Poisson and Exponential Distributions

Poisson describes is the number of discrete events inside a continuous interval.

Ex: The number of donkey kick fatalities in the Prussian army, each year.

How many customers in a store per day

How many pot holes per mile of road

How many imperfections per yd^2 of textiles.

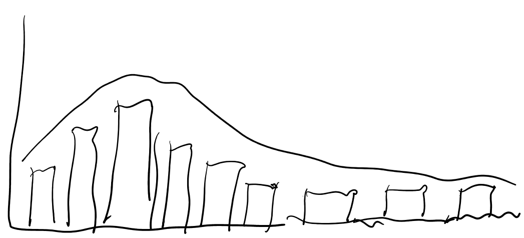
PMF

$$P(X=x) = \begin{cases} \dfrac{\lambda^x e^{-\lambda}}{x!} & x \in \{0,1,2,\dots\} \\ 0 \end{cases}$$

$E[X] = \lambda$

$Var(X) = \lambda$

e is the natural log number ~ 2.718



Exponential Distribution

Describes the wait time between observations in a Poisson process

Exponential distribution is continuous.

Ex: Wait time for the bus

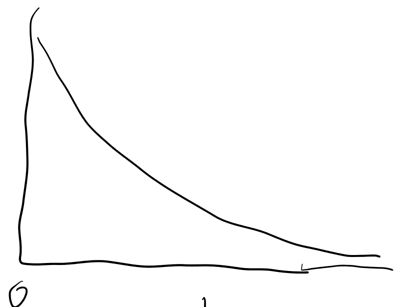
Distance between potholes

Ex. Wait time ...

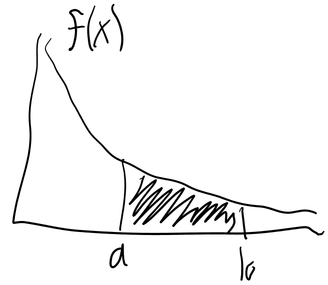
Distance between potholes

Modeling Radioactive decay

Continuous distributions do not have probability mass functions.
We describe continuous distributions by the prob density function (pdf)

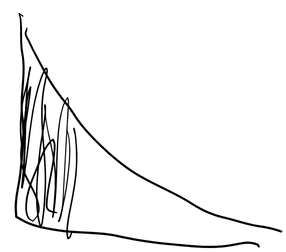


To find probabilities we find areas under the curve



$$P(a<X<b)=\int_a^b f(x)\,dx$$

$$f(x)=\begin{cases}\lambda e^{-x\lambda} & x\geq 0\\ 0 & x<0\end{cases}$$



$$P(X=x)=\int_x^x f(x)\,dx=0$$

f(x) does not describe probability, but instead likelihood.

$$E[X]=\sum_{x\in\mathbb{Z}} P(X=x)\cdot x \qquad E[X]=\int_{\mathbb{R}} f(x)\cdot x\,dx$$

For Exponential distribution

$$\mu=1/\lambda$$

$$V(X)=\sum_{x\in\mathbb{Z}} P(X=x)(x-\mu)^2 \qquad V(X)=\int_{\mathbb{R}} f(x)\cdot(x-\mu)^2\,dx$$

$$\sigma^2=1/\lambda^2$$

Fun Fact

The exponential distribution is <u>memoryless</u>

$$P(X>15) = P(X>30 \mid X>15)$$