Note on $x > 0.01$ extrapolation

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The initial condition fundamental proton dipole is given as

$$\mathcal{N}_{Y_0}(\boldsymbol{x}_\perp) = \exp\left\{ -\left(\frac{x_\perp^2 (Q_{S,0}^p)^2}{4}\right)^\gamma \log\left(\frac{1}{x_\perp \Lambda_{\rm IR}} + {\rm e}\right)\right\}\,, \tag{1}$$

Since $Y = \log x_0/x$, with $x_0 = 0.01$ and so $Y_0 = 0$, $Q_{S,0}^p$ is the initial saturation momentum, γ is the anomalous dimension and $\Lambda_{\rm IR}$ is the IR cutoff of the model. We use the parameter sets

$$\begin{aligned}
&\gamma = 1\,, \qquad (Q_{S,0}^p)^2 = 0.2\,{\rm GeV}^2\,, \qquad \Lambda_{\rm IR} = 0.241\,{\rm GeV}\,.\\
&\gamma = 1.101\,, \qquad (Q_{S,0}^p)^2 = 0.157\,{\rm GeV}^2\,, \qquad \Lambda_{\rm IR} = 0.241\,{\rm GeV}\,.\\
&\gamma = 1.119\,, \qquad (Q_{S,0}^p)^2 = 0.169\,{\rm GeV}^2\,, \qquad \Lambda_{\rm IR} = 0.241\,{\rm GeV}\,.
\end{aligned} \tag{2}$$

The model with $\gamma = 1$ was considered in [1], while the models with $\gamma = 1.101$ and $\gamma = 1.119$ models are obtained from fits to the HERA data [2].

The UGD $\varphi(Y, \boldsymbol{k}_\perp)$ is related to the collinear gluon PDF

$$x f_g(x, Q^2) = \frac{a(x)}{4\pi^3}\int_0^{Q^2} {\rm d}\boldsymbol{k}_\perp^2 \varphi(\boldsymbol{k}_\perp, Y) = (\pi R_p^2)\frac{N_c}{4\alpha_S}\frac{a(x)}{4\pi^3}\int_0^{Q^2} {\rm d}\boldsymbol{k}_\perp^2 \boldsymbol{k}_\perp^2 \mathcal{N}_Y(\boldsymbol{k}_\perp)\,, \tag{3}$$

where an unknown function $a(x)$ is introduced. It is assumed that the following conditions are satisfied $a(x_0) = 1$, $a'(x_0) = 0$, where $x_0 = 0.01$. Using these two conditions one obtains a system of equations for R_p and $Q = Q_0$. We then extrapolate to $x > x_0$ using

$$\mathcal{N}_Y(\boldsymbol{k}_\perp) = a(x)\mathcal{N}_{Y_0}(\boldsymbol{k}_\perp)\,, \tag{4}$$

that is, in such a way that the $\boldsymbol{k}_\perp$-integrated dipole always becomes collinear PDF according to (3). Moreover, the unknown function $a(x)$ is fixed by (3) once we specify the collinear PDF and the dipole. We take (3) and also its derivative and combine them in an equation for $Q = Q_0$ when $x = x_0$

$$\left[f_g(x_0, Q^2) + x_0\left(\frac{df_g(x, Q^2)}{dx}\right)_{x=x_0}\right]\int_0^{Q^2} d\boldsymbol{k}_\perp^2 \boldsymbol{k}_\perp^2 \mathcal{N}_{Y_0}(\boldsymbol{k}_\perp) = x_0 f_g(x_0, Q^2)\int_0^{Q^2} d\boldsymbol{k}_\perp^2 \boldsymbol{k}_\perp^2 \left(\frac{d\mathcal{N}_Y(\boldsymbol{k}_\perp)}{dx}\right)_{Y=Y_0}\,. \tag{5}$$

When we have Q_0 we can determine R_p from (3) when $x = x_0$. Finally, $a(x)$ is also fixed from (3) for $x > x_0$. For the models in (2) we find

$$\begin{aligned}
&\gamma = 1\,, \qquad Q_0 = 2.1541\,{\rm GeV}\,, \qquad R_p = 0.5419\,{\rm fm}\,,\\
&\gamma = 1.101\,, \qquad Q_0 = 2.0651\,{\rm GeV}\,, \qquad R_p = 0.5257\,{\rm fm}\,,\\
&\gamma = 1.119\,, \qquad Q_0 = 2.0494\,{\rm GeV}\,, \qquad R_p = 0.5257\,{\rm fm}\,,
\end{aligned} \tag{6}$$

This large-x extrapolation works only for the proton, because we know the proton PDF. I assume same $a(x)$ also for the nuclei.

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[1] J. L. ALbacete and A. Dumitru, “A model for gluon production in heavy-ion collisions at the LHC with rcBK unintegrated gluon densities,” arXiv:1011.5161 [hep-ph].

[2] J. L. Albacete, N. Armesto, J. G. Milhano, P. Quiroga-Arias, and C. A. Salgado, “AAMQS: A non-linear QCD analysis of new HERA data at small-x including heavy quarks,” *Eur. Phys. J.* **C71** (2011) 1705, arXiv:1012.4408 [hep-ph].