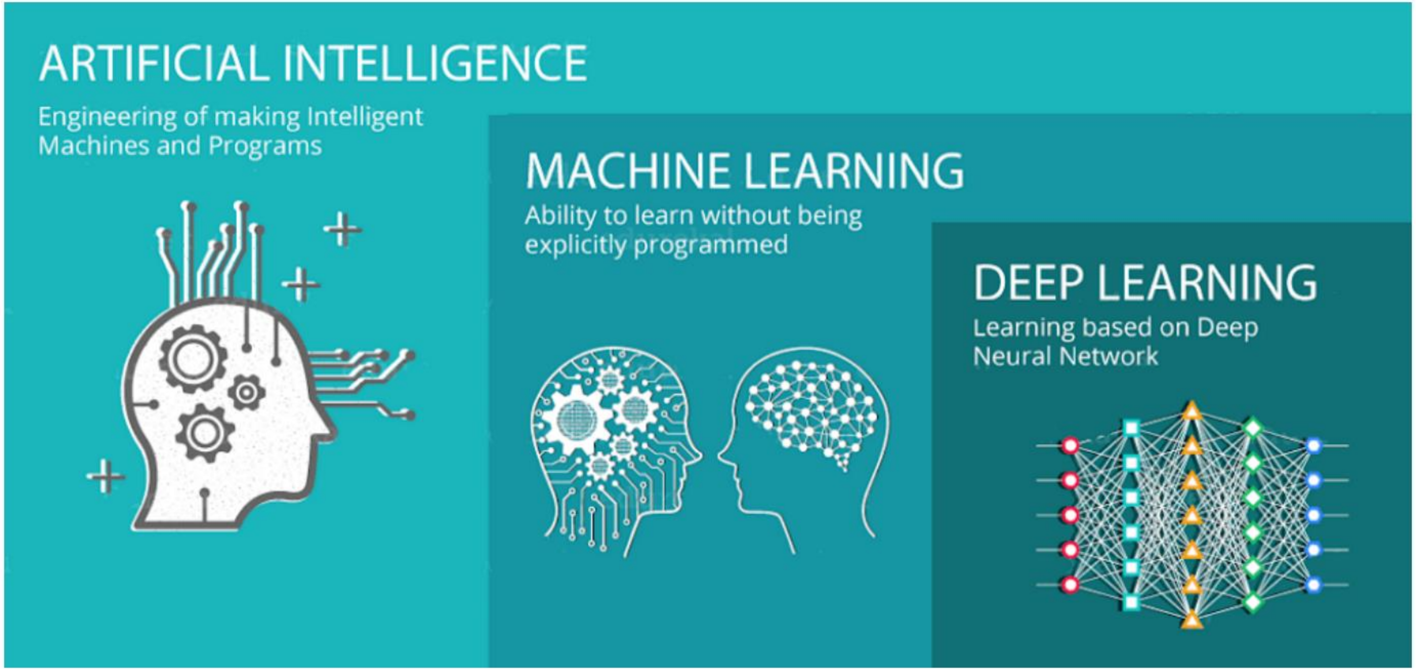
Artificial Neural Network

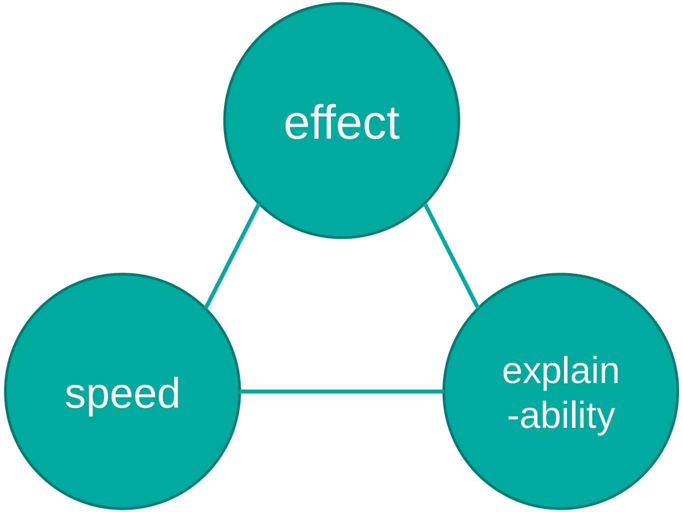
FMMS050 ML PhD course

Lecture 10

Position of Artificial Neural Network



Targets of Artificial Neural Network



Artificial Neural Network (DL)

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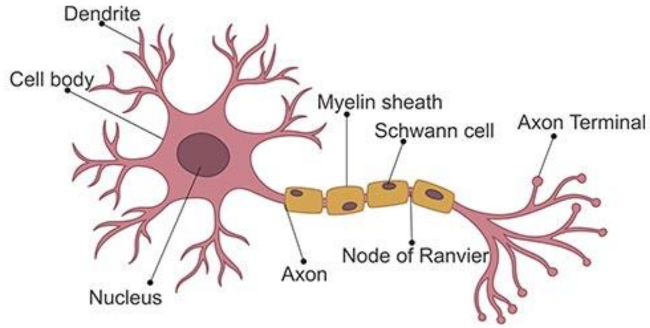
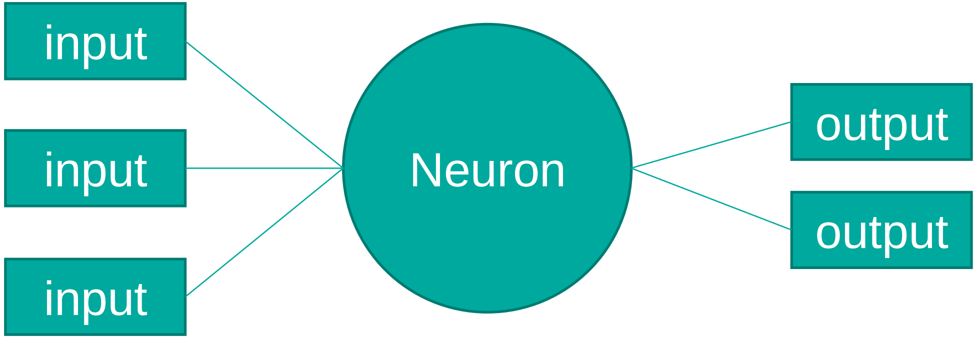
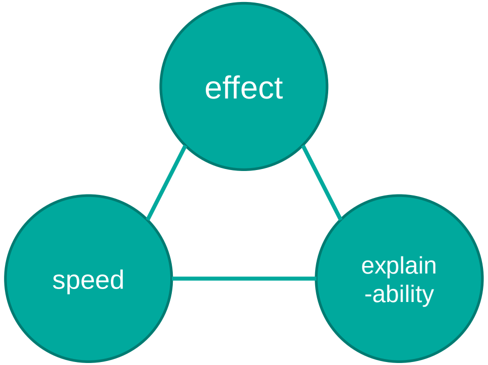
Artificial Neural Network (DL)

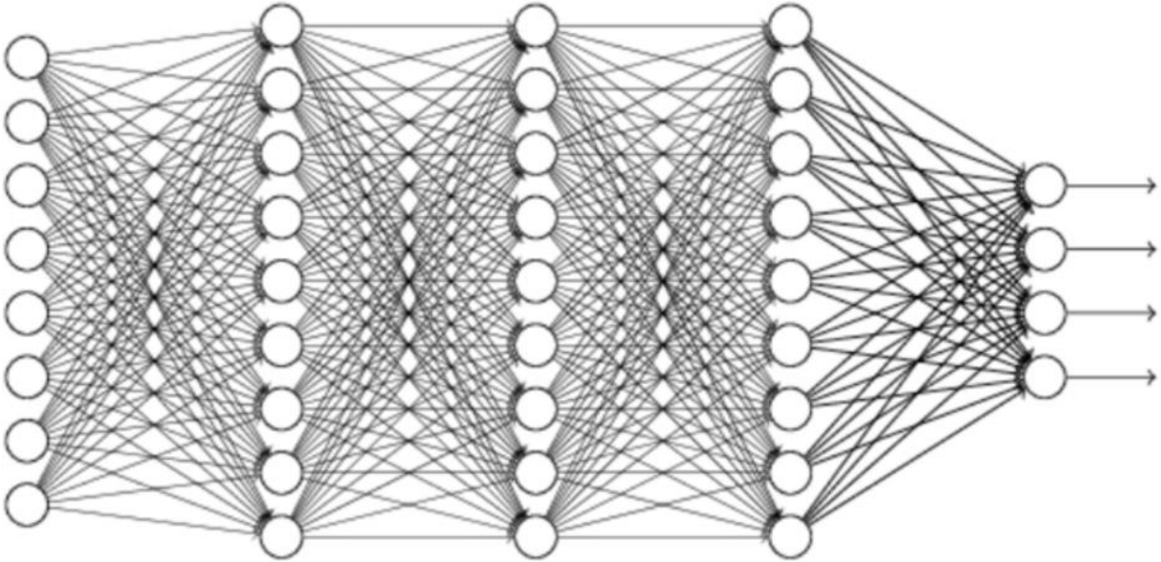
Artificial Neuron





Artificial Neural Network





Artificial Neural Network

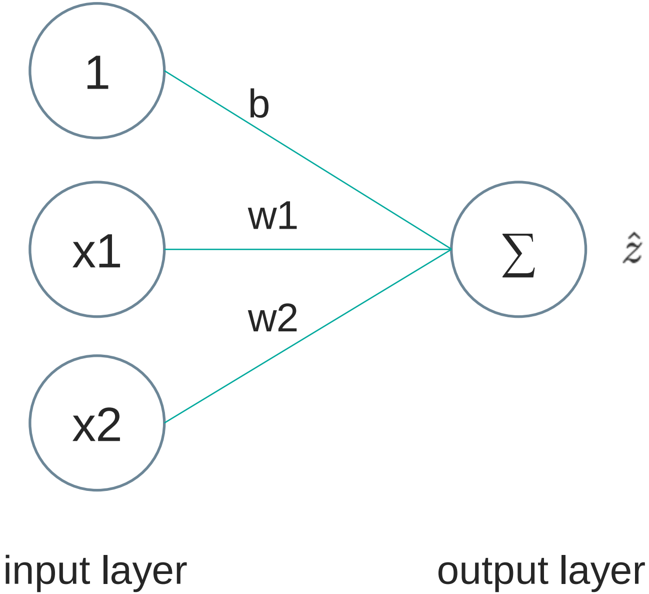
- Boltzmann machine (RBM)
- Multilayer Perceptron Classifier (MLPClassifier)
- Multilayer Perceptron Regressor (MLPRegressor)

Single layer neural network (regression)

$$\hat{z} = \underbrace{b}_{\text{(intercept,bias)}} + x_1 w_1 + x_2 w_2$$

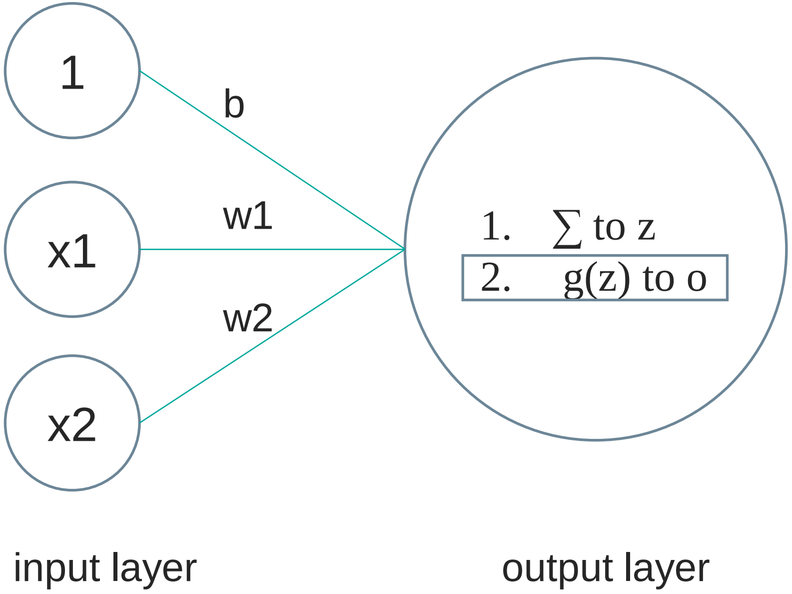
(intercept,bias) (weights)

Single layer neural network



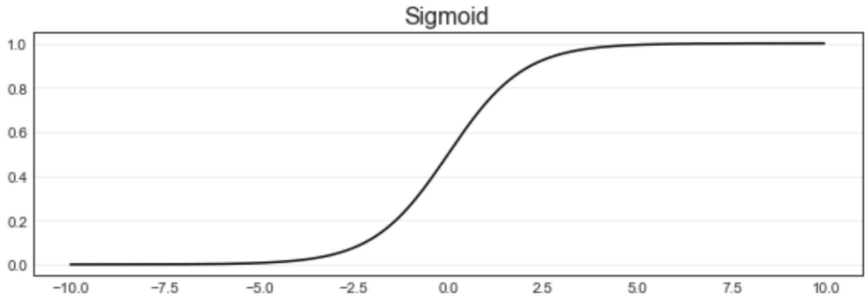
$$\hat{z} = b + x_1 w_1 + x_2 w_2$$

Single layer neural network (binary classification)

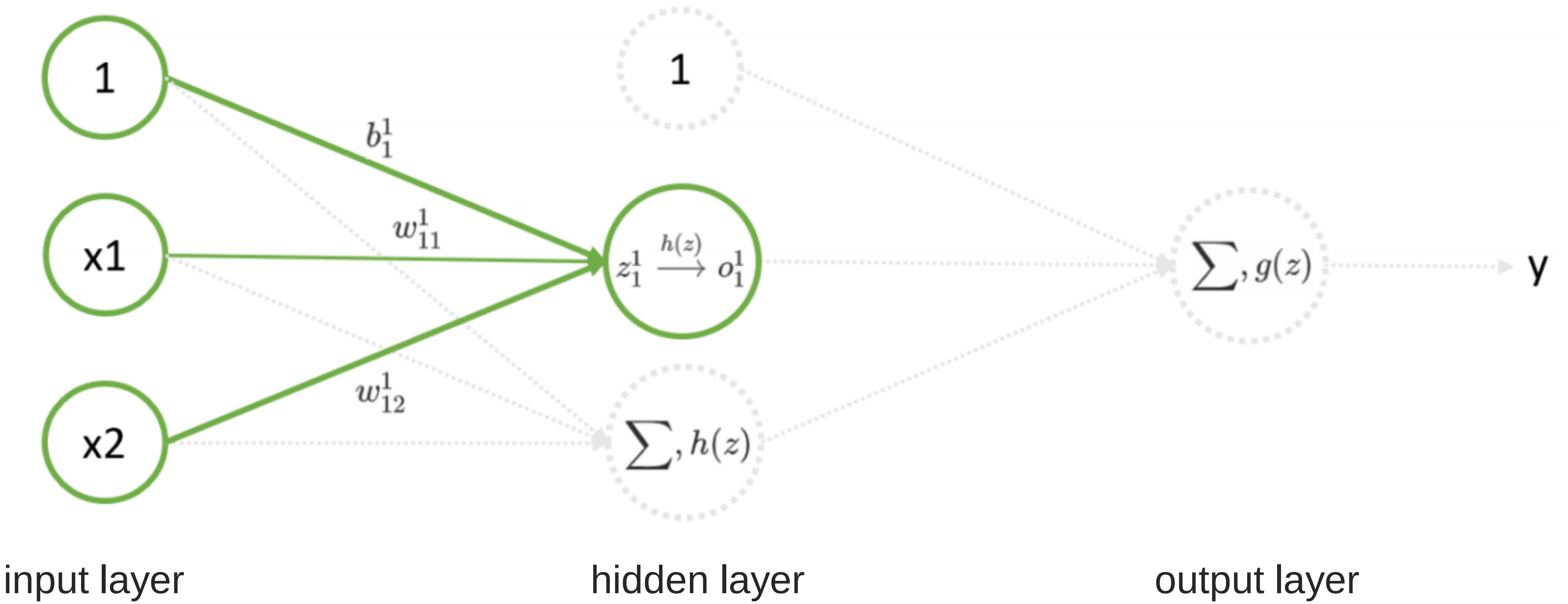


1. Summation: $\hat{z} = b + x_1 w_1 + x_2 w_2$

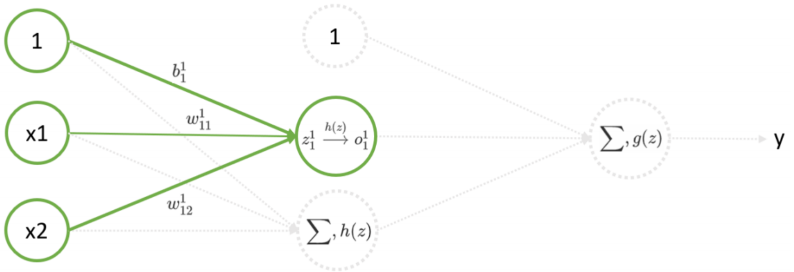
2. Activation: $g(z) = \dfrac{1}{1 + e^{-z}}$



Multi-layer neural network



Multi-layer neural network



$$z_1^1 = 1 * b_1^1 + x_1 w_{11}^1 + x_2 w_{12}^1$$

$$o_1^1 = h(b_1^1 + x_1 w_{11}^1 + x_2 w_{12}^1)$$

layer #

z_1^1 o_1^1

neuron #

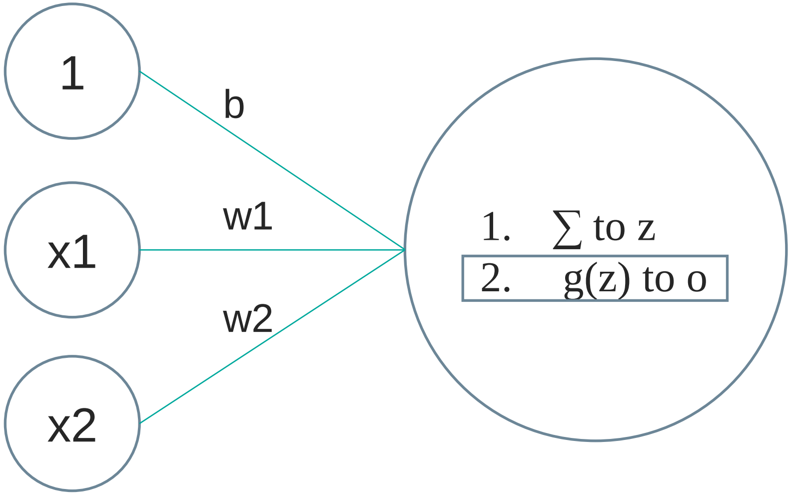
next layer #

w_{12}^1 b_1^1

next layer neuron #

pre layer neuron #

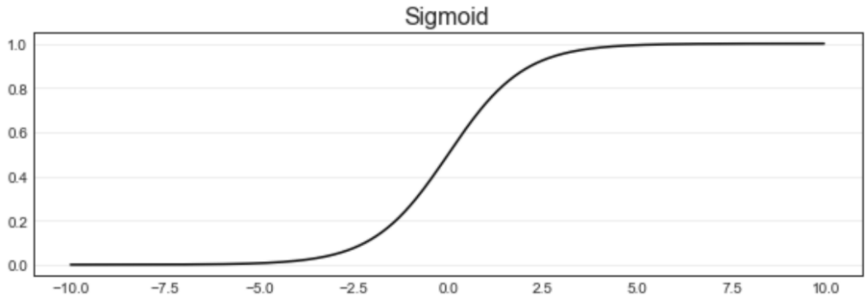
Single layer neural network (binary classification)



input layer

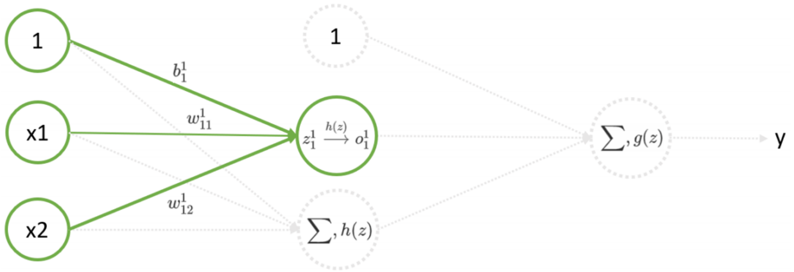
output layer

1. Summation: $\hat{z} = b + x_1 w_1 + x_2 w_2$
2. Activation: $g(z) = \frac{1}{1 + e^{-z}}$



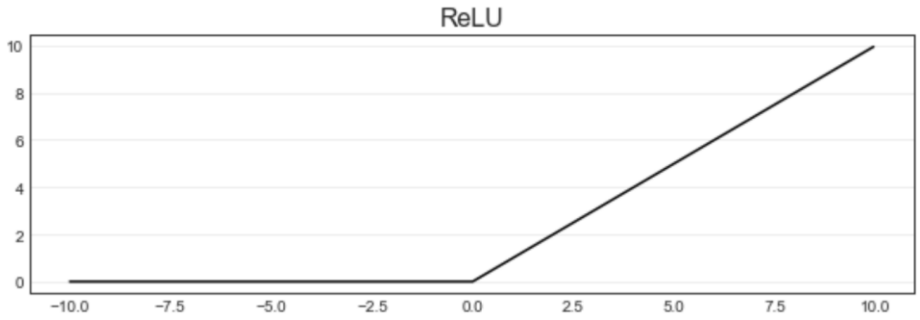
Linear and non-linear

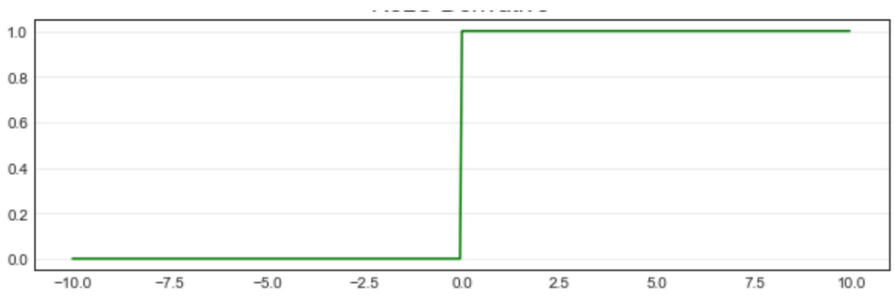
- without h(z), what a layer does is alike affine transformation, (linear in, linear out)
- suitble non-linear h(z) can bring non-linear transformation
- g(z) only relates to the output and is independent with the network
- when we talk activation function, it means h(z)
- g(z) is called out activation function



Activation function (ReLU)

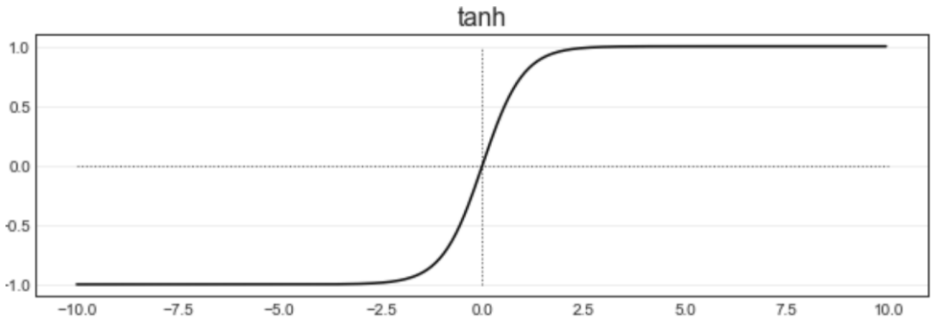
- ReLU (Rectified Linear Unit) $ReLU: h(z) = \begin{cases} z & (z > 0) \\ 0 & (z \leq 0) \end{cases}$

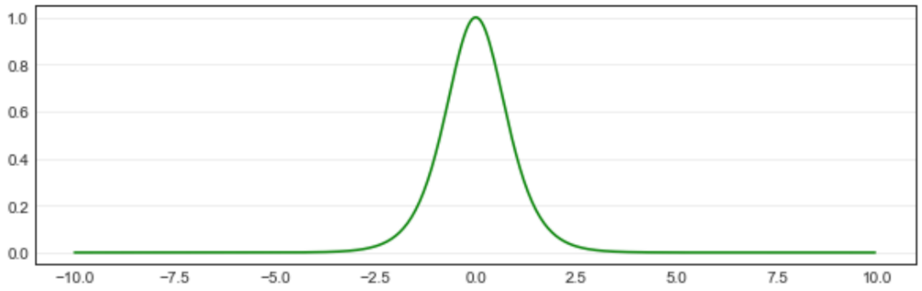




Activation function (tanh)

- tanh (hyperbolic tangent) $$tanh : h(z) = \frac{e^{2z} - 1}{e^{2z} + 1}$$





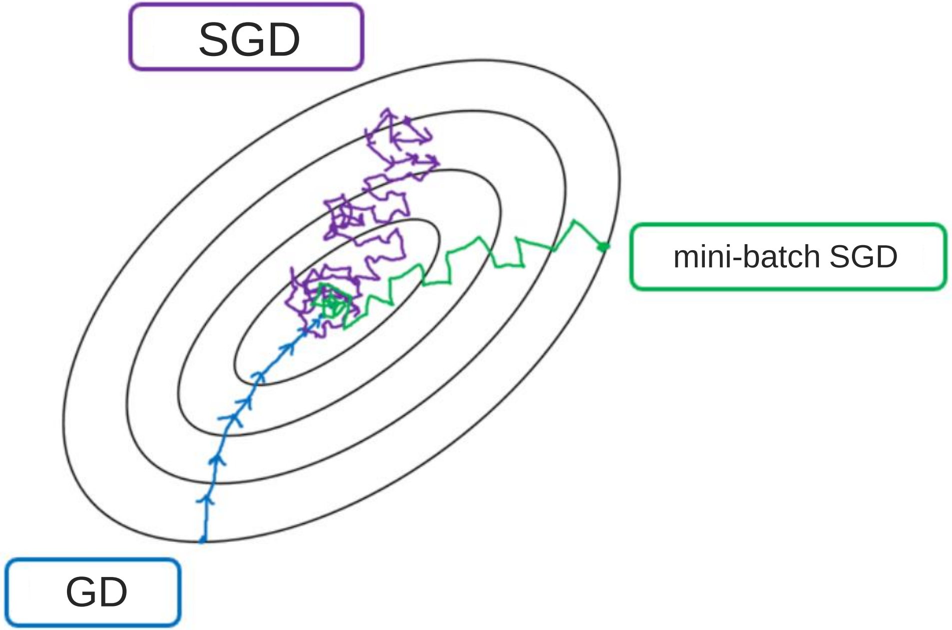
Example #1

- try if one hidden layer works
- 3 to 5 hidden layers
- number of neurons on input > number of neurons on hidden layer > number of neurons on output
- number of neurons on hidden layer = 2/3 * number of neurons on input + number of neurons on output
- number of neurons on hidden layer < 2*number of neurons on input

Learning of ANN parameters

how to determine parameters of “b” and “w” ?

- “lbfgs”,
- “mini-batch S(stochastic)G(gradient)D(descent)” and
- “adam”.



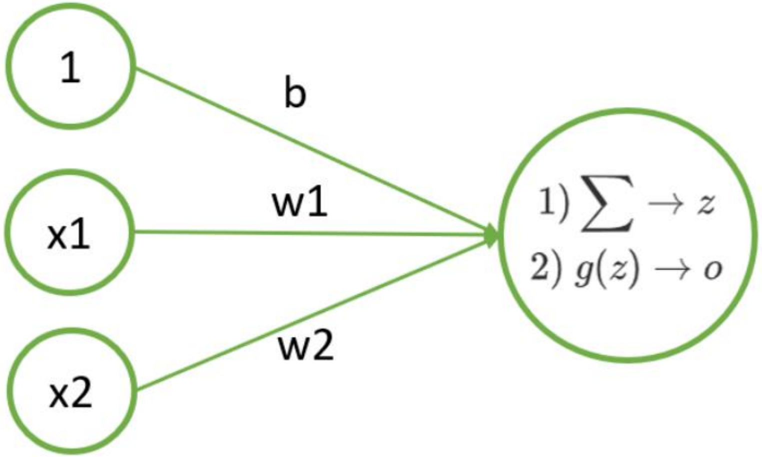
Workflow of learning parameters

Loss functions

- Regression: MSE
- Classification: Cross-Entropy (log loss)

$$L_x(\mathbf{w}, b) = \sum_{i=1}^{N} \frac{1}{2} ||\hat{y}_i - y_i||_2^2 + \frac{\alpha}{2} ||w||_2^2$$

$$\frac{\partial L_x(w_1, w_2, b)}{\partial w}, \frac{\partial L_x(w_1, w_2, b)}{\partial b}$$



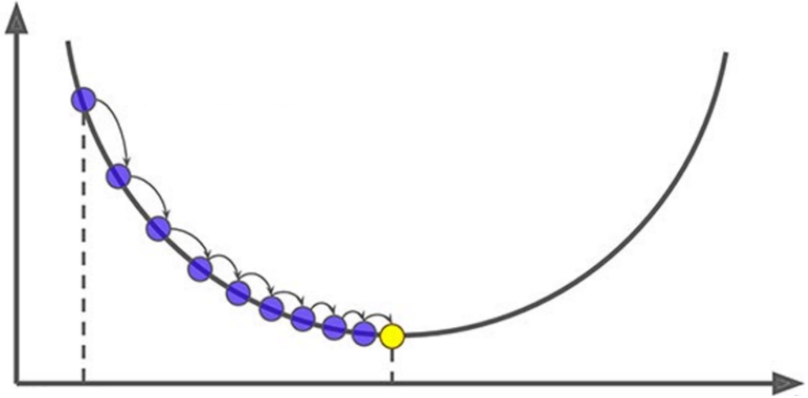
Workflow of learning parameters

$$w_1^{(1)} = w_1^{(0)} - \frac{\boxed{\eta^{(0)}}}{\boxed{N_{\mathrm{B}}}} * \sum_{\boxed{i\in \mathrm{B}_0}} \frac{\partial L_{\boxed{x_i}}(w_1, w_2, b)}{\partial w_1^{(0)}}$$

$$w_2^{(1)} = w_2^{(0)} - \frac{\eta^{(0)}}{N_{\mathrm{B}}} * \sum_{i\in \mathrm{B}_0} \frac{\partial L_{x_i}(w_1, w_2, b)}{\partial w_2^{(0)}}$$

$$b^{(1)} = b^{(0)} - \frac{\eta^{(0)}}{N_{\mathrm{B}}} * \sum_{i\in \mathrm{B}_0} \frac{\partial L_{x_i}(w_1, w_2, b)}{\partial b^{(0)}}$$

$$\eta^{(1)} = \frac{\eta^{(0)}}{1^{power\_t}}$$



Workflow of learning parameters

$$w_1^{(2)} = w_1^{(1)} - \frac{\eta^{(1)}}{N_{\mathrm{B}}} * \sum_{i \in \mathrm{B}_1} \frac{\partial L_{x_i}(w_1, w_2, b)}{\partial w_1^{(1)}}$$

$$w_2^{(2)} = w_2^{(1)} - \frac{\eta^{(1)}}{N_{\mathrm{B}}} * \sum_{i \in \mathrm{B}_1} \frac{\partial L_{x_i}(w_1, w_2, b)}{\partial w_2^{(1)}}$$

$$b^{(2)} = b^{(1)} - \frac{\eta^{(1)}}{N_{\mathrm{B}}} * \sum_{i \in \mathrm{B}_1} \frac{\partial L_{x_i}(w_1, w_2, b)}{\partial b^{(1)}}$$

$$\eta^{(2)} = \frac{\eta^{(0)}}{2^{power\_t}}$$

$$w_1^{(t)} = w_1^{(t-1)} - \frac{\eta^{(t-1)}}{N_{\mathrm{B}}} * \sum_{i \in \mathrm{B}_{t-1}} \frac{\partial L_{x_i}(w_1, w_2, b)}{\partial w_1^{(t-1)}}$$

$$w_2^{(t)} = w_2^{(t-1)} - \frac{\eta^{(t-1)}}{N_{\mathrm{B}}} * \sum_{i \in \mathrm{B}_{t-1}} \frac{\partial L_{x_i}(w_1, w_2, b)}{\partial w_2^{(t-1)}}$$

$$b^{(t)} = b^{(t-1)} - \frac{\eta^{(t-1)}}{N_{\mathrm{B}}} * \sum_{i \in \mathrm{B}_{t-1}} \frac{\partial L_{x_i}(w_1, w_2, b)}{\partial b^{(t-1)}}$$

$$\eta^{(t)} = \frac{\eta^{(0)}}{t^{power\_t}}$$

Workflow of learning parameters

$$w_1^{(2)} = w_1^{(1)} - \frac{\eta^{(1)}}{N_{\mathrm{B}}} * \sum_{i \in \mathrm{B}_1} \frac{\partial L_{x_i}(w_1, w_2, b)}{\partial w_1^{(1)}}$$

$$w_2^{(2)} = w_2^{(1)} - \frac{\eta^{(1)}}{N_{\mathrm{B}}} * \sum_{i \in \mathrm{B}_1} \frac{\partial L_{x_i}(w_1, w_2, b)}{\partial w_2^{(1)}}$$

$$b^{(2)} = b^{(1)} - \frac{\eta^{(1)}}{N_{\mathrm{B}}} * \sum_{i \in \mathrm{B}_1} \frac{\partial L_{x_i}(w_1, w_2, b)}{\partial b^{(1)}}$$

$$\eta^{(2)} = \frac{\eta^{(0)}}{2^{power\_t}}$$

$$w_1^{(t)} = w_1^{(t-1)} - \frac{\eta^{(t-1)}}{N_{\mathrm{B}}} * \sum_{i \in \mathrm{B}_{t-1}} \frac{\partial L_{x_i}(w_1, w_2, b)}{\partial w_1^{(t-1)}}$$

$$w_2^{(t)} = w_2^{(t-1)} - \frac{\eta^{(t-1)}}{N_{\mathrm{B}}} * \sum_{i \in \mathrm{B}_{t-1}} \frac{\partial L_{x_i}(w_1, w_2, b)}{\partial w_2^{(t-1)}}$$

$$b^{(t)} = b^{(t-1)} - \frac{\eta^{(t-1)}}{N_{\mathrm{B}}} * \sum_{i \in \mathrm{B}_{t-1}} \frac{\partial L_{x_i}(w_1, w_2, b)}{\partial b^{(t-1)}}$$

$$\eta^{(t)} = \frac{\eta^{(0)}}{t^{power\_t}}$$

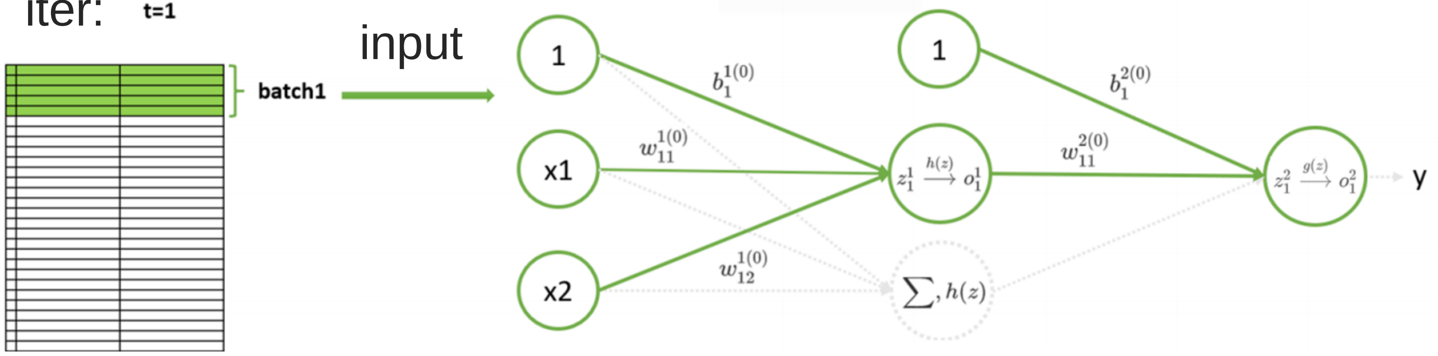
Example #2

Forward broadcast

$$\mathbf{b}^{(0)} = [\, b_1^{1(0)}, b_2^{1(0)}, b_1^{2(0)} \,]$$

$$\mathbf{w}^{(0)} = [\, w_{11}^{1(0)}, w_{12}^{1(0)}, w_{21}^{1(0)}, w_{22}^{1(0)}, w_{11}^{2(0)}, w_{12}^{2(0)} \,]$$

iter: t=1



Forward broadcast

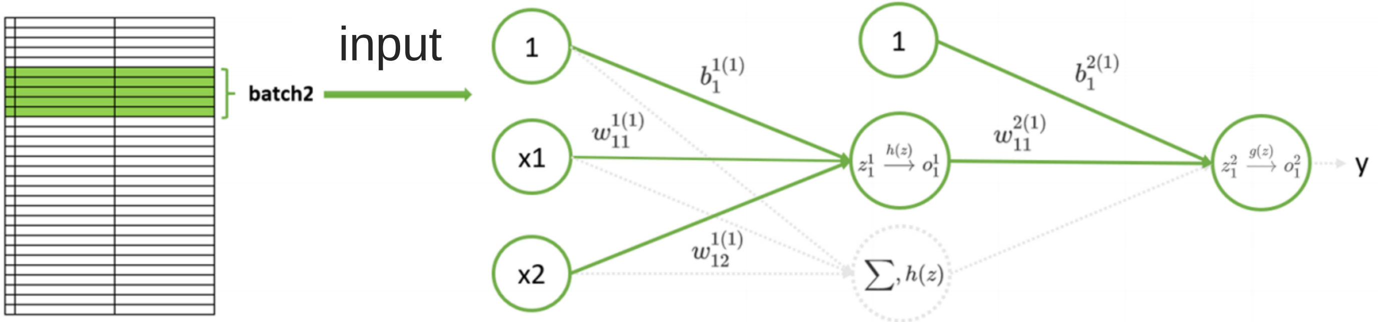
new batch

after first iter

$$\mathbf{b}^{(1)} = [\, b_1^{1(1)}, b_2^{1(1)}, b_1^{2(1)} \,]$$

$$\mathbf{w}^{(1)} = [\, w_{11}^{1(1)}, w_{12}^{1(1)}, w_{21}^{1(1)}, w_{22}^{1(1)}, w_{11}^{2(1)}, w_{12}^{2(1)} \,]$$

iter: t=2



Forward broadcast

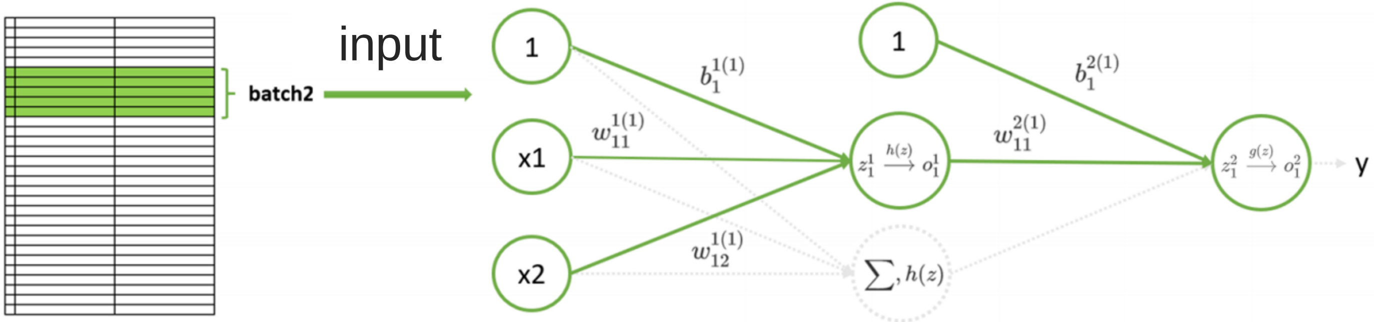
after first iter

new batch

$$\mathbf{b}^{(1)} = [\, b_1^{1(1)}, b_2^{1(1)}, b_1^{2(1)} \,]$$

$$\mathbf{w}^{(1)} = [\, w_{11}^{1(1)}, w_{12}^{1(1)}, w_{21}^{1(1)}, w_{22}^{1(1)}, w_{11}^{2(1)}, w_{12}^{2(1)} \,]$$

iter: **t=2**



Forward broadcast

Forward broadcast

- from input to output layers
- prediction
- generate and save intermediate variables

after sec iter

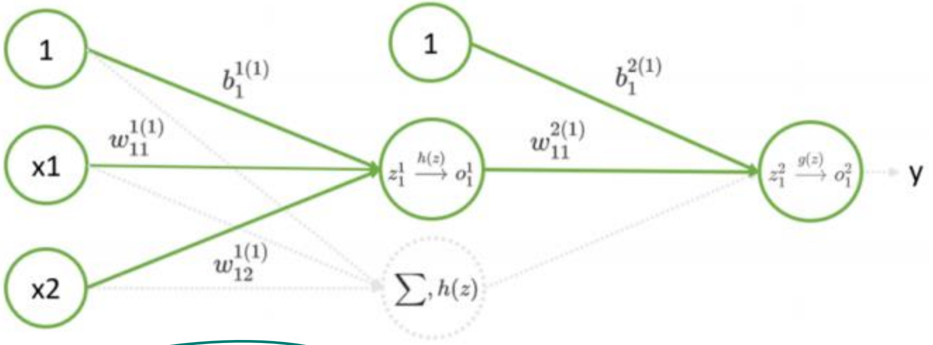
......

after t-th iter

iter: t

$$\mathbf{b^{(t)}} = [\, b_1^{1(t)}, b_2^{1(t)}, b_1^{2(t)} \,]$$

$$\mathbf{w^{(t)}} = [\, w_{11}^{1(t)}, w_{12}^{1(t)}, w_{21}^{1(t)}, w_{22}^{1(t)}, w_{11}^{2(t)}, w_{12}^{2(t)} \,]$$



$$o_1^2 = g(z_1^2)$$

$$\begin{aligned} z_1^2 &= f_{11}^2(o_1^1, o_2^1) \\ &= b_2^{1(0)} + o_1^1 w_{11}^{2(0)} + o_2^1 w_{12}^{2(0)} \end{aligned}$$

$$o_1^1 = h(z_1^1), \;\; o_2^1 = h(z_2^1)$$

$$\begin{aligned} z_1^1 &= f_{11}^1(x_1, x_2) \\ &= b_1^{1(0)} + x_1 w_{11}^{1(0)} + x_2 w_{12}^{1(0)} \\ z_2^1 &= f_{21}^1(x_1, x_2) \\ &= b_2^{1(0)} + x_1 w_{21}^{1(0)} + x_2 w_{22}^{1(0)} \end{aligned}$$

$$\begin{aligned} o_1^2 &= g(z_1^2) \\ &= g(f_{11}^2(o_1^1, o_2^1)) \\ &= g(f_{11}^2(h(z_1^1), h(z_2^1))) \\ &= g\left\{ f_{11}^2 \left[h\left\langle f_{11}^1(x_1, x_2)\right\rangle, h\left\langle f_{21}^1(x_1, x_2)\right\rangle \right] \right\} \end{aligned}$$

$$-y_i ln(o_1^2) = -y_i ln(g\left\{ f_{11}^2 \left[h\left\langle f_{11}^1(x_1, x_2)\right\rangle, h\left\langle f_{21}^1(x_1, x_2)\right\rangle \right] \right\})$$

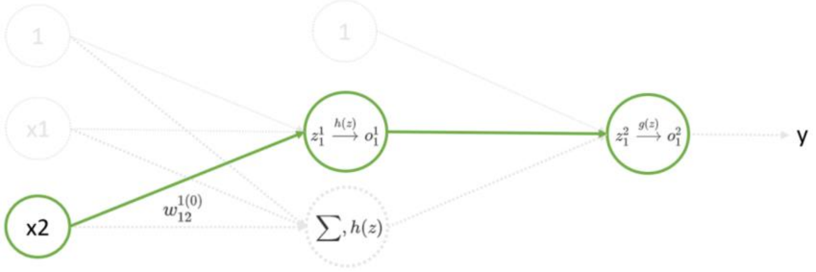
$$L_{x_i}(\mathbf{w}, \mathbf{b}) = -y_i ln(o_1^2) - (1 - y_i) ln(1 - o_1^2)) + \frac{\alpha ||\mathbf{w}||_2^2}{N}$$

Back-propagation

$$L_x(\mathbf{w},\mathbf{b}) = l_x(\mathbf{w},\mathbf{b}) + s(\mathbf{w})$$

$$\begin{aligned}
\frac{\partial l}{\partial w} &= \frac{\partial l(o)}{\partial o_1^2} * \frac{\partial o_1^2}{\partial w} \\
&= \frac{\partial l(o)}{\partial o_1^2} * \frac{\partial(-y_i ln(g\left\{\, f_{11}^2\left[\, h\left\langle f_{11}^1(x_1, x_2)\right\rangle, h\left\langle f_{21}^1(x_1, x_2)\right\rangle \,\right]\right\})}{\partial w} \\
&= \frac{\partial l(o)}{\underline{\partial o_1^2}} * \frac{\partial o_1^2}{\underline{\partial z_1^2}} * \frac{\partial z_1^2}{\partial w} \\
&= \frac{\partial l(o)}{\partial g(z)} * \frac{\partial g(z)}{\partial f_{11}^2(w)} * \frac{\partial f_{11}^2(w)}{\partial w}, \qquad w = w_{11}^{2(0)}
\end{aligned}$$

Back-propagation



$$\frac{\partial l}{\partial w} = \frac{\partial l(o)}{\partial o_1^2} * \frac{\partial o_1^2}{\partial w}$$

$$= \frac{\partial l(o)}{\partial o_1^2} * \frac{\partial(-y_i ln(g\{\, f_{11}^2[\, h\langle f_{11}^1(x_1, x_2)\rangle , h\langle f_{21}^1(x_1, x_2)\rangle \,]\,\})}{\partial w}$$

$$= \frac{\partial l(o)}{\partial o_1^2} * \frac{\partial o_1^2}{\partial z_1^2} * \frac{\partial z_1^2}{\partial o_1^1} * \frac{\partial o_1^1}{\partial z_1^1} * \frac{\partial z_1^1}{\partial w}$$

$$= \boxed{\frac{\partial l(o)}{\partial g(z)} * \frac{\partial g(z)}{\partial f_{11}^2(h)}} * \frac{\partial f_{11}^2(h)}{\partial h(f_{11}^1)} * \frac{\partial h(f_{11}^1)}{\partial f_{11}^1(w)} * \frac{\partial f_{11}^1(w)}{\partial w}, \quad w = w_{12}^{1(0)}$$

$$w_{21}^{1(1)} = w_{21}^{1(0)} - \frac{\eta^{(0)}}{N_{\mathrm{B}}} * \sum_{i \in \mathrm{B}_0} \boxed{\frac{\partial L_{x_i}(\mathbf{w}, \mathbf{b})}{\partial w_1^{(0)}}}$$

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