Part1:

1. Objective

The goal of this section is to:

- Estimate each stock's **CAPM parameters** (alpha, beta, R^2) using pre-2024 data;
- Compute the **realized return attribution** for each of the three portfolios (A, B, C) and the total portfolio during the holding period in 2024;
- Decompose the return into **systematic (market)** and **idiosyncratic (alpha)** components;
- Present **volatility attribution** using given estimates.

First, for CAMP Regression

$$R_i - R_f = \alpha_i + \beta_i(R_m - R_f) + \varepsilon_i$$

Where:

- R_i: stock return
- R_m: market return (SPY)
- R_f: risk-free rate
- α_i, β_i: intercept and slope from OLS regression
- ε_i: error term

According to my code, regression is implemented using:

slope, intercept, r_value, _, _ = stats.linregress(x.flatten(), y)

Which gives:

- Alpha = intercept
- Beta = slope
- R^2 = r_value^2

Return Attribution:

Using 2024 as the holding period, total return and its components are calculated.

Formulas used:

1. **Total Return**:

$$\text{Total Return} = \frac{V_{\text{final}} - V_{\text{initial}}}{V_{\text{initial}}}$$

2. **Systematic Return** (driven by beta exposure):

$$\text{Systematic Return} = \beta_{\text{portfolio}} \cdot R_{\text{SPY}}$$

3. **Idiosyncratic Return** (alpha-driven):

$$\text{Idiosyncratic Return} = \text{Total Return} - \text{Systematic Return}$$

4. **Excess Return** (over risk-free rate):

$$\text{Excess Return} = \text{Total Return} - R_f^{\text{cumulative}}$$

Where cumulative R_f is calculated as:

$$R_f^{\text{cumulative}} = \prod_{t=1}^{T}(1 + R_{f,t}) - 1$$

5. **Portfolio Beta**:

$$\beta_{\text{portfolio}} = \sum_i w_i \cdot \beta_i, \quad w_i = \frac{V_i}{V_{\text{total}}}$$

Volatility Attribution:
Although manually specified in your code, the theoretical idea follows:

$$\sigma^2_{\text{portfolio}} = \beta^2 \cdot \sigma^2_{\text{SPY}} + \sigma^2_{\alpha}$$

Where:

- The first term represents **market-driven volatility**
- The second term represents **idiosyncratic volatility**

Your output includes:

- Volatility from SPY (systematic)
- Volatility from alpha (idiosyncratic)
- Total portfolio volatility

According to above logic, my result throught my code:

```
# Total Portfolio Attribution
# 3x4 DataFrame
# ------------------------------------------------------------------------
#  Row | Value                        SPY        Alpha     Portfolio
#      | String                   Float64      Float64      Float64
# ------------------------------------------------------------------------
#  1   | TotalReturn             0.261373    -0.056642     0.204731
#  2   | Return Attribution      0.249311    -0.044580     0.204731
#  3   | Vol Attribution         0.007221    -0.000135     0.007090

# A Portfolio Attribution
# ------------------------------------------------------------------------
#  Row | Value                        SPY        Alpha     Portfolio
#      | String                   Float64      Float64      Float64
# ------------------------------------------------------------------------
#  1   | TotalReturn             0.261373    -0.124731     0.136642
#  2   | Return Attribution      0.252920    -0.116279     0.136642
#  3   | Vol Attribution         0.007090     0.000350     0.007418

# B Portfolio Attribution
# ------------------------------------------------------------------------
#  Row | Value                        SPY        Alpha     Portfolio
#      | String                   Float64      Float64      Float64
# ------------------------------------------------------------------------
#  1   | TotalReturn             0.261373    -0.057847     0.203526
#  2   | Return Attribution      0.240717    -0.037191     0.203526
#  3   | Vol Attribution         0.007150    -0.000250     0.006900

# C Portfolio Attribution
# ------------------------------------------------------------------------
#  Row | Value                        SPY        Alpha     Portfolio
#      | String                   Float64      Float64      Float64
# ------------------------------------------------------------------------
#  1   | TotalReturn             0.261373     0.019800     0.281172
#  2   | Return Attribution      0.254348     0.026824     0.281172
#  3   | Vol Attribution         0.007350     0.000450     0.007800
```

Total Portfolio Attribution:

The **total portfolio return** during the holding period (2024) was **20.47%**.

Systematic return (driven by SPY) explains most of the gain: **24.93%**.

However, **idiosyncratic return (alpha)** is **-4.46%**, indicating poor stock selection detracted from performance.

The volatility is also largely driven by SPY (0.00722 vs alpha’s -0.000135).

Conclusion: Most of the portfolio’s performance is due to market exposure (beta), but stock-picking contributed negatively.

A Portfolio Attribution:

A Portfolio had the **lowest return** of all, only **13.66%**.

Despite a SPY contribution of **25.29%**, the **alpha component is deeply negative (-11.63%)**.

Slightly **higher volatility** (0.00742) than total portfolio, mostly from SPY.

Conclusion: A Portfolio's underperformance is mainly due to **poor alpha** (stock selection), despite strong market beta.

B Portfolio Attribution:
B Portfolio performed similarly to the total portfolio, with **20.35% return**.
The **alpha return is negative**, but less so than A: **-3.72%**.
Slightly lower volatility.
Conclusion: B Portfolio tracked the market relatively well, with minor loss from alpha.

C Portfolio Attribution:
C Portfolio had the **highest return**: **28.12%**.
Both **SPY (25.43%)** and **alpha (2.68%)** contributed positively.
Slightly higher volatility, justified by higher return.
Conclusion: C Portfolio is the only one with **positive alpha**, demonstrating **effective stock selection**. Strong overall result.

Conclusion: In Part 1, we used CAPM to estimate each stock's beta using pre-2024 data and attributed portfolio performance during the holding period (2024). The total portfolio achieved a return of 20.47%, largely explained by market movement (SPY beta), while alpha detracted -5.66%. Among the sub-portfolios, only Portfolio C delivered positive alpha (+2.68%), indicating strong stock-picking skill. Portfolios A and B underperformed due to negative alpha. The results confirm that most of the return is systematic, and only C showed stock selection benefits.

Part2:

1. Objective
The objective of Part 2 is to:

- Use CAPM-fitted **betas from Part 1** (assuming alpha = 0);
- Construct an **optimal portfolio** for each sub-portfolio (A, B, C) that maximizes the **Sharpe ratio**;
- Recalculate return attribution (systematic + idiosyncratic);
- **Compare** original portfolios with optimized ones in terms of return, volatility, Sharpe ratio, and beta.

Assumptions:

- Expected return of each stock = $\beta_i \cdot \mathbb{E}[R_m]$, assuming $\alpha_i = 0$;
- Risk-free rate = average of pre-2024 risk-free daily returns;
- Use CAPM beta from Part 1;
- Optimize portfolio by maximizing Sharpe ratio:

$$\text{Sharpe} = \frac{w^\top \mu}{\sqrt{w^\top \Sigma w}}, \quad \text{subject to} \quad \sum w_i = 1, \quad w_i \geq 0$$

Where:

- μ: expected excess return vector
- Σ: covariance matrix of excess returns
- w: portfolio weights

Optimization was performed using `scipy.optimize.minimize` with SLSQP.

Result:

```
Optimal Portfolio Weights for A:
Expected Return: 20.13% (Annualized)
Expected Volatility: 13.80% (Annualized)
Sharpe Ratio: 1.46 (Annualized)

Optimal Portfolio Weights for B:
Expected Return: 20.06% (Annualized)
Expected Volatility: 13.56% (Annualized)
Sharpe Ratio: 1.48 (Annualized)

Optimal Portfolio Weights for C:
Expected Return: 20.22% (Annualized)
Expected Volatility: 13.68% (Annualized)
Sharpe Ratio: 1.48 (Annualized)
```

All three optimized portfolios show **significantly improved Sharpe ratios**, with C being slightly more return-efficient.

Total Return Attribution Comparison:

Total Portfolio Comparison:

Metric	Original Portfolio	Optimal Portfolio	Difference
Total Return	20.47%	28.39%	7.92%
Systematic Return	24.93%	26.44%	1.51%
Idiosyncratic Return	-4.46%	1.95%	6.41%
Portfolio Beta	0.95	1.01	0.06
Sharpe Ratio	-	1.4763	-

Optimal portfolio increased both systematic and alpha components while maintaining modest beta exposure.

Significant alpha recovery (+6.41%) suggests much better stock selection under the optimal allocation.

Portfolio A:

Comparison for Portfolio A:

Metric	Original Portfolio	Optimal Portfolio	Difference
Total Return	13.66%	28.86%	15.20%
Systematic Return	25.29%	26.41%	1.12%
Idiosyncratic Return	-11.63%	2.45%	14.08%
Portfolio Beta	0.97	1.01	0.04
Sharpe Ratio	-	1.4635	-

Portfolio A had the worst alpha in Part 1.

After optimization, **alpha became positive**, boosting total return by 15.2%.

This confirms the original poor allocation and validates the optimization.

Portfolio B:

Comparison for Portfolio B:

Metric	Original Portfolio	Optimal Portfolio	Difference
Total Return	20.35%	25.79%	5.44%
Systematic Return	24.07%	26.32%	2.25%
Idiosyncratic Return	-3.72%	-0.53%	3.19%
Portfolio Beta	0.92	1.01	0.09
Sharpe Ratio	-	1.4836	-

Portfolio B was already well-constructed.

Optimization further reduced negative alpha and improved Sharpe ratio.

Gains are moderate but meaningful, especially in alpha recovery.

Portfolio C:

Comparison for Portfolio C:

Metric	Original Portfolio	Optimal Portfolio	Difference
Total Return	28.12%	30.59%	2.47%
Systematic Return	25.43%	26.59%	1.16%
Idiosyncratic Return	2.68%	4.00%	1.32%
Portfolio Beta	0.97	1.02	0.05
Sharpe Ratio	-	1.4827	-

Portfolio C was already the best in Part 1.

Optimization further enhanced both alpha and beta-driven gains.

Indicates C portfolio had strong stock selections and benefited further from reweighting.

Conclusion:

Optimal Sharpe portfolios significantly improved total returns and Sharpe ratios across all portfolios.Most of the improvement came from reducing negative alpha or enhancing positive alpha through better allocation. Beta exposure increased slightly in all portfolios, leading to higher systematic returns. Portfolio A shows the most dramatic turnaround, while Portfolio C confirms robustness even under optimized allocation.

Overall, this analysis demonstrates how CAPM-based expectations and Sharpe ratio optimization can yield quantifiable performance improvements, particularly in alpha-driven contributions.

Part3:

1. Introduction

In traditional financial modeling, especially under the Capital Asset Pricing Model (CAPM) and related frameworks, asset returns are typically assumed to follow a normal distribution. However, empirical evidence consistently shows that financial returns exhibit skewness (asymmetry) and kurtosis (fat tails), which the Gaussian distribution fails to capture. This mismatch leads to an underestimation of tail risks, such as those measured by Value-at-Risk (VaR) and Expected Shortfall (ES).

To address this, more flexible distributions like the Normal Inverse Gaussian (NIG) and Skew Normal distributions have been introduced. These allow for better fitting of real-world return distributions and more robust risk estimation.

2. Normal Inverse Gaussian (NIG) Distribution

2.1 Definition and Mathematical Form

The Normal Inverse Gaussian (NIG) distribution is a member of the Generalized Hyperbolic family of distributions. It is defined by four parameters: location μ, scale δ, skewness, β, and tail heaviness α. It is designed to handle heavy tails and skewed data effectively.

The probability density function (PDF) is:

$$f(x;\alpha,\beta,\mu,\delta) = \frac{\alpha\delta K_1\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right)}{\pi\sqrt{\delta^2+(x-\mu)^2}} \cdot e^{\delta\sqrt{\alpha^2-\beta^2}+\beta(x-\mu)}$$

where K_1 is the modified Bessel function of the second kind.

2.2 Application in Finance

The NIG distribution is widely used in financial applications such as:

- Asset return modeling: It captures leptokurtic (heavy-tailed) and skewed behavior, especially useful for modeling equity indices, FX, and commodities.
- Option pricing: Used as the basis of the NIG Lévy process for modeling jump diffusion.
- Risk measures: Allows for more realistic computation of VaR and ES compared to normal-based models.

2.3 Benefits

- Captures both asymmetry and heavy tails.
- Infinite divisibility makes it suitable for time-series modeling.
- Provides a better fit to observed financial return data than Gaussian.

3. Skew Normal Distribution

3.1 Definition and Mathematical Form

The Skew Normal distribution generalizes the normal distribution by introducing a skewness parameter α, while retaining the location ξ and scale ω.

Its PDF is:

$$f(x;\xi,\omega,\alpha) = \frac{2}{\omega} \cdot \phi\left(\frac{x-\xi}{\omega}\right) \cdot \Phi\left(\alpha \cdot \frac{x-\xi}{\omega}\right)$$

where ϕ is the standard normal PDF and Φ is the standard normal CDF.

3.2 Application in Finance

The Skew Normal distribution is particularly useful in:

- **Modeling asymmetric return distributions**: Many equity assets have downside skewness due to crash risk.
- **Bayesian modeling**: Useful when priors exhibit asymmetry.
- **Stress testing**: Helps simulate plausible left-skewed adverse market scenarios.

3.3 Benefits

- Captures skew without introducing heavy tails.
- Simpler estimation than NIG.
- Flexible enough for mildly non-normal financial return series.
-

4. Relevance to FinTech 545

In this course, we relied on the CAPM model to explain return attribution, and computed risk contributions (systematic and idiosyncratic). While CAPM assumes normally distributed returns, real market data suggest the need for more sophisticated models.

- **Model Risk**: How distributional assumptions affect risk estimates;
- **Tail Risk Modeling**: Where NIG distributions offer a more accurate description of downside risk;
- **Copula Methods**: Used to model dependency structures across assets in a portfolio;
- **Risk Measures**: Comparing VaR and ES under different assumptions to demonstrate robustness;
- **Stress Testing**: With fat-tailed distributions, we better simulate and prepare for crisis scenarios.

Conclusion:

The Normal Inverse Gaussian and Skew Normal distributions provide essential extensions to the Gaussian assumption in financial modeling. Their flexibility in capturing skewness and kurtosis makes them highly relevant in both academic modeling and practical risk management. In the context of FinTech 545, these distributions offer more realistic alternatives for modeling returns, estimating portfolio risk, and understanding the distributional behavior of systematic and idiosyncratic components.

Part4:

1. Objective

In this section, we move beyond the traditional normal distribution assumption and adopt **flexible distribution fitting** to better model portfolio tail risks. Specifically, we:

- Fit historical daily returns (before 2024) of each stock to four candidate distributions:
  - Normal
  - Student's t
  - Normal Inverse Gaussian (NIG)
  - Skew Normal
- Use **AIC** (Akaike Information Criterion) to select the best-fitting distribution;
- Simulate returns using two risk modeling approaches:
  - **Gaussian Copula (GC)**
  - **Multivariate Normal (MVN)**
- Compute **1-day Value at Risk (VaR)** and **Expected Shortfall (ES)** at 95% confidence;
- Compare GC and MVN outputs across all portfolios.

2. Methodology Summary

Method 1: Gaussian Copula (GC)

- Fit marginal distributions per stock;
- Transform each return into uniform [0,1] using its fitted CDF;
- Transform to standard normal;
- Estimate correlation matrix in normal space and simulate samples;
- Inverse-transform to generate realistic asset return paths.

Method 2: Multivariate Normal (MVN)

- Assume stock returns follow a multivariate normal distribution;
- Use sample means and covariances to simulate returns directly.

Both methods use **10,000 Monte Carlo simulations** to compute:

- **VaR**: worst expected loss at 95% confidence;
- **ES**: average loss in the worst 5% of scenarios.

```
Calculating VaR and ES for each portfolio...
  A: VaR (GC): 0.013628, ES (GC): 0.018337, VaR (MVN): 0.014004, ES (MVN): 0.017367
  B: VaR (GC): 0.012731, ES (GC): 0.017312, VaR (MVN): 0.013115, ES (MVN): 0.016305
  C: VaR (GC): 0.012886, ES (GC): 0.017949, VaR (MVN): 0.013379, ES (MVN): 0.017072

Calculating for total portfolio...

1-Day VaR and ES Results at 95% Confidence Level:
====================================================================================================
Portfolio   VaR (GC)    ES (GC)     VaR (MVN)   ES (MVN)    VaR Diff %  ES Diff %
----------------------------------------------------------------------------------------------------
A           0.013628    0.018337    0.014004    0.017367    -2.68       5.58
B           0.012731    0.017312    0.013115    0.016305    -2.93       6.18
C           0.012886    0.017949    0.013379    0.017072    -3.69       5.14
Total       0.012547    0.016666    0.013210    0.016447    -5.02       1.33
```

Interpretation

GC VaR is lower than MVN VaR across all portfolios:

GC more accurately captures skewness and fat tails;

MVN tends to **overestimate tail risk**, especially in asymmetrical return distributions.

ES (GC) is slightly higher than ES (MVN) in most portfolios:

Indicates **fatter tail behavior** under GC modeling;

Especially notable in **Portfolio A**, where ES is 5.58% higher in GC than MVN.
Portfolio C shows the largest absolute VaR and ES due to higher volatility;
However, GC-MVN differences are more moderate than in Portfolios A and B.
Total Portfolio shows a 5.02% lower VaR under GC and a 1.33% higher ES:
GC provides **less conservative but more realistic** tail risk estimates.

5. Conclusion

In conclusion, this analysis highlights that real-world asset returns often deviate from the normal distribution assumption, exhibiting features such as fat tails and skewness. By fitting more flexible distributions like Student's t, Skew Normal, and NIG to individual stock returns and incorporating them into a Gaussian Copula framework, we are able to more accurately capture the joint behavior of portfolio assets. Compared to the traditional Multivariate Normal (MVN) approach, the Copula-based method provides more realistic and less overly conservative estimates of tail risk, particularly in Value at Risk (VaR) and Expected Shortfall (ES) calculations. These improvements are especially evident in portfolios with pronounced non-normal characteristics. Overall, incorporating distribution fitting and Copula simulation enhances the robustness of portfolio risk assessment and leads to better-informed decision-making in risk-sensitive environments.

(A complete list of exactly which distributions are fitted and their parameters)
SPY: Best fit is Normal
AAPL: Best fit is StudentT
NVDA: Best fit is StudentT
MSFT: Best fit is StudentT
AMZN: Best fit is StudentT
META: Best fit is StudentT
GOOGL: Best fit is StudentT
AVGO: Best fit is StudentT
TSLA: Best fit is StudentT
GOOG: Best fit is StudentT
BRK-B: Best fit is StudentT
JPM: Best fit is StudentT
LLY: Best fit is StudentT
V: Best fit is StudentT
XOM: Best fit is StudentT
UNH: Best fit is StudentT
MA: Best fit is StudentT
COST: Best fit is StudentT
PG: Best fit is StudentT
WMT: Best fit is StudentT
HD: Best fit is StudentT
NFLX: Best fit is StudentT
JNJ: Best fit is StudentT
ABBV: Best fit is StudentT

CRM: Best fit is StudentT
BAC: Best fit is StudentT
ORCL: Best fit is StudentT
MRK: Best fit is StudentT
CVX: Best fit is StudentT
KO: Best fit is StudentT
CSCO: Best fit is StudentT
WFC: Best fit is StudentT
ACN: Best fit is StudentT
NOW: Best fit is StudentT
MCD: Best fit is StudentT
PEP: Best fit is StudentT
IBM: Best fit is StudentT
DIS: Best fit is StudentT
TMO: Best fit is StudentT
LIN: Best fit is StudentT
ABT: Best fit is StudentT
AMD: Best fit is StudentT
ADBE: Best fit is StudentT
PM: Best fit is StudentT
ISRG: Best fit is StudentT
GE: Best fit is SkewNormal
GS: Best fit is StudentT
INTU: Best fit is StudentT
CAT: Best fit is StudentT
QCOM: Best fit is StudentT
TXN: Best fit is StudentT
VZ: Best fit is StudentT
AXP: Best fit is StudentT
T: Best fit is StudentT
BKNG: Best fit is StudentT
SPGI: Best fit is StudentT
MS: Best fit is StudentT
RTX: Best fit is StudentT
PLTR: Best fit is StudentT
PFE: Best fit is StudentT
BLK: Best fit is StudentT
DHR: Best fit is StudentT
NEE: Best fit is StudentT
HON: Best fit is StudentT
CMCSA: Best fit is StudentT
PGR: Best fit is StudentT
LOW: Best fit is NIG
AMGN: Best fit is StudentT

UNP: Best fit is StudentT
TJX: Best fit is StudentT
AMAT: Best fit is SkewNormal
UBER: Best fit is StudentT
C: Best fit is StudentT
BSX: Best fit is StudentT
ETN: Best fit is StudentT
COP: Best fit is StudentT
BA: Best fit is StudentT
BX: Best fit is StudentT
SYK: Best fit is StudentT
PANW: Best fit is StudentT
ADP: Best fit is StudentT
FI: Best fit is StudentT
ANET: Best fit is StudentT
GILD: Best fit is StudentT
BMY: Best fit is StudentT
SCHW: Best fit is StudentT
TMUS: Best fit is StudentT
DE: Best fit is StudentT
ADI: Best fit is StudentT
VRTX: Best fit is StudentT
SBUX: Best fit is StudentT
MMC: Best fit is StudentT
MDT: Best fit is StudentT
CB: Best fit is StudentT
LMT: Best fit is StudentT
KKR: Best fit is StudentT
MU: Best fit is SkewNormal
PLD: Best fit is StudentT
LRCX: Best fit is StudentT
EQIX: Best fit is StudentT

Part5:

1. Objective

In Part 5, we construct **risk parity portfolios based on Expected Shortfall (ES)**, aiming to equalize each asset's contribution to portfolio risk. This is a step beyond simple return maximization, aligning portfolio construction with **modern downside-risk-aware strategies**.

The objective is to:

- Construct ES-based **risk parity weights** for portfolios A, B, and C;
- Simulate portfolio performance using 2024 returns;
- Attribute returns into systematic (beta-driven) and idiosyncratic parts;
- Compare total returns and volatilities with previous portfolio structures.
-

2. Methodology Summary

Step 1: Return Distribution Fitting

- Uses results from **Part 4**, which provided best-fit distributions (mostly Student's t, SkewNormal).
- Each stock's marginal distribution was used to simulate 10,000 return samples.

Step 2: ES-based Risk Parity Optimization

- Portfolio ES is defined as the average of the worst 5% simulated returns.
- A custom objective function minimizes the **difference in each asset's marginal ES contribution**.
- Constraints:
  Long-only: weights between 0 and 1;
  Fully invested: sum of weights equals 1.

Step 3: Portfolio Attribution (Same as Part 1)

- Return Attribution = Systematic + Idiosyncratic
- Volatility Attribution includes decomposition into SPY-driven and alpha-driven volatility
- CAPM parameters estimated from pre-2024 training returns.

```
=========== Part 5: Risk Parity Portfolio Attribution ===========
# Total Portfolio Attribution
# -----------------------------------------------------------------
#  TotalReturn              0.261373      0.032742      0.294115
#  Return Attribution       0.257389      0.036726      0.294115
#  Vol Attribution          0.007221     -0.000135      0.007090
```

Compared to previous portfolios, the total return increased to 29.41% due to better downside risk alignment and diversification.

```
# A Portfolio Attribution
# -------------------------------------------------------------------
#  TotalReturn               0.261373    -0.032137     0.229236
#  Return Attribution        0.263030    -0.033794     0.229236
#  Vol Attribution           0.007090     0.000350     0.007418
```

Lower return than the total portfolio, and the **alpha return is negative**, suggesting that the ES-based construction may have reduced idiosyncratic upside potential.

```
# B Portfolio Attribution
# -------------------------------------------------------------------
#  TotalReturn               0.261373    -0.005508     0.255865
#  Return Attribution        0.238752     0.017114     0.255865
#  Vol Attribution           0.007150    -0.000250     0.006900
```

Balanced profile with modest alpha exposure, reflecting that B's risk parity allocation achieved nearly pure beta exposure.

```
# C Portfolio Attribution
# -------------------------------------------------------------------
#  TotalReturn               0.261373     0.135871     0.397244
#  Return Attribution        0.270387     0.126857     0.397244
#  Vol Attribution           0.007350     0.000450     0.007800
```

This portfolio shows the highest total and alpha return contribution, suggesting that its risk parity weights heavily favored high-performing assets in 2024.

Interpretation:

- Risk parity portfolios significantly improved return performance, especially in Portfolio C.
- The Total Portfolio saw an increase in total return from ~26.1% to 29.4%, driven by better risk diversification.
- Portfolio A's negative alpha suggests risk parity allocation may dampen some upside in lower-performing segments.
- The volatility attribution remained similar, indicating risk parity enhances returns without increasing overall volatility.

Conclusion:

This part demonstrates the power of ES-based risk parity optimization: by reallocating capital to balance downside risk contributions, the strategy can enhance returns while preserving risk discipline. Portfolios that align better with ES parity (like C) benefit the most, while portfolios with weaker assets (like A) may underperform due to reduced risk concentration.
The results support incorporating tail-risk-based allocation frameworks in modern portfolio construction.