



3 Types of FL

# Three Types of Federated Learning

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# Horizontal Federated Learning

## 3 Types of FL

$$\text{minimize } f(\theta) = \mathbb{E}_{k \sim \mathcal{P}} [f_k(\theta)] = \sum_{k=1}^K w_k f_k(\theta)$$

$$\text{where } f_k(\theta) = \mathbb{E}_{(x,y) \sim \mathcal{D}_k} [\ell_k(\theta; x, y)]$$

- $\theta$  : learning objective (model parameters),
- $\mathcal{P}$  : distribution of clients,
- $w_k$  : weight of client  $k$ ,
- $\mathcal{D}_k \subseteq \mathcal{X}_k \times \mathcal{Y}_k$  : distribution of data on client  $k$ ,
  - $\mathcal{X}_k$  : feature distribution,
  - $\mathcal{Y}_k$  : label distribution,

**underlying feature spaces and label spaces are the same across clients.**

- $\ell_k$  : loss function.



# Vertical Federated Learning

3 Types of FL

$$\begin{aligned} \text{minimize } f(\theta) &= \sum_{i=1}^N \mathcal{L} \left( \mathcal{F}_K \left( \varphi_K; f_1(\theta_1; x_i^{(1)}), \dots, f_K(\theta_K; x_i^{(K)}), y_i \right) \right) \\ \text{where } \theta &= (\theta_1, \dots, \theta_K), x_i = (x_i^{(1)}, \dots, x_i^{(K)}) \end{aligned}$$

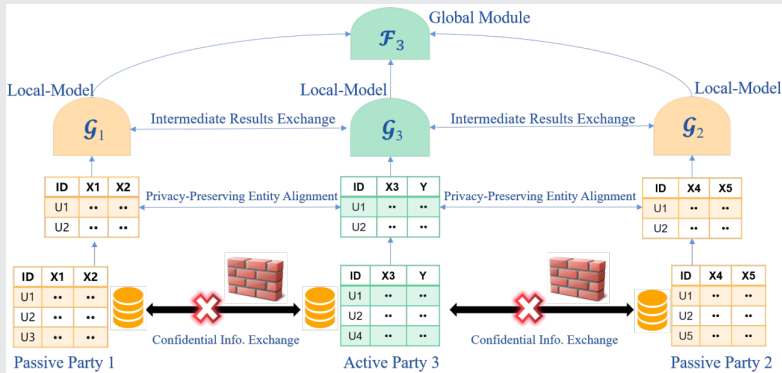
## A Simple Example: Group Lasso

$$\beta^* = \underset{\beta}{\operatorname{argmin}} \left\{ \left\| y - \sum_{k=1}^K X^{(k)} \beta^{(k)} \right\|_2^2 + \lambda \sum_{k=1}^K \|\beta^{(k)}\|_2 \right\}$$



# Vertical Federated Learning

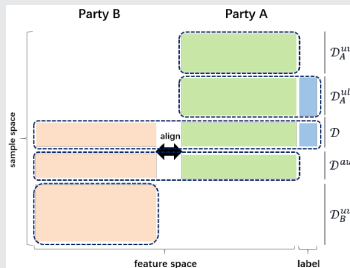
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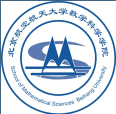
# Federated Transfer Learning

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One choice [3] is to map feature space A and B to some common feature space  $U$ , and solve

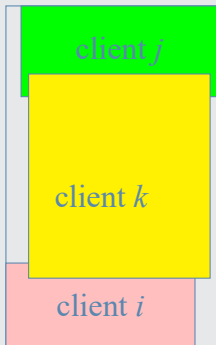
$$\text{minimize} \quad \sum_{k=1}^{K_{AB}} (\ell_1(y_k^A, \varphi(u_k^B)) + \lambda \ell_2(u_k^A, u_k^B))$$



# Visualization

## 3 Types of FL

Horizontal FL



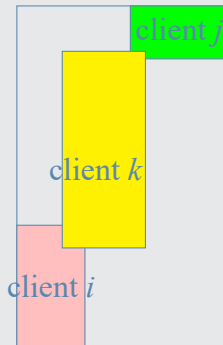
Feature Space

Vertical FL



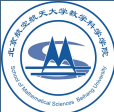
Feature Space

Transfer FL



Feature Space

Sample ID Space



# References I

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- [1] Y. Liu, X. Zhang, Y. Kang, L. Li, T. Chen, M. Hong, and Q. Yang, “FedBCD: A Communication-Efficient Collaborative Learning Framework for Distributed Features,” *IEEE Transactions on Signal Processing*, vol. 70, pp. 4277–4290, 2022.
- [2] Y. Liu, Y. Kang, T. Zou, Y. Pu, Y. He, X. Ye, Y. Ouyang, Y.-Q. Zhang, and Q. Yang, “Vertical Federated Learning,” 2022.
- [3] Y. Liu, Y. Kang, C. Xing, T. Chen, and Q. Yang, “A Secure Federated Transfer Learning Framework,” *IEEE Intelligent Systems*, vol. 35, pp. 70–82, 7 2020.