Program of Thoughts Prompting: Disentangling Computation from Reasoning for Numerical Reasoning Tasks

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Abstract

Recently, there has been significant progress in teaching language models to perform step-by-step reasoning to solve complex numerical reasoning tasks. Chain-of-thoughts prompting (CoT) is by far the state-of-art method for these tasks. CoT uses language models to perform both reasoning and computation in the multi-step 'thought' process. To disentangle computation from reasoning, we propose 'Program of Thoughts' (PoT), which uses language models (mainly Codex) to express the reasoning process as a program. The computation is relegated to an external computer, which executes the generated programs to derive the answer. We evaluate PoT on five math word problem datasets (GSM, AQuA, SVAMP, TabMWP, MultiArith) and three financial-QA datasets (FinQA, ConvFinQA, TATQA) for both few-shot and zero-shot setups. Under both few-shot and zero-shot settings, PoT can show an average performance gain over CoT by around 12% across all the evaluated datasets. By combining PoT with self-consistency decoding, we can achieve SoTA performance on all math problem datasets and near-SoTA performance on financial datasets. All of our data and code are released in Github[1].

1 Introduction

Numerical reasoning is a long-standing task in artificial intelligence. A surge of datasets has been proposed recently to benchmark deep-learning models' capabilities to perform numerical/arithmetic reasoning. Some widely used benchmarks are based on Math word problems (MWP) (Cobbe et al., 2021; Patel et al., 2021; Lu et al., 2022; Ling et al., 2017), where the intelligence systems are supposed to answer math questions expressed with natural text. Besides MWP, some datasets also consider financial problems (Chen et al., 2021b, 2022; Zhu et al., 2021), where the intelligent systems need to answer math-driven financial questions.

Prior work (Ling et al., 2017; Cobbe et al., 2021) has studied how to train models from scratch or fine-tune models to generate intermediate steps to derive the final answer. Such methods are data-intensive requiring a significant amount of training examples with expert-annotated solving steps for these questions. Recently, Wei et al. (2022) have discovered that the large language models (LLMs) (Brown et al., 2020; Chen et al., 2021a; Chowdhery et al., 2022) can be 'prompted' with a few input-output exemplars to solve these tasks without any training or fine-tuning. When demonstrated with a few examples of 'natural language rationale', LLMs can imitate the demonstrations to generate rationale and answer these questions. Such a prompting method is dubbed 'Chain of Thoughts (CoT)', which is able to achieve state-of-the-art performance on a wide spectrum of textual/numerical reasoning datasets.

CoT uses LLMs for both reasoning and computation, i.e. the language model not only needs to 'generate' the mathematical expressions but also needs to 'solve' them step by step. We argue that language models are not ideal for 'solving' these mathematical expressions: 1) LLMs are highly prone to calculation errors, especially when dealing with large numbers, and 2) LLMs cannot solve complex expressions like 'polynomial equations' or even 'differential equations. 3) LLMs are highly redundant at expressing higher-order operations like iterations, especially when the iteration step is enormous. Such limitations restrict CoT's capability to solve complex math problems. PoT greatly decreases the randomness of computation, thus yielding more accurate results.

In order to solve these issues, we propose

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* Work done at University of Waterloo. Wenhu Chen is responsible for paper writing and running Math QA experiments, and Xueguang Ma is responsible for running financial QA experiments and paper polishing.

[1] https://github.com/wenhuchen/Program-of-Thoughts

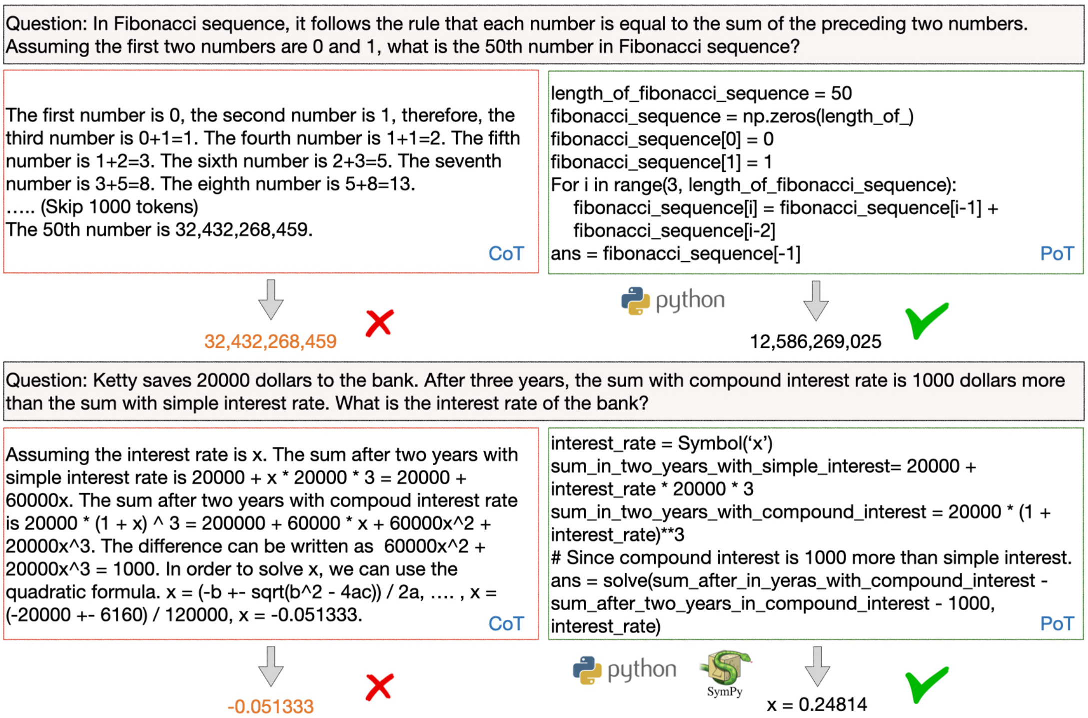

Figure 1: Comparison between Chain of Thoughts and Program of Thoughts.

program-of-thoughts (PoT) prompting to relegate the computation step to an external computer. In essence, LMs are only responsible for expressing thoughts in Python programs, while the computation is accomplished by the an external computer. We depict the difference between CoT and PoT in Figure 1. In the upper example, the iteration runs for 50 times, which not only consumes a massive amount of tokens but also leads to extremely low accuracy[2]. In the lower example, CoT cannot solve the cubic equation with language models and outputs a wrong answer. For the upper example, PoT just needs to express the iteration process with a few lines of code, which can be executed on a Python interpreter to drive the accurate answer. For the lower example, PoT only needs to convert the problem into a program and then resort to the 'SymPy' library to solve it accurately.

We evaluate PoT prompting across five MWP datasets: GSM8K, AQuA, SVAMP, TabMWP, MultiArith, and three financial datasets: FinQA, ConvFinQA, and TATQA. These datasets cover heterogeneous input forms including text, table, and conversation. We give a performance overview in Figure 2. Under both few-shot and zero-shot settings, PoT outperforms CoT significantly across all the evaluated datasets. Under the few-shot setting, the average gain over CoT is around 8% for MWP datasets and 15% for financial datasets. Under the zero-shot setting, the average gain over CoT is also around 12% for MWP datasets. PoT combined with self-consistency (SC) also outperforms CoT-SC (Wang et al., 2022b) by an average of 10% across all datasets. Our PoT+SC can achieve the best-known results on all the evaluated MWP datasets and near SoTA results on financial datasets. Finally, we conduct comprehensive ablation studies to understand different components of PoT.

[2]Assuming each addition is correct with 90% chance, after 50 times of addition, the likelihood of correct output is 0.5%.

2 Program of Thoughts

2.1 Preliminaries

'Prompting' or 'in-context learning' has been proposed in (Brown et al., 2020; Chen et al., 2021a; Chowdhery et al., 2022; Rae et al., 2021). Compared with fine-tuning, in-context learning (1) only takes a few annotations/demonstrations as a prompt, and (2) performs inference without training the model parameters. the LLMs receive the input-output exemplars as the prefix to generate outputs imitating the exemplars. More recently, 'chain of thoughts prompting' (Wei et al., 2022) has been proposed as a specific type of in-context learning where the exemplar's output contains the 'thought process' instead of just an output. This approach has been shown to elicit LLMs' reasoning capabilities by demonstrating reasoning steps.

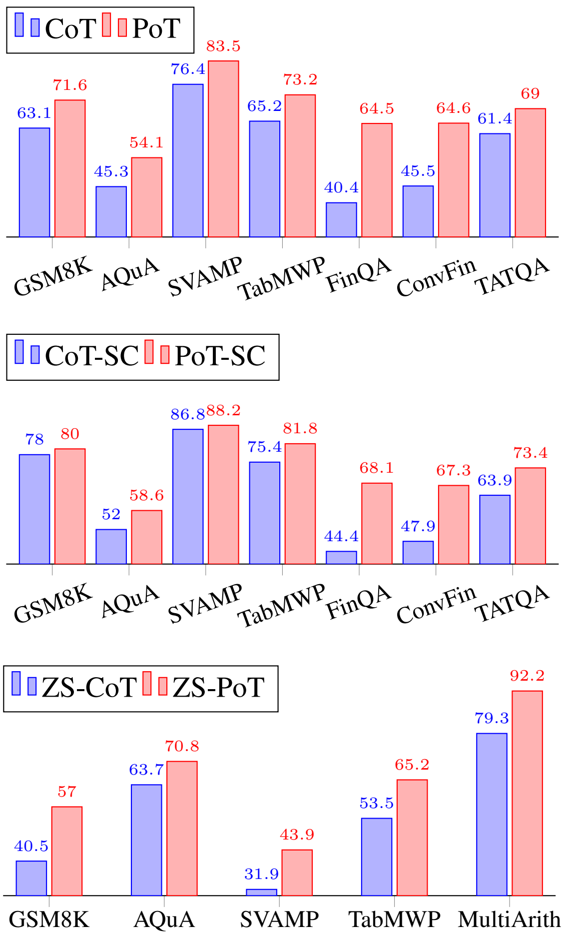

Figure 2: Few-shot (upper), Few-shot + SC (middle) and Zero-Shot (lower) Performance overview of Codex PoT and Codex CoT across different datasets.

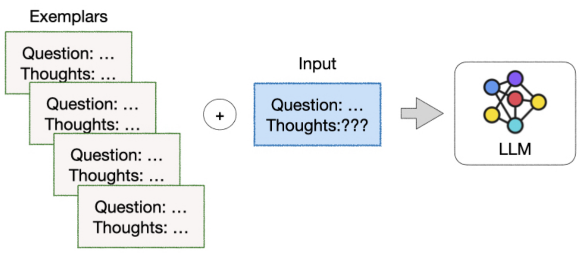

Figure 3: Prompting LLMs to generate outputs with few-shot exemplars.

2.2 Program of Thoughts

Besides natural language, programs can also be used to express our thought processes. By binding semantic meaning to variable names, the program can also be seen as a natural representation to convey human thoughts. For example, in the lower example in Figure 1, we first create an unknown variable of our interest as `interest_rate`. Then we bind 'summation in two years with ... interest rate' to the variable `sum_in_two_years_with_XXX_interest` and write down the equation expressing their mathematical relations with `interest_rate`. These equations are packaged into the 'solve' function provided by 'SymPy'. The program is executed with Python to solve the equations to derive the variable of `interest_rate` as the answer.

Unlike CoT, PoT relegates the 'computation process' or 'solving process' to an external computer. The LLMs are only responsible for binding the 'reasoning process' into the programming language. The computation of the method is relegated to the Python interpreter. In contrast, CoT aims to adopt language models to perform both 'reasoning' and 'computation'. We argue that such an approach is more expressive and accurate.

The 'program of thoughts' is different from generating equations directly, where the generation target would be $\texttt{solve}(20000 * (1+x)^3 - 2000 - x * 20000 * 3 - 1000, x)$. As observed by CoT (Wei et al., 2022), directly generating such equations is challenging for LLMs. PoT differs in two aspects: (1) PoT breaks down the equation into a multi-step 'thought' process, and (2) PoT binds semantic meanings to variables to help the model ground. We found that such a 'thoughtful' process can elicit language models' reasoning capabilities to generate more accurate programs. We provide a detailed comparison in the experimental section.

2.3 PoT Prompting

We show the proposed PoT prompting method in Figure 4 under the few-shot and zero-shot settings. Under the few-shot setting, a few exemplars of (question, 'program of thoughts') pairs will be prefixed as demonstrations to teach the LLM how to generate 'thoughtful' programs. Under the zero-shot setting, the prompt only contains an instruction without any exemplar demonstration. Unlike zero-shot CoT (Kojima et al., 2022), which requires an extra step to extract the answer from the 'chain of thoughts', zero-shot PoT can return the answer straightforwardly without extra steps.

2.4 PoT as Intermediate Step

For certain problems requiring additional commonsense reasoning, we could first utilize PoT to generate a program to compute the answer, which is combined with the question to continue prompting LLM to derive the final answer. Like the left example in Figure 4, the program will generate a float number 2.05. However, adding 2.05 to 11 AM cannot be easily handled by the Python pro-

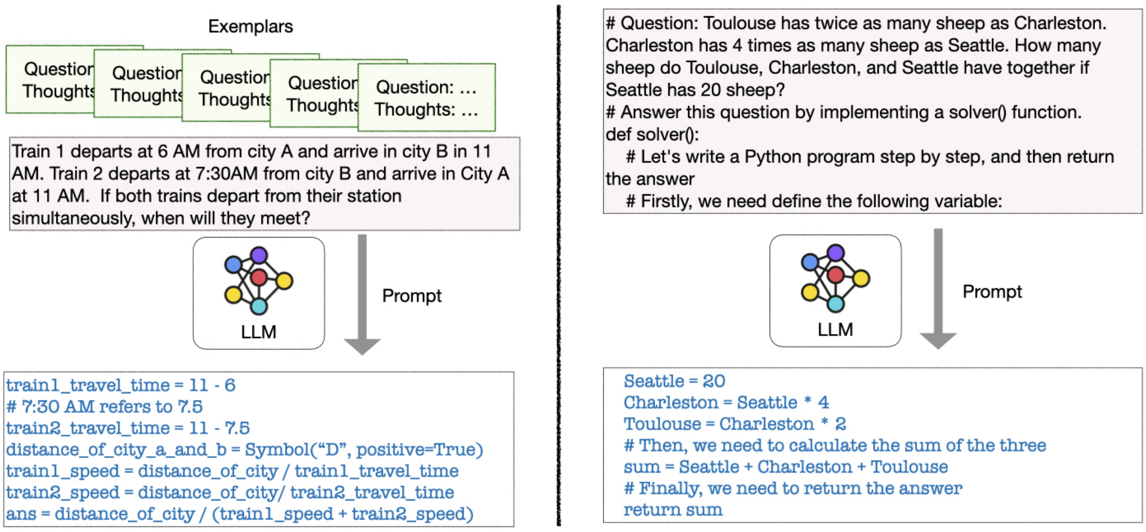

Figure 4: Left: Few-shot PoT prompting, Right: Zero-shot PoT prompting.

gram. Therefore, we continue to prompt LLM to perform an additional step of textual reasoning to derive the final answer. In our experiments, we adopt this approach for the AQuA dataset due to its complexity.

3 Experiments

3.1 Experimental Setup

Datasets We summarize our evaluated datasets in Table 1. We use the test set for all the evaluated datasets except TATQA. These datasets are highly heterogeneous in terms of their input formats. We conduct comprehensive experiments on this broad spectrum of datasets to show the generalizability and applicability of PoT prompting.

To incorporate the diverse inputs, we propose to linearize these inputs in the prompt. For table inputs, we adopt the same strategy as Chen (2022) to linearize a table into a text string. The columns of the table are separated by '|' and the rows are separated by '\n'. If a table cell is empty, it is filled by '-'. For text+table hybrid inputs, we will separate tables and text with '\n'. For conversation history, we can also separate conversation turns by '\n' and then feed them to the language models. The prompt is constructed by the concatenation of task instruction, text, linearized table, and question. For conversation question answering, we simply concatenate all the dialog history in the prompt.

Implementation Details We use the OpenAI Codex (code-davinci-002) API[3] and GPT-3 (text-davinci-002) API[4] model in our experiments. We use Python 3.8 with SymPy library[5] to execute the generated program. For the few-shot setting, we use 4-8 shots for all the datasets based on their difficulty. For simple datasets like SVAMP (Patel et al., 2021) and FinQA (Chen et al., 2021b), we only use 4 shots, while for the other more challenging datasets like AQuA (Ling et al., 2017) and TATQA (Zhu et al., 2021), we adopt 8 shots to cover diverse problems. The examples are taken from the training set. We mostly write 10-20 examples, and then tune the exemplar selection on a small validation set to choose the best 4-8 shots for the full set evaluation.

To elicit LLM's capability to perform multi-step reasoning, we use "Let's write Python program step by step" as our prompt. However, the caveat is that LLM can fall back to generating a reasoning chain in comments rather than in program. Therefore, we suppress the '#' token logits by -2 to decrease its probability. We found that such a simple strategy can greatly improve the success rate.

Metrics We adopt exact match scores as our evaluation metrics for GSM8K, SVAMP, and MultiArith datasets. We will round the predicted number to a specific precision and then compare it with the reference number. For the AQuA dataset, we use PoT to compute the intermediate answer and then prompt the LLM again to output the closest option to measure the accuracy. For TabMWP, ConvFinQA, and TATQA datasets, we use the official evaluation scripts provided in their Github.

[3] https://openai.com/blog/openai-codex/

[4] https://beta.openai.com/

[5] https://www.sympy.org/en/index.html

Dataset	Split	Example	Domain	Input	Output
GSM8K (Cobbe et al., 2021)	Test	1318	MWP	Question	Number
AQuA (Ling et al., 2017)	Test	253	MWP	Question	Option
SVAMP (Patel et al., 2021)	Test	1000	MWP	Question	Number
MultiArith (Roy and Roth, 2015)	Test	600	MWP	Question	Number
TabMWP (Lu et al., 2022)	Test	7861	MWP	Table + Question	Number + Text
FinQA (Chen et al., 2021b)	Test	1147	Finance	Table + Text + Question	Number + Binary
ConvFinQA (Chen et al., 2022)	Test	421	Finance	Table + Text + Multi-Turn Question	Number + Binary
TATQA (Zhu et al., 2021)	Dev	1668	Finance	Table + Text + Question	Number + Text

Table 1: Summarization of all the datasets being evaluated.

For FinQA, we relax the evaluation for CoT because LLMs cannot perform the computation precisely (especially with high-precision floats and large numbers). We adopt 'math.isclose' with relative tolerance of 0.001 to make it more comparable.

Baselines We report results for three different models including Codex (Chen et al., 2021a), GPT-3 (Brown et al., 2020), and PaLM (Chowdhery et al., 2022) and LaMDA (Thoppilan et al., 2022). We consider two types of prediction strategies including direct answer output, and chain of thought to derive the answer. Since PaLM API is not public, we only list PaLM results reported from previous work (Wei et al., 2022; Wang et al., 2022b). We also leverage an external calculator as suggested in Wei et al. (2022) to all the equations generated by CoT, which is denoted as CoT + calc. Besides greedy decoding, we also use self-consistency decoding (Wang et al., 2022b) in CoT and adopt the majority vote of 40 different completions as the final prediction.

3.2 Main Results

Few-shot Results We demonstrate our few-shot results in Table 2 to compare with the baselines. On MWP datasets, PoT with greedy decoding can improve GSM8K/AQuA/TabMWP by more than 8%. On SVAMP, the improvement is 4% mainly due to its simplicity. For financial QA datasets, PoT improves over CoT by roughly 20% on FinQA/ConvFinQA and 8% on TATQA. The larger improvements in FinQA and ConvFinQA are mainly due to miscalculation issues of LLMs for large numbers (e.g. in the million level). CoT adopts LLMs to perform the computation, which is highly prone to miscalculation errors, while PoT adopts a highly precise external computer to solve the problem. As an ablation, we also compare with CoT+calc, which leverages an external calculator to correct the calculation results in the generated 'chain of thoughts'. The experiments show that adding an external calculator only shows mild improvement over CoT on MWP datasets, which is still far behind PoT.

Few-shot + Self-Consistency Results We leverage self-consistency (SC) decoding to understand the upper bound of our method. Such a sampling-based decoding algorithm can greatly reduce the randomness in the generation procedure and boost performance. Specifically, we set a temperature of 0.4 and K=40 throughout our experiments. According to to Table 2, we found that PoT + SC still outperforms CoT + SC significantly on MWP datasets with notable margins. On financial datasets, we observe that self-consistency decoding is less impactful for both PoT and CoT. Similarly, PoT + SC outperforms CoT + SC by roughly 20% on FinQA/ConvFinQA, and 7% on TATQA.

Zero-shot Results Further, we also evaluate the zero-shot performance of PoT and mainly compare with Kojima et al. (2022) in Table 3. As can be seen, zero-shot PoT significantly outperforms zero-shot CoT across all the MWP datasets evaluated. Compared to few-shot prompting, zero-shot PoT outperforms zero-shot CoT by an even larger margin. On the evaluated datasets, PoT's outperforms CoT by an average of 12%. On TabMWP, zero-shot PoT is even higher than few-shot CoT. These results show the great potential to directly generalize to many unseen numerical tasks even without any dataset-specific exemplars.

3.3 Ablation Studies

Here we perform multiple ablation studies under few-shot setting to understand the importance of different factors in PoT.

Backend GPT-3 vs. Codex We further evaluate how GPT-3 (text-davinci-002) will perform with PoT prompting. Unlike Codex, GPT-3 is not opti-

Model	#Params	GSM8K	AQuA	SVAMP	TabWMP	FinQA	ConvFinQA	TATQA	Avg
Fine-tuned or few-shot prompt									
Published SoTA	-	78.0	52.0	86.8	68.2	68.0	**68.9**	**73.6**	70.7
Few-shot prompt (Greedy Decoding)									
Codex Direct	175B	19.7	29.5	69.9	59.4	25.6	40.0	55.0	42.7
Codex CoT	175B	63.1	45.3	76.4	65.2	40.4	45.6	61.4	56.7
GPT-3 Direct	175B	15.6	24.8	65.7	57.1	14.4	29.1	37.9	34.9
GPT-3 CoT	175B	46.9	35.8	68.9	62.9	26.1	37.4	42.5	45.7
PaLM Direct	540B	17.9	25.2	69.4	-	-	-	-	-
PaLM CoT	540B	56.9	35.8	79.0	-	-	-	-	-
Codex CoT$_{calc}$	175B	65.4	45.3	77.0	65.8	-	-	-	-
GPT-3 CoT$_{calc}$	175B	49.6	35.8	70.3	63.4	-	-	-	-
PaLM CoT$_{calc}$	540B	58.6	35.8	79.8	-	-	-	-	-
Codex PoT	175B	71.6	54.1	83.6	73.2	64.5	64.6	69.0	68.6
Few-shot prompt (Self-Consistency Decoding)									
LaMDA CoT-SC	137B	27.7	26.8	53.5	-	-	-	-	-
Codex CoT-SC	175B	78.0	52.0	86.8	75.4	44.4	47.9	63.2	63.9
PaLM CoT-SC	540B	74.4	48.3	86.6	-	-	-	-	-
Codex PoT-SC	175B	**80.0**	**58.6**	**88.2**	**81.8**	**68.1**	67.3	70.2	**73.4**

Table 2: The few-shot results for different datasets. Published SoTA includes the best-known results. on GSM8K, AQuA and SVAMP, the Prior SoTA results are CoT + self-consistency decoding (Wang et al., 2022b). On FinQA, the prior best result is achieved by Wang et al. (2022a). On ConvFinQA, the prior best result is achieved by FinQANet (Chen et al., 2022). On TabWMP (Lu et al., 2022), the prior best result is achieved by Dynamic Prompt Learning (Lu et al., 2022). On TATQA, the SoTA result is achieved by RegHNT (Lei et al., 2022).

Model	#Params	GSM8K	AQuA	SVAMP	TabMWP	MultiArith	Avg
Zero-shot Direct (GPT-3)	150B	12.6	22.4	58.7	38.9	22.7	31.0
Zero-shot CoT (GPT-3)	150B	40.5	31.9	63.7	53.5	79.3	53.7
Zero-shot CoT (PaLM)	540B	43.0	-	-	-	66.1	-
Zero-shot PoT	150B	**57.0**	**43.9**	**70.8**	**66.5**	**92.2**	**66.1**

Table 3: The zero-shot results for different datasets. The baseline results are taken from Kojima et al. (2022).

mized for generating programs, we are curious how much degradation we will experience with GPT-3 as the backend. We choose three datasets: GSM8K, SVAMP, and FinQA to analyze the performance difference of PoT and compare that relative with CoT. We show our experimental results in Table 4. We can see that the gap between Codex and GPT-3 with PoT is consistently smaller than their gap with CoT. We conclude that our prompting approach is still effective for models that are not specifically optimized for program generation. However, we do observe that the gap will increase as the dataset becomes more challenging.

Model	GSM8K	SVAMP	FinQA
Codex CoT	63.1	76.4	40.4
GPT3 CoT	46.9	58.9	26.1
Codex - GPT3 (CoT)	16.2	7.5	14.3
Codex PoT	71.6	83.5	64.5
GPT3 PoT	60.4	79.2	56.7
Codex - GPT3 (PoT)	11.2	4.4	7.8

Table 4: GPT-3 and Codex performance difference under CoT and PoT prompting.

Sensitivity to Exemplars To better understand how sensitive PoT is w.r.t different exemplars, we conduct further sensitivity analysis. Specifically, we write 20 total exemplars. For k-shot learning, we randomly sample k = (2, 4, 6, 8) out of the 20 exemplars three times as v1, v2, and v3. We will use these randomly sampled exemplars as demonstrations for PoT. We demonstrate our sensitivity analysis in Figure 5. First of all, we found that increasing the number of shots helps more for GSM8K than FinQA. This is mainly due to the diversity of questions in GSM8K. By adding more exemplars, the language models can better generalize to diverse questions. Another observation is that: when pre-

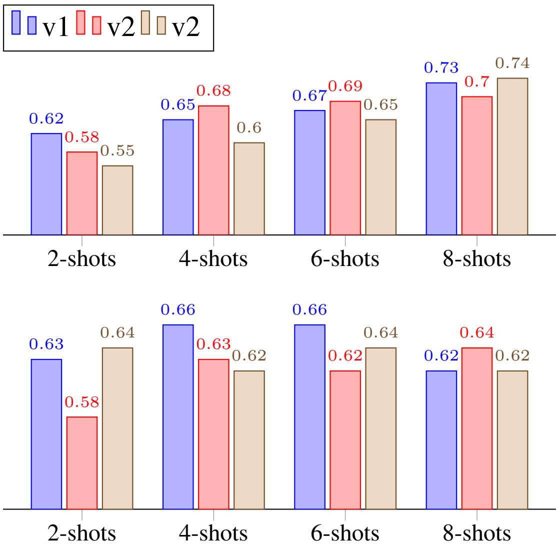

Figure 5: Exemplar sensitivity analysis for GSM8K and FinQA, where v1, v2 and v3 are three versions of k-shot demonstration sampled from the pool.

Method	GSM8K	SVAMP	FinQA
PoT	71.6	83.6	64.5
PoT - Binding	60.2	83.0	61.6
PoT - MultiStep	45.8	81.2	58.9

Table 5: Comparison between PoT and equation generation on three different datasets.

sented with fewer exemplars, PoT's performance variance is larger. When K=2, the performance variance can be as large as 7% for both datasets. By demonstrating more exemplars, LLMs' outputs tend to be more stable with less variance.

Semantic Binding and Multi-Step Reasoning The two core properties of 'program of thoughts' are: (1) MultiStep: breaking down the thought process into the step-by-step program, (2) Binding: associating semantic meaning to the variable names. To better understand how these two properties contribute, we compare them with two variants. One variant is to remove the binding and simply use a, b, c as the variable names. The other variant is to directly predict the final mathematical equation to compute the results. We demonstrate our findings in Table 5. As can be seen, removing the name binding will in general hurt the model's performance. On more complex questions involving more variables like GSM8K, the performance drop is more obvious. Similarly, prompting LLM to directly generate the target equations is also very challenging. Breaking down the target equation into multiple reasoning steps helps the model better solve the tasks.

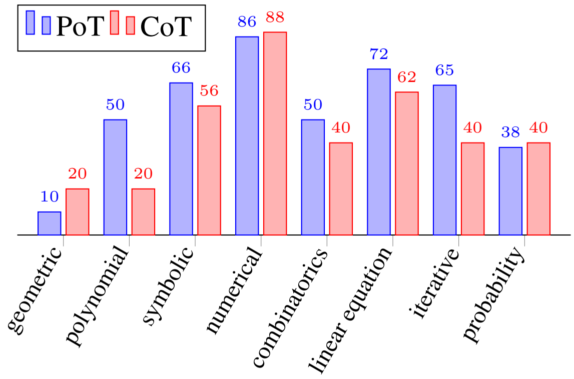

Figure 6: PoT and CoT's breakdown accuracy across different types of questions.

Breakdown Analysis We perform further breakdown analysis to observe where the performance difference between CoT and PoT roots is. We use AQuA (Ling et al., 2017) as our testbed to understand what type of questions PoT can do better. Specifically, we manually classify the questions in AQuA into several categories and show the breakdown accuracy in Figure 6. The major categories are (1) linear equations, (2) numerical calculation, (3) combinatorics, (4) probability, and (5) iterative.

The improvements of PoT are coming from 'linear/polynomial equation', 'iterative', 'symbolic', and 'combinatorics' types. These questions require more complex arithmetic or symbolic operations to solve. In contrast, on 'numerical', 'probability', and 'geometric' questions, PoT and CoT almost are on par with each other.

Error Analysis We perform a detailed error analysis here. We classify the errors into two types: (1) value grounding error, and (2) logic generation error. The first type indicates that the model fails to assign correct values to the variables relevant to the question. The second type indicates that the model fails to generate the correct computation process to answer the question based on the defined variables. Figure 7 shows an example of each type of error. In the upper example, the model fetches the value of the variables incorrectly while the computation logic is correct. In the lower example, the model grounded relevant variables correctly but fails to generate proper computation logic to answer the question. We manually look through the error cases in the TAT-QA results. Among the 198 failure cases of numerical reasoning question with the PoT (greedy) method, 47% has type 1 error and 33% has type 2 error. 15% of them are completely wrong (both errors occurred) and 5% of them we

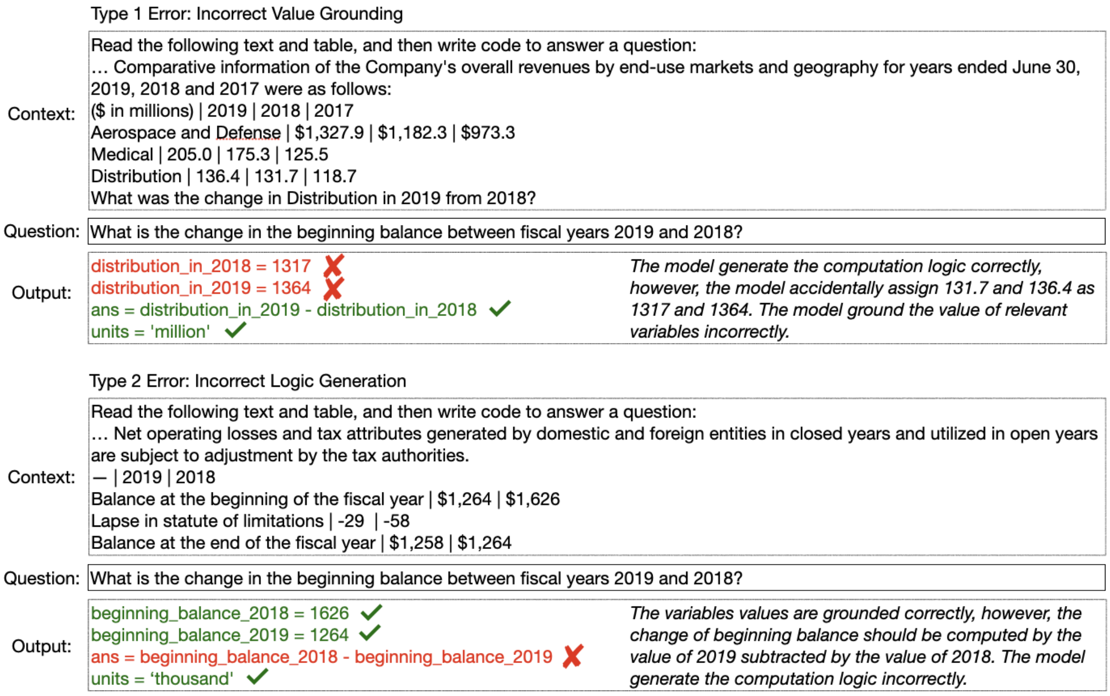

Figure 7: Error cases on TAT-QA dev set using PoT-greedy method.

think the answer is actually correct. We found that the majority part of the errors is type 1 error, which is also common for other methods such as CoT.

4 Related Work

4.1 Mathematical Reasoning in NLP

Mathematical reasoning skills are essential for general-purpose intelligent systems, which have attracted a significant amount of attention from the community. Earlier, there have been studies in understanding NLP models' capabilities to solve arithmetic/algebraic questions (Hosseini et al., 2014; Koncel-Kedziorski et al., 2015; Roy and Roth, 2015; Ling et al., 2017; Roy and Roth, 2018). Recently, more challenging datasets (Dua et al., 2019; Saxton et al., 2019; Miao et al., 2020; Amini et al., 2019; Hendrycks et al., 2021; Patel et al., 2021) have been proposed to increase the difficulty, diversity or even adversarial robustness. One concurrent work (published within 20 days) similar to ours is LiLA (Mishra et al., 2022), which proposes to assemble a large set of mathematical datasets into a unified dataset. LiLA also annotates Python programs as the generation target for solving mathematical problems. However, LiLA is mostly focused on dataset unification. Our work aims to understand how to generate 'thoughtful programs' to best elicit LLM's reasoning capability. Besides, we also investigate how to solve math problems without any exemplars.

4.2 In-context Learning with LLMs

GPT-3 (Brown et al., 2020) demonstrated a strong capability to perform few-shot predictions, where the model is given a description of the task in natural language with few examples. Scaling model size, data, and computing are crucial to enable this learning ability. Recently, (Rae et al., 2021; Smith et al., 2022; Chowdhery et al., 2022; Du et al., 2022) have proposed to train different types of LLMs with different training recipes. Such capability to follow few-shot exemplars to solve unseen tasks is not existent on smaller variants, which is emergent as the model scales up (Kaplan et al., 2020). Recently, there has been several work (Xie et al., 2021; Min et al., 2022) aiming to understand how and why in-context learning works. Another concurrent work similar to ours is BINDER (Cheng et al., 2022), which applies Codex to synthesize 'soft' SQL queries to answer questions from tables.

4.3 Chain of Reasoning with LLMs

Although LLMs have demonstrated remarkable success across a range of NLP tasks, their ability to demonstrate reasoning is often seen as a limitation. Recently, CoT (Wei et al., 2022; Kojima et al., 2022; Wang et al., 2022b) have been proposed to enable LLM's capability to perform reasoning tasks by demonstrating 'natural language rationale'. Suzgun et al. (2022) have shown that CoT can already surpass human performance on challenging BIG-Bench tasks. Later on, several other works (Drozdov et al., 2022; Zhou et al., 2022; Nye et al., 2021) also propose different approaches to utilize LLMs to solve compositional reasoning tasks by allowing intermediate steps.

5 Conclusions

In this work, we mainly investigate how to disentangle computation from reasoning in solving numerical problems. By 'program of thoughts' prompting, we are able to elicit LLMs' capability to generate accurate programs to express the 'thought procedure'. The computation is separately handled by an external computer to deliver accurate answers. We are able to boost the state-of-the-art performance on several math datasets significantly. We believe our work can inspire more work in the area to combine symbolic execution into LLMs.

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A Appendix

A.1 PoT as intermediate step

We demonstrate the workflow in Figure 8. We write

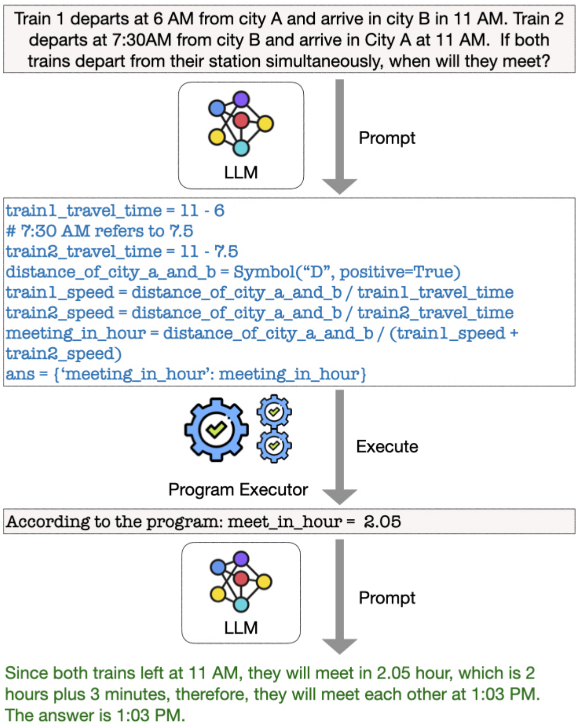

Figure 8: We adopt PoT to prompt language models to first generate an intermediate answer and then continue to prompt large models to generate the final answer.

the pseudo code as follows:

```
# Function PoT(Input) -> Output
# Input: question
# Ouptut: program
# Function Prompt(Input) -> Output
# Input: question + intermediate
# Ouptut: answer
program = PoT(question)
exec(program)
if isintance(ans, dict):
    ans = list(x.items()).pop(0)
    extra = 'according to the program: '
    extra += ans[0] + ' = ' + ans[1]
    pred = Prompt(question + extra)
else:
    pred = ans
return pred
```