

# STAT 512 Homework 2

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A study investigated body fat of  $n = 20$  (female) subjects. The amount of body fat was measured by a cumbersome and expensive procedure requiring immersion of the person in water. For each subject, the following information was recorded:

- BodyFat (Y)
- Triceps (X1) = triceps skinfold thickness
- Thigh (X2) = thigh circumference
- Midarm (X3) = midarm circumference

The data is available from Canvas as “BodyFat.csv”. This data is taken from “Applied Linear Statistical Models” by Neter, Kutner, Nachtsheim and Wasserman.

## Question 1: Correlations and scatterplots

Calculate pairwise (Pearson) correlations between the 4 variables (BodyFat and each of the predictors). You should also briefly examine the pairwise scatterplots, but you do NOT need to include them in your assignment.

rowname	body_fat	triceps	thigh	midarm
body_fat	NA	0.8432654	0.8780896	0.1424440
triceps	0.8432654	NA	0.9238425	0.4577772
thigh	0.8780896	0.9238425	NA	0.0846675
midarm	0.1424440	0.4577772	0.0846675	NA

## Question 2: Multiple regression model

Fit the “full” model using BodyFat as the response and including all 3 predictors. Include the parameter estimate information (“Coefficients” table) and R2 value for the full model in your assignment. Questions 3 through 5 are based on the “full” model from question 2.

```
##
## Call:
## lm(formula = body_fat ~ triceps + thigh + midarm, data = fat_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.7263 -1.6111  0.3923  1.4656  4.1277
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept) 117.085      99.782    1.173    0.258
## triceps      4.334       3.016    1.437    0.170
## thigh       -2.857       2.582   -1.106    0.285
## midarm       -2.186       1.595   -1.370    0.190
##
## Residual standard error: 2.48 on 16 degrees of freedom
## Multiple R-squared:  0.8014, Adjusted R-squared:  0.7641
## F-statistic: 21.52 on 3 and 16 DF,  p-value: 7.343e-06
```

The  $R^2$  value for the multiple linear regression of triceps, thigh, and midarm measurements (X) on body fat percentage (Y) is 0.8014. See below for table of coefficient estimates and corresponding p-values.

term	estimate	std.error	statistic	p.value
(Intercept)	117.084695	99.782403	1.173400	0.2578078
triceps	4.334092	3.015511	1.437266	0.1699111
thigh	-2.856848	2.582015	-1.106441	0.2848944
midarm	-2.186060	1.595499	-1.370142	0.1895628

### Question 3: Confidence intervals (betas)

Based on the “full” model, give 95% confidence intervals for each of the four betas (intercept and three partial regression coefficients).

.rownames	X2.5..	X97.5..
(Intercept)	-94.444550	328.613940
triceps	-2.058507	10.726691
thigh	-8.330476	2.616780
midarm	-5.568367	1.196246

**Intercept**

**Triceps**

**Thigh**

**Midarm**

### Question 4: Hypothesis tests (betas)

Based on the “full” model, test the null hypothesis that all three of the partial regression coefficients are simultaneously zero. In other words, test  $H_0: \beta_1 = \beta_2 = \beta_3 = 0$ . Give the F-statistic and p-value and make a conclusion about the test. (4 pts)

### Question 5: Hypothesis tests (betas)

Based on the “full” model, test the null hypothesis that the partial regression coefficients for Thigh and Midarm are simultaneously zero. In other words, test  $H_0: \beta_2 = 0$  AND  $\beta_3 = 0$ . Give a test statistic, p-value and conclusion. (4 pts)

## Question 6: Parsimony

Now we will sequentially eliminate any terms from the model that are not significant at the 0.05 level. Starting from the “full” model, eliminate the least significant predictor variable (highest p-value) and rerun the regression. Continue that process until all predictor variables are significant at the 0.05 level. Include the parameter estimate information (“Coefficients” table) and R<sup>2</sup> value for the final model in your assignment. (4 pts) We will use this “final” model for the remaining questions.

## Question 7:

In the initial inspection of the pairwise correlations and plots (question 1) it appeared that there was a relationship between BodyFat and Thigh; however, Thigh was dropped from the multiple regression because it was not significant. Speculate about why this is the case.

## Question 8:

Working from the “final” model, look at the residual plots, paying particular attention to the (A) plot of residuals versus fitted values and (B) qqplot of residuals. Discuss each of these plots and whether the regression assumptions appear to be satisfied. You do not need to include the graphs in your assignment, just discuss your findings and conclusions. (4 pts)

Assumptions:

- *Independence*: Unknown. Study design and experimentation are not described.
- *Linearity*: Assumed. The plot of residuals vs. fitted values does not show a trend (e.g., “megaphone” pattern) in residuals.
- *Equal variance*: Assumed. The plot of residuals vs. fitted values shows equal scatter among residuals.
- *Normality*: Assumed/acceptable. The Q-Q plot indicates an acceptable pattern of standardized residuals, considering the small sample size.

## Question 9:

Consider a subject with Triceps = 20 and Midarm = 25. Working from the “final” model, give (A) predicted body fat for this subject, (B) 95% confidence interval for the mean BodyFat of subjects with the same values and (C) 95% prediction interval for the predicted BodyFat for a new subject with these values. (4 pts)

## Question 10:

Working from the “final” model, identify the largest RStudent residual and do an outlier test for that value. Give the test statistic, unadjusted p-value and Bonferonni adjusted p-value. Based on the Bonferonni adjusted p-value, can we conclude this observation is an outlier? Note: The outlierTest() function from the car package can be used for this question, but may return an NA for the Bonferonni p-value. I still want the Bonferonni adjusted p-value! (4 pts)

## Appendix

```
# load packages
library(tidyverse)
library(janitor)
library(GGally)
library(corr)
library(kableExtra)
library(broom)
library(car)
# set global options
knitr::opts_chunk$set(fig.width = 6,
                       fig.height = 4,
                       fig.path = "figs/",
                       echo = FALSE,
                       warning = FALSE,
                       message = FALSE)

# read fat data
fat_data <- readr::read_csv("data/BodyFat.csv") %>% janitor::clean_names()
# 1. pearson correlations between variables of interest
fat_data %>%
  corrr::correlate() %>%
  kableExtra::kable()
# 1. pairwise plot (base) for fat data
plot(fat_data)
# 1. pairwise plot (gg) for fat data
GGally::ggpairs(fat_data, columns = c("body_fat", "triceps", "thigh", "midarm"))
# 2. multiple regression with fat data
fat_multreg <- lm(body_fat ~ triceps + thigh + midarm, data = fat_data)
summary(fat_multreg)
# 2. create tidy lm df and table
fat_multreg_tidy <- broom::tidy(fat_multreg)
kableExtra::kable(fat_multreg_tidy)
fat_multreg %>%
  confint(level = 0.95) %>%
  broom::tidy() %>%
  kableExtra::kable()
```