The Binomial Distribution

Binary distribution (Bernoulli distribution)

→ Defined by p = probability of "success", and takes values 0 and 1

X	0	1
$P(X=x)$	$(1-p)$	p

$p = .5$ "success" of flipping heads

$p = 1/6$ "success" of rolling a 6

$p = .2$ prob that I "successfully" plug a USB drive in correctly.

$$E[X] = 0 \times (1-p) + 1 \times p$$
$$= p$$

$$q = 1 - p$$

$$V(X) = (0-p)^2 \times (1-p) + (1-p)^2 \times p$$
$$= p(1-p)$$

$$\sigma = \sqrt{p(1-p)}$$

Binomial Distribution

The sum of a fixed number of independent Bernoulli distributions.

p = prob of "success" n = number of trials

Consider flipping 5 coins. The number of heads is of interest

outcomes	count
HHHHH	5
HTHHT	3
TTHHH	3
TTTHT	1

For some counts (not 0 or n) there are multiple ways to get the same number of heads.

$$P(HHTTH) = P(TTHHH) = P(HTHTH)$$

TTHHH $\qquad$ $P(HHHTT)=P(HHTHH)=P(\ldots)$

TTTHT $\qquad 1$

For n coin flips, and for x successes, there are $\binom{n}{x}$ n choose x ways of ordering x successes in n total trials.

$$\binom{n}{x} = \frac{n!}{x!(n-x)!} \qquad x! = 1 \times 2 \times \ldots \times x$$

$$1! = 1 \quad 0! = 1$$

$$P(X=x) = \binom{n}{x} \underbrace{p^x (1-p)^{n-x}}$$

$\rightarrow P(HHTTH)$ or $P(TTHHH)\ldots$
we need to count instances

$$E[X] = \binom{n}{0} p^0 (1-p)^n \times 0 + \binom{n}{1} p^1 (1-p)^{n-1} + \ldots\ldots$$
$$= np$$

$E[X] = E\{X_1 + X_2 + \ldots + X_n\}$

Binomial $\qquad$ Bernoulli

$$V(X) = \binom{n}{0} p^0 (1-p)^n \times (0-np)^2 + \ldots\ldots$$
$$= np(1-p)$$

$$V(X) = V(X_1 + X_2 + \ldots + X_n)$$
$$= V(X_1) + V(X_2) + \ldots\ldots$$

$$P(X=x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & x \in \{0, 1, \ldots, n\} \\ 0 & \text{otherwise} \end{cases}$$

Probability Mass Function.

Probability Mass Function.