Min Yang

STAT2112

Project 3: Methods of Quality Improvement

Dr. Vadakkeveetil

Introduction

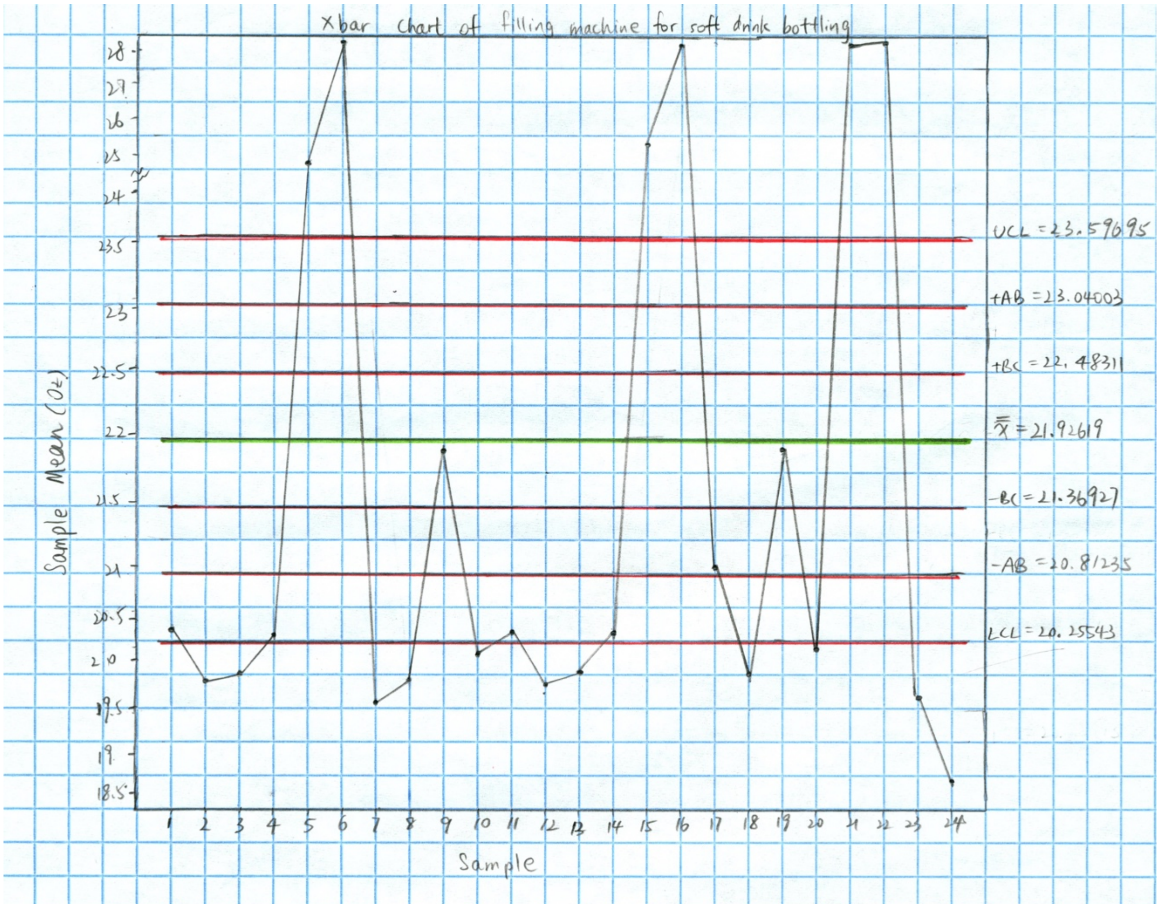
Data picked:

Sample	X1	X2	X3	X4	X5	X6	X7
1	19	17.9	21.2	20.4	20	22.3	21.5
2	18.1	20.8	17.8	19.6	19.8	21.7	20
3	17.1	19.4	18.6	20.9	21.8	21	19.8
4	20.2	22.4	22	19.6	19.6	20	18.5
5	25.1	24.3	26	23.1	25.8	27	26.5
6	28.2	27.5	29.3	30.7	27.6	28	27
7	18.5	19.2	18	20.1	22	20.2	19.5
8	21.4	20.3	22	19.2	18	17.9	19.5
9	20.1	19.8	22.3	22.5	21.8	22.7	23
10	22.3	18.2	21.5	19	19.4	20.5	20
11	19	17.9	21.2	20.4	20	22.3	21.5
12	18.1	20.8	17.8	19.6	19.8	21.7	20
13	19.1	19	22.3	21.5	17.8	19.2	19.4
14	20.2	22.4	22	19.6	19.6	20	18.5
15	25.1	24.3	26	23.1	25.8	27	26.5
16	28.2	27.5	29.3	30.7	27.6	28	27
17	22.1	21.4	23.3	20.5	19.8	20.5	19
18	21.4	20.3	22	19.2	18	17.9	19.5
19	20.1	19.8	22.3	22.5	21.8	22.7	23
20	22.3	18.2	21.5	19	19.4	20.5	20
21	25.8	29.2	28.5	29.1	27.8	29	28
22	28.2	28.6	28.1	26	30	28.5	28.3
23	18.5	19.2	18	20.1	22	20.2	19.5
24	18.4	16.5	18.1	19.2	17.5	20.9	19.6

The data set shows 24 samples (24 hours) of size n=7 collected to test the production process of a filling machine for soft drink bottling. The target filling is 20 fluid ounces.

Data Analysis

1. X Bar Chart
    1. Construct an x-bar chart. Need to use graph paper.



X-bar	LCL	CL	UCL	upperA-B	upperB-C	lowerA-B	lowerB-C
20.32857	20.25543	21.92619	23.59695	23.04003	22.48311	20.81235	21.36927
19.68571	20.25543	21.92619	23.59695	23.04003	22.48311	20.81235	21.36927
19.8	20.25543	21.92619	23.59695	23.04003	22.48311	20.81235	21.36927
20.32857	20.25543	21.92619	23.59695	23.04003	22.48311	20.81235	21.36927
25.4	20.25543	21.92619	23.59695	23.04003	22.48311	20.81235	21.36927
28.32857	20.25543	21.92619	23.59695	23.04003	22.48311	20.81235	21.36927
19.64286	20.25543	21.92619	23.59695	23.04003	22.48311	20.81235	21.36927
19.75714	20.25543	21.92619	23.59695	23.04003	22.48311	20.81235	21.36927
21.74286	20.25543	21.92619	23.59695	23.04003	22.48311	20.81235	21.36927
20.12857	20.25543	21.92619	23.59695	23.04003	22.48311	20.81235	21.36927
20.32857	20.25543	21.92619	23.59695	23.04003	22.48311	20.81235	21.36927
19.68571	20.25543	21.92619	23.59695	23.04003	22.48311	20.81235	21.36927
19.75714	20.25543	21.92619	23.59695	23.04003	22.48311	20.81235	21.36927
20.32857	20.25543	21.92619	23.59695	23.04003	22.48311	20.81235	21.36927
25.4	20.25543	21.92619	23.59695	23.04003	22.48311	20.81235	21.36927
28.32857	20.25543	21.92619	23.59695	23.04003	22.48311	20.81235	21.36927
20.94286	20.25543	21.92619	23.59695	23.04003	22.48311	20.81235	21.36927
19.75714	20.25543	21.92619	23.59695	23.04003	22.48311	20.81235	21.36927
21.74286	20.25543	21.92619	23.59695	23.04003	22.48311	20.81235	21.36927
20.12857	20.25543	21.92619	23.59695	23.04003	22.48311	20.81235	21.36927
28.2	20.25543	21.92619	23.59695	23.04003	22.48311	20.81235	21.36927
28.24286	20.25543	21.92619	23.59695	23.04003	22.48311	20.81235	21.36927

19.64286	20.25543	21.92619	23.59695	23.04003	22.48311	20.81235	21.36927
18.6	20.25543	21.92619	23.59695	23.04003	22.48311	20.81235	21.36927
21.92619							

X-DblBar

2. Calculate and plot the centerline and upper and lower control limits for the charts.
   For n=7, $A_2 = 0.419$
   $\bar{\bar{x}} = \dfrac{\overline{x_1} + \overline{x_2} + \overline{x_3} + \overline{x_4} + \cdots + \overline{x_{24}}}{24} = 21.92619$
   $\bar{R} = \frac{R_1+R_2+\cdots+R_{24}}{24}$ =3.9875
   $upper\ control\ limit = \bar{\bar{x}} + A_2\bar{R} = 21.92619 + 0.419 \times 3.9875 = 23.59695$
   $lower\ control\ limit = \bar{\bar{x}} - A_2\bar{R} = 21.92619 - 0.419 \times 3.9875 = 20.25543$

3. Calculate and plot the A, B and C zone boundaries of the x-bar chart.
   the A–B boundaries are positioned 2 standard deviations from the centerline and the B–C boundaries are 1 standard deviation from the centerline.

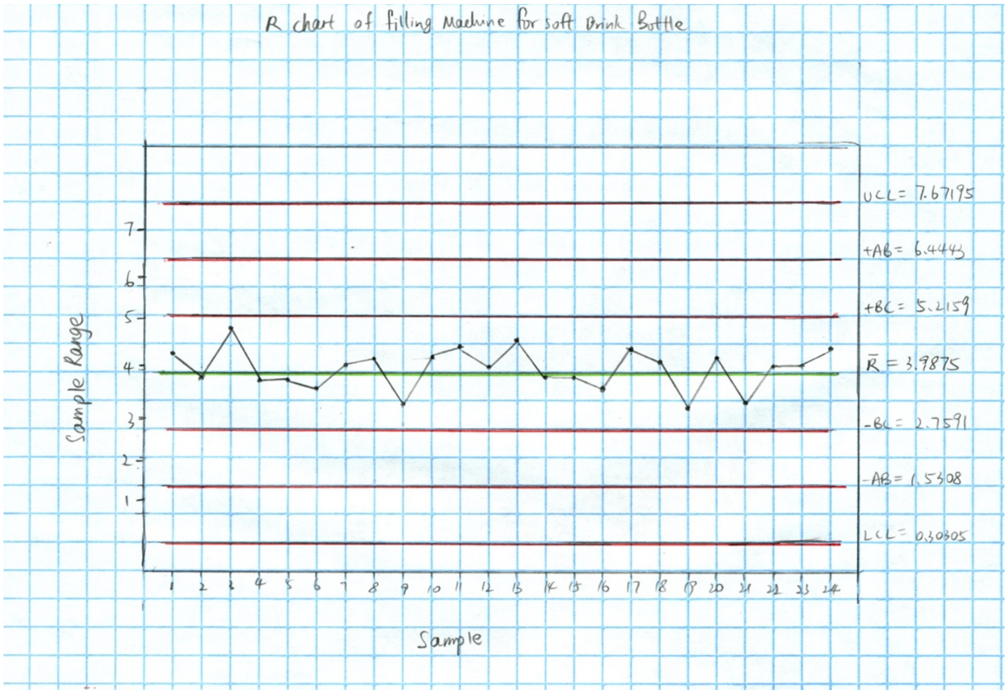
   $upper\ A - B\ boundaries = \bar{\bar{x}} + \dfrac{2}{3}A_2\bar{R} = 21.92619 + \dfrac{2}{3} \times 0.419 \times 3.9875$
   $= 23.04003$
   $lower\ A - B\ boundaries = \bar{\bar{x}} - \dfrac{2}{3}A_2\bar{R} = 21.92619 - \dfrac{2}{3} \times 0.419 \times 3.9875$
   $= 20.81235$
   $upper\ B - C\ boundaries = \bar{\bar{x}} + \dfrac{1}{3}A_2\bar{R} = 21.92619 + \dfrac{1}{3} \times 0.419 \times 3.9875$
   $= 22.48311$
   $lower\ B - C\ boundaries = \bar{\bar{x}} - \dfrac{1}{3}A_2\bar{R} = 21.92619 - \dfrac{1}{3} \times 0.419 \times 3.9875$
   $= 21.36927$

4. Plot the 24 sample means on the *x*-bar chart.
   See above.

5. Based on *x*-bar findings, is the process under statistical control? Explain and justify your answer.

The process is out of control. There are several points located beyond upper and below control limit. With a careful comparison of the six pattern-analysis rules with the sequence of sample means, there is out-of-control signals. The rule 1 (one point beyond zone A), rule 4 (fourteen points in a row alternating up and down), rule 5 (two out of three points in a row in Zone A or beyond) and rule 6 (four out of five point in a row in zone B or beyond) are observed. From the x-bar chart, we find there are six upper out-of-control points and eleven below out-of control points. We can find evidences of a shift in the process mean. Accordingly, we conclude that the process is out of control.

2. R chart
   1. Construct an R-chart. Need to use graph paper.



R Chart

R	LCL	CL	UCL	upperA-B	upperB-C	lowerA-B	lowerB-C
4.4	0.30305	3.9875	7.67195	6.4443	5.2159	1.5307	2.7591
3.9	0.30305	3.9875	7.67195	6.4443	5.2159	1.5307	2.7591
4.7	0.30305	3.9875	7.67195	6.4443	5.2159	1.5307	2.7591
3.9	0.30305	3.9875	7.67195	6.4443	5.2159	1.5307	2.7591
3.9	0.30305	3.9875	7.67195	6.4443	5.2159	1.5307	2.7591
3.7	0.30305	3.9875	7.67195	6.4443	5.2159	1.5307	2.7591
4	0.30305	3.9875	7.67195	6.4443	5.2159	1.5307	2.7591
4.1	0.30305	3.9875	7.67195	6.4443	5.2159	1.5307	2.7591
3.2	0.30305	3.9875	7.67195	6.4443	5.2159	1.5307	2.7591
4.1	0.30305	3.9875	7.67195	6.4443	5.2159	1.5307	2.7591
4.4	0.30305	3.9875	7.67195	6.4443	5.2159	1.5307	2.7591
3.9	0.30305	3.9875	7.67195	6.4443	5.2159	1.5307	2.7591
4.5	0.30305	3.9875	7.67195	6.4443	5.2159	1.5307	2.7591
3.9	0.30305	3.9875	7.67195	6.4443	5.2159	1.5307	2.7591
3.9	0.30305	3.9875	7.67195	6.4443	5.2159	1.5307	2.7591
3.7	0.30305	3.9875	7.67195	6.4443	5.2159	1.5307	2.7591
4.3	0.30305	3.9875	7.67195	6.4443	5.2159	1.5307	2.7591
4.1	0.30305	3.9875	7.67195	6.4443	5.2159	1.5307	2.7591
3.2	0.30305	3.9875	7.67195	6.4443	5.2159	1.5307	2.7591
4.1	0.30305	3.9875	7.67195	6.4443	5.2159	1.5307	2.7591
3.4	0.30305	3.9875	7.67195	6.4443	5.2159	1.5307	2.7591
4	0.30305	3.9875	7.67195	6.4443	5.2159	1.5307	2.7591
4	0.30305	3.9875	7.67195	6.4443	5.2159	1.5307	2.7591
4.4	0.30305	3.9875	7.67195	6.4443	5.2159	1.5307	2.7591

3.9875

R-Bar

2. Calculate and plot the centerline and upper and lower control limits for the charts.

To determine the control limits, we need the constants D_3 and D_4. For n=7, D_3 =0.076, D_4=1.924.
$\bar{R} = \frac{R_1+R_2+\cdots+R_{24}}{24}$ =3.9875
$upper\ control\ limit = \bar{R} \times D_4 = 3.9875 \times 1.924 = 7.67195$
$lower\ control\ limit = \bar{R} \times D_3 = 3.9875 \times 0.076 = 0.30305$

3. Calculate and plot the A, B and C zone boundaries of the R-chart.
   For n=7, $d_3 = 0.833, d_2 = 2.704$

the A–B boundaries are positioned 2 standard deviations from the centerline and the B–C boundaries are 1 standard deviation from the centerline.

$$upper\ A - B\ boundary: \bar{R} + 2d_3\left(\frac{\bar{R}}{d_2}\right) = 3.9875 + 2 \times 0.833\left(\frac{3.9875}{2.704}\right) = 6.4443$$
$$lower\ A - B\ boundary: \bar{R} - 2d_3\left(\frac{\bar{R}}{d_2}\right) = 3.9875 - 2 \times 0.833\left(\frac{3.9875}{2.704}\right) = 1.5307$$
$$upper\ B - C\ boundary: \bar{R} + d_3\left(\frac{\bar{R}}{d_2}\right) = 3.9875 + 0.833\left(\frac{3.9875}{2.704}\right) = 5.2159$$
$$lower\ B - C\ boundary: \bar{R} - d_3\left(\frac{\bar{R}}{d_2}\right) = 3.9875 - 0.833\left(\frac{3.9875}{2.704}\right) = 2.7591$$

4. Plot the sample ranges on the R-chart.
   See above

5. Analyze the pattern of the R-chart.
   All the plotted R value fall below the control limit. The points vary randomly around the center line and are within the control limits for both charts. No trends or patterns are present. Apparently, no significant special causes of variation are influencing the variation of the process. By using the pattern-analysis Rule 1-4, none of the rules signal the presence of special causes of variation. Accordingly, we conclude that it is reasonable to treat the process—in particular, the variation of the process—as being under control during the period in question.

6. Based on R-chart findings, is the process under statistical control? Explain and justify your answer.
   The process is under statistical control. Because the variation of the process appears t be in control during the period when the sample data were collected, the control limits appropriately characterize the variation in R that would be expected when the process is in a state of statistical control.

Theory and Concept

1. Theoretically, what is the difference between x-bar chart and R-chart?
   The x-bar chart depends on R chart, because the assumption to make x-bar chart meaningful is the variation of the process is stable. If the process variation is out of

control, the control limits of the x-bar chart have little meaning. Because when the process variation is changing, any single estimate of the variation is not representative of the process. The appropriate procedure is to first construct and then interpret the R-chart, if it indicates that the process variation is in control, then it makes sense to construct and interpret the x-bar chart.

2. How are they used? That is, what is their purpose?

   X-bar chart is used to detect changes in the process mean/ indicate when changes occur in the control tendency of the process. R chart is used to detect changes in process variation. X-Bar and R charts draw a control chart for subgroup means and a control chart for subgroup ranges in one graphic. Interpreting both charts together allows you to track both process center and process variation and detect the presence of special causes. Generally, a user focuses on the range portion of the chart first, confirming that the process is in control. Finally, the user focuses on the average chart, looking for special cause there.

Conclusion

1. Compare the findings from your x-bar chart and R-chart.

   The control limits for both charts were constructed from k=24 samples of size n=7. The x-bar chart is found to be out of control while the R chart is to be found under control. Because only when R chart is stable, the output of x-bar chart is meaningful.

2. Explain why rational sub-groupings are used in constructing control charts?

   Because we want to choose the subgroups in a way that will give the maximum chance for the measurements in each subgroup to be alike and the maximum chance for the subgroups to be different.

3. What are the major findings from the study?

   Because there is no out of control points on R chart, there is no special causes to be eliminated. There is no value repeated too often. Because the R chart has no out of control point, the variation of the process is stable, so the x-bar chart output is meaningful. scfaThere are a lot of out of control point on x-bar chart. We cannot analyze the capability relative to requirements because the process does not show control relative to the statistical limits and Run test fro a sufficient period of time. Because we cannot predict the outcome of an unstable process.

4. What are your conclusions from the study?

   First, we interpret R chart and conclude that it is stable, so the assumption of x-bar chart is stable. From the analysis, we can conclude that the process is not stable over time. Because the process was found to not be in control during the period in which the samples were drawn, the trial control limit constructed in chart cannot be considered official. So they cannot be extended to eth right or used to monitor future process output.