Modeling urban taxi service with e-hailings: A queuing networks approach

Supplemental Material

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Abstract

The rise of e-hailing taxis have significantly altered urban transportation and resulted in an competitive taxi market with both traditional street-hailing and e-hailing taxis. The new mobility services provide similar door-to-door rides as the traditional one and there is competition across these various services. Meanwhile, the increasing e-hailing supply, together with traditional taxicab flows, influence the urban road network performance, which can also in turn affect taxi mode choice and operation. In this study, we propose an innovative modeling structure for the competitive taxi market and capture the interactions not only within the taxi market but also between the taxi market and urban road system.

The model is built on a network consisting of two types of queueing theoretic approaches for both the taxi and urban road system. Considering both the passenger and vehicle arrivals, we utilize an assembly-like queue $SM/M/1$ for passenger-vehicle matching within the taxi system, which controls how many and how frequently vehicles drive from the taxi system to the urban road system. A common multi-server $M/M/c$ queue that can account for road capacity is proposed for the urban road system and a feedback of network states are sent back to the taxi system. Moreover, within the taxi system, we introduce state-dependent service rate to account for the stochasticity of passenger-vehicle matching efficiency. Then the stationary state distributions, as well as asymptotic properties, of the queueing network are discussed.

An example is designed based on data from New York City. Numerical results show that the proposed modeling structure, together with the corresponding approximation method, can capture dynamics within high demand areas using multiple data sources. Overall, this study shows how the queueing network approach can measure both the taxi and urban road system performance at an aggregate level. The model can be used to estimate not only the waiting/searching time during passenger-vehicle matching but also the delays in the urban road network. Furthermore, the model can be generalized to study the control and management of taxi markets.

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1 Hypothesis Testing of Poisson Assumption

The study proposes $M/M/1$ queue for vehicle-passenger matching of each taxi service type, which requires strong assumption of Poisson arrivals. However, we have not seen any empirical studies for the assumption of taxi flows, regardless of passenger and external vehicle arrivals. With massive taxi trip records, this study employs multiple hypothesis testing methods and performs extensive statistical tests. From the test results, we can expect following experiment setups in one large-scale taxi system, given strong empirical evidence:

- Homogeneous Spatial Units are the spatial divisions of one large city or taxi system. In one specific homogeneous spatial unit, both traditional street-hailing (TTS) and emerging app-based (ATS) taxi services have Poisson passenger and vehicle arrivals;
- Arrival Count Intervals are the time interval to count how many arrivals generate. The set of arrival counts should be from Poisson distribution;
- Hours and Days are of interest. The taxi activities are significantly influenced by time-of-the-day and day-of-the-week. Certain hours and days will be more likely to have Poisson arrivals, other than all days and hours.

The null hypothesis is that the observed taxi arrival events under given spatial division and time interval follow Poisson distribution. The alternative hypothesis is that the observed taxi arrival events under given spatial division and time interval do not follow Poisson distribution. The first hypothesis testing method is adapted from Kolmogorov-Smirnov test, considering the application for discrete events[1]. The correction primarily consider sample size, as equation1. Once D_n is greater than one critical value from Kolmogorov-Smirnov distribution, it rejects the null hypothesis and the observed arrival events can not be assumes as Poisson distribution at confidence level of 95%.

$$D_n = max_{x \in J} \sqrt{n} |H(x) - F_n(x)| - \frac{1}{\sqrt{n}} \tag{1}$$

where, n is sample size; J is levels of arrival count ranging from 0 to maximum count; $H(x)$ is hypothesized cumulative density function of one Poisson distribution with empirical mean estimated from random half of samples; and $F_n(x)$ is empirical cumulative density function for remaining half of samples.

Additionally, we introduce three statistical hypothesis testing method for homogeneous Poisson [2]. The null hypothesis is that taxi arrival events are from Poisson distribution with same rate across time in one given spatial division. The alternative hypothesis is that taxi arrival events are from Poisson distribution but with different rates across time in one given spatial division. Such statistical testing are applied directly on a sequence of arrival counts $\{c_1, c_2, \cdots, c_i, \cdots, c_n\}$ and are generally based on χ^2 distribution. The differences in three methods are in statistics computation. The Anscombe method first transforms original arrival counts then compute statistics with squared errors, as shown in equation 2. The likelihood ratio statistics are computed from ratio to mean value, as shown in equation 3. And the conditional χ^2 statistics are measured with ratio of squared error to mean value, as shown in equation 4. If computed statistics are greater than critical χ^2_{n-1}, it rejects null hypothesis at confidence level of 95% and the taxi arrival events are from Poisson distribution with time-dependent rates.

$$\begin{aligned} y_i &= \sqrt{c_i + \tfrac{3}{8}} \\ T_{anscombe} &= 4 \sum (y_i - \bar{y})^2 \end{aligned} \tag{2}$$

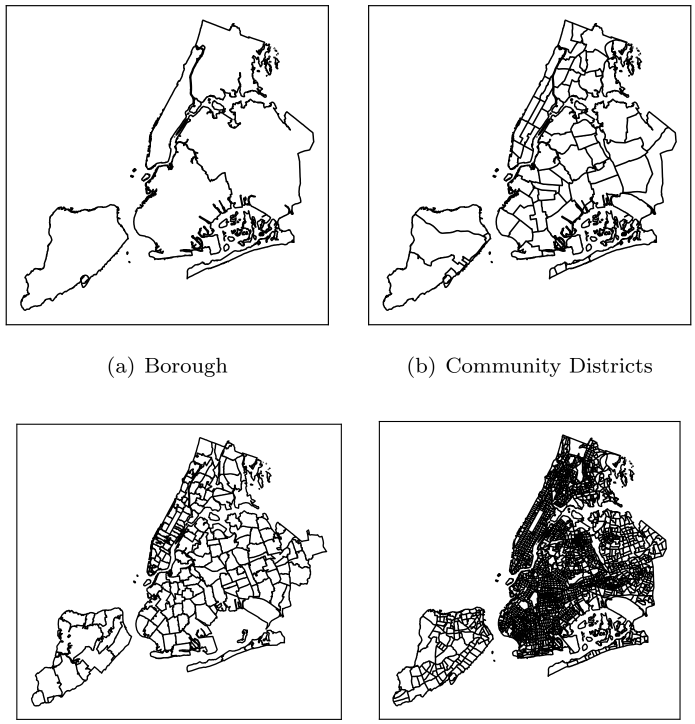
$$T_{likelihood} = 2 \sum_{i=1}^{n} c_i ln \left(\frac{c_i}{\bar{c}} \right) \tag{3}$$

$$T_{anscombe} = \sum_{i=1}^{n} \frac{(c_i - \bar{c})^2}{\bar{c}} \tag{4}$$

where, n is sample size; c_i is arrival count in a specific time interval i and spatial division; and $\bar{c}$ is mean values of all arrival counts.

The potential aggregation scales, as well as study periods of interest, are summarized as follows:

- The potential spatial scales are mainly based on four administrative divisions in NYC, including Borough (Figure 1 (a), 5 in total, $\sim$60.4 mi^2 on average per Borough), Community Districts (Figure 1 (b), 71 in total, $\sim$4.3 mi^2 on average per community district), Zip Code Tabulation Area [ZCTA] (Figure 1 (c), 214 in total, $\sim$1.4 mi^2 on average per zip code tabulation area), and Census Tracts (Figure 1 (d), 2165 in total, $\sim$0.14 mi^2 on average per census tract). Note that



(a) Borough

(b) Community Districts

(c) Zip Code Tabulation Area

(d) Census Tracts

Figure 1: Potential spatial division in New York City

  we use administrative divisions rather than grid based spatial scale, since most socioeconomic variables are only available at administrative divisions and modeling at administrative divisions will be much easier to measure socioeconomic impacts on taxi activities in future studies.

- We test seven different count intervals, that are 1-min, 5-min, 10-min, 15-min, 20-min, 30-min, and 60-min. In other words, we count TTS, ATS, or both TTS and TTS pickups (or vehicle arrivals) every count interval then test whether the flow can be described with Poisson distribution.

- In addition, we also test homogeneous period selection, considering time-of-day and day-of-week effects. Regarding the peak (or off peak) hour, we include three difference cases, including 1-hour period (peak: 6pm to 7pm, or off peak: 10am to 11am), 2-hour period (peak: 5pm to 7pm, or off peak: 9am to 11am), and 3-hour period (peak: 5pm to 8pm, or off peak: 9am to 12pm). Moreover, we classify the weekdays from Mondays to Thursdays, compared to the all seven days case.

2 Spatiotemporal Aggregation

Figure 2 to 9 show the percentage of zones not rejecting Poisson distribution with 4 methods, 7 count intervals, and day of the week. It is apparent that the smaller count interval generally leads to more zones not rejecting Poisson assumptions, across almost all plots. Although several hypothesis tests by corrected KS reveals similar percentages from 1-min to 60-min count interval, the homogeneous Poisson tests generally reject the null hypothesis of arrival counts are from one single homogeneous Poisson distribution if we have a larger count interval. Thus, we select 1-minute as the count interval in this study.

Figure 2 to Figure 5 summarize the percentages of zones where passenger pickups can be assumed

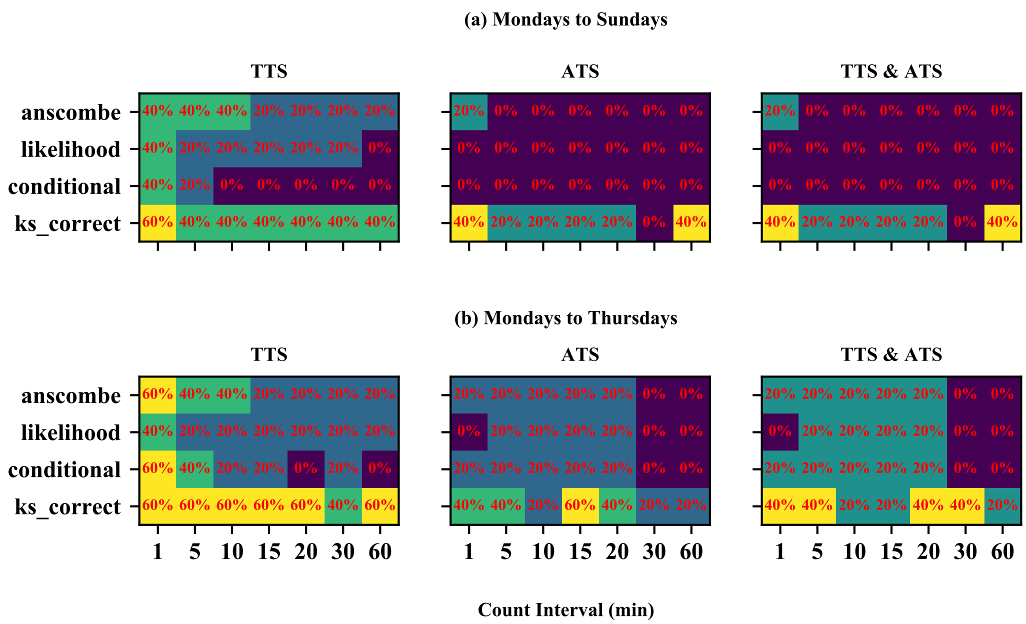


Figure 2: Hypothesis test results for passenger pickups at Boroughs in one-hour peak

as Poisson distribution at four levels of spatial scale in peak hour, respectively. Figure 6 to Figure 9 summarize the percentages of zones where passenger pickups can be assumed as Poisson distribution at four levels of spatial scale in off peak hour, respectively. Within our expectations, the too large or small aggregation does not have perfect output, since large zones have more spatial interactions and heterogeneity and small zones usually do not have any rides. Both community district and ZCTA aggregation have higher percentages. And community district aggregation generally has a slightly higher percentage, regardless of methods, taxi services, count interval and day of the week. One interesting point is that the passenger pickups of TTS and ATS are likely independent considering significant Poisson tests for TTS, ATS, and overall pickups of both TTS and ATS.

Figure 10 to Figure 13 exhibit the percentages of zones where vehicle arrivals (newly online by ATS driver partners or a new shift of TTS driver) can be assumed as Poisson distribution at four levels of spatial scale in peak hour, respectively. Figure 14 to Figure 17 exhibit the percentages of zones where vehicle arrivals (newly online by ATS driver partners or a new shift of TTS driver) can be assumed as Poisson distribution at four levels of spatial scale in off peak hour, respectively. Similar as test results for passenger pickups, both community district and ZCTA aggregation reveals higher percentages. In particular, the percentages resulted from ATS vehicle arrivals, are sometimes close to 100%.

To sum up, we choose community district to aggregate passenger and vehicle arrivals, and select 1 minute to count those arrivals.

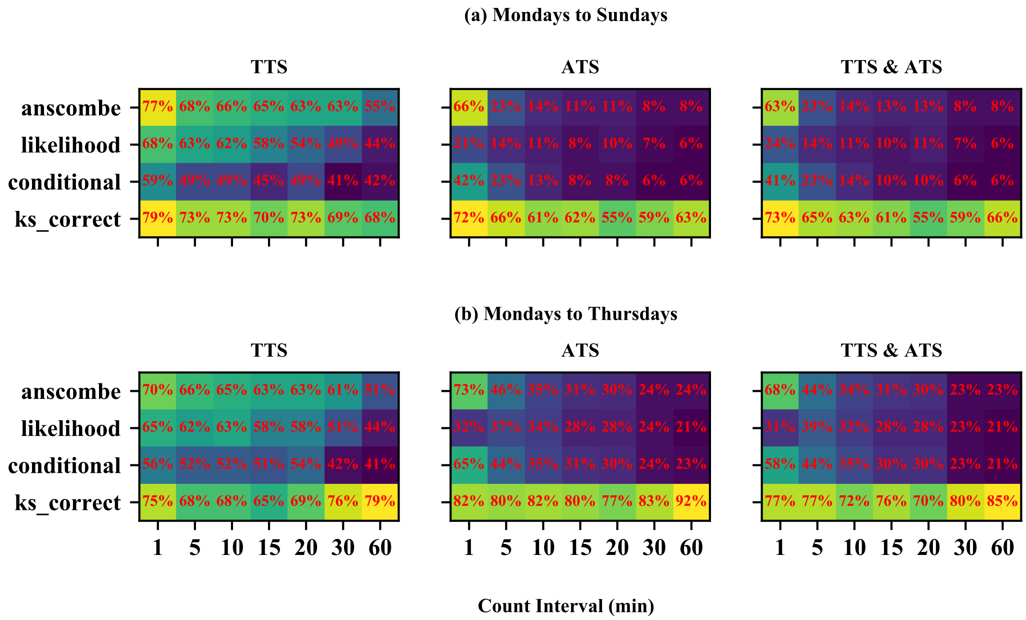


Figure 3: Hypothesis test results for passenger pickups at Community Districts in one-hour peak

3 Hours and Days of Interest

Regarding the hours and days of interest, we would like to shorten our study period to weekdays (Mondays to Thursdays) and one-hour off peak, mainly depending on the following empirical evidences:

Figure 3, 18, and 19 compare the hypothesis results for community district aggregation of passenger pickups across different levels of peak hours, as well as day of the week. As number of hours included into peak hours increase, less community districts are not rejecting Poisson distribution. In addition, limiting to the weekdays from Monday to Thursday can slightly increase percentages of significant zones. Figure 11, 20, and 21 compare the hypothesis results for community district aggregation of vehicle arrivals across different levels of off peak hours, as well as day of the week. There are no big differences in the percentages by week of the day, as well as number of hours in off peak period. However, introducing more hours or focusing on weekdays can lead to very small increases in percentages.

Figure 7, 22, and 23 compare the hypothesis results for community district aggregation of passenger pickups across different levels of off peak hours, as well as day of the week. As number of hours included into off peak increase, less community districts are not rejecting Poisson distribution. In addition, limiting to the weekdays from Monday to Thursday can slightly increase percentages of significant zones. Figure 15, 24, and 25 compare the hypothesis results for community district aggregation of vehicle arrivals across different levels of off peak hours, as well as day of the week. There are no big differences in the percentages by week of the day, as well as number of hours in off peak period. However, introducing more hours or focusing on weekdays can lead to very small increases in percentages.

4 Spatial Distribution

Given the identified aggregation scales, we find the both passenger and vehicle arrivals can be assumed with Poisson distribution, in most community districts, as shown in Figure 7 and 15. In addition, we plot the spatial distribution of community districts not rejecting Poisson distribution in Figure 26 and 27. The both figures indicate that those insignificant (i.e. rejecting Poisson assumption) community

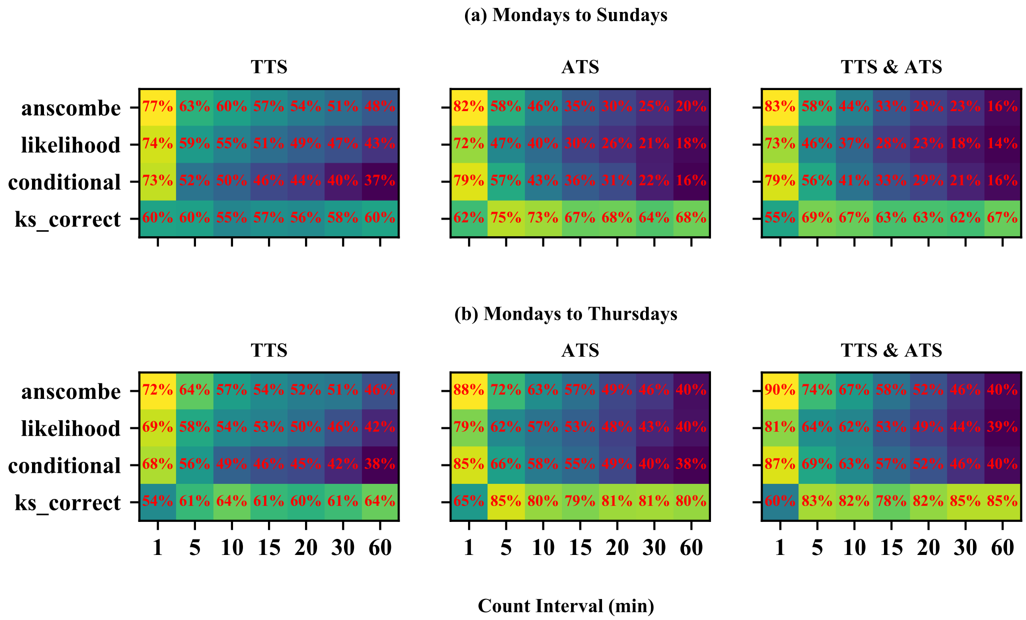


Figure 4: Hypothesis test results for passenger pickups at ZCTA in one-hour peak

districts mainly locate in remote suburban areas, generally with rare TTS and ATS activities. In New York City, most taxi activities concentrate in Manhattan downtown and midtown, as well as two airports, Brooklyn downtown, and Queens downtown. Our hypothesis tests strongly support the Poisson assumption on passenger and vehicle arrivals in those areas. For more details on TTS and ATS activities and facts in NYC, you can refer to 2018 NYC TAXI FACT BOOK (`http://www.nyc.gov/html/tlc/downloads/pdf/2018_tlc_factbook.pdf`).

References

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[2] Lawrence D. Brown and Linda H. Zhao. A test for the poisson distribution. *Sankhy: The Indian Journal of Statistics, Series A (1961-2002)*, 64(3):611–625, 2002.

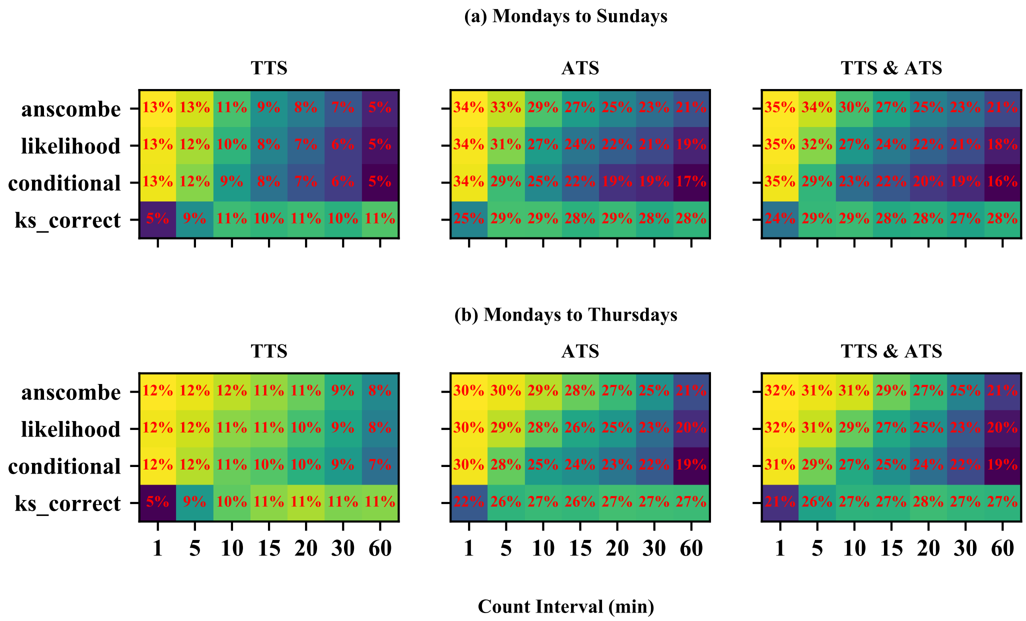


Figure 5: Hypothesis test results for passenger pickups at Census Tracts in one-hour peak

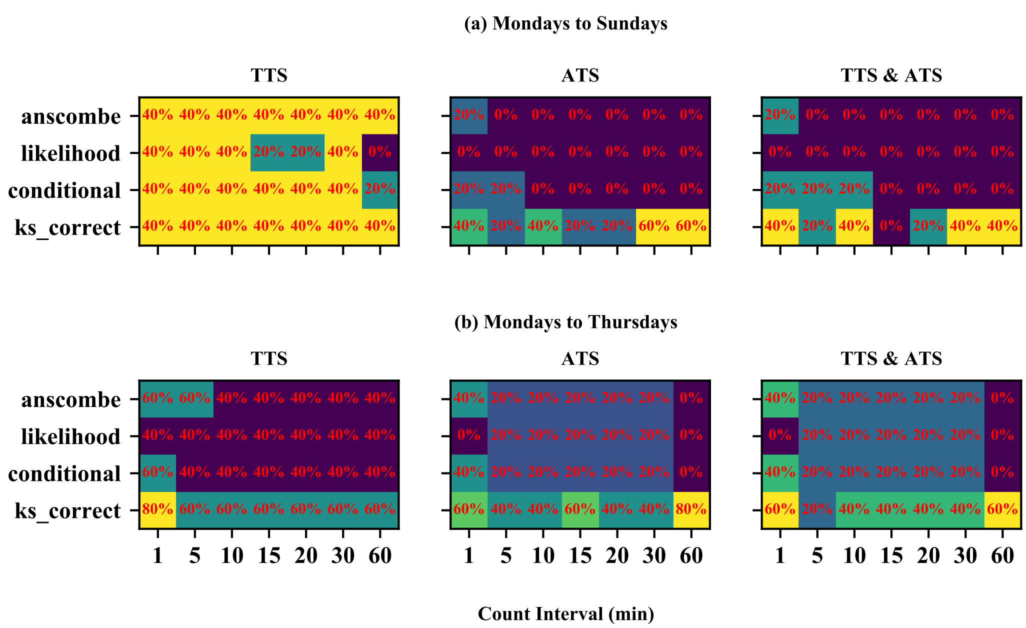


Figure 6: Hypothesis test results for passenger pickups at Boroughs in one-hour off peak

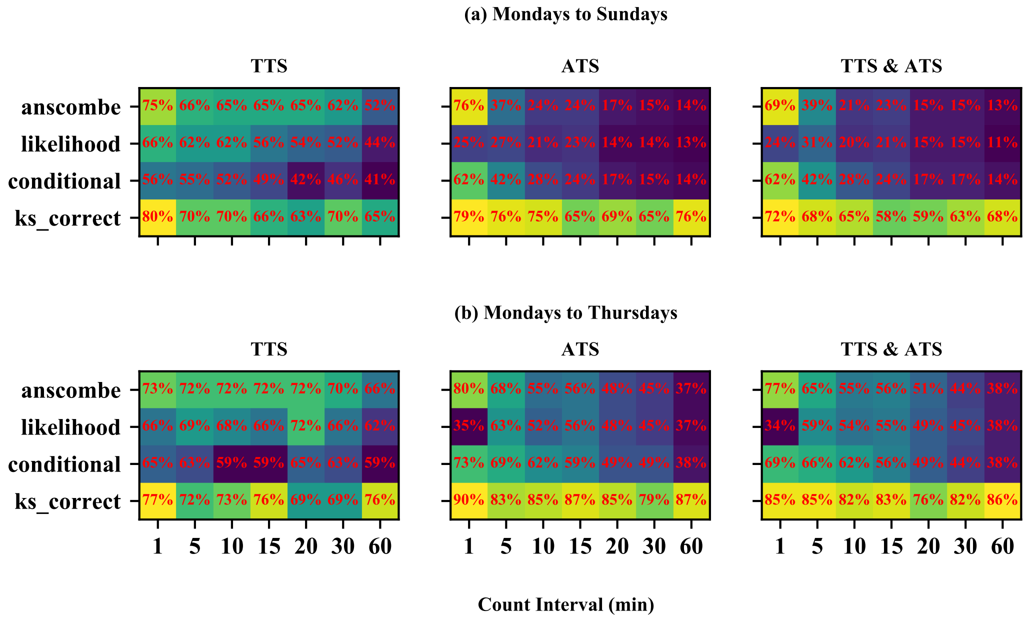


Figure 7: Hypothesis test results for passenger pickups at Community Districts in one-hour off peak

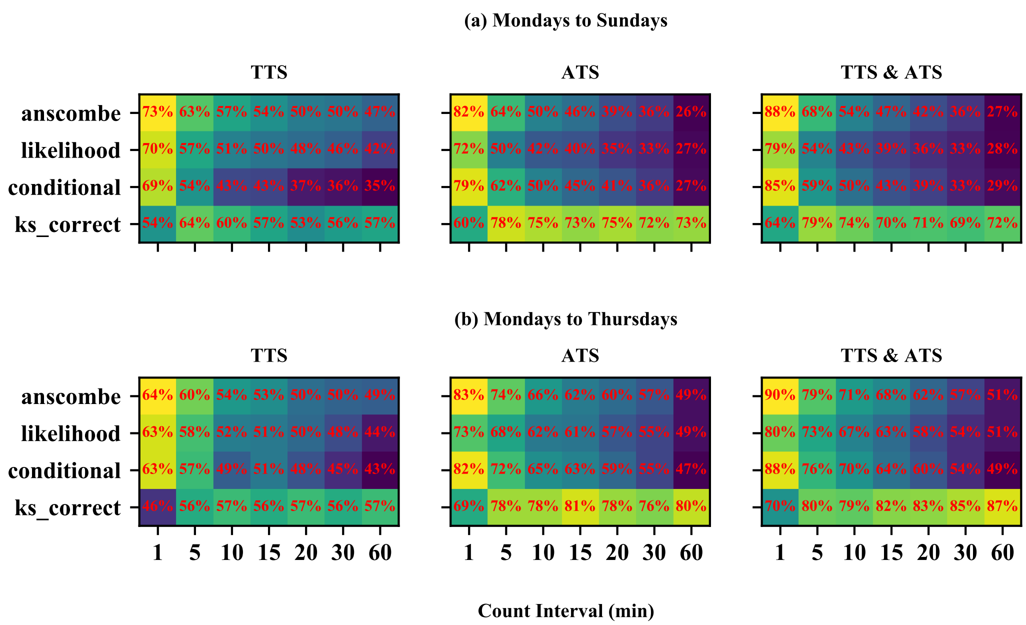


Figure 8: Hypothesis test results for passenger pickups at ZCTA in one-hour off peak

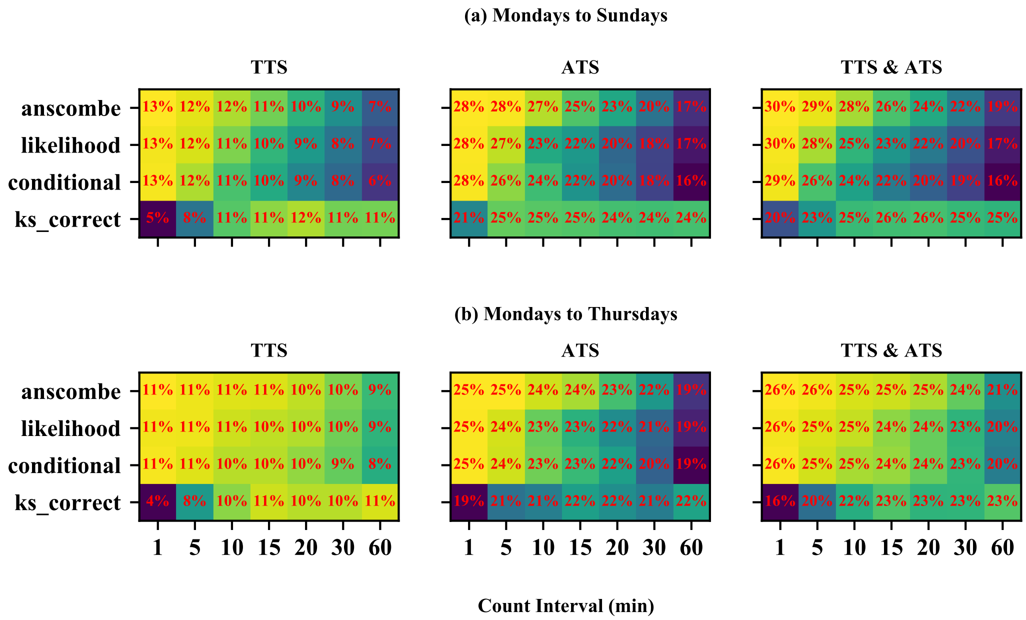


Figure 9: Hypothesis test results for passenger pickups at Census Tracts in one-hour off peak

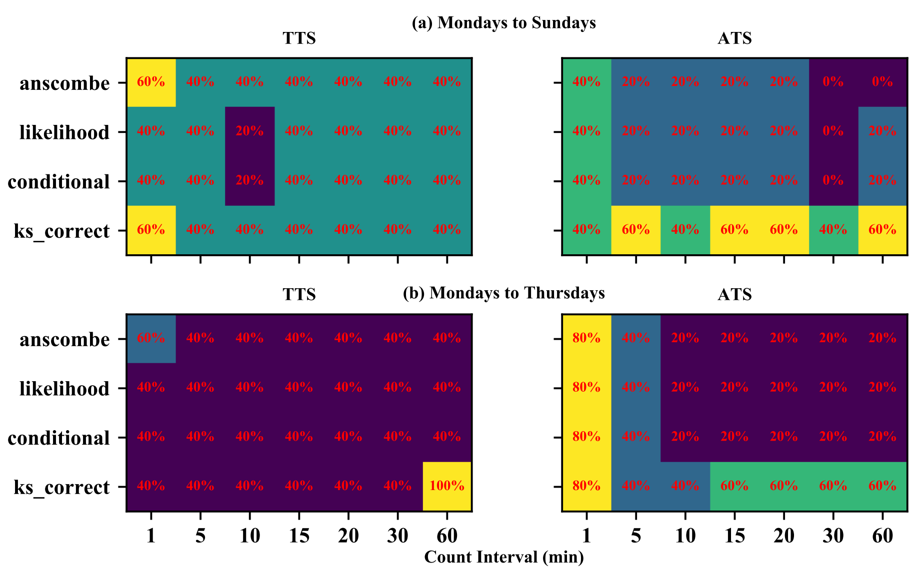


Figure 10: Hypothesis test results for vehicle arrivals at Boroughs in one-hour peak

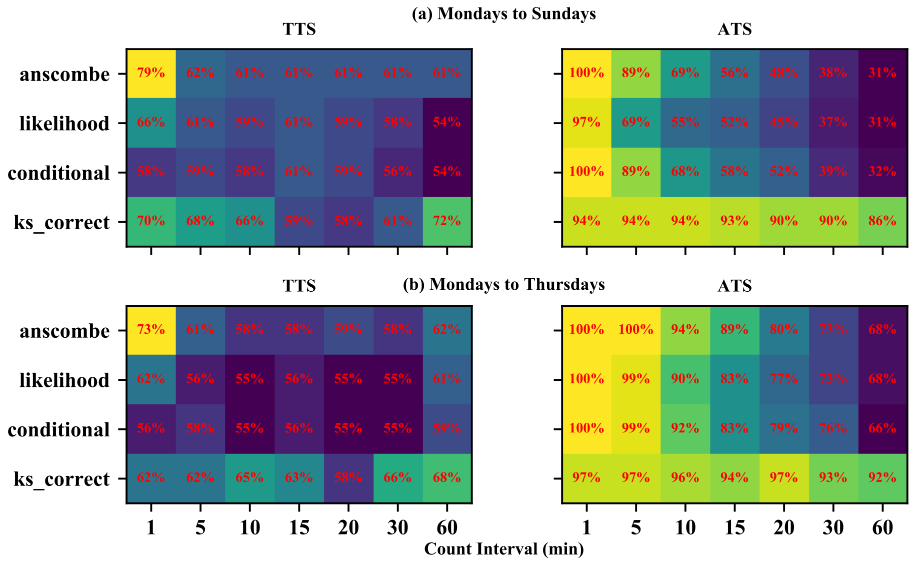


Figure 11: Hypothesis test results for vehicle arrivals at Community Districts in one-hour peak

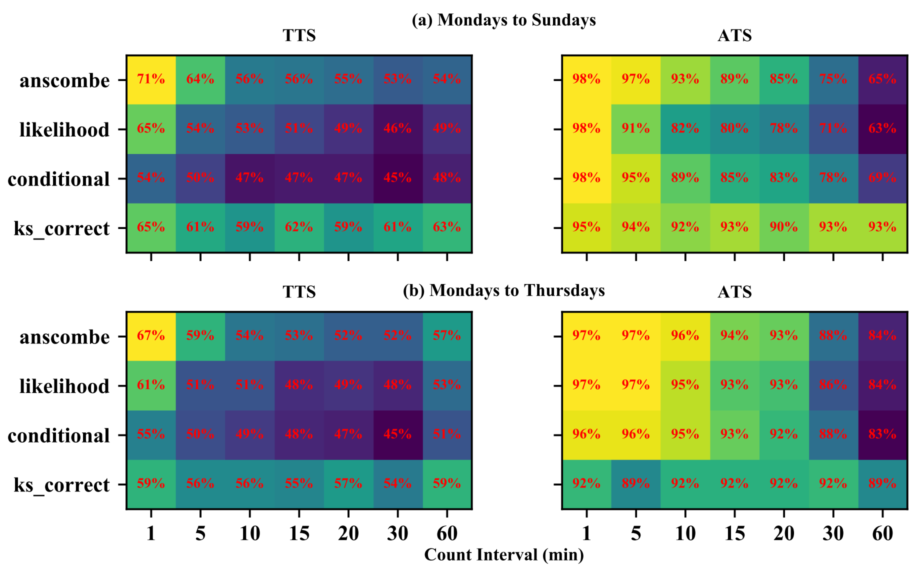


Figure 12: Hypothesis test results for vehicle arrivals at ZCTA in one-hour peak

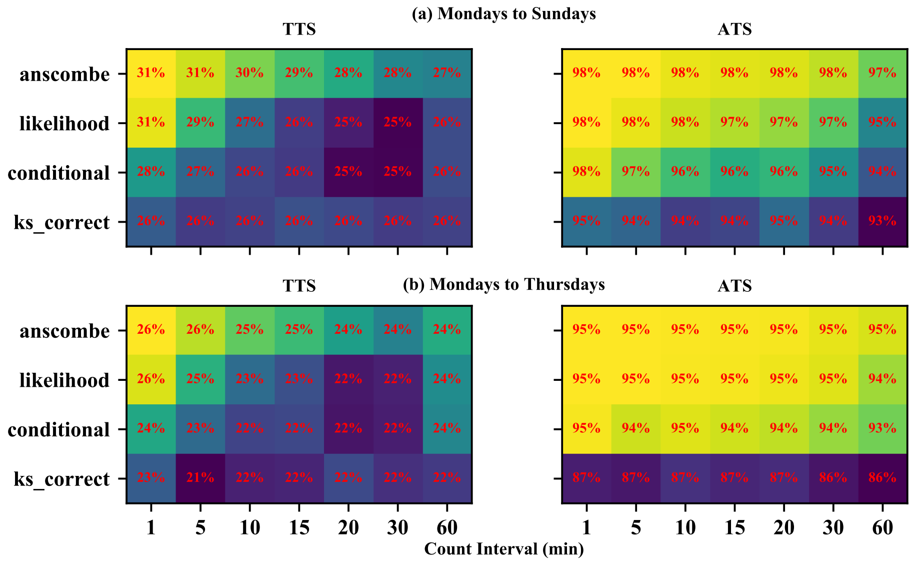


Figure 13: Hypothesis test results for vehicle arrivals at Census Tracts in one-hour peak

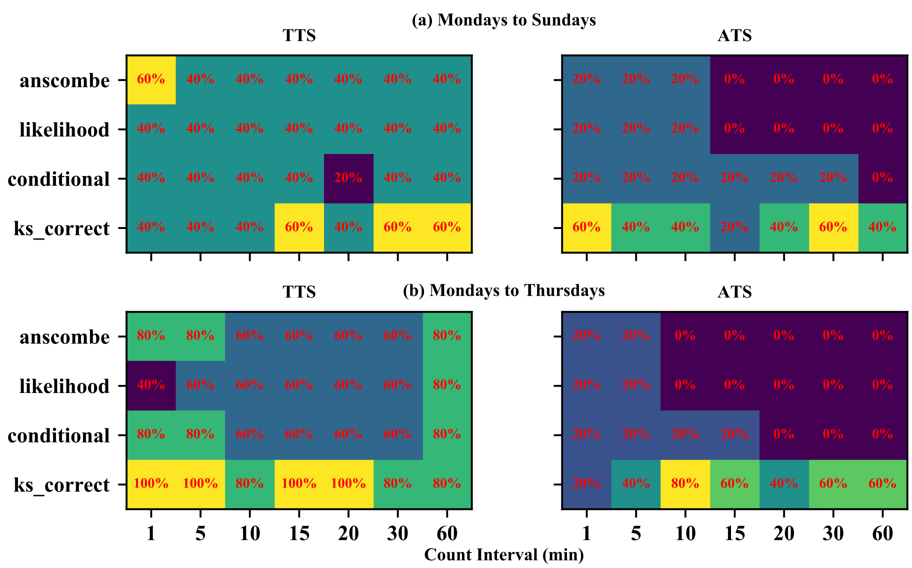


Figure 14: Hypothesis test results for vehicle arrivals at Boroughs in one-hour off peak

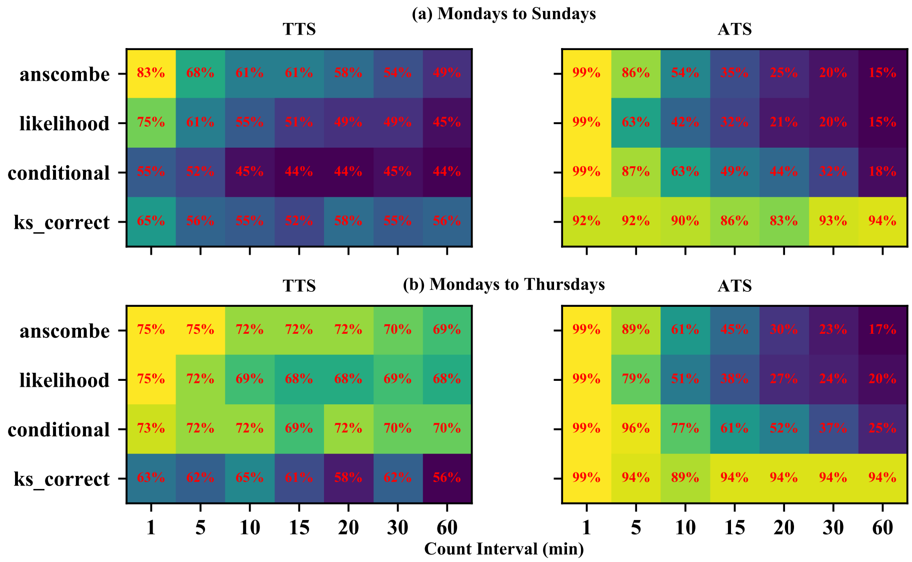


Figure 15: Hypothesis test results for vehicle arrivals at Community Districts in one-hour off peak

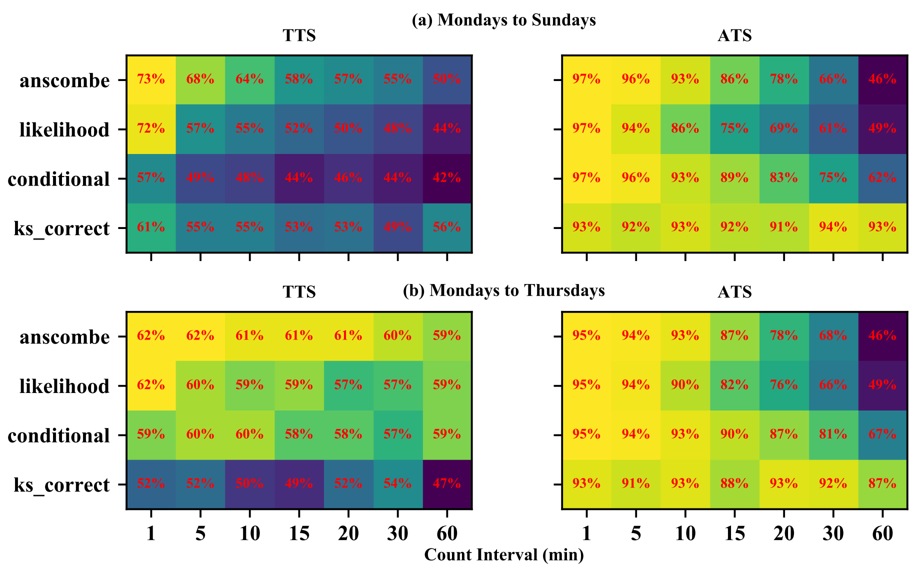


Figure 16: Hypothesis test results for vehicle arrivals at ZCTA in one-hour off peak

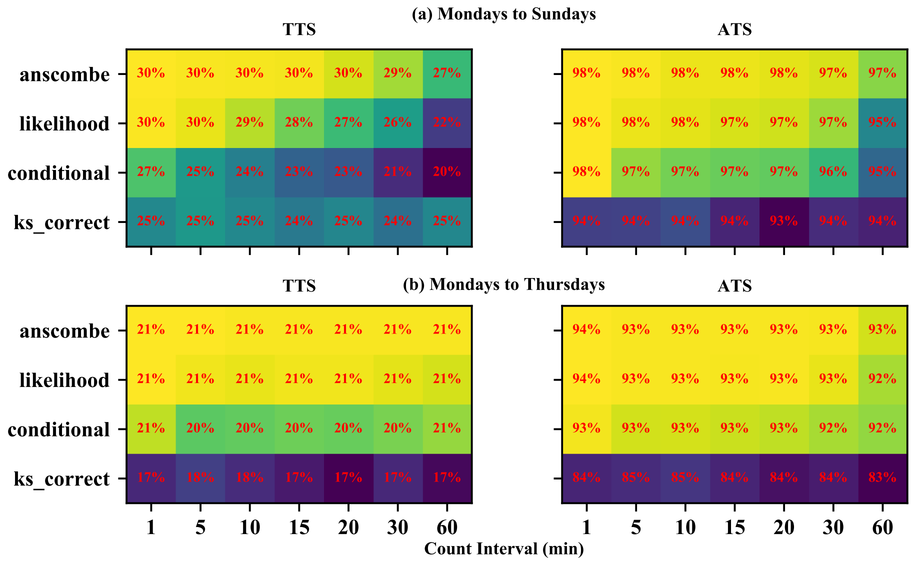


Figure 17: Hypothesis test results for vehicle arrivals at Census Tracts in one-hour off peak

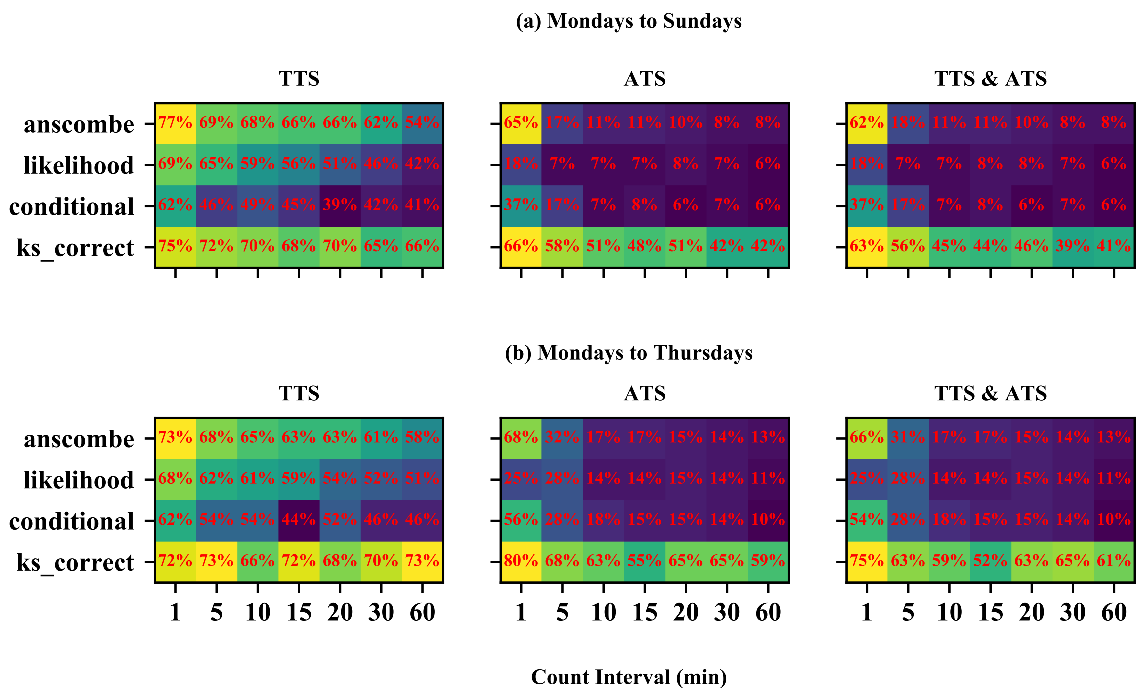


Figure 18: Hypothesis test results for passenger pickups at Community Districts in 2-hour off peak

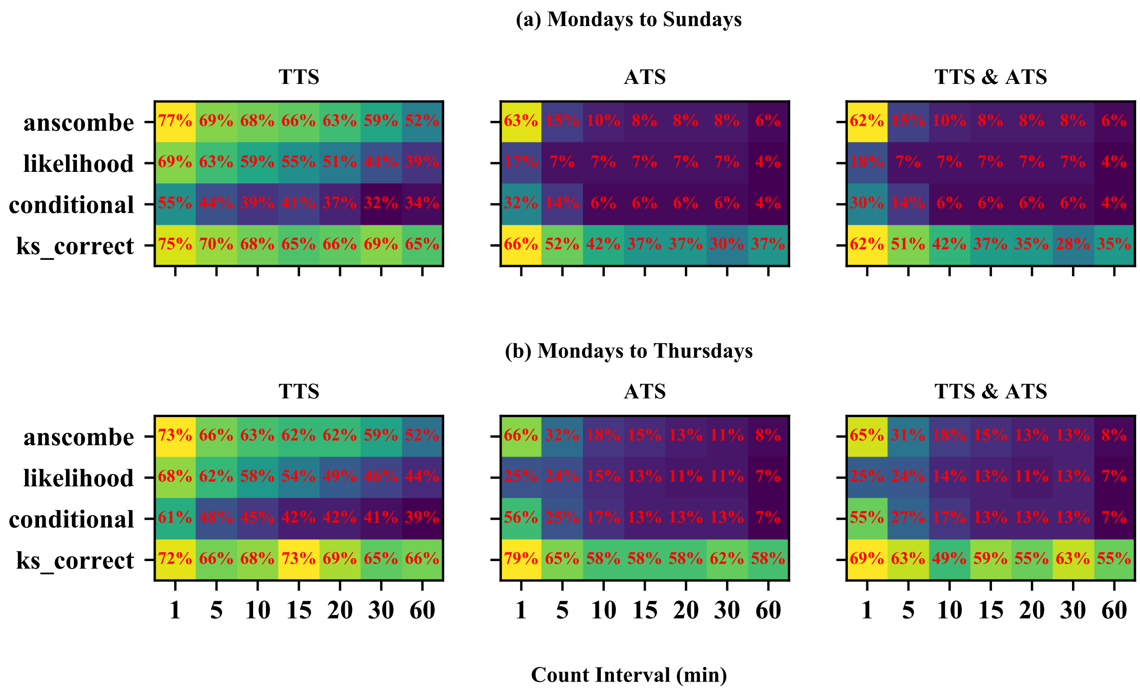


Figure 19: Hypothesis test results for passenger pickups at Community Districts in 3-hour off peak

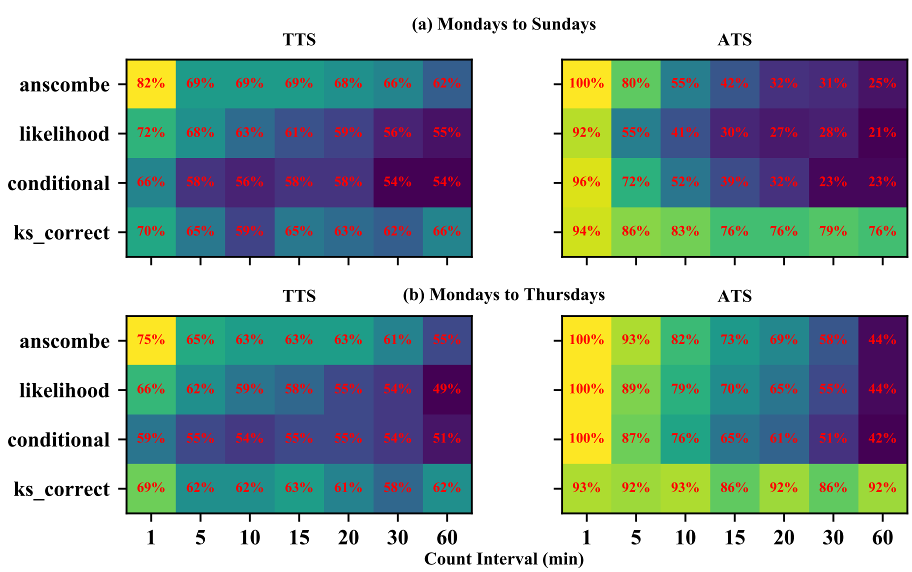


Figure 20: Hypothesis test results for vehicle arrivals at Community Districts in 2-hour off peak

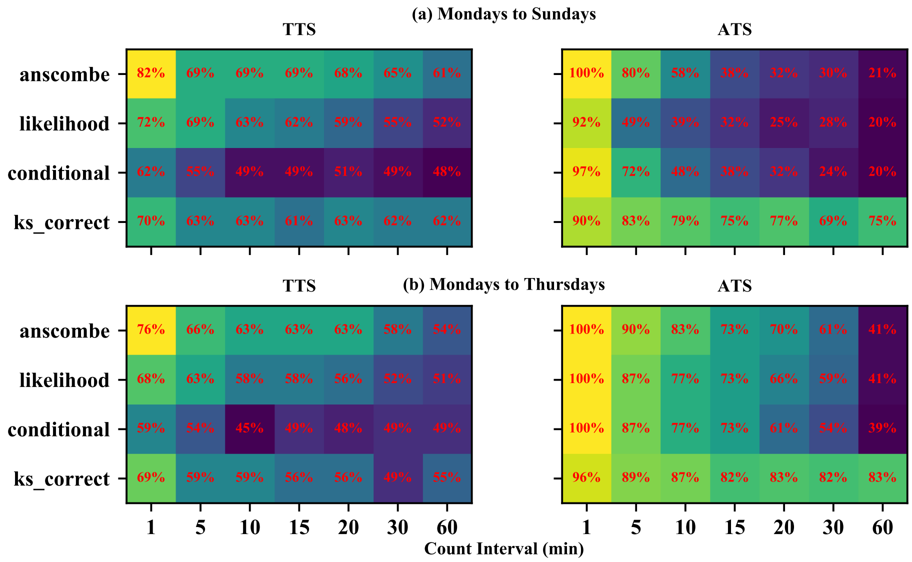


Figure 21: Hypothesis test results for vehicle arrivals at Community Districts in 3-hour off peak

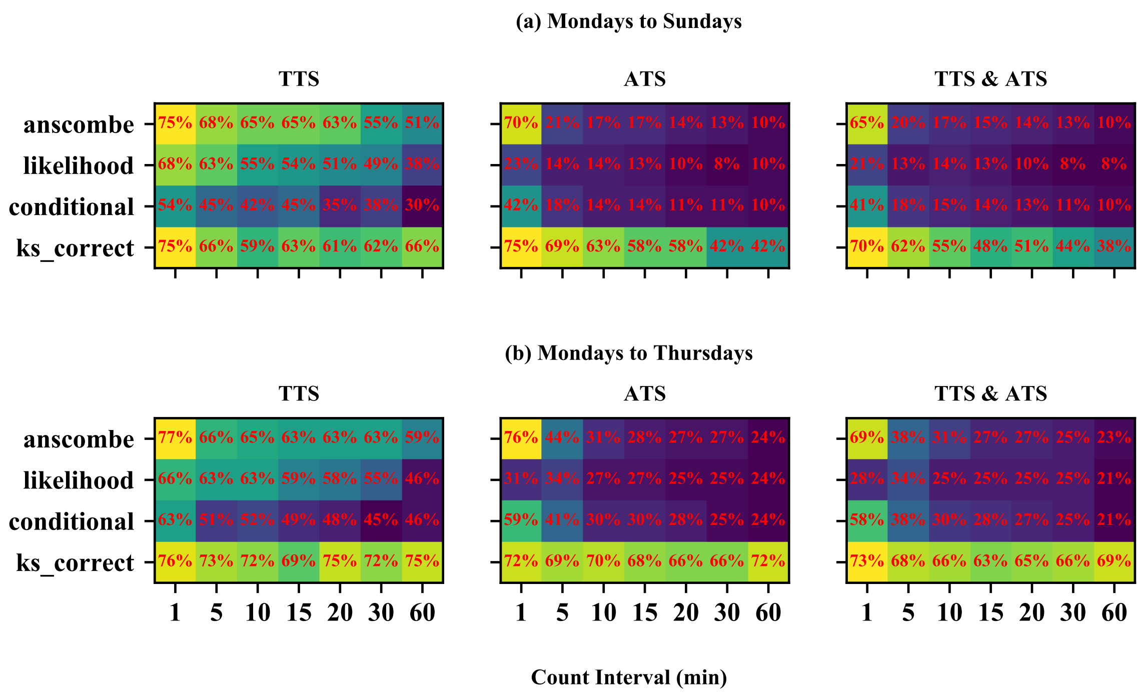


Figure 22: Hypothesis test results for passenger pickups at Community Districts in 2-hour off peak

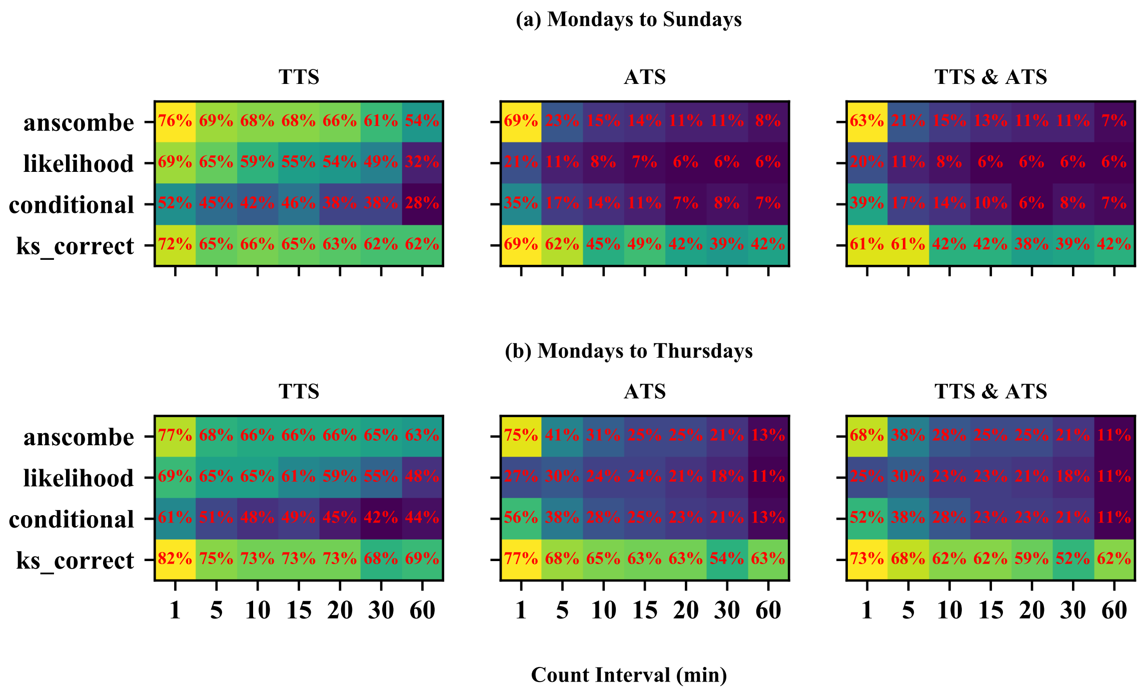


Figure 23: Hypothesis test results for passenger pickups at Community Districts in 3-hour off peak

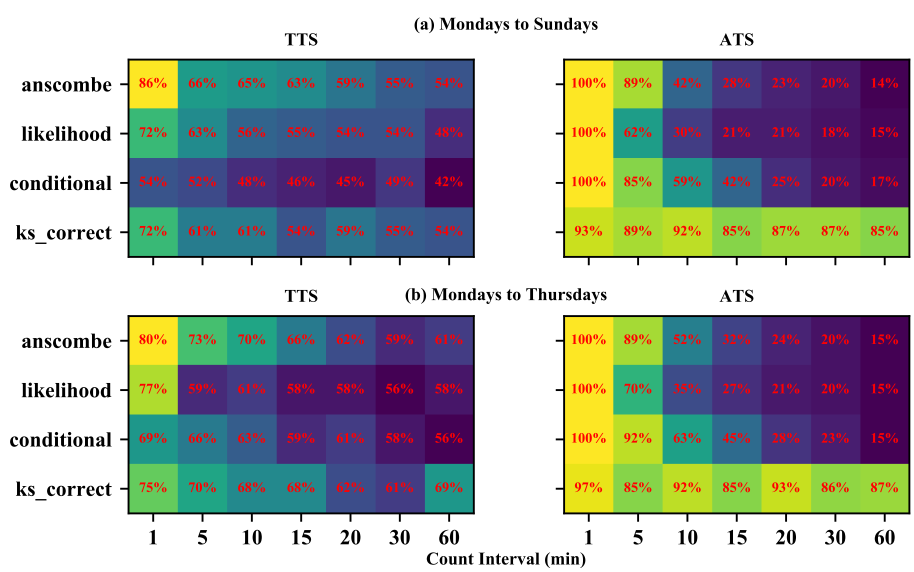


Figure 24: Hypothesis test results for vehicle arrivals at Community Districts in 2-hour off peak

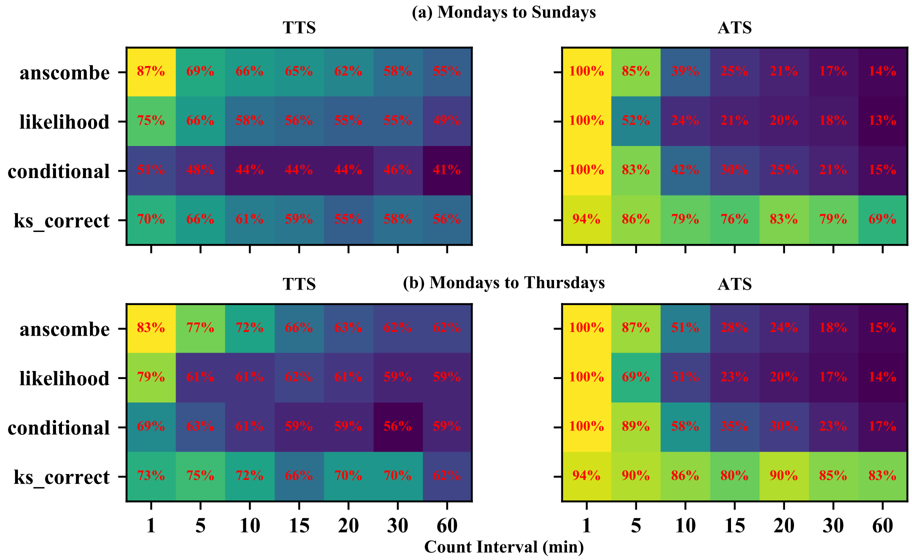
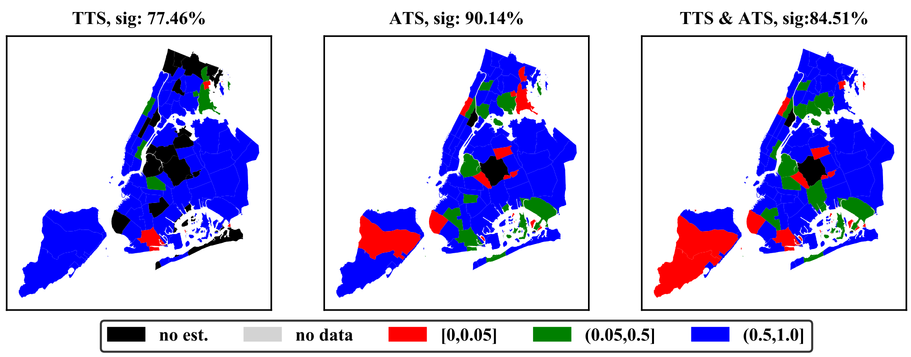
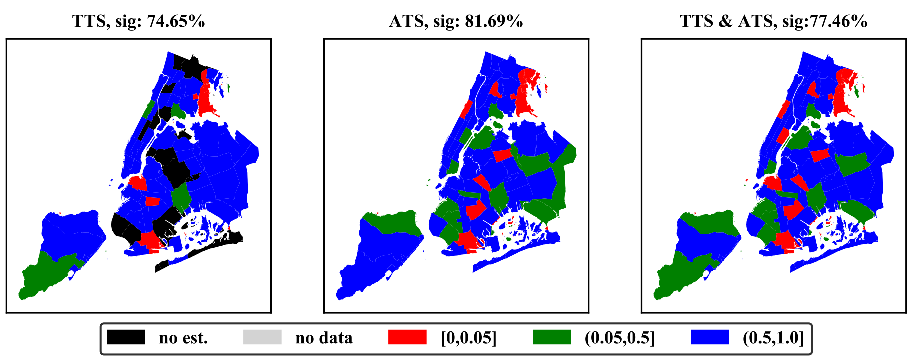


Figure 25: Hypothesis test results for vehicle arrivals at Community Districts in 3-hour off peak

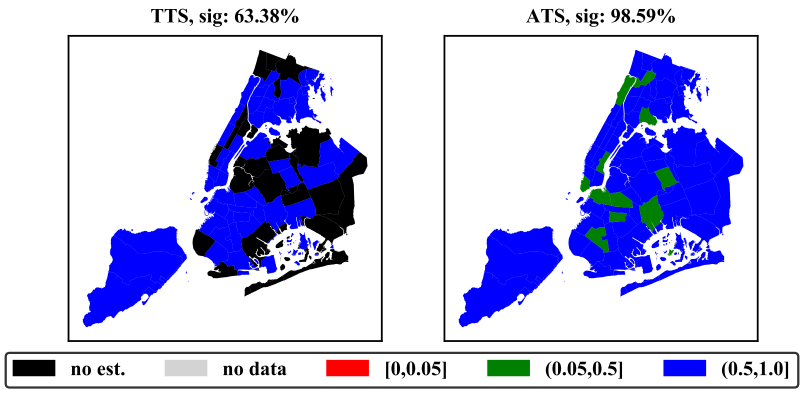


(a) 1-min count interval in 1-hour off peak

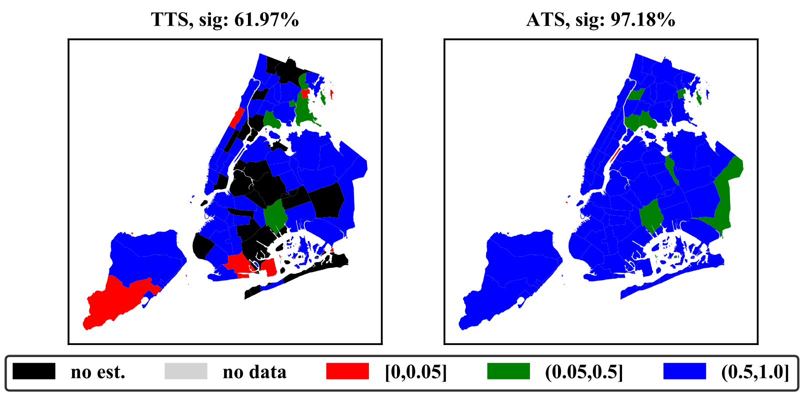


(b) 1-min count interval in 1-hour peak

Figure 26: Hypothesis test results for passenger pickups by Community Districts in weekdays. Note: 'sig' indicates percentage of community districts not rejecting Poisson distribution, represented by blue and green color



(a) 1-min count interval in 1-hour off peak



(b) 1-min count interval in 1-hour peak

Figure 27: Hypothesis test results for vehicle arrivals by Community Districts in weekdays. Note: 'sig' indicates percentage of community districts not rejecting Poisson distribution, represented by blue and green color