

2.

$$\rho(r) = 2 \sum_{i=1}^{N/2} |\psi_i(r)|^2$$

$$= 2 \sum_{i=1}^{N/2} \psi_i^*(r) \psi_i(r)$$

Integrating both sides w/ rsl \int :

$$\int dr \rho(r) = 2 \int \sum_{i=1}^{N/2} \psi_i^*(r) \psi_i(r) dr$$

$$\because \psi_i(r) = \sum_{\mu=1}^K c_{\mu i} \phi_{\mu}(r)$$

$$\text{then } \Rightarrow \int dr \rho(r) = 2 \int \sum_{i=1}^{N/2} \sum_{\mu=1}^K c_{\mu i}^* c_{\mu i} \phi_{\mu}^* \phi_{\mu}$$

$$= \int 2 \sum_{\mu=1}^K \sum_{i=1}^{N/2} c_{\mu i}^* c_{\mu i} \phi_{\mu}^* \phi_{\mu}$$

$$= \int \sum_{\mu=1}^K P_{\mu\mu} \phi_{\mu}^* \phi_{\mu}$$

$$= \sum_{\mu=1}^K P_{\mu\mu} S_{\mu\mu}$$

$$= \text{tr}(P \underline{S}).$$

$$P_{\mu\nu} = 2 \sum_{i=1}^{N/2} c_{\mu i} c_{\nu i}$$

$$S_{\mu\nu} = \int dr \phi_{\mu}(r) \phi_{\nu}(r)$$