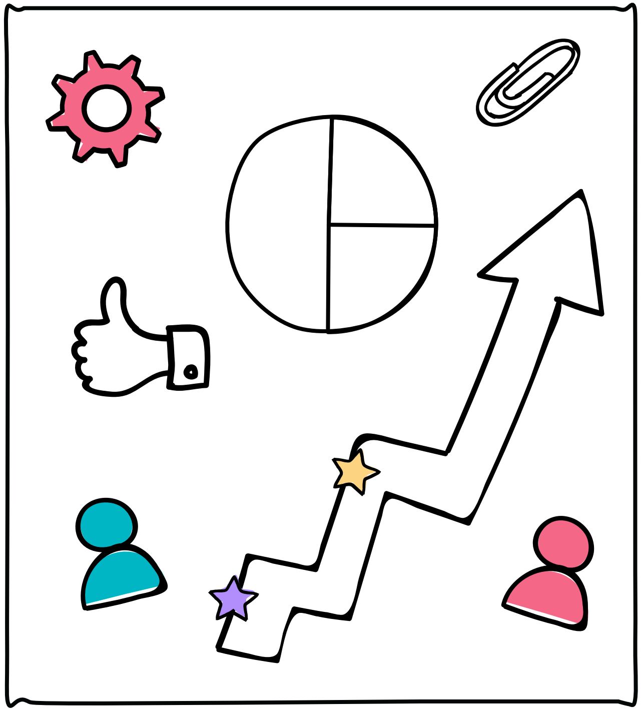
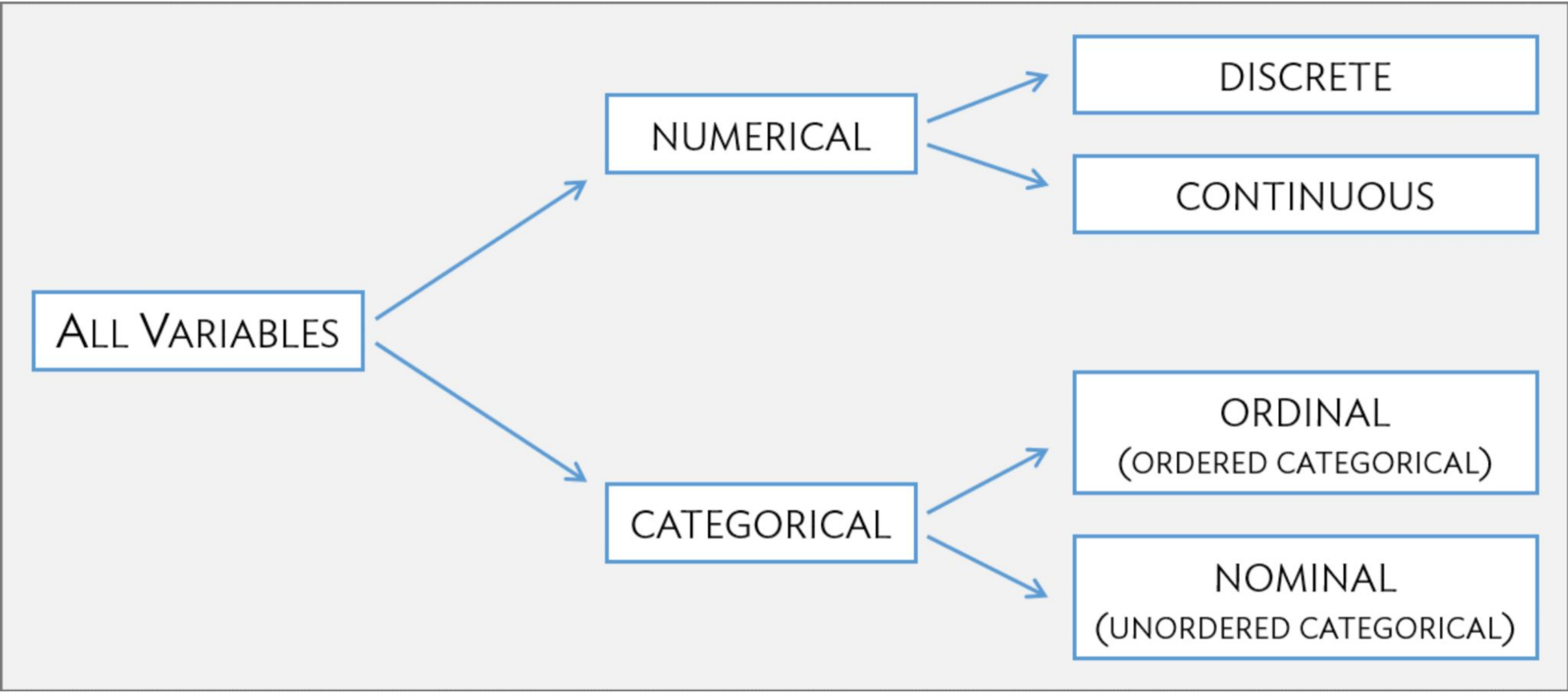


STAT 1110 Recitation 1

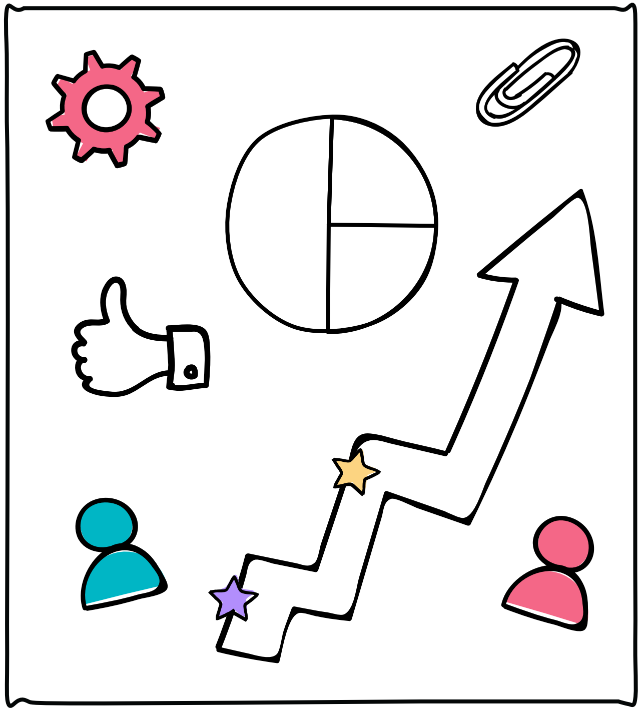
September 6, 2024

Population Parameters	
μ	Population mean
p, θ	Population proportion
σ^2	Population variance
σ	Population standard deviation
ρ	Population correlation

Sample Statistics (point estimates)	
$\bar{x}$	Sample mean
$\hat{p}$	Sample proportion
s^2	Sample variance
s	Sample standard deviation
r	Sample correlation


ALL VARIABLES
NUMERICAL
CATEGORICAL
DISCRETE
CONTINUOUS
ORDINAL
(ORDERED CATEGORICAL)
NOMINAL
(UNORDERED CATEGORICAL)

	POPULATION	SAMPLE
Size	N	n
Mean	$\mu = \frac{\sum_{i=1}^{N} x_i}{N} = \frac{x_1 + x_2 + \cdots + x_N}{N}$	$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{x_1 + x_2 + \cdots + x_n}{n}$
Variance	$\sigma^2 = \frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N} = \frac{\sum_{i=1}^{N} x_i^2 - \frac{\left(\sum_{i=1}^{N} x_i\right)^2}{N}}{N}$	$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1} = \frac{\sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n}}{n-1}$
Standard deviation	$\sigma = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}} = \sqrt{\frac{\sum_{i=1}^{N} x_i^2 - \frac{\left(\sum_{i=1}^{N} x_i\right)^2}{N}}{N}}$	$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n-1}}$



STAT 1110 Recitation 2

September 13, 2024

Conditional Probability

$$Prob(A|B) = \frac{Prob(A \cap B)}{Prob(B)}$$

We don't assume independence unless explicitly stated

Conditional Probability and Independence

$$Prob(A|B) = \boxed{\frac{Prob(A \cap B)}{Prob(B)} = \frac{Prob(A) \times Prob(B)}{Prob(B)}} = Prob(A).$$

Law of Total Probability

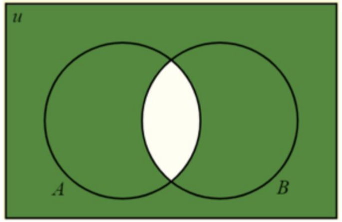
$$\boxed{P(B) = P(A \cap B) + P(A' \cap B)}$$

$$P(A) = \sum_n P(A \cap B_n)$$

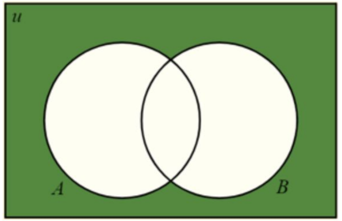
Bayes' Theorem

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{[P(B|A)P(A)] + [P(B|A^c)P(A^c)]}$$

DeMorgan's Law



$(A \cap B)' = A' \cup B'$



$(A \cup B)' = A' \cap B'$

Actual

Predicted

	Disease	No Disease
T^+	True Positive	False Positive
T^-	False Negative	True Negative

$$Sensitivity = \frac{True\ Positive}{True\ Positive + False\ Negative}$$

If you actually have the disease, will the test say you have it?

$$Specificity = \frac{True\ Negative}{True\ Negative + False\ Positive}$$

If you don't have the disease, will the test say you don't?

Random variable: a numerical quantity that takes on different values depending on chance.

A **discrete random variable** is a conceptual and numerical quantity that, in some future experiment involving chance, or randomness, will take one value from some discrete set of possible values. In some cases, the respective probabilities of the members of this set of possible values are known and in other cases they are unknown.

A **continuous random variable** is a conceptual numerical quantity which in some future experiment will take some value in a continuous range of values.

A **probability distribution** consists of all disjoint outcomes and their associated probabilities.

The sum of probabilities associated with each disjoint outcome must total to 1.

Probability mass function (pmf): a mathematical relation that assigns probabilities to all possible outcomes for a *discrete* random variable.

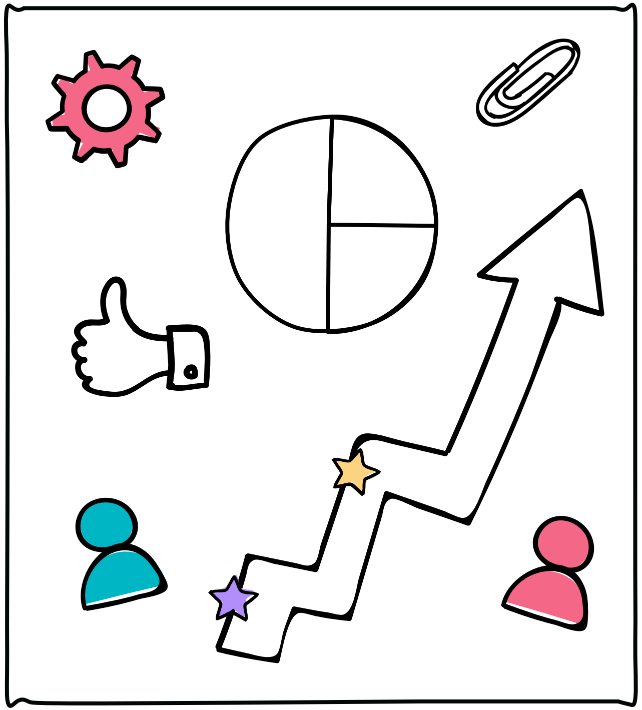
Discrete R.V.

Expected value (or mean) **of X** = $E(X) = \mu_x = \sum_{i=1}^{n} x_i Prob(X = x_i)$

Variance of X = $\sigma_x^2 = \sum_{i=1}^{n}[(x_i - \mu_x)^2 \, Prob(X = x_i)] = \sum_{i=1}^{n}[x_i^2 Prob(X = x_i)] - \mu_x^2$

Standard Deviation of X = $\sqrt{\sigma_x^2} = \sigma_{\mathrm{x}}$

Sign	Keywords
$=$	• Equal • Exactly • The same
$<$	• Less than • Below • Fewer than
$\leq$	• Less than or equal to • At most • Not more than
$>$	• Greater than • Above • More than
$\geq$	• Greater than or equal to • At least • Not less than



STAT 1110 Recitation 3

September 20, 2024

Binomial Distribution

Binomial distribution requirements:

1. **Fixed number of trials**
2. **Two possible outcomes on each trial**
3. **Trials must be independent**
4. **Probability of success must be same on all trials**

$$Prob(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ and $n! = n(n-1)(n-2)\ldots 1$

$$\text{Mean} = \mu = np$$

$$\text{Variance} = \sigma^2 = np(1-p)$$

$$\text{Standard deviation} = \sigma = \sqrt{np(1-p)}$$

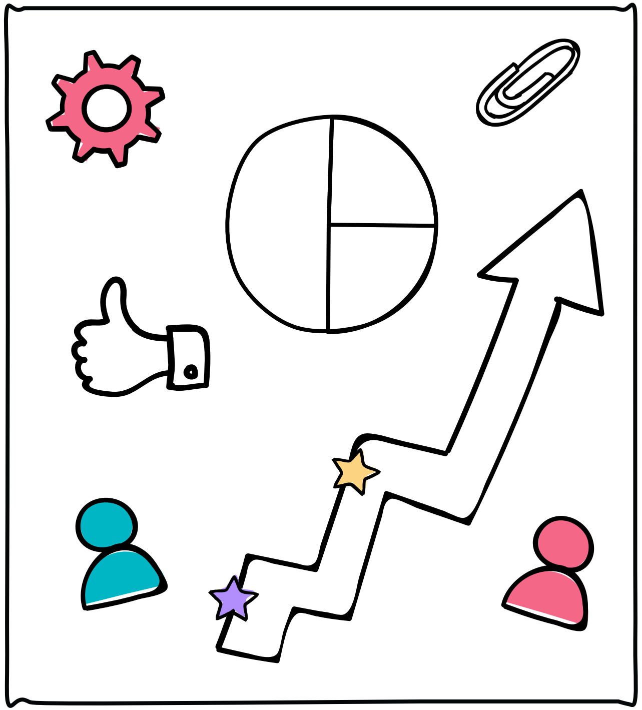
52-card deck

Example set of 52 playing cards; 13 of each suit: clubs, diamonds, hearts, and spades

	Ace	2	3	4	5	6	7	8	9	10	Jack	Queen	King
Clubs													
Diamonds													
Hearts													
Spades													

Midterm Review

https://create.kahoot.it/share/stat-11
10-midterm-1-review-f24/f0756e27-80
8f-4124-8039-e4903718561c



STAT 1110 Recitation 4

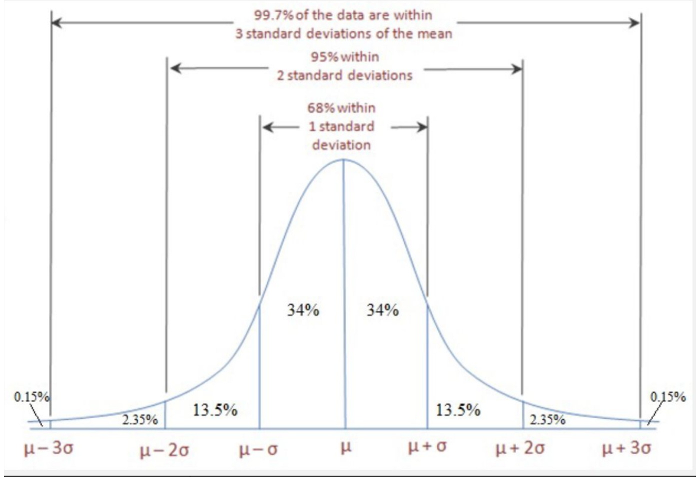
September 27, 2024

Normal Distribution

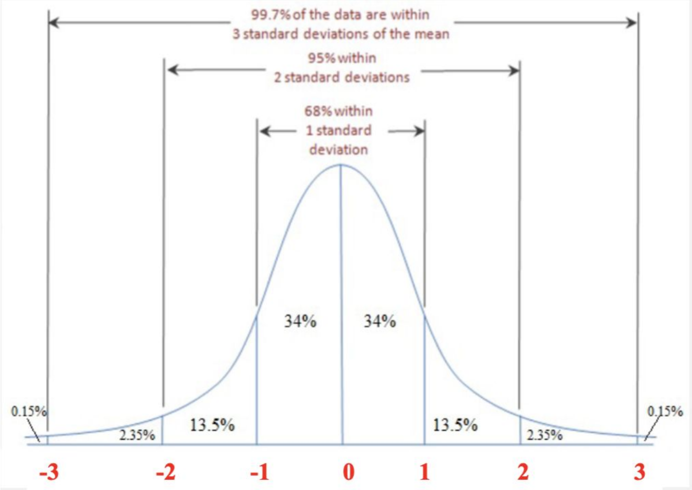
PDF

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

Empirical Rule: 68-95-99.7



Standard Normal Distribution $Z \sim N(0,1)$.



To turn r.v. X to Standard Normal r.v. Z:

$$\textbf{Z-score: } z = \frac{x-\mu}{\sigma} \quad \Longrightarrow \quad x = \mu + z\sigma$$

Normal Distribution Characteristics

- **Unimodal**
- **Symmetric**
- **Asymptotic/Unbounded: tails of the distribution never touches the x-axis**
    - **Because it is a *theoretical* distribution that *approximates* the true population metric**

COMMON CONVENTION ROUNDING RULES FOR Z-SCORES

- Calculate the Z-score and round to the hundredths place ***if*** the thousandths place is <u>not</u> equal to 5:
  - If the thousandths place is 0 – 4, round down (round toward zero for negative Z-scores)
    - i.e., If $z = 1.1445 \rightarrow$ use $z = 1.14$
  - If the thousandths place is 6 – 9, round up (round away from zero for negative Z-scores)
    - i.e., If $z = -2.4792 \rightarrow$ use $z = -2.48$
  - If the thousandths place is **equal to 5**, the Z-score to the thousandths place is used.
    - i.e., If $z = 1.5359 \rightarrow$ use $z = 1.535$

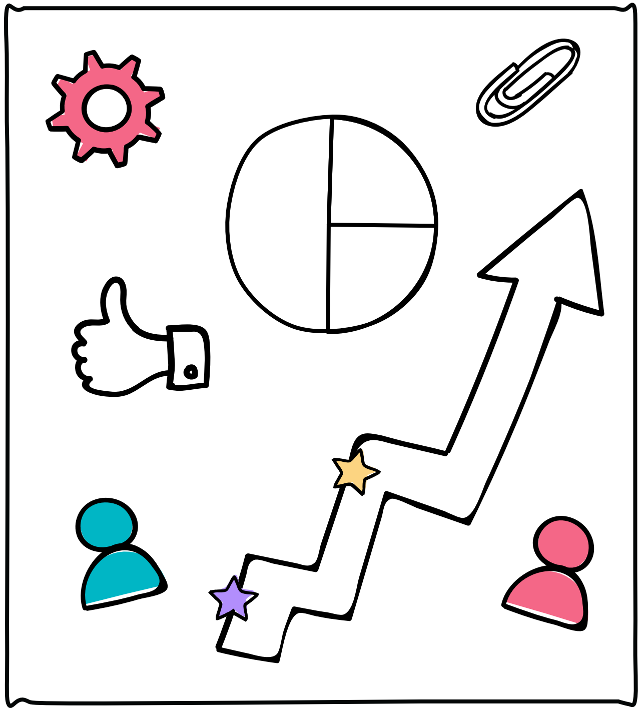
Normal Approximation

LARGE SAMPLE RULE FOR BINOMIAL (NORMAL APPROXIMATION TO THE BINOMIAL)

Rule: When n is large, the binomial distribution of X, the count of successes, gets close to the normal distribution.

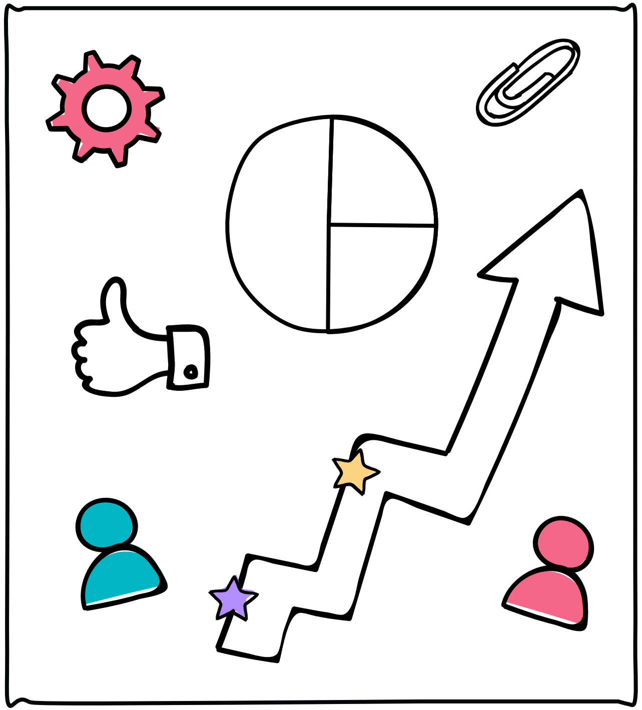
$$X \sim N\left(np, \sqrt{np(1-p)}\right)$$

Note: Need $\boldsymbol{np \geq 10}$ and $\boldsymbol{n(1-p) \geq 10}$



STAT 1110 Recitation 5

October 11, 2024



STAT 1110 Recitation 6

October 18, 2024

Agenda

1. Hypothesis Testing Steps (not in detail)
2. Type I and Type II Errors
3. P-value
    a. Important principles
    b. Interpretation
    c. Misconceptions
    d. Confidence intervals and p-value

Hypothesis Testing

- A “hypothesis” is a statement of predicted value or relationship within a population that can be tested to determine if it is supported or refuted by data

Hypothesis Testing Steps

1. State alpha
2. State and check assumptions
3. State hypotheses (Ho and Ha)
4. Calculate test statistic
5. State degree of freedom
6. Interpret p-value
7. State decision

HYPOTHESES FOR POPULATION MEAN

H_0 Null Hypothesis	Mathematical Statement	H_a Alternative Hypothesis
• Greater than or equal to # • At least # • Not less than #	*Left-tailed:* $\begin{cases} H_0: \mu \geq \# \\ H_a: \mu < \# \end{cases}$	• Less than # • Below # • Fewer than #
• Less than or equal to # • At most # • Not more than #	*Right-tailed:* $\begin{cases} H_0: \mu \leq \# \\ H_a: \mu > \# \end{cases}$	• Greater than # • Above # • More than #
• Equal to # • Exactly # • The same	*Two-tailed:* $\begin{cases} H_0: \mu = \# \\ H_a: \mu \neq \# \end{cases}$	• Not equal to # • Different from # • Changed

Example 5: You have run a statistical test at alpha=0.01 to test the claim that the mean cost of raising a child from birth to age 2 in the United States is $13,120. A random sample of 500 children has a mean cost of $12,925 with a population standard deviation of $1745. You determine a p-value of 0.08. Are the results statistically significant?

$\alpha = 0.01$

p-value = 0.08

p-value = 0.08 $>$ $\alpha = 0.01$

Since the p-value is greater than the significance level (α), our *decision* is to fail to reject the null hypothesis. Therefore, our *conclusion* states, "There <u>is not</u> sufficient evidence at $\alpha = 0.01$ to conclude the population mean cost of raising a child from birth to age 2 in the United States is not $13,120."

Therefore, we can state that **we do not have statistically significant results**.

Type I and Type II Errors

		TRUTH	
		H_0 is TRUE	**H_0 is FALSE**
DECISION	**Fail to reject H_0**	Correct Decision (Confidence level)	Type II Error (*β-level error*)
	Reject H_0	Type I Error (*α-level*)	Correct Decision (Power)

Type I Error/false positive/alpha = P(reject null | null is true)
Type II Error/false negative/beta = P(fail to reject null | alternative is true)

		TRUTH	
		Innocent	**Guilty**
VERDICT	**Not Guilty**	Correct Decision	Type II Error
	Guilty	Type I Error	Correct Decision

A restaurant is comparing two methods for preparing a new chicken dish. The chef steams the chicken for one meal and fries the chicken for another meal. The restaurant wants to know if patrons have a preference of the steamed chicken or fried chicken. State a type I error and type II error in the context of the problem.

(Informal hypotheses)
Null hypothesis: patrons have no preference for steamed chicken vs. fried chicken
Alternative hypothesis: patrons prefer one of the meals over the other

Type I error = P(reject null | null is true)
In the context of this problem: A type I error would occur if patrons truly have no preference for steamed chicken vs. fried chicken, but we determine they do have some preference for one meal over the other.

Type II error = P(fail to reject null | alternative true)
In the context of this problem: A type II error would occur if patrons truly did prefer one of the meals over the other, but we conclude that they may not have any preference.

Important Principles of P-value

- It indicates how incompatible the data is with a specified statistical model
  - The smaller the p-value, the stronger the evidence against the null
- It <u>does not</u> measure the probability that a hypothesis is true
- It <u>does not</u> measure the size of an effect or the importance of a result; it tells us how likely it is that the result is due to chance
- By itself, it <u>does not</u> provide a good measure of evidence regarding a model or hypothesis
  - Proper inferences requires more information than just p-value

P-value Interpretation

If we assume that (*state Ho*) is true, we would expect to see a sample (*state what you're testing i.e. mean, mean difference, proportion, proportion difference, etc.*) of (*state sample value*) or (*smaller/greater/more extreme*) about (*p-value*)% of the time by chance.

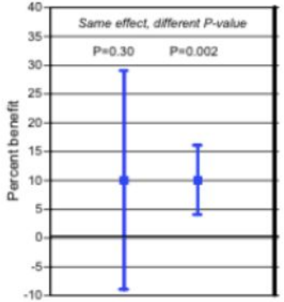
Example 6: Suppose birth weights have $\mu = 112$ oz, $\sigma = 20.6$ oz. We conduct a study with 25 newborns and find $\bar{x} = 126$ oz.

We reject the null hypothesis due to a p-value = 0.0006. Interpret this value.

If the population mean birth weight was truly 112 oz, we'd expect to see a sample mean weight of 126 oz, or something more extreme, about 0.06% of the time by chance.

P-value Misconceptions

- If the p-value = 0.05, the null hypothesis only has a 5% chance of being true.
- A non-significant difference means there's no difference between groups.
- A statistically significant finding is clinically important.
- Studies with p-values on opposite sides of 0.05 are conflicting.



- With a p-value = 0.05, the chance of a type 1 error will be 5%.
- A scientific conclusion or treatment policy should be based on whether or not the p-value is significant

Confidence Interval and P-value

- If the null value falls within the CI range, fail to reject null hypothesis and results are NOT significant (not significantly different from population)

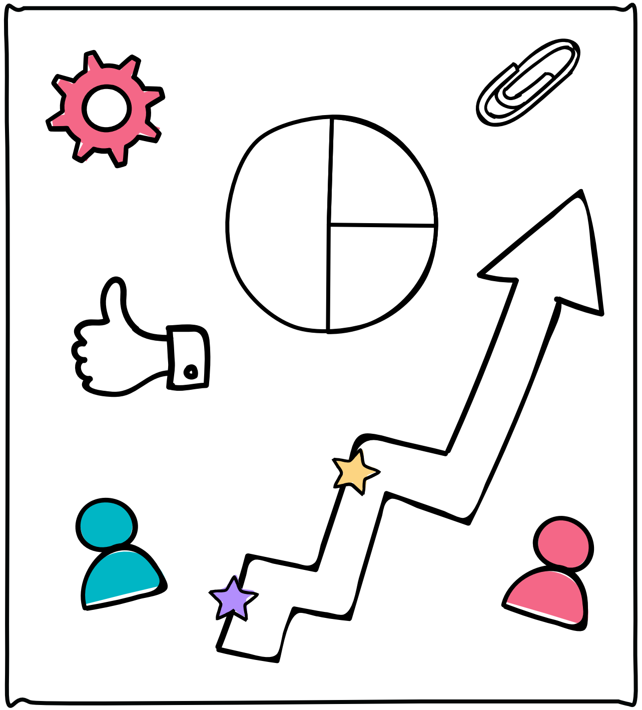
- If null value does NOT fall within the CI range, reject the null hypothesis and results ARE significant (significant difference from population)

Example 7: A two-sided test of H_0: μ=0 yields a p-value of 0.03. Will the 95% confidence interval for μ include 0 in its midst?

Hypothesis test $\Longrightarrow$ $\mathrm{H_0}: \mu = 0$ and $\mathrm{H_a}: \mu \neq 0$
P-value = 0.03

- 95% CI $\Longrightarrow$ $\alpha = 0.05$

  P−value $< \alpha$ $\Longrightarrow$ **Decision:** reject the null hypothesis
  $\Longrightarrow$ The null value of μ=0 does not fall within the 95% confidence interval range.



STAT 1110 Recitation 7

October 25, 2024

One Sample z-Test

Used to test population mean for one sample when population s.d. Is known

Assumptions:

1. Simple random sample (SRS)
2. Independent observations
3. Normal distribution: normally distributed data OR sample size is large ($n \geq 30$)
4. Population standard deviation is known

Hypotheses:

Ho: $\mu \neq$ [population mean]

Ha: $\mu =$ [population mean]

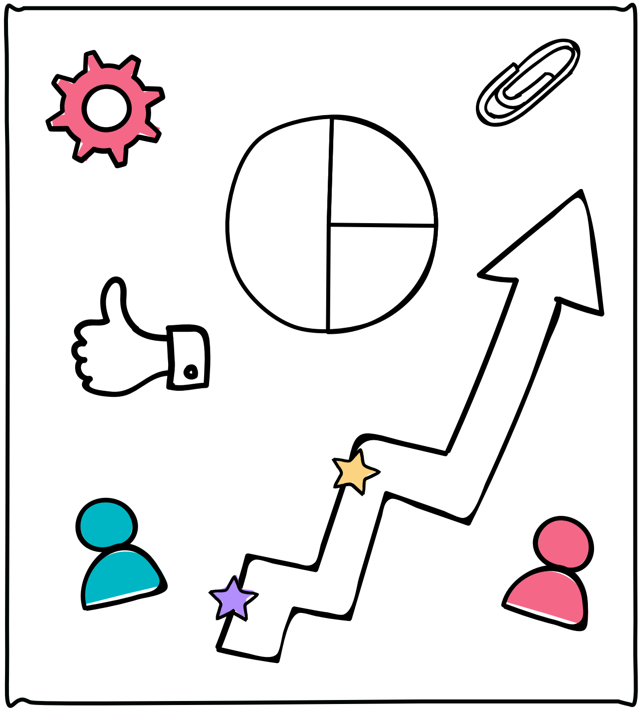
Test statistic: $z = \frac{\bar{x}-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$

z-distribution

This is a left-tailed distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998



STAT 1110 Recitation 8

November 1, 2024

One Sample t-Test

Used to test population mean for one sample when population s.d. Is unknown OR sample size is small (n < 30)

Assumptions:

1. Simple random sample (SRS)
2. Independent observations
3. Normal distribution: normally distributed data OR sample size is large ($n \geq 30$)

Hypotheses:

Ho: $\mu \neq$ [population mean]

Ha: $\mu =$ [population mean]

Test statistic: $$t = \frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)}$$

Degrees of freedom (df) = n - 1

Degrees of Freedom

"The number of independent data that are free to vary"

$x + y + z = 15$

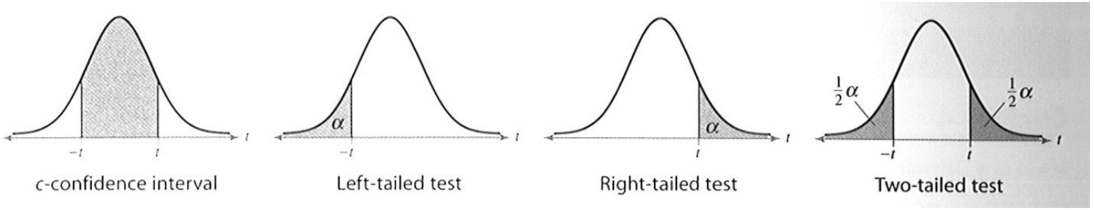
$x = 5$
$5 + y + z = 15$
$y + z = 10$

$y = 7$
$7 + z = 10$
$z = 3$ (no more "freedom" to choose z, z has to be 3)

t-distribution

	Level of confidence, c	0.50	0.80	0.90	0.95	0.98	0.99
	One tail, α	0.25	0.10	0.05	0.025	0.01	0.005
d.f.	Two tails, α	0.50	0.20	0.10	0.05	0.02	0.01
1		1.000	3.078	6.314	12.706	31.821	63.657
2		.816	1.886	2.920	4.303	6.965	9.925
3		.765	1.638	2.353	3.182	4.541	5.841
4		.741	1.533	2.132	2.776	3.747	4.604
5		.727	1.476	2.015	2.571	3.365	4.032
6		.718	1.440	1.943	2.447	3.143	3.707
7		.711	1.415	1.895	2.365	2.998	3.499
8		.706	1.397	1.860	2.306	2.896	3.355
9		.703	1.383	1.833	2.262	2.821	3.250
10		.700	1.372	1.812	2.228	2.764	3.169
11		.697	1.363	1.796	2.201	2.718	3.106
12		.695	1.356	1.782	2.179	2.681	3.055
13		.694	1.350	1.771	2.160	2.650	3.012
14		.692	1.345	1.761	2.145	2.624	2.977
15		.691	1.341	1.753	2.131	2.602	2.947
16		.690	1.337	1.746	2.120	2.583	2.921
17		.689	1.333	1.740	2.110	2.567	2.898
18		.688	1.330	1.734	2.101	2.552	2.878
19		.688	1.328	1.729	2.093	2.539	2.861
20		.687	1.325	1.725	2.086	2.528	2.845
21		.686	1.323	1.721	2.080	2.518	2.831
22		.686	1.321	1.717	2.074	2.508	2.819
23		.685	1.319	1.714	2.069	2.500	2.807
24		.685	1.318	1.711	2.064	2.492	2.797
25		.684	1.316	1.708	2.060	2.485	2.787
26		.684	1.315	1.706	2.056	2.479	2.779
27		.684	1.314	1.703	2.052	2.473	2.771
28		.683	1.313	1.701	2.048	2.467	2.763
29		.683	1.311	1.699	2.045	2.462	2.756
∞		.674	1.282	1.645	1.960	2.326	2.576

This is a right-tailed distribution by default



z-distribution vs t-distribution

Go to 🟥 first, then 🟦

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	[illegible]	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

	Level of confidence, c	0.50	0.80	0.90	0.95	0.98	0.99
	One tail, α	0.25	0.10	0.05	0.025	0.01	0.005
d.f.	Two tails, α	0.50	0.20	0.10	0.05	0.02	0.01
1		1.000	3.078	6.314	12.706	31.821	63.657
2		.816	1.886	2.920	4.303	6.965	9.925
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4		.741	1.533	2.132	2.776	3.747	4.604
5		.727	1.476	2.015	2.571	3.365	4.032
6		.718	1.440	1.943	2.447	3.143	3.707
7		.711	1.415	1.895	2.365	2.998	3.499
8		.706	1.397	1.860	2.306	2.896	3.355
9		.703	1.383	1.833	2.262	2.821	3.250
10		.700	1.372	1.812	2.228	2.764	3.169
11		.697	1.363	1.796	2.201	2.718	3.106
12		.695	1.356	1.782	2.179	2.681	3.055
13		.694	1.350	1.771	2.160	2.650	3.012
14		.692	1.345	1.761	2.145	2.624	2.977
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17		.689	1.333	1.740	2.110	2.567	2.898
18		.688	1.330	1.734	2.101	2.552	2.878
19		.688	1.328	1.729	2.093	2.539	2.861
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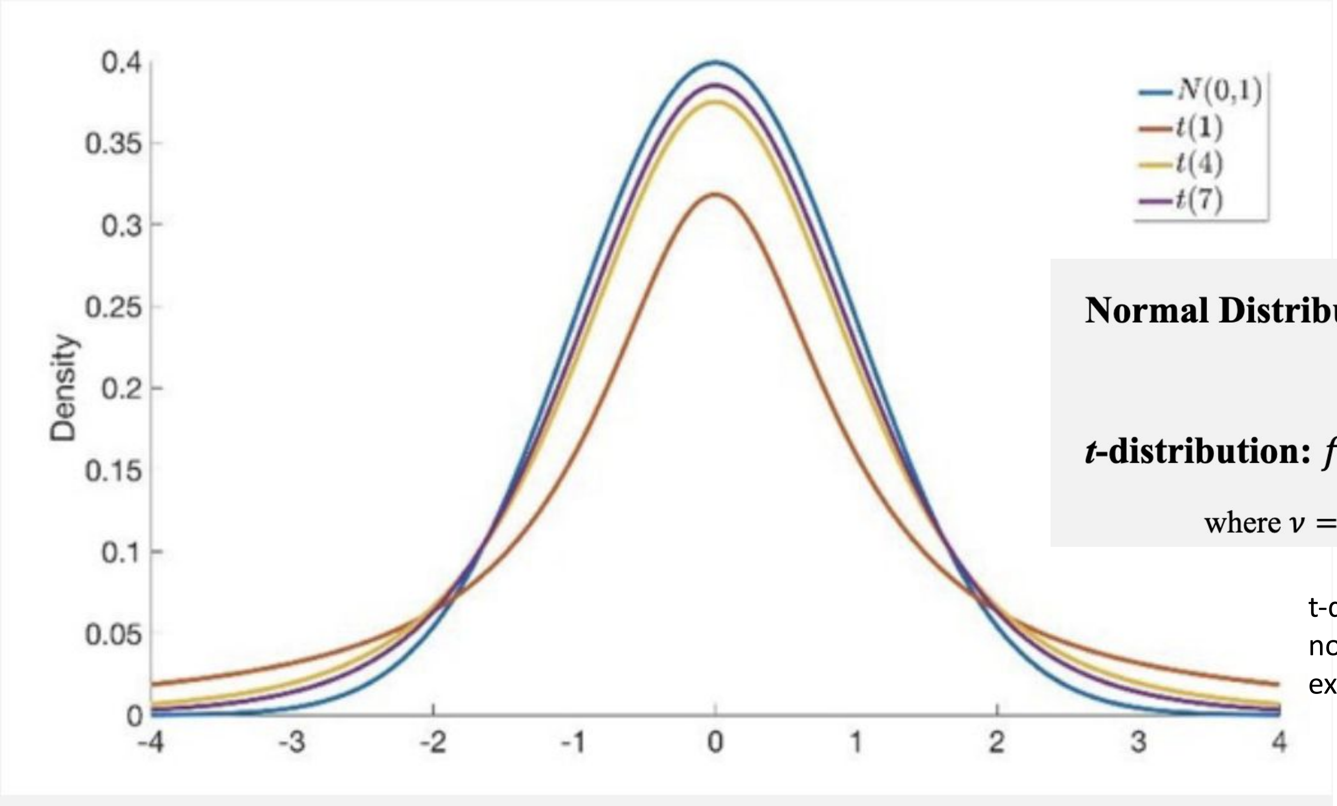
22		.686	1.321	1.717	2.074	2.508	2.819
23		.685	1.319	1.714	2.069	2.500	2.807
24		.685	1.318	1.711	2.064	2.492	2.797
25		.684	1.316	1.708	2.060	2.485	2.787
26		.684	1.315	1.706	2.056	2.479	2.779
27		.684	1.314	1.703	2.052	2.473	2.771
28		.683	1.313	1.701	2.048	2.467	2.763
29		.683	1.311	1.699	2.045	2.462	2.756
∞		.674	1.282	1.645	1.960	2.326	2.576

t-distribution Characteristics

- Unimodal
- Symmetric
- Defined by df, which then determines the heaviness of the tails
  - The higher the df, the thinner the tails
- Asymptotic/Unbounded: tails of the distribution never touches the x-axis
  - Because it is a *theoretical* distribution that *approximates* the normal distribution



Normal Distribution: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

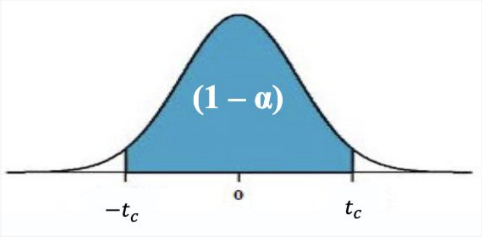
***t*-distribution:** $f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}},$

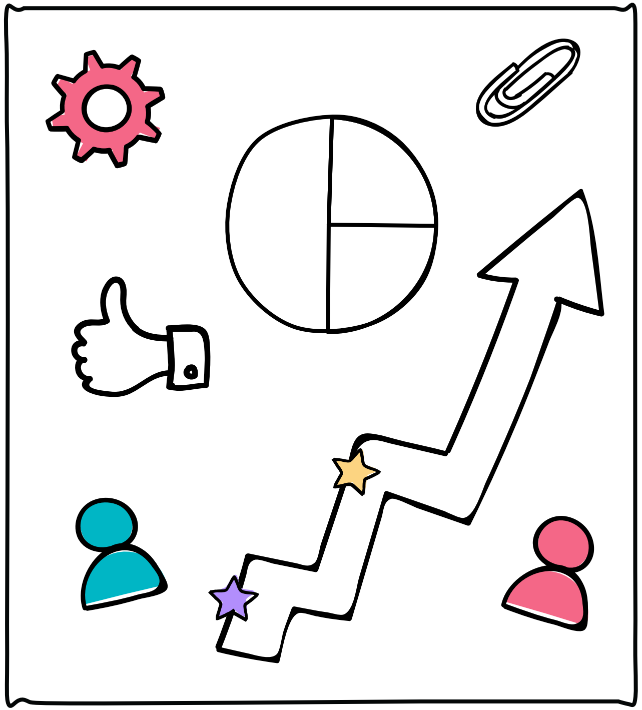
where $\nu = \text{d.f.}$ and $\Gamma(n) = (n-1)!$

t-distributions have heavier tails than normal distributions = greater chance for extreme values

One Sample t Confidence Interval

$$\bar{x} \pm t_c\left(\frac{s}{\sqrt{n}}\right)$$





STAT 1110 Recitation 9

November 8, 2024

Paired Sample t-Test

Used to test population mean for differences (e.g. before/after data or matched samples)

Assumptions:

1. Simple random sample (SRS)
2. Independent observations
3. Normal distribution: normally distributed data OR sample size is large ($n \geq 30$)

Hypotheses:

Ho: μ_d = [population mean difference]

Ha: μ_d ≠ [population mean difference]

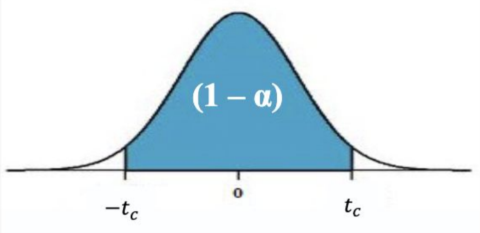
Test statistic:

$$t = \frac{\bar{x}_d - \mu_d}{\left(\frac{s_d}{\sqrt{n}}\right)}, \text{ where } s_d = \sqrt{\frac{\sum x_d^2 - \frac{\left(\sum x_d\right)^2}{n}}{n-1}}, \quad \bar{x}_d = \text{sample mean of the differences, n = number of pairs}$$

Degrees of freedom (df) = n - 1

One Sample Mean of Difference t Confidence Interval

$$\bar{x}_d \pm t_c\left(\frac{s_d}{\sqrt{n}}\right)$$



Two Sample t-Test

Used to test population mean for two independent groups

Assumptions:

1. Simple random sample (SRS)
2. Independent observations
3. Independent groups
4. Normal distribution: normally distributed data OR sample size is large ($n \geq 30$)

Hypotheses:

Ho: $\mu_1 = \mu_2$

Ha: $\mu_1 \neq \mu_2$

Welch Method/Satterthwaite's (unpooled – assumes unequal population variances):

Test statistic: $t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{SE}$, where SE = standard error

$$\boldsymbol{d.f.} = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$$ *(round down)*

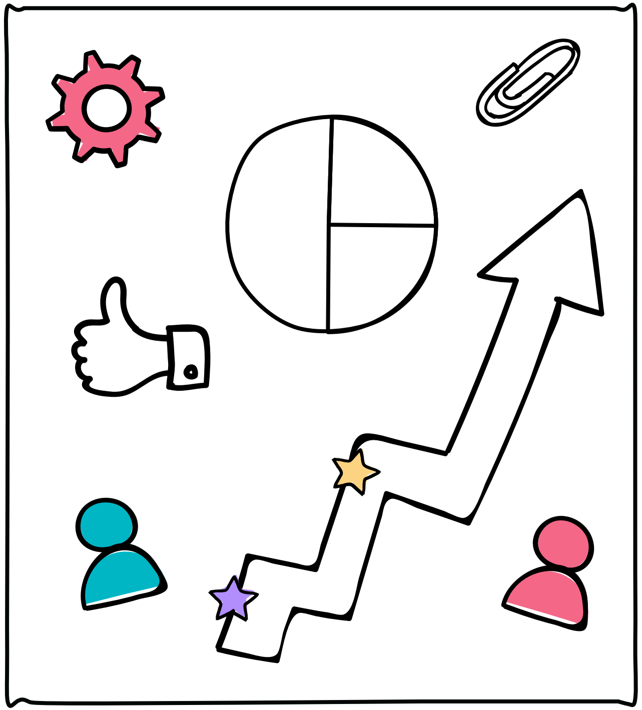
Student *t*-distribution (pooled – assumes equal population variances):

Test statistic: $t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{SE}$, where SE = standard error

$\boldsymbol{d.f.} = n_1 + n_2 - 2$

Two Sample t Confidence Interval

$$(\bar{x}_1 - \bar{x}_2) \pm t_c(SE)$$



STAT 1110 Recitation 10

November 15, 2024

One Sample Proportion Test

Used to test population proportion (binary outcome) for one sample

Assumptions:

1. Simple random sample (SRS)
2. Binomial distribution
   a. Fixed number of trials
   b. Two possible outcomes on each trial
   c. Trials must be independent
   d. Probability of success must be same on all trials
3. Normal distribution: np ≥ 10 and n(1-p) ≥ 10
   a. If this is not met, use exact Binomial method

Hypotheses:

Ho: p = p_0

Ha: p ≠ p_0

Test statistic:

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}, \text{ where } \hat{p} = \text{sample proportion}, n = \text{sample size, and } SE = \sqrt{\frac{p_0(1-p_0)}{n}}$$

One Sample Proportion Confidence Interval

$$\hat{p} \pm z_c \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Chi-Square Test (of Goodness of Fit)

Used to test whether the distribution of a single categorical variable conforms to a null hypothesis

Assumptions:

1. Simple random sample (SRS)
2. Independent observations
3. Sufficient cell count: all expected cell counts are >= 5

Expected counts = $E_i = np_i$, where n = total sample size and p_i = population proportion for group i

Hypotheses:

Ho: The data collected is consistent with the population distribution

Ha: The data collected is not consistent with the population distribution

Test statistic: $\chi^2 = \sum_{i=1}^{m} \frac{(O_i - E_i)^2}{E_i}$, where O_i = observed counts and m = total number of cells

Degrees of freedom: (# of categories - 1)

CHI-SQUARE DISTRIBUTION

$$f(x|k) = \frac{1}{\left(\frac{k}{2}-1\right)!}\left(\frac{1}{2^{k/2}}\right)x^{\left(\frac{k}{2}\right)-1}e^{-\left(\frac{x}{2}\right)},$$

where k = d.f.

Observations: it is always positive because the test statistic is squared

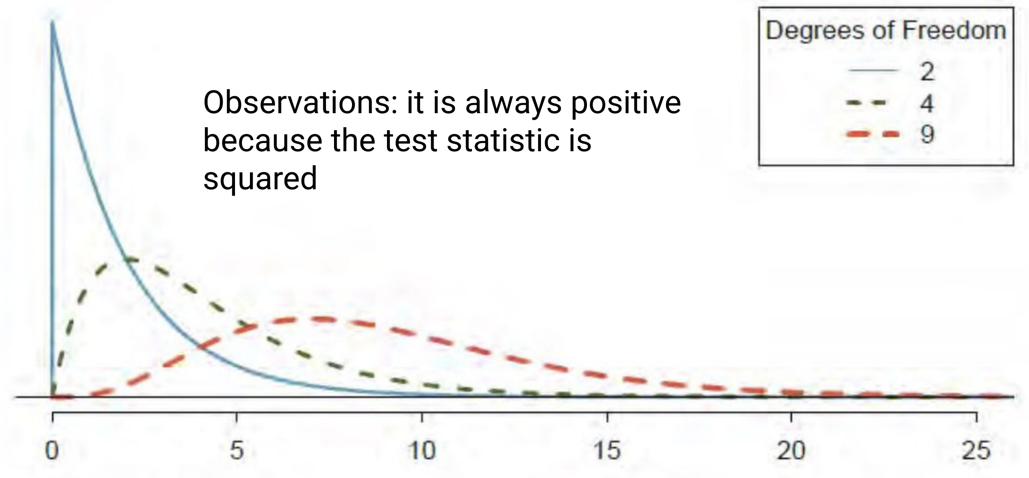


Figure 6.8: Three chi-square distributions with varying degrees of freedom.

Go to $\boxed{}$ first, then $\boxed{}$

Chi-Square Distribution Table

	P										
DF	0.995	0.975	0.2	0.1	0.05	0.025	0.02	0.01	0.005	0.002	0.001
1	.0004	.00016	1.642	2.706	3.841	5.024	5.412	6.635	7.879	9.55	10.828
2	0.01	0.0506	3.219	4.605	5.991	7.378	7.824	9.21	10.597	12.429	13.816
3	0.0717	0.216	4.642	6.251	7.815	9.348	9.837	11.345	12.838	14.796	16.266
4	0.207	0.484	5.989	7.779	9.488	11.143	11.668	13.277	14.86	16.924	18.467
5	0.412	0.831	7.289	9.236	11.07	12.833	13.388	15.086	16.75	18.907	20.515
6	0.676	1.237	8.558	10.645	12.592	14.449	15.033	16.812	18.548	20.791	22.458
7	0.989	1.69	9.803	12.017	14.067	16.013	16.622	18.475	20.278	22.601	24.322
8	1.344	2.18	11.03	13.362	15.507	17.535	18.168	20.09	21.955	24.352	26.124
9	1.735	2.7	12.242	14.684	16.919	19.023	19.679	21.666	23.589	26.056	27.877
10	2.156	3.247	13.442	15.987	18.307	20.483	21.161	23.209	25.188	27.722	29.588
11	2.603	3.816	14.631	17.275	19.675	21.92	22.618	24.725	26.757	29.354	31.264
12	3.074	4.404	15.812	18.549	21.026	23.337	24.054	26.217	28.3	30.957	32.909
13	3.565	5.009	16.985	19.812	22.362	24.736	25.472	27.688	29.819	32.535	34.528
14	4.075	5.629	18.151	21.064	23.685	26.119	26.873	29.141	31.319	34.091	36.123
15	4.601	6.262	19.311	22.307	24.996	27.488	28.259	30.578	32.801	35.628	37.697
16	5.142	6.908	20.465	23.542	26.296	28.845	29.633	32	34.267	37.146	39.252
17	5.697	7.564	21.615	24.769	27.587	30.191	30.995	33.409	35.718	38.648	40.79
18	6.265	8.231	22.76	25.989	28.869	31.526	32.346	34.805	37.156	40.136	42.312
19	6.844	8.907	23.9	27.204	30.144	32.852	33.687	36.191	38.582	41.61	43.82
20	7.434	9.591	25.038	28.412	31.41	34.17	35.02	37.566	39.997	43.072	45.315

Chi-Square Test (of Independence)

Used to test association between two categorical (nominal) variables

Assumptions:

1. Simple random sample (SRS)
2. Independent observations
3. Sufficient cell count: all expected cell counts are >= 5

Expected counts = $E_i = \frac{(row\ total)_i (column\ total)_i}{(overall\ total)_i}$

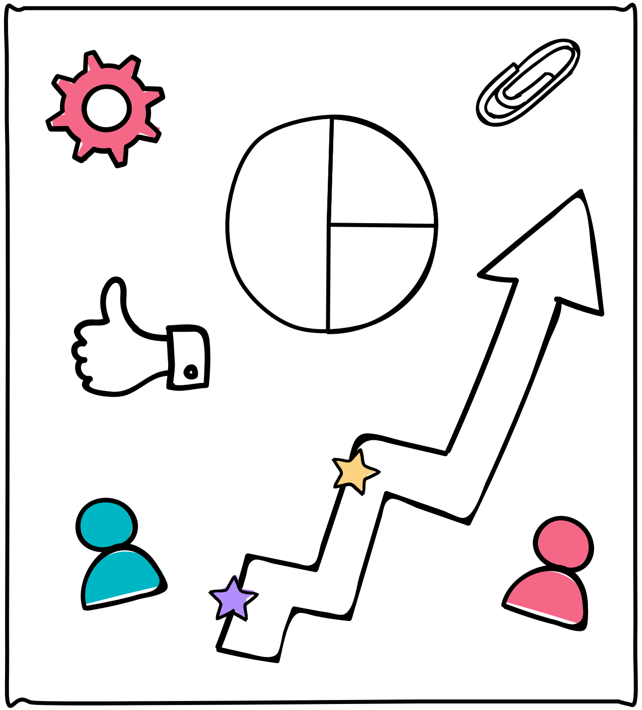
Hypotheses:

Ho: There is no association between the two categorical variables in the population

Ha: There is some association between the two categorical variables in the population

Test statistic: $\chi^2 = \sum_{i=1}^{m} \frac{(O_i - E_i)^2}{E_i}$, where O_i = observed counts and m = total number of cells

Degrees of freedom: (# of rows - 1)(# of columns - 1)

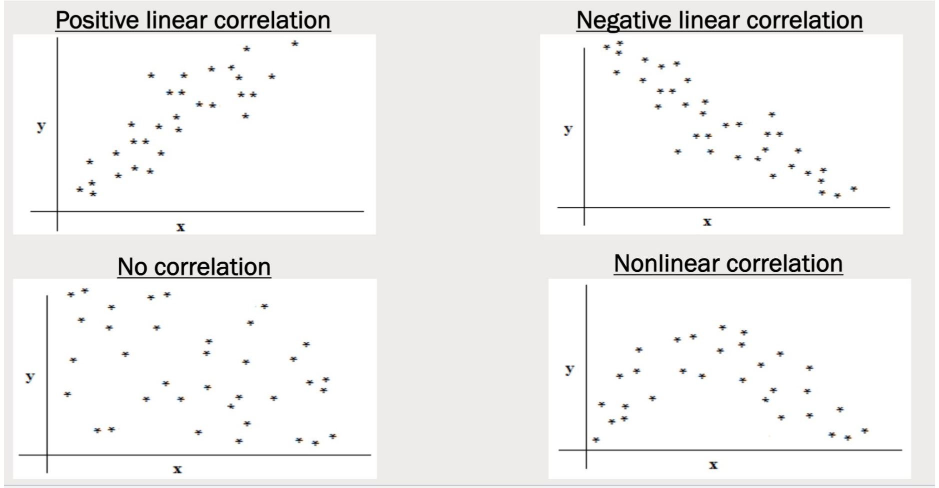


STAT 1110 Recitation 11

November 22, 2024

Correlation

- Definition: a relationship between two numerical variables x and y
  - We only focus on the case where x is independent and y is dependent in this class
  - If you're assessing the correlation between two independent variables, correlation is used to identify multicollinearity between the two variables
  - If you're assessing the correlation between two categorical variables, use chi-square of independence



Pearson Correlation

- **Definition: A measurement of strength and direction of linear relationship between two numerical variables**
- **Notation:**

  ρ = linear correlation coefficient for *population*

  r = linear correlation coefficient for *sample*

- **Formula:**

$$r = \frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{s_x}\right)\left(\frac{y_i - \bar{y}}{s_y}\right),$$

where $s_x = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$, $s_y = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1}}$, $\bar{x} = \frac{\sum x}{n}$, $\bar{y} = \frac{\sum y}{n}$,

n = number of pairs

- **Properties:**
  - **Range:** $-1 \leq r \leq 1$
  - **Measures *strength* and *direction* of *linear* relationship**
  - **Measure of *correlation*, NOT *causation***

    correlation $\neq$ causation

Hypothesis Test for Correlation Coefficient

Used to test if there is any linear relationship between two numerical variables

Assumptions:

1. Simple random sample (SRS)
2. Independent observations
3. Linearity - check scatterplot
4. Bivariate normality - check distribution of X and Y

Hypotheses:

H_0: $\rho = 0$ (There is no linear correlation between x and y in the population.)

H_a: $\rho \neq 0$ (There is some linear correlation between x and y in the population.)

Test statistic:

$$t = \frac{r\sqrt{(n-2)}}{\sqrt{1-r^2}}, \quad \text{where } n = \text{number of pairs}$$

Degrees of freedom: n - 2

Think of this test as y = mx + b, you're trying to look for m and b, which are two parameters

Which Hypothesis Test?

1. A company claims that their battery lasts an average of 300 hours. A researcher tests 50 batteries and finds a mean of 290 hours with a population standard deviation of 20 hours. Can the researcher conclude that the battery life is significantly different from the claimed value?
2. A nutritionist claims that the average sodium content in a brand of soup is 500 mg. You sample 15 cans of soup and find a mean of 520 mg with a standard deviation of 30 mg. Is there enough evidence to support the claim that the mean sodium content differs from 500 mg?
3. A psychologist is studying the effectiveness of two different therapies for treating anxiety. Group A (n=20) receives therapy 1, and group B (n=20) receives therapy 2. After 6 weeks, the mean anxiety score for Group A is 35 with a standard deviation of 8, and for Group B, it is 30 with a standard deviation of 6. Are the therapies equally effective?
4. A sociologist collects data on the relationship between gender (male, female) and preference for a type of movie (comedy, action, drama). Is there a significant association between gender and movie preference?
5. The mean height of adults in a country is claimed to be 170 cm with a population standard deviation of 8 cm. A random sample of 40 adults has a mean height of 168 cm. Does this sample provide enough evidence to refute the claim?

Which Hypothesis Test?

1. One sample z-test
2. One sample t-test
3. Two sample t-test
4. Chi-square test for independence
5. One sample z-test

Which Hypothesis Test (Part 2)?

1. A political analyst claims that 60% of voters support a new policy. A survey of 200 voters shows that 110 voters support the policy. Can the analyst conclude that the proportion of voters supporting the policy is different from 60%?
2. A company sells its products in four regions (North, South, East, West). The expected sales distribution is equal across all regions. The observed sales for a month are as follows:
   - North: 120
   - South: 150
   - East: 130
   - West: 100

   Does the observed sales distribution significantly differ from the expected distribution?
3. A school cafeteria offers four types of snacks: chips, cookies, fruit, and granola bars. The cafeteria manager believes students select each snack equally often. After collecting data for a month, is there evidence that students have a preference for certain snacks?
4. A researcher wants to investigate whether sleep deprivation affects cognitive performance. Ten participants are tested on a cognitive task after a full night's sleep and then again after 24 hours of sleep deprivation. The scores are recorded for both conditions. Can the researcher conclude that sleep deprivation significantly impacts cognitive performance?

Which Hypothesis Test (Part 2)?

1. One sample proportion
2. Chi-square test for goodness of fit
3. Chi-square test for goodness of fit
4. Paired sample t-test