Supplementary: Interrelated Dense Subgraph Detection in Multilayered Networks

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ABSTRACT

We provide detailed information about related algorithms, give more experimental results and analysis, and more case studies of real-world networks, to support the main paper.

A ALGORITHMS AND ANALYSIS

A.1 Densest Subgraph Detection

Here, we provide details of some dense subgraph detection algorithms, which consider the undirected graph $G = (\mathcal{V}, \mathcal{E})$.

Algorithm 1 summarizes the greedy detection algorithm Greedy Solver. It takes the original nodeset $\mathcal V$ as the starting point, then greedily removes the node with the smallest degree from the graph G (i.e., the selection step in Line 3), and returns the densest subgraph among the shrinking sequence of subgraphs created by the procedure. When using the *priority tree* data structure to manage the nodes during the peeling process, the time complexity of the Greedy Solver algorithm is $O(|\mathcal E|\log|\mathcal V|)$.

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Algorithm 1: GreedySolver: Densest Subgraph Detection with Greedy algorithm
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Input: Undirected graph G = (\mathcal{V}, \mathcal{E}); density metric \rho(\cdot)
Output: Nodeset of the optimal densest subgraph of G.

1 \mathcal{V}^* \leftarrow \mathcal{V}
2 while \mathcal{V} \neq \emptyset do

2 in \mathcal{V}^* \leftarrow \mathcal{V}
3 | \mathcal{V}^* \leftarrow \mathcal{V} | \mathcal{V}^* \leftarrow \mathcal{V}^* \mathcal{V}^* \leftarrow \mathcal{V}^* \leftarrow \mathcal{V}^* \leftarrow \mathcal{V}^* | \mathcal{V}^* \leftarrow \mathcal{V}^* \leftarrow \mathcal{V}^* \leftarrow \mathcal{V}^* \leftarrow \mathcal{V}^* | \mathcal{V}^* \leftarrow \mathcal{V}^* \leftarrow \mathcal{V}^* \leftarrow \mathcal{V}^* \leftarrow \mathcal{V}^* \rightarrow \mathcal{V}^*
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Algorithm 2 summarizes how to detect dense subgraph with a MF-Solver over the graph G, and MF-Solver is the decomposition-based approximate detection method, e.g., EigenSolver and NMF-Solver. MF-DSD selects the top-ranked vertices based on the scores of each factor $\mathbf{U}(:,k)$ with a given threshold Δ (Line 5), and returns the subgraph with the highest density derived from those vertices. A commonly used threshold is $\Delta = \frac{1}{|\mathcal{V}|}$, which is from the random graph assumption, i.e., the link between two nodes exists with equal probability. The total time complexity is $O(K|\mathcal{V}|) + O(\mathrm{MF-Solver})$, which is dominated by the second term.

A.2 Density Metric Analysis

We propose the *joint density for the multilayered networks* in Definition 1 as a flexible density metric, where the hyperparameters

Algorithm 2: MF-DSD: Matrix Factorization based Dense Subgraph Detection over an aggregated-graph

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Input: Undirected graph G = (\mathcal{V}, \mathcal{E}); density metric \rho(\cdot); MF-Solver; factorization rank K; vector truncation threshold \Delta.

Output: Nodeset of the optimal densest subgraph of G.
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 $V^* \leftarrow \emptyset$

 $\{\gamma_{i,j}\}$ s control the contribution of the cross-layer $C_{i,j}$ to the total density and different cross-layers can also use distinct γ s (we only adopt the same value for $\gamma_{i,j}$ in our experiments). When $\gamma=0$, the cross-layer links are not considered, and the detected result might be (near-) free dense subgraph (no guarantee for the interrelation), at this point, it is equivalent to applying the DSD algorithms (i.e., DSDSOLVER) layer by layer alone; and when $\gamma\to\infty$, the contribution of within-layer links becomes very weak, the detected dense subgraphs are mainly composed of the cross-layer dependencies (almost equivalent to g+1-partite graph); given a proper value of $0<\gamma\ll\infty$, there is a trade-off of the contribution of within-layers and cross-layers for detecting the interrelated dense subgraph.

B EXPERIMENTAL RESULTS.

B.1 Dataset Information.

We list the description of the real-world multilayered networks as follows, and their statistical information is summarized in Tab. 2.

- AMINER [5]: a three-layered network in the academic collaboration domain, which contains co-authorship, papercitation, and a venue-citation within networks. The crosslayer dependencies are naturally generated from who-writeswhich paper and which venue-publishes-which paper [1].
- BIO [4]: a three-layer CTD (Comparative Toxicogenomics
 Database) network in the biological domain, contains chemicalsimilarity, gene-similarity, and disease-similarity as within
 networks; which chemical-influences-which gene, which
 chemical-cures-which disease, and which gene-related-which
 disease constitute the cross dependencies of different layers.

Table 1: F scores on *sub-BIO* network with different injection density under camouflage ($p_{cam} = 1.00$). $\gamma = 15$ for INTDUEN.

Method	k-th		Inje	ection d	lensity	p_{qt}	
	layer	0.10	0.20	0.30	0.40	0.50	0.60
DESTINE	1,2,3	0.00	0.00	0.00	0.00	0.67	0.67
IntDuen	1	1.00	1.00	0.89	1.00	0.91	0.86
/ Exp.	2	0.99	1.00	1.00	1.00	0.84	0.76
/ Exp.	3	1.00	1.00	1.00	1.00	0.84	0.76
	1	0.98	0.98	0.98	0.98	0.98	0.98
IntDuen	2	1.00	1.00	1.00	1.00	1.00	1.00
	3	0.98	0.98	0.99	0.99	0.99	0.99

* Here, OQC,OQC^a, Fraudar, Fraudar^a, MF-DSD,MF-DSD^a and Greedy are ignored since their detection results are all 0.00.

- DBLP [5]: a three-layered dataset similar to AMINER, which has different sizes and is collected for different time spans.
- INFRA-3 [2]: a three-layered network in the critical infrastructure domain, which contains an airport network, an autonomous system network, and a power grid. The crosslayer dependencies between different layers are generated from geographic proximity.
- INFRA-5 [2]: a five-layered network consists of one internet network and four regional power grids. The regional power grids are partitioned by macro-regions, and the dependencies between these layers are based on the power transfer lines. See detailed information in [2].

Note that *INFRA-5* is only for the comparison with Destine; the (# of nodes, # of within-layer links, # of cross-layer links) of INFA-5 corresponds to (249, 379, 565) respectively. Due to the small size, it was not used in our main experiments.

B.2 Detection performance under camouflage.

As in Sec. 5.5, we apply all detection algorithms to the IDSD problem over different networks under camouflage.

The configuration of *SF network* keeps the same as Sec. 5.4.1; we also follow the same way as for ER network in Sec. 5.3 to generate the camouflage, where the link probability is also p_{cam} . Here, we report the *F score* of the detection results for each layer as well, and merge those layers if they have the same results.

Table 1 and Table 2 show the detection performance with the change of injection probability p_{gt} of the injected interrelated dense subgraphs under $p_{cam} = 1.0$.

For the <code>sub-BIO</code> <code>network</code>, <code>IntDuen</code> with $\gamma=15$ nearly achieves the best result with $F\approx 1.0$ while in some cases for the 1st and 3rd layer, <code>Expander</code> introduces some non-target node, which slightly damages its precision and F score, compared with <code>IntDuen/Exp.</code>; but these two algorithms still outperform all others. Many baseline methods fail to detect any target, which is similar to the results over ER network; <code>Destine</code> can detect some results when the injection density is not less than 0.50, where it cannot distinguish camouflage and the target and gets F=0.67 (precision =0.50, recall =1.0).

For the *SF network*, IntDuen with $\gamma = 20$ achieves F = 1.0 and outperforms all baselines, and the Expander greatly improves its result, compared with IntDuen/Exp.; while the latter is also better than or equal to other baselines; In addition, the underlying network

Table 2: F scores on *SF network* with different injection density under camouflage ($p_{cam} = 1.00$). $\gamma = 20$ for INTDUEN.

Method	k-th		Inje	ection o	lensity	p_{qt}	
Method	layer	0.10	0.20	0.30	0.40	0.50	0.60
	1	0.02	0.02	0.02	0.02	0.04	0.04
OQC	2	0.05	0.06	0.06	0.07	0.08	0.09
	3	0.05	0.06	0.06	0.06	0.09	0.67
OQC^a	1,2,3	0.00	0.00	0.00	0.00	0.00	0.00
Fraudar	1	0.00	0.00	0.00	0.00	0.00	0.00
TRAUDAR	2,3	0.00	0.00	0.00	0.00	0.00	0.67
Fraudar ^a	1,2,3	0.00	0.00	0.00	0.00	0.00	0.00
	1	0.13	0.15	0.23	0.31	0.67	0.67
MF-DSD	2	0.19	0.23	0.29	0.42	0.67	0.67
	3	0.21	0.28	0.35	0.52	0.66	0.67
MF - DSD^a	1,2	0.00	0.00	0.00	0.00	0.00	0.00
MI -D3D	3	0.58	0.64	0.66	0.67	0.67	0.67
Greedy	1,2	0.00	0.00	0.00	0.00	0.00	0.00
Greedy	3	0.00	0.00	0.00	0.00	0.00	0.67
	1	0.65	0.65	0.65	0.65	0.65	0.65
DESTINE	2	0.60	0.60	0.60	0.60	0.60	0.60
	3	0.55	0.55	0.55	0.55	0.55	0.55
IntDuen	1	0.64	0.66	0.67	0.64	0.66	0.67
/ Exp.	2	0.67	0.66	0.68	0.68	0.67	0.67
/ Exp.	3	0.68	0.67	0.69	0.69	0.71	0.70
IntDuen	1,2,3	1.00	1.00	1.00	1.00	1.00	1.00

model also influences the performance of IntDuen/Exp. compared with Tab. 5 in main paper, especially for low injection densities. Fraudar and Greedy cannot detect any targets until $p_{gt}=0.60$; OQC detects few targets for all layers, except for the 3rd layer when $p_{gt}=0.60$; MF-DSD can find some targets for all layers, even for low injection density. For the aggregated graph, OQC and Fraudar do not find any target, MF-DSD can only detect results for the 3rd layer with a larger size. For all layers, the performance of Destine is insensitive to the injection density here, which is consistent with Fig. 3(b); and it has better performance for smaller-size graphs.

Influence of camouflage strength: In addition, to show the influence of the strength of camouflage (i.e., p_{cam}) on the performance and serve as a comparison, we use the ER network and change p_{cam} to be 0.5 while keeping other setting the same as Sec. ??. Then, we show the detection results in Tab. 5.3.1 As we can see, IntDuen still achieves perfect detection performance and outperforms all baselines significantly. When $p_{cam} = p_{qt}$, IntDuen/Exp. has the worst results since that CoNF can not well distinguish the camouflage and target if their density is very similar, and thus the improvement of neighbor booster (Expander) for IntDuen is more prominent; in addition, IntDuen/Exp. drops a lot for low injection density here is also because of distinguishing between p_{cam} and p_{qt} , compared with Tab.5. With the weaken camouflage, other baselines have a little better performance than the case of $p_{cam} = 1.0$, but most of them still only find very few targets when $p_{qt} \leq 0.40$; MF-DSD^a and Destine also have a lower detection density, OQC^a and Fraudar cannot find any target until $p_{at} \ge 0.50$.

Table 3: F scores on ER network with different injection den-
sity under camouflage ($p_{cam} = 0.50$). $\gamma = 15$ for IntDuen.

Method	<i>k</i> -th		Inje	ection d	lensity	p_{gt}	
Wittiou	layer	0.10	0.20	0.30	0.40	0.50	0.60
OQC	1,2,3	0.18	0.18	0.18	0.18	0.18	0.18
OQC^a	1,2,3	0.00	0.00	0.00	0.00	0.00	1.00
FRAUDAR	1,2	0.00	0.00	0.00	0.00	0.00	0.00
TRAUDAR	3	0.00	0.00	0.00	0.00	0.00	1.00
Fraudar ^a	1,2,3	0.18	0.18	0.18	0.18	0.18	0.18
MF-DSD	1,2	0.00	0.00	0.00	0.00	0.00	0.00
MIT-DSD	3	0.18	0.18	0.18	0.18	0.18	0.18
	1	0.00	0.09	0.67	0.67	0.67	0.67
$MF-DSD^a$	2	0.00	0.07	0.67	0.67	0.67	0.67
	3	0.00	0.05	0.67	0.67	0.67	0.67
	1	0.00	0.00	0.00	0.00	1.00	0.66
Greedy	2	0.00	0.00	0.00	0.00	0.00	0.00
	3	0.21	0.23	0.23	0.26	0.67	0.18
Destine	1,2,3	0.00	0.00	0.67	0.67	0.67	0.67
	1	0.62	0.59	0.57	0.58	0.49	0.56
IntDuen/ Exp	2	0.56	0.55	0.58	0.54	0.66	0.56
	3	0.61	0.59	0.57	0.65	0.52	0.58
IntDuen	1,2,3	1.00	1.00	1.00	1.00	1.00	1.00

B.3 Case study: Drug-Disease Networks

We chose another two-layer network from [3] for the 'drug-disease', which includes 894 drugs, 454 diseases, and 2704 drug-disease associations. In the within-layer similarity graphs, we keep an edge if the similarity between two drugs/diseases is not less than 0.30.

Figure 1 shows the detected interrelated dense sub-network of IntDuen with $\gamma = 1.2$. Fig. 1(a) is the global view of the interrelated dense subgraph, Fig. 1(b)-1(c) illustrate the within-layer graph for drugs' and diseases' similarity networks with node labels. We can get the name and detailed information from the databases¹ about the drugs and diseases based on the DrugBank ID and Mesh ID. Tab. 4-5 list the basic information about the drugs and diseases contained in Fig. 1. The dense-connected components like this will benefit related fields, like disease diagnosis, drug discovery, etc.

B.4 Comparison with Destine.

For a fair comparison with DESTINE, we construct the same experiment settings and report the detection results of different methods.

B.4.1 Detection of Injected Interrelated Cliques. We test the performance for detecting the injected interrelated cliques on synthetic networks (ER and SF). The injection strategy here is slightly different from the description in our main text, and see [6] for the detail settings. Table 6 shows the detection results of different methods, and p or m of the underlying model for each layer is provided.

As we can see, IntDuen with $\gamma = 30$ consistently achieves the best results with F = 1.0 for each layer in both networks and outperforms all baselines. Therefore, our method can adapt to networks with different underlying models.

For the *ER networks*, DESTINE, MF-DSD, and NMF also have the best performance for easier detection cases, e.g., p = 0.1 and 0.2,

while becoming inferior for the within layer with more links (e.g., p=0.6), where NMF even closes to the worst; OQC and Fraudar get worse results. For the *SF networks*, the conclusion is similar, Fraudar, Destine and MF-DSD perform as well as IntDuen; NMF just handles those easier cases. In addition, we also use the aggregated graph as the input for OQC, Fraudar, and MF-DSD, they also underperformance for most of this setting, where MF-DSD even misses hitting any target.

Besides, we also test the performance of IntDuen with different DSDSolvers on these synthetic networks, that is, replacing GreedySolver with MaxFlowSolver and MaxCliqueSolver, it turns out that they all archieve the same performance. IntDuen using GreedySolver with Layer-By-Layer detection strategy also gets the results of F=1.0.

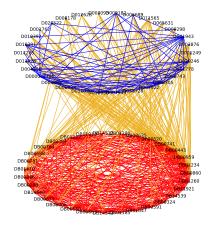
B.4.2 Detection for Real Networks. Here, we use IntDuen with the Layer-By-Layer strategy (also means $\gamma=0$) and apply to detect dense subgraphs in each layer over the same networks used by Destine, i.e., AMINER, BIO, INFRA-3, and INFRA-5. Table 7 shows the detection results, including the edge ratio density and size of the dense subgraphs of each detection method.

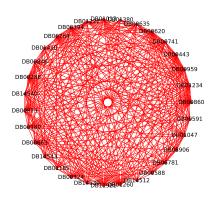
We can see that INTDUEN can always detect the subgraph with the highest density. OQC tends to detect subgraphs with large sizes and small densities. Destine can find near-clique subgraphs but may ignore some important nodes at the *Author* and *Gene* layers compared to INTDUEN.

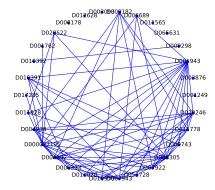
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 $^{^1\}mathrm{Drug}$ information from https://www.drugs.com/, and disease information from U.S. National Library of Medicine https://id.nlm.nih.gov/mesh/.







- (a) Interrelated dense subgraph extracted by ${\tt INTDUEN}.$
- (b) Drugs' similarity within-layer graph in (a).
- (c) Diseases' similarity within-layer graph in (a).

Figure 1: Interrelated dense subgraph patterns in the two-layer Drug-Disease networks. (a) shows the detection result of INTDUEN with $\gamma=1.2$, the edge density of the drug-disease association network is 0.19. (b) and (c) give the within-layer graph view with node labels of the drugs' similarity and diseases' similarity network of (a) with edge density 0.62 and 0.17, respectively. Here, 'DBXXXX' is the Drugbank ID of some drug, e.g., DB00635 denotes Prednisone with the chemical formula as $C_{21}H_{26}O_5$, and 'D0XXXXX' is the Mesh ID of some disease, e.g., D065631 for Rhinitis, Allergic.

Table 4: Drugs Information for the Drug-Disease Network.

		· ·	· ·		
DrugBank ID	Name	Chemical Formula	DrugBank ID	Name	Chemical Formula
DB01260	Desonide	C24H32O6	DB00443	Betamethasone	C22H29FO5
DB00288	Amcinonide	C28H35FO7	DB00741	Hydrocortisone	C21H30O5
DB01222	Budesonide	C25H34O6	DB00324	Fluorometholone	C22H29FO4
DB00764	Mometasone	C22H28Cl2O4	DB00846	Flurandrenolide	C24H33FO6
DB00635	Prednisone	C21H26O5	DB01185	Fluoxymesterone	C20H29FO3
DB01410	Ciclesonide	C32H44O7	DB01380	Cortisone acetate	C23H30O6
DB11921	Deflazacort	C25H31NO6	DB14512	Mometasone furoate	C27H30Cl2O6
DB00180	Flunisolide	C24H31FO6	DB00959	Methylprednisolone	C22H30O5
DB00873	Loteprednol	C21H27ClO5	DB08906	Fluticasone furoate	C27H29F3O6S
DB00663	Flumethasone	C22H28F2O5	DB01013	Clobetasol propionate	C25H32ClFO5
DB01047	Fluocinonide	C26H32F2O7	DB00591	Fluocinolone acetonide	C24H30F2O6
DB00860	Prednisolone	C21H28O5	DB00588	Fluticasone propionate	C25H31F3O5S
DB06781	Difluprednate	C27H34F2O7	DB14539	Hydrocortisone acetate	C23H32O6
DB00620	Triamcinolone	C21H27FO6	DB14540	Hydrocortisone butyrate	C25H36O6
DB01234	Dexamethasone	C22H29FO5	DB14543	Hydrocortisone probutate	C28H40O7
			DB00394	Beclomethasone dipropionate	C28H37ClO7

Table 5: Diseases Information for the Drug-Disease Network.

Mesh ID	Name	Mesh ID	Name	Mesh ID	Name
D001249	Asthma	D011778	Q Fever	D006333	Heart Failure
D011565	Psoriasis	D010930	Plague	D000743	Anemia, Hemolytic
D009298	Nasal Polyps	D001762	Blepharitis	D020522	Lymphoma, Mantle-Cell
D001943	Breast Neoplasms	D010392	Pemphigus	D007943	Leukemia, Hairy Cell
D065631	Rhinitis, Allergic	D007922	Leptospirosis	D055728	Primary Myelofibrosis
D003876	Dermatitis, Atopic	D017285	Polymyositis	D010391	Pemphigoid, Bullous
D006689	Hodgkin Disease	D011928	Raynaud Disease	D008305	Malignant Hyperthermia
D009182	Mycosis Fungoides	D020246	Venous Thrombosis	D011020	Pneumonia, Pneumocystis
D003093	Colitis, Ulcerative	D004938	Esophageal Neoplasms	D008178	Lupus Erythematosus, Cutaneous
D012628	Dermatitis, Seborrheic	D004892	Erythema Multiforme	D000077195	Squamous Cell Carcinoma of Head and Neck

Table 6: F scores for detecting the injected cliques on the synthetic data.

Data	k-th layer	OQC	OQC ^a	Fraudar	Fraudar ^a	MF-DSD	MF-DSD ^a	NMF	DESTINE	IntDuen
	p = 0.1	0.54	0.00	0.18	0.00	1.00	0.00	1.00	1.00	1.00
ER	p = 0.2	0.03	0.00	0.18	0.00	1.00	0.00	1.00	1.00	1.00
	p = 0.6	0.03	0.18	0.18	0.18	0.41	0.00	0.05	0.85	1.00
	m = 20	0.41	0.00	1.00	0.00	1.00	0.00	1.00	1.00	1.00
SF	m = 40	0.29	0.00	1.00	0.00	1.00	0.00	0.90	1.00	1.00
	m = 60	0.25	1.00	1.00	1.00	1.00	0.00	0.85	1.00	1.00

^{*} MF-DSD utilizes SVDSolver; 'a' labeled methods mean the detection results for the aggregated graph.

Table 7: Evaluate over the real-world networks. IntDuen with Layer-By-Layer strategy.

Dataset	Layer	Edge-Ratio Density						Size				
	,	OQC	NMF	SVD	DESTINE	IntDuen	OQC	NMF	SVD	Destine	IntDuen	
	Paper	0.71	0.61	0.01	0.90	1.00	10	8	504	5	5	
AMINER	Venue	0.53	0.94	0.09	0.94	1.00	48	20	172	18	13	
	Author	1.00	1.00	0.00	1.00	1.00	28	27	3,539	26	28	
BIO	Chem	1.00	1.00	0.19	1.00	1.00	91	91	318	91	91	
	Gene	0.43	0.29	0.01	0.97	1.00	73	55	3,469	9	14	
	DZ	0.70	0.89	0.08	1.00	1.00	119	89	460	59	58	
	AP	0.60	0.92	0.05	0.99	1.00	82	44	377	34	31	
INFRA-3	AS	0.51	0.76	0.03	1.00	1.00	13	7	151	4	4	
	Power	0.55	0.80	0.01	0.97	1.00	51	23	1,045	15	13	
	P1	0.53	1.00	0.13	1.00	1.00	6	3	18	3	3	
	P2	0.47	0.18	0.06	0.67	1.00	6	11	36	3	3	
INFRA-5	P3	0.50	1.00	0.13	1.00	1.00	5	3	13	3	3	
	P4	0.67	0.20	0.09	1.00	1.00	4	10	22	3	3	
	Net	0.50	0.25	0.12	0.67	1.00	4	8	15	4	3	