

1. Exercises 15.3-5

Suppose that in the rod-cutting problem of Section 15.1, we also had limit l_i on the number of pieces of length i that we are allowed to produce, for $i = 1, 2, \dots, n$. Show that the optimal-substructure property described in Section 15.1 no longer holds.

2. Exercises 15.5-2

Determine the cost and structure of an optimal binary search tree for a set of $n = 7$ keys with the following probabilities:

i	0	1	2	3	4	5	6	7
p_i		0.04	0.06	0.08	0.02	0.10	0.12	0.14
q_i	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.05

3. Problems 15-5: Edit distance

Given two sequences $x[1..m]$ and $y[1..n]$ and set of transformation-operation costs, the edit distance from x to y is the cost of the least expensive operation sequence that transforms x to y . Describe a dynamic-programming algorithm that finds the edit distance from $x[1..m]$ to $y[1..n]$ and prints an optimal operation sequence. Analyze the running time and space requirements of your algorithm.

- transformation-operations
 - **Insert** a character into x
 - **Delete** a character from x
 - **Replace** a character from x by another character
 - **Twiddle** two adjacent characters from x

4. Find out all LCS of $\langle \text{BADBCBA} \rangle$ and $\langle \text{ABACDBC} \rangle$

5. An algorithm to solve the LCS problem of two strings X and Y has space complexity $O(|X| |Y|)$ typically.

- a. Design an algorithm to find the length of LCS of two string X and Y just using only $2 \cdot \min(|X|, |Y|)$ cells for working space.
- b. Design an algorithm to find the length of LCS of two string X and Y just using only $1 + \min(|X|, |Y|)$ cells for working space.

6. String Alignment

Let σ be an alphabet set, β denote the blank character in σ , and a measure function $F: \sigma \times \sigma \rightarrow \mathbb{R}$. Where F is defined as followings, for any x

and y in σ , $F(x, y) < 0$ if $x \neq y$ and $F(x, y) > 0$ if $x = y$; whereas $F(\beta, \beta) = -\infty$. Given X and Y be two strings of σ^* , let X' and Y' denote two new strings made by inserting some β into X and Y respectively. The similarity of X and Y is defined by measuring the maximal value of $\sum_{a_i \in X, b_i \in Y} F(a_i, b_i)$ among all possible X' and Y' .

- a. Design an algorithm to find the similarity of X and Y .
- b. Design an algorithm that describe where the blank characters are inserted to get the similarity.

7. Exercises 15.4-2

Give pseudocode to reconstruct an LCS from the completed c table and the original sequences $X = \langle x_1, x_2, \dots, x_m \rangle$ and $Y = \langle y_1, y_2, \dots, y_n \rangle$ in $O(m + n)$ time, without using the b table.