~~$r \sin\beta \cos\gamma =$~~
~~$r \sin\beta \sin\gamma$~~

$$\begin{cases} r \sin\beta = 4{,}3 \\ r \cos\beta = 7 \end{cases} \quad \beta = tg^{-1}\left(\frac{4{,}3}{7}\right) = 31{,}56^{0}$$

$$r = \frac{7}{\cos(31{,}56)} = 8{,}21$$

Ponto B $\frac{r \sin\beta \sin\gamma}{r \sin\beta \cos\gamma} \Rightarrow tg^{-1}\left(\frac{\sin\gamma}{\cos\gamma}\right) \to tg^{-1}\left|\right.$

$\left\{ r \sin\beta \cos\gamma = \right.$

b cilíndrica

$$\left.\begin{aligned} r \cos\alpha &= 10 \\ r \sin\alpha &= 4 \end{aligned}\right\} \quad tg^{-1}\left(\frac{4}{10}\right) = 21{,}8$$

$r = 7$

$$r \sin(21{,}8) = 4.$$

$$r = \frac{4}{\sin(21{,}8)} = 10{,}77$$

orientação → 3 últimos juntas (Punho).

Robótica - Introdução a cinemática de orientação Ângulos de Euler e RAG

E. Inversa de matrizes de transformação

$$\underbrace{{}^U T_E \Rightarrow {}^U T_R}_{\text{da base até ?}}\ \underbrace{{}^R T_H}_{\text{ponto até ponto}}\ {}^H T_E = \underbrace{{}^U T_P}\ {}^P T_E$$

da base até ? e há

ponto até ponto

$$\underbrace{({}^U T_R)^{-1}\, {}^U T_R}_{I}\ {}^R T_H\ \underbrace{{}^H T_E\, ({}^U T_E)^{-1}}_{I} = ({}^U T_R)^{-1}\ {}^U T_P\ {}^P T_E\ ({}^H T_E)^{-1}$$

$${}^R T_H = {}^R T_U\ {}^U T_P\ {}^P T_E\ {}^E T_H$$

Se $c\theta = 1 \Rightarrow s\theta = 0 \Rightarrow \theta = 0$

$$R_{zyz} = \begin{bmatrix} c\phi c\psi - s\phi s\psi & -c\phi s\psi - s\phi c\psi & 0 \\ s\phi c\psi + c\phi s\psi & -s\phi s\psi + c\phi c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{zyz} = \begin{bmatrix} c(\phi + \psi) & -s(\phi + \psi) & 0 \\ s(\phi + \psi) & c(\phi + \psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow tg^{-1}\left(\frac{s(\phi + \psi)}{c(\phi + \psi)}\right.$$

Para $c\theta < 1$

$$s\theta^2 + c\theta^2 = 1$$

$$s\theta = \sqrt{1 \pm c\theta^2}$$

Euler e RAG ⇒ causam Gimball lock.

x y z

Roll, Pitch and Yaw

cartesiano - RAG

PPP RRR.

Euler ⇒ ZYZ = correntes

RAG ⇒ xyZ = Fixos

Exercício

① cartesiano + RAG

PPP + RRR →

$$\begin{bmatrix} 1 & 0 & 0 & 4,33 \\ 0 & 1 & 0 & 2,50 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} R_{3\times3} & T_{3\times1} \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{array}{l} P_x = 4,33 \\ P_y = 2,5 \\ P_z = 8 \end{array}$$

cinemática posição

$\theta_a = \text{atan2d}(0,505 ; 0,354) = 55°$

$\theta_o = \text{atan2d}(-(-0,788) \cdot (0,354 \cdot \text{cosd}(55) + 0,505 \cdot \text{sind}(55)))$

$\theta_o = \text{atan2d}(0,788 ; 0,62) = 52°$

$\theta_m = \text{atan2d}(-0,475 \cos(55) + 0,649 \sin(55), \ 0,722 \text{cosd}(55) - (-0,647 \sin(55)))$

$\theta_m = 15,32$

$RAG = (\theta_a, \theta_o, \theta_m) = (55, 52, 15,32)$

$Rot(z,55)\, Rot(y,52)\, Rot(x, 15,32)$

55, 52 15,32

Para RAG → (ϕ, θ, ψ)

$\theta_a \ \theta_o \ \theta_m$

② Cilíndrica + RAG

RPP + RRR

$r c\alpha = 4,33$ $\alpha = 30$ $\theta_a = 55 - 30 = 25°$

$r s\alpha = 2,55 \Rightarrow r = 5$ $\theta_o = 52$

$L = 8$ $L : 8$ $\theta_m = 15,32$

Se cair na prova, não precisa subtrair

Exercício 2) (mudou p/ esférico euler)

Esférico - Euler.

RRP RRR → Euler

$r\,s\beta\,c\gamma = 5$

$r\,s\beta\,s\gamma = 7 \Rightarrow r = 9{,}11°$

$r\,c\beta = 3$

$\beta = 70{,}77°$

$\gamma = 54{,}5°$

$\theta = atan2d\left(\pm\sqrt{1-(0{,}766)^2}\,,\,0{,}76\right)$

$\theta = 40°$

$\phi = atan2d(-0{,}22\,,\,-0{,}604) = -160$

$\psi = atan2d(-0{,}99\,,\,-(0{,}611)) = -162°$

Exercício 3)

Exercício 3)

$${}^{0}A_1 = \quad T(0, c+e, 0)\, T(0, 0, a-d)$$
$$T(0, c+e, a-d)\, Rot(z, 180)$$

Lista 1 de exercícios.

12) a) $Rot(90, z)\, T(3,0,0)\, Rot(90, x)\, B\, T(0,5,0) = B\ final.$

$\therefore B\ final = Rot(90, z)\, T(3,0,0)\, Rot(90, x)\, T(0,5,0)$

b) $p(1,5,4)^T$ $\quad T \quad Rot\, x \quad B$

$P\ final = B\ final\ p$

15 $\ {}^{G}T_{O} = ?\quad \underbrace{{}^{B}T_G,\ {}^{B}T_E,\ {}^{E}T_O}_{\text{conhecidas}}$

${}^{B}T$

16 $R^2_3 = ?\quad R^1_2\ e\ R^1_3$

Lista 2 (ex. fig) | P. 23 aula 3

Cartesiano - RPY

PPP + RRR

$\theta_a = atan2d(ny, nx) = atan2d(0{,}369; 0{,}527) = \boxed{35°}$

$\rightarrow P_x = 4$

$P_y = 6$

$P_z = 9.$

$\theta_n = atan2d((-ay\cos(\theta_a) + ax\sin(\theta_a); [oy\cos(\theta_a) - ox\sin(\theta_a))$

$\theta_o = atan2d(-nz; (nx \cdot cosd(\theta_a) + ny\sin(\theta_a))$

$\theta_n = atan2d((-0{,}439\cos(35°) + 0{,}628\sin(35); 0{,}819\cos(35) + 0{,}...)$

$\theta_o = atan2d(0{,}766; (0{,}527 cosd(35) + 0{,}369 sind(35)) = 50°$

(9) Cartesiano - Euler

PPP RRR

Euler $\theta = atan2(\pm\sqrt{1 - r_{33}^2}, r_{33})$

$\phi = atan2(\pm r_{23}, \pm r_{13})$,

$\psi = atan2d(\pm r_{32}, \mp r_{31})$ cuidado

$\theta = atan2d(\sqrt{1 - 0{,}643^2}; 0{,}633) \Rightarrow \boxed{\theta = 50°}$

$\phi = atan2d(0{,}439; 0{,}628) \Rightarrow \boxed{\phi = 35°}$

$\psi = atan2d(0, -0{,}574) \Rightarrow \boxed{\psi = 0°}$

$\theta \quad \phi \quad \psi$

$\theta \quad \phi \quad \psi$

(10) $30°, 40°, 50°$

(11) $B_{final} = Rot_{x,45}\, Rot_{z,60}\, T(0,0,3) \cdot B \cdot T(0,6,0) \cdot Rot_{n,60}$

$B_{final}: Rot_{x,45}\, Rot_{z,60}\, T(0,0,3)\, B \cdot T(0,6,0)\, Rot_{n,60}$

Lista 3) (Ex fig)

$r \sin\beta \cos y = 3{,}1375$ $\quad tg^{-1}\left(\frac{r \sin\beta \sin y}{r \sin\beta \cos y}\right) = tg^{-1}\left(\frac{\sin y}{\cos y}\right) = \left(\frac{2{,}195}{3{,}1375}\right) tg^{-1} \Rightarrow \boxed{y = 35°}$

$r \sin\beta \sin y = 2{,}195$

$r \cos\beta = 3{,}214 \Rightarrow r = \frac{3{,}214}{\cos\beta}$

$\frac{3{,}214}{\cos\beta} \sin\beta = \frac{3{,}1375}{\cos y} \Rightarrow \frac{\sin\beta}{\cos\beta} = \frac{3{,}1375}{3{,}214 \cdot \cos 35} \Rightarrow tg^{-1}\left(\frac{3{,}1375}{3{,}214 \cos 35}\right) \beta = 50°$

$r = \frac{3{,}214}{\cos 50} = \boxed{5}$

Cinemática desacoplada

Valores das juntas na prova

Sequência ϕ, θ, ψ

⑤ a) $B_{ramo} = Rot_{x,45°} Rot_{z,60} T(0,0,3)\, {}^{u}B \cdot T(0,4,0) Rot_{x,60}$

$B_{mevo} = Rot_{x,45°} Rot_{z,60} T(0,0,3)\, {}^{u}B\, T(0,6,0) Rot_{x,60}$

Desacoplamento Cinemático

quando os 3 últimos eixos se interceptam em 1 ponto

cinemática inversa de posição + cinemática inversa de orientação

Lista 2

⑩ Euler(30,40,50) =
$$\begin{bmatrix} 0,0439 & -0,8296 & 0,5557 & 0 \\ 0,9056 & 0,2633 & 0,3214 & 0 \\ -0,413 & 0,4924 & 0,7660 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

aplicar na matriz de Euler

usar fórmulas RAG

$\theta_n = 32,73 \quad 267,26$

$\theta_o = \cancel{24,4} \; +180 \quad 153,5$

$\theta_a = 82,26 \quad 212,7$

d - translação

d_0^6 - trans de 0 - 6

$d_0^6 = {}^1d_0\, {}^2d_1\, d_3\, d \ldots$

tamanho dos links

$$\boxed{p_c = d_0^6 - d_6 R_0^6 K}$$

d_6 = dist da junta 5 a 6

$$K = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$