ST511 HW #3

Reading: Review Chapter 5 of Ott & Longnecker.
See Canvas Calendar for due date.

34 points total, 2 points per problem part unless otherwise noted.

1. Suppose the mean oxygen level of a certain lake was of interest. A total of **n=10** samples were taken (from randomly selected locations) and oxygen level was measured in ppm. The sample mean oxygen level was found to be 5.3 and the sample standard deviation was found to be 0.5. Use $\alpha = 0.05$.
   A. Calculate the SE (standard error) and the 95% ME (margin of error).
   B. Use your ME from above to construct a 95% confidence interval for μ (population mean).
   C. Use your CI from above to test H0: $\mu=5$ vs HA: $\mu\neq5$. Make a conclusion about the test. Justify your conclusion based on the CI.
   D. Now test H0: $\mu=5$ vs HA: $\mu\neq5$ using a formal hypothesis test. Be sure to define the rejection region, calculate the test statistic and state your conclusion. (**4 pts**)
   E. Now test H0: $\mu\leq5$ vs HA: $\mu>5$ using a formal hypothesis test. Be sure to define the rejection region, calculate the test statistic and state your conclusion. (**4 pts**)
   F. Now suppose that the summary statistics were based on a sample of size **n=51**. Rerun the hypothesis test from part D (H0: $\mu=5$ vs HA: $\mu\neq5$) based on this larger sample size. Be sure to define the rejection region, calculate the test statistic, and state your conclusion. (**4 pts**)
   G. Comparing the tests in part D (n = 10) vs part F (n = 51), which has higher power? No need to justify.

2. Manufacturers must test the amount of the active ingredient in medications before releasing the batch of pills. The data Pills.csv (available from Canvas) represents the amount (in mg) of the active ingredient in n = 24 pills (from a random sample of the same large batch). Use $\alpha = 0.05$.
   A. Create a histogram and qqplot and test for normality of the data (using Shapiro-Wilk test). Based on this evidence, does the data appear to be normally distributed? Justify your response. Be sure to include the plots in your assignment. (**4 pts**)
   B. Give an estimate of the mean and corresponding 95% confidence interval.
   C. For this question, suppose that if there is evidence that the mean is <u>different from</u> 20mg, the batch of pills will be destroyed. Is there evidence that the batch of pills has a mean amount <u>different from</u> 20mg? State your hypotheses, test statistic, p-value and make a conclusion. (**4 pts**)
   D. For this question, suppose that if there is evidence that the mean is <u>less than</u> 20mg, the batch of pills will be destroyed. Is there evidence that the batch of pills has a mean amount <u>less than</u> 20mg? State your hypotheses, test statistic, p-value and make a conclusion. (**4 pts**)

Important note: In both problem groups above we "recycle" the same data to test both two-sided and one-sided alternatives. In practice, the hypotheses should be decided in advance. A two-sided test should be used by default (unless there is some compelling reason to do otherwise).