# Algorithm HW13

**1. Exercises 26.1-7**

Suppose that, in addition to edge capacities, a flow network has **vertex capacities**. That is each vertex $v$ has a limit $l(v)$ on how much flow can pass through $v$. Show how to transform a flow network $G = (V, E)$ with vertex capacities into an equivalent flow network $G' = (V', E')$ without vertex capacities, such that a maximum flow in $G'$ has the same value as a maximum flow in $G$. How many vertices and edges does $G'$ have?

**2. Exercises 26.2-3**

Show the execution of the Edmonds-Karp algorithm on the flow network of Figure 26.1(a).

**3. Exercises 26.2-11**

The ***edge connectivity*** of an undirected graph is the minimum number $k$ of edges that must be removed to disconnect the graph. For example, the edge connectivity of a tree is 1, and the edge connectivity of a cyclic chain of vertices is 2. Show how to determine the edge connectivity of an undirected graph $G = (V, E)$ by running a maximum-flow algorithm on at most $|V|$ flow networks, each having $O(V)$ vertices and $O(E)$ edges.

**4. Exercises 26.2-13**

Suppose that you wish to find, among all minimum cuts in a flow network $G$ with integral capacities, one that contains the smallest number of edges. Show how to modify the capacities of $G$ to create a new flow network $G'$ in which any minimum cut in $G'$ is a minimum cut with the smallest number of edges in $G$.

**5.**

The vertex connectivity $\kappa(G)$ of a graph $G$ is the minimum size of a vertex cut, i.e., a vertex subset $S \subseteq V(G)$ such that $G - S$ is disconnected or has only one vertex.For example, the vertex connectivity of a tree is 1, and the vertex connectivity of a cyclic chain of vertices is 2. Show how to determine the vertex connectivity of an undirected graph $G = (V, E)$ by running a maximum-flow algorithm on at most $|V|$ flow networks, each having O($V$) vertices and O($E$) edges.

**6. EXT 10-1**

There are two extended ways used to find the augmenting path that we have mentioned in class (refers to slides p.14, Unit 10), please design an efficient algorithm with the argument of second method to find the augmenting path. Argue that your algorithm is correct and also analyze the time-complexity.

**7. Exercise 26.2-12**

Suppose that you are given a flow network $G$, and $G$ has edges entering the source $s$. Let $f$ be a flow in $G$ in which one of the edges $(v, s)$ entering the source has $f(v, s)=1$. Prove that there must exist another flow $f'$ with $f'(v, s) = 0$ such that $|f|=|f'|$. Give an $O(E)$-time algorithm to compute $f'$, given $f$ , and assuming that all edge capacities are integers.