

HW6

February 25, 2026

1 Homework 6

1.0.1 Wing Huang PID: A18894844

Date: 2026-02-17

```
[2]: import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import quad
```

1.1 Problem 1: Integration

```
[3]: # (a)
f1 = lambda x: (1 + x**2) / x
print(quad(f1, 1, 20))

(202.495732273554, 1.3443696573563126e-08)
```

```
[4]: # (b)
f2 = lambda x: np.sin(x) * np.exp(-x) / x
print(quad(f2, 1, np.inf))

(0.1793245350394128, 7.620675259525255e-09)
```

```
[6]: # (c)
f3 = lambda x: np.sin(x)/x
print(quad(f3, 0, 1, points=[0]))

(0.946083070367183, 1.0503632079297087e-14)
```

```
[20]: # (d)
def mid(f, a, b):
    return (b - a) * f((a+b) / 2)

def rec(f, a, b, tol, d, md):
    m = mid(f, a, b)
    mm = (a + b) / 2
    if abs(mid(f, a, mm) + mid(f, mm, b) - m) < tol or d >= md:
        return mid(f, a, mm) + mid(f, mm, b)
    return rec(f, a, mm, tol/2, d+1, md) + rec(f, mm, b, tol/2, d+1, md)
```

```
def adaptive_integration(f, a, b, tol, md):
    return rec(f, a, b, tol, 0, md)

f = lambda x: (1 + x**2)/x
print(adaptive_integration(f, 1, 20, 1e-8, 20))
```

202.49573227355057

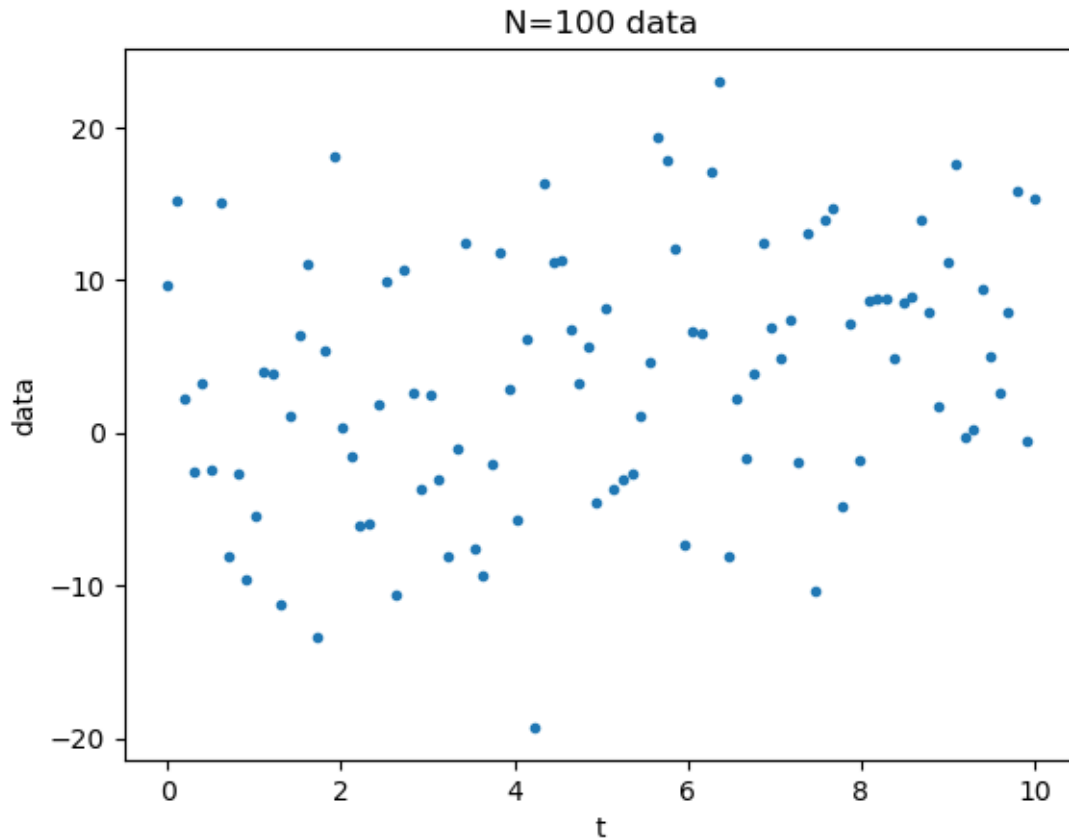
1.2 Problem 2

```
[21]: # Generate your unique numbers s and a from your PID (A18894844 -> 18894844)
student_number = 18894844
np.random.seed(student_number)
s = np.random.uniform(1, 10)
np.random.seed(student_number)
a = np.random.randn()
print(s, a)
```

7.6548296906390165 0.8122727023788983

```
[22]: def make_data(N):
    t = np.linspace(0, 10, N)
    signal = a * t
    noise = np.random.randn(N) * s
    data = signal + noise
    return t, data

# (a)
N = 100
t, data = make_data(N)
plt.plot(t, data, '.')
plt.xlabel("t")
plt.ylabel("data")
plt.title("N=100 data")
plt.show()
```



```
[23]: # (b)
def best_fit_slope(t, data):
    return np.dot(t, data) / np.dot(t, t)

a_hat = best_fit_slope(t, data)
print(a_hat)
```

0.7781379936726632

```
[24]: # (c)
slopes = []
for _ in range(1000):
    t, data = make_data(100)
    slopes.append(best_fit_slope(t, data))

plt.hist(slopes, bins=30)
plt.xlabel("estimated slope")
plt.ylabel("count")
plt.title("1000 fits (N=100)")
plt.show()
```

