Probability Basics

Probability is core to statistics

Selection of a sample, assignment of treatment
↳ Results of statistical tests
↳ Models fit to data
All of this involves randomization

Probability is the understanding of this randomness.
→ We can never predict a specific random outcome.

Probability is the study of random processes in the long run.

Random Process

Roll a die → Events

We describe outcomes of a random process as an event.

Roll 1, Roll 2, ..., Roll 6, ~~Roll 7~~ 0 prob

Roll Prime, Roll odd, Roll even, Roll 4, 5 or 6

Prob tells us how likely these events are in the long run.

Notation

P(Event) → Probability of event occurring in a single random outcome.

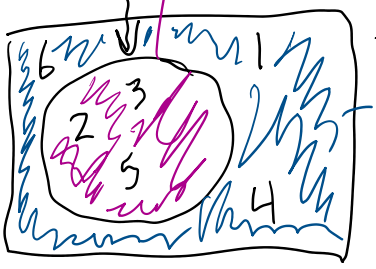
$P(\text{Roll 1}) = 1/6$ $P(\text{Roll Prime}) = 1/2$

When we define an event, we also define its compliment

P(Roll Prime) vs P(Roll non-Prime)

P(Roll Prime) vs P(Roll non-Prime)
P(Roll Prime^c) or $\overline{P(\text{Roll Prime})}$
P(!Roll Prime)

Prob occurs with an outcome space



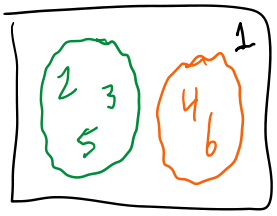
Complementary events are disjoint (Mutually Exclusive)

Disjoint events cannot occur simultaneously

Disjoint
Roll Prime vs Roll non-prime
Roll Prime vs Rolling 4 or 6

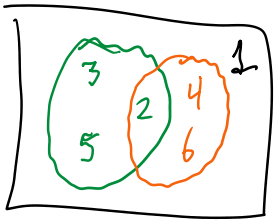
Not-disjoint
Roll Prime vs Roll even

Disjoint Events



Roll prime vs Roll 4 or 6

Non-disjoint



Roll Prime vs Roll Even

Disjoint events are never independent

Two events are independent if the prob that one will occur is not impacted by the fact that the other has occurred.

Events

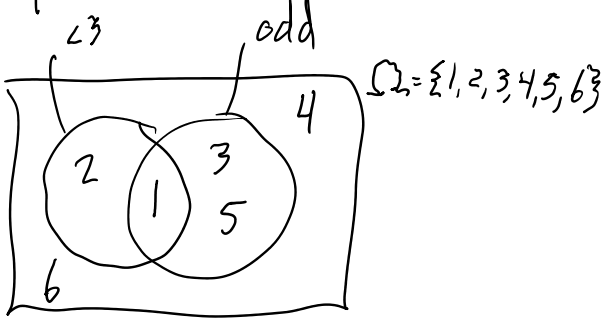
Roll < 3 → 1 or 2

P(Roll < 3) = 1/3 or 2/6

Roll odd → 1, 3, 5

P(Roll odd) = 3/6 = 1/2

If I know one of these has occurred, does it impact the prob of the other?



Conditional Prob

P(A|B) → Prob of A given B, is the prob that A has occurred given B has occurred

the prob that B has occurred

$P(\text{Roll} < 3 \mid \text{Roll odd}) = P(\text{Roll} < 3 \mid \text{Rolled } 1, 3, \text{or } 5) = 1/3 = P(\text{Roll} < 3)$

$P(\text{Roll odd} \mid \text{Roll} < 3) = 1/2 = P(\text{Roll odd})$

The idea of independence is pivotal to a lot the statistics that we will do.

Weight loss drug → Some people lose or gain in both treatment and control

Assume of the trial → Those in treatment group are more likely to lose weight.

Gambler's Fallacy

The idea that the outcomes of past independent events influence future events.

Example

Roullette: We get Red 7 times in a row.

Fallacy: 8 reds in a row is very unlikely
therefore the next spin being red is very unlikely.

Reality: The prob 8 reds in a row is unlikely
but the next spin is $P(\text{Red} \mid \text{Last 7 spins were Red}) = P(\text{Red})$