Grass Example: Random Effects One-Way

Six grass samples were randomly randomly selected from a large area. (We have the idea that each sample represents a genotype that exists in the area.) We then select 5 tillers from each grass sample and measure the chlorophyll content on each tiller.

For illustration, will first consider the fixed effects models. Note that lm() is used to fit the fixed effects (one-way ANOVA) model.

But since grasses were randomly selected, the random effects model is more appropriate. Note that lmer() from the lme4 package is used to fit the random effects model.

Also for illustration, REML and ML estimation are considered. In practice, default REML estimation is preferred.

FIXED Effects (one-way ANOVA) Model (for Comparison)

```
Model1 <- lm(y ~ grass, data = Grasses)
anova(Model1)
## Analysis of Variance Table
##
## Response: y
            Df
                Sum Sq Mean Sq F value Pr(>F)
             5 4.9137 0.98275 1.3184 0.2898
## Residuals 24 17.8896 0.74540
summary(Model1)
##
## lm(formula = y ~ grass, data = Grasses)
##
## Residuals:
##
      Min
                1Q Median
                                3Q
                                       Max
## -1.4860 -0.6195 -0.0390 0.5120 1.3620
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                4.1540
                            0.3861 10.759 1.16e-10 ***
## grass2
                 0.9540
                            0.5460
                                    1.747
                                             0.0934 .
## grass3
                0.3040
                            0.5460
                                     0.557
                                             0.5829
## grass4
                 1.1620
                            0.5460
                                     2.128
                                             0.0438 *
```

```
## grass5
                 0.4420
                            0.5460
                                     0.809
                                             0.4262
                 0.8680
                            0.5460
                                     1.590
                                             0.1250
## grass6
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.8634 on 24 degrees of freedom
## Multiple R-squared: 0.2155, Adjusted R-squared: 0.05204
## F-statistic: 1.318 on 5 and 24 DF, p-value: 0.2898
emout <- emmeans(Model1, ~grass)</pre>
emout
                    SE df lower.CL upper.CL
   grass emmean
##
##
   1
           4.15 0.386 24
                              3.36
                                       4.95
##
           5.11 0.386 24
                              4.31
                                       5.90
## 3
            4.46 0.386 24
                              3.66
                                       5.25
                              4.52
##
  4
            5.32 0.386 24
                                       6.11
  5
            4.60 0.386 24
                              3.80
                                       5.39
##
##
            5.02 0.386 24
                              4.23
                                       5.82
   6
##
## Confidence level used: 0.95
emmeans <- summary(emout)$emmean
```

RANDOM Effects Model (using lmer)

The notation (1|grass) specifies that grass be treated as a random effect. REML estimation is used by default. Because we have a random effects model (with no fixed effects), no ANOVA table is returned. The rand() function gives tests of variance components. The ranef() function can be used to calculate "blups".

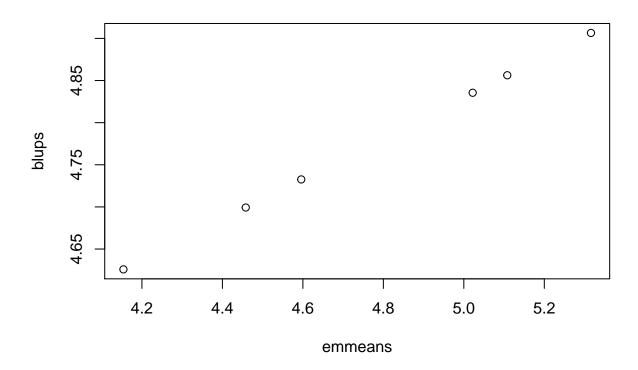
```
library(lme4)
library(pbkrtest)
library(lmerTest)
Model2 <- lmer(y ~ (1|grass), data = Grasses)
summary(Model2)
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: y ~ (1 | grass)
##
     Data: Grasses
##
## REML criterion at convergence: 78.6
##
## Scaled residuals:
##
       Min
                 1Q
                       Median
                                    3Q
                                            Max
## -1.87901 -0.75344 0.05554 0.49831 1.86951
##
## Random effects:
## Groups
                         Variance Std.Dev.
             (Intercept) 0.04747 0.2179
##
   grass
                         0.74540 0.8634
## Number of obs: 30, groups: grass, 6
##
## Fixed effects:
```

```
Estimate Std. Error
                                   df t value Pr(>|t|)
## (Intercept)
                  4.776
                             0.181 5.000
                                           26.39 1.46e-06 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
anova(Model2)
## Type III Analysis of Variance Table with Satterthwaite's method
        Sum Sq Mean Sq NumDF DenDF F value Pr(>F)
rand(Model2)
## ANOVA-like table for random-effects: Single term deletions
## Model:
## y ~ (1 | grass)
##
                                       LRT Df Pr(>Chisq)
              npar logLik
                               AIC
## <none>
                  3 -39.280 84.561
## (1 | grass)
                  2 -39.364 82.728 0.16776 1
                                                  0.6821
ranef(Model2)
## $grass
##
     (Intercept)
## 1 -0.15014125
## 2 0.08026318
## 3 -0.07672097
## 4 0.13049811
## 5 -0.04339203
## 6 0.05949297
##
## with conditional variances for "grass"
blups <- ranef(Model2)$grass + 4.776</pre>
```

Compare emmeans and blups

```
Temp <- data.frame(emmeans, blups)
colnames(Temp) <- c("emmeans","blups")
Temp

## emmeans blups
## 1  4.154  4.625859
## 2  5.108  4.856263
## 3  4.458  4.699279
## 4  5.316  4.906498
## 5  4.596  4.732608
## 6  5.022  4.835493
```



RANDOM Effects Model (using lmer with ML estimation)

Primarily for Illustration. Default REML estimation is standard. Note that the estimated variance component associated with grass is different detpending on whether REML or ML estimation is used.

```
Model3 <- lmer(y ~ (1|grass), data = Grasses, REML = FALSE)
summary(Model3)</pre>
```

```
## Linear mixed model fit by maximum likelihood . t-tests use
##
     Satterthwaite's method [lmerModLmerTest]
  Formula: y ~ (1 | grass)
##
      Data: Grasses
##
##
        AIC
                 BIC
                       logLik deviance df.resid
##
       82.9
                87.1
                        -38.4
                                   76.9
                                              27
##
## Scaled residuals:
##
        Min
                  1Q
                       Median
                                     3Q
                                             Max
## -1.91058 -0.76284 0.07823 0.46725
##
## Random effects:
   Groups
             Name
                         Variance Std.Dev.
    grass
             (Intercept) 0.01472 0.1213
```