9 Exercise: Alternative stress vectors and tensors

9.1 Cauchy stress vectors

Consider the rigid body motion

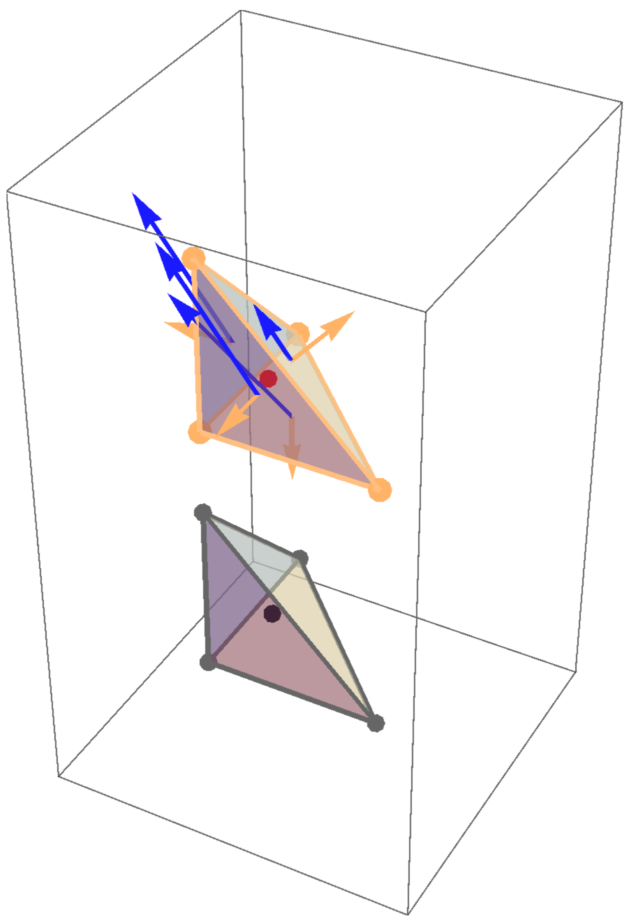
$$[\underline{\underline{\mathbf{R}}}] = \begin{bmatrix} \cos(\frac{2\pi t}{t_{max}}) & -\sin(\frac{2\pi t}{t_{max}}) & 0 \\ \sin(\frac{2\pi t}{t_{max}}) & \cos(\frac{2\pi t}{t_{max}}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad [\underline{\mathbf{b}}] = \begin{bmatrix} 0 \\ 0 \\ \frac{3}{20}\frac{t}{t_{max}} \end{bmatrix}$$

and the contact forces/moments of exercise 8 with $t_{max} = 1/2$.

Plot the resulting Cauchy stress vectors on each face.

$$^{fi}\underline{\mathbf{t}}(t) = \underline{\underline{\mathbf{T}}}(t)^{fi}\underline{\mathbf{n}}(t) - \frac{1}{2}\underline{\mathbf{w}}(t) \wedge {}^{fi}\underline{\mathbf{n}}(t) + \frac{1}{4\,^{fi}A}{}^{con}\underline{\mathbf{f}}(t) \qquad i = 1,4 \tag{1}$$

where the internal stress $\underline{\underline{\mathbf{T}}}$ is undetermined and therefore set to zero.

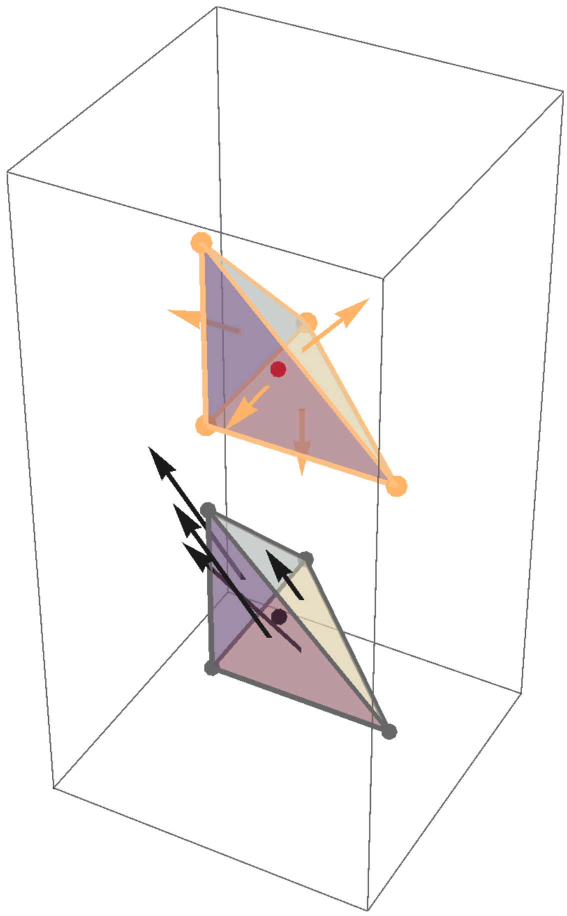


9.2 Nominal stress vectors

Plot the colinear nominal stress vectors for $t = t_{max}/2$ obtained by bringing the actual Cauchy stress vectors on the original faces:

$$^{fi}\underline{\mathbf{p}}(t)^{fi}A = {}^{fi}\underline{\mathbf{t}}(t)^{fi}A_t \qquad i = 1, 4 \tag{2}$$

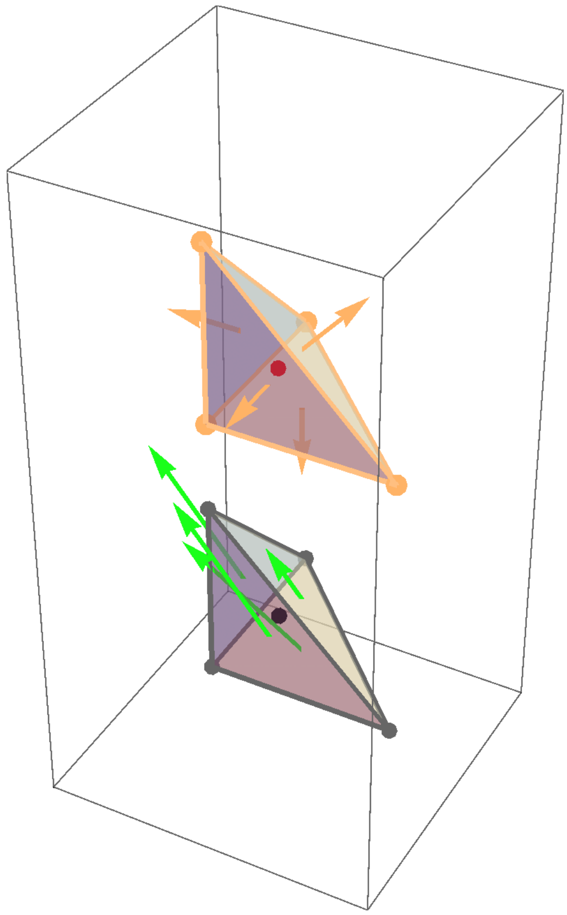
The correction for the change of area is not visible in this case because the rigid body motion preserves areas.



9.3 Material stress vectors

Plot the material stress vectors in the original configuration for $t = t_{max}/2$ obtained from the inverse transformation applied to the nominal stress vectors:

$$^{fi}\underline{\mathbf{s}}(t) = \underline{\underline{\mathbf{F}}}^{-1}(t)\ ^{fi}\underline{\mathbf{p}}(t) \qquad i = 1, 4 \tag{3}$$



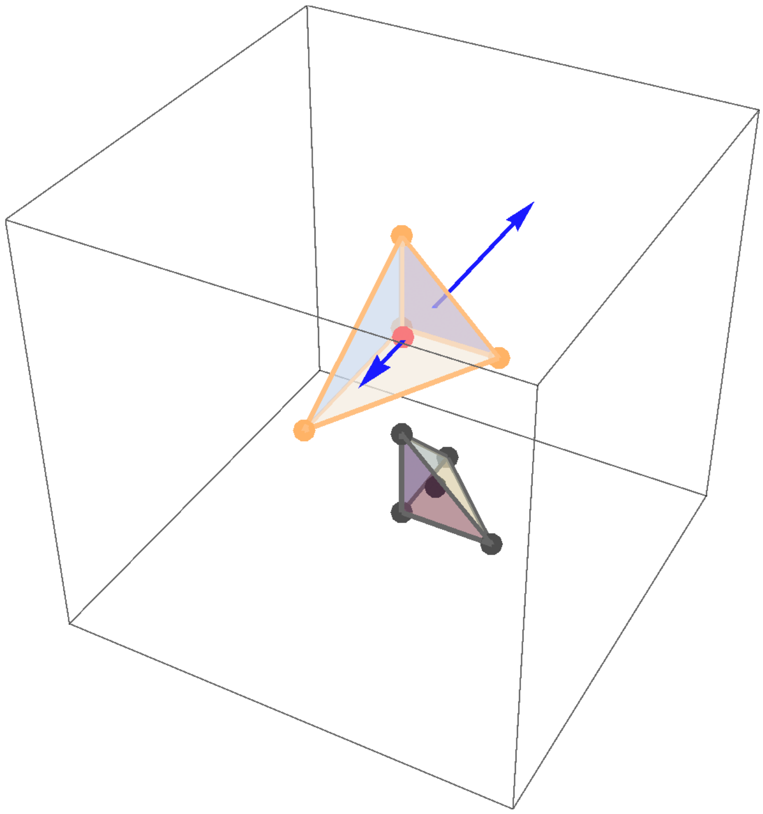
9.4 Cauchy stress tensor

Consider the previous rigid body motion composed to a continuous extension up to a factor 2 along ${}^1\underline{\mathbf{e}}$ at $t_{max} = 3/8$. Neglecting the inertial and volume forces, the resulting Cauchy stress tensor $\underline{\underline{\mathbf{T}}}$ is

$$[\underline{\underline{\mathbf{T}}}] = 10^6 \begin{bmatrix} 2t(t+1)(1+2t)Cos^2(4\pi t) & t(t+1)(1+2t)Sin(8\pi t) & 0 \\ t(t+1)(1+2t)Sin(8\pi t) & 2t(t+1)(1+2t)Sin^2(4\pi t) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Calculate and plot the corresponding stress vectors at $t = t_{max}/4$ on the actual faces of the tetrahedron

$$ {}^{fi}\underline{\mathbf{t}}(t) = \underline{\underline{\mathbf{T}}}(t)\,{}^{fi}\underline{\mathbf{n}}(t) \qquad i = 1,4 \tag{4}$$

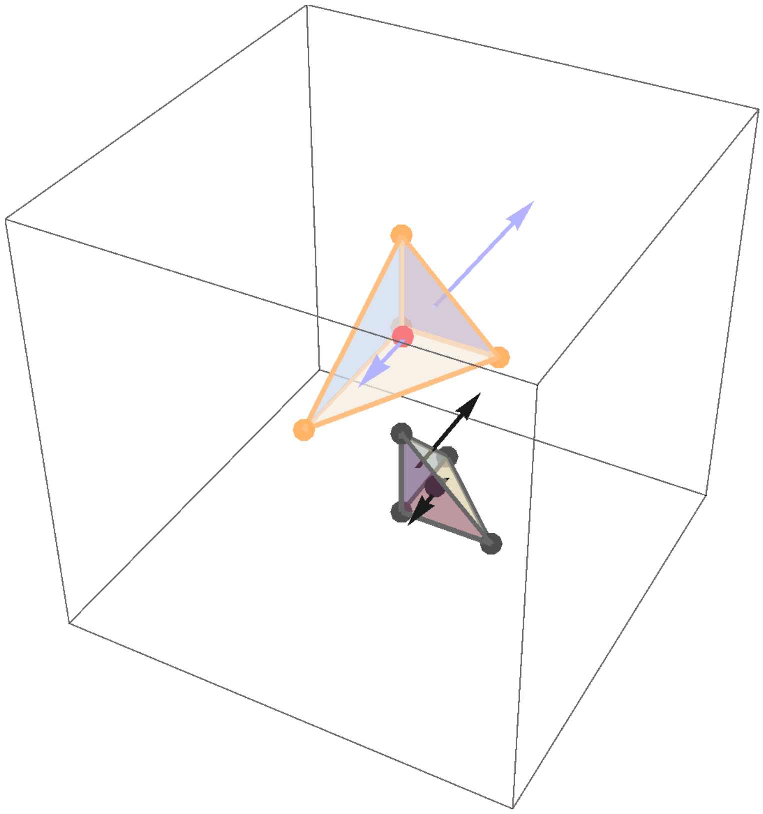


Knowing the relationship between the nominal and Cauchy tensor

$$\underline{\underline{\mathbf{P}}}(t) = \underline{\underline{\mathbf{T}}}(t)\underline{\underline{\mathbf{F}}}^*(t)$$

Calculate the corresponding nominal stress vectors

$$^{fi}\underline{\mathbf{p}}(t) = \underline{\underline{\mathbf{P}}}(t)^{fi}\underline{\mathbf{n}} \qquad i = 1, 4 \tag{5}$$



Similarly, knowing the relationship between the nominal and material stress tensor

$$\underline{\underline{\mathbf{P}}}(t) = \underline{\underline{\mathbf{F}}}(t)\underline{\underline{\mathbf{S}}}(t)$$

Calculate the corresponding material stress vectors

$$^{fi}\underline{\mathbf{s}}(t) = \underline{\underline{\mathbf{S}}}(t)^{fi}\underline{\mathbf{n}} \qquad i = 1, 4 \tag{6}$$