[4] (25 points) Two objects of mass m and negligible size are connected to the origin with springs of equilibrium (unstretched) length $L_1 = L_2 = a$ and to each other by a third spring of equilibrium length $L_3 = a\sqrt{2}$. One mass is constrained to move only in the y direction and the other mass can only move in the x direction as shown. All three springs have negligible mass and the same spring constant k. Answer the following questions using coordinates $q = x - a$ and $r = y - a$ for small deviations from the equilibrium position (i.e. small $|q|$ and $|r|$). You may ignore friction and gravity.
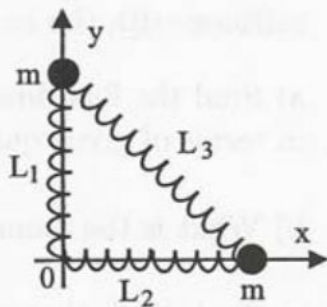
a) What is the potential energy U of this system?

b) What is the kinetic energy T of this system?

c) What are Lagrange's equations of motion for this system in simplified form (i.e. leave your answer as one or more differential equations)?

d) What are the normal modes of this system?

e) What are the natural frequencies of this system?