1.

A.

By formula:

$$\text{Arithmetic Return} = \frac{P_t - P_{t-1}}{P_{t-1}}$$

Total stand deviation is the sum of each stock's deviation.

So we can get:

```
--- Arithmetic Returns (Last 5 Rows) ---
                 SPY      AAPL      EQIX
Date
2024-12-27 -0.011492 -0.014678 -0.006966
2024-12-30 -0.012377 -0.014699 -0.008064
2024-12-31 -0.004603 -0.008493  0.006512
2025-01-02 -0.003422 -0.027671  0.000497
2025-01-03  0.011538 -0.003445  0.015745

--- Arithmetic Returns Standard Deviation ---
SPY     0.008077
AAPL    0.013483
EQIX    0.015361
dtype: float64
```

B. Basic the same logic as part A, but I use formula:

$$\text{Log Return} = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

Drop first line because there is no return data previous the first day.

```
--- Log Returns (Last 5 Rows) ---
                 SPY      AAPL      EQIX
Date
2024-12-27 -0.011515 -0.014675 -0.006867
2024-12-30 -0.012410 -0.014696 -0.007972
2024-12-31 -0.004577 -0.008427  0.006602
2025-01-02 -0.003392 -0.027930  0.000613
2025-01-03  0.011494 -0.003356  0.015725

--- Log Returns Standard Deviation ---
SPY     0.008078
AAPL    0.013446
EQIX    0.015270
dtype: float64
```

2.

A. I use `Portfolio Value = 100 * SPY + 200 * AAPL + 150 * EQIX`

portfolio_value = 251862.4969482422

B.

Normal-Distribution

1. Calculate the arithmetic returns for each stock:

$$\text{Return} = \frac{\text{Price Today} - \text{Price Yesterday}}{\text{Price Yesterday}}$$

2. The covariance matrix was calculated using exponentially weighted moving average (EWMA)

$$\text{Covariance Matrix} = \text{EWMA}(\text{Returns}, \lambda)$$

:

3. Standard deviation was calculated using the portfolio weights and covariance matrix:

$$\text{Portfolio Std} = \sqrt{\mathbf{w}^T \cdot \text{Covariance Matrix} \cdot \mathbf{w}}$$

VaR and ES:

$$\text{VaR} = \text{Mean} + \text{Portfolio Std} \cdot \Phi^{-1}(\alpha)$$

$$\text{ES} = \text{Mean} - \text{Portfolio Std} \cdot \frac{\phi(\Phi^{-1}(\alpha))}{\alpha}$$

T-Distrribution

1. Generating multivariate normally distributed random samples using correlation coefficient matrices

$$\mathbf{Z} \sim \mathcal{N}(\mathbf{0}, \text{Correlation Matrix})$$

The samples were converted to a uniform distribution using the cumulative distribution function (CDF) of the standard normal distribution:

$$\mathbf{U} = \Phi(\mathbf{Z})$$

Converting uniformly distributed samples to T-distributed samples using the t-distributed quantile function (PPF):

$$\text{Portfolio Returns} = \mathbf{T} \cdot \mathbf{w}$$

VaR and ES:

VaR:

$$\text{VaR} = \text{Percentile}(\text{Portfolio Returns}, \alpha)$$

ES:

$$\text{ES} = \text{Mean}(\text{Portfolio Returns} \leq \text{VaR})$$

3. Historical

Calculates VaR and ES directly using historical yield data.

No assumptions about the distribution of yields, complete reliance on historical data

$$\text{Portfolio Returns} = \text{Returns} \cdot \mathbf{w}$$

VaR and ES:

VaR：

$$\text{VaR} = \text{Percentile}(\text{Portfolio Returns}, \alpha)$$

ES：

$$\text{ES} = \text{Mean}(\text{Portfolio Returns} \leq \text{VaR})$$

Therefore, Result:

```
--- Individual Stock VaR & ES ---

SPY:
  VaR (Normal): $827.85
  VaR (T-Dist): $774.88
  VaR (Historical): $872.40
  ES (Normal): $1,056.91
  ES (T-Dist): $1,018.48
  ES (Historical): $1,080.10

AAPL:
  VaR (Normal): $946.08
  VaR (T-Dist): $1,030.11
  VaR (Historical): $1,067.11
  ES (Normal): $1,324.49
  ES (T-Dist): $1,409.73
  ES (Historical): $1,437.79

EQIX:
  VaR (Normal): $2,933.51
  VaR (T-Dist): $3,406.99
  VaR (Historical): $3,635.08
  ES (Normal): $4,255.14
  ES (T-Dist): $4,669.60
  ES (Historical): $4,714.89

--- Portfolio VaR & ES ---
VaR (Normal): $3,856.32
VaR (T-Dist): $5,959.16
VaR (Historical): $4,575.03
ES (Normal): $5,449.90
ES (T-Dist): $7,425.65
ES (Historical): $6,059.39
```

3.

A.

Implied Volatility:

I fuse folsove, ccording to formula:

$$C_{BS}(S, K, r, T, \sigma) = C_{\text{obs}}$$

I get: Implied Volatility = 33.51%

B.

Delta: The amount by which an option price changes when the asset price changes by 1 dollar.

Call：$\Delta = N(d_1)$

Put：$\Delta = N(d_1) - 1$

I get Call Delta = 0.6659
I get Put Delta = -0.3341

Vega: The amount of change in the option price for a 1% change in volatility.

$$V = SN'(d_1)\sqrt{T}$$

I get: Vega = 5.6407

Theta: the amount of change in the option price as time decreases by 1 day.

Call:

$$\Theta = -\frac{SN'(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT}N(d_2)$$

Put:

$$\Theta = -\frac{SN'(d_1)\sigma}{2\sqrt{T}} + rKe^{-rT}N(-d_2)$$

I get Call Theta = -5.5446
I get Put Theta = -2.6186

C.
Verify Put-Call Parity:

$$C - P = S - Ke^{-rT}$$

Put price = 1.1045
Since LHS = 1.8955, RHS = 1.8956
The calculated error is close to 0 (0.0001), indicating that Put-Call parity holds.

D. Delta Normal Approximation

$$\Delta_{\text{portfolio}} = \Delta_{\text{call}} + \Delta_{\text{put}} + 1$$

$$\Theta_{\text{portfolio}} = \Theta_{\text{call}} + \Theta_{\text{put}}$$

I get Mean Portfolio Change = -0.6403
Std Portfolio Change = 2.1704

Stock price volatility over 20 days:

$$\sigma_{\text{20-day}} = \sigma_{\text{annual}} \times \sqrt{\frac{20}{255}}$$

Portfolio VaR at the 5% confidence level:

$$\text{VaR}_\alpha = -(\text{Theta contribution} + \Delta \times \sigma_{\text{20-day}} \times z_\alpha)$$

VaR(5%) = 4.2103

$$\mathrm{ES}_\alpha = -(\text{Theta contribution} + \Delta \times \sigma_{\text{20-day}} \times \frac{N(z_\alpha)}{\alpha})$$

ES(5%) = 3.8367

(E). Monte Carlo Simulation

$$S_{\text{final}} = S_0 \times \exp\left(-\frac{1}{2}\sigma^2 dt + \sigma\sqrt{dt} \cdot Z\right)$$

S_0 = Initial stock price

$dt = \frac{1}{255}$ (single trading day step)

$Z \sim N(0,1)$ (standard normal random variable)

VaR (5%) is the 5th percentile of the sorted returns: VaR(5%) = 3.3695
ES (5%) is the average of losses beyond VaR: ES(5%) = 4.1637

The Delta Normal approximation assumes that asset returns are normally distributed and only considers the effects of Delta and Vega. Monte Carlo simulation, on the other hand, estimates the value distribution of an option by modeling a large number of possible paths, and is able to capture more nonlinear effects. Typically, Monte Carlo simulation is more accurate, but more computationally expensive.