Polynomial Regression: Linear and Quadratic Terms

Why include the linear term?

It is illuminating to notice that a quadratic relationship can be written in two ways:

𝑦=𝑎0+𝑎1𝑥+𝑎2𝑥2=𝑎2(𝑥−𝑏)2+𝑐y=a0+a1x+a2x2=a2(x−b)2+c

(where, equating coefficients, we find −2𝑎2𝑏=𝑎1−2a2b=a1 and 𝑎2𝑏2+𝑐=𝑎0a2b2+c=a0 The value 𝑥=𝑏𝑥=𝑏x=b corresponds to a global extremum of the relationship (geometrically, it locates the vertex of a parabola).

If you do not include the linear term 𝑎1𝑥𝑎1𝑥, the possibilities are reduced to

𝑦=𝑎0+𝑎2𝑥2=𝑎2(𝑥−0)2+𝑐y=a0+a2x2=a2(x−0)2+c

(where now, obviously, 𝑐=𝑎0𝑐=𝑎0 and it is assumed the model contains a constant term 𝑎0a0). That is, you force 𝑏=0

In light of this, question #1 comes down to whether you are *certain* that the global extremum must occur at 𝑥=0. If you are, then you may safely omit the linear term 𝑎1𝑥𝑎1𝑥. Otherwise, you *must* include it.