# Normal Distribution

What is "normal"?

Human adult heights are approximately normal with mean of $\mu = 66''$ or 168cm, and std dev of $\sigma = 4''$ or 10cm

Specifically height $\sim N(66'', 4'')$

Normal dist — mean — sd

Some will use var in place of SD

Defined by $\mu$ and $\sigma$

Is it normal for someone to be 5'6" (168cm)? Y

— — — — — — — — 6'10 (208cm)? Y

— — — — — — — — 4'10 (147cm)? Y

Is it normal for a room full of people to be 5'6" N

— — — — — — — — 6'10 N

— — — — — — — — 4'10 N

Normal distribution is defined by its mean ($\mu$) and standard deviation ($\sigma$)

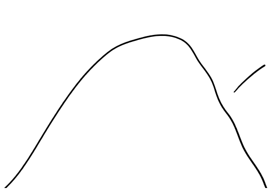
Specifically $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$

for $x \in \mathbb{R}$

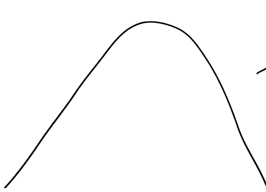
Axiom: All models are wrong, but some models are useful

$\mu$ (mean) tells us location

$\sigma$ (std dev) tells us spread

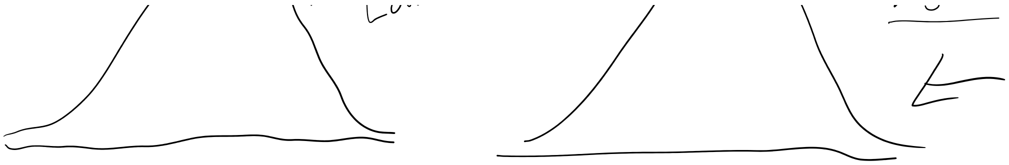


Low mean
Low SD



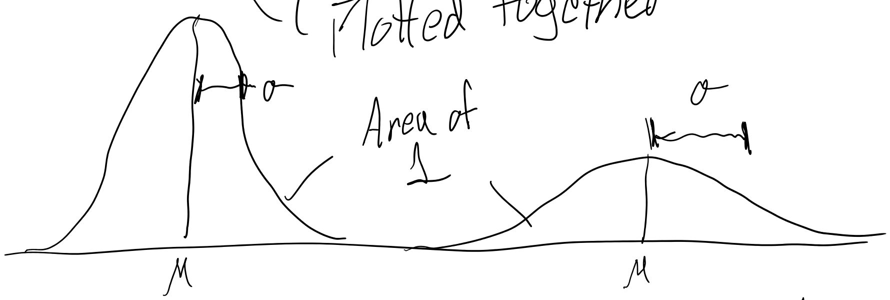
high mean
high SD

← Look identical



Look identical
Are identical
(given $\mu, \sigma$)

Plotted together



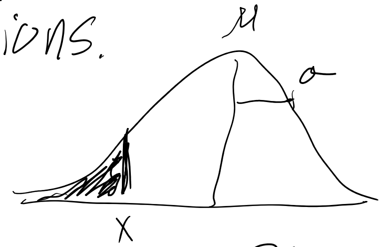

Given the mean and sd, every normal dist is identical if we frame it around the standard deviation and mean.

Standardization allows us to compare values from different normal distributions.

some point, mean

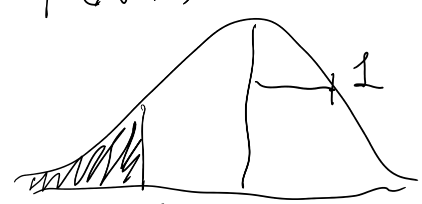
$$Z = \frac{X - \mu}{\sigma}$$

sd




$$P(X < x) = P(Z < z)$$

$N(0,1)$ - standard normal distribution




| | Std Norm Z | Any normal |
|---|---|---|
| 68% of obs are between | $-1, 1$ | $\mu - \sigma, \mu + \sigma$ |
| 95% of obs are between | $-2, 2$ | $\mu - 2\sigma, \mu + 2\sigma$ |
| 99.7% of obs are between | $-3, 3$ | $\mu - 3\sigma, \mu + 3\sigma$ |

$Z = 2 = \frac{X - \mu}{\sigma}$ $\quad$ $X = \mu + 2\sigma$

A standard score (z) represents the number

$$Z = 2 = \frac{X-\mu}{\sigma}$$

$$X = \mu + 2\sigma$$

represents the number of standard deviations that we are away from the mean.

$$P(Z < z) = P(X < x)$$

$$z = \frac{x-\mu}{\sigma}$$