Biomass Example: Three-way ANOVA with Unequal Variance

In this example, there are 2 Types (grass or forb), 3 Herbicide treatments (A, B or C) and 2 Water regimes (Low or High) for a total of 12 treatment combinations. There are three reps per treatment combination.

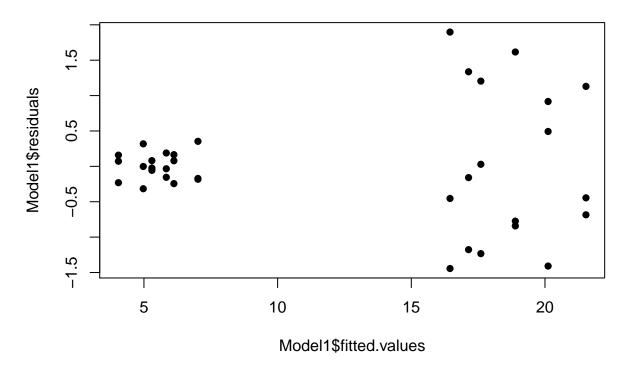
Based on the summary statistics and residual diagnostic plot, it appears that the variance is different for grass versus forb. We can deal with this two ways: (1) running a 2way analysis separately for grass and forb (2) using gls() from nlme to allow different variances by Type.

```
library(XLConnect)
## Warning: package 'XLConnect' was built under R version 3.4.4
library(dplyr)
library(emmeans)
excel.file <- file.path("C:/hess/STAT512/RNotes/ExtraTopics/Biomass.xlsx")</pre>
InData <- readWorksheetFromFile(excel.file, sheet = "Sheet1")</pre>
str(InData)
  'data.frame':
                   36 obs. of 4 variables:
                   "Grass" "Grass" "Grass" ...
   $ Type
           : chr
                   "A" "A" "A" "A" ...
   $ Herb
             : chr
                   "Low" "Low" "High" ...
   $ Water : chr
## $ Biomass: num 18.8 16.4 17.6 20.5 18.1 ...
SumStats <- summarize(group_by(InData, Type, Herb, Water),</pre>
                 n = n(),
                 mean = mean(Biomass),
                 sd = sd(Biomass))
SumStats
## # A tibble: 12 x 6
## # Groups: Type, Herb [?]
##
      Type Herb Water
                            n mean
##
      <chr> <chr> <chr> <int> <dbl> <dbl>
##
   1 Forb A
                 High
                            3 6.12 0.216
##
   2 Forb A
                 Low
                            3 4.97 0.317
##
   3 Forb B
                 High
                            3 5.30 0.0718
                            3 4.05 0.205
##
   4 Forb B
                 Low
##
   5 Forb
           С
                 High
                            3 7.02 0.306
##
   6 Forb C
                 Low
                            3 5.84 0.174
  7 Grass A
                 High
                            3 18.9 1.40
##
  8 Grass A
                            3 17.6 1.22
                 Low
## 9 Grass B
                            3 20.1 1.24
                 High
## 10 Grass B
                 Low
                            3 17.1 1.26
## 11 Grass C
                 High
                            3 16.4 1.72
## 12 Grass C
                 Low
                            3 21.5 0.986
```

3Way Model

```
options(contrasts =c("contr.sum", "contr.poly"))
Model1 <- lm(Biomass ~ Type*Herb*Water, data = InData)
emmeans(Model1, ~ Herb*Water*Type)
##
    Herb Water Type
                        emmean
                                       SE df
                                              lower.CL
                                                        upper.CL
##
   Α
         High Forb
                      6.120484 0.5480887 24
                                              4.989285
                                                        7.251684
##
   В
         High
                      5.296773 0.5480887 24
                                                        6.427973
               Forb
                                             4.165574
##
   C
         High
               Forb
                      7.019339 0.5480887 24
                                             5.888139
                                                        8.150538
##
   Α
         Low
               Forb
                      4.974372 0.5480887 24
                                              3.843172
                                                        6.105571
##
   В
         Low
               Forb
                      4.045132 0.5480887 24
                                             2.913933
                                                        5.176332
##
   C
         Low
               Forb
                      5.835487 0.5480887 24
                                             4.704287
         High Grass 18.891215 0.5480887 24 17.760016 20.022415
##
   Α
##
         High
               Grass 20.117282 0.5480887 24 18.986083 21.248482
##
   C
         High Grass 16.448119 0.5480887 24 15.316919 17.579318
##
               Grass 17.601195 0.5480887 24 16.469996 18.732395
   В
         Low
               Grass 17.142260 0.5480887 24 16.011060 18.273459
##
##
               Grass 21.533570 0.5480887 24 20.402371 22.664770
##
## Confidence level used: 0.95
plot(Model1$residuals ~ Model1$fitted.values, main = "3way Model: Resids vs Fitted Values", pch = 16)
```

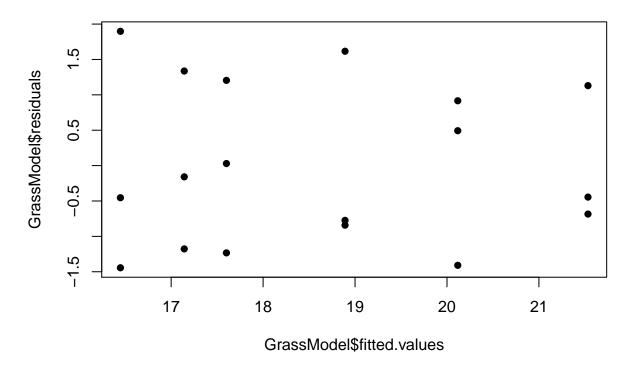
3way Model: Resids vs Fitted Values



Grass: 2way Model

```
GrassModel <- lm(Biomass ~ Herb*Water, data = InData[InData$Type == "Grass",])</pre>
emmeans(GrassModel, ~ Herb*Water)
   Herb Water
                 emmean
                               SE df lower.CL upper.CL
##
         High 18.89122 0.7636266 12 17.22742 20.55501
##
         High 20.11728 0.7636266 12 18.45348 21.78108
   С
         High 16.44812 0.7636266 12 14.78432 18.11192
##
               17.60120 0.7636266 12 15.93740 19.26499
               17.14226 0.7636266 12 15.47846 18.80606
   В
         Low
##
##
               21.53357 0.7636266 12 19.86977 23.19737
##
## Confidence level used: 0.95
plot(GrassModel$residuals ~ GrassModel$fitted.values,
     main = "Grass 2way Model: Resids vs Fitted Values", pch = 16)
```

Grass 2way Model: Resids vs Fitted Values

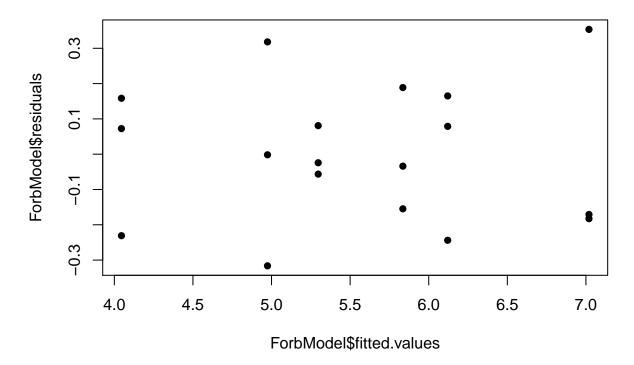


Forb: 2way Model

```
ForbModel <- lm(Biomass ~ Herb*Water, data = InData[InData$Type == "Forb",])
emmeans(ForbModel, ~ Herb*Water)
## Herb Water emmean SE df lower.CL upper.CL</pre>
```

```
##
   Α
         High 6.120484 0.1329548 12 5.830801 6.410168
##
   В
         High 5.296773 0.1329548 12 5.007090 5.586457
               7.019339 0.1329548 12 6.729655 7.309022
##
   C
##
         Low
               4.974372 0.1329548 12 4.684688 5.264055
   Α
##
         Low
               4.045132 0.1329548 12 3.755448 4.334816
               5.835487 0.1329548 12 5.545803 6.125171
##
         Low
## Confidence level used: 0.95
plot(ForbModel$residuals ~ ForbModel$fitted.values,
     main = "Forb 2way Model: Resids vs Fitted Values", pch = 16)
```

Forb 2way Model: Resids vs Fitted Values



3way Analysis allowing Unequal variance

Important Note: Most people recommend using Satterthwaite (or Kenward-Roger) df for this scenario. This option is available in SAS and other programs, but I do not know of a way to do this in R. The lmerTest package uses Satterthwaite df but does not work with nlme.

```
library(nlme)

##

## Attaching package: 'nlme'

## The following object is masked from 'package:dplyr':

##

## collapse
```

Model4 <- gls(Biomass ~ Type*Herb*Water, weights=varIdent(form = ~1 | Type), data = InData) emmeans(Model4, ~ Herb*Water*Type)

```
Herb Water Type
                                              lower.CL
##
                                       SE df
                                                        upper.CL
                        emmean
                      6.120484 0.1329548 24
                                                        6.394890
##
   Α
         High Forb
                                              5.846079
##
   В
         High
               Forb
                      5.296773 0.1329548 24
                                              5.022368
                                                        5.571179
##
   С
         High
               Forb
                      7.019339 0.1329548 24
                                              6.744933
                                                        7.293744
##
         Low
               Forb
                      4.974372 0.1329548 24
                                              4.699966
                                                        5.248777
   Α
##
   В
         Low
               Forb
                      4.045132 0.1329548 24
                                              3.770727
                                                        4.319537
                      5.835487 0.1329548 24
                                             5.561082
##
   C
         Low
               Forb
                                                       6.109892
##
   Α
         High Grass 18.891215 0.7636265 24 17.315168 20.467263
##
   В
               Grass 20.117282 0.7636265 24 18.541234 21.693330
   С
               Grass 16.448119 0.7636265 24 14.872071 18.024166
##
         High
               Grass 17.601195 0.7636265 24 16.025148 19.177243
##
   Α
         Low
##
   В
         Low
               Grass 17.142260 0.7636265 24 15.566212 18.718308
               Grass 21.533570 0.7636265 24 19.957523 23.109618
##
   С
         Low
##
## Confidence level used: 0.95
```

plot(Model4)

