

3. (40 points). A particle with an initial total energy  $E$  moves under gravity in an inertial frame of reference on a surface described by  $z(r) = Ar^n$  where  $z$  is the particle height and  $r$  is the radius from the origin.  $A > 0$

$z$  axis

- (a) Write down the kinetic and potential energies for this system.
- (b) Define the Lagrangian and use the constraint to reduce the variables to  $r$  and  $\theta$ .
- (c) Identify the generalized momenta conjugate to  $r$  and  $\theta$ .
- (d) Is the Hamiltonian equal to the total energy,  $E$ ? Why?
- (e) For what radius,  $r_0$  does stable circular motion occur? Detail, with reasons, any restrictions on the value of  $n$  at which this may be achieved?

