$$\langle r_4^2\rangle = \frac{3}{2\times 4}\frac{1}{(m_1+m_2+m_3+m_4)\omega}(m_1(\frac{1}{m_2}+\frac{1}{m_3}+\frac{1}{m_4})+m_2(\frac{1}{m_3}+\frac{1}{m_4}+\frac{1}{m_1}) +m_3(\frac{1}{m_4}+\frac{1}{m_1}+\frac{1}{m_2})+m_4(\frac{1}{m_1}+\frac{1}{m_2}+\frac{1}{m_3}))$$

$$\begin{pmatrix} R \\ y_1 \\ y_2 \\ y_3 \end{pmatrix} = J_4 \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{pmatrix} \text{ with}$$

$$J_4 = \begin{pmatrix} \frac{m_1}{m_1+m_2+m_3+m_4} & \frac{m_2}{m_1+m_2+m_3+m_4} & \frac{m_3}{m_1+m_2+m_3+m_4} & \frac{m_4}{m_1+m_2+m_3+m_4} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \\ \sqrt{2/3}\frac{m_1}{m_1+m_2} & \sqrt{2/3}\frac{m_2}{m_1+m_2} & -\sqrt{2/3} & 0 \\ \sqrt{3/4}\frac{m_1}{m_1+m_2+m_3} & \sqrt{3/4}\frac{m_2}{m_1+m_2+m_3} & \sqrt{3/4}\frac{m_3}{m_1+m_2+m_3} & -\sqrt{3/4} \end{pmatrix}$$

its inverse

$$J_4^{-1} = \begin{pmatrix} 1 & \frac{m_2}{m_1+m_2}\sqrt{2} & \frac{m_3}{m_1+m_2+m_3}\sqrt{\frac{3}{2}} & \frac{m_4}{m_1+m_2+m_3+m_4}\sqrt{\frac{4}{3}} \\ 1 & -\frac{m_1}{m_1+m_2}\sqrt{2} & \frac{m_3}{m_1+m_2+m_3}\sqrt{\frac{3}{2}} & \frac{m_4}{m_1+m_2+m_3+m_4}\sqrt{\frac{4}{3}} \\ 1 & 0 & -\frac{m_1+m_2}{m_1+m_2+m_3}\sqrt{\frac{3}{2}} & \frac{m_4}{m_1+m_2+m_3+m_4}\sqrt{\frac{4}{3}} \\ 1 & 0 & 0 & -\frac{m_1+m_2+m_3}{m_1+m_2+m_3+m_4}\sqrt{\frac{4}{3}} \end{pmatrix},$$

$$\begin{pmatrix} K \\ k_{y_1} \\ k_{y_2} \\ k_{y_3} \end{pmatrix} = J_4^{-,+} \begin{pmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{pmatrix}$$

The Wigner density is

$$\rho^W(y_1, y_2, y_3, k_{y_1}, k_{y_2}, k_{y_3}) = 8^3 \exp(-\frac{y_1^2}{\sigma_1^2} - \frac{y_2^2}{\sigma_2^2} - \frac{y_3^2}{\sigma_3^2} - \sigma_1^2 k_{y_1}^2 - \sigma_2^2 k_{y_2}^2 - \sigma_3^2 k_{y_3}^2)$$

$$= \begin{pmatrix} 1 & 1 & 1 & 1 \\ \frac{m_2}{m_1+m_2}\sqrt{2} & -\frac{m_1}{m_1+m_2}\sqrt{2} & 0 & 0 \\ \frac{m_3}{m_1+m_2+m_3}\sqrt{\frac{3}{2}} & \frac{m_3}{m_1+m_2+m_3}\sqrt{\frac{3}{2}} & -\frac{m_1+m_2}{m_1+m_2+m_3}\sqrt{\frac{3}{2}} & 0 \\ \frac{m_4}{m_1+m_2+m_3+m_4}\sqrt{\frac{4}{3}} & \frac{m_4}{m_1+m_2+m_3+m_4}\sqrt{\frac{4}{3}} & \frac{m_4}{m_1+m_2+m_3+m_4}\sqrt{\frac{4}{3}} & -\frac{m_1+m_2+m_3}{m_1+m_2+m_3+m_4}\sqrt{\frac{4}{3}} \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{pmatrix};$$

i.e.

$$K = k_1 + k_2 + k_3 + k_4,$$
$$k_{y_1} = \sqrt{2}\left(\frac{m_2}{m_1+m_2}k_1 - \frac{m_1}{m_1+m_2}k_2\right)$$
$$k_{y_2} = \sqrt{\frac{3}{2}}\left(\frac{m_3}{m_1+m_2+m_3}k_1 + \frac{m_3}{m_1+m_2+m_3}k_2 - \frac{m_1+m_2}{m_1+m_2+m_3}k_3\right)$$
$$k_{y_3} = \sqrt{\frac{4}{3}}\left(\frac{m_4}{m_1+m_2+m_3+m_4}k_1 + \frac{m_4}{m_1+m_2+m_3+m_4}k_2 + \frac{m_4}{m_1+m_2+m_3+m_4}k_3 - \frac{m_1+m_2+m_3}{m_1+m_2+m_3+m_4}k_4\right)$$

$\sigma_1^2 = (\mu_1\omega)^{-1}$, $\sigma_2^2 = (\mu_2\omega)^{-1}$ and $\sigma_3^2 = (\mu_3\omega)^{-1}$
with

$$M = m_1 + m_2 + m_3 + m_4,$$
$$\mu_1 = 2(\frac{1}{m_1} + \frac{1}{m_2})^{-1}$$
$$\mu_2 = \frac{3}{2}(\frac{1}{m_1+m_2} + \frac{1}{m_3})^{-1}$$
$$\mu_3 = \frac{4}{3}(\frac{1}{m_1+m_2+m_3} + \frac{1}{m_4})^{-1}$$