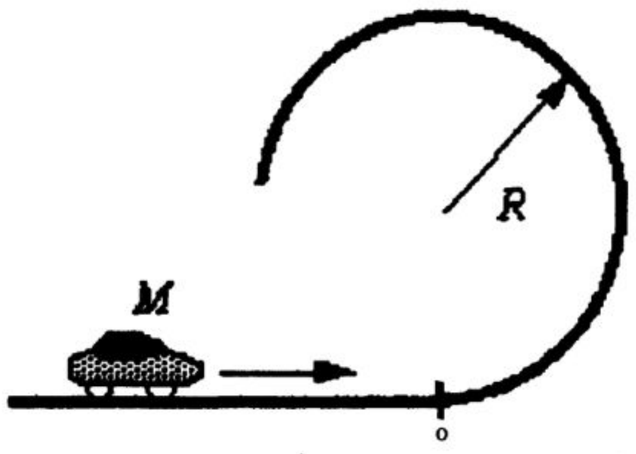
Mechanics

3. A small car of mass M travels at a constant speed v_0 along a straight, horizontal track. As shown in the figure, the track then bends into a vertical circle of radius R.

   (a) Calculate the minimum value of v_0 for the car to remain in contact with the track (ignore friction).

   (b) Assume now that there is friction in the circular track, i.e., add a frictional force $f = -\mu N$, where μ is the coefficient of kinetic friction and N is the normal force. Find an integral expression for the minimum speed v_0 required to reach the top of the track. Assume friction starts at point 0 as shown.

   (c) Show that the integral in (b) reduces to the answer in part (a) in the limit $\mu \to 0$.



ANSWER: $v_0^2 = 2gR \int_0^\pi \sin\theta e^{2\mu\theta} d\theta$