Inference for Means

For inference for proportions (p) we rely on the fact

$$Z = \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} \approx \frac{\hat{p}-p}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}$$

If we know p (IE: from H_0) we use that value

$Z \sim N(0,1)$ we expect if H_0 is true.

Ex: Solve this for p: $\quad p: \hat{p} \pm Z \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

CI for p.

So what if we want to do inference for a mean μ.

We know that $\bar{X} \sim N(\mu, \sigma/\sqrt{n})$ from the Central Limit Theorem.

$$Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \text{ is Normal}(0,1)$$

But we don't know σ, and can't estimate it from μ for a specific H_0.

We can estimate σ with S, the sample standard deviation.

$$S = \sqrt{\frac{1}{n-1}\sum (X_i - \bar{X})^2}$$

Could use μ but ... consider here...

Could use μ but we won't consider here...

S is uncertain, it follows approximately a χ^2 dist and the variability in estimating S impacts our test statistic.

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim \text{Student T with degrees of freedom of } \underline{df = n-1}$$

Student T has mean of 0, and is defined only by df.

CI 90%

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \rightarrow \mu: \bar{X} \pm T_{df,.95} \cdot S/\sqrt{n}$$

This is not $\underline{\sigma}$ maybe higher or lower.

Wider than $Z_{.95}$ ✓

T distribution is <u>very close</u> to a Normal dist, but is a bit wider. As $df \rightarrow \infty$, $T_{df} \rightarrow N(0,1)$

When we do inference for a population mean (compared to inference for p), we utilize the Student T distribution instead of the standard normal.

For p

$$CI: \hat{p} \pm Z \cdot \hat{SE}_{\hat{p}}$$

$\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

For μ

$$CI: \bar{X} \pm T_{df} \cdot \hat{SE}_{\bar{X}}$$

$S/\sqrt{n}$

is true

CI: $\hat{p} \pm Z \cdot SE_{\hat{p}}$

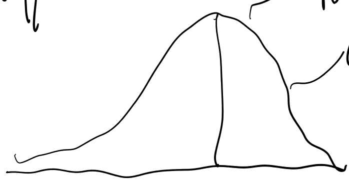
Hypothesis Testing

If H_0 true

Normal(0,1)

dist of possible

$Z = \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}}$



CI: $\bar{X} \pm T_{df} \cdot SE_{\bar{X}}$

HT

H_0 is true

student T

dist of possible

$T = \frac{\bar{X}-\mu}{s/\sqrt{n}}$