May-Happen-in-Parallel Analysis of X10 Programs

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Abstract

X10 is a modern object-oriented programming language designed for high performance, high productivity programming of parallel and multi-core computer systems. Compared to the lower-level thread-based concurrency model in the JavaTM language, X10 has higher-level concurrency constructs such as async, atomic and finish built into the language to simplify creation, analysis and optimization of parallel programs. In this paper, we introduce a new algorithm for May-Happen-in-Parallel (MHP) analysis of X10 programs. The analysis algorithm is based on simple path traversals in the Program Structure Tree, and does not rely on pointer alias analysis of thread objects as in MHP analysis for Java programs. We introduce a more precise definition of the MHP relation than in past work by adding condition vectors that identify execution instances for which the MHP relation holds, instead of just returning a single true/false value for all pairs of executing instances. Further, MHP analysis is refined in our approach by using the observation that two statement instances which occur in atomic sections that execute at the same X10 place must have MHP = false. We expect that our MHP analysis algorithm will be applicable to any language that adopts the core concepts of places, async, finish, and atomic sections from the X10 programming model. We also believe that this approach offers the best of two worlds to programmers and parallel programming tools — higher-level abstractions of concurrency coupled with simple and efficient analysis algorithms.

Categories and Subject Descriptors D.1.3 [Programming Technique]: Concurrent Programming—Parallel Programming

General Terms Languages, Verification

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1. Introduction

The current trends towards multi-core processors is expanding the demand for languages and tools that simplify parallel programming. X10 [4] is a modern object-oriented programming language designed for high performance, high productivity programming of parallel and multi-core computer systems. X10 offers various concurrency control constructs to the programmers: multiple parallel activities can be created using the async construct, their termination can be coordinated using the finish construct, mutual exclusion can be enforced using atomic blocks, and barrier based synchronization among activities can be performed using the clock construct. X10 also supports partitioning of data and activities across places through the use of distributions.

The goal of May-Happen-in-Parallel (*MHP*) analysis is to statically determine if it is possible for execution instances of two given statements (or the same statement) to execute in parallel. *MHP* analysis serves as a key foundation for concurrent static and dynamic debugging tools including tools for data race detection [5, 10, 14, 16]. Past research on *MHP* analysis has been conducted for parallel programming languages ranging from Ada [6, 15, 17] to Java [2, 13, 18]. In general, the problem of precise *MHP* analysis for all pairs of statements in a given program is undecidable. If all control flow paths in all threads are assumed to be executable, Taylor [23] has proved that, under certain assumptions, computation of *MHP* information is an NP-complete problem. Given the complexity of the problem, a scalable and efficient *MHP* analysis algorithm remains a significant challenge in the area of analysis of parallel programs.

In this paper, we focus on computing *MHP* information for concurrent X10 programs. The main contributions of this work compared to past *MHP* analysis algorithms are as follows:

- We introduce a more precise definition of the MHP relation than in past work by adding condition vectors that identify execution instances for which the MHP relation holds, instead of just returning a single true/false value for all pairs of executing instances.
- 2. Compared to past work, the availability of basic concurrency control constructs in X10 such as async and finish enable the use of more efficient and precise analysis algorithms based on simple path traversals in the Program Structure Tree, and does not rely on interprocedural pointer alias analysis of thread objects as in MHP analysis for the Java language. Note that Taylor's NP-hardness results [23] for MHP analysis applies to programs that use lower-level synchronization primitives such

- as Ada's rendezvous, and are not applicable to the X10 constructs considered in this paper.
- 3. Finally, *MHP* analysis is refined in our approach by using the observation that two statement instances which occur in atomic sections that execute at the same X10 place must have *MHP* = false.

2. X10 Overview

This section provides a brief summary of a core subset of v0.41 of the X10 programming language [4]. The goal of X10 is to introduce a core set of new language constructs that address the fundamental requirements for high productivity programming of parallel systems at all scales — multi-core processors, symmetric shared-memory multiprocessors (SMPs), commodity clusters, high end supercomputers, and even embedded processors like Cell. The key features of X10 include:

- Explicit reification of locality in the form of *places*, with support for a *partitioned global address space* (PGAS) across places
- Lightweight activities embodied in async, foreach, and ateach constructs which subsume communication and multithreading operations in other languages
- A finish construct for termination detection and rooted exception handling of descendant activities
- Support for lock-free synchronization with atomic blocks

With a view to mainstream adoption, X10 uses a serial subset of the Java language as its foundation, but replaces the Java language's current support for concurrency by new constructs that are motivated by high-productivity high-performance parallel programming.

The remainder of this section briefly describes the three core constructs of X10 — async, atomic, finish. An important safety result in X10 is that any program written with async, finish, and atomic can never deadlock. Section 2.1 outlines how these three constructs are used to write single-place parallel programs. Section 2.2 then discusses how the async and finish constructs extend to the multi-place case.

2.1 Single-Place Programming in X10 using async, atomic, finish

2.1.1 async (stmt)

Async is the X10 construct for creating or forking a new asynchronous activity. The statement, *async* (*stmt*), causes the parent activity to create a new child activity to execute (*stmt*). Execution of the async statement returns immediately i.e., the parent activity can proceed immediately to its next statement.

Consider the following X10 code example of an async construct. The goal of this example is to use two activities to compute in parallel the sums of the odd and even numbers in the range $1 \dots n$. This is accomplished by having the main program activity use the *async* statement to create a child activity to execute the foriloop and print oddSum, while the main program activity proceeds in parallel to execute the for-j loop and print evenSum.

```
public static void main(String[] args) {
  final int n = 100;
  async { // Compute oddSum in child activity
    double oddSum = 0;
    for (int i = 1; i <= n; i += 2) oddSum += i;
    System.out.println("oddSum = " + oddSum);
}
// Compute evenSum in parent activity
  double evenSum = 0;</pre>
```

```
for (int j = 2; j <= n; j += 2) evenSum += j;
System.out.println("evenSum = " + evenSum);
} // main()</pre>
```

2.1.2 finish (**stmt**)

The X10 statement, finish \(stmt \), causes the parent activity to execute \(\stmt \) and then wait till all sub-activities created within \(\stmt \) have terminated globally. If async is viewed as a fork construct, then finish can be viewed as a join construct restricted to only the activities created within the scope of the finish. X10 distinguishes between local termination and global termination of a statement. The execution of a statement by an activity is said to terminate locally when the activity has completed all the computation related to that statement. For example, the creation of an asynchronous activity terminates locally when the activity has been created. A statement is said to terminate globally when it has terminated locally and all activities that it may have spawned (if any) have, recursively, terminated globally.

Consider a variant of the previous example in which the main program waits for its child activity to finish so that it can print the result obtained by adding oddSum and EvenSum:

```
public static void main(String[] args) {
   final int n = 100;
   final BoxedDouble oddSum = new BoxedDouble();
   double evenSum = 0;
   finish {
     async { // Compute oddSum in child activity
        for (int i = 1 ; i <= n ; i += 2 )
           oddSum.val += i;
     }
     // Compute evenSum in parent activity
     for (int j = 2 ; j <= n ; j += 2 ) evenSum += j;
   } // finish
   System.out.println("Sum = " + (oddSum.val + evenSum));
} // main()</pre>
```

The finish statement guarantees that the child activity terminates globally before the print statement is executed. Note that the result of the child activity is communicated to the parent in a shared object, oddSum, since X10 does not permit a child activity to update a local variable in its parent activity. In this case, the local variable oddSum contains a pointer to an object with a val field, thereby enabling oddSum.val to be updatable even though oddSum is a constant pointer. It is also worth noting that the X10 memory model is weak enough to allow oddSum.val to be allocated to a register during the execution of the entire for-i loop.

2.1.3 atomic $\langle stmt \rangle$

The atomic construct in X10 is used to coordinate accesses by multiple activities to shared data located at the same place. The X10 statement, atomic $\langle stmt \rangle$, causes $\langle stmt \rangle$ to be executed atomically i.e., its execution occurs as if in a single step during which (stmt) executes and terminates locally while all other concurrent activities in the same place are suspended. Compared to user-managed locking, the X10 user only needs to specify that a collection of statements should execute atomically and leaves the responsibility of lock management and alternative mechanisms for enforcing atomicity to the language implementation. Commutative operations, such as updates to histogram tables and insertions in a shared data structure, are a natural fit for atomic blocks when performed by multiple activities. An atomic block may include method calls, conditionals, and other forms of sequential control flow. For scalability reasons, blocking operations like finish and force are not permitted in an atomic block. An async statement is not permitted either because the atomicity guarantee would only apply to local (not global) termination of the child async activity.

2.2 Multi-Place Programming in X10

Current programming models use two separate levels of abstraction for shared-memory thread-level parallelism (e.g., Java threads, OpenMP, pthreads) and distributed-memory communication (e.g., Java messaging, RMI, MPI, UPC) resulting in significant complexity when trying to combine the two. In this section, we show how the three core X10 constructs introduced earlier can be extended to multiple places. A *place* is a collection of resident (non-migrating) mutable data objects and the activities that operate on the data. Every X10 activity runs in a place; the activity may obtain a reference to this place by evaluating the constant *here*.

X10 v0.41 takes the conservative decision that the number of places is fixed at the time an X10 program is launched. Thus, there is no construct to create new places. This is consistent with current programming models, such as MPI, UPC, and OpenMP, that require the number of processes to be specified when an application is launched. This design decision may be revisited in future versions of the language as more experience is gained with adaptive computations which may naturally require a hierarchical, dynamically varying notion of places.

Places are virtual — the mapping of places to physical locations is performed by a deployment step that is separate from the X10 program. Though objects and activities do not migrate across places in an X10 program, an X10 deployment is free to migrate places across physical locations based on affinity and load balance considerations. While an activity executes at the same place throughout its lifetime, it may dynamically spawn activities in remote places.

X10 supports a partitioned global address space (PGAS) that is partitioned across places. Each mutable location and each activity is associated with exactly one place. A scalar object in X10 is allocated completely at a single place. In contrast, the elements of an array, may be distributed across multiple places. We now discuss how the async and finish constructs discussed earlier in a single-place context, extend directly to the multi-place case. A key constraint for atomic constructs is that an atomic block is only permitted to access place-local data (Locality Rule).

The statement, async ($\langle place-expr \rangle$) $\langle stmt \rangle$, causes the parent activity to create a new child activity to execute $\langle stmt \rangle$ at the place designated by $\langle place-expr \rangle$. The async is local if the destination place is same as the place where the parent is executing, and remote if the destination is different. Local async's are like lightweight threads, as discussed earlier in the single-place scenario. A remote async can be viewed as an active message, since it involves communication of input values as well as remote execution of the computation specified by $\langle stmt \rangle$. The semantics of the X10 finish operator is identical for local and remote async's i.e., a finish ensures global termination of all asyncs (local and remote) created in the scope of the finish.

3. Program Structure Tree and MHP Analysis of X10 Programs

The May-Happen-In-Parallel analysis problem addressed in this paper deals with analysis of multi-place X10 programs that use async, finish, and atomic constructs, which are higher-level forms of the thread-based start, join, and synchronized constructs in the Java language. In this section, we introduce the Program Structure Tree (PST) representation for X10 procedures, which will be used in later sections as the foundation for performing MHP analysis:

DEFINITION 3.1. A Program Structure Tree PST(N,E) for a procedure is a rooted tree where

1. N is a set of nodes such that each node $n \in N$ has one of the following types: root, statement, loop, async, finish, atomic. The root node designates the start of the procedure.

- Each async node is annotated with a place expression that designates the X10 place executing the async.
- 2. E is set of tree edges obtained by collapsing the abstract syntax tree representation of the procedure into the six node types listed above. PST.parent(N) returns the parent of node N as defined by E.

The X10 language semantics ensures that an atomic node will not be an ancestor of finish or async node. In addition, all statement nodes must be leaf nodes in the PST. \Box

Figure 1 contains a simple example of an X10 code fragment and its *PST*.

Having obtained the *PST* from the abstract syntax tree, the high level steps involved in *MHP* analysis for X10 programs are outlined below:

- 1. First, Never-Execute-In-Parallel (NEP) analysis is performed as described in Section 4. This analysis considers the occurrences of finish and async nodes in the PST and determines statements that can never execute in parallel. For soundness, the NEP analysis conservatively errs on the side of returning NEP = false when it is unable to perform a precise analysis of the input X10 program. In the case of loop nodes in the PST, we use condition vectors (defined in Section 4) to qualify the instances of execution that can never happen in parallel. Note that X10's foreach and ateach constructs for parallel loops can be represented in the PST by an equivalent pair of loop and async nodes.
- 2. Next, a Place-Equivalence (PE) analysis is performed as described in Section 5. The output of this analysis is a predicate, PE(S1,S2), which is set to true if all instances of S1 and S2 are guaranteed to execute at the same place. For soundness, the PE analysis conservatively errs on the side of returning PE = false when it is unable to perform a precise analysis of the input X10 program. Similar to NEP analysis, we use condition vectors to qualify the instances of execution of statements that are place equivalent.
- 3. In the final step of *MHP* analysis as defined in Section 6, we combine the *NEP* and *PE* analysis to obtain the *MHP* information for atomic constructs. The basic intuition is that for all those instances of execution of pair of statements where *NEP* is *true*, *MHP* is assigned *false*. In addition, if the statements are executed atomically, then *MHP* is assigned *false* for all those instances of execution which happen at same place.

4. Never-Execute-in-Parallel Analysis

In this section, we describe our approach for determining if two statements will *never execute in parallel (NEP)*. The *NEP* relation is the complement of the *May-Happen-in-Parallel (MHP)* relation that has been introduced in past work for Java and other concurrent programming languages [2, 18]. *NEP* is used instead of *MHP* in this section for the sake of convenience in presentation. In addition, the *NEP* relation will be used to compute a refined *MHP* relation later in Section 6.

The significant differences between the *NEP* analysis presented in this paper and past work on *MHP* analysis are as follows:

- The availability of basic concurrency control constructs in X10 such as async and finish enables a more efficient and precise NEP analysis algorithm compared to past work on MHP analysis for Java. Our algorithm is based on simple path traversals in the PST.
- 2. Past work on *MHP* analysis resulted in a simple true/false value for a given pair of statements. Our work makes the *NEP*

```
for ( i = 1 ; i \le n ; i ++ )
                                                             LOOP
       finish
                                                             FINISH
          for (j = 1; j \le n; j++)
                                                             LOOP
              for (k = 1; k \le n; k++)
                                                             LOOP
                 async (A.distribution[i,j,k])
                                                             ASYNC
                    atomic {
                                                            ATOMIC
S1:
                        temp = f(A[i,j,k]);
                        A[i,j,k] = temp;
S2:
```

Figure 1. Example X10 Program and its *PST*

relation more precise by adding *condition vectors* that are able to identify execution instances for which the *NEP* relations hold

As discussed later in Section 6, we show how the NEP information can be further refined by using the atomicity properties of atomic sections in X10.

DEFINITION 4.1. Two statements S_1 and S_2 are said to never execute in parallel, written as $NEP(S_1, S_2) = true$, with condition vector set CS if the following conditions hold:

- 1. S_1 and S_2 have exactly k loop nodes, $L_1, \ldots L_k$ as common ancestors in the PST (where $k \ge 0$).
- 2. Each element $\langle C_1, \dots C_k \rangle$ in CS is a vector of k functions of type int \times int \rightarrow boolean. In this paper, we will restrict our attention to three possible functions "=", " \neq ", and "*". The symbol * denotes the function that returns true for all inputs\(^1\).
- 3. Let $S_1[i_1, \ldots i_k]$ denote any execution instance of S_1 in iteration $i_1, \ldots i_k$ of loops L_1, \ldots, L_k , and likewise for $S_2[j_1, \ldots j_k]$. If $C_x(i_x, j_x) = \text{true } \forall 1 \leq x \leq k$ for some condition vector $\langle C_1, \ldots C_k \rangle$ in CS, then it is guaranteed that statement instances $S_1[i_1, \ldots i_k]$ and $S_2[j_1, \ldots j_k]$ cannot execute in parallel. \square

To summarize Definition 4.1, if $NEP(S_1, S_2) = false$ then there are no pairs of instances of S_1 and S_2 that can be guaranteed to not execute in parallel. If $NEP(S_1, S_2) = true$ then the instances of S_1 and S_2 that can be guaranteed to not execute in parallel are determined by the condition vectors in CS.

The algorithm for computing the *NEP* relation is given in Figure 2. The algorithm takes two inputs: PST for the X10 procedure being analyzed, and two statements, S_1 and S_2 , for which we want to compute whether *NEP* is *true* or *false*. Note that the algorithm also accepts the case where $S_1 = S_2$. The first step is to find the least common ancestor of the two statements, denoted by $A = LCA(S_1, S_2)$. This gives us the common scope of execution of the two statements. In Steps 2 and 3, it is established for S_1 and S_2 respectively whether they execute within an "unfinished" async created within A. Depending on this information, there are 4 cases that arise for NEP analysis for the $\langle \text{``="}, \dots, \text{``="} \rangle$ condition vector, as described in Steps 5d - 5g:

- Case 1 (Step 5d): If both S_1 and S_2 do not execute in an async construct under A then we can conclude they will never execute in parallel.
- Case 2 and 3 (Steps 5e and 5f): If exactly one of S_1 or S_2 executes in an async scope, then the dominator relation can be used to compute the $NEP(S_1,S_2)$ relation. In the algorithm, the dominator relation is checked on ancestors of S_1 and S_2 (AS_1 and AS_2 respectively) that are immediate children of $LCA(S_1, S_2)$. If the PST path from S_1 upto $LCA(S_1, S_2)$ contains an async node which is not followed by any finish node and AS_1 dominates AS_2 , then S_1 and S_2 will never execute in parallel.
- Case 4 (Step 5g): If both S_1 and S_2 execute in a async scope, then we have to conservatively assume that NEP = false.

Finally, we come to Step 6 which is performed in the case when S_1 and S_2 have $k \geq 1$ common loops. This step examines all nodes in the PST starting from A, the least common ancestor of S_1 and S_2 , and ending at L_1 , the outermost common loop that encloses S_1 and S_2 . Note that the algorithm uses the fact whether a loop contains a finish or async node in the PST to restrict the set of iterations for which NEP = true. If (say) loop L_x contains a finish node that is an ancestor of both S_1 and S_2 statements and there is no intervening async node in PST path from the finish node to L_x , we observe that instances of S_1 and S_2 from two distinct iterations of L_x (but created in the same iteration of outer loops L_1, \ldots, L_{x-1}) can never execute in parallel. This property is captured by a condition vector in which C_x is set to " \neq ", C_1, \ldots, C_{x-1} are set to "=", and C_{x+1}, \ldots, C_k are set to "**".

The algorithm in Figure 2 assumes that the *PST* has already been constructed, which is a one-time O(N) cost. In addition, Steps 5e and 5f use the *dominator* relation on the original control flow graph, which can be computed using algorithms that vary in execution time complexity from O(NlogN) [12] to O(N) [8] as a one-time cost. We observe that the *NEP* algorithm takes O(H) time for determining if a given pair of nodes, S_1 and S_2 , satisfy $NEP(S_1, S_2) = true$, where H is the *height* of the *PST*. Note that the condition vector set CS, can contain at most L+1 condition vectors — one contributed by Step 5h and L by Step 6(b)iiiB - each of which has O(L) size, where $L \le H$ is the maximum nesting of *loops* in the PST. Step 6(b)iii can be considered a constant time operation. If used to compute the NEP relation for all pairs of statements, the total execution time will be $O(N^2H)$, which is more efficient than the $O(N^3)$ time of the MHP algorithm in [18]. However, we

¹ These three operators have been also used in past work on *direction vectors* [24]. We may choose to extend C_x in the future to represent *distance vectors* or other more general boolean functions.

Inputs:

- 1. A Program Structure Tree (PST) for the procedure being analyzed
- 2. Two statement nodes S_1 and S_2 in the PST with $k \ge 0$ common loop node ancestors in the PST, L_1, \ldots, L_k .

Outputs:

- 1. $NEP(S_1, S_2)$, a boolean value that indicates if instances of S_1 and S_2 never execute in parallel.
- 2. CS, a set of condition vectors that is used only if $NEP(S_1, S_2)$ = true. Given statement instances $S_1[i_1, \dots i_k]$ and $S_2[j_1, \dots j_k]$, if $C_x(i_x,j_x) = true \ \forall 1 \leq x \leq k$ for some condition vector $\langle C_1, \dots C_k \rangle$ in CS then it is guaranteed that the two statement instances cannot execute in parallel.

Algorithm:

```
1. A := LCA(S_1, S_2), the Lowest Common Ancestor of S_1 and S_2 in the PST
2. /* Determine if an instance of S_1 can be executed in a new async activity that escapes a given execution instance of A^*/
   async\_S_1 := false
   for (N := S_1; N \neq A; N := PST.parent(N)) do
  (a) if N is an async node then async_S_1 := true end if
  (b) if N is a finish node then async_S_1 := false end if
   end for
3. /* Repeat the previous step for S_2 */
   \operatorname{async}_{-}S_2 := \operatorname{false}
   for ( N := S_2 ; N \neq A ; N := PST.parent(N) ) do
  (a) if N is an async node then async_S_2 := true end if
  (b) if N is a finish node then async_S_2 := false end if
   end for
4. CS:=\emptyset /* Initialize CS to an empty set */
```

```
5. if (S_1 \neq S_2) then
```

```
/* Analyze four cases for async_S_1 and async_S_2 */
```

```
(a) AS_1 := PST ancestor of S_1 that is a child of A; Note that AS_1 := S_1 if S_1 is a child of A;
```

(b) $AS_2 := PST$ ancestor of S_2 that is a child of A; Note that $AS_2 := S_2$ if S_2 is a child of A;

```
(c) flag := false /* \langle =, ..., = \rangle will be added to CS if flag = true */
```

(d) if (\neg async_ $S_1 \land \neg$ async_ S_2) then flag := true end if /* Case 1 */

(e) if $(\neg async_S_1 \land async_S_2)$ then $flag := (AS_2 \text{ does not dominate } AS_1)$ end if f Case 2 */

(f) if (async_ $S_1 \land \neg$ async_ S_2) then $flag := (AS_1 \text{ does not dominate } AS_2)$ end if /* Case 3 */

(g) if (async_ $S_1 \land async_S_2$) then flag := false end if /* Case 4 */

```
(h) if (flag) then CS := CS \cup \{ \langle "=", \dots, "=" \rangle \} end if
```

6. if ($k \ge 1$) then /* S_1 and S_2 have at least one common loop */

```
(a) segloop := true ; x := k + 1
```

(b) for $(N := A; N \neq L_1; N := PST.parent(N))$ do

i. if N is an async node then seqloop := false end if

ii. if N is a finish node then segloop := true end if

iii. if N is a loop node then

```
A. x := x - 1;
B. if ( seqloop ) then CS:=CS\cup\{\langle C_1=\text{``='},\ldots,C_x=\text{``\neq''},C_{x+1}=\text{``*''},\ldots,C_k=\text{``*''}\rangle\} end if
end if
```

end for

end if

7. $NEP(S_1, S_2) := (CS \neq \emptyset) /* Return NEP = true if CS is non-empty */$

Figure 2. Algorithm for computing Never-Execute-in-Parallel (NEP) relations

```
Main thread:
S1: ExternalHelper1.start();
S2: ...
S3: ExternalHelper1.join();
S4: ... // MHP algorithm concludes that
        // S11 and S12 may happen in parallel with S4
ExternalHelper1 thread:
S5: ...
S6: InternalHelper1_1.start();
S7: InternalHelper1_2.start();
S8: InternalHelper1_1.join();
S9: InternalHelper1_2.join();
S10: ... // MHP algorithm concludes that
         // S11 and S12 cannot happen in parallel with S10
InternalHelper1_1 thread:
S11: ...
InternalHelper1_2 thread:
S12: ...
```

Figure 3. Java example program to illustrate *MHP* algorithm

expect that the execution time overhead of the NEP algorithm will be much smaller than $O(N^2H)$ in practice, since it can be used in a demand-driven fashion to only compute the NEP relation for pairs of statements that are of interest to an interactive tool or compiler transformation.

We will first use the example program in Figure 1 to illustrate the algorithm. The example was intentionally chosen to be as simple as possible to illustrate the core ideas in this paper. In this example, we are interested in determining which pairs of execution instances of statements S_1 and S_2 will never execute in parallel with each other, so the algorithm in Figure 2 will be invoked to compute $NEP(S_1,S_2)$. The output of this algorithm will be $NEP(S_1,S_2)$ = true, with condition vector set $CS = \{\langle =, =, = \rangle, \langle \neq, *, * \rangle\}$. This implies that two instances of S_1 and S_2 are guaranteed to never execute in parallel if they belong to the same i-j-k iteration, or if they come from iterations with distinct values of i.

Finally, we use the following X10 code fragment

```
{ S1 ; async S2 ; S3 ; async S4 ; }
```

to illustrate the four cases in Step 5 in the NEP algorithm as follows:

```
Case 1 NEP(S_1, S_3) = true, in accordance with Step 5d.
Case 2 NEP(S_1, S_2) = true, in accordance with Step 5e.
Case 3 NEP(S_2, S_3) = false, in accordance with Step 5f.
Case 4 NEP(S_2, S_4) = false, in accordance with Step 5g.
```

4.1 Comparison with MHP Analysis of Java programs

In this section, we briefly compare the *NEP* algorithm from Figure 2 with the *MHP* data flow analysis algorithm developed by Naumovich et al [18]. The later algorithm was designed to address all concurrency features in Java threads, including wait/notify/notifyall operations in synchronized blocks. In this comparison, we will restrict our attention to the *MHP* algorithm's handling of the *start*, *join*, and *synchronized* constructs in Java threads, which are comparable, but not equivalent, to *async*, *finish*, and *atomic* in X10.

Figure 3 contains the skeleton of a Java program that represents the parallel control flow in the SplitRendererNested example

```
S0: finish {
 S1: async { // ExternalHelperThread1.start()
   finish {
      S5: ...
      S6: async S11
                      // InternalHelperThread1_1.start()
                      // InternalHelperThread1_2.start()
     S7: async S12
    S8: ... // finish subsumes InternalHelper1_1.join()
    S9: ... // finish subsumes InternalHelper1_2.join()
    S10: ... // NEP algorithm concludes that
             // NEP(S10,S11) = NEP(S10,S12) = false
 S2: ...
S3: ... // S0's finish subsumes ExternalHelperThread1.join()
S4: ... // NEP algorithm concludes that
        // NEP(S4,S11) = NEP(S4,S12) = false
```

Figure 4. X10 example program to illustrate *NEP* algorithm

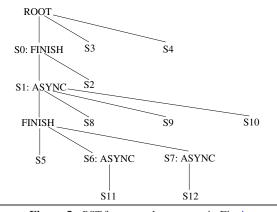


Figure 5. *PST* for example program in Fig 4

used in [18]. As discussed in [18], the MHP algorithm is conservative in its analysis of nested parallelism and concludes that S11 and S12 may happen in parallel with S4, even though it is able to conclude that S11 and S12 cannot happen in parallel with S10.

As a comparison, Figure 4 contains the skeleton of an X10 program that is equivalent to the Java program in Figure 3. If the NEP algorithm from Figure 2 is invoked to compute NEP(S4,S11), it will perform the following steps to conclude that NEP(S4,S11) = true:

```
Step 1: A := LCA(S4,S11) = ROOT
Step 2: async_S4 := false
Step 3: async_S11 := false
Step 5(a): AS4 := S4
Step 5(b): AS11 := S0
Step 5(d): flag := true
Step 5(h): CS := { ⟨ =, ..., = ⟩ }
```

• Step 7: NEP(S4,S11) := true

Thus, the NEP algorithm is able to establish that S11 and S12 cannot happen in parallel with S4, while the MHP algorithm from Figure 3 conservatively concludes that S11 and S12 may happen in parallel with S4.

The above discussion focused on the conservativeness in analysis of nested parallelism in the *MHP* algorithm. As mentioned

earlier, another dimension of conservativeness in MHP analysis of Java programs is the necessity to perform interprocedural pointer alias analysis of thread objects to establish accurate parallel control flow relationships among threads. For example, the MHP analysis must establish that all thread objects (e.g., ExternalHelper1, InternalHelper1_1, InternalHelper1_2) are distinct before it can even conclude that S11 and S12 cannot happen in parallel with S10 in Figure 3. As observed in past work on static data race detection, interprocedural alias analysis of thread objects can pose a significant challenge in practice. In contrast, the analysis of X10's async, finish, and atomic constructs is simpler because it does not rely on alias analysis of thread objects.

5. Place Equivalence Analysis

In this section, we describe our approach for determining if two statements are *place equivalent (PE) i.e.*, if they will definitely execute at the same place. Most parallel programming models that are currently used for distributed-memory multiprocessors follow a Single Program Multiple Data (*SPMD*) model in which one thread is executed per place. However, the X10 programming model is more general since it integrates thread-level parallelism and cluster-level parallelism by allowing multiple activities to be created at different *places*. Place equivalence analysis therefore becomes important for more general parallel programming models such as X10.

DEFINITION 5.1. Two statements S_1 and S_2 are said to be place equivalent, written as $PE(S_1, S_2) = true$, with condition vector set CS if the following conditions hold:

- 1. S_1 and S_2 have exactly k loop nodes, $L_1, \ldots L_k$ as common ancestors in the PST (where $k \ge 0$)
- 2. Let $S_1[i_1,\ldots i_k]$ denote any execution instance of S_1 in iteration $i_1,\ldots i_k$ of loops L_1,\ldots ,L_k , and likewise for $S_2[j_1,\ldots j_k]$. If $C_x(i_x,j_x)=$ true $\ \forall 1\leq x\leq k$ for some condition vector $\langle C_1,\ldots C_k\rangle$ in CS, then it is guaranteed that statement instances $S_1[i_1,\ldots i_k]$ and $S_2[j_1,\ldots j_k]$ must execute at the same place. \square

To summarize Definition 5.1, if $PE(S_1,S_2) = false$ then there are no pairs of instance of S_1 and S_2 for which place equivalence is guaranteed. If $PE(S_1,S_2) = true$ then the instances of S_1 and S_2 that can be guaranteed to execute at the same place are determined by the condition vectors in CS.

The algorithm for computing the *PE* relation is given in Figure 6. The algorithm needs two additional pre-passes as inputs along with the *PST*. First, a global place-value numbering prepass for place expressions to determine place local information for statements. Second, a global loop-invariant analysis (also known as *LoopSet* analysis) pre-pass to determine loops for which a given place expression is place variant. Global value numbering can be performed using an SSA-based algorithm as in [1]. Let us describe the how *LoopSet* information is computed.

Consider the following code fragment as an example:

To compute LoopSet information for the place expression, A.distribution[f(i,j),k], in the async statement in the above code fragment, we need to know the data distribution of array A. In X10 [4], array A can be distributed using a wide range of standard and user-defined distributions such as UNIQUE, RANDOM, CYCLIC, and BLOCK. As an example, let us assume that A is distributed in (BLOCK, *) fashion so that A[p,*] is guaranteed to re-

side at the same place, where p is the index of the first dimension. The async activity in the above code fragment with distribution (BLOCK,*) will be mapped to different places based on indices i and j, but not k i.e., place-variant with respect to loops L1 and L2. Hence, $LoopSet(A.distribution[f(i,j),k]) := \{L1,L2\}.$

As shown in Step 4a of Figure 6, a pair of statements S_1 and S_2 associated with same global place-value numbers i.e., $V(S_1) = V(S_2)$ are always going to execute at the same place. If $V(S_1) \neq V(S_2)$ and there are no intervening async nodes within the innermost common scope of S_1 and S_2 , then these statements are also bound to execute at the same place.

Step 5 traverses the common ancestors (only loop and async *PST* nodes) to compute *condition vector* using *LoopSet*. For loops that are *placeLocalLoops*, the condition vector entries are set to *. Note that *LoopSet* keeps track of the place-variant loops and *placeLocalLoops* keeps track of place-invariant loops.

The algorithm in Figure 6 assumes that the PST is constructed in O(N) time. The pre-passes for the other inputs to the algorithm, Global Value Numbering and LoopSet analysis, can also be computed in linear time. We observe that Step 2 takes O(H) time. The condition vector set CS, can at most have two entries – one obtained from Step 4a and another from Step 5c – each of which has O(L) size, where $L \leq H$. For all pairs of statements in the X10 program, the overall complexity of PE analysis is bounded by $O(N^2H)$, which is the same complexity as that of NEP analysis.

Let us now see how the algorithm works for the example program in Figure 1, assuming that array A has a (BLOCK, BLOCK, *) distribution. This means that elements A[i,j,*] of array A are guaranteed to be mapped to the same place, and the async statement in the example will follow the same distribution. Hence, the PE algorithm will compute $placeLocalLoops = \{L_3\}$, which in turn results in a place condition vector set of $CS = \{\langle =, =, = \rangle, \langle =, =, * \rangle\}$. This implies that S_1 and S_2 with same values of i and j are guaranteed to execute in the same place. Note that, in general, the algorithm in Figure 6 does not require that the number of dimensions in an array reference match the number of loops in the loop nest or that the index ordering for the array access match the ordering of the loop nesting.

6. May-Happen-In-Parallel Analysis using Atomic Sections

In this section, we show how the Never-Execute-in-Parallel (NEP) analysis from Section 4 can be combined with the Place-Equivalence (PE) information analysis from Section 5 to obtain a more precise May-Happen-in-Parallel (MHP) analysis from X10 programs by using atomic sections. The simple approach to computing MHP would be to simply invert the NEP relation i.e., to return $MHP(S_1,S_2)$ = false when $NEP(S_1,S_2)$ = true. The key insight leveraged in this section is that two execution instances of statements S_1 and S_2 in an X10 program are guaranteed to not happen in parallel if they both occur in atomic sections that are executed at the same place. This enables us to broaden the number of executions for which we can assert that MHP = false. Note that instances of S_1 and S_2 can indeed happen in parallel if they occur in atomic sections that execute at different places.

The algorithm for computing the MHP relation is given in Figure 7. For a pair of statements S_1 and S_2 , Steps 2 and 3 check if they are nested in atomic blocks. In case both S_1 and S_2 are nested in atomic blocks, $MHP(S_1,S_2)$ is computed by combining the NEP and PE results in Step 4. Otherwise, the MHP relation is computed directly from the NEP relation.

² For an SSA-based algorithm such as [1], the complexity is technically linear in the size of the SSA form, which in turn is observed to be linear in the size of the input program in practice.

Inputs:

- 1. A Program Structure Tree (*PST*) for the procedure being analyzed.
- 2. Two statement nodes S_1 and S_2 in the PST with $k \geq 0$ common loop node ancestors in the PST, L_1, \ldots, L_k .
- 3. A value number V(e), for each place expression e that is the target of an async (e) statement. For convenience, we also assume the availability of V(N) for each async node N in the PST, where V(N) denotes the value of here for the activity executing S. V(e) and V(S) can be computed by a global value numbering analysis [1] pre-pass on place expressions.
- 4. For each place expression e, LoopSet(e) = subset of loops $\{L_1, \ldots, L_m\}$ for which the value of place expression e is place-variant, where L_1, \ldots, L_m are the loops surrounding expression e (counting from outer to inner). LoopSet(e) can be computed by a global loop-invariant analysis pre-pass.

Outputs:

- 1. $PE(S_1, S_2)$, a boolean value that indicates if instances of S_1 and S_2 must execute at the same place.
- 2. CS, a set of condition vectors that is used only if $PE(S_1, S_2) = \text{true}$. Given statement instances $S_1[i_1, \dots i_k]$ and $S_2[j_1, \dots j_k]$, if $C_x(i_x, j_x) = \text{true} \ \forall 1 \leq x \leq k$ for some condition vector $\langle C_1, \dots C_k \rangle$ in CS then it is guaranteed that the two statement instances must execute at the same place.

Algorithm:

```
1. A := LCA(S_1, S_2), the Lowest Common Ancestor of S_1 and S_2 in the PST
2. Compute async_S_1, and async_S_2 as in Figure 2
3. CS := \emptyset /* Initialize CS to an empty set */
4. if (S_1 \neq S_2) then
   (a) if ( V(S_1) = V(S_2) ) then
        i. CS := CS \cup \{ \langle *, \dots, * \rangle \} / * S_1 and S_2 always execute at the same place */
       ii. PE(S_1, S_2) := \text{true}
       iii. return
       else if (\neg \operatorname{async}\_S_1 \land \neg \operatorname{async}\_S_2) then
       CS := CS \cup \{ \langle =, \dots, = \rangle \} /* Instances of S_1 and S_2 that come from the same iteration of L_1, \dots, L_k must execute in the same
       activity and hence at the same place. */
   end if
5. if ( k \ge 1 \land \neg async_S_1 \land \neg async_S_2 ) then /* S_1 and S_2 have at least one common loop */
   (a) placeLocalLoops := \{L_1, \dots, L_k\}; x := k + 1
   (b) for (N := A; N \neq L_1; N := PST.parent(N)) do
        i. if N is an async node with destination place expression e then
           placeLocalLoops := placeLocalLoops - LoopSet(e) end if
        ii. if N is a loop node then
           A. x := x - 1;
           B. if ( L_x \in placeLocalLoops ) then C_x := "*" else C_x := "=" end if
           end if
   (c) if (placeLocalLoops \neq \emptyset) then CS := CS \cup \{\langle C_1, \ldots, C_k \rangle\} end if
   end if
6. PE(S_1, S_2) := (CS \neq \emptyset) /* Return PE = true if CS is non-empty */
```

Figure 6. Algorithm for computing Place Equivalence (*PE*) relations

Inputs:

- 1. A Program Structure Tree (PST) for the procedure being analyzed
- 2. Two statement nodes S_1 and S_2 in the PST with $k \geq 0$ common loop node ancestors in the PST, L_1, \ldots, L_k .

Outputs:

- 1. $MHP(S_1, S_2)$, a boolean value that indicates if instances of S_1 and S_2 may happen in parallel.
- 2. CS, a set of condition vectors that is used only if $MHP(S_1, S_2) = \text{false}$. Given statement instances $S_1[i_1, \dots i_k]$ and $S_2[j_1, \dots j_k]$, if $C_x(i_x, j_x) = true \ \forall 1 \leq x \leq k$ for some condition vector $\langle C_1, \dots C_k \rangle$ in CS then it is guaranteed that the two statement instances cannot happen in parallel.

Algorithm:

- 1. Compute $NEP(S_1, S_2)$ and its associated condition vectors, CS_{NEP} using the NEP algorithm in Figure 2
- 2. Set atomic_ S_1 := true if S_1 has an atomic node as an ancestor in the *PST*
- 3. Set atomic_ S_2 := true if S_2 has an atomic node as an ancestor in the PST
- 4. if (atomic_ $S_1 \wedge \text{atomic}_S_2$) then

/* Combine NEP and PE analysis results */

- (a) Compute $PE(S_1, S_2)$ and its associated condition vectors, CS_{PE} using the PE algorithm in Figure 6
- (b) $MHP(S_1, S_2) := \neg (NEP(S_1, S_2) \lor PE(S_1, S_2))$
- (c) $CS := CS_{NEP} \cup CS_{PE}$

else

/* Just return NEP analysis results */

- (a) $MHP(S_1, S_2) := \neg NEP(S_1, S_2)$
- (b) $CS := CS_{NEP}$

end if

Figure 7. Algorithm for computing May-Happen-in-Parallel (MHP) relation using place equivalence and atomic sections in X10

The complexity of computing the MHP relation is bounded by $O(N^2H)$. This is due to the complexity of computing both NEP relation and PE relation.

As discussed earlier, the NEP solution computed using the analysis described in Section 4 for the example in Figure 1 was $NEP(S_1,S_2) = true$ with condition vector set $CS_{NEP} = \{ \langle =, = \} \}$ $,= \rangle, \langle \neq, *, * \rangle$. This indicates that for different values of loop index i, S_1 and S_2 can not execute in parallel. However, this information can be refined using the PE solution due to the presence of atomic PST node in the loop body. The PE solution computed using the analysis described in Section 5 was $PE(S_1,S_2) = true$ with condition vector set $CS_{PE} = \{ \langle =, =, = \rangle, \langle =, =, * \rangle \}$. This indicates that S_1 and S_2 will execute at the same X10 place for different values of loop index k but with same values for loop indices i and j. Using the above two results, the algorithm in this section is able to determine that $MHP(S_1, S_2) = false$ with condition vector set $CS = \{\langle =, =, = \rangle, \langle \neq, *, * \rangle, \langle =, =, * \rangle\}$. i.e., $MHP(S_1, S_2) =$ false if they belong to the same i-j-k iteration, or if they come from iterations with distinct values of i or with the same values of i and j.

7. Related Work

Several approaches for computing May Happen in Parallel (*MHP*) information for programs have been suggested in the past. Callahan and Sublok [3] proposed a data flow algorithm that computes, for each statement in a concurrent program with post-wait synchronization, the set of statements that must be executed before this statement can be executed (*B4* analysis). Deusterwald and Soffa [6] applied *B4* analysis approach to the Ada rendezvous model and extended *B4* analysis to be interprocedural. Masticola and Ryder [15] proposed an iterative approach of non-concurrent analysis that computes a conservative estimate of the set of pairs of com-

munication statements that can never happen in parallel in a concurrent Ada program (the complement of this set is a conservative approximation of the set of pairs that may occur in parallel). In that work, it is assumed initially that any statement from a given process can happen in parallel with any statement in any other process. This pessimistic estimate is then improved by a series of refinements that are applied iteratively until a fixed point is reached. Naumovich and Avrunin [17] proposed a data flow algorithm for computing the MHP information for programs with a rendezvous model of concurrency. Thereafter, Naumovich, Avrunin and Clarke [18] proposed an algorithm for computing MHP information for concurrent Java programs. Their algorithm uses a data flow framework to compute a conservative estimate of MHP information and is shown to be more efficient than reachability analysis based algorithms that determines 'ideal' static MHP information. A practical implementation of the algorithm from [18] is described in [13]. Barik [2] proposed an alternative algorithm to compute MHP information for concurrent Java programs based on the thread creation tree. This algorithm works with abstract threads rather than concrete thread instance of threads and is shown to be more efficient than the algorithm in [18].

Program Dependence Graphs (PDGs) [7] have been used to analyze the intrinsic parallelism in a sequential program, but are not designed to represent explicitly parallel programs. Parallel Program Graphs (PPGs) [20, 21, 22] are a generalization of PDGs and CFGs that can be used as a foundation for analyzing flow-sensitive properties of parallel programs that go beyond the PST-based analysis presented in this papers. Srinvasan et al. [9] proposed a Parallel Flow Graph (PFG) for optimizing explicitly parallel programs. They provided data flow equations for the reaching definitions analysis and used a copy-in/copy-out semantics for accessing shared variables in parallel constructs. Concurrent Control Flow Graphs

(CCFGs) [11] are similar to PPGs and PFGs, with the addition of conflict edges in addition to synchronization and control flow edges that can be used for analysis of programs with a sequentially consistent memory model.

8. Conclusions and Future Work

In this paper, we introduced a new algorithm for May-Happenin-Parallel (*MHP*) analysis that is applicable to any language that adopts the core concepts of places, async, finish, and atomic sections from the X10 programming model. The main contributions of this work compared to past *MHP* analysis algorithms are as follows:

- We introduced a more precise definition of the MHP relation than in past work by adding condition vectors that identify execution instances for which the MHP relation holds, instead of just returning a single true/false value for all pairs of executing instances.
- 2. Compared to past work, the availability of basic concurrency control constructs such as async and finish enabled the use of more efficient and precise analysis algorithms based on simple path traversals in the Program Structure Tree, and did not rely on interprocedural pointer alias analysis of thread objects as in MHP analysis for the Java language.
- 3. We introduced place equivalence (*PE*) analysis to identify execution instances that happen at the same place. The *PE* analysis helps us in leveraging the fact that two statement instances which occur in atomic sections that execute at the same X10 place must have *MHP* = false.

For future work, we plan to implement this *MHP* algorithm in an Eclipse-based X10 Development Toolkit (X10DT) being developed at IBM and included in the X10 release on SourceForge [19], so as to be able to respond to interactive *MHP* queries. As motivation, the X10DT screen shot in Figure 8 shows how closely the *outline view* in X10DT mirrors the Program Structure Tree. We will also investigate how the algorithms in this paper can be extended to support additional concurrency constructs in X10, notably clocks and futures. Finally, we will explore how the algorithms can be enriched using distance vectors and can be applied in an interprocedural context.

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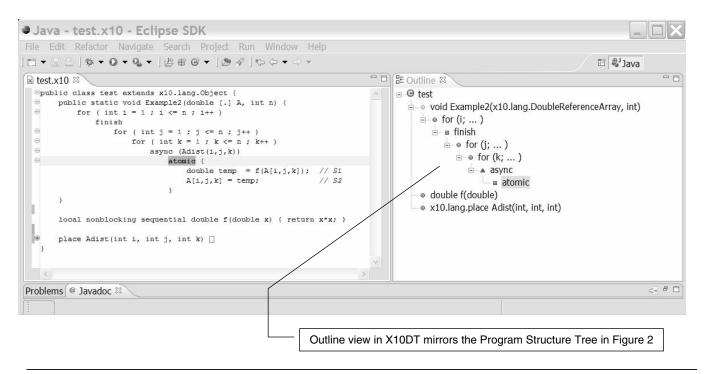


Figure 8. Sample X10DT screenshot for example from Figure 1

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