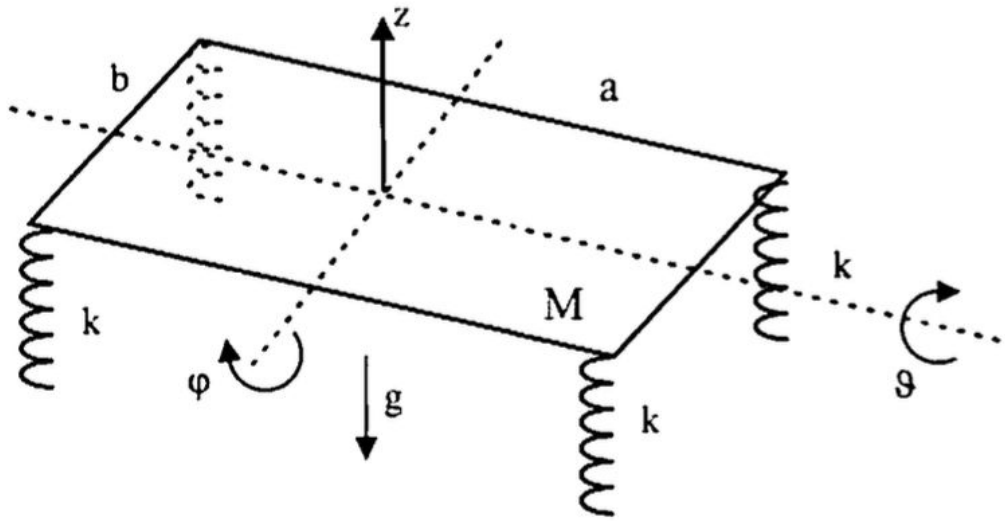
[4] (25 points) A thin uniform rectangular plate of mass M. with length a and width b is supported at its corners by four equal length springs with spring constants k. The springs are restricted to move only in the vertical direction, but the plate may tilt. Answer the following for small oscillations, in terms of generalized coordinated z, the height of the center of mass of the plate, θ the rotation about the a axis. and ϕ the rotation about the b axis (both through the center of mass), and their derivatives. Gravity is acting in the negative z direction. This system might model a vibration isolation table.



a) What is the potential energy U of this system?

b) What is the kinetic energy T of this system?

c) What is the Lagrangian L of this system?

d) What are Lagrange's equations of motion for this system in simplified form (i.e. leave your answer as one or more differential equations)?

e) What are the natural frequencies of the three normal modes of this system?