Probability Rules and ~~Bayes~~ Theorem

Conditional Probability

Recap

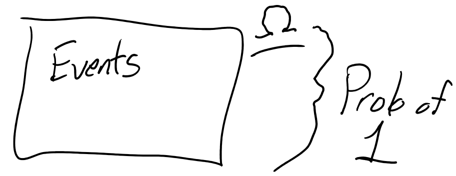
$P(\text{Event}) \to$ Probability of an event

$P(A|B) \to$ Prob of A given B

Disjoint Events cannot occur simultaneously (at same time)

Independent Events do not affect the prob of one another if we know the outcome of one of them.

Roll a die

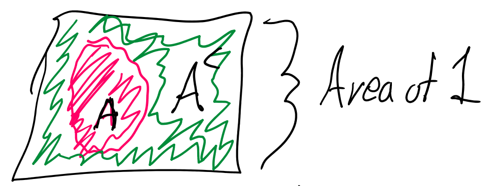


Rules for Probability

1) $0 \le P(A) \le 1$

$0\% \le P(A) \le 100\%$

P(Roll 7) P(Roll 1-6)

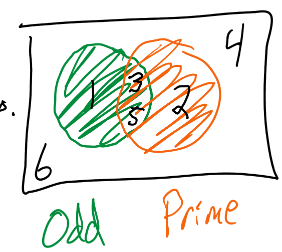


2) $P(A) = 1 - P(A^c)$

$P(A^c) = 1 - P(A)$ $\quad 1 = P(A) + P(A^c)$

$P(A|B) = 1 - P(A^c|B)$

Compliment rules apply for conditional prob.



3) **Addition rule** $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(\text{Odd or Prime}) = P(\text{Odd}) + P(\text{Prime}) - P(\text{odd and Prime})$

$P(\text{Roll } 1, 3, \text{or } 5) + P(\text{Roll } 2, 3, \text{or } 5) - P(3 \text{ or } 5) = 2/3$

$\quad 1/2 \qquad\qquad 1/2 \qquad\qquad 1/3$

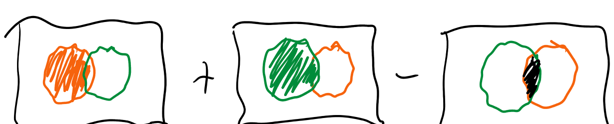
$P(1, 2, 3, \text{or } 5) = 4/6 = 2/3$

These two overlap on 3 and 5

If we add P(odd) + P(Prime) we double count 3 and 5

So we subtract P(odd and Prime)

P(3 or 5)



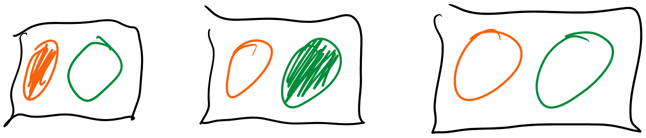
$P(A) + P(B) - P(A \text{ and } B)$

★ If two events are disjoint $\quad P(A \text{ or } B) = P(A) + P(B)$

3. If two events are disjoint

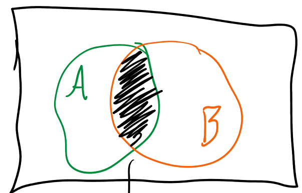
$P(A \text{ or } B) = P(A) + P(B)$

$P(A \text{ and } B) = 0$



$P(A) + P(B) - 0$

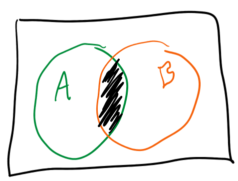
Consequence: $P(A) = P(A \text{ and } B) + P(A \text{ and } B^c)$

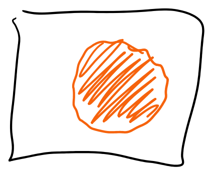


$P(A \text{ and } B)$

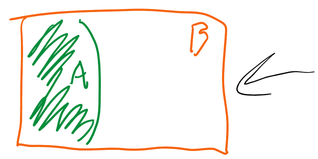
4) Multiplication Rule

$$P(A \text{ and } B) = P(A)P(B|A)$$
$$= P(B)P(A|B)$$





$P(B)$



$P(A|B)$

$P(\text{Odd and Prime})$
$= P(\text{odd})\,P(\text{Prime}|\text{odd})$
$= P(\text{Prime})\,P(\text{odd}|\text{prime})$

$P(\text{odd}) = 1/2$

$2/3 = P(\text{prime}|\text{odd})$

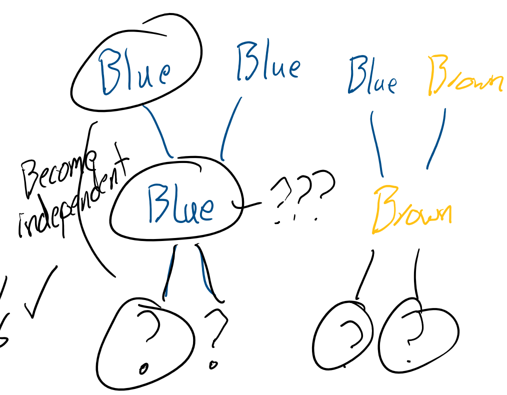
$P(\text{odd and prime}) = \frac{2}{3} \times \frac{1}{2}$
$= 1/3$

$P(3 \text{ or } 5) = 1/3$

★ For independent events

$$P(A \text{ and } B) = P(A)P(B)$$
$$= P(B)P(A)$$

$$P(A|B) = P(A)$$
$$P(B|A) = P(B)$$



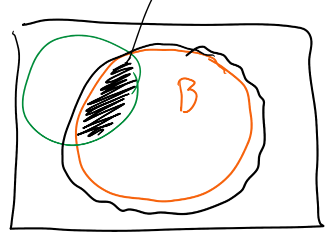
$P(\text{Roll} < 3 \text{ and Roll Prime}) = \underbrace{P(\text{Roll} < 3)}_{1/3}\,\underbrace{P(\text{Prime})}_{1/2} = 1/6$ ✓

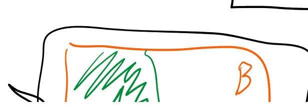
$P(\text{Roll } 2) = 1/6$ ✓

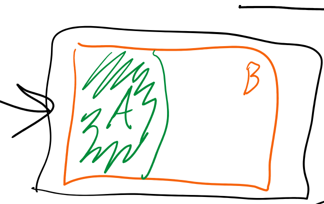
5) Bayes Theorem
Conditional Prob theorem

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

what prop of $P(B)$ is $P(A \text{ and } B)$







Using our rules

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{P(B)P(B|A)}{P(B \text{ and } A) + P(B \text{ and } A^c)} = \frac{P(B)P(B|A)}{P(B)P(B|A) + P(B)P(B|A^c)}$$

P(Disease | + test result for disease)

P(+ | Disease)

P(+ | No disease)