# Two-Step Optimal Advertisement Strategy on Social Network System

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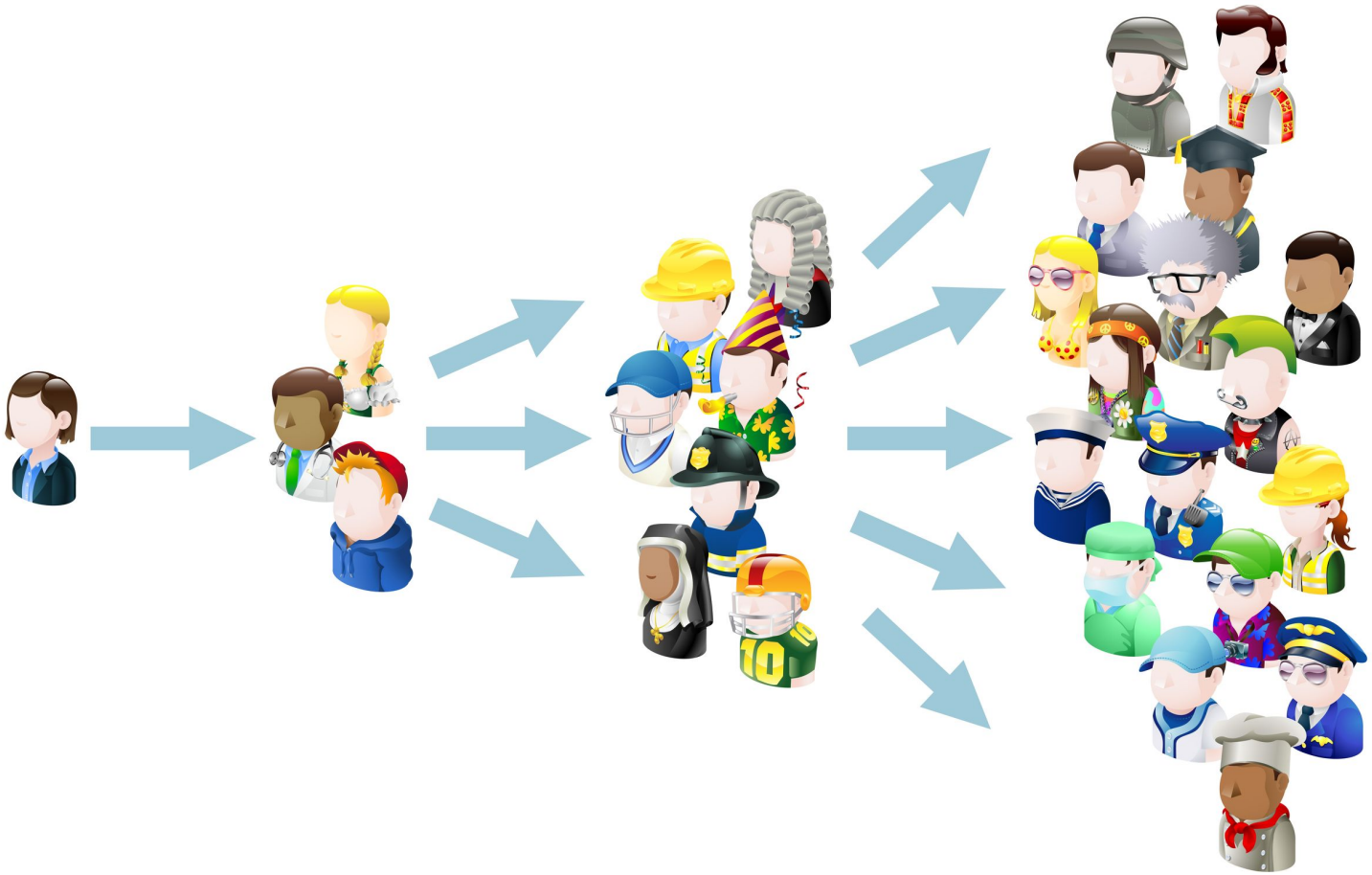
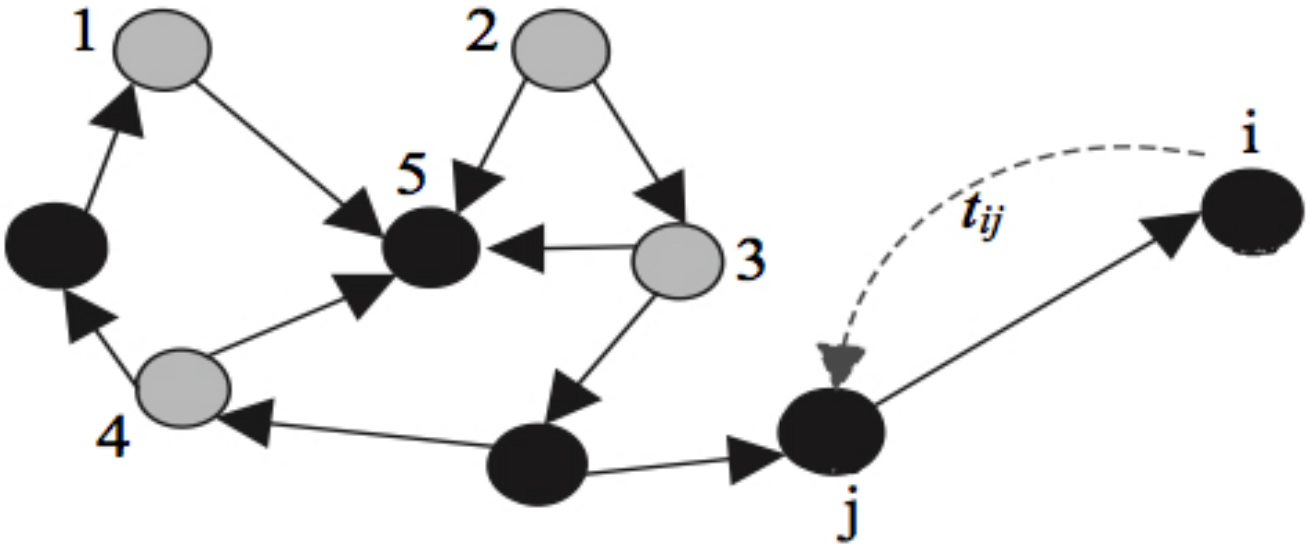
# Advertisement Network



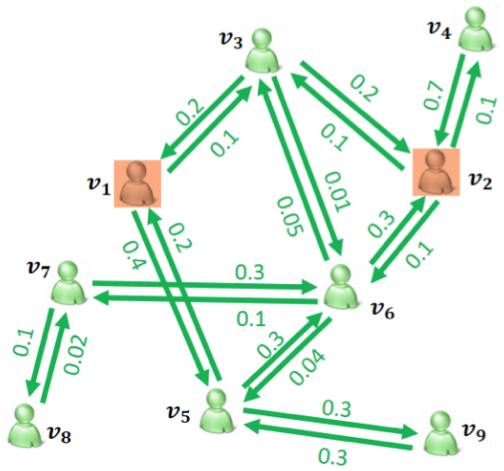
# Problem Formulation

- $G = (V, \mathbf{E}, \mathbf{T})$
- $\mathbf{T} = [t_{ij}]_{n \times n}$ transmission matrix for influence propagation

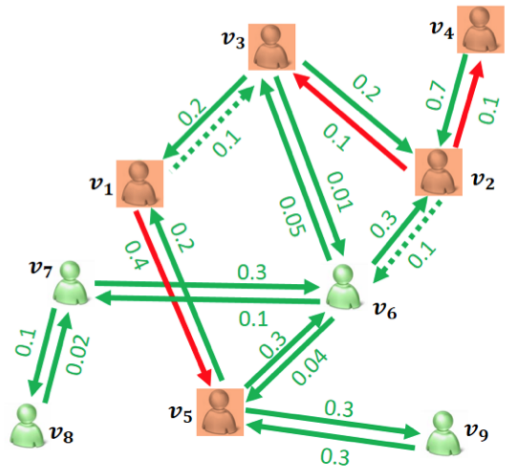


# Literature Review

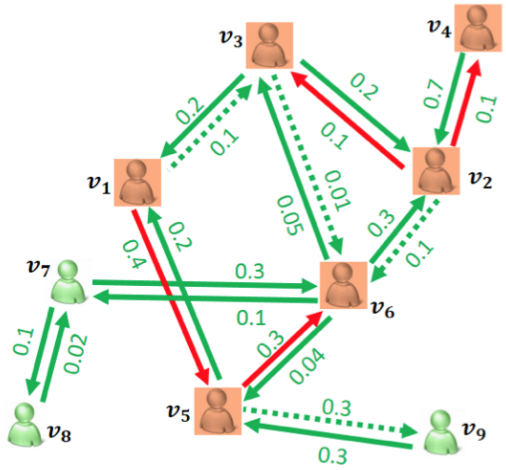
- Independent Cascade Model (ICM) (Goldenberg, 2001)



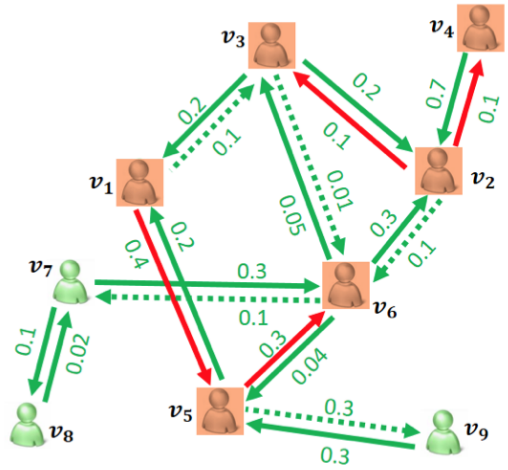

(a) $t = 0$




(b) $t = 1$




(c) $t = 2$




(d) $t = 3$

# Literature Review

- Influence Maximization (IM)

$$S = \underset{S \subset V, |S| \leq K}{\arg\max} \ \mathbb{E}\big[\sigma(S)\big]$$

# My Model

- Denote the influence from $i$ to $j$ by $f_{i\to j}$.

$$f_{i\to i} = \alpha_i,\ \alpha_i > 0$$

$$f_{i\to j} = \frac{1}{1+\lambda_j}\sum_{k\in N_j} t_{kj} f_{i\to k},\ for j \neq i$$

- Constrainted optimization problem

$$S = \underset{S\subset V, |S|\leq K}{\arg\max} \sum_{i\in S} f_{i\to V}$$

# My Work — Two-step algorithm

- First step — Screening
  1. Objective Function

$$g(U) = \lambda \sum_{u \in U} r(u) + (1 - \lambda) H_0(U)$$

  2. Relevance—$r(u)$: Jaccard coefficient

$$r(u) = \frac{|I_u \cap I_a|}{|I_u \cup I_a|}$$

  3. Diversity—$H_0(U)$: normalized entropy of the $U$
  4. Greedy Algorithm: Submodularity of $g(U)$

- Second Step — $L_1$ relaxation

1. Spread coverage matrix $\mathbf{R}$
2. Constrainted Optimization Problem

$$\vec{p} = \underset{\vec{p} \in \{0,1\}^{\tilde{n}}}{\arg\max} \|\mathbf{R}\vec{p}\|_0$$
$$\text{s.t. } \mathbf{1}^T \vec{p} = K \leq \tilde{n}$$

3. $L_1$ relaxation
4. Discrete constraints relaxation
5. Linear programming

# Major Reference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# Thank you