STAT 511A Homework 10

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Load packages

```
library(broom)
library(lawstat)
library(metafor)
library(readr)
```

Question 1

Bacillus Calmette-Guerin (BCG) is a vaccine for preventing tuberculosis. For this question, we will examine data from 3 studies (Vandiviere et al 1973, TPT Madras 1980, Coetzee & Berjak 1968). The data is summarized below.

A note about the BCG vaccine from Wikipedia: The most controversial aspect of BCG is the variable efficacy found in different clinical trials that appears to depend on geography. Trials conducted in the UK have consistently shown a protective effect of 60 to 80%, but those conducted elsewhere have shown no protective effect, and efficacy appears to fall the closer one gets to the equator.

Create data array

```
tb <- array(c(619, 2537, 10, 8,
              87892, 87886, 499, 505,
              7232, 7470, 45, 29),
            dim = c(2, 2, 3),
            dimnames = list(group = c("ctrl", "trt"),
                            response = c("tbneg", "tbpos"),
                            study = c("1", "2", "3")))
```

Part 1A

Calculate the odds ratio (corresponding to TBpos for Trt vs Ctrl) for each study separately. (4 pts)

```
tb_cmh <- broom::tidy(lawstat::cmh.test(tb))
```

Study 1 odds ratio

```
tb_cmh$`Odd Ratio of level 1`
```

```
## [1] 0.1951912
```

Study 2 odds ratio

```
tb_cmh$`Odd Ratio of level 2`
```

```
## [1] 1.012093
```

Study 3 odds ratio

```
tb_cmh$`Odd Ratio of level 3`
```

```
## [1] 0.6239119
```

Part 1B

Use the Breslow-Day test to test for equality of odds ratios across the 3 studies. State your p-value and conclusion. Can we conclude that the odds ratios are equal across the 3 studies? Based on this test, should we combine information across studies? (4 pts)

```
cmh <- metafor::rma.mh(ai = tb[1, 1,],
                       bi = tb[1, 2,],
                       ci = tb[2, 1,],
                       di = tb[2, 2,])
cmh$BDp
```

```
## [1] 0.0001456754
```

The Breslow-Day Test for equality of odds ratios of three tuberculosis studies yielded a p-value $< .001$. There is evidence to suggest a difference between odds ratios by study; it is not appropriate to combine information across studies.

Question 2

Problem 10.36 involves bomb hits during WWII. Bomb hits were recorded in n = 576 grids in a map of a region of South London.

Create data list

```
# observations
obs <- c(229, 211, 93, 35, 8)
y <- seq(from = 0, to = 4, by = 1)
bombs <- cbind(y, obs)
```

Part 2A

Find the sample mean ($\hat{\mu}$) bomb hits per grid.

```
# mean
muhat <- sum(obs*y)/sum(obs)
muhat
```

```
## [1] 0.9270833
```

$\hat{\mu} = 0.9270833$ bomb hits per grid.

Part 2B

Use the GOF test to test whether the number of bomb hits per grid follows the Poisson distribution. Calculate the GOF test statistic, df, p-value and give a conclusion using $\alpha = 0.05$. (6 pts)

```
# calculate the corresponding Poisson probabilities
prob <- dpois(y, muhat)
prob
```

```
## [1] 0.39570617 0.36685260 0.17005146 0.05255063 0.01217970
```

```
length(prob)
```

```
## [1] 5
```

```
sum(prob)
```

```
## [1] 0.9973406
```

```
# "fix" the final entry so that the probabilities sum to 1
prob[5] <- 1 - sum(prob[1:4])
prob
```

```
## [1] 0.39570617 0.36685260 0.17005146 0.05255063 0.01483914
```

```
length(prob)
```

```
## [1] 5
```

```
sum(prob)
```

```
## [1] 1
```

```
# calculate expected values and contributions to chi-square test statistic
exp <- prob*576
x2 <- (obs-exp)^2/exp
cbind(y, obs, prob, exp, x2)
```

```
##      y obs       prob        exp           x2
## [1,] 0 229 0.39570617 227.926755 0.0050536183
## [2,] 1 211 0.36685260 211.307096 0.0004463069
## [3,] 2  93 0.17005146  97.949643 0.2501180049
## [4,] 3  35 0.05255063  30.269161 0.7393941810
## [5,] 4   8 0.01483914   8.547345 0.0350502750
```

```
# run GOF test
gof_chi <- sum(x2)
gof_chi
```

```
## [1] 1.030062
```

```
gof_df <- 5-2
```

```
gof_p <- 1 - pchisq(gof_chi, gof_df)
gof_p
```

```
## [1] 0.7939783
```

We fail to reject the null hypothesis that data are from a Poisson distribution, $p = 0.7939783 > \alpha = 0.05$.

Question 3

The data "PoissonData.csv" gives observations Y (counts or events) for n = 50 (units) generated from the Poisson distribution, using the `rpoiss()` function.

```
poisson <- readr::read_csv("PoissonData.csv")
```

```
## Parsed with column specification:
## cols(
##   Y = col_double()
## )
```

Part 3A

Calculate the sample mean and sample standard deviation. Also construct a histogram and qqplot of the data and include them in your assignment. (4 pts)

Sample mean

```
mu <- mean(poisson$Y)
mu
```

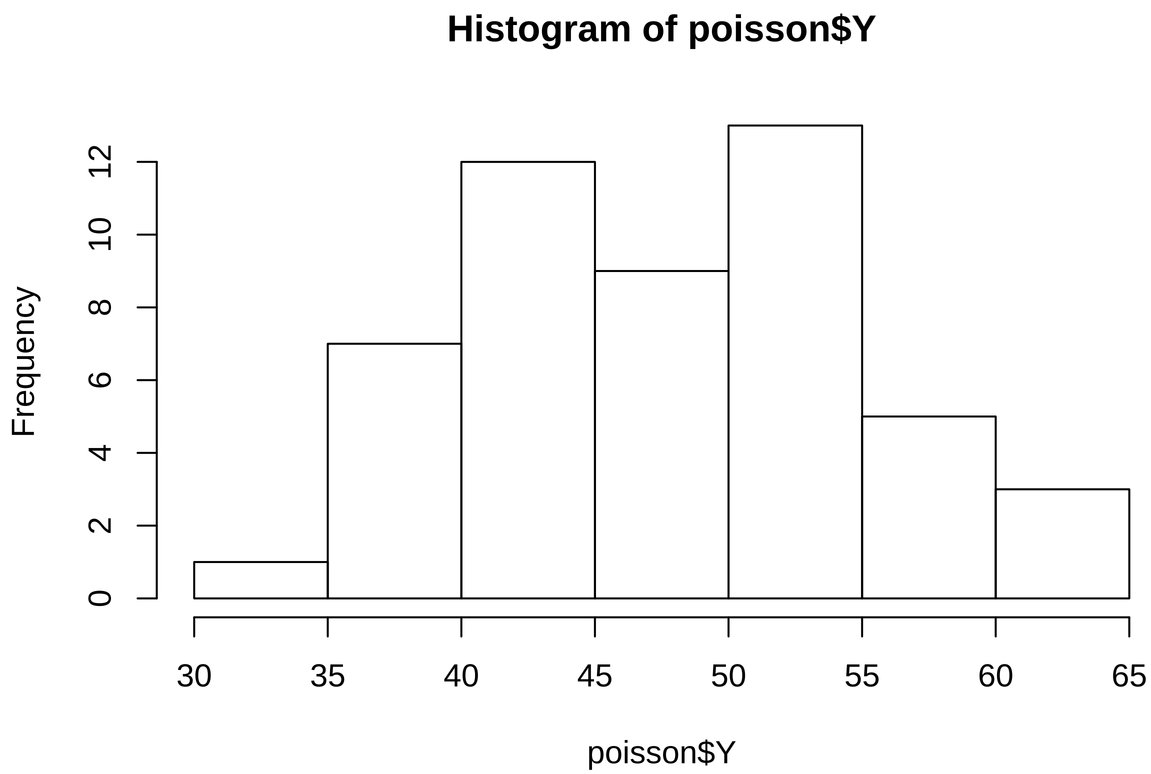
```
## [1] 48.38
```

Sample standard deviation

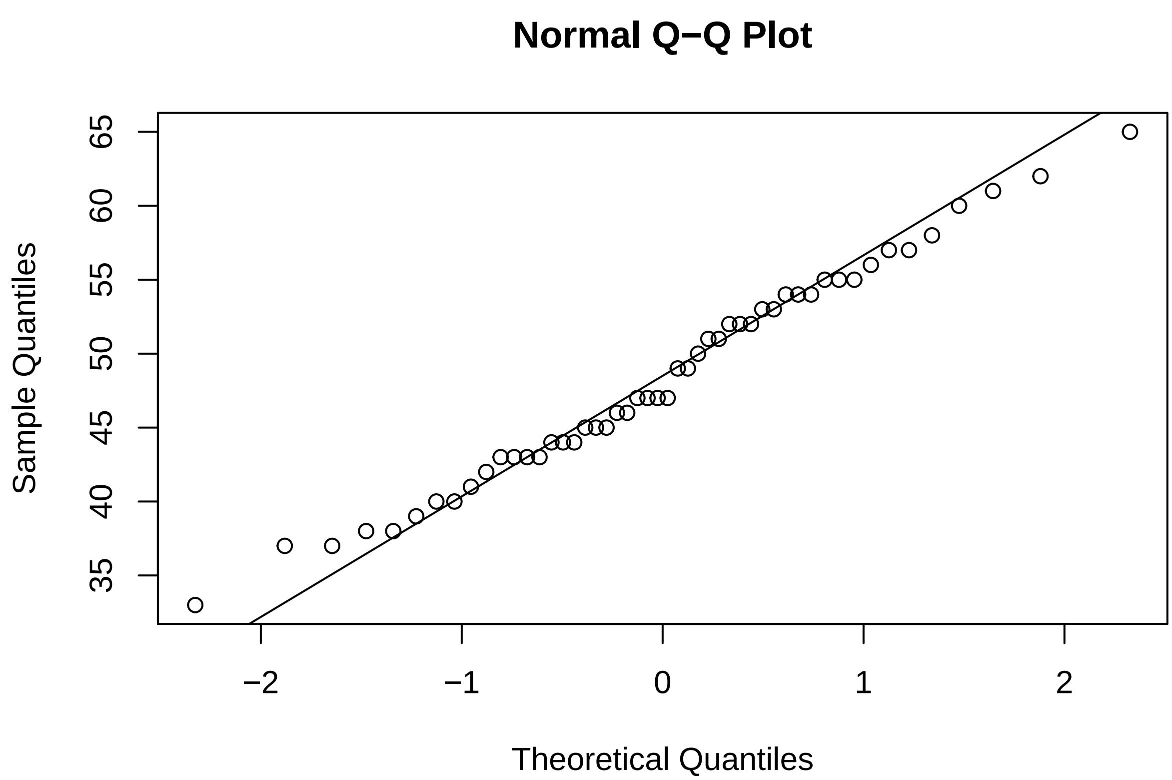
```
s <- sqrt(mu)
s
```

```
## [1] 6.955573
```

Histogram



Q-Q Plot



NOTE: Because the data comes from the Poisson distribution, you should find that the mean and the sample variance (s^2) are close; however, you should also find from the histogram and qqplot that the data looks approximately normal.

Part 3B

Give a standard t-based 95% confidence interval for μ.

```
t <- 2.009
n <- nrow(poisson)
t_ci_ub <- mu + t*(s/(sqrt(n)))
t_ci_lb <- mu - t*(s/(sqrt(n)))
```

(46.4038138, 50.3561862)

Part 3C

Following the example on CH10 Slide 106 (Death by Mule Kick CI), construct a 95% confidence interval for µ based on the normal approximation to the Poisson distribution. (4 pts) In order to do this, you will start by constructing a CI on the total number of events, then divide by the number of units.

NOTE: The CIs from parts B and C should be similar.

```
z <- 1.96
y <- sum(poisson$Y)
z_ci_ub <- (y + z*(sqrt(y)))/50
z_ci_lb <- (y - z*(sqrt(y)))/50
```

(46.4520134, 50.3079866)