STAT511 – Exam 1
Fall 2017

Honor Pledge: I have not given, received, or used any unauthorized assistance on this exam.

Signature:____

Printed Name: KEY

Instructions:

- **Open book, open notes, calculator required. No computers or cell phones.**
- **Time limit is 1 hour 50 minutes - strictly enforced!**
- If an answer is in the computer output, use it; don't calculate it by hand.
- Show your work where appropriate. Put your final answer in the box (if provided).
- Make explanations brief and legible.
- All questions are worth 4 points except where noted. Maximum score is 100.
- Computer input/output is provided at the end of the exam.
- The exam contains a total of 8 pages (including computer input/output).
- If you run out of space, you may use the blank area on page 6 or extra paper.

Questions 1 through 3 (Fill Weights): The weight of cans filled by a certain machine is normally distributed with mean 16.12 oz and standard deviation 0.18 oz. In other words, let Y be the random variable representing fill weight and assume $Y \sim N(\mu = 16.12, \sigma = 0.18)$. Giver your final answers to two decimal places.

1. What proportion of cans contain of 16oz or less? In other words, find $P(Y \leq 16)$.

$P(Y \leq 16) = P(Z \leq -0.67)$
$= 0.2514$

0.2546 OK

$\boxed{0.2514}$

-1 for 0.1335

2. Find the weight such that 2% of cans will weigh less than this amount.

$P(Z < z) = 0.02 \rightarrow \boxed{z = -2.055}$

$y = \mu + z \cdot \sigma$
$= 16.12 - 2.055 * 0.18 = 15.75$

-2 for wrong z

$\boxed{15.75 \text{ oz}}$

3. The process mean can be adjusted through calibration. To what value should the mean (μ) be set so that 2% of cans will weigh less than 16oz.

$y = \mu + z \cdot \sigma$
$\mu = y - z \cdot \sigma$
$= \boxed{16} + 2.055 \cdot 0.18$
$= 16.37$

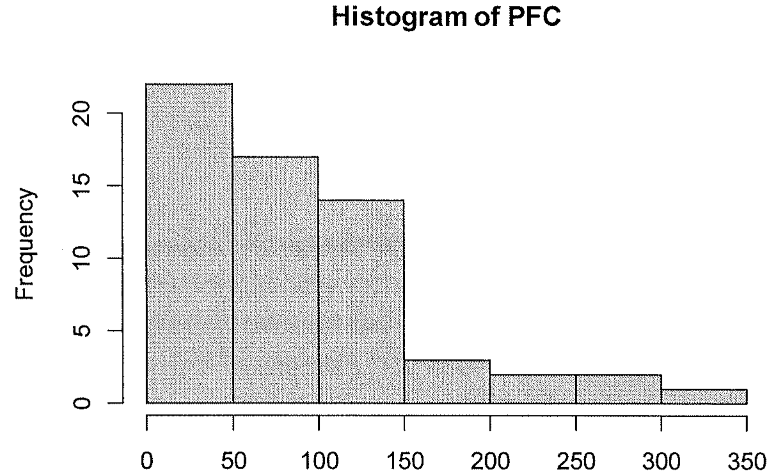
$\boxed{16.37 \text{ oz}}$

-2 for 15.63

12

Questions 4 through 9 (PFC): A recent Denver Post article was titled "State mulls limits for PFCs" (9/18/17). (For the purposes of this analysis, suppose the regulations are already in place.) The maximum allowable level of PFC is 70 ppt (parts per trillion) in groundwater. State agencies will take regulatory action if it can be shown that the mean PFC is greater than 70 ppt. Suppose n = 61 ground water samples were randomly selected from a large aquifer. Select R output is shown below. Use alpha = 0.05.

```
> length(PFC)
[1] 61
> mean(PFC)
[1] 87.46973
> sd(PFC)
[1] 73.72118
> min(PFC)
[1] 2.079265
> median(PFC)
[1] 62.08594
> max(PFC)
[1] 349.2375
```



-2 for Skewed left.

4. Based on the histogram, what is the shape of the distribution? Circle one answer, no need to justify.
   A. Bimodal
   B. Symmetric
   C. Skewed Left
   D. Skewed Right
   E. Other

5. Based on the histogram, if the Shapiro-Wilk test had been run for this data, what would you expect the result to be? Just circle one answer, no need to justify. **(2 pts)**
   A. Shapiro-Wilk p-value < 0.05
   B. Shapiro-Wilk p-value ≥ 0.05.

   Large p-values Support normality.

6. If cumulative exposure to PFC was of interest, would the mean or median be of more interest. Just circle one answer, no need to justify. **(2 pts)**

   Mean   Median

7. Regardless of your answer to the previous question, considering that "state agencies will take regulatory action if it can be shown that the mean PFC is greater than 70 ppt", what are the hypotheses of interest? Be specific!

   H0: $\mu \leq 70$

   HA: $\mu > 70$ Take Action!

12

8. Using the summary statistics in the output, run a hypothesis test corresponding to the previous question. Give the test statistic, df, rejection rule and conclusion. Note: Run this test regardless of your answers to previous questions!

   A. (**4 pts**) Test Statistic = $t = \frac{87.47-70}{(73.72/\sqrt{61})} = +1.85$

   B. (**2 pts**) df = 60

   C. (**2 pts**) Reject H0 if $t > t_\alpha = 1.671$

   D. (**2 pts**) Conclusion: Reject H_0

9. Give the name of a method that could be used to construct a confidence interval for <u>mean</u> PFC <u>without assuming normality</u>. Just state the method – no need to justify. (**2 pts**)

   Bootstrap

<u>Questions 10 through 12 (Aldosterone 1):</u> In the description below, the investigator has made several errors. An investigator is interested in the aldosterone blood levels in healthy dogs. She obtains a random sample of n = 10 healthy dogs for her study. She takes a 60ml blood sample from each dog, then runs lab work separately on three 20ml subsamples for each dog (for a total of 30 aldosterone values). She finds a sample mean of 100.3 and a sample standard deviation of 21.1 based on 30 values. After looking at the lab results, she decides to test H0: $\bar{y}$ =100 versus HA: $\bar{y} \neq 100$. The 95% CI with df = 29 is found to be (93.3, 107.2). Based on the CI she concludes that the true population mean is 100.

<u>Identify and briefly discuss</u> (or correct) <u>3 different errors</u> in the design, analysis, and conclusions above. Note: There may be more than 3 errors, but you only need to discuss 3 of them.

10. Error:

    Discuss/Correct:

11. Error:

    Discuss/Correct:

12. Error:

    Discuss/Correct:

#1 n = 10 dogs vs 30 obs
Not independent obs

#2 Hypotheses determined <u>after</u> looking at data.

#3 Hypotheses should be about pop params not sample values
Correct: $H_0: \mu = 100$ vs $H_A: \mu \neq 100$.

#4 Cannot conclude $\mu = 100$ based on CI

−4 for small sample size
Not ideal, but not wrong!

24

Questions 13 through 15 (Aldosterone 2): Now suppose a new study is being planned to compare mean aldosterone levels in diseased dogs versus healthy controls. The investigator conjectures that the within group standard deviation is 15. They want 80% power to detect a meaningful difference (between means) of 20 with alpha = 0.05.

13. Using the information above, fill in the power.t.test code: **(8 pts)**

```
power.t.test(
power=0.80,
sig.level=0.05,
   (A)   delta=  20

   (B)   sd=     15

   (C)   alternative = (Circle one answer)

         one.sided      two.sided

   (D)   type= (Circle one answer)

         one.sample     two.sample
```

14. If the investigator wanted 90% power (instead of 80%), would the required sample size be higher or lower? **(2 pts)**

Lower   Higher

15. If the standard deviation was 10 (instead of 15), would the required sample size be higher or lower? **(2 pts)**

Lower   Higher

Questions 16 through 23 (Tips): A recent Denver Post article was titled "Why you tip as much, or as little, as you do" (8/30/17). The article describes a study where diners were randomly assigned to either receive a chocolate candy from their waiter or not. The data presented here is loosely based on that article. The **Tip** amount (as a percent of total bill) from each diner was recorded and compared for the two **Treatment** groups (Choc or NoChoc). Randomization was done using a standard deck of cards. We have observations for n_{Choc} = 28 diners and n_{NoChoc} = 31 diners. For convenience, let μ_{Choc} and μ_{NoChoc} represent the population mean tip for the two treatment groups. The R input and output are labeled **Tips**. Use $\alpha = 0.05$.

16. Is this an experiment or an observational study? Circle one answer, no need to justify. **(2 pts)**

Experiment   Observational Study

17. Based on summary statistics provided, would it be reasonable to assume equal variances (for purposes of the t-test)? Discuss.

Since $\frac{5.68}{4.65} = 1.22 < 2$

Reasonable to assume equal variances

18

18. A colleague looks at the summary statistics and suggests that you drop some of the observations for the NoChoc group to make the sample sizes for Choc and NoChoc the same. Do you agree with this suggestion?

Do you agree with this suggestion? Yes No

Discuss: Never want to drop data without some compelling reason!
Equal sample sizes not required. Larger sample sizes better (higher power).

19. Without assuming normality, test whether there is a difference between the distributions of tip amount for Choc vs NoChoc. This should be a two-sided alternative.

p-value: 0.0795

Conclusion: Fail to Reject Ho.

20. True or False: The t.test() output (in **Tips** analysis) requires the assumption of equal variances. Just circle one answer – no need to justify. **(2 pts)**

TRUE FALSE Welch-Satt t-test (default). does NOT require assumption of equal var's.

21. Using R output and assuming normality, test H0: $\mu_{Choc} - \mu_{NoChoc} = 0$.

p-value: 0.1072

Conclusion: Fail to Reject Ho.

22. Considering the t.test() output and your answer to the previous question, what can be said about the 95% CI for $\mu_{Choc} - \mu_{NoChoc}$. Note that this CI has been "hidden" from the R output. Just circle one answer – no need to justify.

A. The 95% CI is completely below 0.
B. The 95% CI is completely above 0.
C. The 95% CI includes 0.
D. Not enough information to say.

Since we Fail to Reject Ho. the CI must include 0.

23. Assuming normality, test HA: $\mu_{Choc} - \mu_{NoChoc} > 0$. Give the p-value corresponding to this one-sided alternative.

DNG = Did Not Grade

$0.1072/2 = 0.0536$

$\boxed{0.0536}$

Questions continue on the next page.....

22.

Questions 24 through 26: An investigator is interested in estimating some true population mean (μ). Based on a random sample of size n = 15 they find a 95% CI of (80, 90).

24. What is the sample mean ($\bar{y}$)?

$$\bar{y} = \frac{(LB+UB)}{2} = \frac{(80+90)}{2}$$

80 = 85-5 85 90 = 85+5

$\boxed{85}$

25. What is the 95% ME?

$$ME = \frac{(UB-LB)}{2} = \frac{(90-80)}{2}$$

$\boxed{5}$

26. What information would you want to look at in order to decide if the confidence interval is valid? Give at least 2 specific diagnostics you would construct (if you had the raw data available).

Since sample size is small, assumption of normality is required.

Diagnostics

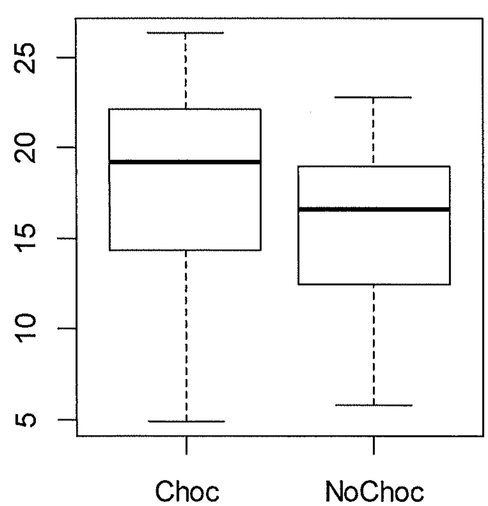
- Histogram
- QQ plot
- Shapiro-Wilk test.

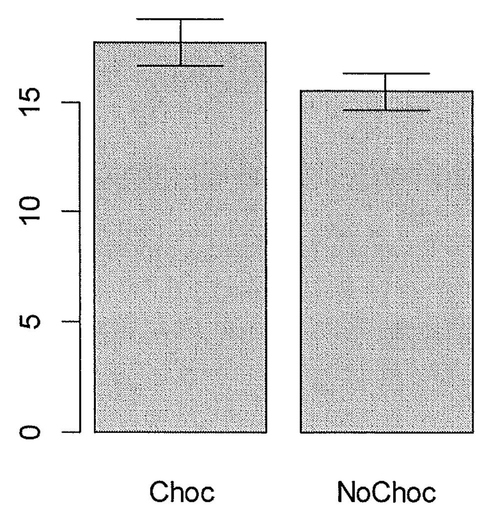
12

Tips (Questions 16 through 23)

```
library(plyr)
library(gplots)
str(TipsData)
SumStats <- ddply(TipsData, c("Trt"), summarize,
                  n = length(Tip),
                  mean = mean(Tip),
                  sd = sd(Tip),
                  SE = sd/sqrt(n))
SumStats
par(mfrow=c(1,2))
boxplot(Tip ~ Trt, data = TipsData)
with(barplot2(mean, plot.ci = TRUE, ci.l = mean-SE, ci.u = mean+SE, names
= Trt), data = SumStats)
t.test(Tip ~ Trt, data = TipsData)
wilcox.test(Tip ~ Trt, data = TipsData)
```

```
> str(TipsData)
'data.frame':	59 obs. of  2 variables:
 $ Trt: Factor w/ 2 levels "Choc","NoChoc": 1 1 1 1 1 1 1 1 1 1 ...
 $ Tip: num  12.3 22.1 24.3 23.6 21.3 ...
> SumStats <- ddply(TipsData, c("Trt"), summarize,
+                   n = length(Tip),
+                   mean = mean(Tip),
+                   sd = sd(Tip),
+                   SE = sd/sqrt(n))
> SumStats
     Trt  n     mean       sd       SE
1   Choc 28 17.70500 5.687775 1.074888
2 NoChoc 31 15.47377 4.651644 0.835460
> par(mfrow=c(1,2))
> boxplot(Tip ~ Trt, data = TipsData)
> with(barplot2(mean, plot.ci = TRUE, ci.l = mean-SE, ci.u = mean+SE, name
s = Trt), data = SumStats)
```





Tips continued (Questions 16 through 23)

```
> t.test(Tip ~ Trt, data = TipsData)

        Welch Two Sample t-test

data:  Tip by Trt
t = 1.6389, df = 52.298, p-value = 0.1072
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 [redacted]
sample estimates:
  mean in group Choc mean in group NoChoc
            17.70500             15.47377

> wilcox.test(Tip ~ Trt, data = TipsData)

        Wilcoxon rank sum test

data:  Tip by Trt
W = 550, p-value = 0.0795
alternative hypothesis: true location shift is not equal to 0
```

Exam 1 Extra Output

Note: This information was NOT provided during the original exam!

#1 - 3 (Fill Weights)

```
> pnorm(16, mean = 16.12, sd = 0.18)

[1] 0.2524925

> qnorm(0.02, mean = 16.12, sd = 0.18)

[1] 15.75033

> pnorm(16, mean = 16.37, sd = 0.18)

[1] 0.01991269
```

#5, #8 (PFC)

```
> shapiro.test(PFC)

      Shapiro-Wilk normality test
data:  PFC
W = 0.8738, p-value = 1.437e-05

> t.test(PFC, mu = 70, alternative = "greater")

      One Sample t-test
data:  PFC
t = 1.8508, df = 60, p-value = 0.03456
alternative hypothesis: true mean is greater than 70
95 percent confidence interval:
 71.70043      Inf
sample estimates:
mean of x
 87.46973
```

#22 (Tips)

```
95 percent confidence interval:
 -0.5002231  4.9626870
```

#23 (Tips)

```
>t.test(Tip ~ Trt, data = TipsData, alternative = "greater")

t = 1.6389, df = 52.298, p-value = 0.05361
```