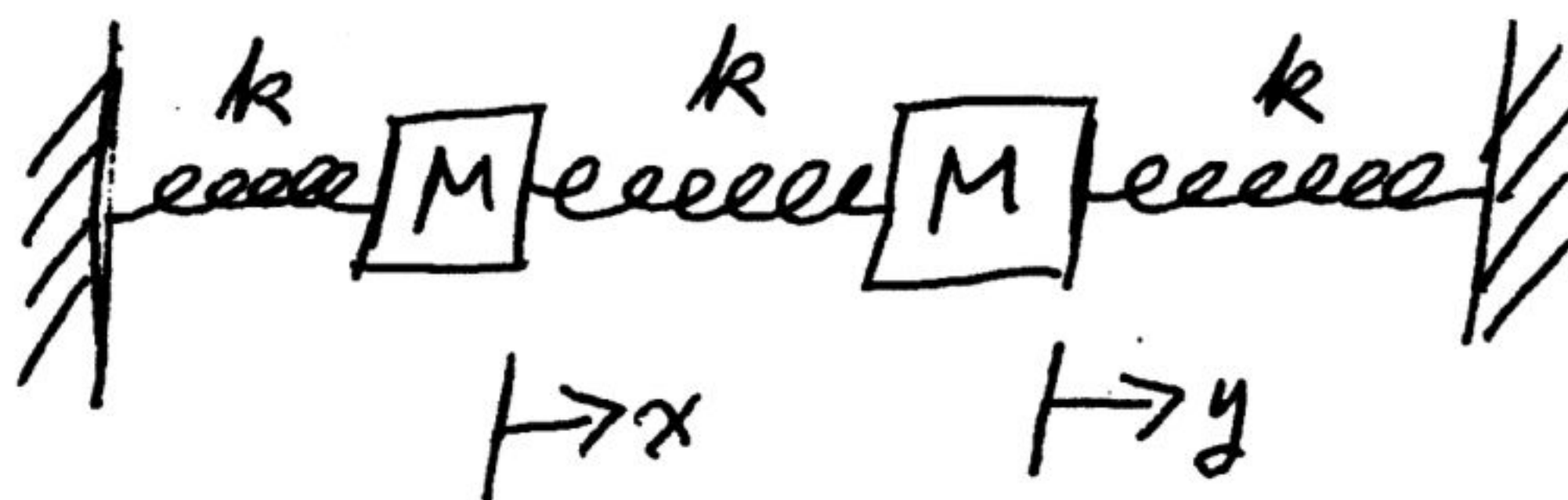


Mechanics

3. Two harmonic oscillators with mass M are coupled by adding another spring to the system as shown in the diagram below. For this system, find the equations of motion and the frequencies of the normal modes of the system. All three springs have spring constant k .



SOLUTION:

Let $\eta = x - y$
 subtract the two equations:
 $(\ddot{x} - \ddot{y}) + \left(\frac{k}{M}\right)(x - y) + \omega_0^2(x - y) = 0$
 $(1 - \mu/M)\ddot{\eta} + \omega_0^2\eta = 0$
 $\omega^2 = \omega_0^2 / (1 - \mu/M) \Rightarrow \boxed{\omega = \omega_0 (1 - \mu/M)^{-1/2}}$

now let $\xi = x + y$, add the equations:
 $(1 + \mu/M)\ddot{\xi} + \omega_0^2\xi = 0$
 $\omega^2 = \omega_0^2 / (1 + \mu/M)$
 $\boxed{\omega = \omega_0 (1 + \mu/M)^{-1/2}}$