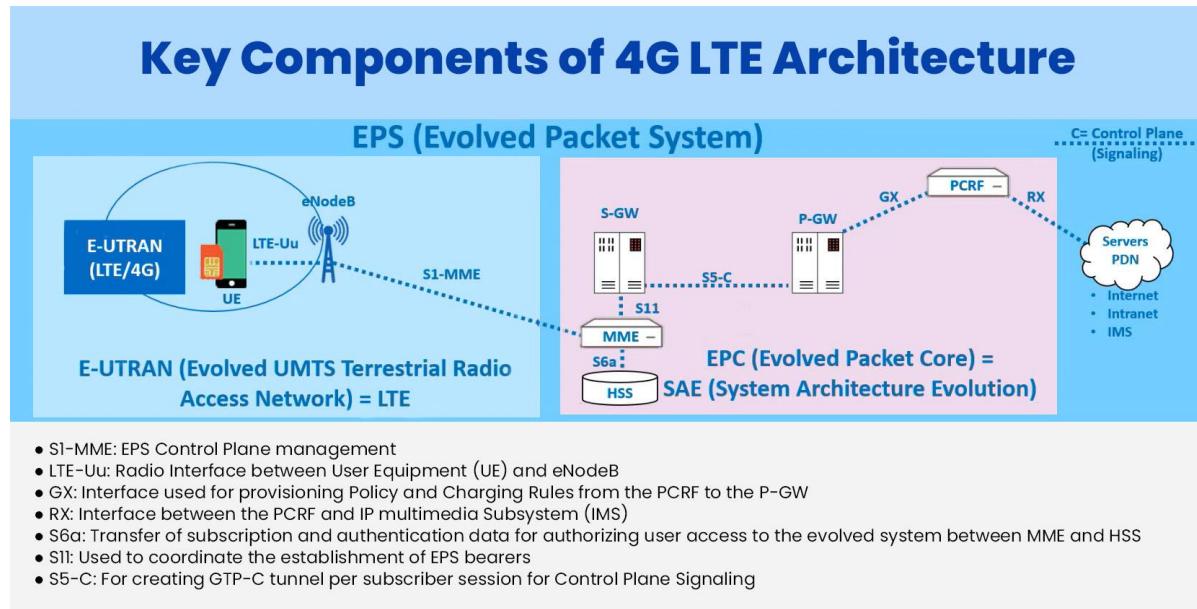


Figure 5.7: Key Components of 4G LTE Architecture

5G – Fifth-generation Communication System

5G network is not just about providing huge data rates. It can create an adaptive, flexible network that can connect virtually everything, including machines, objects and devices. Also, it can provide different features to different customers with many potential and possibilities.

The Radio Access Network belonging to 5G is known as New Radio (NR). The connectivity of 5G NR to a 5G core network is provided by Standalone architecture. The network architecture, known as Non-Standalone architecture, is based on tight interworking with LTE and NR, allowing a smooth evolution towards an end-to-end 5G system.

What Makes 5G Exciting?

The 5G NR technology, including millimeter wave (mmWave) and massive Multiple-Input Multiple-Output (MIMO) with beamforming, enables a network to deliver very high speed, reduced low latency and more data capacity.

The peak data rate of the network for the download link is 20 Gbps and 10 Gbps for uplink and offers a latency of less than 1 millisecond. The new use cases of 5G networks include enhanced Mobile Broadband (eMBB), ultra Reliable Low Latency Communication (uRLLC) and mMTC massive Machine Type Communication (mMTC).

As traffic volume increases exponentially, eMBB can deliver speeds in multi-Gbps peak data rates much faster than its previous generation. With uRLCC, latency can be minimal for applications like self-driving cars. In such cases, the response time can make much difference, facilitating decisions in real-time. Further, it is easy to connect Many smart devices to the network for an extended period, and mMTC delivers a network capable of handling this type of demand.

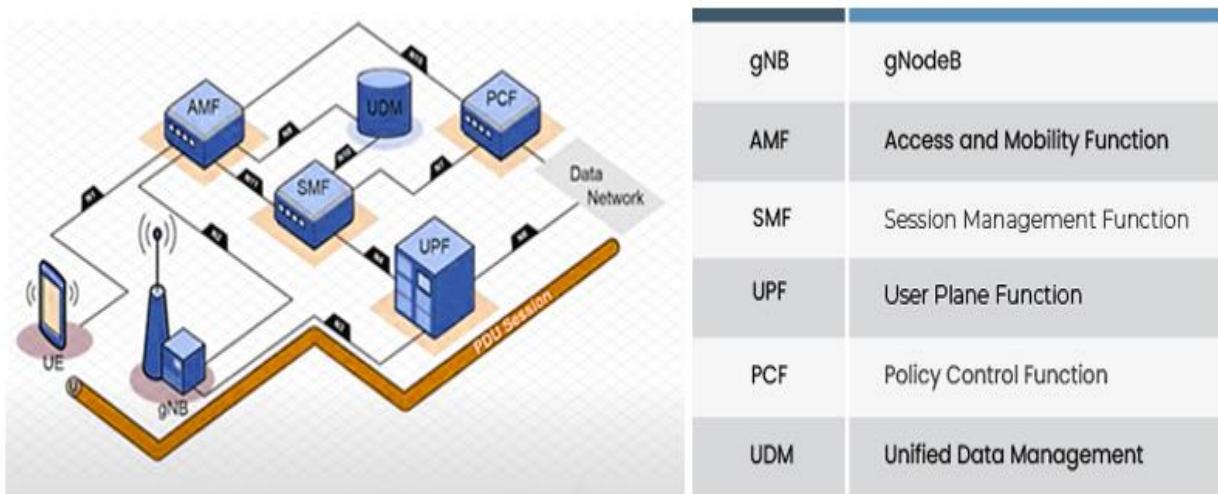


Figure 5.8: 5G Network Architecture

Access and Mobility Function (AMF) knows the cell or tracking area where the subscriber is located. AMF ensures that the subscriber is allowed on the network and authenticates the subscriber. Also, AMF allocates the user equipment with a Globally Unique Temporary ID (GUTI) during mobility and periodic updates.

The Session Management Function (SMF) is responsible for the user equipment's session management and IP address allocation. Directions based on decisions related to creating, modifying or terminating a session to the UPF are given.

SMF liaisons with PCF for policy and QoS enforcement. Also, SMF performs the selection and control of UPF. Unified Data Management (UDM) stores subscriber profiles and data network profiles. User Plane Function (UPF) is responsible for processing and forwarding data. If the user moves from one g-Node B to another, the traffic continues on the same connected UPF. Based on the rules from SMF, UPF ensures the quality of service. Packet Data Unit (PDU) sessions provide connectivity between the device and

the Data Network. Quality of service (QoS) flow within a PDU session offers different QoS levels for different services. The Policy Control Function (PCF) takes dynamic decisions based on network conditions. Hence, it decides the correct resource allocation for a user to access a particular service.