

# Lecture 14: Model-Based RL

## CS486/686 Intro to Artificial Intelligence

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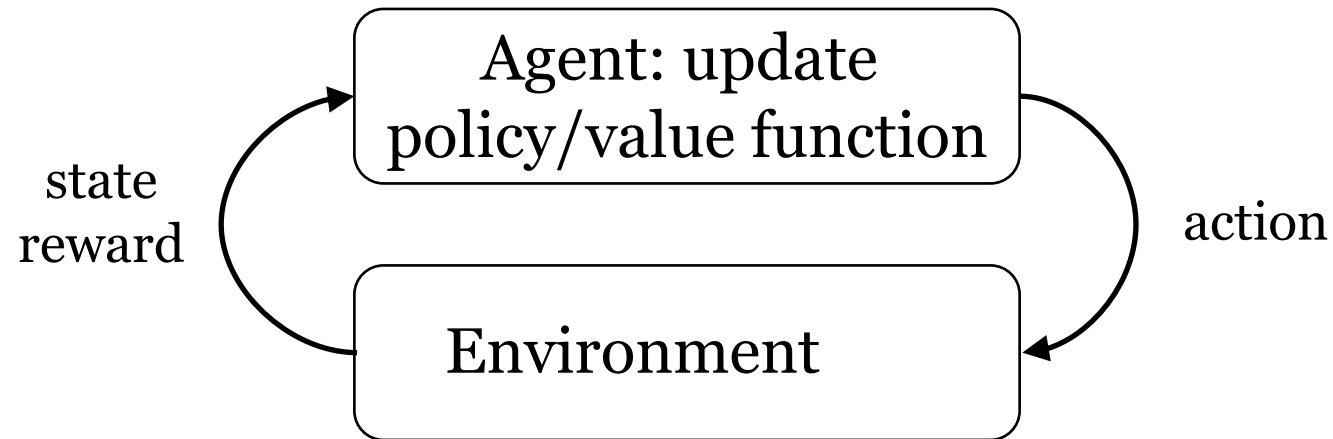
# Outline

- Model-based RL
- Dyna
- Monte-Carlo Tree Search

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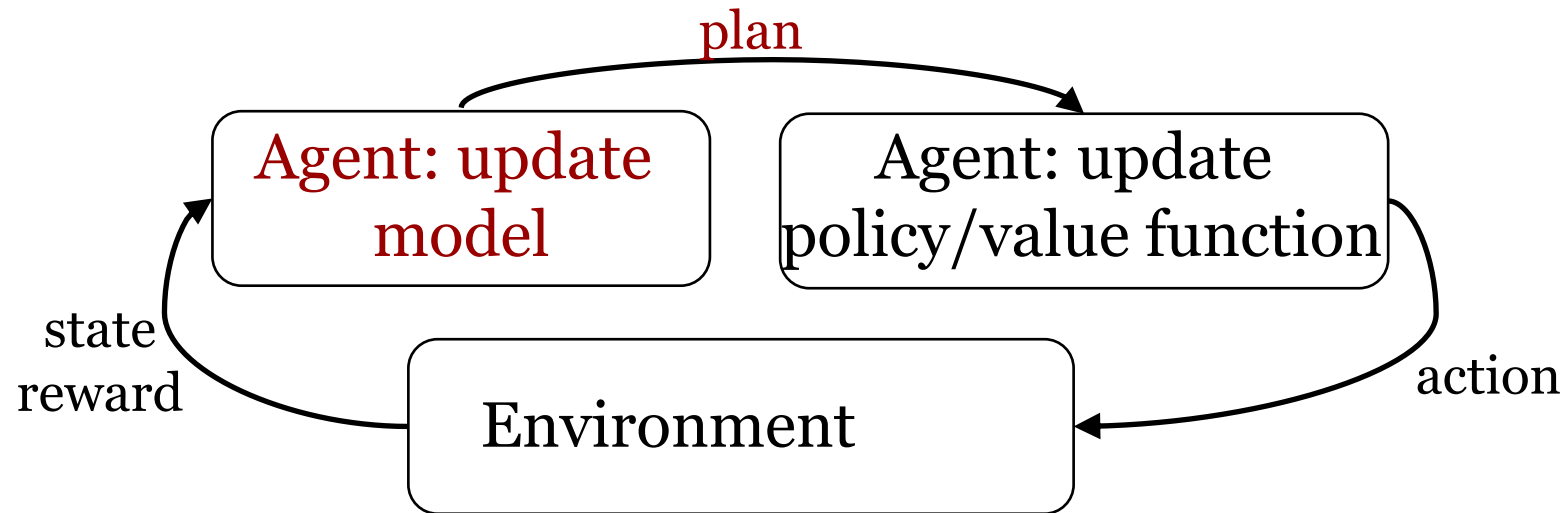
# Model-free Online RL

- No explicit transition or reward models
  - Q-learning: **value-based method**
  - Policy gradient: **policy-based method**



# Model-based Online RL

- Learn explicit transition and/or reward model
  - Plan based on the model
  - **Benefit: Increased sample efficiency**
  - **Drawback: Increased complexity**



# Maze Example

3	<b>r</b>	r	r	+1
2	u		u	-1
1	u	l	l	l
	1	2	3	4

$$\gamma = 1$$

Reward is -0.04 for non-terminal states

**We need to learn all the transition probabilities!**

$(1,1) \rightarrow (1,2) \rightarrow \mathbf{(1,3)} \rightarrow (1,2) \rightarrow \mathbf{(1,3)} \rightarrow (2,3) \rightarrow (3,3) \rightarrow (4,3)_{+1}$   
 $(1,1) \rightarrow (1,2) \rightarrow \mathbf{(1,3)} \rightarrow (2,3) \rightarrow (3,3) \rightarrow (3,2) \rightarrow (3,3) \rightarrow (4,3)_{+1}$   
 $(1,1) \rightarrow (2,1) \rightarrow (3,1) \rightarrow (3,2) \rightarrow (4,2)_{-1}$

$$\left. \begin{array}{l} P((2,3)|(1,3),r) = 2/3 \\ P((1,2)|(1,3),r) = 1/3 \end{array} \right\} \text{Use this information in}$$

$$V^*(s) = \max_a R(s, a) + \gamma \sum_{s'} \Pr(s'|s, a) V^*(s')$$

# Model-based RL

- Idea: at each step
  - Execute action
  - Observe resulting state and reward
  - Update transition and/or reward model
  - Update policy and/or value function

# Model-based RL (with Value Iteration)

## ModelBasedRL( $s$ )

Repeat

    Select and execute  $a$

    Observe  $s'$  and  $r$

    Update counts:  $n(s, a) \leftarrow n(s, a) + 1$ ,  
                                     $n(s, a, s') \leftarrow n(s, a, s') + 1$

    Update transition:  $\Pr(s'|s, a) \leftarrow \frac{n(s, a, s')}{n(s, a)} \quad \forall s'$

    Update reward:  $R(s, a) \leftarrow \frac{r + (n(s, a) - 1)R(s, a)}{n(s, a)}$

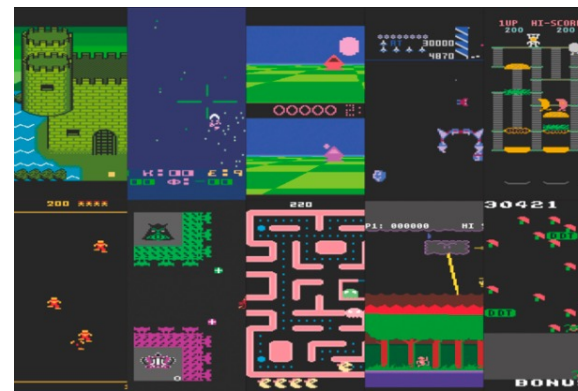
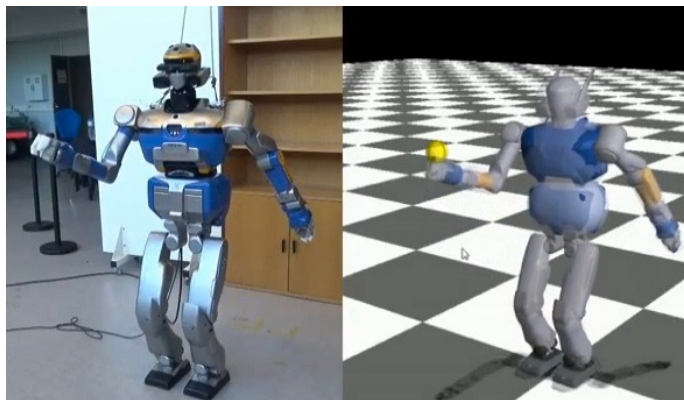
    Solve:  $V^*(s) = \max_a R(s, a) + \gamma \sum_{s'} \Pr(s'|s, a) V^*(s') \quad \forall s$

$s \leftarrow s'$

Until convergence of  $V^*$

Return  $V^*$

# Complex Models



- Use function approximation for transition and reward models
  - Linear model:  $pdf(s'|s, a) = N(s'|w^T \begin{bmatrix} s \\ a \end{bmatrix}, \sigma^2 I)$
  - Non-linear models:
    - Stochastic (e.g., Gaussian process):  $pdf(s'|s, a) = GP(s|w^T \begin{bmatrix} s \\ a \end{bmatrix}, \sigma^2 I)$
    - Deterministic (e.g., neural network):  $s' = T(s, a) = NN(s, a)$



# Partial Planning

- In complex models, fully optimizing the policy or value function at each time step is intractable
- Consider partial planning
  - A few steps of Q-learning
  - A few steps of policy gradient

# Model-based RL (with Q-learning)

## ModelBasedRL( $s$ )

Repeat

Select and execute  $a$ , observe  $s'$  and  $r$

Update transition:  $w_T \leftarrow w_T - \alpha_T (T(s, a) - s') \nabla_{w_T} T(s, a)$

Update reward:  $w_R \leftarrow w_R - \alpha_R (R(s, a) - r) \nabla_{w_R} R(s, a)$

Repeat a few times:

sample  $\hat{s}, \hat{a}$  arbitrarily

$\delta \leftarrow R(\hat{s}, \hat{a}) + \gamma \max_{\hat{a}'} Q(T(\hat{s}, \hat{a}), \hat{a}') - Q(\hat{s}, \hat{a})$

Update  $Q$ :  $w_Q \leftarrow w_Q - \alpha_Q \delta \nabla_{w_Q} Q(\hat{s}, \hat{a})$

$s \leftarrow s'$

Until convergence of  $Q$

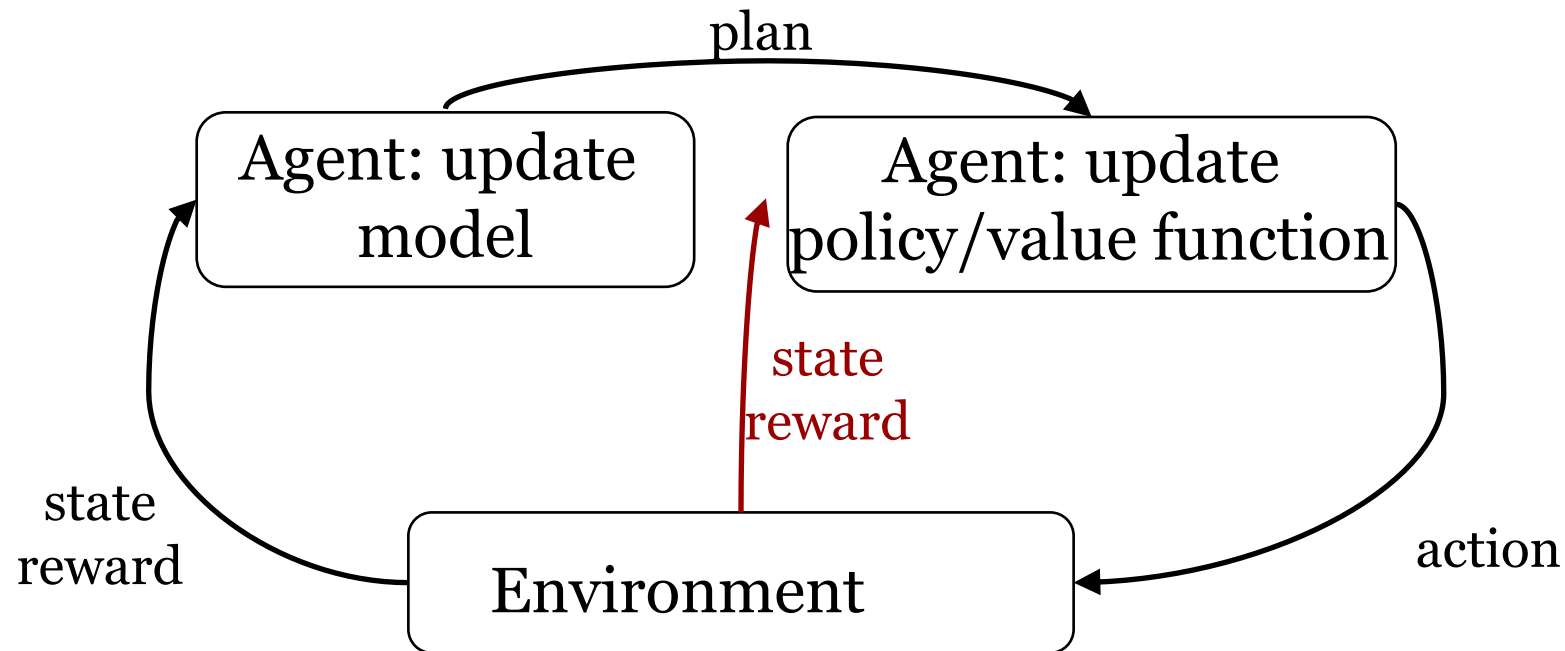
Return  $Q$

# Partial Planning vs Replay Buffer

- Previous algorithm is very similar to Model-free Q-learning with a replay buffer
- Instead of updating Q-function based on samples from replay buffer, generate samples from model
- Replay buffer:
  - Simple, real samples, no generalization to other state-action pairs
- Partial planning with a model
  - Complex, simulated samples, generalization to other state-action pairs (can help or hurt)

# Dyna

- **Learn explicit transition and/or reward model**
  - Plan based on the model
- **Learn directly from real experience**



# Dyna-Q

## Dyna-Q( $s$ )

Repeat

Select and execute  $a$ , observe  $s'$  and  $r$

Update transition:  $w_T \leftarrow w_T - \alpha_T (T(s, a) - s') \nabla_{w_T} T(s, a)$

Update reward:  $w_R \leftarrow w_R - \alpha_R (R(s, a) - r) \nabla_{w_R} R(s, a)$

$\delta \leftarrow r + \gamma \max_{a'} Q(s', a') - Q(s, a)$

Update  $Q$ :  $w_Q \leftarrow w_Q - \alpha_Q \delta \nabla_{w_Q} Q(s, a)$

Repeat a few times:

sample  $\hat{s}, \hat{a}$  arbitrarily

$\delta \leftarrow R(\hat{s}, \hat{a}) + \gamma \max_{\hat{a}'} Q(T(\hat{s}, \hat{a}), \hat{a}') - Q(\hat{s}, \hat{a})$

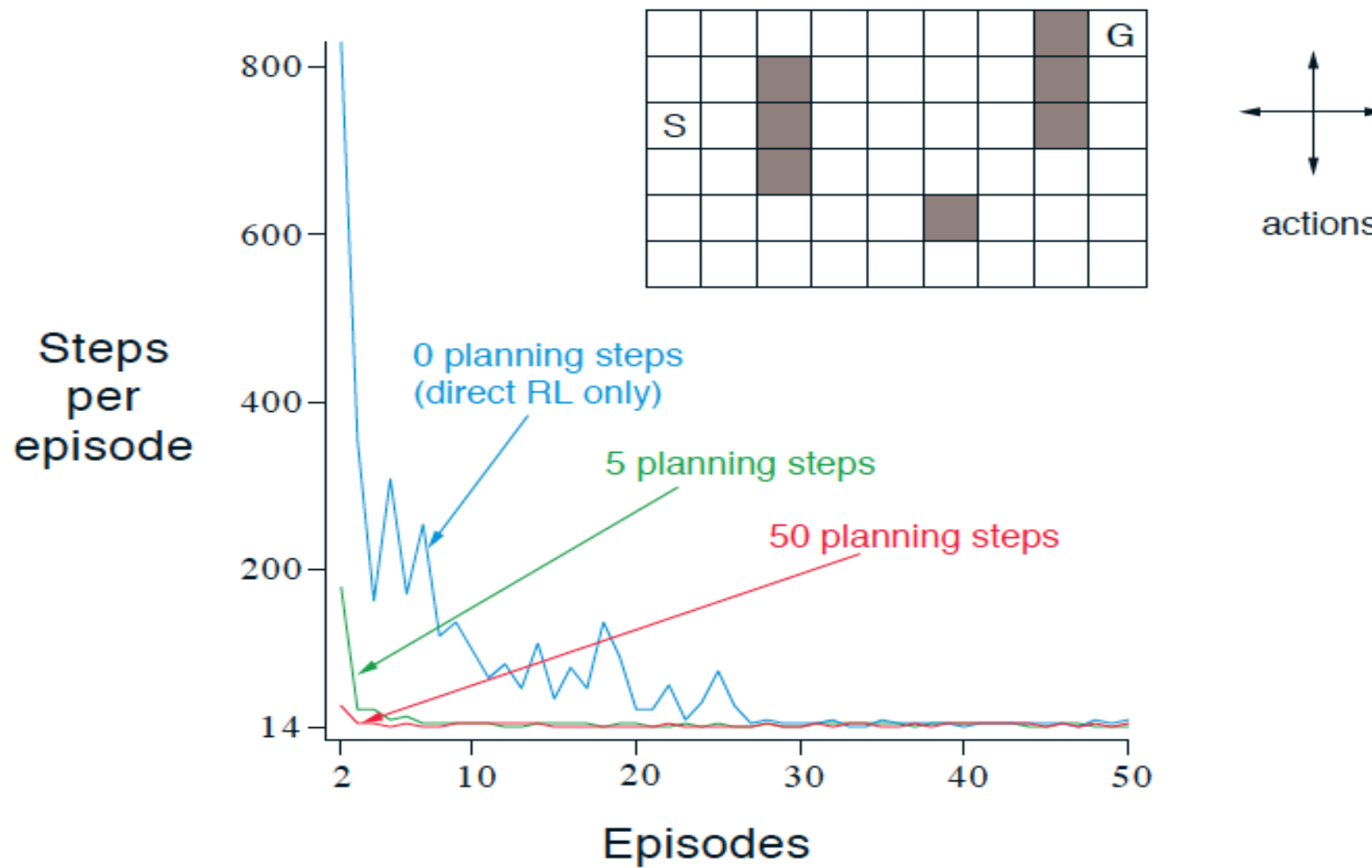
Update  $Q$ :  $w_Q \leftarrow w_Q - \alpha_Q \delta \nabla_{w_Q} Q(\hat{s}, \hat{a})$

$s \leftarrow s'$

Return  $Q$

# Dyna-Q

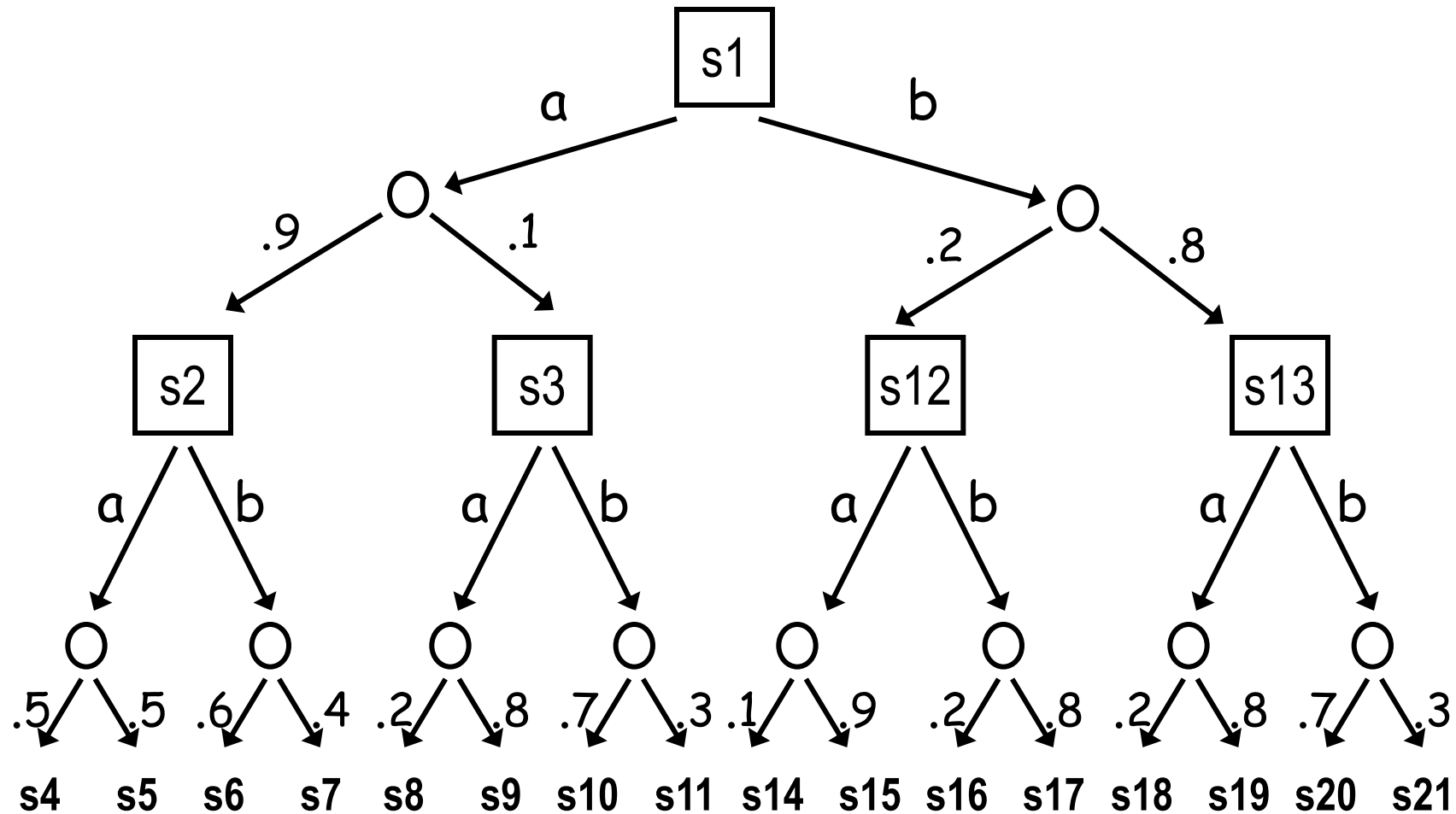
Task:  
reach G  
from S



# Planning from Current State

- Instead of planning at arbitrary states, plan from the current state
  - This helps improve next action
- Monte Carlo Tree Search

# Tree Search





# Tractable Tree Search

- Combine 3 ideas:

- Leaf nodes: **approximate leaf values with value of default policy  $\pi$**

$$Q^*(s, a) \approx Q^\pi(s, a) \approx \frac{1}{n(s, a)} \sum_{k=1}^n G_k$$

- Chance nodes: **approximate expectation by sampling from transition model**

$$Q^*(s, a) \approx R(s, a) + \gamma \frac{1}{n(s, a)} \sum_{s' \sim \text{Pr}(s'|s, a)} V(s')$$

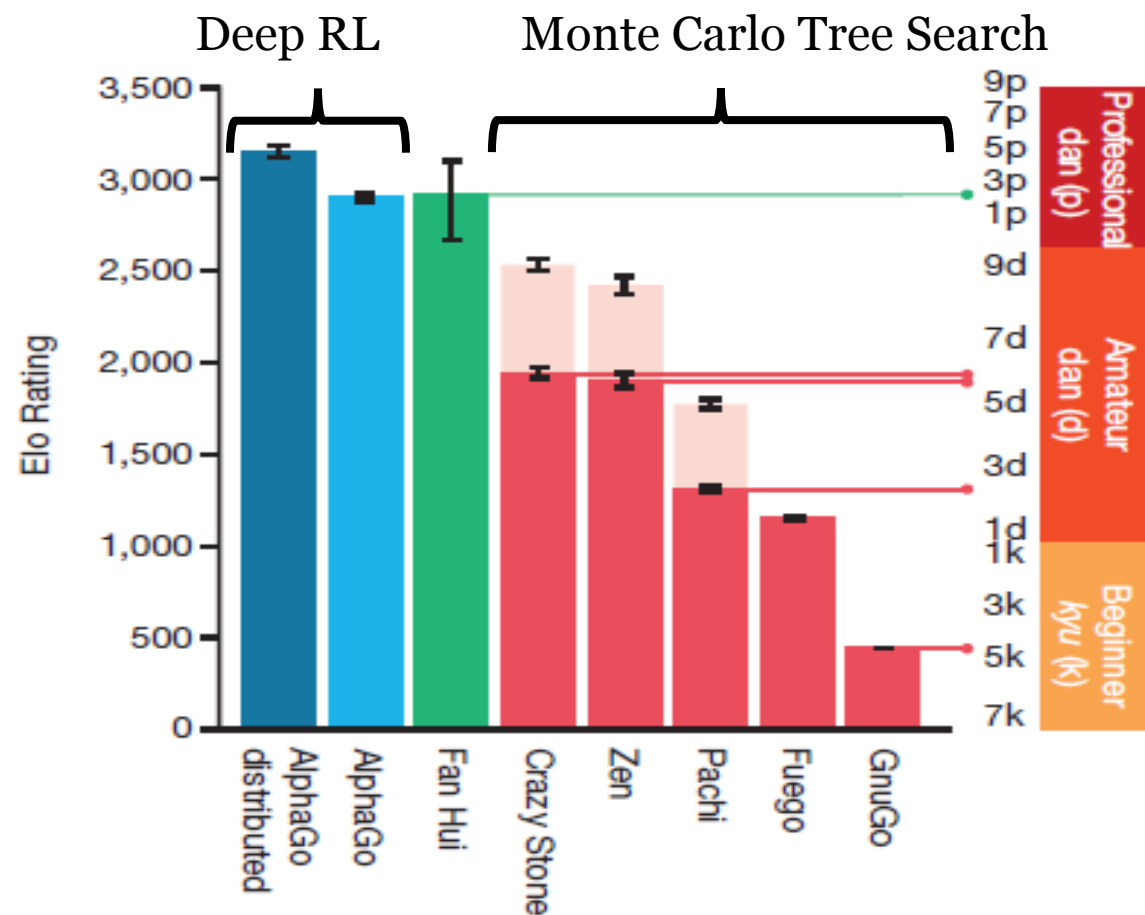
- Decision nodes: **expand only most promising actions**

$$a^* = \operatorname{argmax}_a Q(s, a) + c \sqrt{\frac{2 \ln n(s)}{n(s, a)}} \quad \text{and} \quad V^*(s) = Q(s, a^*)$$

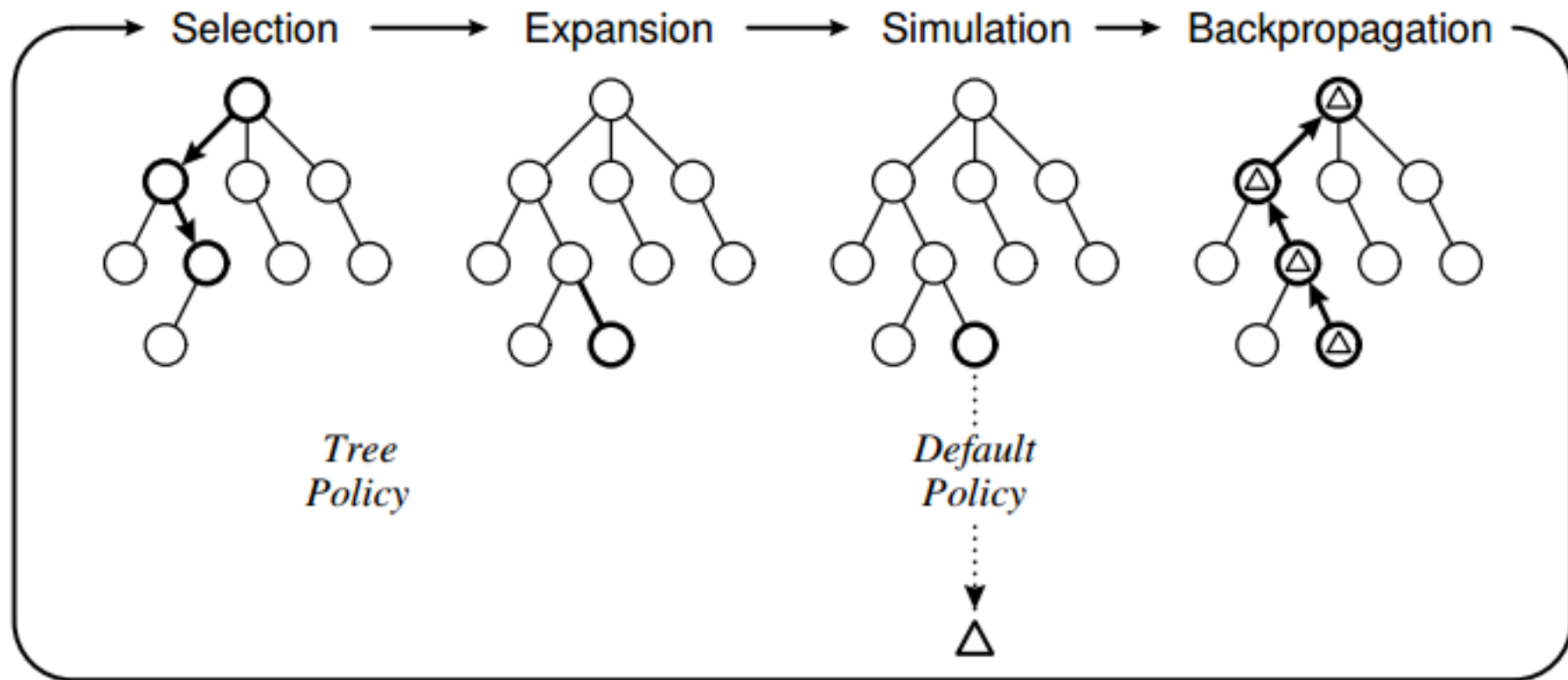
- Resulting algorithm: Monte Carlo Tree Search

# Computer Go

- Oct 2015:



# Monte Carlo Tree Search



# Monte Carlo Tree Search (with upper confidence bound)

**UCT( $s_0$ )**

```
create root  $node_0$  with state  $state(node_0) \leftarrow s_0$   
while within computational budget do  
   $node_l \leftarrow TreePolicy(node_0)$   
   $value \leftarrow DefaultPolicy(state(node_l))$   
   $Backpropagate(node_l, value)$   
return  $action(SelectBestChild(node_0, 0))$ 
```

**TreePolicy( $node$ )**

```
while  $node$  is nonterminal do  
  if  $node$  is not fully expanded do  
    return  $Expand(node)$   
  else  
     $node \leftarrow SelectBestChild(node, C)$   
return  $node$ 
```

# Monte Carlo Tree Search (continued)

## Expand(*node*)

choose  $a \in$  untried actions of  $A(\text{state}(\text{node}))$   
add a new child  $\text{node}'$  to  $\text{node}$   
with  $\text{state}(\text{node}') \leftarrow T(\text{state}(\text{node}), a)$   
return  $\text{node}'$

deterministic  
transition

## SelectBestChild(*node*, *c*)

return  $\arg \max_{\text{node}' \in \text{children}(\text{node})} V(\text{node}') + c \sqrt{\frac{(2 \ln n(\text{node}))}{n(\text{node}')}}$

## DefaultPolicy(*node*)

while  $\text{node}$  is not terminal do  
sample  $a \sim \pi(a | \text{state}(\text{node}))$   
 $s' \leftarrow T(\text{state}(\text{node}), a)$   
return  $R(s, a)$

# Monte Carlo Tree Search (continued)

## Single Player

**Backpropagate(*node*, *value*)**

while *node* is not null do

$$V(\textit{node}) \leftarrow \frac{n(\textit{node})V(\textit{node}) + \textit{value}}{n(\textit{node}) + 1}$$

$$n(\textit{node}) \leftarrow n(\textit{node}) + 1$$

$$\textit{node} \leftarrow \textit{parent}(\textit{node})$$

## Two Players (adversarial)

**BackpropagateMinMax(*node*, *value*)**

while *node* is not null do

$$V(\textit{node}) \leftarrow \frac{n(\textit{node})V(\textit{node}) + \textit{value}}{n(\textit{node}) + 1}$$

$$n(\textit{node}) \leftarrow n(\textit{node}) + 1$$

$$\textit{value} \leftarrow -\textit{value}$$

$$\textit{node} \leftarrow \textit{parent}(\textit{node})$$

# AlphaGo

Four steps:

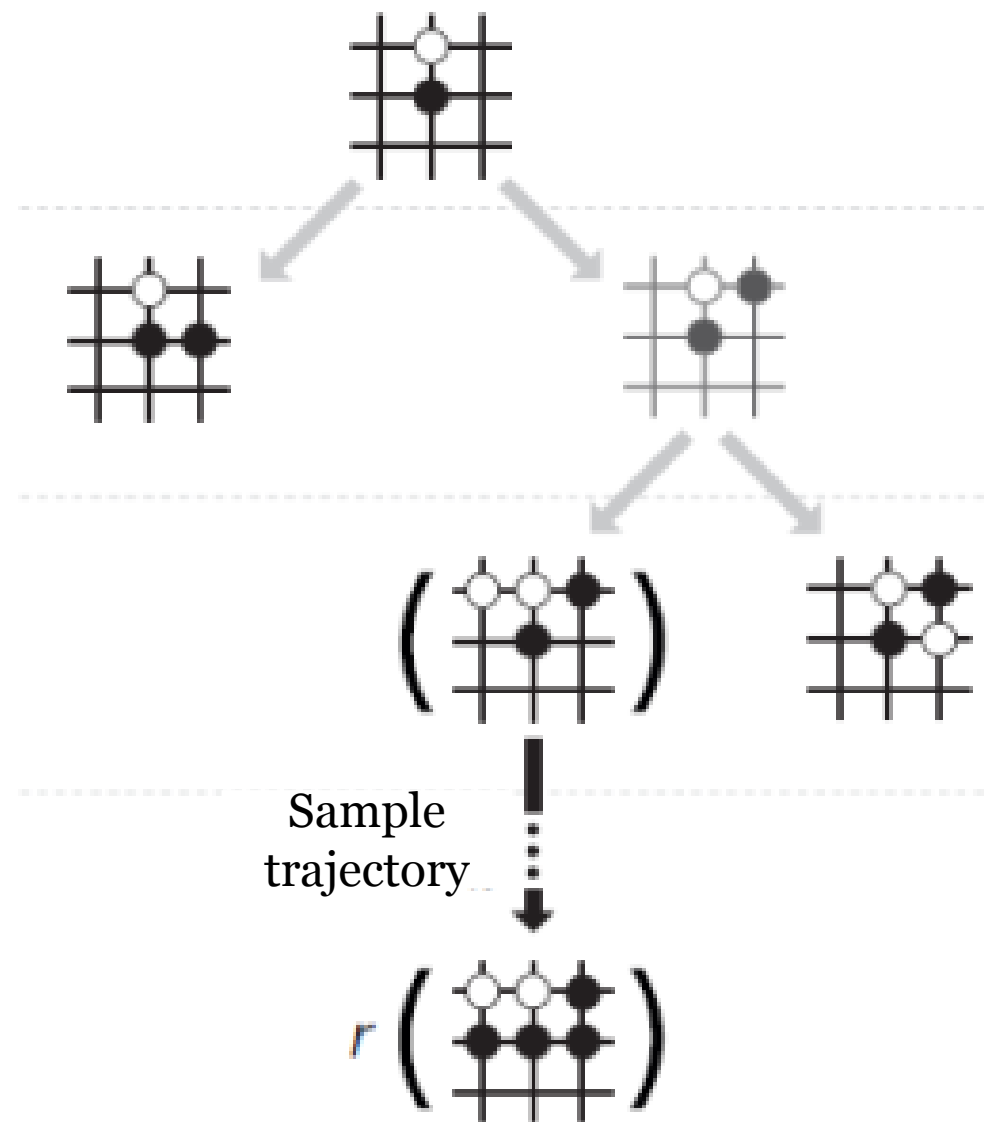
1. Supervised Learning of Policy Networks
2. Policy gradient with Policy Networks
3. Value gradient with Value Networks
4. Searching with Policy and Value Networks
  - Monte Carlo Tree Search variant

# Search Tree

- At each edge store

$$Q(s, a), \pi(a|s), n(s, a)$$

- Where  $n(s, a)$  is the visit count of  $(s, a)$





# Simulation

- At each node, select edge  $a^*$  that maximizes
$$a^* = \operatorname{argmax}_a Q(s, a) + u(s, a)$$

- where  $u(s, a) \propto \frac{\pi(a|s)}{1+n(s,a)}$  is an exploration bonus

$$Q(s, a) = \frac{1}{n(s,a)} \sum_i 1_i(s, a) [\lambda V_w(s) + (1 - \lambda)G_i]$$

$$1_i(s, a) = \begin{cases} 1 & \text{if } (s, a) \text{ was visited at iteration } i \\ 0 & \text{otherwise} \end{cases}$$