Negative Binomial Distribution

Expands the geometric distribution (iid)

The Negative Binomial distribution represents the probability distribution of the number of trial necessary for the k^{th} "success".

Ex: How many free throws before you make five

How many flips of a coin to get 10 heads

We can solve the pmf by starting with:

If we have n flips of a coin with prob p and we want k successes:

Prob of $n-k$ failures followed by k success is...

$$(1-p)^{n-k} p^{k}$$

There are a number of ways we can order the $k-1$ success and $n-k$ failures.

$\mu = k + \frac{k(1-p)}{p}$ $\binom{n-1}{k-1}$ ways of ordering the $k-1$ and $n-k$ successes and failures.

Total # of flips

$$P(X=n) = \begin{cases} \binom{n-1}{k-1} p^{k} (1-p)^{n-k} & n \in \{k, k+1, k+2, \dots\} \\ 0 & \text{otherwise} \end{cases}$$

Notation for R

$$P(X=x) = \begin{cases} \binom{k+x-1}{k-1} p^k (1-p)^x & x \in \{0,1,2,\ldots\} \\ 0 & \text{otherwise} \end{cases}$$

# of failures

$$\mu = \frac{k(1-p)}{p} \qquad \sigma^2 = \frac{k(1-p)}{p^2}$$