9.1 If $L(x)$ and $U(x)$ satisfy $P_\theta(L(X) \leq \theta) = 1 - \alpha_1$ and $P_\theta(U(X) \geq \theta) = 1 - \alpha_2$, and $L(x) \leq U(x)$ for all x, show that $P_\theta(L(X) \leq \theta \leq U(X)) = 1 - \alpha_1 - \alpha_2$.

9.3 The independent random variables $X_1, \ldots, X_n$ have the common distribution

$$P(X_i \leq x) = \begin{cases} 0 & \text{if } x \leq 0 \\ (x/\beta)^\alpha & \text{if } 0 < x < \beta \\ 1 & \text{if } x \geq \beta. \end{cases}$$

(a) In Exercise 7.10 the MLEs of α and β were found. If α is a known constant, α_0, find an upper confidence limit for β with confidence coefficient .95.

(b) Use the data of Exercise 7.10 to construct an interval estimate for β. Assume that α is known and equal to its MLE.

9.5 In Example 9.2.5 a lower confidence bound was put on p, the success probability from a sequence of Bernoulli trials. This exercise will derive an upper confidence bound. That is, observing $X_1, \ldots, X_n$, where $X_i \sim \text{Bernoulli}(p)$, we want an interval of the form $[0, U(x_1, \ldots, x_n))$, where $P_p(p \in [0, U(X_1, \ldots, X_n))) \geq 1 - \alpha$.

(a) Show that inversion of the acceptance region of the test

$$H_0 : p = p_0 \qquad \text{versus} \qquad H_1 : p < p_0$$

will give a confidence interval of the desired confidence level and form.

(b) Find equations, similar to those given in (9.2.8), that can be used to construct the confidence interval.

9.7 (a) Find the $1 - \alpha$ confidence set for a that is obtained by inverting the LRT of $H_0 : a = a_0$ versus $H_1 : a \neq a_0$ based on a sample $X_1, \ldots, X_n$ from a $\text{n}(\theta, a\theta)$ family, where θ is unknown.

(b) A similar question can be asked about the related family, the $\text{n}(\theta, a\theta^2)$ family. If $X_1, \ldots, X_n$ are iid $\text{n}(\theta, a\theta^2)$, where θ is unknown, find the $1 - \alpha$ confidence set based on inverting the LRT of $H_0 : a = a_0$ versus $H_1 : a \neq a_0$.

9.11 If T is a continuous random variable with cdf $F_T(t|\theta)$ and $\alpha_1 + \alpha_2 = \alpha$, show that an α level acceptance region of the hypothesis $H_0 : \theta = \theta_0$ is $\{t : \alpha_1 \leq F_T(t|\theta_0) \leq 1 - \alpha_2\}$, with associated confidence $1 - \alpha$ set $\{\theta : \alpha_1 \leq F_T(t|\theta) \leq 1 - \alpha_2\}$.

9.17 Find a $1 - \alpha$ confidence interval for θ, given $X_1, \ldots, X_n$ iid with pdf

(a) $f(x|\theta) = 1,\ \theta - \frac{1}{2} < x < \theta + \frac{1}{2}$.
(b) $f(x|\theta) = 2x/\theta^2,\ 0 < x < \theta,\ \theta > 0$.

9.23 (a) Let $X_1, \ldots, X_n$ be a random sample from a Poisson population with parameter λ and define $Y = \sum X_i$. In Example 9.2.15 a confidence interval for λ was found using the method of Section 9.2.3. Construct another interval for λ by inverting an LRT, and compare the intervals.

(b) The following data, the number of aphids per row in nine rows of a potato field, can be assumed to follow a Poisson distribution:

$$155, \quad 104, \quad 66, \quad 50, \quad 36, \quad 40, \quad 30, \quad 35, \quad 42.$$

Use these data to construct a 90% LRT confidence interval for the mean number of aphids per row. Also, construct an interval using the method of Example 9.2.15.

9.25 If $X_1, \ldots, X_n$ are iid with pdf $f(x|\mu) = e^{-(x-\mu)} I_{[\mu,\infty)}(x)$, then $Y = \min\{X_1, \ldots, X_n\}$ is sufficient for μ with pdf

$$f_Y(y|\mu) = ne^{-n(y-\mu)} I_{[\mu,\infty)}(y).$$

In Example 9.2.13 a $1-\alpha$ confidence interval for μ was found using the method of Section 9.2.3. Compare that interval to $1-\alpha$ intervals obtained by likelihood and pivotal methods.

9.29 Let $X_1, \ldots, X_n$ are a sequence of n Bernoulli(p) trials.

(a) Calculate a $1-\alpha$ credible set for p using the conjugate beta(a, b) prior.

(b) Using the relationship between the beta and F distributions, write the credible set in a form that is comparable to the form of the intervals in Exercise 9.21. Compare the intervals.

9.35 Let $X_1, \ldots, X_n$ be a random sample from a $\mathrm{n}(\mu, \sigma^2)$ population. Compare expected lengths of $1-\alpha$ confidence intervals for μ that are computed assuming

(a) σ^2 is known.

(b) σ^2 is unknown.