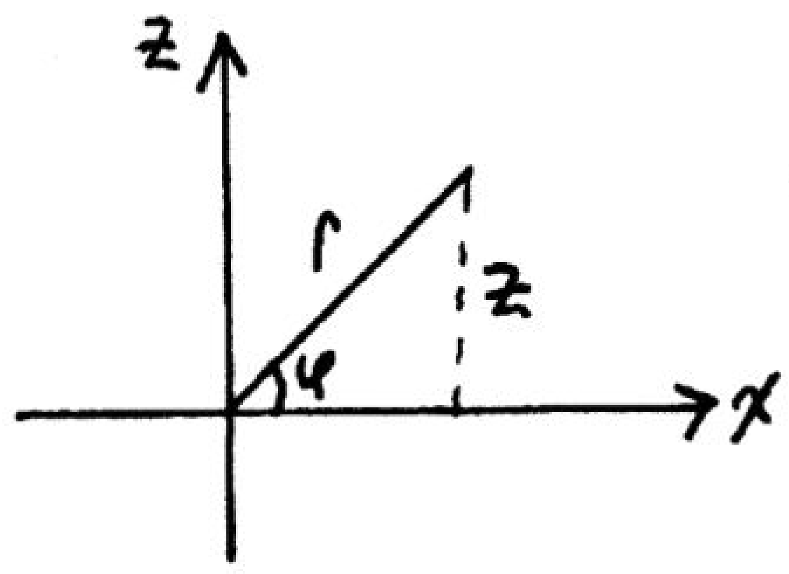
MECHANICS

2. Consider a body that is confined to move in a vertical plane, the x-z plane, with a gravitational field (a gravitational force in the $-\hat{z}$ direction). The body has mass m and moves in the plane subject to the (constant) gravitational force g and an additional "central" force of the form $f = -Ar^{-1/2}$, where $r^2 = x^2 + z^2$. This additional force is thus directed toward the origin. Choose the appropriate generalized coordinates and let the gravitational potential be zero along a horizontal line through the origin ($z = 0$).

   (a) Find the Lagrangian equations of motion for this system.

   (b) Show whether or not angular momentum about the origin is conserved.

SOULTION:



$$V_1 = mgz = mgr\sin\varphi$$

$$V_2 = -\int^r f\,dr = +\int^r A r^{-1/2}\,dr = 2Ar^{1/2}$$

(a) $\mathcal{L} = T - V = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\varphi}^2) - mgr\sin\varphi - 2Ar^{1/2}$

r eq $-$ $m\ddot{r} = \frac{\partial \mathcal{L}}{\partial r} = mr\dot{\varphi}^2 - mg\sin\varphi - Ar^{-1/2}$

$$\boxed{\ddot{r} = r\dot{\varphi}^2 - g\sin\varphi - (A/m)\,r^{-1/2}}$$

φ eq $-$ $m\,\partial_t(r^2\dot{\varphi}) = \frac{\partial \mathcal{L}}{\partial \varphi} = -mgr\cos\varphi$

$$\boxed{\frac{d}{dt}(r^2\dot{\varphi}) = -gr\cos\varphi}$$

(b) specific angular momentum $j \equiv r^2\dot{\varphi}$

φ equ $\Rightarrow$ $\frac{d}{dt}(j) = -gr\cos\varphi \neq 0$ ✓

ang. momentum <u>NOT</u> conserved