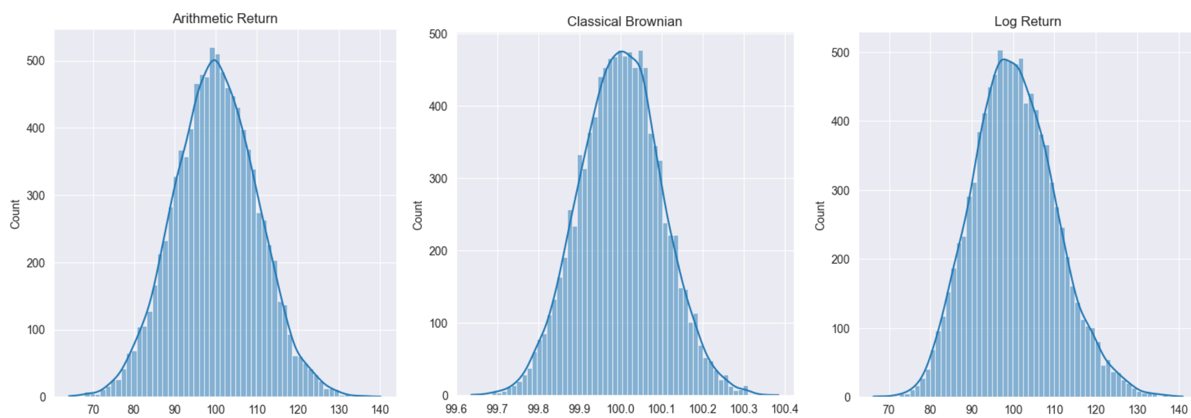
Problem1:

My process:

1. Set initial value p_{t-1}=100, $\sigma = 0.1$
2. Using math formula to calculate the expected mean and standard deviation:
   Classcial Brownian: $mean = p_{t-1}$; $std = \sigma$
   Arithmetic Return: $mean = p_{t-1}$; $std = p_{t-1} * \sigma$
   Log return: $mean = p_{t-1} * e^{\frac{1}{2}\sigma^2}$; $std = p_{t-1} * \sqrt{(e^{\sigma^2} - 1) * e^{\sigma^2}}$
3. Do the simulation for return, $r \sim N(0, \sigma^2)$.
4. Calculate the mean and standard deviation for the simulated p_t and compare.

Result:

	Expected Mean	Actual Mean	Expected Std	Actual Std
Classcial Brownian	100.00	100.00	0.10	0.10
Arithmetic Return	100.00	99.97	10.00	10.14
Log return	100.50	10.08	100.40	10.07



Conclusion:

The simulated result and the expected values is nearly the same for Classical Brownian return, and pretty close for arithmetic return and log normal return. For $\sigma = 0.1$, log return is closer to expected value than arithmetic return. But I found that if the σ is set to a larger value, there will be more discrepancy between expected value and simulated result for log return.
From the distribution of simulated results, we can see that the distribution for Classical Brownian and arithmetic are normal and log return is close to a log normal distribution (It will be more obvious if std is larger).

Problem2:

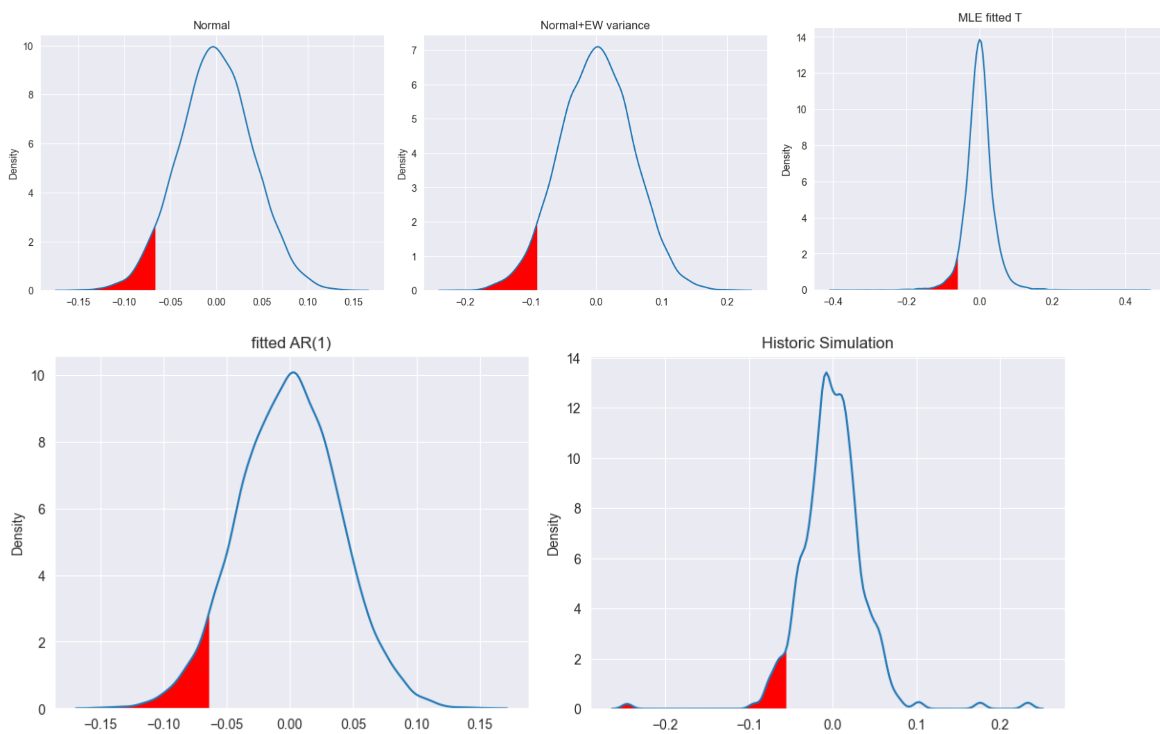
My process:

1. Calculate the returns for META and subtracts the mean
2. Generate simulated returns using the distribution defined. For each distribution, get the parameters by fitting the historical returns. For normal distribution, we need std; For T distribution, we need degrees of freedom, μ and σ; For AR(1), we need the estimators for AR(1) equation and the std of e; For historical simulation, we do not need a distribution for returns. We can draw N returns from the historical data.
3. Sort the simulated returns and get the $N*\alpha$ value as VaR.

Result:

Var (return):

1. Normal Distribution: 0.0667
2. Normal Distribution with exponentially weighted variance: 0.090
3. MLE fitted T: 0.056
4. AR(1): 0.065
5. Historic Simulation: 0.056



Conclusion:

The Normal Distribution with EW variance is significantly larger than others, which implies that the volatility of recent time periods may be larger and further enforced by the weights and make the total variance larger. All the other four methods have close results, with T distribution and Historical simulation slightly smaller. Except historic simulation, all the other four has the distribution of simulated returns as expected. For historic simulation, there is more randomness and do not follow a specific distribution. Since the times of draw is large, it still reflects information from historic returns statistically.

Therefore, we can say that MLE T has the best fit for historic patterns in this case because it is closest to the result of historic simulation.

Problem3:

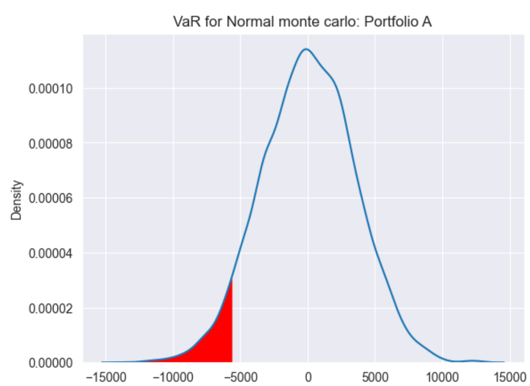
My process:

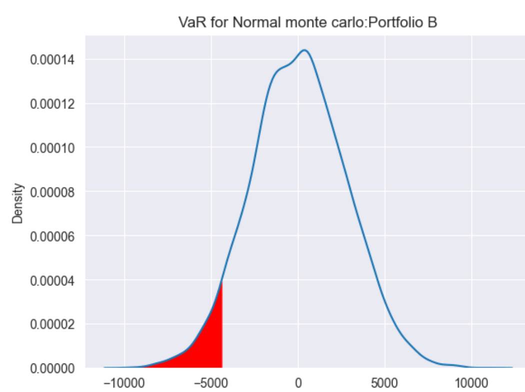
1. Bases on the datasets, get historic prices, current prices and current value for the portfolio
2. Calculate returns using arithmetic method.
3. Calculate exponentially weighted covariance using function from last week
4. Do delta normal calculation
5. Use simulation method: PCA to do Monte Carlo and Historic Simulation

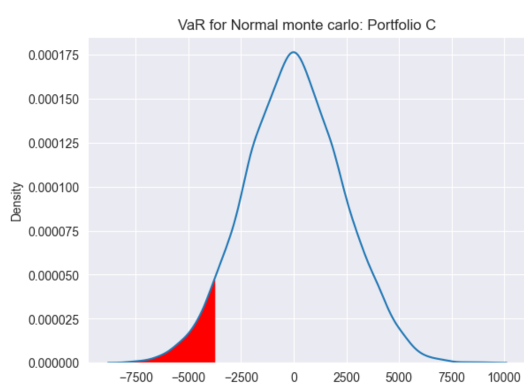
Result:

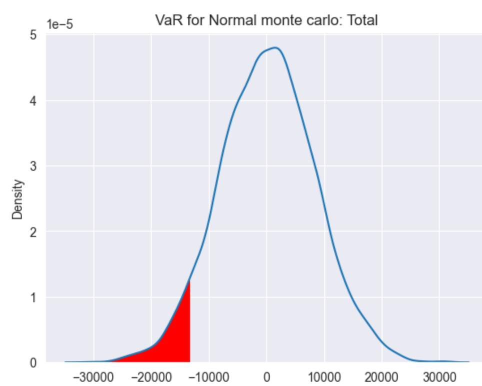
	A	B	C	All
Delta Normal	5670.2	4494.6	3786.59	13577.08
Monte Carlo	5605.31	4337.6	3769.5	13304.87
Historic	9138.87	7001.12	5558.72	20564.36

Monte Carlo Simulated Profit/Loss:

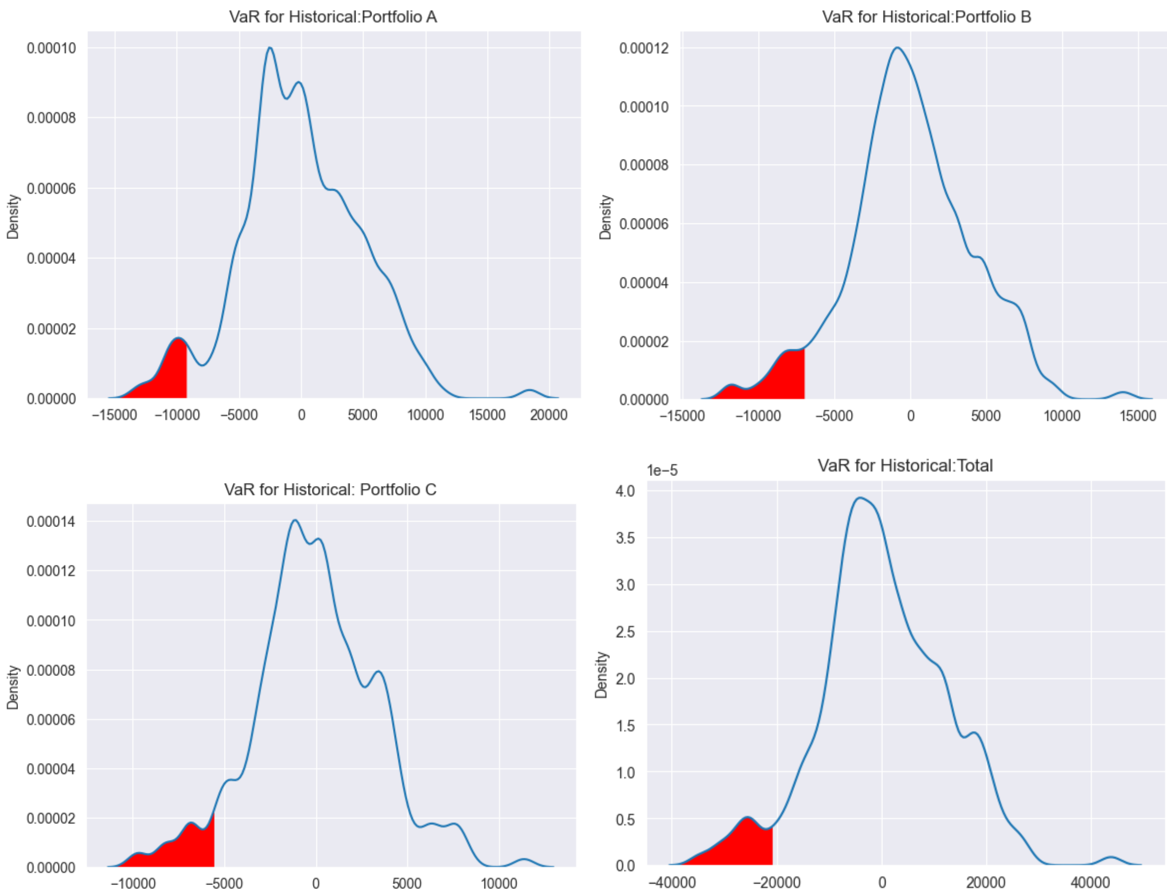








Historic Simulated Profit/Loss:



Conclusion:

The result of Delta Normal and Monte Carlo Simulation is close. For delta normal method we assumes that the returns follow a normal distribution and calculate the VaR directly; For Monte Carlo simulation, we add random variable to simulate returns instead of assuming a distribution. Although results close, the methods differ.

Historic simulation has the largest VaR. Historic simulation has more randomness because we draw directly from existing data with replacement N times. There is no distribution to follow. The difference of historic and normal method may due to the fact that the returns does not follow the normal distribution we assumed and the bias of historic method due to the small sample size.

From my perspective, I will choose historic simulation if we have a longer history data to simulate from. Because this method does not need any assumption and distribution, and it reflects more information about the historical pattern. But for small samples, there is significant bias. For small sample like this, I will choose Monte Carlo Simulation method.