Hypothesis Testing and Power

Power is our ability to correctly reject the null hypothesis. Power = P(Reject Null | Null is false)

Example: Unfair Coin Assume P(Heads) = p = .5

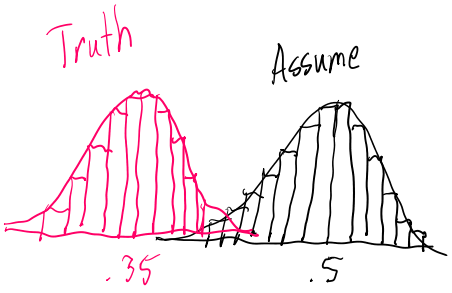
Flip coin 100 times. $\hat{p} = 41\%$ 41/100 heads

If p is actually $p = 35\%$, what is the prob we correctly reject H_0.

H_0: The coin is fair $p = .5$

H_a: The coin is not fair $p \neq .5$

$\alpha = 5\%$



What is my p-value.

41/100 or something as or more extreme

↳ $\frac{40}{100}, \frac{39}{100}, \frac{38}{100}$ ... also $\frac{59}{100}, \frac{60}{100}, \frac{61}{100}$ ...

Binomial(n=100, p=.5)

$P(\hat{p} \leq 41\% \mid p = .5) + P(\hat{p} \geq 59\% \mid p = .5)$

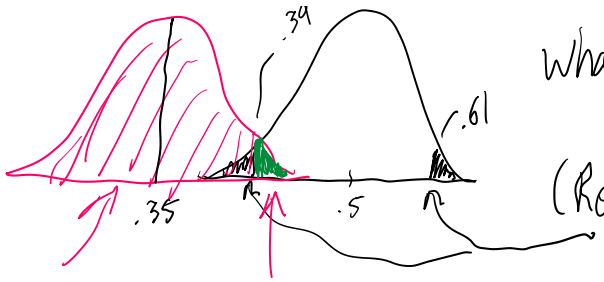
.0443 + .0443

p-value = 8.86%

Based on this p-value we fail to reject the null hypothesis

(Type II error, H_0 is false)

What values of $\hat{p}$ are



What values of $\hat{p}$ are significant? (Rejection Region)

Power

Type II error

$P(\hat{p} \le 39\% \mid p=.5) = 0.0176$

Power + P(Type II) = 1

$P(\hat{p} \ge 61\% \mid p=.5) = 0.0176$

$P(\hat{p} \ge 60\% \text{ or } \hat{p} \le 40\%) = 5.6\%$

$P(\hat{p} \ge 61\% \text{ or } \hat{p} \le 39\% \mid p=.5) = 3.52\%$

If $p=.35$, what is the prob we correctly reject H_0

- - - - - we get a $\hat{p} \le 39\%$ (or $\ge 61\%$)

$P(\hat{p} \le 39\% \mid p=.35) = 0.827585$

$P(\hat{p} \ge 61\% \mid p=.35) = .000000105$

If p was 50%; we have about a 5% chance of incorrectly rejecting the null. ↳ (3.5% due to discrete dist) ↗ Type I error

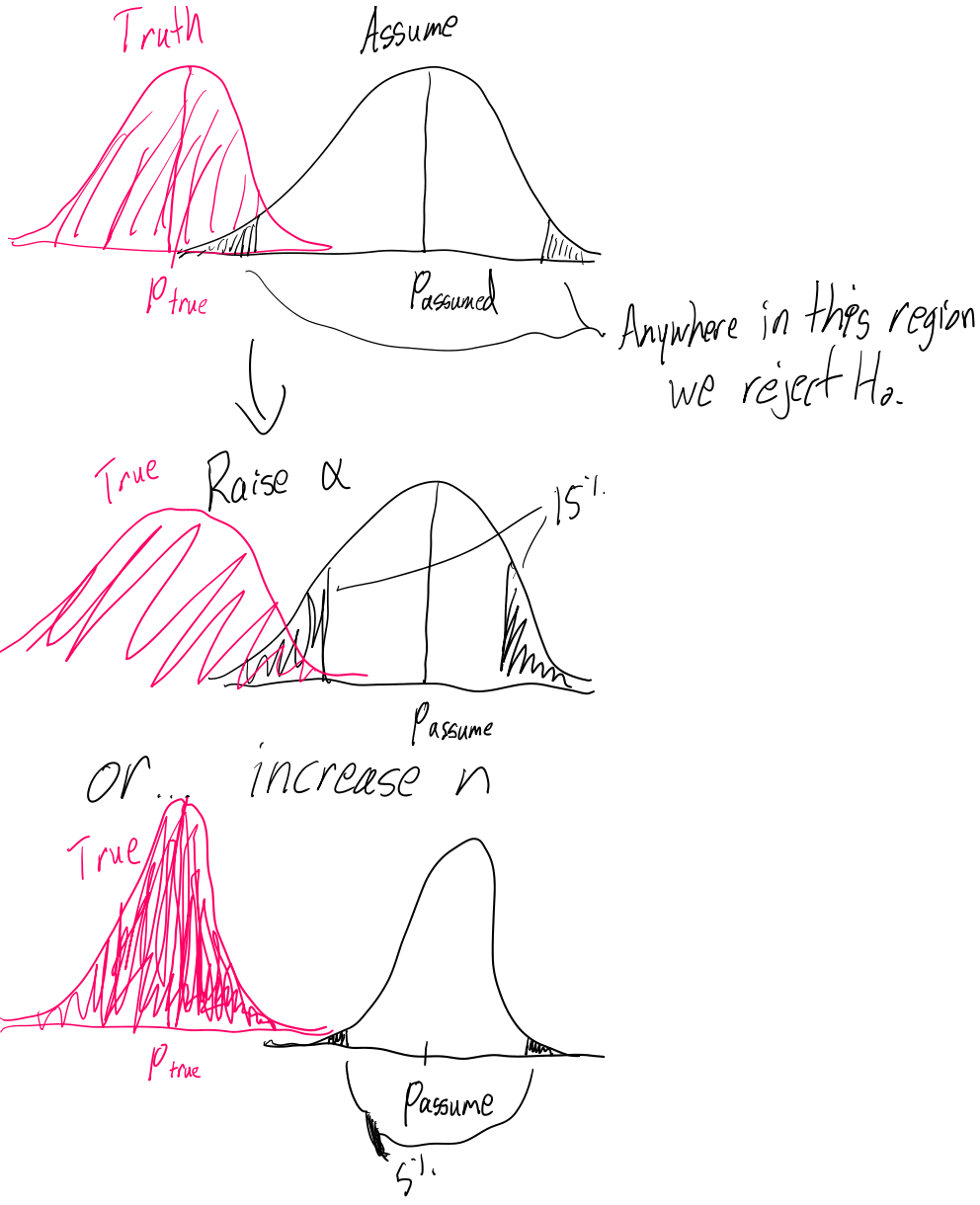
95% of correctly failing to reject.
(96.5%)

Because p is actually 35%; We have a 82.7% chance of correctly rejecting H_0.

We have a 17.3% chance of failing to reject H_0 (Type II error)

Truth

Assume


Truth
Assume
p_{true}
$p_{assumed}$
Anywhere in this region we reject H_a.
True
Raise α
15%
p_{assume}
or... increase n
True
p_{true}
p_{assume}
5%