Project 1: Martingale

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***Abstract*—** In this project, we analyze the results of a betting strategy on an American Roulette wheel. Various experiments were conducted through a Python program simulation, with thousands of attempts documented. We demonstrate through each experiment how the results of the betting strategy changes with varying factors such as limited and unlimited bankroll.

1 EXPERIMENT ONE

In Experiment one, we perform a Monte Carlo simulation on the betting strategy outlined by Professor Balch. We define one thousand spins of the roulette wheel as one episode. We use a probability of winning of his betting strategy on an American Roulette wheel of 47%, which is roughly the estimated probability of winning when betting on black. Below shows various simulations based on an unlimited bankroll.

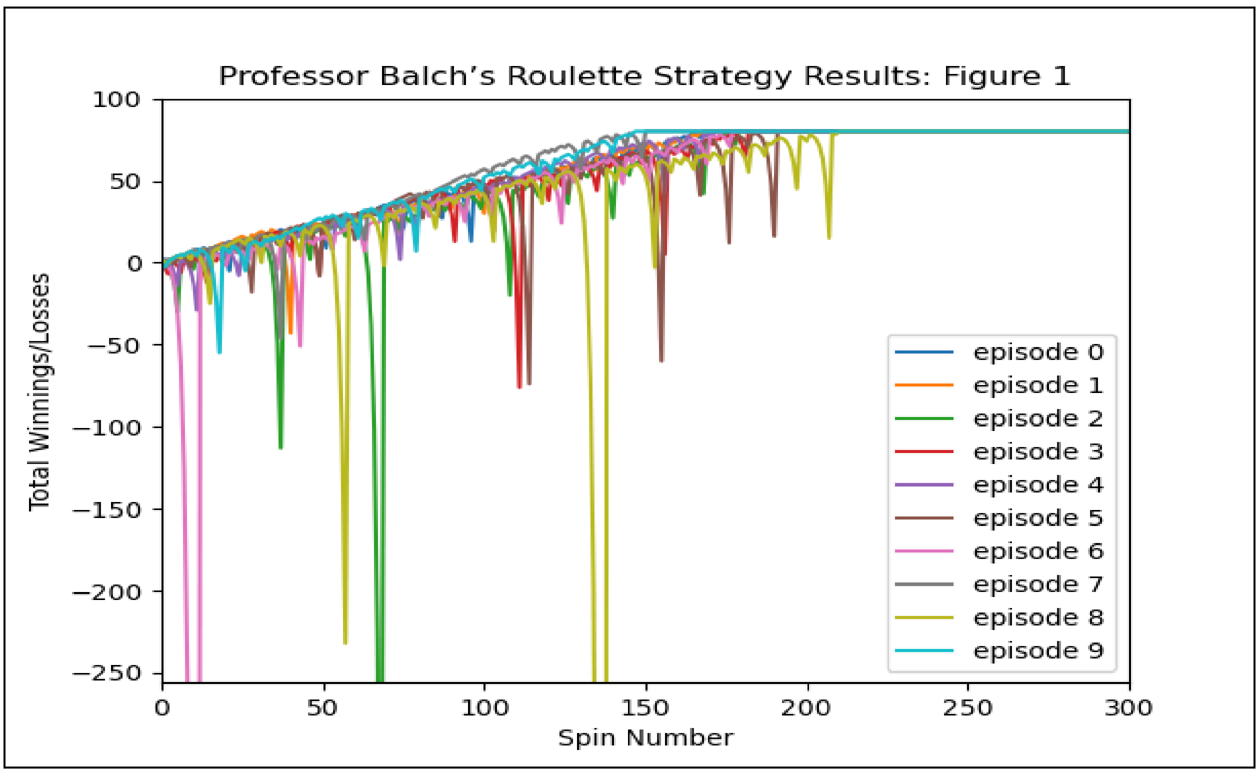

***Figure 1*—**Ten episodes randomly simulated with an unlimited bankroll

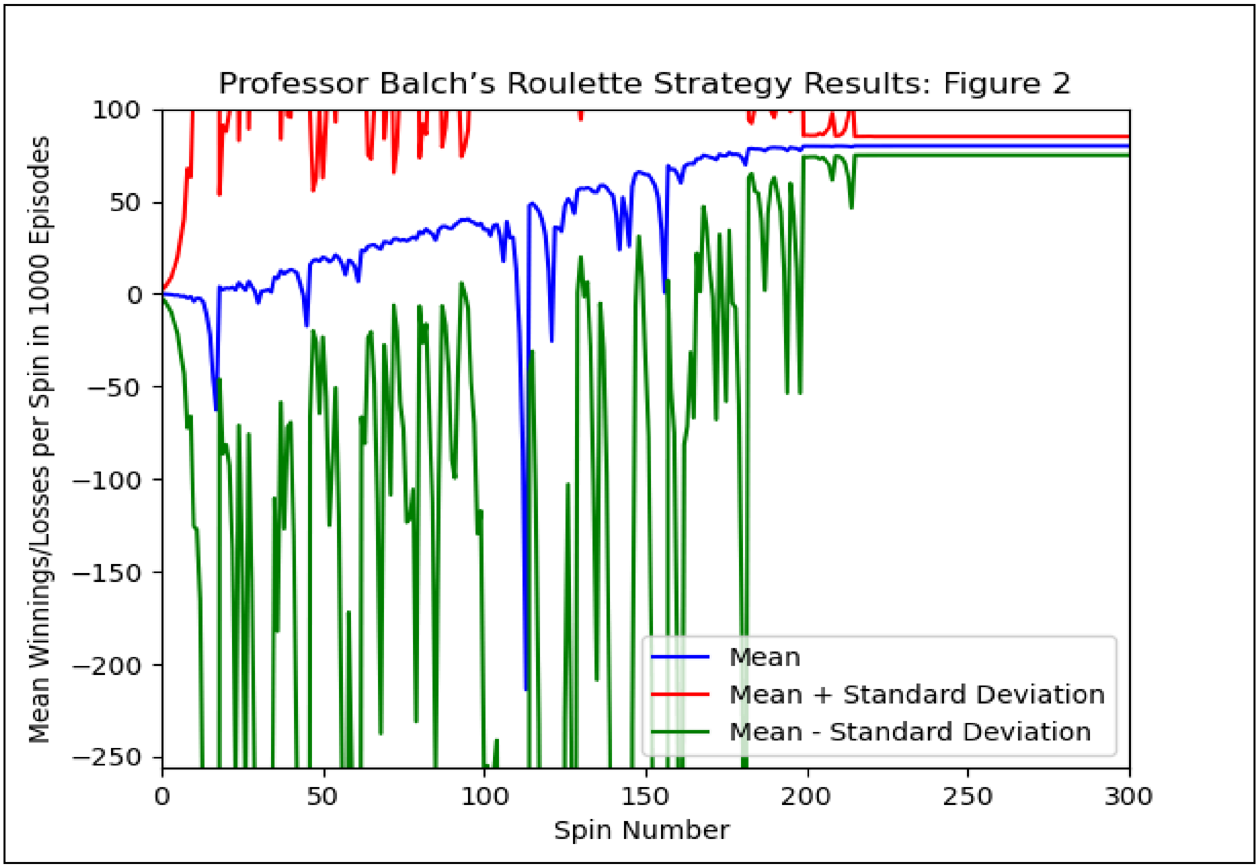

Figure 2—One thousand episodes randomly simulated, with each corresponding spin numbers’ mean calculated and plotted. Standard deviation is also compared to the mean.

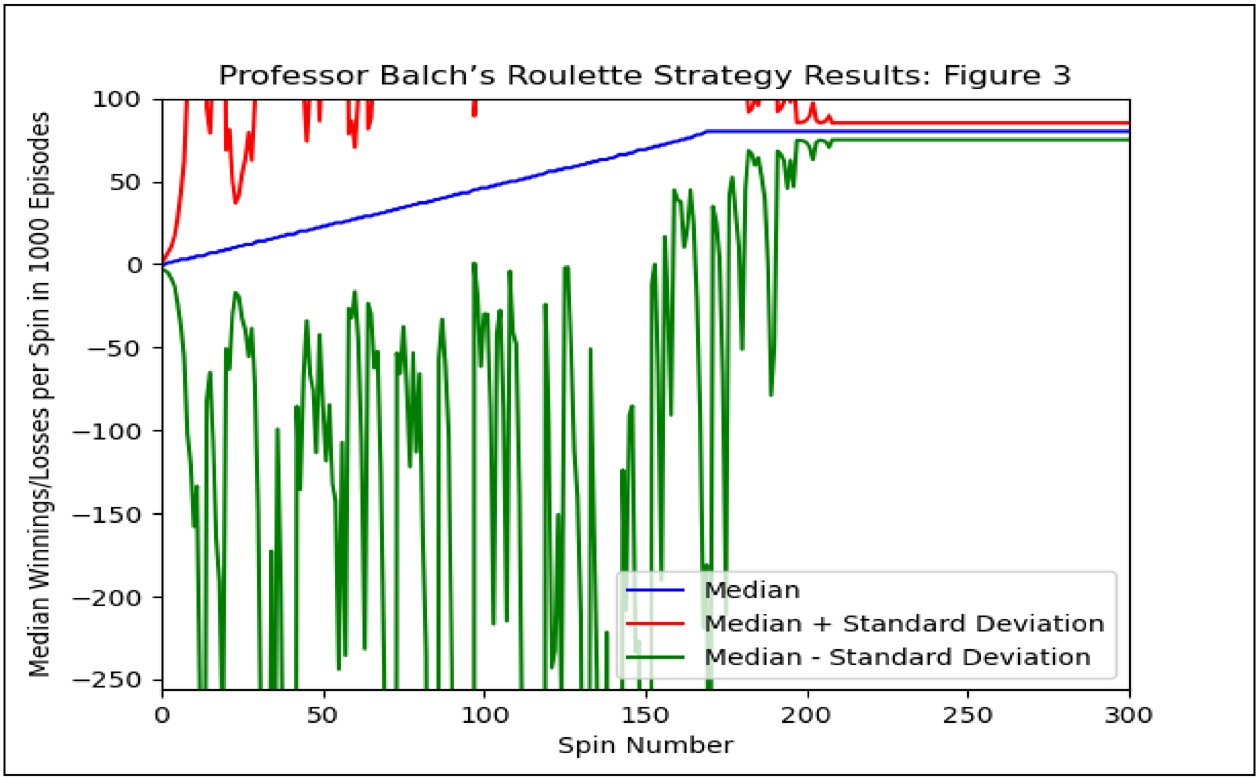

Figure 3—One thousand episodes randomly simulated, with each corresponding spin numbers' median calculated and plotted. Standard deviation is also compared to the median.

1.1 Question Set 1 Response

Our experiment results have shown that there is a 100% chance of winning $80 within 1000 successive bets for this betting strategy with unlimited bankroll. This is calculated as:

$$P(Winning\ \$80) = \frac{Number\ of\ times\ \$80\ is\ won}{1000\ Spins} = \frac{1000}{1000} = 1 = 100\%$$

The Monte Carlo simulation has returned 1000 episodes where the strategy has won $80. This is demonstrated on a smaller scale in Figure 1, where all ten episodes have successfully reached $80 in winnings.

1.2 Question Set 2 Response

We can calculate the expected value of Experiment 1, with one thousand episodes as:

$$E[X] \ = X \cdot P(X) \ = \ \$80 \ \cdot 1 \ = \ \$80$$

Since we already know the probability to be 100%, the expected value of winnings is $80 after one thousand episodes.

1.3 Question Set 3 Response

In Figure 2, The standard deviation lines both reach a maximum value and stabilize in parallel with the mean. This demonstrates that the mean for each spin in the one thousand episodes have reached a winning amount of $80. As shown in figure 2, the standard deviations do converge as the number of spins increase. Since over time, there will be episodes that have reached the $80 winnings, the mean will stabilize around the $80 mark and the standard deviations will have less velocity and begin to converge on the mean line.

2 EXPERIMENT TWO

2.1 Question Set 4 Response

In this case, we now have a bankroll of $256. Once the bankroll has reached zero, we are unable to continue betting. From our results in running the Monte Carlo Simulation in experiment 2, we had a success rate of 63.1%. Success is defined as an Episode reaching a winning amount of $80 within the success one thousand spins. This probability was calculated as below:

$$P(Winning\ \$80) \ = \ \frac{Number\ of\ times\ \$80\ is\ won}{1000\ Spins} \ = \ \frac{631}{1000} \ = \ 0.631 \ = \ 63.1\%$$

2.2 Question Set 5 Response

We can calculate the expected value of Experiment 2, with one thousand episodes and a limited bankroll as:

$$E[X] \ = X_1 P(X_1) \ + X_2 P(X_2) \ = \ \$80 \ \cdot 0.631 \ + \ (-\ \$256) \cdot 0.369 = \ -\ \$43.98$$

We have two outcomes for the strategy, which are winning $80 and losing the bankroll, -$256. They represent X1 and X2, respectively. The probability as previously calculated for X1 is 63.1%, which makes the probability of losing the bankroll: 1 - .631 = 0.369 or 36.9%. These represent P1 and P2, respectively. By summing the expected values of playing this strategy, a better would expect to lose $43.98. This is also outlined in Figures 4 and 5 below, where the mean of one thousand episodes levels out towards -$43.98

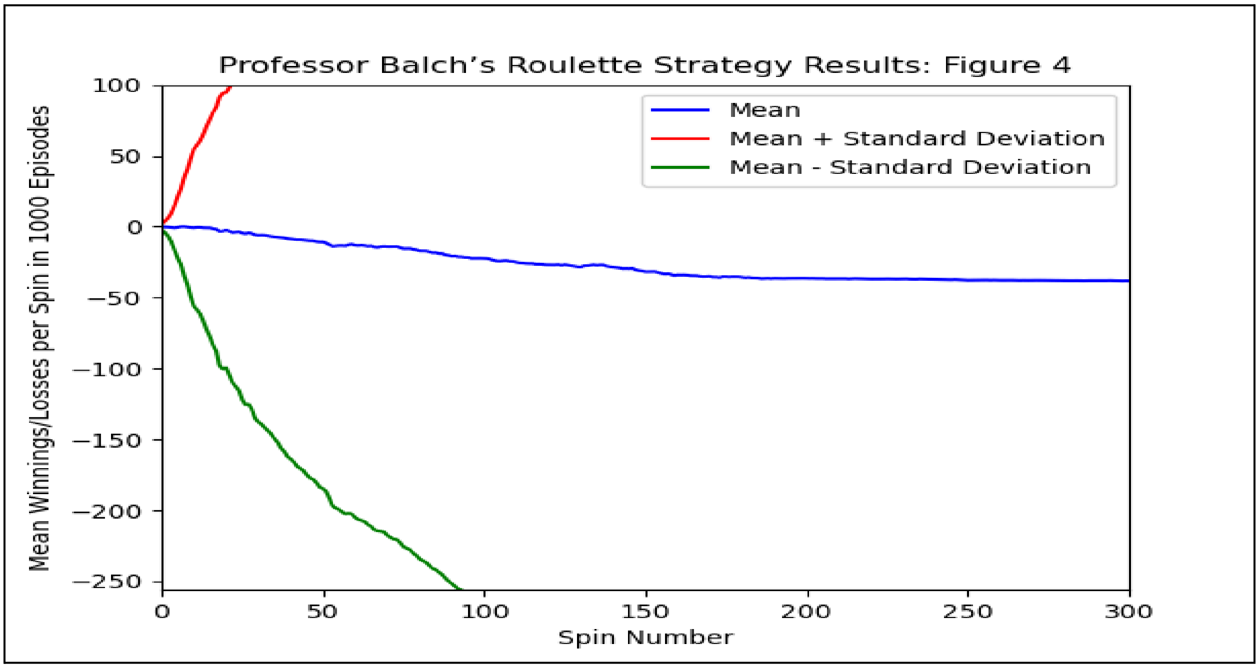

Figure 4—One thousand episodes randomly simulated, with each corresponding spin numbers' mean calculated and plotted. Bankroll of $256.

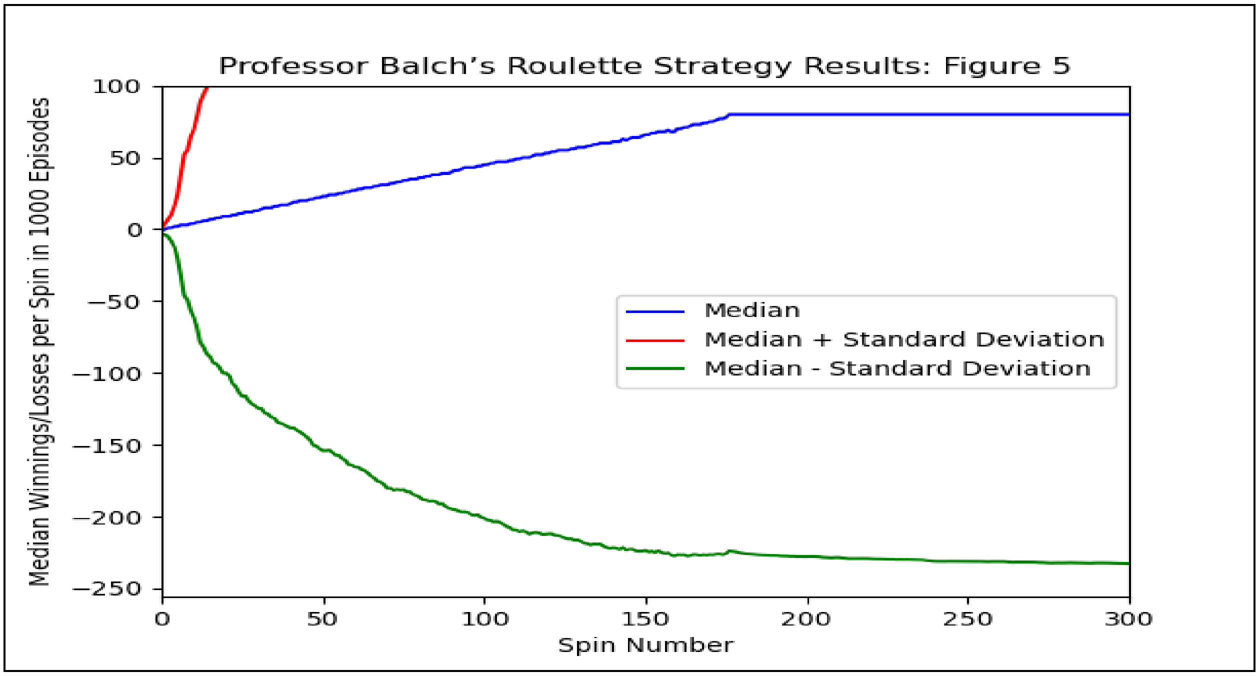

Figure 5—One thousand episodes randomly simulated, with each corresponding spin numbers' median calculated and plotted. Bankroll of $256.

2.3 Question Set 6 Response

In Figure 4, The standard deviation values are widely larger than the mean, cutting out from the graph. The standard deviations do not appear to stabilize or converge at all. This shows that the values of each spin were much different in each episode. Since we now have a bankroll and that failing the betting strategy is an option, the probability of winning $80 in an episode differs wildly. They all average out to the -$43 mark, however.

2.4 Question Set 7 Response

The benefits of using expected value over one single instance is numerous. The expected value can tell you what outcome you should expect from a particular scenario. If the scenario is applied 10 times or 1000 times, you will get the expected value times the number of times the scenario is applied, helping you to decide how many times to repeat the scenario. A single instance will mislead you into thinking what the real outcome will become. While it can be different from the expected value, over many iterations will the expected value become more true than a single instance.

3 REFERENCES

1. Murphy, K. P. (2022). *Probabilistic machine learning: An introduction*. The MIT Press.
2. Wikimedia Foundation. (2024, August 23). *Roulette*. Wikipedia. https://en.wikipedia.org/wiki/Roulette