Algorithm HW6

1. **Exercises 15.3-5**

   Suppose that in the rod-cutting problem of Section 15.1, we also had limit l_i on the number of pieces of length i that we are allowed to produce, for i = 1, 2, …, n. Design a DP algorithm for this problem.

2. **Exercises 16.1-3**

   Not just any greedy approach to the activity-selection problem produces a maximum-size set of mutually compatible activities. Give an example to show that the approach of selecting the activity of least duration from among those that are compatible with previously selected activities does not work. Do the same for the approaches of always selecting the compatible activity that overlaps the fewest other remaining activities and always selecting the compatible remaining activity with the earliest start time.

3. **Exercises 16.1-5**

   Consider a modification to the activity-selection problem in which each activity a_i has, in addition to a start and finish time, a value v_i. The objective is no longer to maximum the number of activities scheduled, but instead to maximize the total value of the activities scheduled. That is, we wish to choose a set A of compatible activities such that $\sum_{a_k \in A} v_k$ is maximized. Give a polynomial-time algorithm for this problem. (Hint: refer to Exercise 16.1-1)

4. **A variation from Exercises 16.2-2**

   Given a 0-1 knapsack problem with the knapsack size K and n items, where each item has its weight in integer and its value in real.

   a. Design an algorithm to find the most valuable load of the items that fit into the knapsack.

   b. Design a pseudo-polynomial time algorithm to determine the optimal solution that the total weight **exactly equals** to K.

5. Exercises 16.2-4

Professor Gekko has always dreamed of inline skating across North Dakota. He plans to cross the state on highway U.S. 2, which runs from Grand Forks, on the eastern border with Minnesota, to Williston, near the western border with Montana. The professor can carry two liters of water, and he can skate m miles before running out of water. (Because North Dakota is relatively flat, the professor does not have to worry about drinking water at a greater rate on uphill sections than on flat or downhill sections.) The professor will start in Grand Forks with two full liters of water. His official North Dakota state map shows all the places along U.S. 2 at which he can refill his water and the distances between these locations.

The professor's goal is to minimize the number of water stops along his route across the state. Give an efficient method by which he can determine which water stops he should make. Prove that your strategy yields an optimal solution, and give its running time.

6. Exercises 16.2-6

Show how to solve the fractional knapsack problem in O(n) time.

7. Exercises 16.3-7

Generalize Huffman's algorithm to ternary codewords (i.e., codewords using the symbols 0, 1, and 2), and prove that it yields optimal ternary codes.

8. EXT 6-4

Find the Huffman codes of the data given below by drawing the tree like the figure 16.5 in the page 432. You should write down each step of the Huffman's algorithm.

a:22 b:2 c:5 d:3 e:5 f:8 g:7 h:11