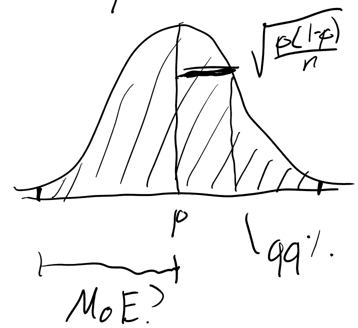
Confidence Interval Examples

Migraine in Females aged 25-34

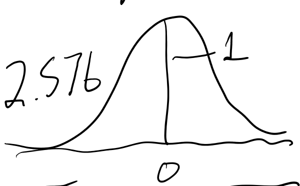
Start with low income ($\leq \$22,500$ HH income)

$\hat{p} \sim N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$



$n = 202 \quad \hat{p} = 37\%$ Calculate a 99% CI

$Z = 2.576$



$P(Z < z) = .995$
$P(Z < -z) = .005$

$SE = \sqrt{\frac{p(1-p)}{n}} \approx \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{.37 \cdot (1-.37)}{202}}$
$= 3.4\%$

99% CI: $\hat{p} \pm Z_{.995} \cdot \hat{SE}$
$.37 \pm 2.576 \cdot .034$
$(28.24\%, 45.75\%)$

We are 99% Confident that the prop of females with low income households in ages 25-34 experience migrains with a proportion between $(28.24\%, 45.75\%)$

$p = .3?$

For mid income ($\$22,500 - \$60,000$)

$n = 439 \quad \hat{p} = 29\%$ Want to calculate a 90% CI

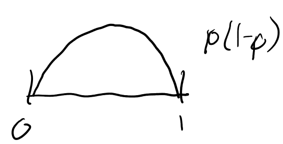
$P(Z < z) = .95 \quad Z = 1.645$
$P(Z < -1.645) = .05$

$\hat{SE} = \sqrt{\frac{.29(1-.29)}{439}} \simeq 2.17\%$

90% CI: $.29 \pm 1.645 \times .0217 = (25.43\%, 32.57\%)$

$p = .3?$

For high income



$n = 369 \quad \hat{p} = 20\%$ We want an 80% CI.

$P(Z < z) = .9 \quad Z = 1.287 \quad \hat{SE} = \sqrt{\frac{.2 \cdot (1-.2)}{369}} = 2.08\%$

$P(Z < z) = .9 \quad z = 1.282 \quad \hat{SE} = \sqrt{\frac{.2 \cdot (1-.2)}{369}} = 2.08\%$

80% CI: $.2 \pm 1.282 \cdot .0208$: $(17.33\%, 22.67\%)$

Consider: How does level of confidence affect the width of the interval?

$\hat{p} \pm z \cdot SE$

How does sample size affect the width of the interval?

$\hat{p} \pm z \cdot SE \qquad \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

I lied about sample sizes! They're all 10 times bigger than values used here. How does that affect intervals?

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{10 \times n}} = \frac{1}{\sqrt{10}} SE_{lies}$$