

# ALGORITHM PROBLEMS FOR DUMMIES: PETR MITRICHEV'S BLOG

MONDAY, MAY 27, 2013

## Fenwick tree range updates

A *Fenwick tree* is a wonderful data structure that supports two operations on an array: increment a given value by a given amount, and find the sum of a segment of values, both in  $O(\log n)$  time. What's more, the Fenwick tree is represented as just an array of the same size as the array being updated and queried, and we don't need to store the original array itself! In other words, we can support the above operations without any additional memory. Yesterday I've discovered that it's capable of even more!

For a start, here is the entire code for a Fenwick tree:

```
private void update(int at, int by) {
    while (at < data.length) {
        data[at] += by;
        at |= (at + 1);
    }
}

private int query(int at) {
    int res = 0;
    while (at >= 0) {
        res += data[at];
        at = (at & (at + 1)) - 1;
    }
    return res;
}
```

Here, `query(at)` returns the sum of all elements from the first element to `at`-th element. Finding a sum of arbitrary segment can be done via `query(right)-query(left-1)`. You can find more details using your favorite search engine.

The standard Fenwick tree only supports updating one element at a time.

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However, it is quite natural to expect it to handle range updates, too - the update command now becomes "increment each value between left-th and right-th by the given amount". It turns out that we can modify the tree to handle such updates in  $O(\log n)$  time, too! Here's how (warning, code untested - but I hope it works):

```
private void update(int left, int right, int by) {
    internalUpdate(left, by, -by * (left - 1));
    internalUpdate(right, -by, by * right);
}

private void internalUpdate(int at, int mul, int add) {
    while (at < dataMul.length) {
        dataMul[at] += mul;
        dataAdd[at] += add;
        at |= (at + 1);
    }
}

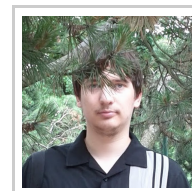
private int query(int at) {
    int mul = 0;
    int add = 0;
    int start = at;
    while (at >= 0) {
        mul += dataMul[at];
        add += dataAdd[at];
        at = (at & (at + 1)) - 1;
    }
    return mul * start + add;
}
```

In other words, we implement a Fenwick tree with range updates via a normal (point-update) Fenwick tree that stores linear functions instead of just values. Is this a well-known trick?

POSTED BY PETR MITRICHEV AT 11:51 PM

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**karthik tunga** · 8 months ago

Very cool. Topcoder also has a nice tutorial on this <http://>



**Petr Mitrichev** · 8 months ago +1

+**Robert King** What you describe is adding a linear function  
a quadratic function, so It is possible to do by maintaining



**Robert King** · 8 months ago · Shared publicly

I've implemented this in python here: <https://gist.github.com/rob>

+**Niyaz Nigmatullin** I used "right - 1" and it seemed to work for the

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**Andrias Meisyal** · 2 months ago · Shared publicly

Hi, Petr, by the way thank you for this post. I can understand abo  
I love your blog, I have bookmarked your blog. Follow your updat

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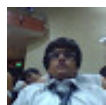


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<http://arxiv.org/abs/1311.6093>


This paper explains the algorithm for all dimensions.

 · Reply**Andrey Naumenko** 8 months ago - Shared publicly

One more way is here <http://habrahabr.ru/post/170933/>  
The code there is longer, but the the idea is a little bit simpler

 · Reply**Niyaz Nigmatullin** 8 months ago - Shared publicly

Shoudn't it be "internalUpdate(right, -by, by \* (right - 1));" instead c

 · Reply**Petr Mitrichev** 8 months ago +1

It depends on the meaning of right, if I'm not missing some inclusive, too, and I believe my code works for that case :)

**Niyaz Nigmatullin** 8 months ago

Oh, at first I thought that in case of inclusiveness it should matter.

**nitish bhagat** 6 months ago - Shared publicly

thanks for such easy explanation

 · Reply**Tigran Hakobyan** 6 months ago - Shared publicly

Hi, Petr. Could you please describe idea which you used for imple

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**Felix Halim** 8 months ago - Shared publicly

<http://apps.topcoder.com/forums/?module=Thread&threadID=71>

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**Riatre Foo** 8 months ago - Shared publicly

This is a well-known trick among Chinese competitive programm

And for those who want to test their codes, try <http://poj.org/prob>

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**Robin Lee** 8 months ago (edited)Another one here: <http://www.spoj.com/problems/PYRSU>**John Smith** 7 months ago

The first link seems dead.

**Leandro Coutinho** 8 months ago - Shared publicly

Hi Petr!

Could you please explain the codes:

1)  $at |= (at + 1);$ 2)  $at = (at \& (at + 1)) - 1;$ 

+5

1

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**Shubham Agarwal** 4 months ago (edited) - Shared publicly

plz help in extending it to 2-d array

<http://www.spoj.com/problems/USUBQSUB/>

want to solve this prob using Fenwick tree

+7

1

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**Surya Kiran** 8 months ago (edited) - Shared publicly

Hey Here is a one problem on fenwick trees which requires som

problem Link : <https://docs.google.com/file/d/10e9mX9vzay7Zs>Editorial Link : <https://docs.google.com/file/d/1rzDKzH8lxKoA2C>

Problem Name : Lets Destroy Them.

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**Nitish Sharma** 6 months ago

he just 'hopes' that it work :p

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**Lei Zhang** via Google+ 7 months ago - Shared publicly

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