Rebalancing Human-Influenced Ecosystems

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Summary

In Task 1, we establish a Volterra predator-prey model with three biological populations, and we specify the steady-state numbers of the three populations. Then, based on the Analytic Hierarchy Process and a competition model, we obtain the ratio of different species in the second population, predict that the steady-state level of water quality is not high, and make the water quality satisfactory by adjusting the numbers of six species.

In Task 2, when milkfish farming suppresses other animal species, we set up a logistic model, and predict that the water quality at steady-state is awful, the same as in the fish pens—insufficient for the continued healthy growth of coral species. When other species are not totally suppressed, with an improved predator-prey model we simulate the water quality of Bolinao (making it match current quality), obtain predicted numbers of populations, and discuss changes to the predator-prey model aimed at making the numbers of the populations agree more closely with observations.

In Task 3, we establish a polyculture model that reflects an interdependent set of species, introduce mussels and seaweed growing on the sides of the pens, and obtain the numbers of populations in steady state and the outputs of our model.

In Tasks 4 and 5, we differentiate the monetary values of different kinds edible biomass and define the total value as the sum of the values of each species harvested, minus the cost of milkfish feed. Under circumstances

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of acceptable water quality, we build a nonlinear equilibrium optimization model, from which we obtain an optimal strategy and harvest.

In Task 6, we put forward a strategy to improve the water quality in Bolinao. With the ratio between feed cost and net income as the index, the index value of the model is smaller than that of Bolinao area, which signifies the leverage of the strategy. Also, we analyze the polyculture system in terms of ecology.

Introduction

To improve the situation in Bolinao, we need to establish a practicable polyculture system and introduce it gradually. So our goal is pretty clear:

- Model the original Bolinao coral reef ecosystem before fish farm introduction.
- Model the current Bolinao milkfish monoculture.
- Model the remediation of Bolinao via polyculture.
- Discuss the outputs and economic values of species.
- Write a brief to the director of the Pacific Marine Fisheries Council summarizing the relationship between biodiversity and water quality for coral growth.

Our approach is:

- Deeply analyze data in the problem, gradually establishing a model of the coral reef foodweb.
- With available data as evaluation criteria, confirm the water quality based on elements in the sediment.
- Establish models, and interpret the actual situation with data, with the purpose of improving water quality.
- Do further discussion based on our work.

Solutions

Task 1

Aiming toward a coral reef foodweb model, we assume that all the species grow in the same fish pen. We divide the species into three populations:

- one alga species (Population 1);

- one herbivorous fish, one mollusc species, one crustacean species, and one echinoderm species (Population 2); and
- the sole predator species, milkfish (Population 3).

The interrelationships among the species are presented in **Figure 1**.

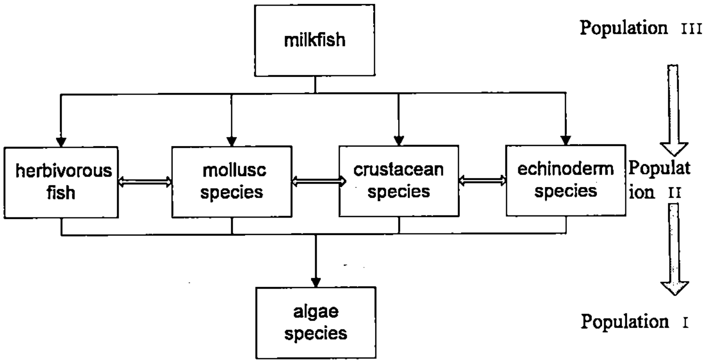

Figure 1. Interrelationships among three populations.

On this basis, we can establish a Volterra predator-prey model with three populations [Shan and Tang 2007]. Let the number of the ith population be $x_i(t)$. If we do not take into consideration the restrictions of natural resources, the algae species of Population 1 growing in isolation will follow an exponential growth law with relative growth rate r_1, so that $\dot{x}(t) = r_1 x_1$. However, species of Population 2 feeding on the alga species will decrease the growth rate of the algae, so the revised model of the alga species is

$$\dot{x}_1(t) = x_1(r_1 - \lambda_1 x_2),$$

where the proportionality coefficient λ_1 reflects the feeding capability of species in Population 2 for the alga species.

Assume that the death rate of the species in Population II is r_2 when existing in isolation; then $\dot{x}_2(t) = -r_2 x_2$, so based on the foodweb we conclude that

$$\dot{x}_2(t) = x_2(-r_2 + \lambda_2 x_1),$$

where the proportionality coefficient λ_2 reflects the support capability of the alga species for Population 2—which in turn provide food for the milkfish. The milkfish reduce the growth rate of the species in Population 2, so we must subtract their feeding effect to get

$$\dot{x}_2(t) = x_2(-r_2 + \lambda_2 x_1 - \mu x_3).$$

Likewise, the model for the milkfish is

$$\dot{x}_3(t) = x_3(-r_3 + \lambda_3 x_2).$$

Altogether, we have an interdependent and mutually-restricting mathematical model of the three populations:

$$\begin{aligned}
\dot{x}_1(t) &= x_1(r_1 - \lambda_1 x_2), \\
\dot{x}_2(t) &= x_2(-r_2 + \lambda_2 x_1 - \mu x_3), \\
\dot{x}_3(t) &= x_3(-r_3 + \lambda_3 x_2).
\end{aligned}$$

Since this system of differential equations has no analytic solution, we need to use Matlab to get its numerical solution.

Ecologists point out that a periodic solution cannot be observed in most balanced ecosystems; in a balanced ecosystem, there is an equilibrium. In addition, some ecologists think that the long-existing and periodically-changing balanced ecosystems in nature tend toward a stable equilibrium; that is, if the system diverges from the former periodic cycle because of disturbance, an internal control mechanism will restore it. However, the periodically-changing state described by the Volterra model is non-structured stability, and even subtle adjustments to the parameters will change the periodic solution.

So we improve the model by letting the alga species follow logistic growth if in isolation:

$$\dot{x}_1(t) = r_1 x_1 \left(1 - \frac{x_1}{N_1}\right),$$

where N_1 is the maximum population of the alga species allowed by the environmental resources. The alga species provides food for the species of Population 2, so the model for the algae species is

$$\dot{x}_1(t) = x_1 r_1 \left(1 - \frac{x_1}{N_1} - \sigma_1 \frac{x_2}{N_2}\right),$$

where N_2 is the maximum capacity of the species in Population 2 and σ_1 refers to the quantity of the algae (compared to N_1) eaten by the unit quantity species in Population 2 (compared to N_2).

Without the algae, the species in Population 2 will perish; let its death rate be r_2, so that in isolation we will have:

$$\dot{x}_2(t) = -r_2 x_2.$$

The algae provide food for Population 2, so we should add that effect; and the growth of the species in Population 2 is also influenced by internal blocking action; so we get

$$\dot{x}_2(t) = r_2 x_2 \left(-1 - \frac{x_2}{N_2} + \sigma_2 \frac{x_1}{N_1}\right),$$

where σ_2 is analogous to σ_1. Analogously, we can get a full model of the species in Population 2 via

$$\dot{x}_2(t) = r_2 x_2 \left(-1 - \frac{x_2}{N_2} + \sigma_2 \frac{x_1}{N_1} - \sigma_3 \frac{x_3}{N_3}\right).$$

Without the species in Population 2, milkfish will disappear; we set their death rate as r_3. The species in Population 2 provide food for the milkfish, and the growth of milkfish is also restricted by internal blocking action. Here the model is

$$\dot{x}_3(t) = r_3 x_3 \left(-1 - \frac{x_3}{N_3} + \sigma_4 \frac{x_2}{N_2}\right).$$

Summarizing, we have simultaneous equations constituting an interdependent mathematical model for the three populations:

$$\begin{aligned}
\dot{x}_1(t) &= x_1 r_1 \left(1 - \frac{x_1}{N_1} - \sigma_1 \frac{x_2}{N_2}\right), \\
\dot{x}_2(t) &= r_2 x_2 \left(-1 - \frac{x_2}{N_2} + \sigma_2 \frac{x_1}{N_1} - \sigma_3 \frac{x_3}{N_3}\right), \\
\dot{x}_3(t) &= r_3 x_3 \left(-1 - \frac{x_3}{N_3} + \sigma_4 \frac{x_2}{N_2}\right).
\end{aligned}$$

We obtain the values of some parameters in the model, and through nonlinear data fitting of the original data of the local three populations [Shan and Tang 2007; Sumagaysay-Chavoso 1998; Chen and Chou 2001], we get their natural growth rates:

$$\begin{aligned}
&\sigma_1 = 0.6, &&\sigma_2 = 0.5, &&\sigma_3 = 0.5, &&\sigma_4 = 2; \\
&N_1 = 150 \times 10^3, &&N_2 = 30 \times 10^3, &&N_3 = 2.2 \times 10^3.
\end{aligned}$$

According to the volume of local fish pens and relevant materials, we get the original numbers of the three populations:

$$x_1(0) = 121.5 \times 10^3, \qquad x_2(0) = 27 \times 10^3, \qquad x_3(0) = 2 \times 10^3.$$

Then we use Matlab to implement the model, with the results of **Figure 2**, where we see that can see that with the passage of time, the $x_i(t)$ tend to the steady-state values 69,027, 27,015, and 1,760.

The number 27,015 of the species in Population 2 is made up of herbivorous fish, molluscs, crustaceans, and echinoderms. Now we confirm the numbers of all the species in Population 2, which stay at the same trophic level, coexisting and mutually competing.

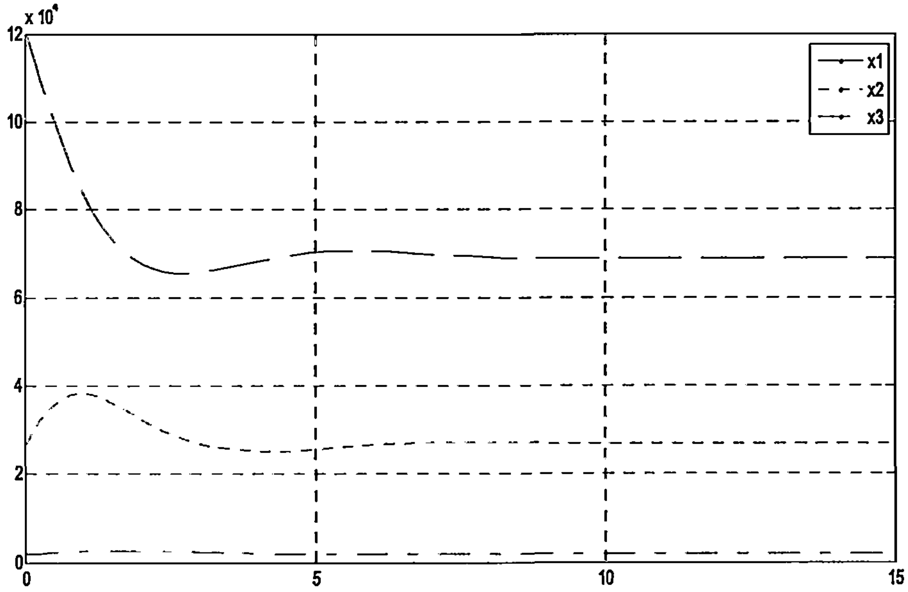

Figure 2. Numerical solutions for $x_i(t)$.

We apply expert system and group decision theory to determine the weights of the species in Population 2. We have a multi-attribute decision problem, where the aim is to select the optimal solution from many alternatives or to sort the available alternatives.

Assume that the finite solution set is $Y = \{y_1, \ldots, y_n\}$, the attribute set is $C = \{c_1, \ldots, c_q\}$, and the decision expert set is $E = \{e_1, \ldots, e_m\}$. Let $S = \{s_1, \ldots, s_g\}$ be a predefined set consisting of odd-chain elements. Expert e_k selects one element from S as the value of solution y_i under attribute c_j; let it be denoted as $p_{ij}^k \in S$, and let

$$p^k = \left(p_{ij}^k\right)_{n \times q}$$

denote the judgment matrix of expert e_k on all the solutions for all the attributes. The attribute weight vector in evaluating information given by expert e_k is

$$W^k = \left(w_1^k, \ldots, w_q^k\right)^T,$$

where w_j^k is the weight of attribute c_j selected by expert e_k from set S, $w_j^k \in S$.

This theory can be actualized through the Analytical Hierarchy Process (AHP), first put forward by American operational researcher T.L. Saaty in the 1970s. AHP is a method for decision-making analysis that combines qualitative and quantitative methods. Using this method, decision-makers can separate complex problems into several levels and factors, and compare and find the weights for different solutions, and provide the basis for the optimum solution.

AHP first classifies the problem into different levels based on the nature and the purpose of the problem, constructing a multilevel structure model ranked as the lowest level (program for decision making, measures etc.), compared with the highest level (the highest purpose). Based on AHP, we can establish the stratification diagram shown in **Figure 3**.

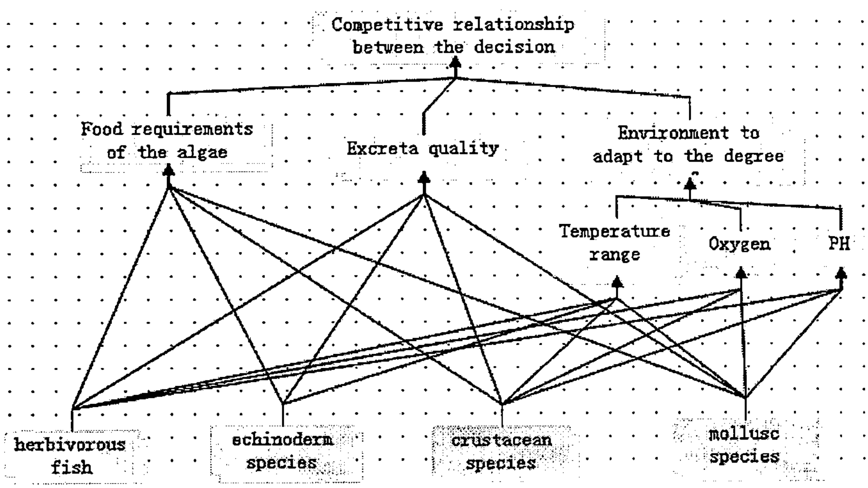

Figure 3. AHP stratification diagram.

At last, we make consistency check of the result, finding that the consistency ratio of each expert's judgment matrix is below 1, so the consistency of the judgment matrix is acceptable. Finally we figure out the weight of the numbers of all the species in Population 2, as shown in **Table 1**:

Table 1.
Weight of each species in Population 2 as measured by AHP.

Species	Weight
Herbivorous fish	.21
Crustaceans	.23
Molluscs	.31
Echinoderms	.24

Here we adopt population competition model to confirm the weight of each species in Population 2:

$$\dot{N}_1 = N_1(\varepsilon_1 + \gamma_1 N_2),$$
$$\dot{N}_2 = N_2(\varepsilon_2 + \gamma_2 N_1),$$

where ε_i are birthrates and γ_i are coefficients of species interaction.

According to these equations, we find that the ratio between different species is almost consistent with that obtained by AHP, which also confirms the correctness of our method.[1]

In this way, we find that herbivorous fish, crustaceans, molluscs, and echinoderms can coexist and also compete. So the number of each species can be figured out based on the data in the steady state from the previous models, as shown in **Table 2.**

Table 2.
Number per pen of each species in steady state.

Organism	Number
Algae	69,027
Herbivorous fish	5,638
Crustaceans	6,305
Molluscs	8,483
Echinoderms	6,589
Milkfish	1,760

Now we use the model to check the water quality, and make clear whether it is suitable for the continued healthy growth of the coral. First, we calculate the current concentration of chlorophyll in a fish pen. With help of relevant references, we find the regression equation between the number of algae and chlorophyll:

$$N = 1.2785 + 0.7568C, \tag{1}$$

where the units are 10^4/ml for N (algae) and μg/L for C (chlorophyll). For $N = 6.9027$ (from **Table 2**), we get $C = 7.43$, a concentration of chlorophyll that is far beyond 0.25 μg/L, the highest suitable concentration for the growth of coral.

From the available data in the problem, we figure out the mass of organic particles in the fish pen, and then work out the mass of each element.

- The dry weight of echinoderms in the pen is 45.5 kg, the dry weight of milkfish excrement is 0.4–0.9 kg, so the total dry weight of excrement in the pen is 1.0–1.4 kg.
- The pen is 10 m × 10 m × 8 m, for a volume of 800 m^3 = 800 × 10^3 L.
- Finally, we get the concentration of organic particles is 1186–1738 μg/L. Based on the percentage of elements given in the problem, we figure out then concentration of carbon C (10%), nitrogen N (0.4%), and phosphorus P (0.6%) (**Table 3**).

[1]EDITOR'S NOTE: The authors' paper does not give further details of the AHP calculation nor of the population competition model.

Table 3.
Concentrations of elements in a pen.

Element	Concentration (μg/L)
C (10%)	119–174
N (0.4%)	5– 7
P (0.6%)	7– 10

Comparing the water quality in Sites A, B, C, and D, we find that the concentration of organics is between A and B, which is suitable for the growth of coral (here the concentration of elements is calculated only based on the excrement of milkfish and echinoderm), so the concentration of the microbes meets the reproduction needs of the coral. But the concentration of chlorophyll is seriously out of limits. So we have to adjust the numbers of some species to make the concentration of chlorophyll reach the standard.

We reason backward from the desired concentration (0.25 μg/L) of chlorophyll suitable for the growth of the coral, using the regression equation **(1)**. With the estimated steady-state value, we can assume the initial values as: (10000, 5500, 350). From relevant references, we get the maximum volume of fish pens : $(N_1, N_2, N_3) = (30000, 6000, 400)$, and through resimulation finally find the positive revised results for the steady-state values: (13732, 5432, 320).

We work out the estimated steady-state number of algae $N = 1.4677$, and then derive the numbers of the three populations: (14677, 5744, 350). After revision, we get the actual steady-state number of the algae: $N = 1.3732$. Putting this value into the regression equation, we get $C = 0.125$, that is, the concentration of chlorophyll is 0.125 μg/L, which means that the water quality after adjustment completely meets the standard demanded. Moreover, the total number of milkfish and echinoderm is smaller than that before revision, so the index of the organics can certainly reach the growing demands of the coral, as shown in **Figure 4**.

In the retroregulation process, which is the feedback mechanism of this model, with known water quality, we reason backwards to the estimated steady-state numbers of all the species, make positive simulation after estimating the initial introducing value of all the species, and get the revised steady-state values. With this mechanism, we can find out the steady-state number of each species based on water quality, which provides great convenience to the solution to the following problems.

Task 2a: Establishment of Logistic Model

In this task, with all the herbivorous fish, crustaceans, molluscs, and echinoderms excluded, we are required to find out the changes to the species and the circumstances of water quality. Based on our analysis, we make

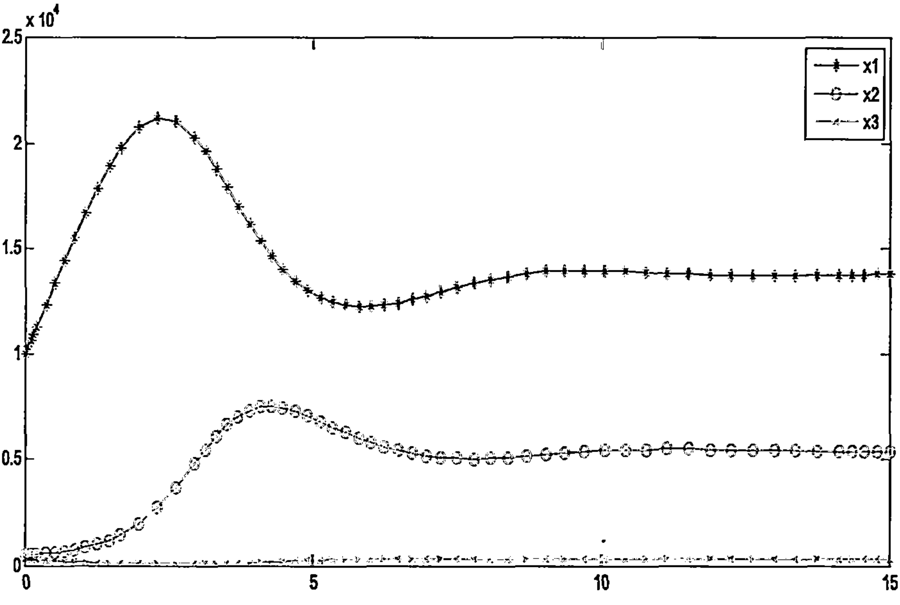

Figure 4. The numbers of species meeting the demands after adjustment.

clear the reasons why the growth rate will decrease after the milkfish increase. Factors such as natural resources and environmental conditions restrict the growth of milkfish; and with their growth, the blocking effect will become greater and greater. The blocking effect is expressed in terms of the influence on the growth rate r of milkfish, making r decrease with the increase in the number x of milkfish. If we express r as $r(x)$, a function of x, it should be a decreasing function, so we have:

$$\dot{x} = r(x), \qquad x(0) = x_0.$$

The simplest assumption of that $r(x)$ is a linear function:

$$r(x) = r - sx \qquad (r > 0,\ s > 0),$$

where r is the intrinsic growth rate. To confirm the meaning of the coefficient s, we introduce the maximum quantity x_m that is allowed by natural resources and environmental conditions, which we regard as the milkfish capacity. When $x = x_m$, then x will stop increasing, that is, the growth rate $r(x)$ will be 0. That occurs for $s = r/x_m$, so that we have

$$r(x) = r\left(1 - \frac{x}{x_m}\right). \tag{2}$$

Another interpretation of (2) is that the growth rate $r(x)$ is in direct proportion to the unsaturated part of the milkfish capacity $x = (x_m - x)/x_m$, where the proportionality coefficient is the intrinsic growth rate r. Putting **(2)** into **(1)**, we get

$$\dot{x} = rx\left(1 - \frac{x}{x_m}\right), \qquad x(0) = x_0. \tag{3}$$

The factor rx on the right side expresses the internal growing tendency of the milkfish, and the factor $(1 - x/x_m)$ expresses the blocking effect of resources and environment on milkfish growth. Obviously, the bigger x is, the bigger rx is, and the smaller $(1 - x/x_m)$ is. The growth of milkfish is the result of the co-action of the two factors. Equation **(3)** can be solved by separation of variables to yield

$$x(t) = \frac{x_m}{1 + \left(\frac{x_m}{x_0} - 1\right) e^{-rt}}. \tag{4}$$

We use linear least squares to estimate the parameters r and x_m of this model, and express **(3)** as

$$\frac{\dot{x}}{x} = r - sx, \qquad s = \frac{r}{x_m}.$$

We consult relevant data in Sumagaysay-Chavoso [1998] (where the amount of milkfish is the amount harvested over the entire Philippines), insert these data into Matlab, and get $r = 0.5$ and $x_m = 1.9 \times 10^5$. Putting these into **(4)**, we get the changes to the function shown in **Figure 5**.

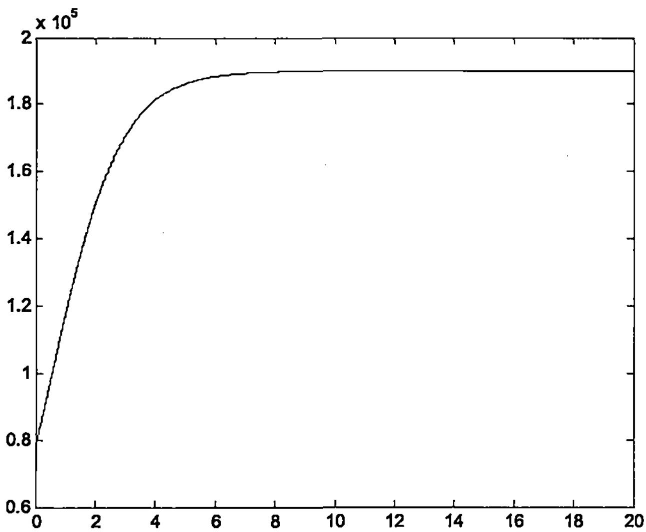

Figure 5. Milkfish changes.

Further, we get the weight and number of the milkfish respectively as 172×10^6 kg and $25–34 \times 10^6$.

The land area of the Philippines is 300,000 km^2, the sea area is 27.6 mi^2. The Philippines is surrounded by the sea, and has lots of islands; the depth of the sea between islands is mostly within 50 m.

Based on the sea area, we calculate the sediment per square meter to be 0.12–0.33 g/m^2. Since the sediment is usually not very thick, we assume that the depth is 0.1 m, so that the sediment per cubic meter is 1.2–3.3 g/m^3. Then based on the information given in the problem, we get the results of **Table 4**.

Table 4.
Element concentrations.

Element	Concentration (μg/L)
C (10%)	117–333
N (0.4%)	47–133
P (0.6%)	70–200

From the table, we can see eutrophication is very serious, and the coral cannot grow. The water quality is very poor, which almost matches the environment in the pens.

Task 2b: Simulating Comparison of the Current Situation

In Task 2a, we discussed the independent farming of the milkfish; but actually in the pen, there are more than just milkfish and algae. So here we have to introduce the removed species as the middle strata, and according to the requirements of the problem, adjust the numbers of the species in the middle strata to simulate the water quality in the Bolinao area until the water quality matches the one currently observed.

The concrete practices are as follows: Simulate the water quality (in Site D, for example) and solve the problem according to the model in Task 1. It is easier to find out the water quality from the initial values of algae, milkfish, and other species than vice versa.

We adopt brute-force random search:

- Set the initial values of algae, other species, and milkfish to 100×10^3, 10×10^3, and 1.3×10^3.
- According to the introductory ratio between the milkfish and the algae, and the requirements for the capacity of the pen obtained from Task 2a, we introduce the algae and the milkfish respectively as 72×10^3 and 1.3×10^3, and at the same time have the introductory numbers of other species come from a random distribution between 8×10^3 and 10×10^3, with the aim of searching for the theoretical value matching the observed water quality.

- Simulate the model in Task 1 1,000 times, and finally output the water quality in steady state that is consistent with the actually observed value.

  We set out the criteria for judging water quality:

- Chlorophyll a $\equiv (0.0001x_1 - 1.2785)/0.7568$.
- Total concentration of organics =

$$x_2 \times 0.2438 \times 6.9 \times [0.2, 11.5] + x1 \times [242, 493].$$

- Percentage of different elements in the excrement: C 10%, N 0.4%, P 0.6%.
- C meets $|c(1) - c1(1)| \leq 100$ and $|c(2) - c1(2)| \leq 100$.
- N meets $|n(1) - n1(1)| \leq 10$ and $|n(2) - n1(2)| \leq 10$.
- Chlorophyll meets $|c_a - 4.5| \leq 0.15$.

We sort out results meeting the above requirements, that is, the numbers of three species when the water quality obtained through simulation similar to the observed one, and show the result in **Table 5.**[2]

Table 5.
Simulation results.

		Pop. 1	Pop. 2	Pop. 3
Simulation results	Initial number $\times 10^3$	70.0	[8.01,9.00]	1.10
	Number in steady state $\times 10^3$	46.1	9.0	1.04
Estimated from data	Number in steady state $\times 10^3$	45.7	9.3	0.9

To make the numbers of the species close to those predicted in the model, we compare the numbers of existing species with those observed in Bolinao area. Here we take into account that the added feedstuff for milkfish can revise the model in Task 1, that is, we can add a constant λ to the the third equation of the model in Task 1 to express the influence of feedstuff on the numbers of the species. The revised model is:

$$\dot{x}_1(t) = x_1 r_1 \left(1 - \frac{x_1}{N_1} - \sigma_1 \frac{x_2}{N_2}\right),$$

$$\dot{x}_2(t) = r_2 x_2 \left(-1 - \frac{x_2}{N_2} + \sigma_2 \frac{x_1}{N_1} - \sigma_3 \frac{x_3}{N_3}\right),$$

$$\dot{x}_3(t) = r_3 x_3 \left(-1 - \frac{x_3}{N_3} + \sigma_4 \frac{x_2}{N_2}\right) + \lambda.$$

We set initial values (70000, [8008,8995], 1100), and calculate the steady-state numbers of all the species: (46062, 8989, 1051), as shown in **Figure 6.**

[2]EDITOR'S NOTE: The accompanying Matlab code does not impose the constraints indicated above on N and C.

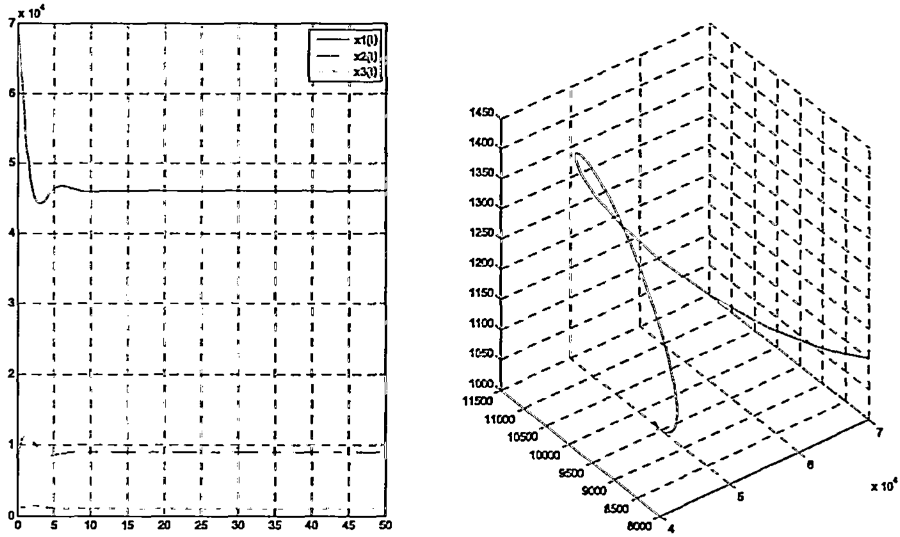

Figure 6. Comparison between observed values and simulated values.

Task 3

Task 3a: Develop a commercial polyculture to remediate Bolinao

We start from the model of Task 1 (the Bolinao coral reef ecosystem model before farming), introduce filter feeders, and revise the model. We renumber the species, with algae as 1, filter feeders as 2, herbivores as 3, and milkfish as 4. Following the same modeling principles as earlier, we arrive at the system:

$$\begin{aligned}
\dot{x}_1(t) &= x_1 r_1 \left(1 - \frac{x_1}{N_1} - \sigma_{12}\frac{x_2}{N_2} - \sigma_{13}\frac{x_3}{N_3}\right), \\
\dot{x}_2(t) &= r_2 x_2 \left(-1 - \frac{x_2}{N_2} + \sigma_2\frac{x_1}{N_1} - \sigma_7\frac{x_4}{N_4}\right), \\
\dot{x}_3(t) &= r_3 x_3 \left(-1 - \frac{x_3}{N_3} + \sigma_3\frac{x_1}{N_1} - \sigma_8\frac{x_4}{N_4}\right), \\
\dot{x}_4(t) &= r_4 x_4 \left(-1 - \frac{x_4}{N_4} + \sigma_4\frac{x_2}{N_2} + \sigma_6\frac{x_3}{N_3} + \sigma_5 k\right),
\end{aligned} \tag{5}$$

where we now use k for the constant of feedstuff.

We solve this system in Matlab to obtain the numbers of algae, filter feeders, herbivorous fish, and milkfish: (14314, 6092, 6129, 6979). **Figure 7** shows the system tending toward equilibrium.

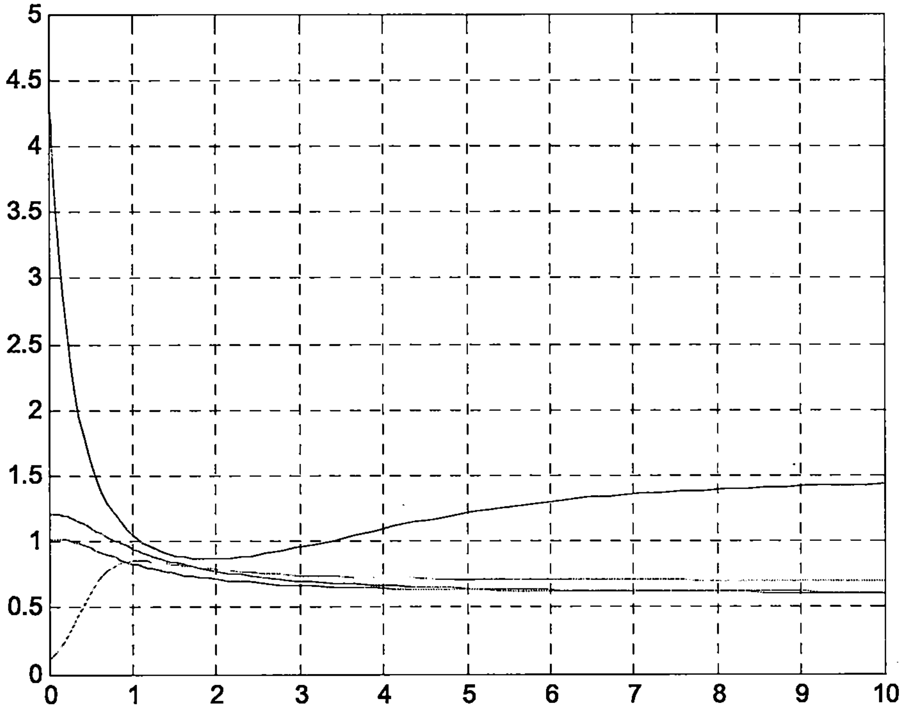

Figure 7. The changes in the numbers of algae, filter feeders, herbivorous fish, and milkfish.

Report on the outputs of the model

Based on **(6)**, we find:

- This model optimize the water quality, since only when the water quality reaches a certain standard, can it provide the ideal growing environment for a species, and only in the viable environment, is it meaningful to talk about the number of each species.
- We establish a newly-born coral reef habitat without the help of man, that is, without feedstuff casting, with least leftover nutriment and particles (foodstuff and excrement) sediment.
- According to Task 3a, we get the steady-state numbers of algae, filter feeders, herbivorous fish, and milkfish. We regard those as the initial values and determine the concentration of chlorophyll as 0.202 μg/L. Based on the information about the elements percentage given in problem, we calculate the content of different elements, as shown in **Table 6.**
- Assume that the total income is $K = \sum x_i v_i$, where v_i is the market value of a unit of species i.
- Based on market investigation and relevant online data, we get the aver-

Table 6.
Concentrations of elements in a pen.

Element	Concentration (μg/L)
C (10%)	35 –72
N (0.4%)	1.4– 2.9
P (0.6%)	2.1– 4.3

age weight and price of each species, and finally figure out the income: $K = \$114 \times 10^3/\text{pen}$.

- To calculate the cost of improving water quality, assume that we introduce 1,000 mussels into the pen. We investigate such factors as weight and market price of mussels, and put them into the model in Task 1 to figure out all the indexes.

Table 7.
Steady-state numbers ($\times 10^4$) of species before adjustment.

	Algae	Molluscs (mussels)	Herbivorous Fish	Milkfish
Before adjustment	1.43	0.61	0.61	0.70
After adjustment	1.37	0.62	0.61	0.70

Table 8.
Concentrations of elements (μg/L) before and after adjustment.

	Chlorophyll	C	N	P
Before adjustment	0.202	35–71	1.4–2.9	2.1–4.3
After adjustment	0.125	33–69	1.3–2.8	2.0–4.1

From **Table 8**, it is easy to see that the water quality has improved. For one thing, the introduced mussels feed on the algae for one thing, and for another they decompose the organic particles.

- The 1,000 introduced mussels cost \$361 or so, scarcely making a dent in the income.

Task 4

From Task 3a, we know the numbers of algae, filter feeders, herbivorous fish, and milkfish: (14314, 6092, 6129, 6979). The algae are the most numerous, and the numbers of the other species are roughly equal. In such a steady state:

- According to the relationship between supply, demand, the price of milkfish is higher that that of seaweed. In addition, although the amount of seaweed is large, it is light, so we cannot pursue maximizing weight.
- Measuring harvest with the price of each species harvested, we have to differentiate the values of the species. Since it costs to feed the milkfish, we should take these costs into consideration when calculating the values of each species. We define the value of edible biomass as the sum of the values of each species harvested, minus the cost of milkfish feed.

Task 5

When evaluating a commercial polyculture scheme, we usually consider not only the economic benefits of farming, but also try to ensure reaching a win-win between economy and environment under the premiss of keeping the ecological environment and water quality in good condition.

Hence, we establish the following optimal model to pursue the maximum commercial benefits, with the premiss of not having water quality worsen. Combined with the previous polyculture system model, we establish the following nonlinear optimization model of balance to maximize the total values of harvest. It is a complex nonlinear single-objective optimization model, since nonlinear differential equations are embedded into the constraints:

Objective function: $\max f = ax_1 + bx_2 + cx_3 + dx_4 - \mu$,
where a, b, c, d are the unit market prices of the species and μ is the feedstuff price.

The constraints on water quality are:

- concentration of of chlorophyll ≤ 0.28 mg/mL,
- concentration of C ≤ 196 μg/L, and
- concentration of N ≤ 39 μg/L.

We can express these conditions in the equations involving the x_i as follows:

$$\frac{0.0001x_1 - 1.2785}{0.756} \leq 0.28,$$
$$1.68222x_2[0.2, 11.5] + 0.1x_4[242, 493] \leq 196,$$
$$1.68222x_2[0.2, 11.5] + 0.004x_4[242, 493] \leq 39.$$

In addition, we have the equality relations among the x_i in **(6)**.

Such a complex optimization problem cannot be solved directly with any software, so first we make a cycle simulation search (actually still a brute-force search) to find enough solutions meeting water quality conditions, and obtain intervals for the steady-state numbers of the species that meet the demands of water quality, as shown in **Table 9**.

Table 9.
Steady-state numbers ($\times 10^4$) of species.

	Algae	Molluscs (mussels)	Herbivorous Fish	Milkfish
Maximum	1.3922	0.6249	0.6233	0.7061
Minimum	1.3286	0.6152	0.6174	0.7018

Therefore, we can replace the equality conditions among the x_i by intervals for the steady-state numbers:

$$\begin{aligned} 1.3286 &\le x_1 \le 1.3922, \\ 0.6152 &\le x_2 \le 0.6249, \\ 0.6174 &\le x_3 \le 0.6233, \\ 0.7108 &\le x_4 \le 0.7061. \end{aligned}$$

We can now use Lingo to solve the equivalent model, with the results of **Table 10.**

Table 10.
Optimal steady-state numbers ($\times 10^4$) of species.

	Algae	Molluscs (mussels)	Herbivorous Fish	Milkfish
Optimal	1.39	0.62	0.62	0.71

The corresponding the maximum harvest value is \$115 $\times 10^3$, and the corresponding water quality is shown in **Table 11.**

Table 11.
Concentrations of elements (μg/L) after optimization.

	Chlorophyll	C	N	P
At optimal	0.15	17–36	0.7–1.4	1.0–2.2

Compared to the water quality required by coral growth, the water quality obtained here is obviously satisfactory, and we reap relatively high economic benefits at the same time.

In order to prove the results of our model are correct, we define:

$$\text{fishing/harvest index} = \frac{\text{feed cost}}{\text{net income}}$$

Then the result we obtained is: fishing/harvest index = 0.06%. The actual result is: fishing/harvest index = 2.8%.

Based on analyses of the model, for the optimal solution we find the feeding cost for one unit of net income is obviously less than the current cost, so our feeding strategy can produce better harvest.

Ecological Perspective on Polyculture

Adding herbivorous fish as the middle stratum

- contributes to the decomposition of solid particles,
- can suppress the over-multiplication of the algae,
- can improve water quality,
- can enable the coral to grow normally, and
- thereby can restore the ecosystem and biodiversity.

However, in our model we don't take into account the soluble POC released by the algae, the accumulation of which is likely to hinder the improvement of water quality. In view of this, some may doubt the restorative ability of our polyculture system. But bacteria in the waters can process POC and rational measures can be taken to control the concentration of microbes, thus ensuring the improvement of water quality. So, in terms of ecology, our polyculture system bears the potential of improving water quality and promoting favorable development of the ecosystems.

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