

Consider the coalescence of two particles with wave functions  $\phi_1(x)$  and  $\phi_2(x)$  to a bound state with wave function  $\Phi(x)$ .

If the two particles are non-identical, then the initial and final states are

$$\begin{aligned}\langle x_1, x_2 | i \rangle &= \phi_1(x_1) \phi_2(x_2), \\ \langle x_1, x_2 | f \rangle &= \frac{1}{\sqrt{V}} e^{iK(x_1+x_2)} \Phi(x_1 - x_2).\end{aligned}$$

Their coalescence probability  $P = |\langle f | i \rangle|^2$  is then

$$\begin{aligned}P &= \left| \frac{1}{\sqrt{V}} \int dx_1 \int dx_2 e^{-iK(x_1+x_2)} \Phi(x_1 - x_2) \phi_1(x_1) \phi_2(x_2) \right|^2 \\ &= \int dK \int dx_1 dk_1 dx_2 dk_2 W_1(x_1, k_1) W_2(x_2, k_2) W(x_1 - x_2, (k_1 - k_2)/2) \\ &\quad \times \delta(K - k_1 - k_2),\end{aligned}$$

where the Wigner function is defined as

$$W(x, k) = \int dy \phi^*(x + y/2) \phi(x - y/2) e^{-iky}.$$

If the two particles are identical, then the initial state is

$$\langle x_1, x_2 | i \rangle = \frac{1}{\sqrt{2}} [\phi_1(x_1) \phi_2(x_2) \pm \phi_2(x_1) \phi_1(x_2)].$$

Then

$$\begin{aligned}\langle f | i \rangle &= \frac{1}{\sqrt{2V}} \int dx_1 \int dx_2 e^{-iK(x_1+x_2)} \Phi(x_1 - x_2) [\phi_1(x_1) \phi_2(x_2) \pm \phi_2(x_1) \phi_1(x_2)] \\ &= \sqrt{\frac{2}{V}} \int dx_1 \int dx_2 e^{-iK(x_1+x_2)} \Phi(x_1 - x_2) \phi_1(x_1) \phi_2(x_2),\end{aligned}$$

where the second line is obtained by interchange  $x_1$  and  $x_2$  and assuming  $\Phi(x)$  is an even (odd) function if the total wave function of the two identical particles is symmetric (anti-symmetric). Hence the coalescence probability of two identical particles is twice that for non-identical particles. **The coalescence of identical particles can thus be treated as non-identical particles.** The same consideration applies to the coalescence of identical particles to form bound state of more than two particles.