Probability Problems with Tree Diagrams

Monty Hall problem

3 door prize problem

Contestant offered 3 doors.
↳ 2 doors contain goats
↳ 1 door contains a new car*

Choose a door
P(Choose car) = 1/3

After the choice is made
Monty Hall opens 1 door → Ask: Do you want to swap doors
Not car, not choice

Question: Should you switch doors?

It's 50/50 now, choice doesn't matter (not true)

Option 1
You choose a goat
Prob 2/3

Option 2
You choose the car 1/3

(Door opens showing a goat)

Choose switch
If we're on goat switch wins
If we're on car switch loses

Choose stay
If we're on car, we win
If we're on the goat, we lose

If we switch, we win 2/3 of the time, If we stay we win 1/3 of the time.

We want to test for tuberculosis (TB)
The incidence rate for people getting tested for TB is .5% = .005
P(TB) = 0.005.

If someone has TB, test comes back Positive (+) 93% of the time.
If someone does not have TB, test comes back positive (+) 16% of the time (Error).

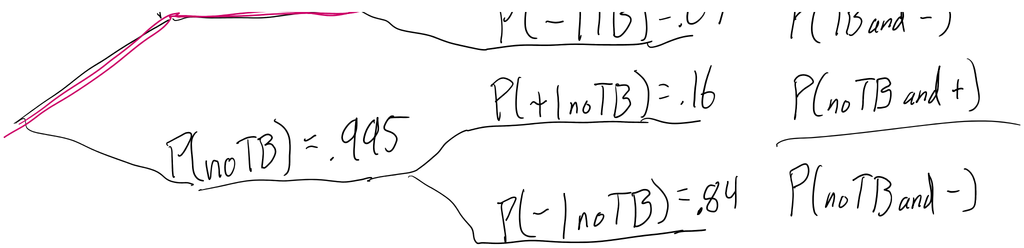
Tree diagram
Break down conditional probabilities into a tree structure

P(TB) = .005

P(+|TB) = .93 P(+)P(+|TB) = P(TB and +)

P(−|TB) = .07 P(TB and −)

P(+|no TB) = .16 P(no TB and +)



$P(TB \text{ and } +) = P(TB)\cdot P(+|TB) = .005 \cdot .93$ $\qquad 1 = P(+|TB) + P(-|TB)$

$P(-|TB) = 1 - P(+|TB) = 1 - .93 = .07$

$P(+ \text{ result}) = P(+ \text{ and } TB) + P(+ \text{ and } noTB) - \cancel{P(+ \text{ and } TB \text{ and } noTB)}$ 0

Disjoint

$= P(+|TB)P(TB) + P(+|noTB)P(noTB)$

$= 0.93 \times 0.005 + 0.16 \times 0.995$

$= 16.385\%$

$P(A|B) = \dfrac{P(A \text{ and } B)}{P(B)}$

$P(TB|+) = \dfrac{P(+ \text{ and } TB)}{P(+)} = \dfrac{P(+|TB)P(TB)}{P(+ \text{ and } TB) + P(+ \text{ and } noTB)}$

$P(A \text{ and } B) = P(A|B)P(B)$
$= P(B|A)P(A)$

$= \dfrac{P(+|TB)P(TB)}{P(+|TB)P(TB) + P(+|noTB)P(noTB)} = \dfrac{0.93 \times 0.005}{0.93 \times 0.005 + 0.16 \times 0.995}$

$= 2.838\%$

If results were independent what is prob a person with TB tests positive twice?

$P(+ \text{ and } + | TB) = P(+|TB)P(+|TB)$
$= 0.93 \times 0.93 \approx 86.5\%$

$P(+ \text{ and } + | noTB)$
$= 0.16 \times 0.16$
$= 0.0256$
2.56%

$P(TB | + \text{ and } +) = \dfrac{P(TB \text{ and } + \text{ and } +)}{P(+ \text{ and } +)}$

In medical testing it is common to retest, as multiple + results increase our belief that patient has the disease.

$P(A) = P(A \text{ and } B) + P(A \text{ and } B^c)$ $\qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ or } B)$

or A^c)

$P(A) = P(A \text{ and } B) + P(A \ldots$

$P(A) = 1 - P(A^c)$

$P(A|B) = \dfrac{P(A \text{ and } B)}{P(B)}$