SOC-GA 2332 Intro to Stats Lab 4

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Logistics & Announcement

- **Problem Set 1** is due on Monday Sep. 30th, 11:59 pm. Remember to submit both your coding file (the R Markdown) and a PDF file knitted from the R Markdown
- Load packages to your environment (today we will use a new package `ipumsr`. Please install the package (`install.packages('ipumsr')`) before you run the following chunks)

```
knitr::opts_chunk$set(echo = TRUE)

library(tidyverse)
library(kableExtra)
library(ipumsr)
```

Part 1: The OLS Estimator

- Regression is one of the most common tool social scientists use to understand the relationships between variables.
- The three perspectives about regression:
  - Regression as describing a relationship
  - Regression as prediction (for conditional means)
  - Regression as a causal relationship
- This week we started on the simplest regression that consists of one outcome (Y_i) and one predictor (X_i) with the parameters β_0, β_1, and the error term ϵ_i. This is called the **bivariate regression**. Consider the underlying **Data Generating Process** (DGP) in the **population**, where

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

- We never observe the population. Instead, we have a **sample** and statistical methods to estimate the value of β_0 and β_1 given observed values of y_i and x_i in the **sample**. This method of estimation is called the **Ordinary Least Squares** (OLS) estimation.
- We assume a linear relation between the observed y_i and x_i in the sample, the same as our DGP.

$$y_i = \hat{\beta}_0 + \hat{\beta}_1 x_i + e_i$$

- The OLS estimators of β_0 and β_1 given the sample *minimize* the sum of squared errors (SSE).

$$SSE = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} \left(y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i)\right)^2$$

$$\hat{\beta}_0, \hat{\beta}_1 = \underset{\hat{\beta}_0, \hat{\beta}_1}{\operatorname{argmin}} \sum_{i=1}^{n} \left(y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i)\right)^2$$

- We use partial derivative to derive our $\hat{\beta}_0$ and $\hat{\beta}_1$.

$$\begin{aligned}\hat{\beta}_1 &= \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \\ &= \frac{Cov(y_i, x_i)}{Var(x_i)}\end{aligned}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

- Does the error term e_i have a mean value of zero?
- Is the covariance between e_i and x_i zero? (*e_i* and *x_i* are not correlated)
  - Does it mean e_i and x_i is independent?
- What happens when we have more than 1 predicting variable?

$$\begin{aligned}\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2 ..., \hat{\beta}_k &= \underset{\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2 ..., \hat{\beta}_k}{\operatorname{argmin}} \sum_{i=1}^{n} \left(y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i + \hat{\beta}_2 x_2 + ... + \hat{\beta}_k x_k)\right)^2 \\ \hat{\beta}_k &= \frac{Cov(y_i, \tilde{x}_{ki})}{Var(\tilde{x}_{ki})}\end{aligned}$$

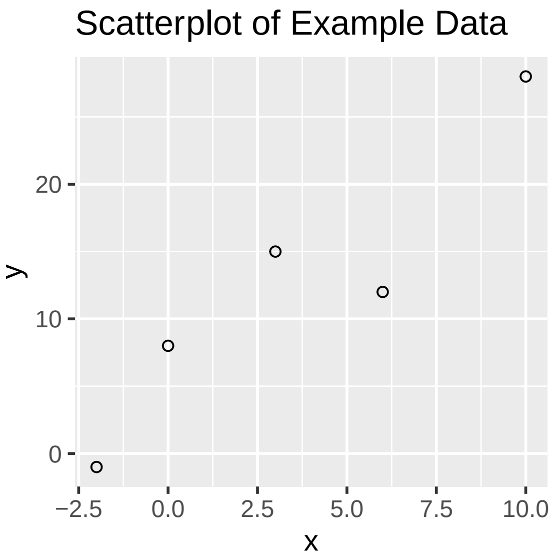
where $\tilde{x}_{ki}$ is the residual from the regression of x_{ki} on all other covariates.

- Now we create a toy example with only five data points in our sample, and calculate the OLS estimators by hand.

```r
## value of x and y are observed
x <- c(-2, 0, 3, 6, 10)
y <- c(-1, 8, 15, 12, 28)

## create a dataframe of x and y
ols_df <- data.frame(
  x = x,
  y = y
)

## scatter plot of x and y
ols_df %>%
  ggplot() +
  geom_point(aes(x, y), shape = 1) +
  labs(title = "Scatterplot of Example Data")
```



Part 1 Exercise: Calculate OLS estimators by hand

Let us apply the formulas we learned in the lecture to work out the value of $\hat{\beta}_0$ and $\hat{\beta}_1$ in the above toy example using OLS.

1. Hand-calculate the value of $\hat{\beta}_0$ and $\hat{\beta}_1$ using the formulas below.
2. Calculate $\hat{\beta}_0$ and $\hat{\beta}_1$ using `R`. *Hint:* You can perform vector operations in `R`.

```
## your code here
```

3. In `R`, you can get the variance of a value vector using the function `var()`, and the covariance of two value vectors using the function `cov()`.

```
## your code here
```

4. You can use the `cor()` function in R to calculate the correlation coefficient ρ of two value vectors. Now calculate $\hat{\beta}_1$ using the formula below.

$$\begin{aligned}\hat{\beta}_1 &= \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}\\ &= \frac{Cov(y_i, x_i)}{Var(x_i)}\\ &= \rho_{x,y}\cdot\frac{s_y}{s_x}\end{aligned}$$

```
## your code here
```

5. Create a new dataframe based on `ols_df` that has a new variable called `fitted_y` that equals to your predicted value of y given your OLS regression equation. *Hint:* Use `mutate()` in `tidyverse` to create a new variable.

```
## your code here
```

6. Calculate your OLS regression's **SSE (sum of squared errors)**.

```
## your code here
```

7. Draw your OLS regression line on the scatter plot created earlier. *Hint:* You can add a line with customized intercept and slope value using the `geom_abline()` function:

```
your_plot +
  geom_abline(intercept = your_intercept, slope = your_slope)
```

```
## your code here
```

Estimate OLS Regression Using R

Estimating linear models using OLS is very simple in `R`. We can use the linear model function `lm()` to estimate regression models. However, you should always remember that OLS estimation entails a set of assumptions about your data that you should always check before estimating your models.

Gauss-Markov Assumptions:

1. Linear in parameters
2. Random sampling
3. No perfect collinearity
4. Zero conditional mean (Do we force this to be true in estimating OLS? No! We hope this is true in the DGP such that our OLS estimation is consistent and unbiased)
5. Homoskedasticity (error term has a constant variance across the value of x): $\text{Var}(\epsilon \mid x_1, x_2, \dots, x_k) = \sigma^2$

```
## estimate a linear model
ols_m1 <- lm(y ~ x, data = ols_df)

## check model result
ols_m1
```

Estimate OLS in R

```
## 
## Call:
## lm(formula = y ~ x, data = ols_df)
## 
## Coefficients:
## (Intercept)            x  
##       5.384        2.064
```

```
## for details, use the summary() function
summary(ols_m1)
```

```
## 
## Call:
## lm(formula = y ~ x, data = ols_df)
## 
## Residuals:
##      1      2      3      4      5 
## -2.257  2.616  3.425 -5.765  1.980 
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)  
## (Intercept)   5.3838     2.5740   2.092   0.1276  
## x             2.0636     0.4715   4.376   0.0221 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.503 on 3 degrees of freedom
## Multiple R-squared:  0.8646, Adjusted R-squared:  0.8194
## F-statistic: 19.15 on 1 and 3 DF,  p-value: 0.02208
```

```
## the SSE is not directly displayed, but we can calculate it by its definition
sum(ols_m1$residuals^2)
```

```
## [1] 60.83114
```

Part 2: Getting Started on the Replication Project

Now let's move on to use real-world data for estimating regression models. Since we will need to use IPUMS for the final replication project, let's download data from IPUMS.

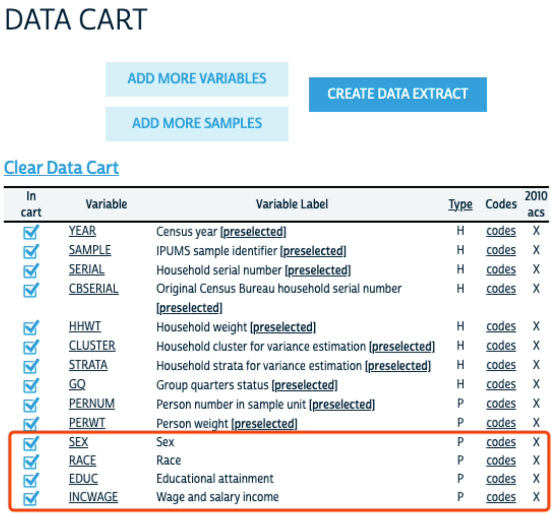
IPUMS

IPUMS is a "data resource center" that stores various census and survey data. It harmonizes survey measures across the years and provide detailed documentations for researchers.

For the replication project, we will use **IPUMS USA** that stores the U.S. Census and American Community Survey data.

Part 2 Exercise 1: Download Data from IPUMS

1. Register an account for using IPUMS-USA here
2. Once your account is created, log in to IPUMS-USA and create a data extract request by selecting the **2010 ACS** sample and the **SEX**, **EDUC**, and **INCWAGE** variables. You can keep the preselected variables. Your "Data Cart" should be similar to this:

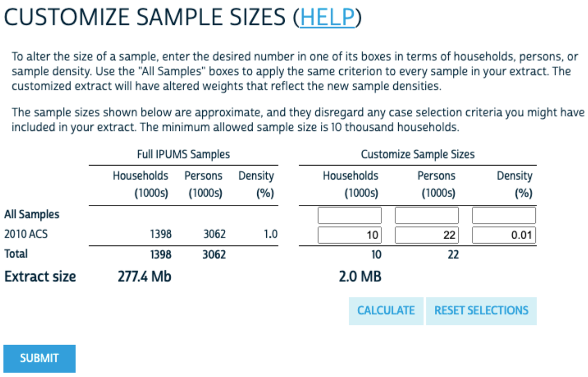


In cart	Variable	Variable Label	Type	Codes	2010 acs
☑	YEAR	Census year [preselected]	H	codes	X
☑	SAMPLE	IPUMS sample identifier [preselected]	H	codes	X
☑	SERIAL	Household serial number [preselected]	H	codes	X
☑	CBSERIAL	Original Census Bureau household serial number [preselected]	H	codes	X
☑	HHWT	Household weight [preselected]	H	codes	X
☑	CLUSTER	Household cluster for variance estimation [preselected]	H	codes	X
☑	STRATA	Household strata for variance estimation [preselected]	H	codes	X
☑	GQ	Group quarters status [preselected]	H	codes	X
☑	PERNUM	Person number in sample unit [preselected]	P	codes	X
☑	PERWT	Person weight [preselected]	P	codes	X
☑	SEX	Sex	P	codes	X
☑	RACE	Race	P	codes	X
☑	EDUC	Educational attainment	P	codes	X
☑	INCWAGE	Wage and salary income	P	codes	X

Figure 1: Data Cart

3. To save time, create a small data extract with a customized sample size of **0.01%**. As shown in the following screenshot. Submit your data extract request.

4. While IPUMS is processing our data extract request, you can prepare the file needed for loading the IPUMS data to R. Open the "DDI" page in your browser, it should be a `.xml` page. Save this `.xml` file



CUSTOMIZE SAMPLE SIZES (HELP)

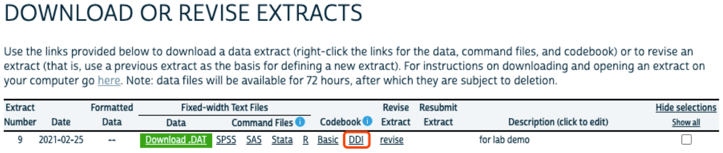
To alter the size of a sample, enter the desired number in one of its boxes in terms of households, persons, or sample density. Use the "All Samples" boxes to apply the same criterion to every sample in your extract. The customized extract will have altered weights that reflect the new sample densities.

The sample sizes shown below are approximate, and they disregard any case selection criteria you might have included in your extract. The minimum allowed sample size is 10 thousand households.

	Full IPUMS Samples			Customize Sample Sizes		
	Households (1000s)	Persons (1000s)	Density (%)	Households (1000s)	Persons (1000s)	Density (%)
All Samples						
2010 ACS	1398	3062	1.0	10	22	0.01
Total	1398	3062		10	22	
Extract size	277.4 Mb			2.0 MB		

Figure 2: Customize

to your current lab folder, or you can save it in a sub-folder devoted to data files, such as a sub-folder called “data” within your current lab folder.



DOWNLOAD OR REVISE EXTRACTS

Use the links provided below to download a data extract (right-click the links for the data, command files, and codebook) or to revise an extract (that is, use a previous extract as the basis for defining a new extract). For instructions on downloading and opening an extract on your computer go here. Note: data files will be available for 72 hours, after which they are subject to deletion.

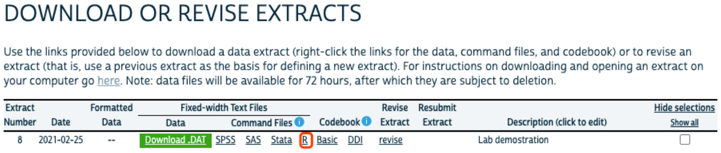
Extract		Formatted	Fixed-width Text Files			Revise	Resubmit		Hide selections
Number	Date	Data	Data	Command Files	Codebook	Extract	Extract	Description (click to edit)	Show all
9	2021-02-25	--	Download .DAT	SPSS SAS Stata R	Basic DDI	revise		for lab demo	□

Figure 3: Download

5. Once your data is ready on IPUMS, you will receive an email with a download link. (The wait time varies from a couple minutes to a couple days, depending on how large your files are and how busy the server is.) Follow the link in your email and download your data (the default format is `.dat.gz`). **Make sure to save it at the same directory with your `.xml` file**

6. Unzip the `.gz` data file using your computer’s unzip applications (for Mac users, if the default extractor doesn’t work, try the Unarchiver application). Now you should have the `.dat` file and the `.xml` file in the same directory.

7. Load the data to your R environment, by using the code shown in the webpage when you click the `R` Command File link in your IPUMS data downloading page.



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Extract		Formatted	Fixed-width Text Files			Revise	Resubmit		Hide selections
Number	Date	Data	Data	Command Files	Codebook	Extract	Extract	Description (click to edit)	Show all
8	2021-02-25	--	Download .DAT	SPSS SAS Stata R	Basic DDI	revise		Lab demostration	□

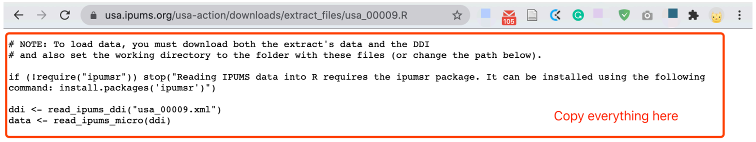
Figure 4: Download 2

```r
## your code here

## NOTE: To load data, you must download both the extract's data and the DDI
## and also set the working directory to the folder with these files (or change the path below).

if (!require("ipumsr")) stop("Reading IPUMS data into R requires the ipumsr package. It can be installe

ddi <- read_ipums_ddi("data/usa_00043.xml")
data <- read_ipums_micro(ddi)
```



usa.ipums.org/usa-action/downloads/extract_files/usa_00009.R

```
# NOTE: To load data, you must download both the extract's data and the DDI
# and also set the working directory to the folder with these files (or change the path below).

if (!require("ipumsr")) stop("Reading IPUMS data into R requires the ipumsr package. It can be installed using the following
command: install.packages('ipumsr')")

ddi <- read_ipums_ddi("usa_00009.xml")
data <- read_ipums_micro(ddi)
```

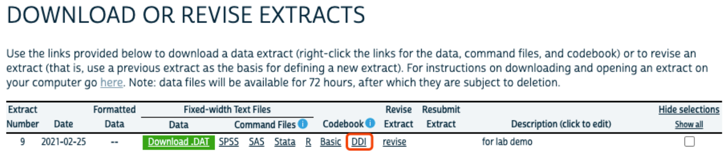
Copy everything here

Figure 5: Command

```
## Use of data from IPUMS USA is subject to conditions including that users should cite the data approp
```

Read IPUMS Documentations

Before working with the data, we should be clear about how the variables are coded. You can access the documentation of each variables through the `.xml` webpage you opened earlier.



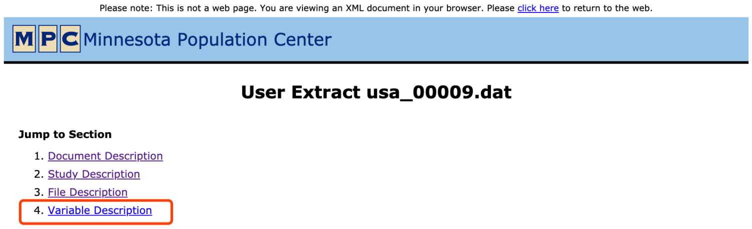
DOWNLOAD OR REVISE EXTRACTS

Use the links provided below to download a data extract (right-click the links for the data, command files, and codebook) or to revise an extract (that is, use a previous extract as the basis for defining a new extract). For instructions on downloading and opening an extract on your computer go here. Note: data files will be available for 72 hours, after which they are subject to deletion.

Extract		Formatted	Fixed-width Text Files							Revise	Resubmit		Hide selections
Number	Date	Data	Data	Command Files				Codebook		Extract	Extract	Description (click to edit)	Show all
9	2021-02-25	--	Download .DAT	SPSS	SAS	Stata	R	Basic	DDI	revise		for lab demo	□

Figure 6: Documentation 1

When you have that page open in your browser, click "Variable Description" to access the codebook for each variables we just downloaded.



Please note: This is not a web page. You are viewing an XML document in your browser. Please click here to return to the web.

MPC Minnesota Population Center

User Extract usa_00009.dat

Jump to Section

1. Document Description
2. Study Description
3. File Description
4. Variable Description

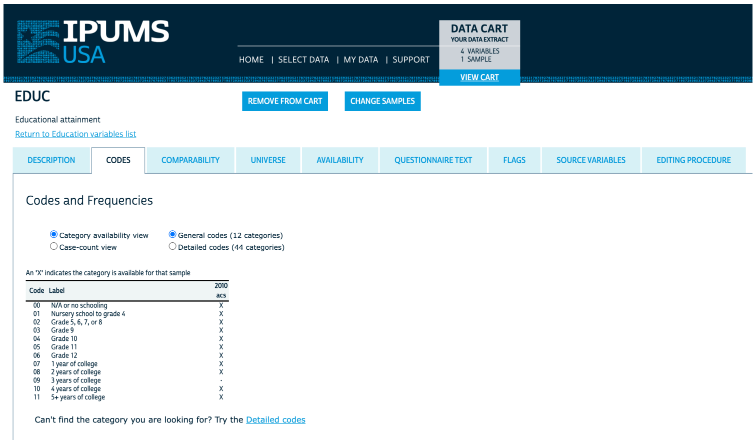
Figure 7: Documentation 2

The other way is to read the documentation of each variable directly in the online system. This is my preferred approach.

Part 2 Exercise 2: Read Data Codebook

Going through the variable description, and answer the following question:

1. What does the variable "PERNUM" represents? How can you uniquely identify each person within the IPUMS with this variable?
2. What does the variable "PERWT" represents? When should you consider using this variable?
3. For the variable "SEX", what are the possible values it can take? What does each value represent? Try run `str(data$SEX)`, what do you see?
4. What does the variable "EDUC" represents? How many values this variable can take? What is the value that represents N/A?
5. For the variable "INCWAGE", what are the codes for N/A and missing data?



EDUC

Educational attainment

Return to Education variables list

Codes and Frequencies

● Category availability view ● General codes (12 categories)
○ Case-count view ○ Detailed codes (44 categories)

An 'X' indicates the category is available for that sample

Code	Label	2010 acs
00	N/A or no schooling	X
01	Nursery school to grade 4	X
02	Grade 5, 6, 7, or 8	X
03	Grade 9	X
04	Grade 10	X
05	Grade 11	X
06	Grade 12	X
07	1 year of college	X
08	2 years of college	X
09	3 years of college	·
10	4 years of college	X
11	5+ years of college	X

Can't find the category you are looking for? Try the Detailed codes

Figure 8: Codebook

Part 3: Analyzing IPUMS Data

We will try fit an OLS regression on the relationship between **education and wage income**. However, before doing that, we need to make sure we know how the variables are distributed in the sample and remove N/A and missing values (we do not consider imputing missing values at this point).

Note: Since we each extracted a 0.01% random sample, it is normal if your results are slightly different from this demo.

First, make sure your data is loaded in your environment.

```r
## load ipums data
ddi <- read_ipums_ddi("data/usa_00043.xml")
data <- read_ipums_micro(ddi)
```

```
## Use of data from IPUMS USA is subject to conditions including that users should cite the data appropr
```

Then, we will keep the following variables for subsequent data analysis: SAMPLE, SERIAL, PERNUM, PERWT, SEX, EDUC, INCWAGE

```r
## select variables
data_clean <- data %>%
  select(PERWT, SEX, EDUC, INCWAGE)
```

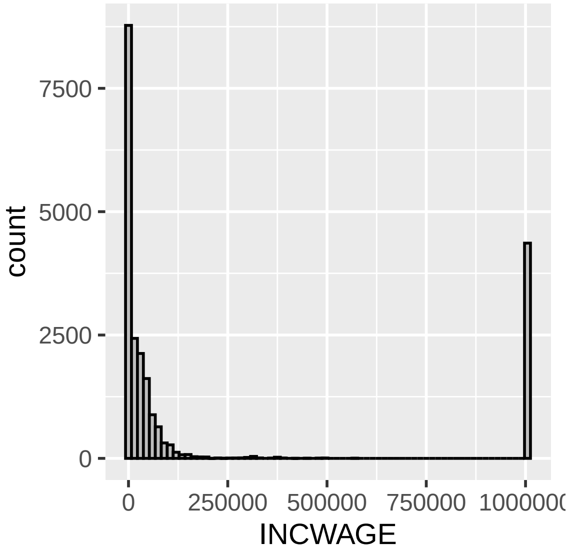
Remove missing value

Since we are running regression for **education and wage income**, we need to remove observations that do not have any of the two values. You can either remove then by filtering out the values that represent N/A or missing based on the variable codebook, or you can recode these values to `NA` so that you keep all the observations. We will cover this latter method in the future.

```r
## check number of observations for each value of EDUC
table(data_clean$EDUC)
```

```
##
##    0    1    2    3    4    5    6    7    8   10   11
## 1509 1865 1725  599  657  674 6527 2639 1265 2842 1651
```

```r
## check coding scheme for EDUC in codebook
## --> we need to remove value 0, which means N/A or no schooling
## check distribution of INCWAGE
data_clean %>%
  ggplot() +
  geom_histogram(aes(INCWAGE), binwidth = 15000, color = "black", fill = "grey")
```



```
## check coding scheme for INCWAGE in codebook
## --> we need to remove value 999998 (missing) and value 999999 (N/A)
max(data_clean$INCWAGE)
```

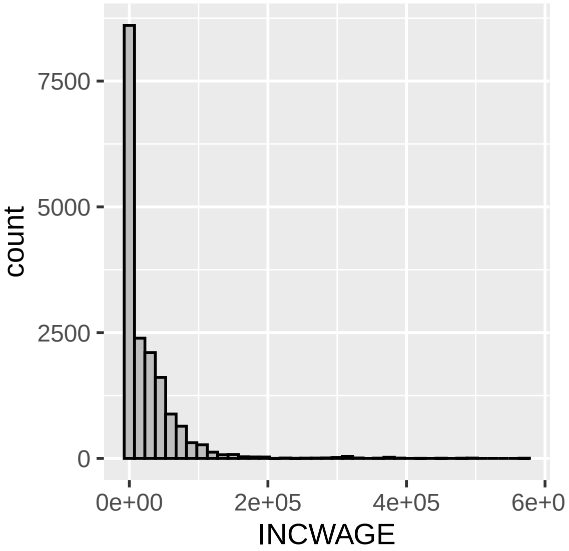
```
## <labelled<double>[1]>: Wage and salary income
## [1] 999999
##
## Labels:
##    value   label
##   999998 Missing
##   999999     N/A
```

```
## remove missing value use filter()
data_clean <- data_clean %>%
  filter(EDUC != 0 & INCWAGE < 999998)

## check distribution after removing missing values
## EDUC
table(data_clean$EDUC)
```

```
##
##    1    2    3    4    5    6    7    8   10   11
##  114  609  413  616  671 6525 2639 1265 2842 1651
```

```
## INCWAGE
data_clean %>%
  ggplot() +
  geom_histogram(aes(INCWAGE), binwidth = 15000, color = "black", fill = "grey")
```



Estimate Regression

Now, let's try estimate a OLS regression model for wage income and education levels. Let's make an unrealistic assumption that we can treat the EDUC variable as a discrete numeric variable, that the value 1, 2, 3, …, 11 correspond to the years of education one receives. (Of course, if you read the codebook carefully, you know this is not what each value represents. We will learn how to correctly use categorical predictors in future lectures.)

Part 3 Exercise: Estimate OLS Model Using IPUMS Data

1. Clean your data using the code above (create `unique_id` and remove missing values for INCWAGE and EDUC)

```
## your code here
```

2. Plot a scatter plot with EDUC on the x axis and INCWAGE on the y axis

```
## your code here
```

3. Use `lm()` to fit a OLS regression model with INCWAGE as the dependent variable and EDUC as the independent variable. Report your $\hat{\beta}_0$, $\hat{\beta}_1$, and SSE. No need to weight individuals at this point.

```
## your code here
```

4. Use `+ geom_smooth(aes(x = your_x, y = your_y), method = "lm")` to plot the fitted regression line on top of your scatter plot.

```
## your code here
```

5. Plot the following diagnostic plots. What patterns do you see about the residual terms relative to the value of EDUC and fitted y? Do you think the data and the model set up fulfill the Gauss-Markov assumptions?

Hint: Use the base R function `plot(x = your_x_vector, y = your_y_vector)` for quick plotting.

- ϵ_i vs x_i: Make a scatter plot with the predictor on the x axis and the model residuals on the y axis.

```
## your code here
```