Robótica

Planejamento de Trajetórias

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Introduction

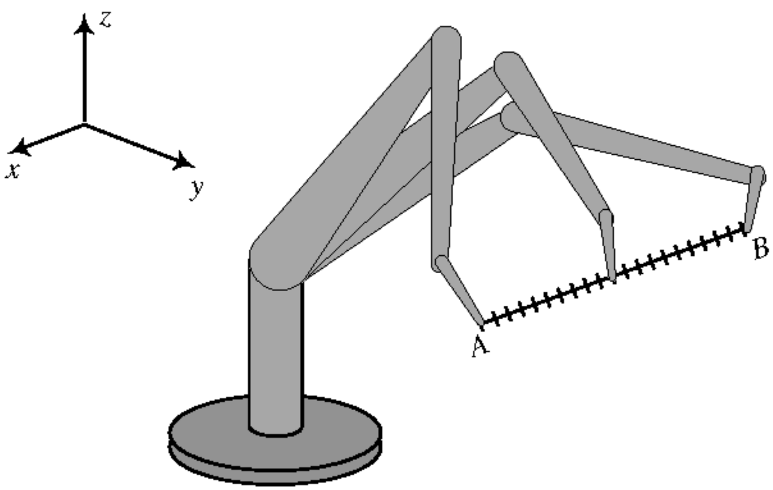
- In this chapter we discuss the joint-space and Cartesian-space trajectory planning.
- We also discuss the methods that can be used for achieving desired velocity and accelerations limits and maximum values and blending motion portions.

Path versus Trajectory

- A path is the collection of positions and orientations that a robot (or its joints) goes through between different locations.
- A trajectory is the time history of the locations and orientations that the robot (or its joints) goes through between different points.
- A trajectory in the time history of a path; therefore, it includes how the joint values change in time.

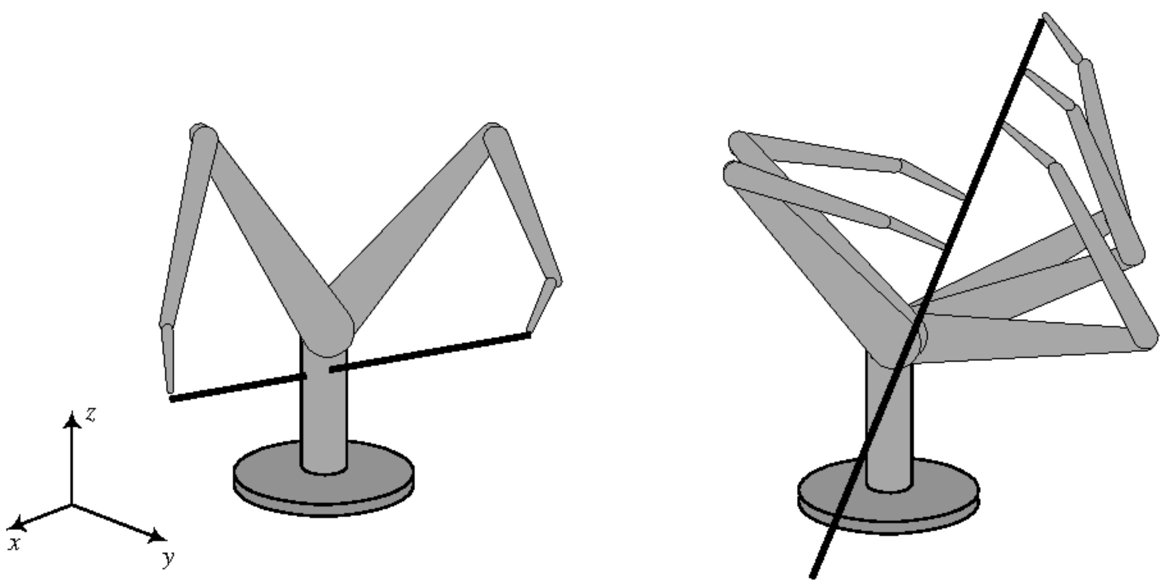
Joint-Space versus Cartesian-Space

- Cartesian-space describes the location and orientation of robot's end effector in space.
- Joint-space is the description of joint values in time. The collection of joint-space values creates the Cartesian-space values.



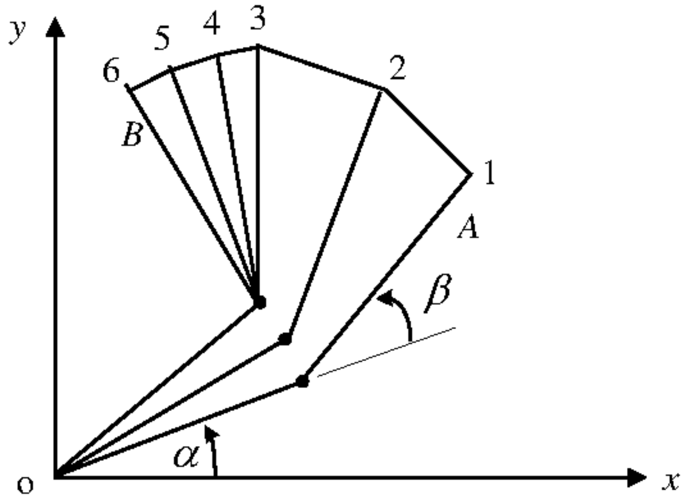
Cartesian-Space Trajectory Problems

- Robot running into itself
- Sudden changes in orientation while making a motion.



Basics of Trajectory Planning

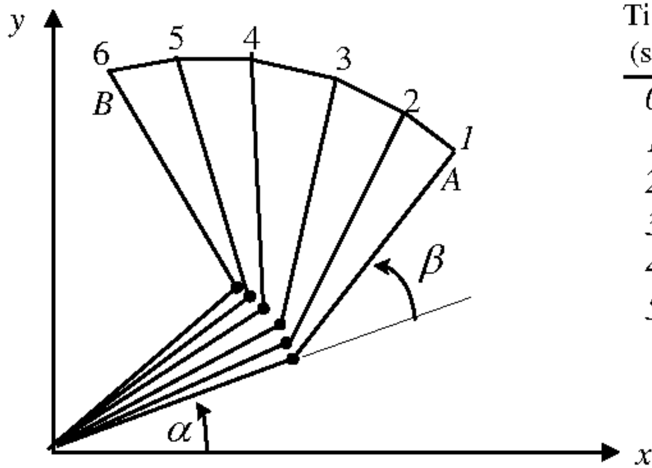
- Assume joints moves at their maximum value.
- The resulting motion is non-uniform and unpredictable.



Time (sec)	α	β
0	20	30
1	30	40
2	40	50
3	40	60
4	40	70
5	40	80

Basics of Trajectory Planning: Cont.

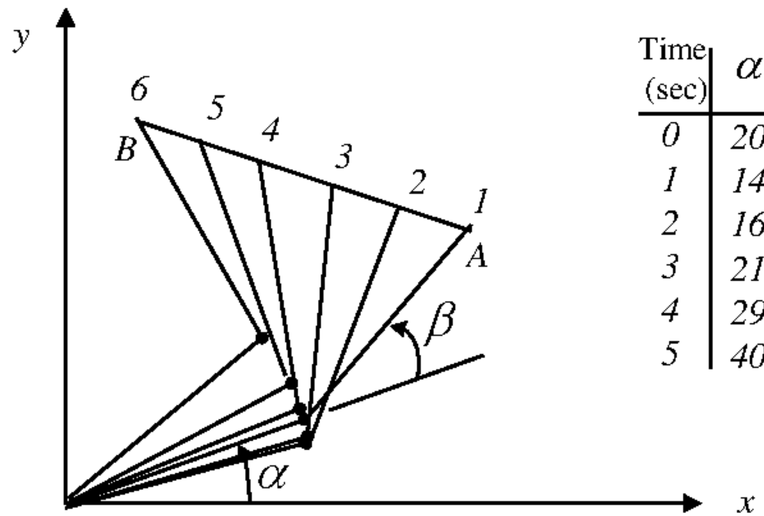
- Normalizing the joints to start and stop together improves the motion.
- The path is unpredictable.



Time (sec)	α	β
0	20	30
1	24	40
2	28	50
3	32	60
4	36	70
5	40	80

Basics of Trajectory Planning: Cont.

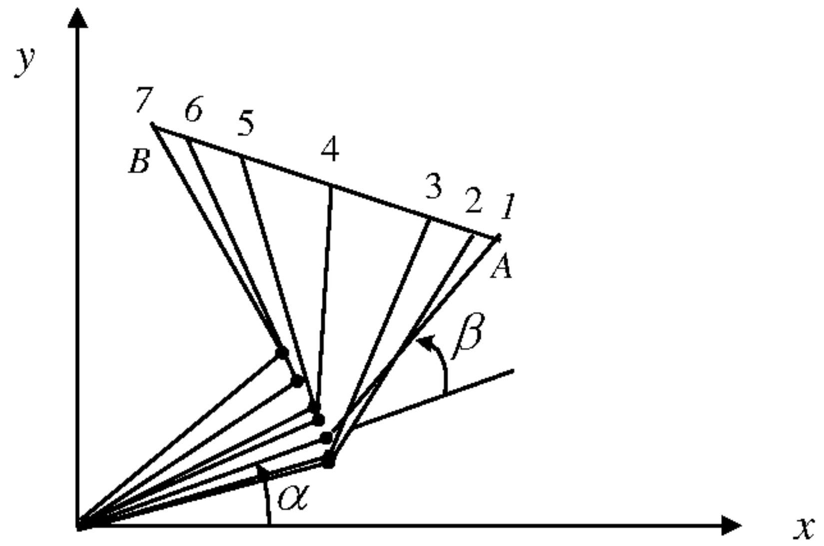
- Planning a desired path (such as a straight line) improves the outcome.
- Acceleration or velocities may be unachievable.



Time (sec)	α	β
0	20	30
1	14	55
2	16	69
3	21	77
4	29	81
5	40	80

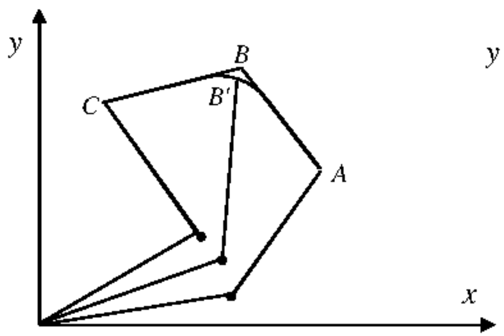
Basics of Trajectory Planning: Cont.

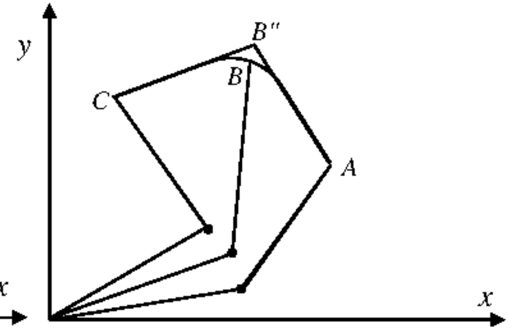
- Dividing the path (becomes a trajectory) into variable portions to control velocities and accelerations improves the outcome.
- Boundary velocities and accelerations are specified.

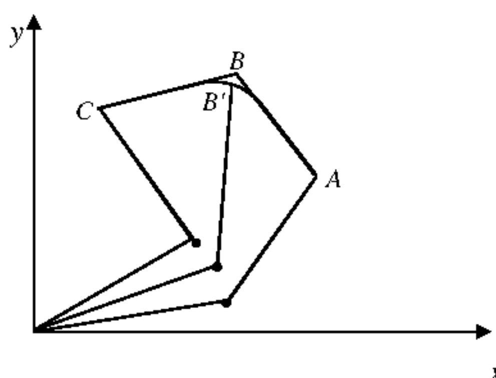


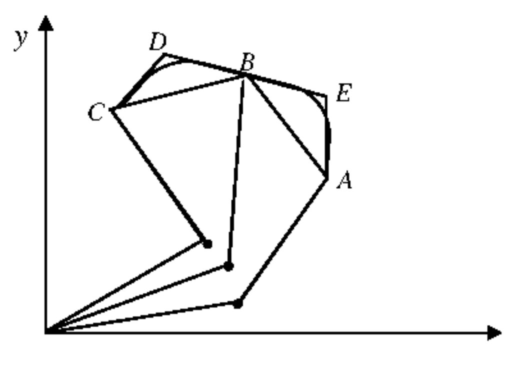
Blending of Motion Portions

- To prevent stop/start regimes at every point, motion portions may be blended together.
- There may be a need for additional via points to ensure passage through a desired location.









Trajectory Planning in Joint-Space

- It is more desirable and more practical to plan a trajectory in joint-space.
- We will use the joint-space trajectories to create a motion that is specified in Cartesian-space.
- In joint-space trajectory planning, it is the motions of each individual joint that is planned based on some regime (higher-order polynomials, controlled acceleration limits, blended motions, and so on).
- Each joint's motion is planned individually based on the values calculated from the inverse kinematic equations.

Third-Order Polynomial Trajectory Planning

- Initial and final positions and velocities are known; 4 values

$$\theta(t_i) = \theta_i \qquad \theta(t_f) = \theta_f$$

$$\dot{\theta}(t_i) = 0 \qquad \dot{\theta}(t_f) = 0$$

- 4 unknowns may be calculated; therefore, a third-order polynomial.

$$\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$$

$$\dot{\theta}(t) = c_1 + 2c_2 t + 3c_3 t^2$$

Third-Order Polynomial Trajectory Planning: Cont.

- Substitute the initial and final conditions into these equations to get:

$$\theta(t_i) = c_0 = \theta_i$$

$$\theta(t_f) = c_0 + c_1 t_f + c_2 t_f{}^2 + c_3 t_f{}^3$$

$$\dot{\theta}(t_i) = c_1 = 0$$

$$\dot{\theta}(t_f) = c_1 + 2c_2 t_f + 3c_3 t_f{}^2 = 0$$

- Or in matrix form:

$$\begin{bmatrix} \theta_i \\ \dot{\theta}_i \\ \theta_f \\ \dot{\theta}_f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & t_f & t_f{}^2 & t_f{}^3 \\ 0 & 1 & 2t_f & 3t_f{}^2 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

Third-Order Polynomial Trajectory Planning: Cont.

- From the inverse kinematic equations the boundary values are determined.
- Each joint is driven based on the third-order polynomial.
- The resulting motion follows the desired trajectory.

Exercícios:

1) É desejável que a primeira articulação de um robô de 6 eixos vá do ângulo inicial de 30º para um ângulo final de 75º em 5 segundos. Usando um polinômio de terceira ordem, calcule o ângulo da articulação em 1,2,3 e 4 segundos. Esboce graficamente os ângulos e velocidades ao longo do tempo.

2) Suponha que o mesmo braço robótico deva continuar para o próximo ponto, em que a articulação deva chegar a 105º em mais 3 segundos. Projete as curvas de posição e velocidade para o movimento.

Fifth-Order Polynomial Trajectory Planning

- Initial and final positions, velocities, and accelerations are known; 6 values
- 6 unknowns may be calculated; therefore, a fifth-order polynomial.

$$\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 + c_5 t^5$$

$$\dot{\theta}(t) = c_1 + 2c_2 t + 3c_3 t^2 + 4c_4 t^3 + 5c_5 t^4$$

$$\ddot{\theta}(t) = 2c_2 + 6c_3 t + 12c_4 t^2 + 20c_5 t^3$$

Fifth-Order Polynomial Trajectory Planning: Cont.

- Substitute the initial and final conditions into these equations and solve for the constants.
- Use the equation to run each joint based on the data calculated from the inverse kinematic equations.

Exercícios:

3) Repita o exemplo anterior mas supondo que a aceleração inicial e a desaceleração final serão de 5º/s2

4) Seja um manipulador antropomórfico que necessita mover todas as juntas em 7 segundos ate um ponto especificado

- A primeira junta deve ir de 0º a 50º por um polinômio de ordem 3
- A segunda junta deve ir de 0º a 90º por um polinômio de ordem 5, com aceleração e desaceleração igual a 0.
- A terceira junta deve ir de 0º a 10º por um polinômio de ordem 5, com aceleração e desaceleração igual a 0.

Esboce graficamente os ângulos, velocidades e aceleração ao longo do tempo.

Higher-order Polynomials

- When more segments are present, more information is available.
- Higher-order polynomials may be used, but more difficult to solve higher-order polynomials:

$$\theta(t) = c_0 + c_1 t + c_2 t^2 + \cdots + c_{n-1} t^{n-1} + c_n t^n$$

- use combinations of lower-order polynomials for different segments of the trajectory and blend them together to satisfy all required boundary conditions.
- Can use a 4-3-4 trajectory, a 3-5-3 trajectory, and a 5-cubic trajectory to replace a 7th-order polynomial

Higher-Order Polynomials: Cont.

- In a 4-3-4 trajectory we use:
  - a 4th-order polynomial to plan a trajectory between the initial point and the first via point (e.g. lift-off),
  - a 3rd-order polynomial between two via points (e.g. lift off and set-down),
  - and a 4th-order polynomial between the last via point (e.g. set-down) and the final destination.
- A 3-5-3 trajectory is planned between the initial and the first via point, between the successive via points, and between the last via point and the final destination.

Higher-Order Polynomials: Cont.

- We must solve for 4 coefficients for a third-order polynomial, 5 for a fourth-order polynomial, and 6 for a fifth-order polynomial.
- Both 4-3-4 and 3-5-3 trajectories require solving for a total of 14 coefficients. For the 4-3-4 trajectory, the unknown coefficients are in the form:

$$\theta(t)_1 = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4$$

$$\theta(t)_2 = b_0 + b_1 t + b_2 t^2 + b_3 t^3$$

$$\theta(t)_3 = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4$$

Higher-Order Polynomials: Cont.

1. Initial position of is known.
2. Initial velocity may be specified.
3. Initial acceleration may be specified.
4. Position of the first via point is known, and is the same as the final position of the first fourth-order segment.
5. The first via point's position is the same as the initial position of the third-order segment for continuity.
6. Continuous velocity must be maintained at the via point.
7. Continuous acceleration must be maintained at the via point.

Higher-Order Polynomials: Cont.

8. Position of a second (and other) via point is specified and is the same as the final position of the third-order segment.
9. The position of the second (and other) via point is the same as the initial position of the next segment for continuity.
10. Continuous velocity must be maintained at the next via point.
11. Continuous acceleration must be maintained at the next via point.
12. Position of destination is specified.
13. Velocity of the destination is specified.
14. Acceleration of the destination is specified

Higher-Order Polynomials: Cont.

- The result is:

$$
\begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \\ \ddot{\theta}_1 \\ \theta_2 \\ \theta_2 \\ 0 \\ 0 \\ \theta_3 \\ \theta_3 \\ 0 \\ 0 \\ \theta_4 \\ \dot{\theta}_4 \\ \ddot{\theta}_4 \end{bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & \tau_{1f} & \tau_{1f}^{2} & \tau_{1f}^{3} & \tau_{1f}^{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 2\tau_{1f} & 3\tau_{1f}^{2} & 4\tau_{1f}^{3} & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 6\tau_{1f} & 12\tau_{1f}^{2} & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & \tau_{2f} & \tau_{2f}^{2} & \tau_{2f}^{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 2\tau_{2f} & 3\tau_{2f}^{2} & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 6\tau_{2f} & 0 & 0 & -2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \tau_{3f} & \tau_{3f}^{2} & \tau_{3f}^{3} & \tau_{3f}^{4} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2\tau_{3f} & 3\tau_{3f}^{2} & 4\tau_{3f}^{3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 6\tau_{3f} & 12\tau_{3f}^{2}
\end{bmatrix}
\times
\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_0 \\ b_1 \\ b_2 \\ b_3 \\ c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}
$$

Higher-Order Polynomials: Cont.

- Or simply:

$$[\theta] = [M][C]$$

$$[C] = [M]^{-1}[\theta]$$

- This can be used to run a joint based on a 4-3-4 polynomial.
- Similar results may be obtained for 5-3-5 or 5-cubic regiments.