

STAT511 – Exam 2

Fall 2018

Honor Pledge: I have not given, received, or used any unauthorized assistance on this exam.

Signature: _____

Printed Name: KEY

Instructions:

- Open book, open notes, calculator required. No computers or cell phones.
- Time limit is 1 hour 50 minutes - strictly enforced!
- If an answer is in the computer output, use it; don't calculate it by hand.
- Show your work where appropriate. Put your final answer in the box (if provided).
- Make explanations brief and legible.
- All questions are worth 4 points except where noted. Maximum score is 100.
- Computer input/output is provided at the end of the exam.
- The exam contains a total of 9 pages (including computer input/output).
- If you run out of space, you may use your own paper.

Questions 1 through 3: Consider two analysis scenarios. Both Scenarios have $n = 6$ observations per trt and $s_W^2 = \text{MSResid} = 3$.

Scenario A: $t = 3$ treatments, $\text{dfResid} = 15$

Scenario B: $t = 8$ treatments, $\text{dfResid} = 40$.

1. Complete the table below. (8 pts) Recall that the LSD value is a 95% unadjusted ME for pairwise comparisons of means. The HSD value is a 95% Tukey adjusted ME for pairwise comparisons of means.

	LSD _{0.05}	HSD _{0.05}
Scenario A $t = 3, \text{dfResid} = 15$	2.13	2.60
Scenario B $t = 8, \text{dfResid} = 40$	2.02	3.20



$$\text{HSD} = q_{\alpha} \sqrt{\frac{s_W^2}{n}} = 3.67 \sqrt{\frac{3}{6}} = 2.595$$

$$\text{LSD} = t_{\alpha/2} \sqrt{\frac{2s_W^2}{n}} = 2.021 \sqrt{\frac{2 \cdot 3}{6}} = 2.02$$

2. Based on your completed table above, using the LSD (unadjusted) method which scenario has higher power? Circle one answer, no need to justify. (2 pts)

Scenario A Scenario B

3. Based on your completed table above, using the HSD (Tukey) method which scenario has higher power? Circle one answer, no need to justify. (2 pts)

Scenario A Scenario B

Questions 4 through 7 (Fish): An investigator is planning a study where $n = 6$ fish will be grouped into each tank. Prior to grouping into tanks, a large number of fish are held in a large holding vessel (consider this the population). Assume that in this holding vessel (population), **half of the fish are female**. It is not possible to tell the sex of the fish before grouping them into tanks. Let Y be the random variable representing the number of females in a tank (with a total of $n = 6$ fish). A partial table of probabilities is shown.

k	0	1	2	3	4	5	6
$P(Y = k)$	0.016	0.094	0.234	?	?	0.094	0.016

4. Y has a binomial distribution. Give the values of n (size) and π (prob). (2 pts)

n : 6
 π : 0.5

5. Give the mean of Y . (2 pts)

$$6 * 0.5 = 3$$

3

6. What is the probability that exactly 3 fish in a tank will be female? In other words, find $P(Y = 3)$.

NOTE: $0! = 1$, $1! = 1$, $2! = 2$, $3! = 6$, $4! = 24$, $5! = 120$, $6! = 720$.

$$P(Y=3) = \frac{6!}{3! \cdot 3!} (0.5)^3 (0.5)^3 = 0.3125$$

0.3125

7. What is the probability that at least 2 of the fish in a tank will be female? In other words, find $P(Y \geq 2)$.

$$\begin{aligned} P(Y \geq 2) &= 1 - P(Y \leq 1) \\ &= 1 - 0.094 - 0.016 \\ &= 0.89 \end{aligned}$$

0.89

Questions 8 through 9 (Virus1): A study was done to compare two different preparation methods (Prep1, Prep2) for a certain virus. There are $n = 11$ independent preparations using each method. Let the response (Y) represent the viral abundance.

```
> str(Virus1)
'data.frame':      22 obs. of  2 variables:
 $ Method: Factor w/ 2 levels "Prep1","Prep2": 1 1 1 ..
 $ Y      : int  18 20 9 14 38 26 15 10 25 7 ...
> var.test(Y ~ Method, data = Virus1)

      F test to compare two variances
data:  Y by Method
F = 0.82025, num df = 10, denom df = 10, p-value = 0.7601
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
 0.2206879 3.0487019
sample estimates:
ratio of variances
 0.8202509
```

Virus1 continued.....

8. State the hypotheses for this test. Be specific.

$$\begin{array}{l} H_0: \sigma_1^2 / \sigma_2^2 = 1 \\ H_A: \sigma_1^2 / \sigma_2^2 \neq 1 \end{array} \quad \left\{ \begin{array}{l} \sigma_1^2 = \sigma_2^2 \\ \sigma_1^2 \neq \sigma_2^2 \end{array} \right.$$

9. The test above requires the assumption of normality. (2 pts)

☒ TRUE

☐ FALSE

Questions 10 through 13 (Virus2): Similar to above, suppose a study was done to compare two different preparation methods (Prep1, Prep2) for a certain virus. This time, the methods were compared by applying the virus to leaves of tobacco plants and counting the number of virus spots. A total of $n = 22$ leaves were used. Let the response (Y) represent the number of viral spots. The R input and output are labeled **Virus2**.

10. State the hypotheses for **Test1**. Be specific.

$$H_0: \mu_1 - \mu_2 = 0$$

$$H_A: \mu_1 - \mu_2 \neq 0$$



11. The hypotheses for **Test2** are the same as the previous question. (2 pts)

☒ TRUE

☐ FALSE



12. Suppose that the study had been conducted using $n = 11$ plants. For each plant, one leaf has Prep1 applied and another leaf has Prep 2 applied. Which test is appropriate for this study design? (2 pts)

Test1

☒ Test2

13. Beyond being appropriate for the design, what is the benefit of using the test that you selected in the previous question?

Increased Power using Paired t-test (Test 2).
"Better able to detect true difference!"
"Differences have smaller variability!"
-4 for "does not require equal variances"

Questions 14 through 20 (Wheat): A study was conducted to look at the effect of fertilizer on the yield of wheat. Four different treatments ($t = 4$) were considered: control (C), nitrogen only (N), phosphorous only (P) and both nitrogen and phosphorus (B). Treatments were applied to a total of 16 equally sized plots of wheat (four randomly selected plots per treatment). At the end of the experiment the yield was recorded. The R input and output are labeled **Wheat**. Let μ_i represent the population mean for treatment i . Let σ_i^2 represent the population variance for treatment i .

14. Test $H_0: \mu_B = \mu_C = \mu_N = \mu_P$. Provide the test statistic, p-value and conclusion.

Test Statistic: $F = 10.241$

p-value: $p = 0.001$

Conclusion: $\text{Reject } H_0$.

15. The test above requires the assumption of normality. (2 pts)

☒ TRUE

☐ FALSE

16. Test $H_0: \sigma_B^2 = \sigma_C^2 = \sigma_N^2 = \sigma_P^2$. Provide a p-value and conclusion.

p-value: $p = 0.7649$

Conclusion: $\text{Fail to Reject } H_0$.

17. The test above requires the assumption of normality. (2 pts)

☐ TRUE

☒ FALSE

18. The investigator is interested in estimating and testing $H_0: \mu_C - \mu_N = 0$. 2 pts per question.

A. Give an estimate of the difference.

$$4.35 - 6.3 = -1.95$$

-1.95

B. Without adjusting for multiple testing, give the p-value for the comparison.

0.0539

C. Using Tukey's method, give the p-value for the comparison.

0.1965

D. Now suppose Bonferonni's method had been used. What can be said about the Bonferonni adjusted p-value for this comparison?

$p < 0.05$

☒ $p \geq 0.05$

19. Construct a CLD display based on Tukey's method. Recall that a CLD display uses lines (or numbers) to indicate which pairs are not significantly different from each other. Hint: You do NOT need to calculate an HSD value to answer this question!

4.35 5.62 6.3 9.22
C P N B



20. Before the experiment was started, the investigator was interested in testing whether the combined treatment (B) yielded higher mean yield than the average of the two single treatments means (N and P). Provide appropriate contrast coefficients.

B	C	N	P
1	0	-0.5	-0.5

Questions 21 through 23: Consider an analysis where an investigator is comparing $t = 7$ treatments using one-way ANOVA. There are $n = 7$ observations per treatment, for a total of $7 \times 7 = 49$ observations.

21. Suppose that the investigator is interested in testing all pairwise comparisons. Name two methods that could be used to control the experiment wise error rate (EER). Just name two methods, no need to justify.

Method 1: Tukey

Method 2: Bonferoni

22. Considering your answer to the previous question, which of the two methods has higher power (while controlling EER)? No need to justify. (2 pts)

Tukey

23. Suppose that ANOVA assumptions had NOT been satisfied. Give an alternate approach that could be used for the analysis.

Transformation or Kruskal-Wallis

Questions 24 and 25: An investigator is interested in planning a study to estimate the proportion of CO who support Amendment 74. They will be making inference for a single population proportion using the large sample normal approximation.

24. They would like the 95% ME (margin of error) to be 15% or less ($95\%ME \leq 0.15$). They conjecture that 30% of voters support Amendment 74. What sample size is required? Give the sample size as an integer value.

$$n = \frac{(1.96)^2 (0.3)(0.7)}{(0.15)^2} = 35.86$$

36

25. Suppose they used a conjectured value of 50% (instead of 30%). Would the required sample size be larger or smaller than your answer from the previous question? Just circle one answer, no need to justify. (2 pts).

Larger

Smaller

Questions 26 through 29: An investigator is interested in making inference for a single proportion (π). Specifically, they want to test $H_0: \pi \geq 0.2$ vs $H_A: \pi < 0.2$. They are working from a random sample of size $n = 40$.

```
> prop.test(6, 40, p = 0.2, alternative = "less", correct = FALSE)

1-sample proportions test without continuity correction
data: 6 out of 40, null probability 0.2
X-squared = 0.625, df = 1, p-value = 
alternative hypothesis: true p is less than 0.2
95 percent confidence interval:
 0.0000000 0.2647439
sample estimates:
p
0.15
```

26. Calculate the Z test statistic. Note: This is different from the X-squared statistic in the output. Watch out for the sign! Give your answer to two decimal places.

$$Z = \frac{(0.15 - 0.20)}{\sqrt{\frac{0.2 \times 0.8}{40}}} = -0.79$$

-2 for +0.79
-0.79

27. Give the (one-sided) p-value.

$$P(Z \leq -0.79) = 0.2148$$

-2 for 0.7852
0.2148

28. The sample size is large enough for the test above to be valid. Note: $3 \times SE = 0.17$. (2 pts)

TRUE

FALSE

No, since $\hat{\pi} < 3 \cdot SE$

29. Regardless of your answer to the previous question, if the sample size was NOT large enough, give an alternative method that could be used. You can either give the name of the method or the name of the R function.

Exact Binomial Test
binom.test()
-2 for Continuity Correction

Virus 2 (Questions 10 through 13)

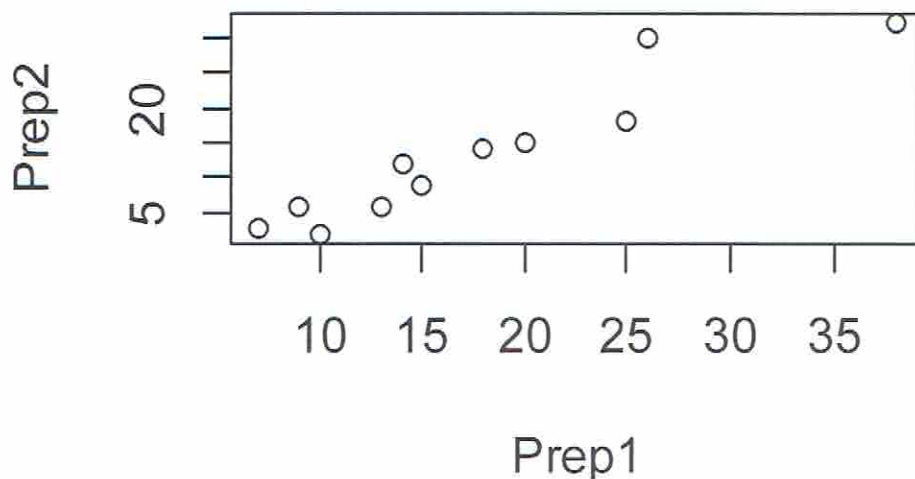
```
> str(Virus2)
'data.frame':      11 obs. of  2 variables:
 $ Prep1: int  18 20 9 14 38 26 15 10 25 7 ...
 $ Prep2: int  14 15 6 12 32 30 9 2 18 3 ...
> #Test1
> t.test(Virus2$Prep1, Virus2$Prep2)

Welch Two Sample t-test
data:  Virus2$Prep1 and Virus2$Prep2
t = 1.065, df = 19.807, p-value = 0.2997
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -4.188671 12.915944
sample estimates:
mean of x mean of y
 17.72727  13.36364

> #Test2
> t.test(Virus2$Prep1, Virus2$Prep2, paired = TRUE)

Paired t-test
data:  Virus2$Prep1 and Virus2$Prep2
t = 4.3529, df = 10, p-value = 0.001437
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
  2.129980 6.597293
sample estimates:
mean of the differences
      4.363636

> plot(Prep2 ~ Prep1, data = Virus2)
```



Wheat (Questions 14 through 20)

```
> library(dplyr)
> library(car)
> library(emmeans)
> str(Wheat)
'data.frame':   16 obs. of  2 variables:
 $ Trt  : Factor w/ 4 levels "B","C","N","P": 2 2 2 2 3 3 3 3 4 4 ...
 $ Yield: num  3.5 4.2 3.4 6.3 6.2 5.3 6.5 7.2 3.4 6.3 ...

> SumStats <- summarise(group_by(Wheat, Trt),
+                         n = n(),
+                         mean = mean(Yield),
+                         sd = sd(Yield),
+                         SE = sd/sqrt(n))
> SumStats
# A tibble: 4 x 5
   Trt      n mean    sd    SE
  <fct> <int> <dbl> <dbl> <dbl>
1 B         4  9.22  1.22  0.609
2 C         4  4.35  1.35  0.674
3 N         4  6.3   0.787 0.394
4 P         4  5.62  1.66  0.828
> Model <- lm(Yield ~ Trt, data = Wheat)
> anova(Model)
```

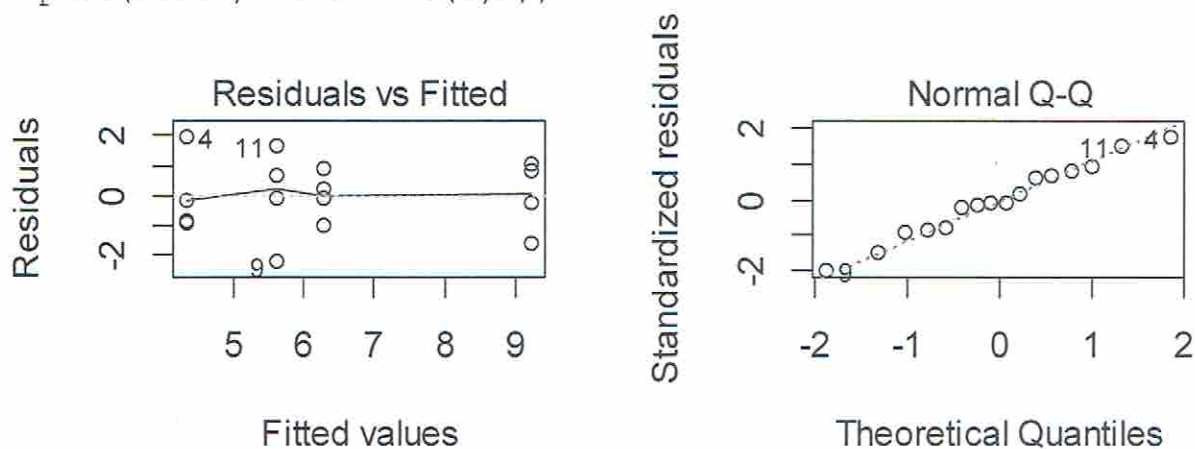
Analysis of Variance Table

Response: Yield

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Trt	3	51.165	17.055	10.241	0.001255 **
Residuals	12	19.985	1.6654		

```
> par(mfrow = c(1,2))
```

```
> plot(Model, which = c(1,2))
```



Wheat continued (Questions 14 through 20)

```
> leveneTest(Yield ~ Trt, data = Wheat)
```

Levene's Test for Homogeneity of Variance (center = median)

	Df	F value	Pr(>F)
group	3	0.3863	0.7649
	12		

```
> shapiro.test(Model$residuals)
```

Shapiro-Wilk normality test

data: Model\$residuals

W = 0.98008, p-value = 0.9642

```
> emout <- emmeans(Model, "Trt")
```

```
> emout
```

Trt	emmean	SE	df	lower.CL	upper.CL
B	9.225	0.6452551	12	7.81911	10.63089
C	4.350	0.6452551	12	2.94411	5.75589
N	6.300	0.6452551	12	4.89411	7.70589
P	5.625	0.6452551	12	4.21911	7.03089

Confidence level used: 0.95

```
> pairs(emout, adjust = "none")
```

contrast	estimate	SE	df	t.ratio	p.value
B - C	4.875	0.9125285	12	5.342	0.0002
B - N	2.925	0.9125285	12	3.205	0.0076
B - P	3.600	0.9125285	12	3.945	0.0019
C - N		0.9125285	12	-2.137	0.0539
C - P	-1.275	0.9125285	12	-1.397	0.1876
N - P	0.675	0.9125285	12	0.740	0.4737

```
> pairs(emout)
```

contrast	estimate	SE	df	t.ratio	p.value
B - C	4.875	0.9125285	12	5.342	0.0009
B - N	2.925	0.9125285	12	3.205	0.0332
B - P	3.600	0.9125285	12	3.945	0.0091
C - N		0.9125285	12	-2.137	0.1965
C - P	-1.275	0.9125285	12	-1.397	0.5242
N - P	0.675	0.9125285	12	0.740	0.8792

P value adjustment: tukey method for comparing a family of 4 estimates

Exam2 Extra Output

Note: This information was not provided in the original exam!

#1

```
> #Scenario A
> qt(0.975, df = 15)*sqrt(3*2/6)
[1] 2.13145
> qtukey(0.95, 3, df = 15)*sqrt(3/6)
[1] 2.59747
> #Scenario B
> qt(0.975, df = 40)*sqrt(3*2/6)
[1] 2.021075
> qtukey(0.95, 8, df = 40)*sqrt(3/6)
[1] 3.196501
```

#6 (Fish)

```
> dbinom(3, size = 6, prob = 0.5)
```

```
[1] 0.3125
```

#7 (Fish)

```
> 1-pbinom(1, size = 6, prob = 0.5)
```

```
[1] 0.890625
```

#19 (Wheat)

```
> CLD(emout)
```

Trt	emmean	SE	df	lower.CL	upper.CL	.group
C	4.350	0.6452551	12	2.94411	5.75589	1
P	5.625	0.6452551	12	4.21911	7.03089	1
N	6.300	0.6452551	12	4.89411	7.70589	1
B	9.225	0.6452551	12	7.81911	10.63089	2

Confidence level used: 0.95

P value adjustment: tukey method for comparing a family of 4 estimates

#27

```
> prop.test(6, 40, p = 0.2, alternative = "less", correct = FALSE)
```

1-sample proportions test without continuity correction

data: 6 out of 40, null probability 0.2

X-squared = 0.625, df = 1, p-value = 0.2146

alternative hypothesis: true p is less than 0.2

95 percent confidence interval:

0.0000000 0.2647439

sample estimates:

p

0.15