# Machine Learning Process #2

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Huawei / IFCE

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# Agenda

1. Model Validation
   - Training and Generalization Error
   - Underfitting and Overfitting
   - Model capacity
   - Bias and Variance
   - Model Complexity and Error
2. Model Learning Performance Evaluation
   - Classification
   - Regression

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### Generalization

- The goal of machine learning is that the model obtained after learning should perform well on new samples
- The capability of applying a model to new samples is called generalization or robustness

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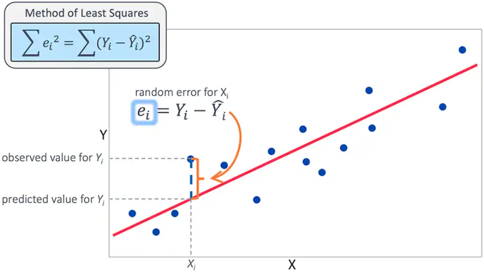

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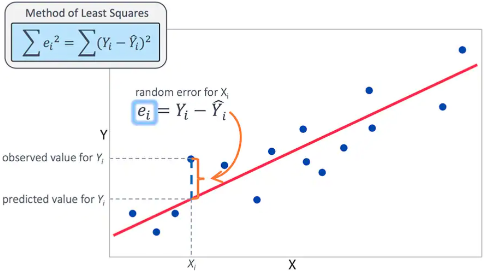


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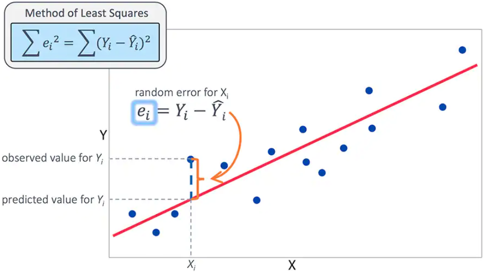


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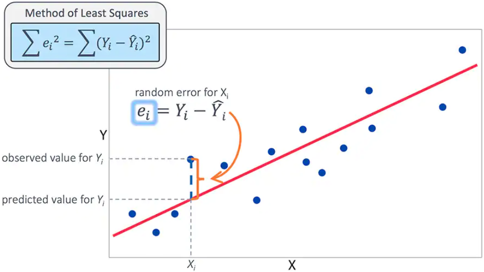


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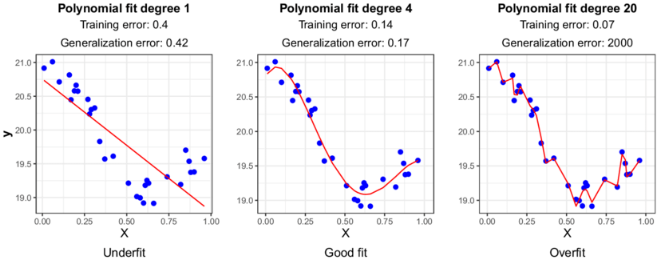

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- When the capacity suits the task complexity and the amount of training data provided, the algorithm effect is usually optimal.
- Models with insufficient capacity cannot solve complex tasks and underfitting may occur.
- A high-capacity model can solve complex tasks, but overfitting may occur if the capacity is higher than that required by a task.

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# Bias and Variance

- Let $\mathcal{D} = \{\boldsymbol{x}_n, y_n\}_{n=1}^{N}$ a data set. We assume there is a relationship between a $\boldsymbol{x}_n$ and a $y_n$ is

$$y_n = f(\boldsymbol{x}) + \epsilon \tag{1}$$

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$$\begin{aligned} \mathrm{Error}(\boldsymbol{x}) &= \mathsf{E}\left[(y - \hat{f}(\boldsymbol{x}))^2\right] \qquad (2) \\ &= \underbrace{\left(\mathsf{E}\left[f(\boldsymbol{x})\right] - \hat{f}(\boldsymbol{x})\right)^2}_{\mathrm{bias}^2} + \underbrace{\mathsf{E}\left[\left(\hat{f}(\boldsymbol{x}) - \mathsf{E}\left[\hat{f}(\boldsymbol{x})\right]\right)^2\right]}_{\mathrm{variance}} \\ &+ \underbrace{\sigma_e^2}_{\text{irreducible error}} \end{aligned}$$

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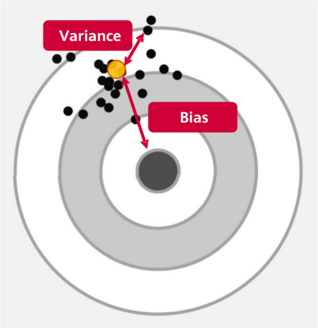

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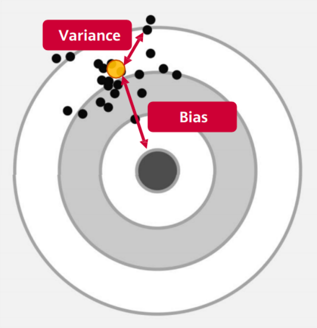


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- Offset of the prediction result from the average value

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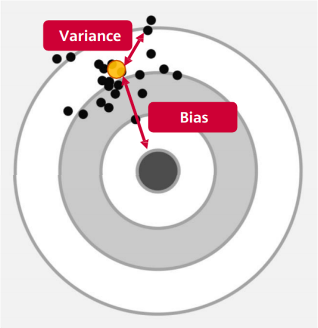


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- Offset of the prediction result from the average value

**Bias**

- Difference between the expected prediction value and the correct

# Bias and Variance

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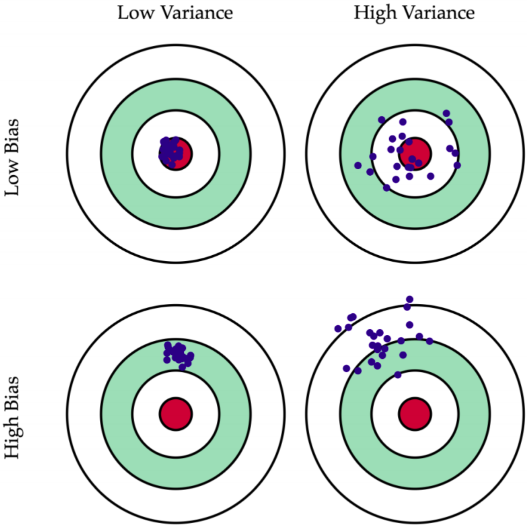
| *bias* | *variance* |
|---|---|
| $\downarrow$ | $\downarrow$ |
| $\downarrow$ | $\uparrow$ |
| $\uparrow$ | $\downarrow$ |
| $\uparrow$ | $\uparrow$ |



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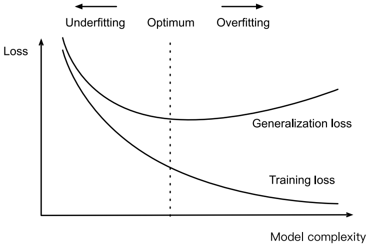

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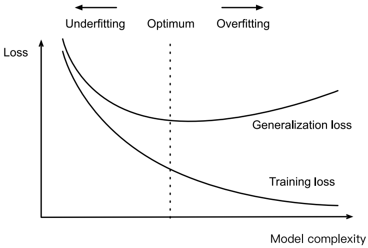


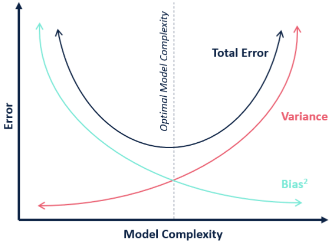

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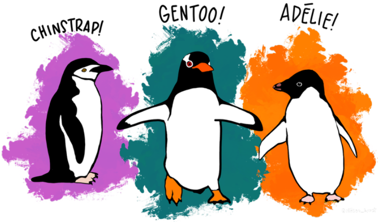

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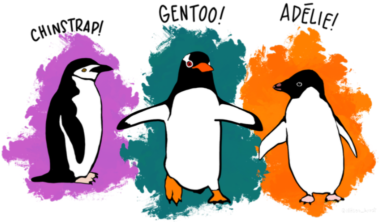


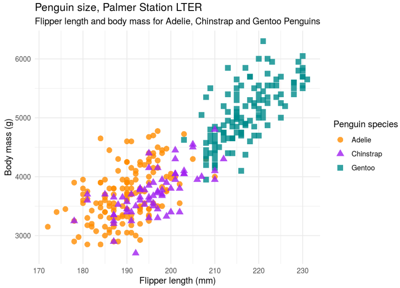

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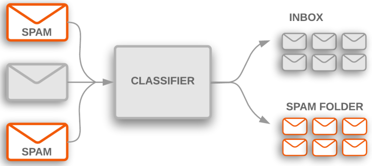

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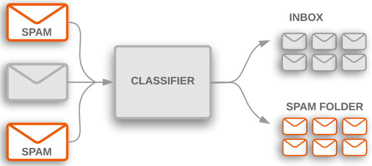


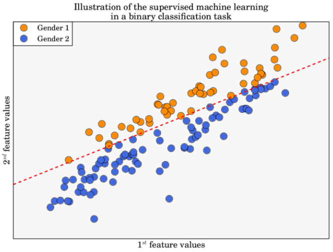

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| | | **Predicted Class** | | |
|---|---|---|---|---|
| | | **Positive** | **Negative** | |
| **Actual Class** | **Positive** | True Positive (TP) | False Negative (FN) **Type II Error** | **Sensitivity** $\frac{TP}{(TP+FN)}$ |
| | **Negative** | False Positive (FP) **Type I Error** | True Negative (TN) | **Specificity** $\frac{TN}{(TN+FP)}$ |
| | | **Precision** $\frac{TP}{(TP+FP)}$ | **Negative Predictive Value** $\frac{TN}{(TN+FN)}$ | **Accuracy** $\frac{TP+TN}{(TP+TN+FP+FN)}$ |

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## Confusion matrix example and accuracy rate

| | | Predicted Class | |
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| | | **Spam** | **Non-Spam** |
| **Actual Class** | **Spam** | TP=45 | FN=20 |
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## Confusion matrix example and accuracy rate

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$$\mathrm{ACC}\left(\hat{f}\left(\mathcal{D}_{\mathsf{test}};\mathcal{D}_{\mathsf{train}},\boldsymbol{\theta}\right)\right) = \frac{TP+TN}{TP+FP+TN+FN} \tag{3}$$

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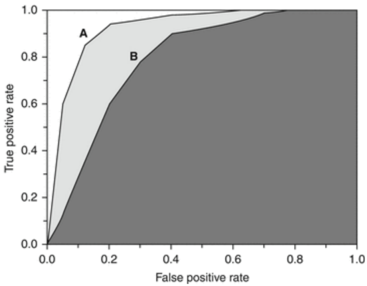
**Area under a ROC curve**

- Abbreviated as AUC, is a single scalar value that measures the overall performance of a binary classifier



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### Performance evaluation example

- We have trained a machine learning model to identify whether the object in an image is a cat. Now we use 200 pictures to verify the model performance. Among the 200 images, objects in 170 images are cats, while others are not. The identification result of the model is that objects in 160 images are cats, while others are not.

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- Precision? $\frac{140}{140+20} = 0.875$
- Recall? $\frac{140}{140+30} = 0.824$

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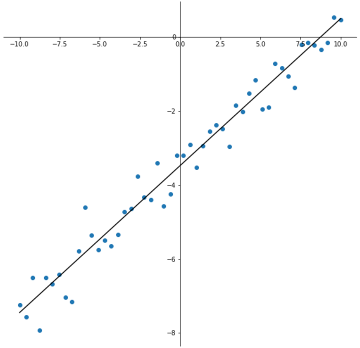

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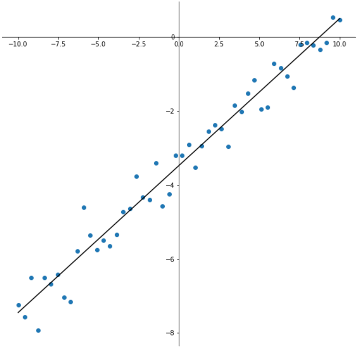


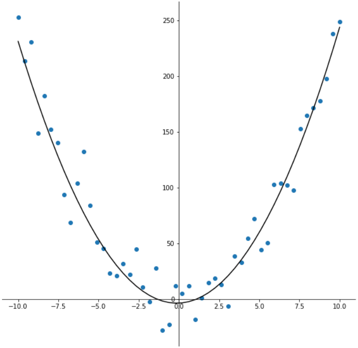

**Regression metrics**

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| Mean Squared Error (MSE) | $\mathrm{MSE} = \frac{1}{N}\sum_{n=1}^{N}(y_n - \hat{y}_n)^2$ |
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| Mean Absolute Error (MSE) | $\mathrm{MAE} = \frac{1}{N}\sum_{n=1}^{N}\lvert y_n - \hat{y}_n\rvert$ |
| R-squared ($R^2$) | $R^2 = 1 - \frac{\sum_{n=1}^{N}(y_n - \hat{y}_n)^2}{\sum_{n=1}^{N}(y_n - \bar{y}_n)^2}$ |