Algorithm HW12

1. Ext. 9-6

Consider that we set $d(u, u) = \infty$ initially while running Floyd-Warshall algorithm, what does it mean if we finally find some u such that $d(u, u) < \infty$.

2. Exercises 24.4-2

Find a feasible solution or determine that no feasible solution exists for the following system of difference constraints:

$$
\begin{aligned}
x_1 - x_2 &\le 4 , \\
x_1 - x_5 &\le 5 , \\
x_2 - x_4 &\le -6 , \\
x_3 - x_2 &\le 1 , \\
x_4 - x_1 &\le 3 , \\
x_4 - x_3 &\le 5 , \\
x_4 - x_5 &\le 10 , \\
x_5 - x_3 &\le -4 , \\
x_5 - x_4 &\le -8 .
\end{aligned}
$$

3. Exercises 25.1-10

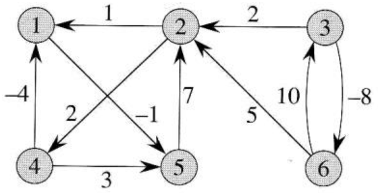
Give an efficient algorithm to find the length (number of edges) of a minimum-length negative-weight cycle in a graph.

4. Exercises 25.2-1

Run the Floyd-Warshall algorithm on the weighted, directed graph of Figure 25.2. and answer the following questions:

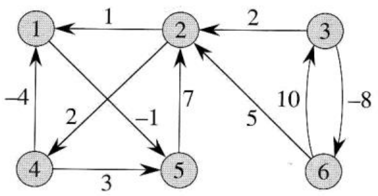
a. Show the matrix $D^{(k)}$ that results for each iteration of the outer loop.

b. List the vertices of one such shortest path from v_6 to v_1.



5. Exercises 25.3-1

Use Johnson's algorithm to find the shortest paths between all pairs of vertices in the graph of Figure 25.2. Show the values of h and $\hat{w}$ computed by the algorithm.



6. Problem 24-3 *Arbitrage*

Arbitrage is the use of discrepancies in currency exchange rates to transform one unit of a currency into more than one unit of the same currency. For example, suppose that 1 U.S. dollar buys 49 Indian rupees, 1 Indian rupee buys 2 Japanese yen, and 1 Japanese yen buys 0.0107 U.S. dollars. Then, by converting currencies, a trader can start with 1 U.S. dollar and buy $49\times2\times0.0107 = 1.0486$ U.S. dollars, thus turning a profit of 4.86 percent.

Suppose that we are given n currencies $c_1, c_2, \ldots c_n$ and an $n\times n$ table R of exchange rates, such that one unit of currency c_i buys $R[i, j]$ units of currency c_j .

a. Give an efficient algorithm to determine whether or not there exists a sequence of currencies $\langle c_{i1}, c_{i2}, \ldots, c_{ik}\rangle$ such that $R[i_1, i_2]\times R[i_2, i_3]\times\ldots\times R[i_{k-1}, i_k]\times R[i_k, i_1] > 1$. Analyze the running time of your algorithm.

b. Give an efficient algorithm to print out such a sequence if one exists. Analyze the running time of your algorithm.

7. Problem 25-1: Transitive closure of a dynamic graph

Suppose that we wish to maintain the transitive closure of a directed graph $G= (V, E)$ as we insert edges into E. That is, after each edge has been inserted, we want to update the transitive closure of the edges inserted so far. Assume that the graph G has no edges initially and that we represent the transitive closure as a Boolean matrix.

a. Show how to update the transitive closure $G^{*}= (V, E^{*})$ of a graph $G= (V, E)$ in $O(V^2)$ time when a new edge is added to G.

b. Give an example of a graph G and an edge e such that $\Omega(V^2)$ time is required to update the transitive closure after the insertion of e into G, no matter what algorithm is used.

c. Describe an efficient algorithm for updating the transitive closure as edges are inserted into the graph. For any sequence of n insertions, your algorithm should run in total time $\sum_{i=1}^{n} t_i = O(V^3)$, where t_i is the time to update the transitive closure upon inserting the ith edge. Prove that your algorithm attains this time bound.