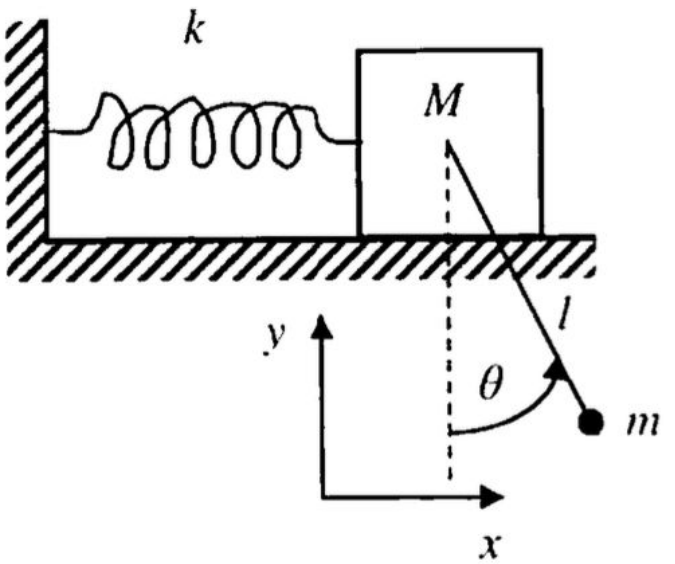
Problem 3: Analytical Mechanics

A simple pendulum consisting of a mass m and weightless string of length l is mounted on a support of mass M which sits on a frictionless surface and is attached to a horizontal spring with force constant k as show in the figure below. (The acceleration due to gravity g points down.)



a) Set up Lagrange's equations. Please use coordinates where the origin is placed at the location of m when the system is in equilibrium.
   - Location of M is (x, l)
   - Location of m is $(x + l\sin\theta,\ l(1-\cos\theta))$

b) Find the frequencies of small oscillations.