STAT511 – Exam 1
Fall 2018

Honor Pledge: I have not given, received, or used any unauthorized assistance on this exam.

Signature:____

Printed Name: KEY

Instructions:

- **Open book, open notes, calculator required. No computers or cell phones.**
- **Time limit is 1 hour 50 minutes - strictly enforced!**
- If an answer is in the computer output, use it; don't calculate it by hand.
- Show your work where appropriate. Put your final answer in the box (if provided).
- Make explanations brief and legible.
- All questions are worth 4 points except where noted. Maximum score is 100.
- Computer input/output is provided at the end of the exam.
- The exam contains a total of 8 pages (including computer input/output).
- If you run out of space, you may use the blank area on page 6 or extra paper.

Questions 1 through 4 (IQ scores): The intelligence quotient (IQ) score, as measured by the Stanford-Binet IQ test, is normally distributed in a certain population of adults. The mean IQ score is 100 points and the standard deviation is 16 points. In other words, let Y be the random variable representing IQ and assume Y ~ N(μ = 100, σ = 16).

1. What proportion of adults have an IQ between 80 and 120? In other words, find P(80 ≤ Y≤ 120). Give your answer to two decimal places.

$P(80 \le Y \le 120) = P(-1.25 \le Z \le +1.25)$
$= 0.8944 - 0.1056$
$= 0.7888$

$\boxed{0.79}$

2. Give an interval such that 95% of adults have IQ scores within this interval. Give your bounds to an integer value.

Empirical Rule: $\mu \pm 2\sigma$
$100 \pm 2 \cdot 16$

(69, 131) OK

$\boxed{(68, 132)}$

(0, 126) OK

3. Mensa is a "high IQ society". Mensa's requirement for membership is an IQ score at or above the 98th percentile. Find the IQ score such that only 2% of adults have scores higher than this value (corresponding to Mensa eligibility). For this question, either provide the answer or provide the R code you could use to find the answer.

qnorm(0.98, mean=100, sd=16)

$P(Z \le z) = 0.98 \rightarrow z = +2.055$

$y = \mu + z \cdot \sigma$
$= 100 + 2.055 \cdot 16$
$= 132.88$

$\boxed{133}$

−2 for 67
−2 for wrong "z"

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IQ score questions continued….

4. Consider a party where all n = 20 guests are part of Mensa. Suppose we have the IQs for each guest at the party. Remember Mensa is a "high IQ society". No need to justify for these questions. **(2pts each)**
   A. Recall that the mean IQ for the general population is 100. Would you expect the mean of IQs at the Mensa party to be higher or lower than 100?

      Higher

   B. Recall that the standard deviation of IQ for the general population is 16. Would you expect the standard deviation of IQs at the Mensa party to be higher or lower than 16? For reference, Albert Einstein and Stephen Hawking supposedly had extremely high IQs at 160.

      Lower

   C. Recall that IQs for the general population is normally distributed. Would you expect the distribution of IQ scores at the Mensa party to be normally distributed? If not, what "shape" would you expect?

      Skewed right

Questions 5 through 9 (Chicken Feed Proposal): Suppose you are the PI planning a study to compare two chicken feeds (A and B). Prior to starting the study, you are required to submit a proposal to your committee. The study will start with some number of chicks 6 weeks in age. The chicks will then be divided into two groups, with half of the chicks provided Feed A and the other half provided Feed B. At 12 weeks of age, the weight gain (in grams) for each chick will be recorded. The "better" feed is the one with the larger average weight gain. Use alpha = 0.05.

5. For the purposes of the proposal, you need to include an analysis plan (indicating what statistical analysis will be done at the end of the study). Give the name of an analysis that could be used to compare the two chicken feeds. **Notes:** Be specific, but no need to justify. There may be more than one correct answer to this question. You may need to make some preliminary "assumptions" here.

   Two-Sample t-test   CI for $\mu_1 - \mu_2$

   Note: Use 2sided test by default!!

6. Considering the analysis you proposed in the previous question, briefly describe how you would choose (or justify) the sample size for the study.

   -4 for largest possible   Power Calculation   CI/ME width

   -2 for ME/CI width if 2 sample t-test above.

7. Considering the sample size justification you proposed in the previous question, what information would you need to do the calculation? List all required info.

   Conj for $\mu_A - \mu_B$ Conj for σ

   Conj for σ

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8. What is the benefit of randomly assigning the chicks to the two Feed groups?

Protects against bias.
Allows causal conclusions.

9. Suggest one approach that could be used to reduce (or control) variability in this study. **Note:** This is a common sense question with many possible correct answers, not something you will find in our text book.

Use fixed breed of chicken.
This is just one example.

-2 for increase sample size

Questions 10 through 12 (Chicken Feed Analysis): The study was started with a total of n = 50 chicks. But during the study two of the chicks on Feed A died leaving sample sizes of $n_A = 23$ and $n_B = 25$. After the study is completed, the resulting data is entered into a CSV file. Suppose the CSV file contains two columns: Feed (A or B) and Gain (in grams). Suppose you are now working on data analysis and preparing a presentation.

10. Name a graph that would be appropriate for summarizing the results in your presentation. No need to justify.

Boxplots
Bar chart w/error bars

11. Considering the fact that two of the chicks on Feed A died (by accident, deaths not related to nutrition), what (if any) changes would be required for analysis (compared to what you proposed in #5)?

No changes required due to $n_A \neq n_B$

Note: Decision to use W-S vs Pooled t-test depends on s_1, s_2 not n_1, n_2 !!

12. What type of numerical results would you present in your write up? Consider your proposed analysis (from #5), but also consider what practical information would help the reader.

Test Statistic / p-value or CI
Summary statistics (n, mean, SE)

Questions continue on the next page……

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<u>Questions 13 through 16 (Sodium):</u> Daily sodium consumption (in mg) was measured for n = 26 adult Americans. Assume these values were obtained from a random sample. For convenience, let μ (mu) represent the true population mean. According to "Dietary Guidelines for Americans 2005", it is suggested that adults should consume 2,300 mg of sodium or less per day.

```
#Summary Statistics
> length(Sodium)
[1] 26
> mean(Sodium)
[1] 2317.8
> sd(Sodium)
[1] 159.6
```

```
        One Sample t-test
data:   Sodium
t = 0.5676, df = [redacted], p-value = 0.2877
alternative hypothesis: true mean is greater
than 2300
95 percent confidence interval:
 2264.303       Inf
sample estimates:
mean of x
 2317.8
```

13. Calculate the 95% Margin of Error (95% ME) for the mean.

$t_{\alpha/2} \cdot s/\sqrt{n} = 2.060 \left(\frac{159.6}{\sqrt{26}}\right)$

$= 64.478$

-2 for 53.5

$\boxed{64.48}$

14. A p-value (p = 0.2877) is shown in the t.test() output. What hypotheses are being tested? Be specific.

H0: $\mu \leq 2300$

HA: $\mu > 2300$

15. The American Heart Association recommends that adults consume 1,500 mg of sodium or less per day. For this question, suppose the hypotheses from the previous question were revised to use this as the null hypothesized value. In other words, run a hypothesis test similar to the one above, but use a null hypothesized value of 1500.

A. **(4 pts)** Test statistic = $t = \frac{(\bar{y} - \mu_0)}{(s/\sqrt{n})} = \frac{2317.8 - 1500}{(159.6/\sqrt{26})} = 26.13$

B. **(2 pts)** df = $n - 1 = 25$

C. **(2 pts)** Reject H0 if $t > t_\alpha = +1.708$

D. **(2 pts)** Conclusion: Reject H_0

16. What <u>assumption</u> is required for the test from above to be <u>valid</u>? **(2 pts)**

Normality.

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Questions 17 through 23 (Brains): Studies have linked brain volume in toddlers to a number of future ailments, including autism. One study looked at the brain sizes of $n_A = 7$ **Autistic** boys and $n_C = 5$ **Control** (non-autistic) boys who all had MRI scans as toddlers. The whole-brain **Volume** (in mL) was recorded for each child. For convenience, let μ_A (muA) and μ_C (muC) represent the population mean brain volumes for the two groups. The R input and output is labeled **Brains.** Use alpha = 0.05. (This data is a subset from an article published in Neurology (2001) by Courchesne et al.)

17. Is this an experiment or an observational study? Circle one answer, no need to justify. **(2 pts)**

Experiment   Observational Study

18. Using the Test1 output, interpret the 95% confidence interval.

We can be 95% confident that true diff between pop means ($\mu_A - \mu_C$) is between -6.19 and +178.25.

19. Using the Test1 output, test H_0: $\mu_A - \mu_C = 0$ vs H_A: $\mu_A - \mu_C \neq 0$. Briefly justify your response.

Conclusion: Fail to Reject Ho

Justification: p-value $= 0.06366 > \alpha = 0.05$
CI includes zero

20. At least one of the tests shown in the R output requires the assumption of normality.
A. Which of the tests require the assumption of normality? Circle all that apply. **(2 pts)**

Test1   Test2   Test3

B. Is the assumption of normality reasonable for this data? Briefly justify your response.

QQplots look OK but not great.
Large p-values for S-W tests support normality.

21. At least one of the tests shown in the R output requires the assumption of equal variances.
A. Which of the tests require the assumption of equal variances? Circle all that apply. **(2 pts)**

Test1   Test2   Test3(?)

B. Is the assumption of equal variances reasonable for this data? Briefly justify your response.

$71.8/63 = 1.14 < 2$
Assumption of equal variances is reasonable.

Questions continue on the next page……

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Brains questions continued….

22. A colleague suggests that you should "always use non-parametric test with the fewest assumptions". Regardless of your answers above, assume needed assumptions are satisfied; give one benefit of using a two-sample t-test instead of a non-parametric alternative.

Two-Sample t-test offers increased power compared to Two-Sample Wilcoxon

(OK) Wilcoxon assumes shifted/identical distb'ns. OR does not provide CI

23. Considering the results of Tests 1, 2 and 3 (and using alpha = 0.05), is there any practical difference between the conclusions for these tests?

Test 1 p-value = 0.064

Test 2 p-value = 0.052

Test 3 p-value = 0.106

All p-values > $\alpha = 0.05$

So conclusions for all tests are the same.

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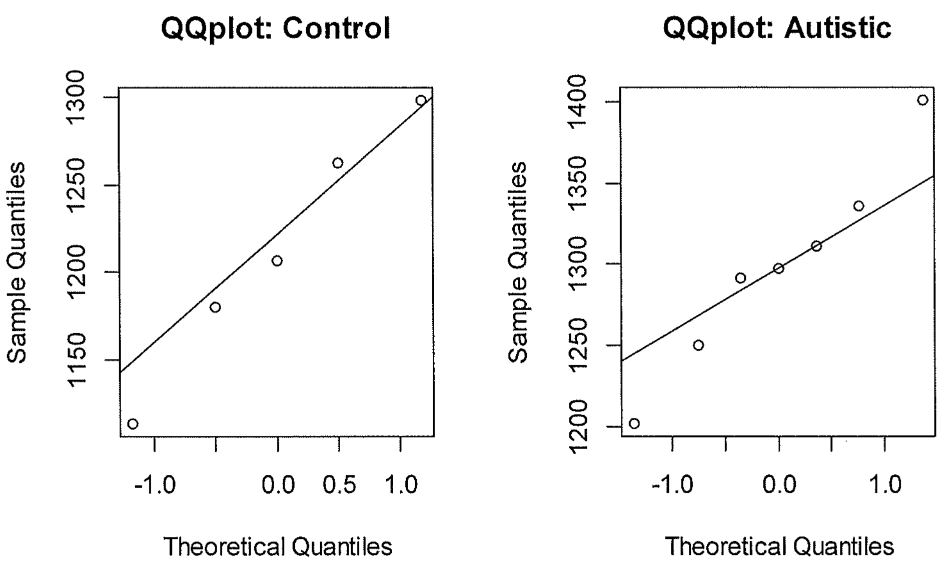
Brains (Questions 17 through 23)

```
> str(Brains)
'data.frame':      12 obs. of  2 variables:
 $ Group : Factor w/ 2 levels "autistic","control": 1 1 1 1 1 1 1 2 2 ...
 $ Volume: int  1311 1250 1292 1401 1297 1202 1336 1114 1180 1207 ...
> SumStats <- summarize(group_by(Brains, Group),
+                       n = n(),
+                       mean = mean(Volume),
+                       sd = sd(Volume),
+                       se = sd/sqrt(n))
> SumStats
# A tibble: 2 x 5
  Group        n  mean    sd    se
1 autistic     7  1298  63.0  23.8
2 control      5  1212  71.8  32.1

> Control <- subset(Brains, Group == "control")$Volume
> Autistic <- subset(Brains, Group == "autistic")$Volume
> par(mfrow = c(1,2))
> qqnorm(Control, main = "QQplot: Control");qqline(Control)
> qqnorm(Autistic, main = "QQplot: Autistic");qqline(Autistic)
```




```
> shapiro.test(Control)
      Shapiro-Wilk normality test
data:  Control
W = 0.981, p-value = 0.9399

> shapiro.test(Autistic)
      Shapiro-Wilk normality test
data:  Autistic
W = 0.98114, p-value = 0.9649
```

```
> #Test1
> t.test(Volume ~ Group, data = Brains)

        Welch Two Sample t-test

data:  Volume by Group
t = 2.1517, df = 7.9882, p-value = 0.06366
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
  -6.194889 178.252032
sample estimates:
mean in group autistic  mean in group control
              1298.429               1212.400

> #Test2
> t.test(Volume ~ Group, data = Brains, var.equal = TRUE)

        Two Sample t-test

data:  Volume by Group
t = 2.2043, df = 10, p-value = 0.05206
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
  -0.9319084 172.9890512
sample estimates:
mean in group autistic  mean in group control
              1298.429               1212.400

> #Test3
> wilcox.test(Volume ~ Group, data = Brains)

        Wilcoxon rank sum test

data:  Volume by Group
W = 28, p-value = 0.1061
alternative hypothesis: true location shift is not equal to 0
```

Exam1 Extra Output

Note: This information was not provided in the original exam!

#1-4 (IQs)

#1

```
> pnorm(120, mean = 100, sd = 16) - pnorm(80, mean = 100, sd = 16)
[1] 0.7887005
```

#2

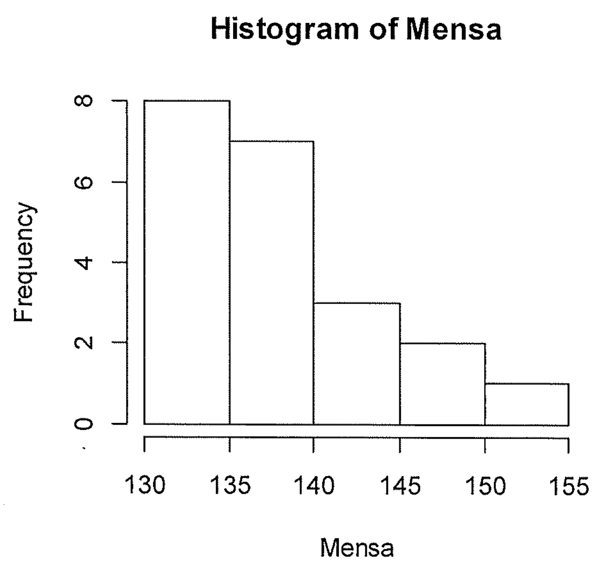
```
> pnorm(132, mean = 100, sd = 16) - pnorm(68, mean = 100, sd = 16)
[1] 0.9544997
> pnorm(131, mean = 100, sd = 16) - pnorm(69, mean = 100, sd = 16)
[1] 0.9473157
```

#3

```
> qnorm(0.98, mean = 100, sd = 16)
[1] 132.86
```

#4

```
> set.seed(5883)
> Temp <- rnorm(1000, mean = 100, sd = 16)
> Mensa <- Temp[Temp >=133]
> mean(Mensa)
[1] 139.4648
> sd(Mensa)
[1] 5.827978
> hist(Mensa)
```



#15 (Sodium)

```
> t.test(Sodium, mu = 1500, alternative = "greater")
      One Sample t-test
data:  Sodium
t = 26.128, df = 25, p-value < 2.2e-16
alternative hypothesis: true mean is greater than 1500
95 percent confidence interval:
 2264.303      Inf
```