COMPSCI/SFWRENG 2FA3
Discrete Mathematics with Applications II
Winter 2019

Assignment 9

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Assignment 9 consists of two problems. You must write your solutions to the problems using LaTeX.

Please submit Assignment 9 as two files, `Assignment_9_`*`YourMacID`*`.tex` and `Assignment_9_`*`YourMacID`*`.pdf`, to the Assignment 9 folder on Avenue under Assessments/Assignments. *`YourMacID`* must be your personal MacID (written without capitalization). The `Assignment_9_`*`YourMacID`*`.tex` file is a copy of the LaTeX source file for this assignment (`Assignment_9.tex` found on Avenue under Contents/Assignments) with your solution entered after each problem. The `Assignment_9_`*`YourMacID`*`.pdf` is the PDF output produced by executing

```
pdflatex Assignment_9_YourMacID
```

This assignment is due **Sunday, March 31, 2019 before midnight.** You are allow to submit the assignment multiple times, but only the last submission will be marked. **Late submissions and files that are not named exactly as specified above will not be accepted!** It is suggested that you submit your preliminary `Assignment_9_`*`YourMacID`*`.tex` and `Assignment_9_`*`YourMacID`*`.pdf` files well before the deadline so that your mark is not zero if, e.g., your computer fails at 11:50 PM on March 31.

Although you are allowed to receive help from the instructional staff and other students, your submission must be your own work. Copying will be treated as academic dishonesty! If any of the ideas used in your submission were obtained from other students or sources outside of the lectures and tutorials, you must acknowledge where or from whom these ideas were obtained.

Problems

1. **[10 points]** Let $G = (N, \Sigma, P, S)$ be the CFG where $N = \{S\}$, $\Sigma = \{a, b\}$, and P contains the following productions:

   $$S \to aSb \mid \epsilon.$$

   For $x \in \Sigma^*$, let $P(x)$ be the property that $S \xrightarrow[G]{*} x$ iff $x = a^n b^n$ for some $n \geq 0$. Prove $\forall\, x \in \Sigma^* \,.\, P(x)$ by weak induction on the length of the derivation $S \xrightarrow[G]{*} x$ for the $(\Rightarrow)$ direction and by strong induction on the length of x for the $(\Leftarrow)$ direction.

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   Proof. $S \xrightarrow[G]{n} x \Rightarrow x = a^n b^n$ for all $n \in \mathbb{N}$ by weak induction.

   Base case: $n = 0$.
   $S \xrightarrow[G]{0} \epsilon \Rightarrow x = \epsilon$

   Induction step: $n \geq 0$. Assume $P(n)$. We must show $P(n + 1)$.
   $S \xrightarrow[G]{n+1} x_{n+1} \Leftarrow S \xrightarrow[G]{1} aSb \wedge aSb \xrightarrow[G]{n} x_{n+1}$ (definition)
   $= S \xrightarrow[G]{1} aSb \wedge aSb \xrightarrow[G]{n} ax_n b$ (Induction hypothesis)
   $= S \xrightarrow[G]{1} aSb \wedge aSb \xrightarrow[G]{n} aa^n b^n b$ (Induction hypothesis)
   $= S \xrightarrow[G]{1} aSb \wedge aSb \xrightarrow[G]{n} a^{n+1} b^{n+1}$ (Arithmetic)
   $= S \xrightarrow[G]{n+1} a^{n+1} b^{n+1}$ (definition)

   □

   Proof. $S \xrightarrow[G]{*} x \Leftarrow x = a^n b^n$ for all $n \in \mathbb{N}$ by strong induction.

   Base case: $n = 0$.
   x $= a^0 b^0 = \epsilon = S \xrightarrow[G]{0} \epsilon$

   Induction step: $n \geq 0$. Assume $\forall i \in \mathbb{N} | i \leq n . P(n)$. We must show $P(n + 1)$.
   x $= a^{n+1} b^{n+1}$
   $= aa^n b^n b$ (Arithmetic)
   $= aSb$ (Induction hypothesis)
   $\leftarrow$ S (Definition)

   □

2. **[10 points]** Let $\Sigma = \{a, b\}$ and $L = \{x \in \Sigma^* \mid x \neq \mathsf{rev}(x)\}$. Construct a simple grammar for L as well as grammars in Chomsky and Greibach normal form for L.

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simple grammar:

Let G = (N,$\{a, b\}$,P,S)

N = S, R, T

P =

$S \to aSa|bSb|T$

$T \to aRb|bRa$

$R \to aRa|bRb|T|a|b|\epsilon$

Chomsky:

N = $S, S_a, S_b, T, R, R_a, R_b, A, B, E$

P =

$S \to AS_a|AS_b|TE$

$S_a \to SA$

$S_b \to SB$

$T \to AR_b|BR_a$

$R_a \to RA$

$R_b \to RB$

$R \to AR_a|BR_b|TE|a|b|\epsilon$

$A \to a$

$B \to b$

$E \to \epsilon$

Greibach:

N = S, T, R, A, B

P =

$S \to aSA|aSB|aTB|bTA$

$T \to aRB|bRA$

$R \to aRA|bRB|bRA|aRB|a|b|\epsilon$

$A \to a$

$B \to b$