

[3] (25 points) Two objects of mass  $m_1$  and  $m_2$  and negligible size interact through a model potential  $V(r) = \frac{a^2}{4r^4} - \frac{b^2}{3r^3}$  where  $a$  and  $b$  are constants with the appropriate units.  $r = |\vec{r}|$  is the distance between the two objects and  $\vec{r}$  is along the line connecting the centers of each object. In a particular reference frame, the Lagrangian for this isolated system can be written  $L = \frac{1}{2}\mu|\dot{\vec{r}}|^2 - V(r)$ .

- Specify the reference frame in which  $L$  has this form.
- What is the equation determining  $\mu$ ?
- Briefly explain why the angular momentum,  $l$ , must be conserved in this system.

Because angular momentum is conserved, we can assume the motion is in a plane and use polar coordinates, so that  $L = \frac{1}{2}\mu\dot{r}^2 + \frac{l^2}{2\mu r^2} - V(r)$

- What is the maximum angular momentum at which the masses can move in stable circular orbits around each other?
- What is the relative separation  $r$  in a circular orbit with angular momentum infinitesimally below the breakup value?
- Find the equilibrium separation of the objects in the case where they don't rotate around each other, and compare to the result in (e).