3. The FRW equations

$$\begin{aligned} \frac{\dot{a}^2}{a^2} &= \frac{8\pi G}{3}\rho \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho + 3p) \end{aligned}$$

control the evolution of homogeneous cosmologies with scale factor $a(t)$ and flat spatial sections. a) Assuming $p = w\rho$, where w is a constant, determine the dependence of the energy density ρ on the scale factor for radiation dominated ($w = 1/3$) , matter dominated ($w = 0$), and dark energy dominated ($w = -1$) cosmologies. b) Find the time-dependence of the scale factor for each of the three equations of state.