

3. (25 points) A cylinder of mass  $m_2$ , radius  $r$ , and moment of inertia  $I_2$  (about its longitudinal axis) rolls down a wedge shaped block of mass  $m_1$  (as shown in Fig. 3). The block ( $m_1$ ) is free to slide horizontally without friction and the left hand side of the block has coordinate  $q_1 = x_1$ . The cylinder rolls without slipping, and its distance from the top of the wedge is  $q_2$ . Both the cylinder and the block are initially at rest and are influenced by gravity  $g$  in the vertical direction. You may assume that the block and the cylinder are always in contact. For each of the following, express your answer in terms of generalized coordinates  $q_1$  and  $q_2$  and their time derivatives.

a) What is the potential energy  $U$  of this system? Define the position of the cylinder when  $U = 0$ . You may ignore terms that do not vary with time.

b) What is the kinetic energy  $T$  of this system?

c) What is the Lagrangian  $L$  of this system?

d) What are Lagrange's equations of motion for this system in simplified form (i.e. leave your answer as one or more differential equations)?

