

Learning to Solve PDEs on Neural Shape Representations

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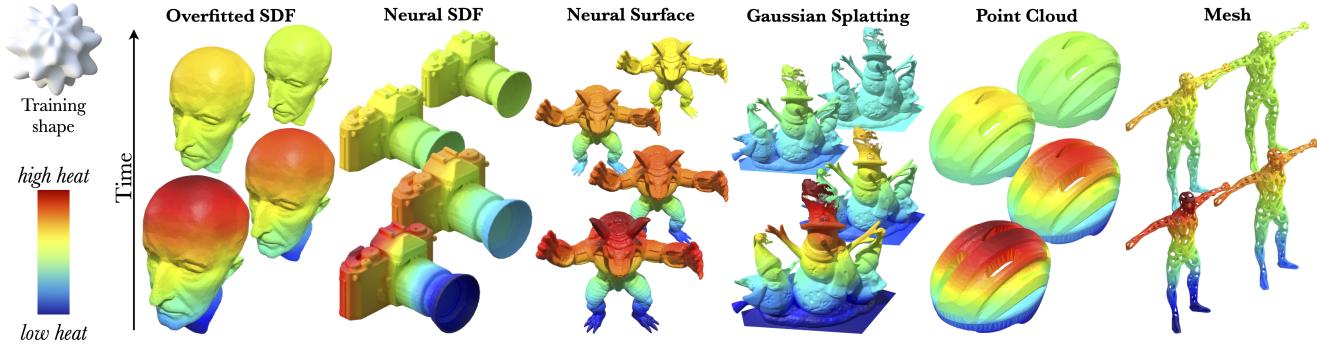


Figure 1. Our mesh-free, geometry-conditioned learned operator solves surface PDEs *directly in the neural domain* on multiple modalities, without mesh extraction, or per-instance optimization. Trained once on a single exemplar (SPIKE, top-left), the learned operator generalizes across unseen geometries, topologies, and input modalities. See supplemental for heat flow videos and also Poisson solves.

Abstract

001 Solving partial differential equations (PDEs) on shapes underpins many shape analysis and engineering tasks; yet, pre-
 002 vailing PDE solvers operate on polygonal/triangle meshes
 003 while modern 3D assets increasingly live as neural represen-
 004 tations. This mismatch leaves no suitable method to solve
 005 surface PDEs directly within the neural domain, forcing
 006 explicit mesh extraction or per-instance residual training,
 007 preventing end-to-end workflows. We present a novel, mesh-
 008 free formulation that learns a local update operator condi-
 009 tioned on neural (local) shape attributes, enabling surface
 010 PDEs to be solved directly where the (neural) data lives.
 011 The operator integrates naturally with prevalent neural sur-
 012 face representations, is trained once on a single represen-
 013 tative shape, and generalizes across shape and topology
 014 variations, enabling accurate, fast inference without explicit
 015 meshing or per-instance optimization while preserving dif-
 016 ferentiability. Across analytic benchmarks (heat equation
 017 and Poisson solve on sphere) and real neural assets across
 018 different representations, our method slightly outperforms
 019 CPM while remaining reasonably close to FEM, and, to
 020 our knowledge, delivers the first end-to-end pipeline that
 021 solves surface PDEs on both neural and classical surface
 022 representations. Code will be released on acceptance.
 023

1. Introduction

024 Solving partial differential equations (PDEs) on surfaces is
 025 central to geometry processing, shape analysis, and many engi-
 026 neering tasks; examples include heat flow on surfaces, Pois-
 027 son equation, and harmonic interpolation. Classical FEM-
 028 based solvers [11, 42] operate on (discretized) polygonal or
 029 triangle meshes, with well-understood accuracy and stability
 030 behavior. However, they cannot directly handle contempo-
 031 rary 3D assets that are increasingly represented as *neural*
 032 *shape representations* (e.g., point clouds or splats [36, 44],
 033 neural surfaces [29, 43], overfitted implicit shapes [15, 40],
 034 neural implicit fields [32, 44]). These representations are
 035 popular as they are differentiable, often topology-agnostic,
 036 and integrate naturally with modern learning and generative
 037 systems. This creates a mismatch: *mesh-centric PDE solvers*
 038 *do not operate in the domain where neural data lives*.
 039

040 Handling surface PDEs *directly in the neural shape do-*
 041 *main* removes mesh extraction, preserves end-to-end differ-
 042 entiability (crucial for inverse problems and PDE priors), and
 043 handles topology changes without intermediate re-meshing
 044 or reparameterization, while avoiding round-trip errors and
 045 engineering overhead. However, current workarounds ei-
 046 ther *extract a mesh* (e.g., marching cubes [23], dual con-
 047 tourning [14] and their neural variants [8, 9]) and shuttle
 048 results back, hindering differentiable pipelines; or *rely on*
 049 *per-instance residual training* (e.g., surface PINNs [12]),

050 which generalizes poorly across shape variation.

051 We introduce a geometry-aware neural PDE solver that
052 targets the surface-structure component of PDEs and operates
053 directly inside neural representations while retaining
054 classical-solver fidelity and applying equally to classical sur-
055 faces. Inspired by the *Closest Point Method* (CPM) [39], our
056 approach learns how surface geometry governs the extension
057 of a surface field into a *narrow band*—the core operation
058 of embedding-based solvers—allowing PDEs to be solved
059 directly on their surface representations. A lightweight neural
060 operator captures local geometric context (e.g., normals and
061 principal curvature directions) and produces this extension.
062 Trained once on a single shape, it generalizes across
063 unseen geometries, modalities, and topologies, requires no
064 meshing or per-instance optimization, and remains fully dif-
065 ferentiable, making it suitable as a new neural PDE layer
066 into existing neural training setups.

067 We demonstrate our neural PDE operator on different
068 popular representations: neural surfaces, spherical neural
069 surfaces [43], point clouds [16], overfitted occupancy
070 fields [26], Gaussian splatting [16], as well as deep implicit
071 fields [32]. We validate our results in two ways: (i) On
072 spheres, we evaluate the heat and Poisson equations against
073 analytical ground truth, and compare with both FEM and
074 CPM [11, 39], to assess accuracy. (ii) For general shapes
075 and modalities, we use dense-mesh FEM as reference to
076 assess generalization and robustness. Our solver achieves
077 competitive accuracy with zero meshing overhead and no
078 extend-restrict shuttling, as required by CPM. Most impor-
079 tantly, we find that our method, once trained, generalizes
080 surprisingly well across surface variations, topology, and
081 meshing changes. As shown in Figure 1, a model trained on
082 a single shape (the SPIKE) generalizes across diverse shapes
083 and neural representations. We also ablate design choices
084 and hyperparameters.

085 In summary, our main contributions are:

- Introducing a novel mesh-free, end-to-end differentiable solver for surface PDEs operating directly on both neural and classical surface representations.
- A lightweight, shape-conditioned network trained on a single shape that implicitly learns the narrow-band extension without per-shape optimization.
- Extensive evaluation showing generalization across unseen shapes, topologies, and representations, with competitive accuracy and speed on heat and Poisson equation.

095 2. Related Work

096 **Classical and mesh-based methods.** In Euclidean do-
097 mains, PDEs are classically discretized by finite differences
098 and Galerkin finite elements [25, 42]. Extending finite dif-
099 ferences to curved manifolds typically requires embedding
100 strategies, whereas Galerkin methods naturally generalize
101 to arbitrary geometries via mesh-based formulations. On

102 surfaces, *surface finite elements* (SFEM) discretize the man-
103 ifold and apply intrinsic schemes on a triangulation, offer-
104 ing strong accuracy and stability guarantees under standard
105 regularity and shape-regular mesh assumptions; see the sur-
106 vey [11]. Discrete differential geometry operators (e.g., the
107 cotangent Laplacian) are also widely used for geometry pro-
108 cessing and harmonic problems on meshes [10, 27]. The
109 main limitations are geometric and practical: performance
110 hinges on mesh quality, evolving or noisy geometries often
111 require (re)meshing, and distortion/tangling can degrade
112 conditioning, accuracy, and robustness. The main restriction
113 being that such methods cannot directly be applied to current
114 neural representations, without meshing.

115 **Embedding and unfitted methods.** Embedding methods
116 solve surface PDEs in the ambient domain while enforc-
117 ing surface constraints. We build on the *Closest Point*
118 *Method* (CPM), which alternates extension and Cartesian
119 updates on a narrow band and is valued for simplicity
120 and robustness [39]; accuracy/flexibility have been boosted
121 with high-order and meshfree RBF-FD stencils and least-
122 squares implicit variants, including moving surfaces [33, 34].
123 Stochastic *Projected Walk on Spheres* offers discretization-
124 free Monte Carlo solutions via repeated manifold projec-
125 tions [41], and CPM has been adapted to interior bound-
126aries [17]. Unfitted FEM avoids explicit surface meshes
127 by solving on a background grid: *CutFEM* stabilizes cut
128 cells with ghost penalties [2, 3], while *TraceFEM* restricts
129 spaces to an implicit level set and extends to evolving in-
130 terfaces [20, 30]. Despite reduced meshing effort, these ap-
131 proaches still shuttle information between surface and grid,
132 introducing overhead and potential bias, especially with im-
133 plicit surfaces. In contrast, our method performs *grid-to-grid*
134 updates without extend-restrict loops.

135 **Learning-based solvers.** Physics-Informed Neural Net-
136 works (PINNs) [38] impose PDE residuals and bound-
137 ary terms in the training loss, enabling mesh-free for-
138 ward/inverse solves but typically requiring *per-instance* opti-
139 mization; they are sensitive to stiffness, boundary enforce-
140 ment, residual weighting, and training stability at scale. Sur-
141 face extensions (e.g., [12]) demonstrate feasibility on man-
142 ifolds without meshing yet inherit the same optimization
143 and runtime burdens. A complementary direction, *neural*
144 *operator* (DeepONet, FNO) [18, 21, 24] and manifold vari-
145 ants [5, 35], amortizes solution maps across problem families
146 but generally relies on supervision from classical solvers,
147 assumes fixed discretizations/charts, and does not natively
148 target neural implicit geometry. Closer to our goals, *implicit*
149 *neural spatial representations* treat an implicit neural rep-
150 resentation (INR) as the spatial discretization and evolve
151 its weights over time to solve time-dependent PDEs [6, 7].
152 These methods show strong accuracy–memory trade-offs but
153 still operate via global weight evolution and per-problem
154 time integration, and cannot be directly used to unseen

155 shapes. In contrast, we learn a *local, geometry-conditioned*
156 *update operator* that works directly in a *narrow band* around
157 the surface. It takes geometric cues from diverse neural
158 shape representations and performs a single forward update,
159 avoiding per-instance training and mesh dependencies while
160 retaining solver-level accuracy.

161 **Neural shape representations.** Modern 3D pipelines
162 increasingly favor neural implicit/explicit representations over
163 traditional meshes. Point clouds (e.g., PointNet/PointNet++
164 or splats [16]) provide a mesh-free sampling interface but
165 lack continuity and differential structure by default [36, 37].
166 Neural *implicit* fields capture geometry as continuous functions:
167 signed distance fields (DeepSDF [32]) and occupancy
168 networks [26] model surfaces at effectively infinite resolution
169 and are widely used for reconstruction and analysis.
170 Overfitted implicit neural representations, such as
171 SIREN [40], fit a single shape/scene as a coordinate MLP
172 and expose smooth values and derivatives. For genus-0 surfaces,
173 spherical neural surfaces map \mathbb{S}^2 to embedded shapes
174 and expose intrinsic operators without meshing [43]. Scene
175 appearance and volume are commonly modeled by neural
176 radiance fields (NeRF) [28], with real-time explicit variants
177 via 3D Gaussian splatting [16]. Triplane feature layouts (e.g.,
178 EG3D [4]) factor 3D into three orthogonal 2D feature planes
179 that are both expressive and efficient for reconstruction and
180 generation. Finally, recent latent encodings for neural fields,
181 such as 3DShape2VecSet [44], represent shapes as sets of
182 vectors tailored for generative modeling and downstream
183 learning. These representations are differentiable and often
184 topology-agnostic, making them suitable for our PDE solver
185 that operates without mesh extraction (see Section 4).

186 3. Method

187 We propose a representation-agnostic solver that computes
188 surface PDEs on *neural surfaces*, while remaining compatible
189 with other geometric representations. Given a surface,
190 which may be neural (e.g., Spherical Neural Surface, SDF
191 or occupancy INR, overfitted implicit, or point cloud with
192 normals), we first extract local geometric context such as
193 normals and curvature tensor at sampled surface points. We then
194 build a narrow Cartesian band around the embedded surface,
195 following the principle of the *Closest Point Method* [39],
196 and reformulate the surface PDE as a volumetric one defined
197 within this neighborhood. Surface functions are extended
198 to the band through a closest point extension that enforces
199 normal constancy — the main assumption underlying
200 embedding-based solvers. *This extension is modeled*
201 *by a lightweight geometry-conditioned neural operator that*
202 *learns it implicitly.* The operator acts locally across the
203 surface, recognizing the underlying geometry from local features
204 and grid stencils to produce local band functions, which
205 are then assembled into a single global solution. This design

206 helps generalize across a wide range of shape modalities,
207 topologies, and surface functions.

208 Our training is local and data-efficient: patches from a
209 single representative shape (see SPIKE in Figure 1) suffice
210 to learn the operator. Two aspects are central to its construction:
211 (i) structuring the architecture with geometric
212 conditioning, and (ii) ensuring generalization to unseen
213 functions at test time. At inference, the method first produces
214 the extended band function by applying the local operator across
215 all patches — each acting locally but contributing to a single
216 global update of the field. The PDE is then solved directly
217 within the band, as in [39], and for time-dependent problems,
218 this process reduces to repeated global updates over time
219 steps. The approach requires no meshing or per-instance
220 optimization, remains fully differentiable, and integrates as
221 a drop-in neural PDE layer.

222 **Closest Point Method (CPM).** The original method [39]
223 embeds surface PDEs in a thin Cartesian *narrow band*
224 around the surface \mathcal{S} . The equivalent volumetric PDE
225 is then solved inside this band using standard numerical
226 methods such as finite differences (FD), Runge–Kutta (RK)
227 schemes and time integrators such as forward Euler for time-
228 dependent problems. For the solution of the volumetric PDE
229 to coincide with that of the original surface PDE when re-
230 stricted to \mathcal{S} , the surface function must be extended into the
231 band through a *closest point extension*, ensuring constancy
232 along surface normals.

233 The CPM alternates between two simple operations: (i) a
234 standard discrete volumetric *solve* in the *narrow band*, and
235 (ii) a *re-extension* step to ensure normal constancy. At the
236 core is the closest-point map cp sending a band point $x \in \mathbb{R}^3$
237 to its nearest point on the surface $cp(x)$. A surface function
238 $u_{\mathcal{S}}$ is extended to the band by $u(x) := u_{\mathcal{S}}(cp(x))$. When
239 u is (approx.) constant along normals, ambient derivatives
240 restricted to surface \mathcal{S} coincide with intrinsic ones, allowing
241 replacement of surface operators (e.g., gradient/Laplacian)
242 by standard finite-difference stencils during *solve* step.

243 However, each time step (involving a FD *solve*) generally
244 breaks normal constancy, causing the field to vary along
245 surface normals. Hence, CPM *re-extends* by overwriting the
246 band value at x with the value at $cp(x)$ (which is interpo-
247 lated from the neighbouring grid points). In practice, CPM
248 requires (i) constructing a band of width ε , (ii) efficient eval-
249 uation of closest points, and (iii) *re-extension* operation, and
250 (iv) consistent handling of boundary conditions by tagging
251 band cells whose projections lie on any boundary $\partial\mathcal{S}$. For
252 details, see the original paper [39].

253 The CPM is simple, robust, and reuses
254 off-the-shelf Cartesian solvers; however, it requires
255 *extend-solve-reextend*, using surface information, at
256 every iteration. Instead, we design a neural network that
257 replaces CPM’s *re-extension* step implicitly and naturally
258 accommodates neural shape representations.

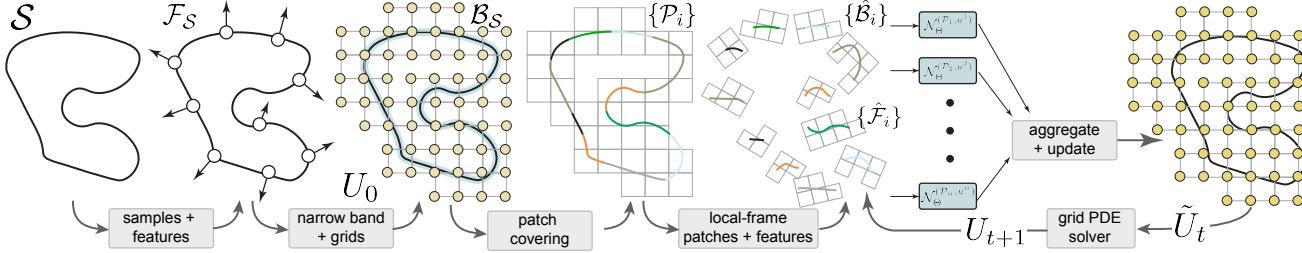


Figure 2. Pipeline overview. From a surface \mathcal{S} , we sample points and geometric features (normals, local features). Around an ε -narrow band around the shape, we gather Cartesian grids $\mathcal{B}_\mathcal{S}$ to store an initial field U_0 extended from surface values and covered by overlapping, surface-centred patches $\{\mathcal{P}_i\}$. Each patch is reoriented to its local frame, yielding $\{\hat{\mathcal{B}}_i, \hat{\mathcal{F}}_i\}$, which are processed by our lightweight geometry-conditioned operators $\{\mathcal{N}_\Theta\}$ to produce local updates (see Figure 3). The local updates are smoothly aggregated to form the global band update \tilde{U}_t and advanced with a standard grid PDE time step to get U_{t+1} . Iterating this loop yields surface PDE solutions directly in the neural domain via grid-to-grid updates, *without* mesh extraction or extend-restrict shuttling.

259

3.1. Algorithm Steps

We learn a local neural solver that operates around a *narrow band* of implicit or explicit surface and produces band updates that keep the function constant along surface normals. Our pipeline is representation-agnostic and applies to any surface that supports (i) sampling on the surface, and (ii) estimating local differential cues. In Section 4, we discuss how to estimate these for different neural shape representations.

Inputs and notation. Let $\mathcal{S} \subset \mathbb{R}^3$ be a smooth surface representation, either neural (e.g., SNS, SDF/occupancy INR) or traditional (e.g., point cloud, mesh). We denote the unit normal at $x \in \mathcal{S}$ by $\mathbf{n}(x)$, and the principal curvature directions by $(\mathbf{t}_1(x), \mathbf{t}_2(x))$, with \mathbf{t}_1 aligned to the maximum curvature and \mathbf{t}_2 to the minimum; when curvatures are equal (umbilic points), any orthogonal tangent pair is chosen and the order doesn't matter. A regular Cartesian grid G provides samples in a narrow ε -band around \mathcal{S} . We now describe our full pipeline (see Figure 2).

Pipeline Overview. (i) *Surface feature extraction.* Compute geometric descriptors on the surface on a set of surface samples as: $\mathcal{F}_{\mathcal{S}} := \{(x, \mathbf{n}(x), \mathbf{t}_1(x), \mathbf{t}_2(x)) \mid x \in \mathcal{S}\}$.

(ii) *Narrow-band construction.* Define a uniform volumetric grid G in a bounding box of \mathcal{S} and retain grid nodes within distance ε of the surface: $\mathcal{B}_{\mathcal{S}} := \{y \in G \mid \text{dist}(y, \mathcal{S}) \leq \varepsilon\}$.

(iii) *Overlapping local patches.* Cover \mathcal{S} and its *narrow band* $\mathcal{B}_{\mathcal{S}}$ with surface-centered patches. Each patch is anchored at a surface point $p_i^c \in \mathcal{S}$, which serves as its center, and is defined as: $\mathcal{P}_i := (\mathcal{L}_i, \mathcal{B}_i, \mathcal{F}_i)$, where $\mathcal{L}_i = (p_i^c, \mathbf{n}(p_i^c), \mathbf{t}_1(p_i^c), \mathbf{t}_2(p_i^c))$ defines a local frame at p_i^c used to express all subsequent quantities. $\mathcal{B}_i \subset \mathcal{B}_{\mathcal{S}}$ collects nearby band samples and \mathcal{F}_i gathers surrounding surface features (points and normals in our implementation). All quantities within a patch are expressed in the local frame \mathcal{L}_i . Accordingly, coordinates of local band samples and local surface features (i.e., $\mathcal{B}_i, \mathcal{F}_i$) are transformed into this intrinsic coordinate system, and we denote their local-frame representations with a hat symbol (i.e., $\hat{\mathcal{B}}_i, \hat{\mathcal{F}}_i$).

(iv) *Learned band-to-band update (neural operator).*

Given a scalar band field $U_t : \mathcal{B}_{\mathcal{S}} \rightarrow \mathbb{R}$ at time t , we denote by $u_t^i := U_t|_{\mathcal{B}_i}$ its restriction to the local band associated with patch \mathcal{P}_i . A lightweight neural operator \mathcal{N}_Θ consumes per-patch stencils (band values and local geometry) and updates the sampled field to enforce normal constancy: $\tilde{u}_t^i = \mathcal{N}_\Theta^{(\mathcal{P}_i, u_t^i)}(\hat{\mathcal{B}}_i)$. We denote by a tilde ($\tilde{\cdot}$) functions that are approximately constant along surface normals.

(v) *Aggregation of local predictions.* Local predictions are combined to reconstruct a global band field via smooth, proximity-weighted averaging as,

$$\tilde{U}_t(x) = \frac{\sum_{i:x \in \mathcal{B}_i} \exp(-\|x - p_i^c\|^2/T) \tilde{u}_t^i(x)}{\sum_{i:x \in \mathcal{B}_i} \exp(-\|x - p_i^c\|^2/T)},$$

with temperature $T > 0$ controlling the blending.

(vi) *Time evolution in the band.* Since \tilde{U}_t is nearly constant along normals, intrinsic surface operators can directly be accurately approximated. We evolve \tilde{U}_t using standard finite differences and forward Euler scheme (e.g., for heat/diffusion):

$$U_{t+dt}(x) = \tilde{U}_t(x) + dt \Delta \tilde{U}_t(x), \quad x \in \mathcal{B}_{\mathcal{S}},$$

where Δ is the discrete Laplacian on G restricted to the band. Boundary conditions are imposed on band nodes whose projections lie on $\partial\mathcal{S}$, as in [39]. Note that U_{t+dt} carries no tilde, as there is no guarantee that the updated function remains constant along the surface normals.

(vii) *Iterate or reconstruct.* If additional steps are needed, return to the *learned band-to-band update* (step iv) and repeat the aggregate–evolve cycle. At any time, we ‘readout’ the surface solution by restricting the band field to \mathcal{S} , by interpolating with radial basis functions (Gaussian kernels in ours) for a smooth surface field.

3.2. Overlapping Local Patches

We decompose the surface \mathcal{S} and its *narrow band* $\mathcal{B}_{\mathcal{S}}$ into overlapping, surface-centered local patches, each aggregating nearby band samples for grid-based updates and nearby

331 surface samples with geometric features for conditioning. A
 332 patch \mathcal{P}_i is centered at a surface point $p_i^c \in \mathcal{S}$. Around p_i^c , we
 333 gather the k nearest band nodes to form a local stencil \mathcal{B}_i . In
 334 Section 4, we discuss choice of k for good accuracy–locality
 335 trade-off. Next, we take tight axis-aligned bounding box
 336 of \mathcal{B}_i and dilate it by a small margin. All surface sam-
 337 ples whose coordinates lie inside this enlarged box, together
 338 with their normal, constitute the surface-conditioning set
 339 \mathcal{F}_i , providing a broader geometric context around the local
 340 surface region. Thus, the tuple $\mathcal{P}_i := (\mathcal{L}_i, \mathcal{B}_i, \mathcal{F}_i)$, with
 341 $\mathcal{L}_i = (p_i^c, \mathbf{n}(p_i^c), \mathbf{t}_1(p_i^c), \mathbf{t}_2(p_i^c))$, defines one such patch.
 342 Using \mathcal{L}_i , we express all quantities of \mathcal{B}_i and \mathcal{F}_i in the lo-
 343 cal frame centered at p_i^c with basis $(\mathbf{n}(p_i^c), \mathbf{t}_1(p_i^c), \mathbf{t}_2(p_i^c))$,
 344 ensuring invariance to translation and rotation. Degener-
 345 ate cases where curvature directions are ambiguous (e.g.,
 346 umbilic regions where principal curvatures coincide) are nat-
 347 urally present in the training data and are further handled
 348 through data augmentation: random rotations of the local
 349 patch (encompassing both normal and tangent directions)
 350 enforce the network to learn rotational invariance.

351 We progressively generate patches across \mathcal{S} , expanding
 352 outward from an initial (surface) seed so that coverage nat-
 353 urally propagates over the surface (similar to floodfill re-
 354 stricted to the surface). This strategy yields a family of over-
 355 lapping patches whose union covers the entire band, while
 356 maintaining controllable redundancy. The degree of overlap
 357 is controlled by a spacing parameter in our patch-placement
 358 procedure, which determines how far each new center is
 359 placed from the previous ones while ensuring that adjacent
 360 band regions still overlap. Smaller spacing increases redun-
 361 dancy and overlap, whereas larger spacing yields sparser
 362 coverage. Increasing either improves robustness but adds
 363 computational cost. In our implementation, nearest-neighbor
 364 queries on $\mathcal{B}_{\mathcal{S}}$ and \mathcal{S} are accelerated with a KD-Tree.

365 **Coverage condition.** A potential issue arises from using
 366 a fixed number of neighbours k to define each patch: for
 367 small grid spacing Δx or large band width ε , some band
 368 points may lie too far from any surface center p_i^c , leading to
 369 incomplete coverage of the band $\mathcal{B}_{\mathcal{S}}$. This motivates us to
 370 seek a relation linking ε , Δx , and k .

371 This setting is closely related to the classical *Gauss circle problem* (and its three-dimensional analogue, the *Gauss sphere problem*, see [19]), which counts the number of lattice
 372 points contained in a ball of radius r . In three dimensions,
 373 neglecting higher-order terms, the number of grid points N_3
 374 within a band of radius ε and spacing Δx is well approxi-
 375 mated by the volume of a ball of radius $\varepsilon/\Delta x$:

$$378 N_3(\varepsilon/\Delta x) \approx \frac{4}{3}\pi \left(\frac{\varepsilon}{\Delta x} \right)^3.$$

379 Ensuring every band point is covered by at least one patch
 380 yields the condition, we arrive at:

$$381 \varepsilon \leq \Delta x \left(\frac{3k}{4\pi} \right)^{1/3}.$$

3.3. Learning a Neural Update Operator

3.3.1. Neural geometry encoder

Our neural geometry encoder is a lightweight network built
 384 from small MLPs with an attention-like interaction for local
 385 geometry adherence. It operates locally in the *narrow band*
 386 around the surface and updates the target function in a single
 387 step, *directly* on grid values.

Network architecture. As illustrated in Figure 3, our neu-
 389 ral solver acts on local geometry attributes and updates the
 390 field within each narrow band \mathcal{B}_i . The inputs are the query
 391 point, the band points $\hat{\mathcal{B}}_i$, and their current field values u^i , to-
 392 gether with the associated surface features $\hat{\mathcal{F}}_i$, all expressed
 393 in the local frame \mathcal{L}_i . The query point q attends to its neigh-
 394 boring band samples to obtain spatial weights, while $\hat{\mathcal{F}}_i$
 395 modulates this aggregation so the update is *conditioned* on
 396 local geometry. The output is a weighted sum producing the
 397 updated value at q . To handle a variable number of surface
 398 features per patch, we apply mean pooling (similar to [36]),
 399 yielding a fixed-size descriptor. Formally, we encode the
 400 neural update as:

$$\mathcal{N}_{\Theta} : \mathbb{R}^3 \times \mathbb{R}^{k \times 3} \times \mathbb{R}^{N_i \times 6} \times \mathbb{R}^k \longrightarrow \mathbb{R}, \\ 402 (q, \hat{\mathcal{B}}_i, \hat{\mathcal{F}}_i, u^i) \longmapsto \mathcal{N}_{\Theta}(q, \hat{\mathcal{B}}_i, \hat{\mathcal{F}}_i, u^i)$$

where Θ contains the weights of the three MLPs encoder
 403 ($\theta_1, \theta_2, \theta_3$) and a learnable scalar λ . For convenience, we
 404 define a compressed form of the network where the patch
 405 $\mathcal{P}_i = (\mathcal{L}_i, \hat{\mathcal{B}}_i, \hat{\mathcal{F}}_i)$ and its current field u^i are fixed:

$$406 \mathcal{N}_{\Theta}^{(\mathcal{P}_i, u^i)} : q \longmapsto \mathcal{N}_{\Theta}(q, \hat{\mathcal{B}}_i, \hat{\mathcal{F}}_i, u^i)$$

In practice, our implementation evaluates multiple queries
 408 simultaneously. Let $Q = \{q_1, \dots, q_b\} \subset \mathbb{R}^3$; then
 409 $\mathcal{N}_{\Theta}^{(\mathcal{P}_i, u^i)}(Q) = \{\mathcal{N}_{\Theta}^{(\mathcal{P}_i, u^i)}(q)\}_{q \in Q}$. When solving PDE,
 410 we set $Q = \hat{\mathcal{B}}_i$ as band values are updated at band samples.

3.3.2. Training setup

Dataset construction. We train on a single representative
 413 surface, the SPIKE, represented by an SNS [43] $S_{\omega} : \mathbb{S}^2 \subset$
 414 $\mathbb{R}^3 \rightarrow \text{SPIKE}$. Note that since our network only depends
 415 on first and second order quantities (i.e., normal and curva-
 416 tures), the single SPIKE shape, having a good distribution of
 417 curvature profiles, is sufficient to train on – we test its gen-
 418 eralization behavior in Sec. 4. Surface is split into patches
 419 $\{\mathcal{P}_i\}$, as Sec. 3.2. For each patch, we apply random rotations.
 420 Each band \mathcal{B}_i is paired with closest points as,

$$421 \text{cp}(x) := S_{\omega} \left(\arg \min_{y \in \mathbb{S}^2} \|x - S_{\omega}(y)\|_2^2 \right), \quad x \in \mathcal{B}_i,$$

422 forming $\Pi_i := \{\text{cp}(x) \mid x \in \mathcal{B}_i\}$. For supervision, we use
 423 monomials:

$$424 \mathcal{M} := \{(x, y, z) \mapsto x^i y^j z^k \mid i+j+k \leq 5\}.$$

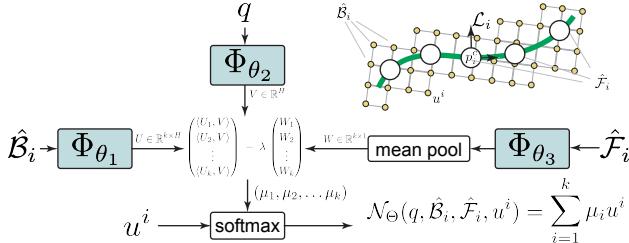


Figure 3. Neural update operator (overview). Given a query location q , the local band $\hat{\mathcal{B}}_i$ expressed in the local frame \mathcal{L}_i centered around p_i^c for patch \mathcal{P}_i , with locally-transformed surface features $\hat{\mathcal{F}}_i$ (e.g., positions, normals), and current band values u^i at grid sites (time index t omitted for brevity; full notation u_t^i), our operator predicts updated function value at location q . Trainable components include compact MLP blocks ($\Phi_{\theta_1}, \Phi_{\theta_2}, \Phi_{\theta_3}$) and a scalar λ . The full network \mathcal{N}_{Θ} produces the updated band value at q , yielding a single geometry-conditioned grid-to-grid step.

Note that we rely on any (unseen) function to be sufficiently approximated by only their top few Taylor coefficients since PDE solutions are smooth; hence, learning over monomial functions turns out to be sufficient in our tests. For each such $g \in \mathcal{M}$, we evaluate $g(\mathcal{B}_i)$ and $g(\Pi_i)$, where the first serves as the network input and the second as the ground truth target, yielding pairs:

$$\mathcal{E}_i := \{ (g(\mathcal{B}_i), g(\Pi_i)) \mid g \in \mathcal{M} \}.$$

Thus, the training dataset is $\mathcal{D} := \{ (\hat{\mathcal{B}}_i, \hat{\mathcal{F}}_i, \mathcal{E}_i) \}_{i=1}^p$.

Loss functions. We use two loss functions. The primary mean squared error enforces accurate function reconstruction:

$$L_{\text{MSE}} = \frac{1}{k|\mathcal{D}||\mathcal{M}|} \sum_{\substack{(\hat{\mathcal{B}}_i, \hat{\mathcal{F}}_i, \mathcal{E}_i) \in \mathcal{D} \\ (g, g^{\text{GT}}) \in \mathcal{E}_i}} \left\| \mathcal{N}_{\Theta}^{(\mathcal{P}_i, g)}(\hat{\mathcal{B}}_i) - g^{\text{GT}} \right\|_2^2.$$

To check geometric consistency, we monitor a normal-consistency term enforcing constancy along surface normals, evaluated over different patches and functions (\mathcal{P}, u):

$$L_{\text{NC}} := \sum_q |\langle \nabla_q \mathcal{N}_{\Theta}^{(\mathcal{P}, u)}(q), \mathbf{n}(\text{cp}(q)) \rangle|.$$

This term encourages the field gradient to remain orthogonal to the surface normals; when the dot products are close to zero then the field is nearly constant along the surface normals. The gradient $\nabla_q \mathcal{N}_{\Theta}^{(\mathcal{P}, u)}(q)$ is computed via automatic differentiation with respect to the query location q (other terms are detached). The overall objective is:

$$L = L_{\text{MSE}} + \alpha L_{\text{NC}}.$$

We conduct an ablation study (see supplemental) to assess the contribution of the normal-consistency term L_{NC} and to determine an appropriate weighting α . This analysis highlights how enforcing normal-aligned consistency improves accuracy across surfaces.

4. Evaluation

Baselines. We evaluate against two established families: (i) *Surface FEM (SFEM)* discretizes the PDE intrinsically on an explicit triangle mesh: unless stated otherwise, we use linear elements with the cotangent Laplacian and a consistent mass matrix [11, 42]. Meshes come either from the ground-truth surface or from marching cubes [22] on the same implicit surface used by our method, with multiple resolutions to probe convergence and mesh-quality effects. We do not aim to outperform FEM, whose solvers are highly mature and extensively optimized. Instead, we use FEM to provide reliable reference solutions and expected error levels, illustrating the behavior and capability of our method. Comparisons to FEM should therefore be viewed as grounding rather than competition.

(ii) *Closest Point Method (CPM)* solves in an Eulerian *narrow band* around the surface using standard Cartesian stencils and alternates *solve* and *re-extend* steps [39]. Closest-point projections and normals are computed from the same implicit geometry used by our method to ensure parity. We tested different interpolation schemes for the *re-extension* step (trilinear versus polynomial) and retained the polynomial one, consistent with the original CPM formulation, as it yielded the best accuracy-efficiency trade-off.

For fairness, all baselines share the same right-hand sides, initial data, and boundary conditions; we also matched resolution schedules and aligned stopping criteria (final time or steady-state residual).

PDEs. We benchmark *heat diffusion* ($\partial_t u = \Delta_S u$) and *Poisson* ($\Delta_S u = f$) on closed surfaces. For well-posedness, we initialize the heat equation with a prescribed initial condition and let it evolve until reaching a steady state. For Poisson, we choose a zero-mean function f over the surface and select the zero-mean solution on the surface, since the kernel of the Laplace–Beltrami operator on closed surfaces corresponds to constant functions. We report both boundary-free cases and settings with Dirichlet conditions on embedded curves (imposed identically for all methods).

Metrics. We report normalized mean absolute error (NMAE), normalized max error (NMaxE), normalized root mean square error (NMRSE). Input probe function ranges were normalized to $[-0.5, 0.5]$. See supplemental for details.

Shape representations. We evaluate across common shape encodings and derive the geometric cues needed by our operator in a consistent way. (i) *Meshes*: normals are area-weighted averages of incident face normals; mean curvature normals follow the cotangent discretization, and principal curvatures are obtained from discrete differential operators [10, 27]. (ii) *Point clouds*: we estimate normals via PCA of k -NN neighborhoods with sign disambiguation along a coarse viewpoint field. More advanced point normal prediction [13] may be used; we do not use curvature features in

507 this case. (iii) *Spherical Neural Surfaces (SNS)*: The method
 508 provides direct access to normals and first/second fundamental forms by differentiating the mapping $S_\omega : \mathbb{S}^2 \subset \mathbb{R}^3 \rightarrow \mathbb{R}^3$; 509 principal curvatures follow from the Weingarten map [43].
 510 (iv) *Implicit SDF fields (DeepSDF/overfitted INRs)*: normals are $\nabla\phi/\|\nabla\phi\|$ for these implicit fields [32]. However,
 511 although we could have used curvatures using level-set formulas [31], we found the estimates to be noisy; hence, we
 512 did not use curvature estimates in these cases. For occupancy fields [26, 40], we compute normals from the implicit
 513 gradient of the network near the isosurface. (iv) *GSplats*: after the training we treat all splats as a point cloud and filter
 514 out those that have high depth error. We extract the features
 515 using the same protocol as in (ii). See Figure 1.
 516

517 **Accuracy and Convergence on Spheres.** We begin on the
 518 unit sphere, where closed-form solutions for heat diffusion
 519 and Poisson problems are available via spherical harmonics,
 520 enabling precise accuracy and convergence studies (see
 521 Chapter 6 [1]). We also compare across four sphere mesh
 522 resolutions—*coarse*, *medium*, *fine*, and *very fine* with ap-
 523 proximately $0.1k$, $1k$, $10k$, $100k$ vertices, respectively.

524 Across resolutions, our solver matches CPM in accuracy
 525 (Table 1), even when CPM benefits from dense meshes, and
 526 follows similar error trends. Experiments further show that
 527 high-resolution SFEM provides a reliable proxy for ground
 528 truth (used later when analytic solutions are unavailable).
 529 Unlike mesh-centric pipelines, our errors are notably stable
 530 under remeshing and connectivity changes (see supplemen-
 531 tal), indicating reduced sensitivity to sampling irregularities
 532 and local topology. Most importantly, our method operates
 533 natively in the neural implicit domain and can be used as
 534 a drop-in *neural PDE layer* within standard deep-learning
 535 frameworks.

536 **Table 1. Poisson on the sphere (analytic GT).** Error vs. resolution
 537 for SFEM, CPM, and our method. We report normalized mean
 538 (NMAE) and max (NMaxE) errors (lower is better); all methods
 539 use identical right-hand sides and evaluation grids. See the sup-
 540 plemental for the corresponding heat equation table.

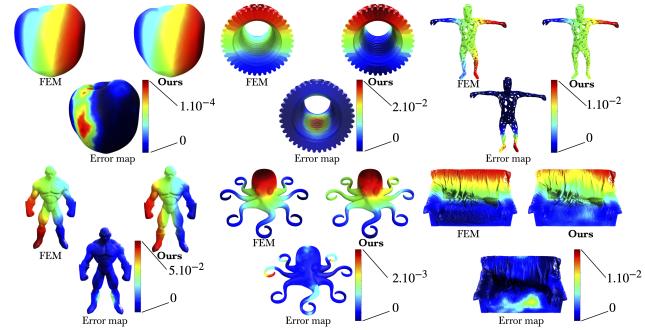
Solver	Resolution	NMAE ↓	NMaxE ↓
SFEM	Coarse	1.05×10^{-2}	2.32×10^{-2}
	Medium	6.48×10^{-4}	1.90×10^{-3}
	Fine	2.84×10^{-4}	6.30×10^{-4}
	Very fine	1.11×10^{-4}	1.29×10^{-4}
CPM	Coarse	4.56×10^{-2}	1.36×10^{-1}
	Medium	1.49×10^{-2}	3.59×10^{-2}
	Fine	1.46×10^{-2}	3.49×10^{-2}
	Very fine	1.48×10^{-2}	3.52×10^{-2}
Ours	Coarse	2.75×10^{-2}	9.14×10^{-1}
	Medium	1.24×10^{-2}	2.99×10^{-2}
	Fine	1.33×10^{-2}	3.17×10^{-2}
	Very fine	1.32×10^{-2}	3.23×10^{-2}

541 **Handling Different Shape Representations.** We run our
 542 solver on multiple shape encodings (mesh, point cloud, SNS,
 543 SDF/occupancy INRs, Gaussian Splatting), using the same
 544 local features described above (positions and normals, with
 545 optional curvature), illustrated in Figure 1. Evaluation is per-
 546 formed on the steady-state solution, whose ground truth is
 547 analytic (given by the mean of the initial condition over the
 548 surface). Although a direct comparison is not strictly mean-
 549 ingful across different shapes and representations, similar
 550 trends are observed (see Table 2), with SNS yielding the best
 551 results—consistent with its superior geometric estimates.
 552 Generalization across modalities also indicates robustness
 553 to noisy geometric quantities, as different representations
 554 provide features of varying quality.

555 **Table 2. Comparison of different surface representations** for the
 556 heat equation on the steady state. The metric is the Normalized
 557 Root Mean Squared Error (NRMSE), computed against the analytic
 558 solution.

Representation / Shape	NRMSE ↓
Neural SDF / Camera	2.17×10^{-2}
Overfitted SDF / Max Planck Face	3.03×10^{-2}
Spherical Neural Surface / Armadillo	1.88×10^{-2}
Gaussian Splatting / Snowman	6.15×10^{-2}
Point Cloud / Hat	2.94×10^{-2}
Mesh / Holey Human	1.92×10^{-2}

559 **Generalization across shapes.** Our neural operator,
 560 trained once on a single shape, transfers seamlessly to *unseen*
 561 shapes and topologies across input modalities. As illustrated
 562 in Figure 4 and quantified in Table 3, it closely matches
 563 SFEM reference solutions for Poisson across diverse geometries
 564 (e.g., organic, CAD parts with sharp transitions, and
 565 thin-structure cases) with consistently low NRMSE. This
 566 amortized, geometry-conditioned behavior underpins cross-
 567 shape generalization. Nonetheless, errors tend to appear
 568 near regions where closest points are not unique (e.g., sharp
 569 corners).



570 **Figure 4. Comparison to SFEM on diverse shapes.** For each
 571 object, left shows SFEM and right shows ours; the small inset below
 572 visualizes the pointwise error (ours vs. SFEM) with a hot–cold
 573 colormap. See color bar for error scale and the supplemental for
 574 per-shape statistics. (Error colormaps are normalized per instance.)

564 edges, thin parts, near the medial axis), a limitation inherited
 565 from the CPM formulation.
 Table 3. Poisson equation results on different shapes (NRMSE \downarrow).
 Errors are computed against the FEM as ground truth solution.

Shape	NRMSE	Shape	NRMSE
Jared	5.12×10^{-2}	Sofa	2.65×10^{-2}
Octopus	1.13×10^{-2}	Holey Human	2.67×10^{-2}
Apple	3.20×10^{-3}	Fastener	3.95×10^{-2}

566 **Boundary handling.** As in CPM, Neumann conditions
 567 are naturally satisfied since our update enforces normal con-
 568 sistency. For exterior Dirichlet boundaries, we follow the
 569 CPM practice of clamping boundary values and updating
 570 only in the band. On the Max Planck head (see Fig. 5),
 571 we solve heat with homogeneous and sinusoidal Dirichlet
 572 data and compare to a high-resolution SFEM reference on
 573 the corresponding open surface. Errors remain low and sta-
 574 ble; detailed plots and per-case statistics are provided in the
 575 supplemental. These results confirm that exterior Dirichlet
 576 conditions are handled effectively, leveraging CPM’s bound-
 577 ary treatment, which fits naturally within our method.



578 **Figure 5. Dirichlet boundaries on an open surface.** Heat diffusion
 579 on the Max Planck head cut at the neck (left to right) with boundary
 580 values clamped on the cut. See supplemental.

581 **Ablations.** Unless noted, all ablations use identical data,
 582 schedules, and hyperparameters; results across tables are not
 583 directly comparable. Overall, the studies support a simple,
 584 data-efficient design (see supplemental). (i) *Geometric con-*
 585 *sistency vs. data.* A small weight on L_{NC} reliably lowers er-
 586 rors; large weights bias toward trivial normal-invariant fields
 587 and hurt fidelity. Even without L_{NC} , the operator largely
 588 maintains normal consistency. (ii) *Local features.* Positions
 589 and normals matter most; accurate curvature adds marginal
 590 benefit. (iii) *Band receptive field.* Larger k yields dimin-
 591 ishing returns: errors drop mildly then plateau, consistent
 592 with convex aggregation. A mid-range k (~ 300) balances
 593 accuracy, stability, and cost. (iv) *Model capacity.* Shallow,
 594 narrow MLPs suffice; deeper/wider variants bring marginal
 595 gains and may overfit. (v) *Learnable attention strength.* A
 596 single learnable scalar λ modestly but consistently improves
 597 accuracy, providing lightweight adaptation.

598 5. Conclusion

599 We presented a mesh-free solver that computes surface
 600 PDEs *directly* on neural implicit surfaces. Our geometry-
 601 conditioned local operator performs a direct grid-to-grid
 602

603 update in a *narrow band* around the target surface, remains
 604 fully differentiable, and generalizes across shapes and topolo-
 605 gies without requiring meshing or per-instance optimization.
 606 On analytic benchmarks and real neural assets (across dif-
 607 ferent representations), our method achieves competitive
 608 accuracy and runtime while eliminating mesh extraction and
 609 extend-restrict shuttling as required in the classical CPM
 610 method. The approach integrates naturally into learning
 611 pipelines, opening up options to directly add PDE neural lay-
 612 ers for analysis, editing, and reconstruction involving neural
 613 shape representations. While our focus is on the *geometric*
 614 component of PDE solving—rather than optimizing high-
 615 order numerical schemes—this design makes our operator
 616 complementary to stronger discretizations and suitable for
 617 integration into more advanced solvers. This opens promis-
 618 ing directions for coupling our geometry-aware update with
 619 advanced solvers, enabling PDE layers operating directly on
 620 neural surfaces while remaining compatible with classical
 621 surface representations.

622 Limitations and Future Work

623 *Self-intersections and medial-axis neighborhoods.* Near self-
 624 intersections and/or close to the medial axis, SDF gradients
 625 may become unreliable, which can degrade update quality. A
 626 pragmatic remedy is a *hybrid fallback*: detect ill-conditioned
 627 patches (e.g., via $|\nabla\phi|$ or curvature thresholds) and locally
 628 hand off to a classical local FEM/embedded solver; the
 629 operator remains applicable elsewhere. Eventually, ours relies on
 630 the neural implicit surface to be accurate on/near the surface.

631 *Evolving surfaces and rebanding.* When the underlying
 632 surface moves under the PDE (e.g., surface evolution under
 633 curvature flow), the *narrow band* must be rebuilt and re-
 634 sampled, reducing any amortization benefits. Incremental
 635 band updates, and reusing cached features can mitigate cost;
 636 extending the operator to predict both updates and band
 637 maintenance is also an interesting future direction.

638 *Grid dependence and scale effects.* Our operator is not
 639 fully discretization-invariant: performance can vary with
 640 grid spacing and band thickness. Scale-aware condition-
 641 ing and multi-resolution training may improve robustness.
 642 Learning on a modest range of resolutions generalizes in
 643 practice. Extending our coverage condition, it will be in-
 644 teresting to derive a relation between the sampling density
 645 and implicit surface quality to further guide the grid and
 646 sampling processes.

647 Finally, in this work, we focus on Poisson solves and heat
 648 equation; strongly anisotropic or stiff systems may require
 649 tailored stabilization or implicit time-stepping. Incorporat-
 650 ing learnable preconditioners, implicit updates, or operator
 651 splitting within our framework is a promising future work.

648

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